QUANTUM SIGNATURES AND DECOHERENCE DURING INFLATION FROM PRIMORDIAL GRAVITATIONAL WAVES

Arxiv: 2503.23150

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Benasque ,The Dawn of Gravitational wave cosmology,
01/05/2025

THE THEORY OF INFLATION

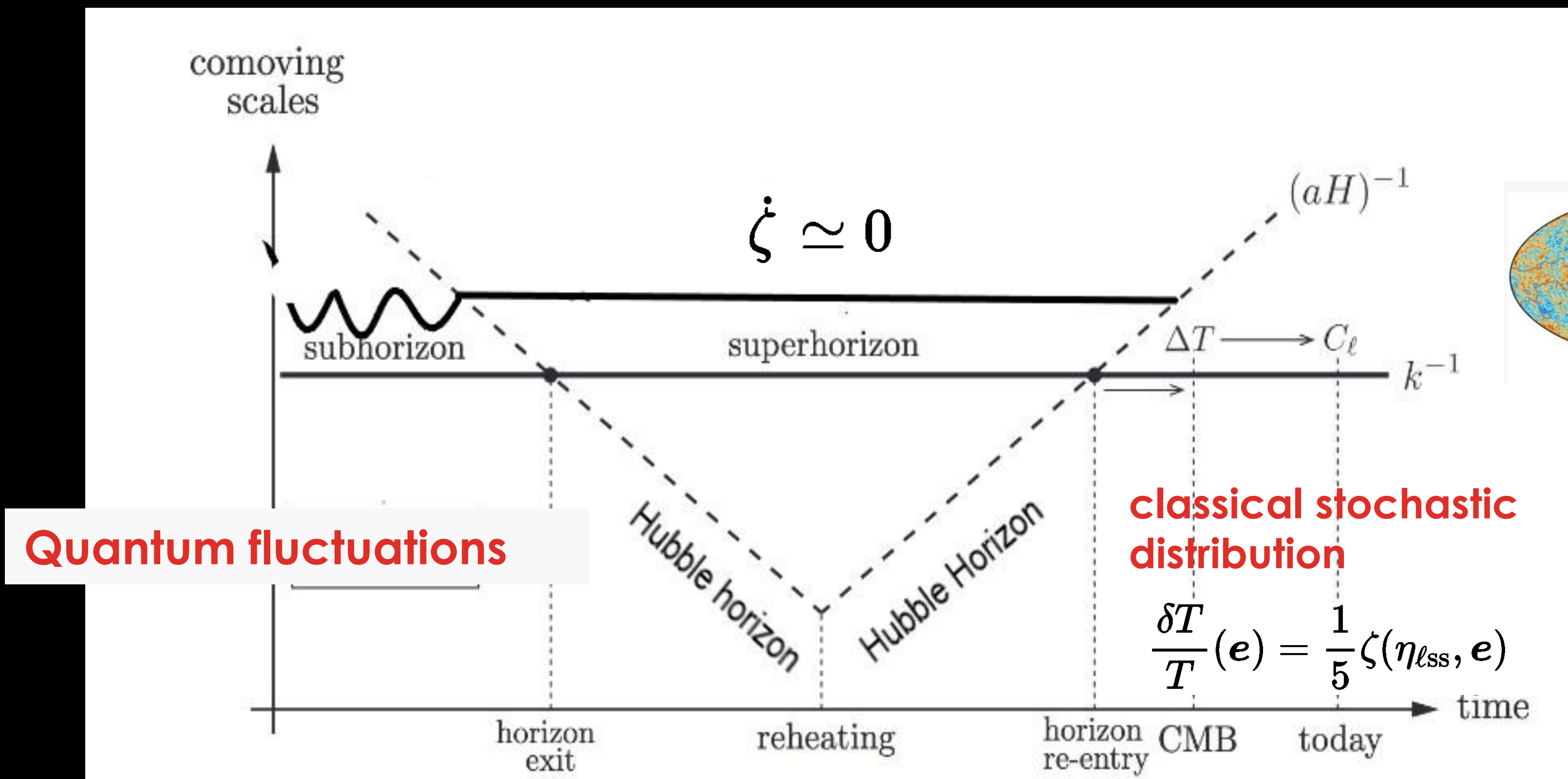
- Accelerated expansion driven by one (or more) quantum scalar field(s)
- Solves the shortcomings in the Hot Big Bang Theory;
- Provides a mechanism to **explain anisotropies and inhomogeneities** from the tiny quantum fluctuations of the scalar field.

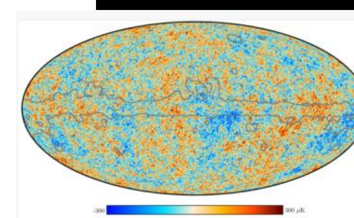
$$g_{ij}(\vec{x},t) = a^2(t)e^{2\zeta(\vec{x},t)}(\delta_{ij} + h_{ij}(\vec{x},t))$$

- Tensor perturbation h_{ij} (Stochastic Gravitational Waves Background)
- Scalar (curvature) perturbations ζ

QUANTUM TO CLASSICAL TRANSITION IN COSMOLOGY

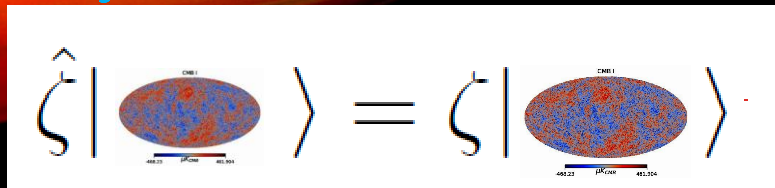
- Quantum fluctuations of the scalar field driving the accelerated expansion are the seeds for the anisotropies we observe in CMB.



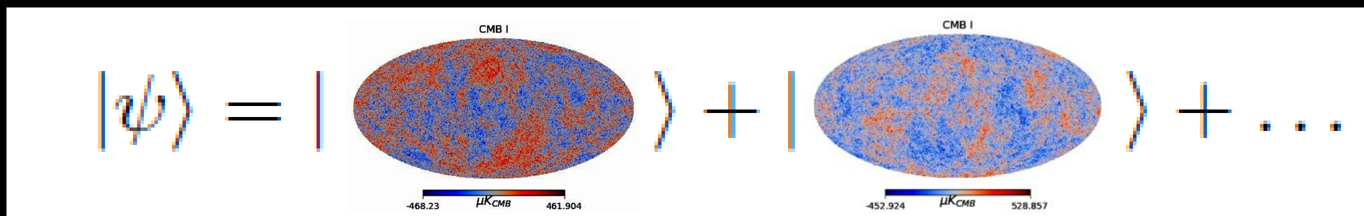


- Credits:Coles and Lucchin,Cosmology,,D.Baumann, Lectures on Inflation.

How could quantum fluctuations become classical objects?



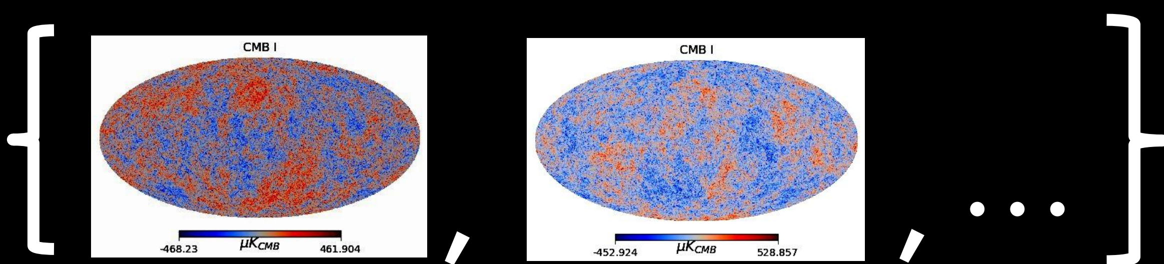
- ζ quantum operator;
- Configuration of perturbations (~CMB Maps) are eigenvectors of ζ



COHERENT **SUPERPOSITION**

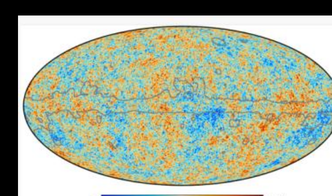
DECOHERENCE
(after interaction, and entanglement with unobservable environment)

Quantum to classical transition!



STATISTICAL ENSEMBLE

BUT ONLY ONE REALIZATION!



Credits:mock maps,Claudio Ranucci; real map, Planck 2018

WHAT DOES DECOEHERENCE DO?

Interaction with an unobsevable environment → Entanglement, suppress quantum coherence between different possible outcomes

Interference terms in red

$$\rho_{sys} = \begin{pmatrix} |\zeta_1\rangle\langle\zeta_1| & |\zeta_1\rangle\langle\zeta_2| \\ |\zeta_2\rangle\langle\zeta_1| & |\zeta_2\rangle\langle\zeta_2| \end{pmatrix} \xrightarrow{\text{decoherence}} \rho_r = \mathrm{Tr}_{ENV}\, \rho_{sys+env} = \begin{pmatrix} |\zeta_1\rangle\langle\zeta_1| & 0 \\ 0 & |\zeta_2\rangle\langle\zeta_2| \end{pmatrix}$$

↓

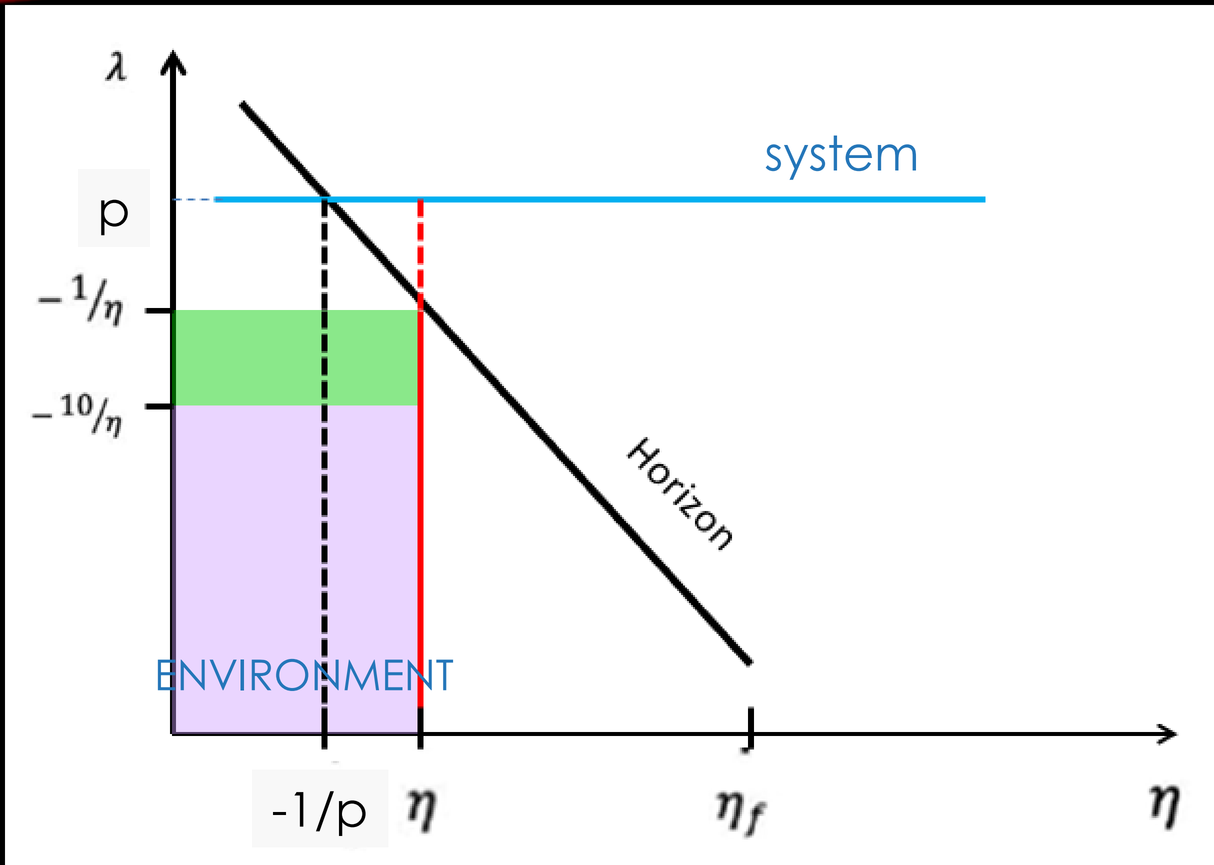
Statistical ensemble!

How to quantify? Purity!

$$\gamma = \mathrm{Tr}\,\rho^2 = 1 \xrightarrow{\text{decoherence}} \gamma = \mathrm{Tr}\,\rho_r^2 \to 0$$

OUR MODEL: ENVIRONMENT

Time dependent DEEP SUBHORIZON environment:



Short Correlation time!

$$\tau_{env} \ll \tau_{sys}$$

- The light blue line is the mode of interest (i.e. the system);
- Purple: our environment at a certain time η, deep subhorizon modes;
- Green: horizon crossing modes to be analyzed in further studies;

(Nonlinear) **gravitational Interaction (GR)**

- **superhorizon system;**
- **subhorizon GWs environment.**

Entanglement

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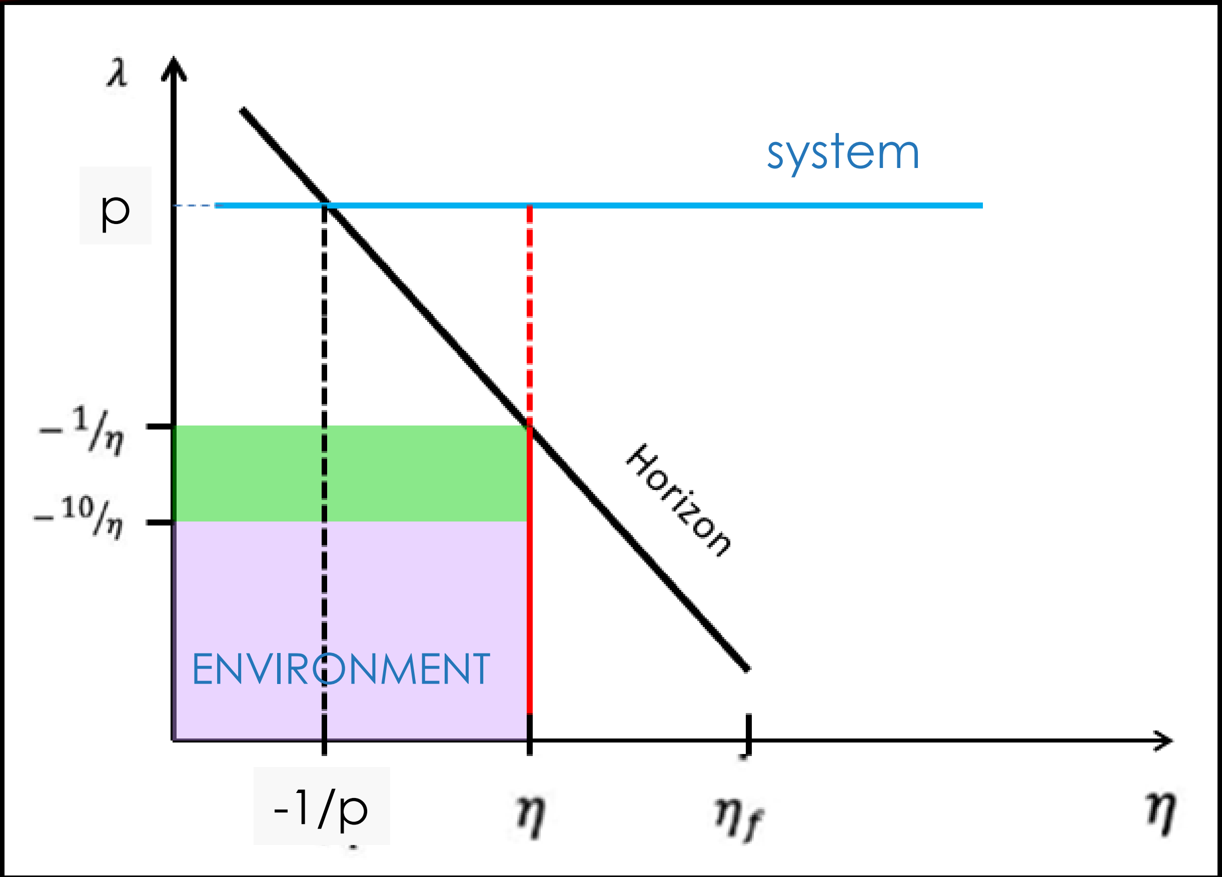

(Nonlinear) **gravitational Interaction (GR)**

- **superhorizon system;**
- **subhorizon GWs environment.**

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QUANTUM MASTER EQUATION

- "Equation of motion" for the Density Matrix of the System

$$\frac{d\rho_r}{d\eta} = -\mathrm{i}\left[H + H_{LS}, \rho_r(\eta)\right] + \sum_{\boldsymbol{p}} D_{11}(\eta)\left(v_{\boldsymbol{p}}(\eta)\rho_r(\eta)v_{\boldsymbol{p}}^{\dagger}(\eta) - \frac{1}{2}\left\{v_{\boldsymbol{p}}^{\dagger}(\eta)v_{\boldsymbol{p}}(\eta), \rho_r(\eta)\right\}\right)$$

Many approximations to derive it….but most important is the Markovian Approximation! $\rho_r(\eta') \to \rho_r(\eta)$ $v_p\left(\eta'\right) \to v_p(\eta)$

D_{11} (η) "canonical decay rate": contains environmental MEMORY

$$D_{11} = g(\eta)\int_{-\frac{1}{p}}^{\eta} d\eta' g\left(\eta'\right) 2\Re K\left(\eta, \eta'\right)$$

If $\tau_{env} \ll \tau_{sys}$ ***Fast*** ENVIRONMENTAL CORRELATION FUNCTIONS **(peaked around η'~η)**

↓

Markovianity; $D_{11}>0$

If long tails of memory → **Non Markovianity!** $\delta D_{11}<0$

CANONICAL FORM QME

- Canonical form

$$\frac{d\rho_r}{d\eta} = -\mathrm{i}\left[H + H_{LS}, \rho_r(\eta)\right] + \sum_{\boldsymbol{p}} D_{11}(\eta)\left(v_{\boldsymbol{p}}(\eta)\rho_r(\eta)v_{\boldsymbol{p}}^{\dagger}(\eta) - \frac{1}{2}\left\{v_{\boldsymbol{p}}^{\dagger}(\eta)v_{\boldsymbol{p}}(\eta), \rho_r(\eta)\right\}\right)$$

D_{11} (η) "canonical decay rate"

$$D_{11} = g(\eta)\int_{-\frac{1}{p}}^{\eta} d\eta' g\left(\eta'\right) 2\Re K\left(\eta, \eta'\right)$$

POSITIVITY: if **$D_{11}>0$**

- **Dynamics is physical for sure!**
- **Markovian approximation** works well

But there may be some small **non-Markovian corrections** ($\delta D_{11}<0$) **as long as the total D_{11} is positive**

CANONICAL FORM QME

- Canonical form

$$\frac{d\rho_r}{d\eta} = -\mathrm{i}\left[H + H_{LS}, \rho_r(\eta)\right] + \sum_{\boldsymbol{p}} D_{11}(\eta)\left(v_{\boldsymbol{p}}(\eta)\rho_r(\eta)v_{\boldsymbol{p}}^{\dagger}(\eta) - \frac{1}{2}\left\{v_{\boldsymbol{p}}^{\dagger}(\eta)v_{\boldsymbol{p}}(\eta), \rho_r(\eta)\right\}\right)$$

Lamb Shift Hamiltonian
-finite **renormalization of the unitary** hamiltonian
-corrections to the **mass of perturbations and spectral index of** Power Spectrum.

Non Unitary part:
-Decoherence;
-non-Unitary correction to observables.

TIME DEPENDENT ENVIRONMENT

•Also used in: ('19 Gong-Seo, '21-'22 Brahma et al.,…)

'23 Burgess: nobody computed the effect

$$\mathrm{Tr}_{ENV(\eta)}\,\frac{\mathrm{d}}{\mathrm{d}\eta}\rho(\eta) \not\to \frac{\mathrm{d}}{\mathrm{d}\eta}\mathrm{Tr}_{ENV(\eta)}\,\rho(\eta) = \frac{\mathrm{d}}{\mathrm{d}\eta}\rho_{\mathrm{r}}(\eta)$$

Additional term appears in the quantum master equation:

- **THE CANONICAL FORM IS RESPECTED!!**
- **Just a (small) modification of D_{11}**

$$\Delta D_{11} \simeq -0.02\frac{H^2\epsilon}{8\pi^2 M_{pl}^2}$$

DECOHERENCE IN SINGLE FIELD INFLATION

- GR non linear gravitational interactions (Gangui+1993,Maldacena 2003)

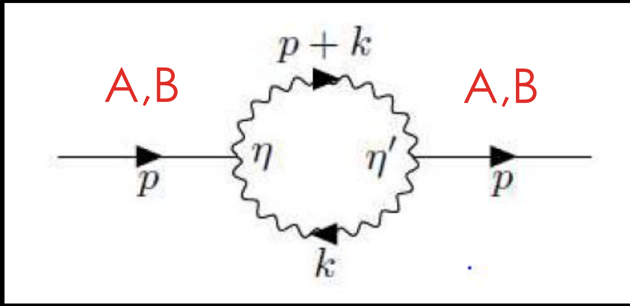
$$S = \frac{\epsilon M_{pl}^2}{8} \int dt d^3x \left(a^3 \zeta \dot{h}_{ij} \dot{h}_{ij} + a \zeta \partial_l h_{ij} \partial_l h_{ij} - 2a^3 \dot{h}_{ij} \partial_l h_{ij} \partial_l \left(\nabla^2 \right)^{-1} \dot{\zeta} \right)$$

Also considered in Burgess+'22, but only one interaction and different environment!

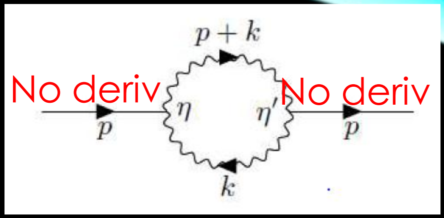
WE CONSIDER THE INTERPLAY BETWEEN ALL INTERACTIONS!

A)DERIVATIVELESS interaction,more important contributions.

B) DERIVATIVE interactions (just like the circled one).



DERIVATIVELESS INTERACTIONS



- $D_{11}>0$ **positive!** $$D_{11}^{int11}=\frac{\epsilon H^2}{4\pi^2 M_{pl}^2\eta^2}\left(\frac{\pi}{2}-1.52\right)\simeq\frac{\epsilon H^2}{4\pi^2 M_{pl}^2\eta^2}0.05$$

- We can achieve decoherence when $$\gamma\to 0 \quad \Leftrightarrow \frac{1}{\gamma}\to\infty$$

$$\frac{1}{\gamma^2}\simeq\frac{\epsilon H^2}{\pi^2 M_p^2}1.25\times 10^{-3}\left(\frac{aH}{p}\right)^3\lesssim 10^{-18}e^{3(N_{\rm end}-N_\star)}$$

$$\frac{\epsilon H^2}{M_{pl}^2}\lesssim 10^{-13}$$

$$\left(\frac{\lambda_{phys}}{R_H}\right)^3=e^{3(N_{end}-N_*)}$$

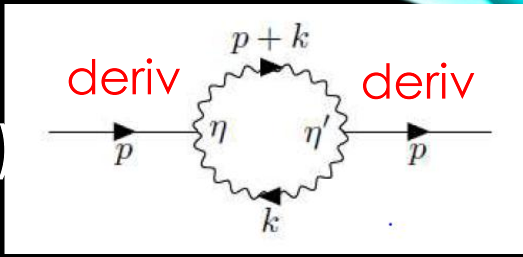
Decoherence is a superhorizon phenomenon!

If saturating the bounds, then at least: $N_{\rm end}-N_*\simeq 15$ efolds

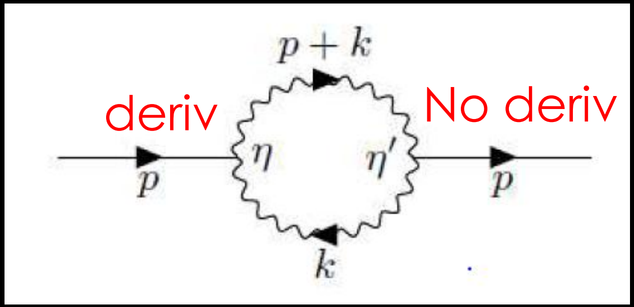
CMB well decohered!! But what about smaller scales?

DERIVATIVE INTERACTIONS

- NEGLIGIBLE!! (Just for deep subhorizon modes)



MIXED DERIVATIVE-DERIVATIVELESS INTERACTIONS



•**Mixed terms give** NEGATIVE CONTRIBUTIONS

$$D_{11}^{int1-23} < 0$$

Non-Markovianity

$$D_{11}^{SMAR} = D_{11}^{int11} + D_{11}^{int1-23} \simeq \frac{\epsilon H^2}{4M_{pl}^2\eta^2} 5 \times 10^{-4}$$

$$N_{end} - N_* \simeq 17 efolds$$

LAMB SHIFT:UNITARY RENORMALIZATION

- **Weinberg '05**: compute corrections to Power Spectrum due to interaction at second order:"**loop quantum correction**";
- Method: in-in formalism, just a **unitary** dynamics;
- But…late time secular effects: very large logarithms in corrections

$$\Delta P_{vv} \propto \log(-k\eta_f), \log^2(-k\eta_f), \ldots \log^n(-k\eta_f) \ldots = \log \frac{k}{aH}, \log^2 \frac{k}{aH}, \ldots \log^n \frac{k}{aH} \ldots$$

•**May break perturbation theory!**

•Many methods in the past for resummation.

LAMB SHIFT:UNITARY RENORMALIZATION

- Entanglement with environment "renormalizes" in a finite way energy levels of the system (in this case, mass);

- **NON-Perturbative resummation!!**
- Quantum "loop" corrections, but **"authomatically" produce resummation by considering the interaction with an environment for decoherence.**

$$\mathcal{P}(k,\eta) \propto e^{\frac{2\varepsilon H^2}{4\pi^2 M_{pl}^2}\ln(-k\eta)}$$

Modifies spectral index in Curvature Power Spectrum:

$$\mathcal{P}(k) = A_s \left(\frac{k}{k_0}\right)^{n_s - 1}$$

- Blue Correction to spectral index n_s
- No secular corrections

$$\delta n_S \simeq \frac{\varepsilon H^2}{4M_{pl}^2\pi^2}2$$

Lamb Shift: unitary corrections

Modifies spectral index in Curvature Power Spectrum:

$$\delta n_S \simeq \frac{\varepsilon H^2}{4M_{pl}^2\pi^2}2$$

- Blue Correction to spectral index n_s
- No secular corrections.

Corrections from non unitary part:

Same order, different form

$$\frac{\Delta P_{vv}}{P_{vv}} = \left(\frac{\pi}{2} - 1.5\right)\frac{\epsilon H^2}{432\pi^2 M_{pl}^2}$$

Of course, too little (Gravitational interactions)!

$$\frac{\epsilon H^2}{M_{pl}^2} \lesssim 10^{-13}$$

What about specific models with stronger non Gaussianity?

Lamb Shift for the Bispectrum

Tensorial environment on **scalar system**

$$\left\langle \zeta(\vec{k_1})\zeta(\vec{k_2})\zeta(\vec{k_3}) \right\rangle = (2\pi)^3 B(k_1, k_2, k_3)\delta^3\left(\vec{k_1}+\vec{k_2}+\vec{k_3}\right)$$

Important for the investigation of non gaussianity

•Dealt with(in a different way) in Daddi Hammou+'22, Martin+'18, Colas+24

- Three modes k_1, k_2, k_3 superhorizon at the end of inflation
- Since the smallest wavelentgh mode k_1 crosses the horizon there is a decay in time

$$B\left(k_1, k_2, k_3, \eta\right) = B\left(k_1, k_2, k_3, -\frac{1}{k_1}\right) e^{\frac{3.6\epsilon H^2}{4\pi^2 M_{pl}^2}\ln(-k_1\eta)}$$

CONCLUSIONS

- We computed **decoherence and quantum corrections to observables, in single field inflation,** in an environment only of subhorizon modes;
- For the first time we considered more than just one interaction at a time, but also the interplay between them;
- We introduced a method to evaluate the impact of a **time dependent environment** onto the quantum master equations;
- We found Lamb Shift corrections to the **Bispectrum**(decay in time) and to the Power Spectrum.

FUTURE PROJECTS

- Non-Markovianity in an expanding background: how to deal with it?

- ***Non unitary evolution is needed for quantum to classical transition***

And should be there even in a minimal setting. can this change any conclusion?

(e.g. in-in formalism calculations,…)

- Can we prove, either directly, or indirectly, e.g. through corrections by decoherence, the quantum nature of inflationary perturbations?

Small-scale modes
-If modes cross the horizon in the last e-folds of inflation
may not have the time to decohere?

-There may be genuine quantum features! Gravitational waves?

- .

THANK YOU FOR YOUR ATTENTION!

BACK UP

MARKOVIAN APPROXIMATION(S)

Actually, on the RHS we should have….

$$g(\eta)\int_{-\frac{1}{p}}^{n} d\eta' g\left(\eta'\right) 2\Re K\left(\eta,\eta'\right) v_p\left(\eta'\right)\rho_r\left(\eta'\right) + (\leftrightarrow \text{perm})$$

But if: $\tau_{env} \ll \tau_{sys}$ then you can take system variables out of the integral:

1)Born-Markov approximation $\rho_r\left(\eta'\right) \rightarrow \rho_r(\eta) + O(g^2)$

2)Strong Markovian Approximation (Burgess+'22,Kaplanek+'21)

$v_p\left(\eta'\right) \rightarrow v_p(\eta)$ More on this in Nicola's talk next week…

$$D_{11} = g(\eta)\int_{-\frac{1}{p}}^{\eta} d\eta' g\left(\eta'\right) 2\Re K\left(\eta,\eta'\right)$$

$$\frac{d\rho_r}{d\eta} = -\mathrm{i}\left[H + H_{LS}, \rho_r(\eta)\right] + \sum_{\boldsymbol{p}} D_{11}(\eta)\left(v_{\boldsymbol{p}}(\eta)\rho_r(\eta)v_{\boldsymbol{p}}^{\dagger}(\eta) - \frac{1}{2}\left\{v_{\boldsymbol{p}}^{\dagger}(\eta)v_{\boldsymbol{p}}(\eta), \rho_r(\eta)\right\}\right)$$

OUR WORK

ORIGINAL POINTS:

•Decoherence from a time dependent environment of **subhorizon modes;**

•We compute **all the interactions,** underlining the importance **of the interplay between mixed couples.**

•**Quantum corrections to correlators:**
-Lamb Shift (unitary corrections);
-Non unitary corrections.

WHAT WE FOUND IN THE MEANTIME:

- Comment on **the Strong Markovian approximation validity;**
- Evaluate effects on the quantum master equation of **time dependent environment**.

BURGESS+'22;PART TWO

D_{11}>0? No!!! **Is the model unphysical?**

$$\rho_r'(\eta) = -g(\eta)\int_{\eta_0}^{\eta} \mathrm{d}\eta' g(\eta') \sum_k v_{\boldsymbol{k}}(\eta) v_{-\boldsymbol{k}}(\eta')\rho_r(\eta) K(k,\eta,\eta') + (\leftrightarrow) \textit{similar terms}$$

Adopt "**Strong Markovian approximation**": $v_k(\eta') \to v_k(\eta)$

Stronger than the usual one $\rho_r(\eta') \to \rho_r(\eta) + O(g^2)$

CONDITIONS:

- ENVIRONMENTAL CORRELATION FUNCTIONS K(η,η') HAVE REALLY SHORT MEMORY; $T_{ENV} << T_{SYS}$
- SYSTEM OPERATORS v_k (η), EVOLVE SLOWLY

Then, D_{11} >0!!!

$$D_{11}^{FIX\,ENV} = \frac{\varepsilon H^2}{1024\pi^2 M_{\mathrm{p}}^2}\frac{80\pi}{\eta^2} \simeq 0.98\frac{\varepsilon H^2}{4\pi^2 M_{\mathrm{p}}^2}$$

Claim: Remove "unphysical" memory, extract Only the markovian part of the quantum master equation

QUANTUM MASTER EQUATION AND TCL$_2$

- "Equation of motion" for the Density Matrix of the System **MEMORY!**

$$\mathrm{Tr}_{\mathcal{E}} \frac{\mathrm{d}}{\mathrm{d}\eta}\rho(\eta) = \frac{\mathrm{d}\rho_{\mathrm{r}}}{\mathrm{d}\eta}(\eta) = -g^2 \int_{\eta_{\mathrm{in}}}^{\eta} \mathrm{d}\eta' \, \mathrm{Tr}_{\mathcal{E}}\left[H_{\mathrm{int\,i}}(\eta), \left[H_{\mathrm{int\,j}}(\eta'), \rho_r(\eta')\right]\right] \quad i,j = 1,2,3$$

- BORN-MARKOVIAN APPROXIMATION: memory corrections are higher order in the coupling constant;

$$\rho_r(\eta') \to \rho_r(\eta) + O(g^2)$$

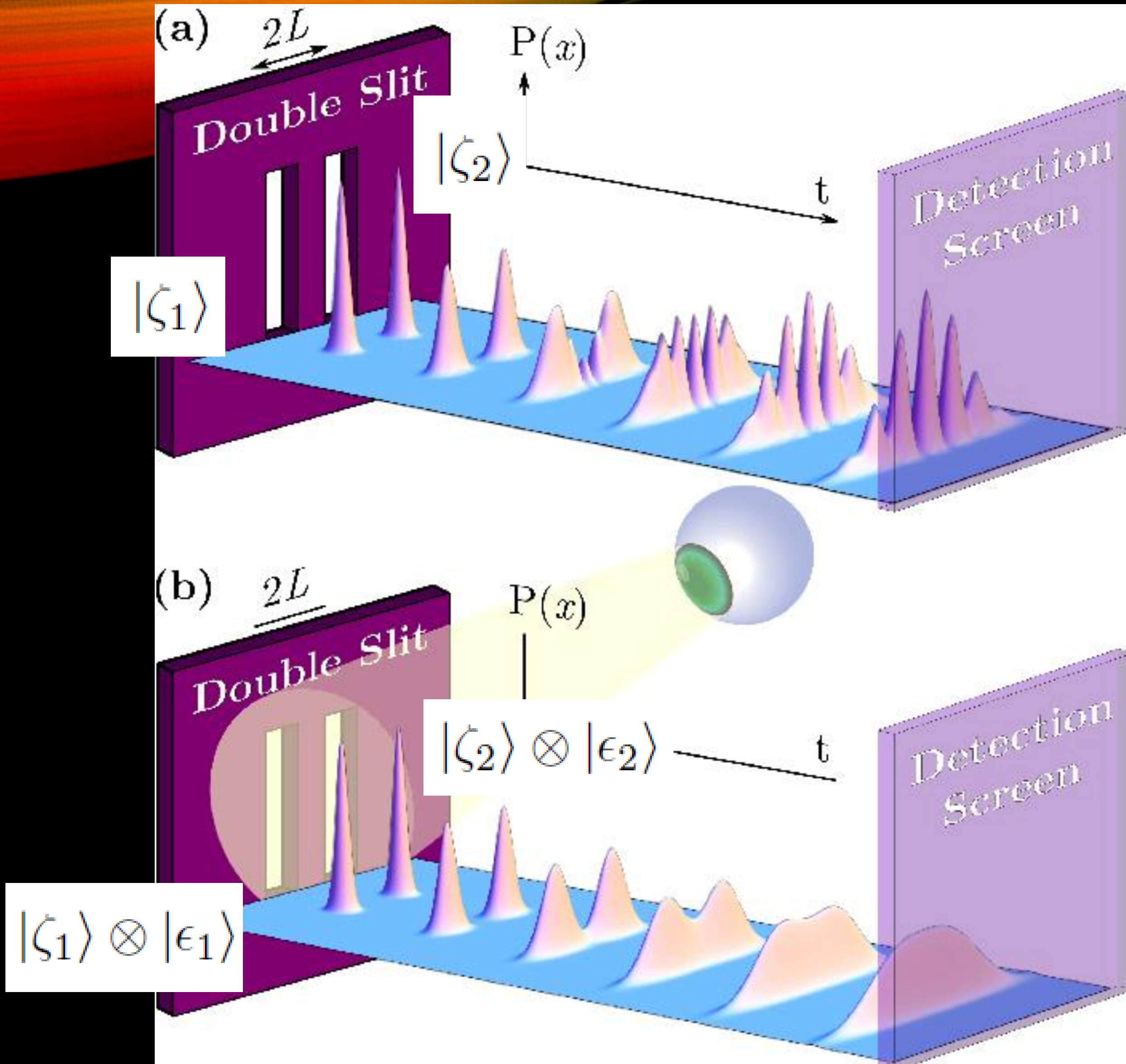
Convolution!

TCL$_2$: TIME CONVOLUTIONLESS EQUATION(at 2° order)

$$\rho_r'(\eta) = -g(\eta)\int_{\eta_0}^{\eta} \mathrm{d}\eta' g(\eta') \sum_{\boldsymbol{k}} \left[v_{\boldsymbol{k}}(\eta) v_{-\boldsymbol{k}}(\eta') \rho_r(\eta) K(k,\eta,\eta') + \rho_r(\eta) v_{-\boldsymbol{k}}(\eta') v_{\boldsymbol{k}}(\eta) K^*(k,\eta,\eta') - v_{\boldsymbol{k}}(\eta)\rho_r(\eta) v_{-\boldsymbol{k}}(\eta') K^*(k,\eta,\eta') - v_{-\boldsymbol{k}}(\eta')\rho_r(\eta) v_{\boldsymbol{k}}(\eta) K(k,\eta,\eta')\right]$$

MEMORY!

ENVIRONMENTAL CORRELATION FUNCTIONS



Interference!

Waves of probability

DECOHERENCE

GAUSSIAN
DISTRIBUTION

DECOHERENCE: TOY MODEL

Of course, we can have infinite possible configurations all over the Universe for scalar perturbations.
(Infinite dimensional Hilbert space)

As a TOY MODEL, consider ζ as a TWO STATES OPERATOR, with only two possibile eigenvalues:

$$\hat{\zeta}\,|\zeta_1\rangle = \zeta_1\,|\zeta_1\rangle \qquad \hat{\zeta}\,|\zeta_2\rangle = \zeta_2\,|\zeta_2\rangle$$

Consider a coherent superposition:

$$|\psi\rangle = c_1\,|\zeta_1\rangle + c_2\,|\zeta_2\rangle$$

But what we usually have to consider is the density matrix:

$$\rho = |\psi\rangle\langle\psi|$$

Expanding (interference terms in red):

$$\rho = |c_1|^2|\zeta_1\rangle\langle\zeta_1| + |c_2|^2|\zeta_2\rangle\langle\zeta_2| + c_1^* c_2|\zeta_1\rangle\langle\zeta_2| + c_1 c_2^*|\zeta_2\rangle\langle\zeta_1|$$

Purity γ:

$$\gamma = \mathrm{Tr}\,\rho^2 = (|c_1|^4 + |c_2|^4 + 2|c_1|^2|c_2|^2) = (|c_1|^2 + |c_2|^2)^2 = 1$$

Full quantum coherence

Introduce an Environment (TWO STATES)

$$|\epsilon_1\rangle, |\epsilon_2\rangle \quad such\ that \quad \langle\epsilon_1|\epsilon_2\rangle = 0$$

System environment interaction → Creates entanglement

$$|\zeta_1\rangle \otimes |\epsilon_1\rangle, \qquad |\zeta_2\rangle \otimes |\epsilon_2\rangle$$

We CANNOT observe the environment, so we trace over it:

$$\rho_{reduced} = \mathrm{Tr}_\epsilon\, \rho = \mathrm{Tr}_\epsilon \left(|c_1|^2 |\zeta_1\rangle|\epsilon_1\rangle\langle\epsilon_1|\langle\zeta_1| + |c_2|^2 |\zeta_2\rangle|\epsilon_2\rangle\langle\epsilon_2|\langle\zeta_2| \right.$$
$$\left. + c_1^* c_2 |\zeta_1\rangle|\epsilon_1\rangle\langle\epsilon_2|\langle\zeta_2| + c_1 c_2^* |\zeta_2\rangle|\epsilon_2\rangle\langle\epsilon_1|\langle\zeta_1| \right)$$

$$\rho_r = \left(|c_1|^2 |\zeta_1\rangle\langle\epsilon_1|\epsilon_1\rangle\langle\zeta_1| + |c_2|^2 |\zeta_2\rangle\langle\epsilon_2|\epsilon_2\rangle\langle\zeta_2| \right.$$
$$\left. + c_1^* c_2 |\zeta_1\langle\epsilon_1|\epsilon_2\rangle\langle\zeta_2| + c_1 c_2^* |\zeta_2\rangle\langle\epsilon_2|\epsilon_1\rangle\langle\zeta_1| \right)$$

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$$\left. + c_1^* c_2 |\zeta_1\rangle|\epsilon_1\rangle\langle\epsilon_2|\langle\zeta_2| + c_1 c_2^* |\zeta_2\rangle|\epsilon_2\rangle\langle\epsilon_1|\langle\zeta_1| \right)$$

$$\rho_r = \left(|c_1|^2 |\zeta_1\rangle\langle\epsilon_1|\epsilon_1\rangle\langle\zeta_1| + |c_2|^2 |\zeta_2\rangle\langle\epsilon_2|\epsilon_2\rangle\langle\zeta_2| \right.$$
$$\left. + c_1^* c_2 |\zeta_1\rangle\langle\epsilon_1|\epsilon_2\rangle\langle\zeta_2| + c_1 c_2^* |\zeta_2\rangle\langle\epsilon_2|\epsilon_1\rangle\langle\zeta_1| \right)$$

Purity:

$$0 < \gamma = \mathrm{Tr}_{system}\, \rho_r^2 = |c_1|^4 + |c_2|^4 + NO\,INTERFERENCE < 1$$

Take Home message: **Purity<<1, No Interference** → Decoherence!

During inflation:

Sasaki-Mukhanov variable for curvature perturbations (during inflation, canonically normalized):

$$\hat{v} = a\sqrt{2\epsilon}M_{pl}\hat{\zeta}$$

Quantum operators during Inflation:

$$\hat{v}_{\boldsymbol{k}} = u_{\boldsymbol{k}}\hat{c}_{\boldsymbol{k}} + u^*_{\boldsymbol{k}}\hat{c}^\dagger_{-\boldsymbol{k}}$$

Possible configurations of perturbations:

$$\hat{v}|v\rangle = v|v\rangle$$

State of perturbations during inflation

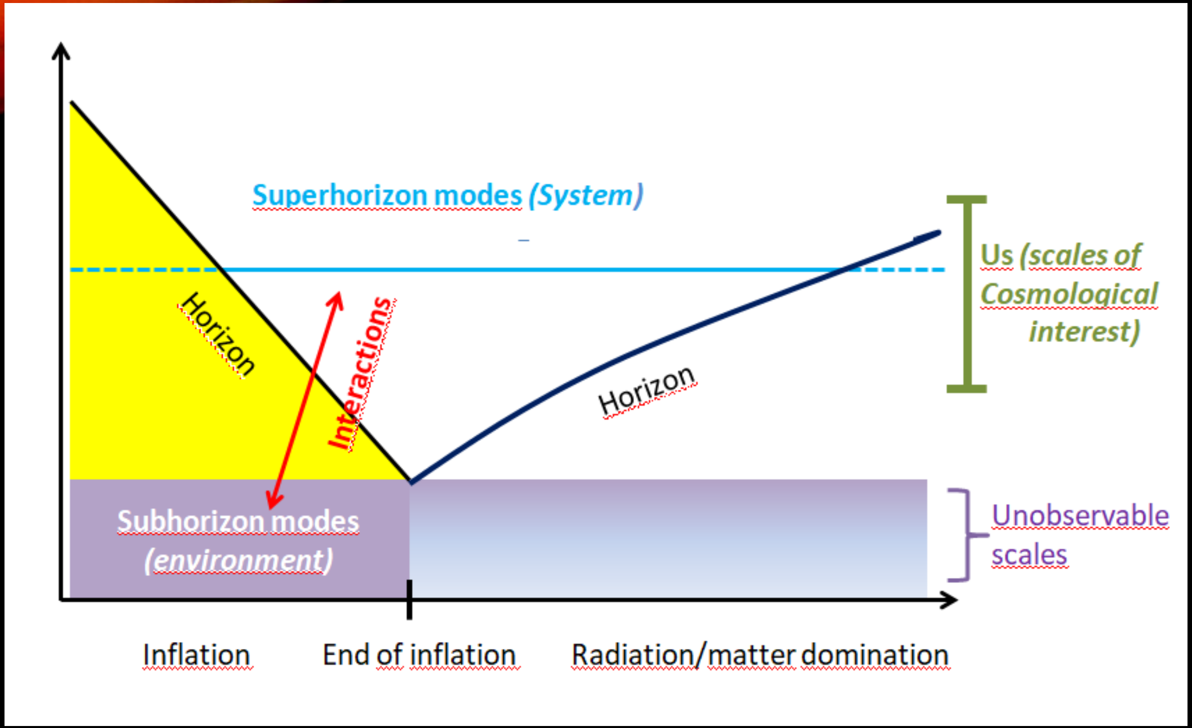
$$|\psi\rangle = c_1|v_1\rangle + c_2|v_2\rangle + \ldots$$

After inflation:

Stochastic (quasi) gaussian distribution of Temperature anisotropies in CMB:

$$\frac{\delta T}{T}(\boldsymbol{e}) = \frac{1}{5}\zeta(\eta_{\ell ss}, \boldsymbol{e})$$

OPEN QUANTUM SYSTEMS

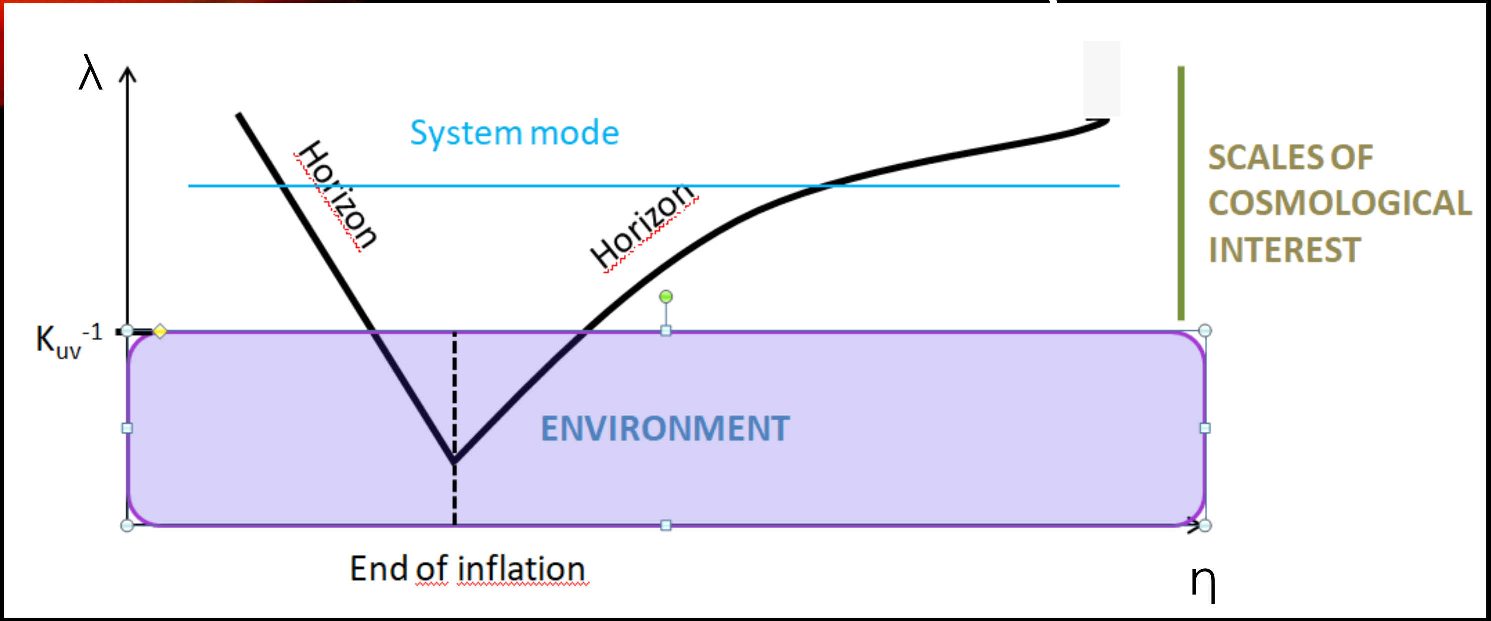


Decoherence :

- already during inflation, after Horizon crossing: Superhorizon phenomenon.

System	Environment	N. e-folds	Authors
Scalar	Scalar	10-20	Nelson,'16;Burgess+,22
Tensor	Tensor	5-10	Seo et al., 2019
Scalar	Tensor+Scalar	13	Burgess et al. , 2022

MINIMAL DECOHERENCE(BURGESS+,22)



•**Fixed** Environment: k>125/Mpc $=k_{UV}$ System: Scalar perturbation, $k<k_{UV}$.

ASSUMPTION: most of decoherence comes from the Superhorizon modes in the environment.

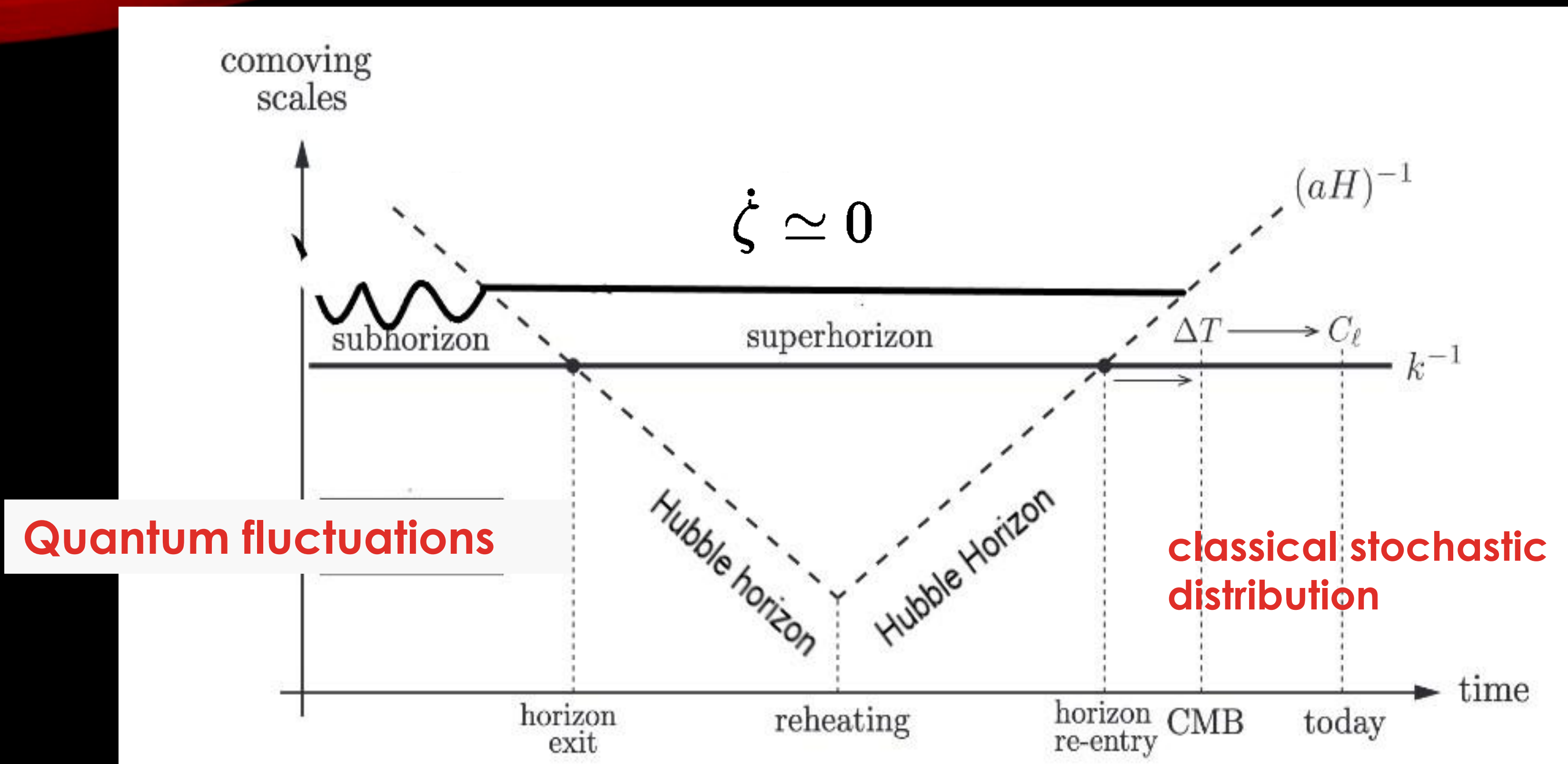
•Time derivative interactions are suppressed!

• GR nonlinear gravitational interactions (Maldacena, 2003);

$$S=\frac{\epsilon M_{pl}^2}{8}\int dtd^3x\left(a^3\zeta\dot{h}_{ij}\dot{h}_{ij}+a\zeta\partial_l h_{ij}\partial_l h_{ij}-2a^3\dot{h}_{ij}\partial_l h_{ij}\partial_l\left(\nabla^2\right)^{-1}\dot{\zeta}\right)$$

QUANTUM TO CLASSICAL

Guth, Pi(1985), Polarski,Starobinski (1996++),…

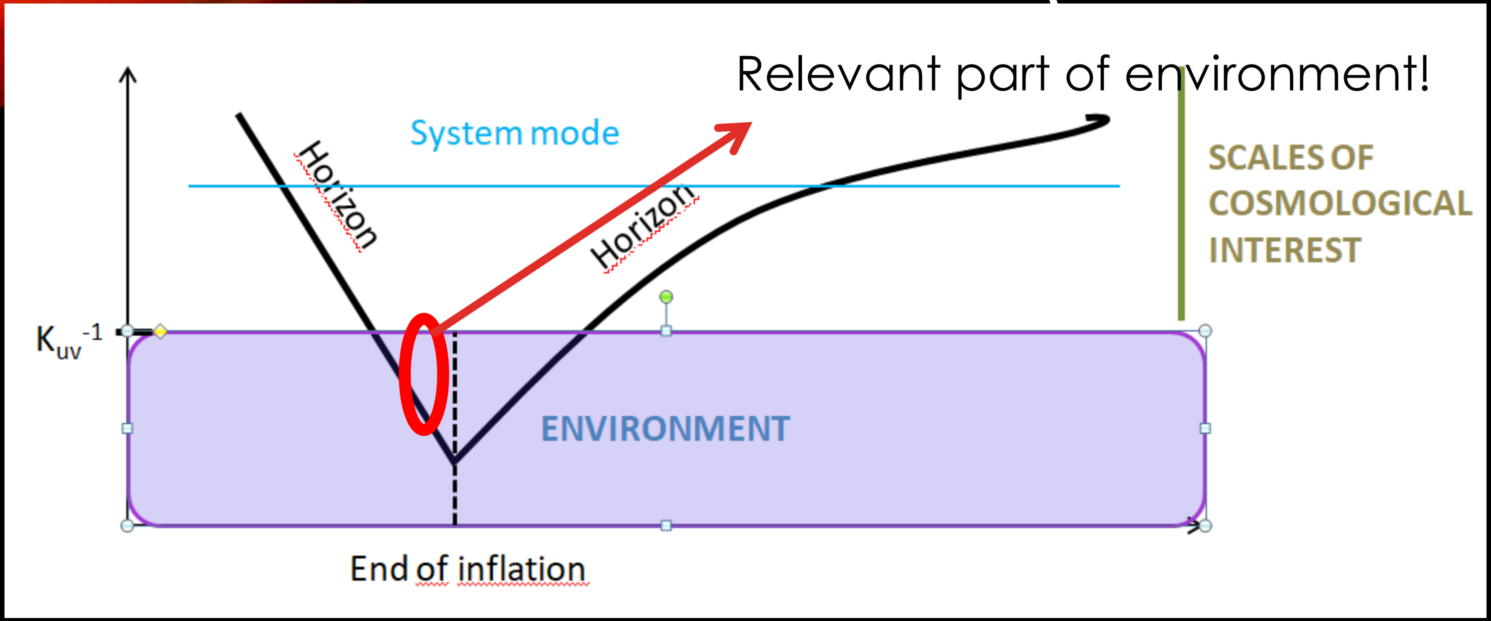


$$\frac{\delta T}{T}(\boldsymbol{e}) = \frac{1}{5}\zeta(\eta_{\ell ss}, \boldsymbol{e})$$

- Credits:D.Baumann, Lectures on Inflation; Coles and Lucchin,Cosmology.

How could quantum fluctuations become classical objects?

MINIMAL DECOHERENCE(BURGESS+,22)



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DO WE NEED INTERACTIONS?

No: "Decoherence without decoherence"
(Starobinski et al.,, 1996)

Yes: Only Interactions with an **unobservable environment** induce **decoherence**

OPEN QUANTUM SYSTEMS

QUANTUM OR CLASSICAL PERTURBATIONS?

"Decoherence without decoherence"(Starobinski et al, 1996): after horizon crossing, quantum states freely evolve into "squeezed" quantum states

$$ {}_S\left\langle 0,\eta \middle| G(v(\vec{k}))\, G^\dagger(v(\vec{k})) \middle| 0,\eta \right\rangle_S = \iint d\Re v(\vec{k})\; d\Im v(\vec{k})\; \rho(|v(\vec{k})|)\, |G(v(\vec{k}))|^2 $$

Quantum vacuum expectation value, in squeezed quantum states

Statistical average with a Gaussian stochastic distribution

They are indistinguishable! *(in the free case)*

How is it possible to prove the quantum origin of inflation primordial perturbations?

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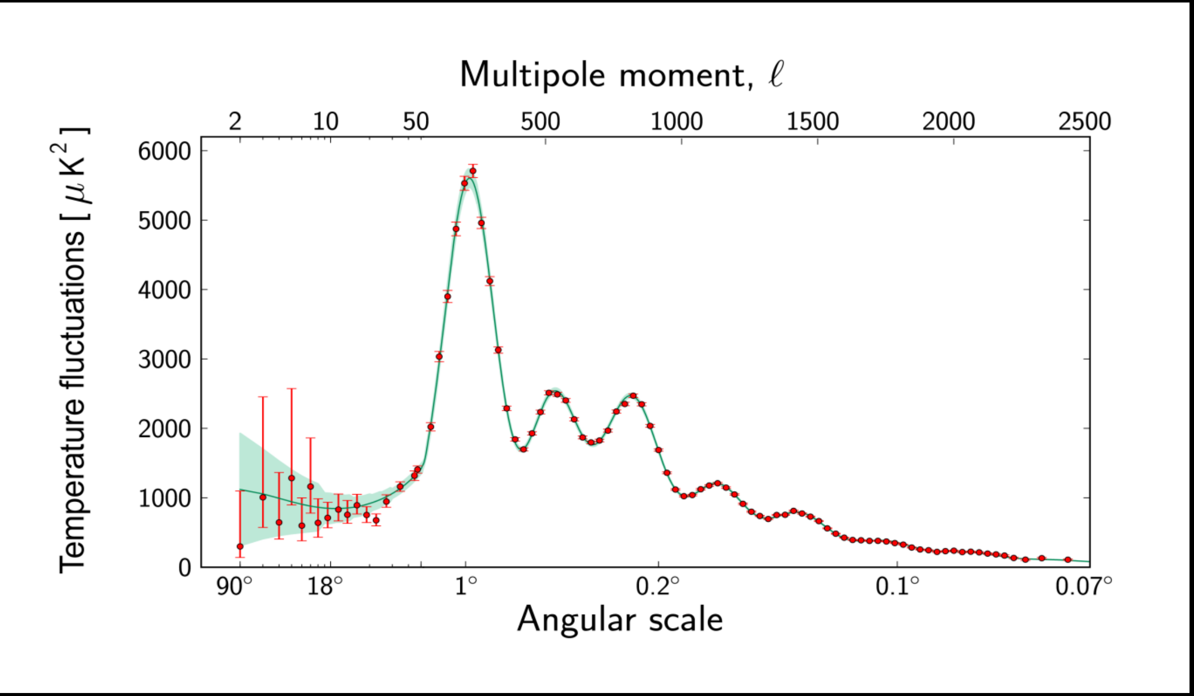
How is it possible to prove the quantum origin of inflation primordial perturbations?

How to connect the ζ perturbations to the temperature anisotropies in CMB:

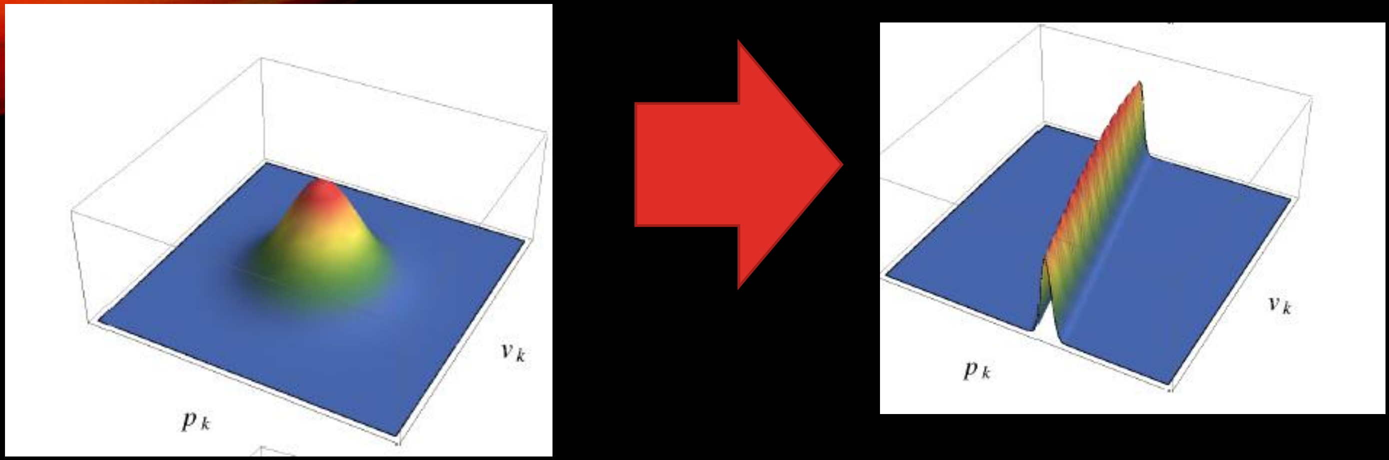
$$\frac{\delta T}{T}(e) = \frac{1}{5}\zeta[\eta_{\ell ss}, -e(\eta_{\ell ss} - \eta_0) + x_0]$$

$$\left\langle \Psi \left| \frac{\widehat{\delta T}}{T}(e_1)\frac{\widehat{\delta T}}{T}(e_2) \right| \Psi \right\rangle = \frac{1}{4\pi}\sum_{\ell=2}^{\infty}(2\ell+1)C_\ell P_\ell(e_1 \cdot e_2)$$

$$C_\ell = \frac{1}{2a^2 M_{\mathrm{Pl}}^2 \epsilon_1}\frac{4\pi}{25}\int \frac{\mathrm{d}k}{k} j_\ell^2[k(\eta_0 - \eta_{\ell ss})]\mathcal{P}_v(k)$$



Squeezing is in the field momentum, broadening in the field amplitude



All quantum predictions, in the squeezed limit, can be reproduced if one assumes that the system always followed classical laws but had random initial conditions with a given probability density function.

$|\Psi_{2sq}\rangle$ Quantum pure state: superposition of all the field amplitudes, not a Stochastic ensemble.

Highly squeezed states are extremely sensitive to each environment; this is the Reason why they are so diffficult to reproduce in the laboratory.

1) This is indistinguishability is not sufficient for QtoCl, Unitary evolution does not break symmetries!

~~"Decoherence without decoherence"(Starobinski et al.,, 1996)~~

Only Interactions with an **unobservable environment** induce **decoherence**

OPEN QUANTUM SYSTEMS

2)How is it possible to prove the quantum origin of inflation primordial perturbations?

THE THEORY OF INFLATION

- Accelerated expansion driven by one (or more) quantum scalar field(s) in the very first instants of the universe

•(quasi) de Sitter metric, but de Sitter approximation for the scale factor:

$$g_{ij}(\vec{x},t) = a^2(t)e^{2\zeta(\vec{x},t)}(\delta_{ij} + h_{ij}(\vec{x},t))$$

Scalar (curvature) perturbations ζ ⟷ Quantum fluctuations of the scalar field

$$\hat{\zeta}|\zeta\rangle = \zeta|\zeta\rangle$$

$$\hat{v} = a\sqrt{2\epsilon}M_{pl}\hat{\zeta}$$

Tensor perturbation h_{ij} (Stochastic Gravitational Waves Background)

WHAT DOES DECOEHERENCE DO?

Interaction with an unobsevable environment → Entanglement, suppress quantum coherence between different possible outcomes

Interference terms in red

$$\rho_{sys} = \begin{pmatrix} |\zeta_1\rangle\langle\zeta_1| & |\zeta_1\rangle\langle\zeta_2| \\ |\zeta_2\rangle\langle\zeta_1| & |\zeta_2\rangle\langle\zeta_2| \end{pmatrix} \xrightarrow{\text{decoherence}} \rho_{sys} = \mathrm{Tr}_{ENV}\,\rho_{sys+env} = \begin{pmatrix} |\zeta_1\rangle\langle\zeta_1| & 0 \\ 0 & |\zeta_2\rangle\langle\zeta_2| \end{pmatrix}$$

↓

Statistical ensemble!

How to quantify? Purity!

$$\gamma = \mathrm{Tr}\,\rho^2 = 1 \xrightarrow{\text{decoherence}} \gamma = \mathrm{Tr}\,\rho_r^2 \to 0$$

MARKOVIAN APPROXIMATION(S)

- "Equation of motion" for the Density Matrix of the System

$$\frac{d\rho_r}{d\eta} = -\mathrm{i}\left[H + H_{LS}, \rho_r(\eta)\right] + \sum_{\boldsymbol{p}} D_{11}(\eta)\left(v_{\boldsymbol{p}}(\eta)\rho_r(\eta)v_{\boldsymbol{p}}^{\dagger}(\eta) - \frac{1}{2}\left\{v_{\boldsymbol{p}}^{\dagger}(\eta)v_{\boldsymbol{p}}(\eta), \rho_r(\eta)\right\}\right)$$

D_{11} (η) "canonical decay rate": contains environmental MEMORY

$$D_{11} = g(\eta)\int_{-\frac{1}{p}}^{\eta} d\eta' g\left(\eta'\right) 2\Re K\left(\eta, \eta'\right)$$

Actually, on the RHS we should have….

$$g(\eta)\int_{-\frac{1}{p}}^{n} d\eta' g\left(\eta'\right) 2\Re K\left(\eta, \eta'\right) v_{\boldsymbol{p}}\left(\eta'\right)\rho_r\left(\eta'\right)$$

But if: $\tau_{env} \ll \tau_{sys}$

1)Born-Markov approximation $\rho_r\left(\eta'\right) \to \rho_r(\eta) + O(g^2)$

2)Strong Markovian Approximation (Burgess+'22,Kaplanek+'21)

$$v_p\left(\eta'\right) \to v_p(\eta)$$

More on this in Nicola's talk next week…

INTERACTIONS

Effective Lagrangian Interactions (Maldacena, 2003):

$$S = \frac{\epsilon M_{pl}^2}{8} \int dt d^3x \left(a^3 \zeta \dot{h}_{ij} \dot{h}_{ij} + a\zeta \partial_l h_{ij} \partial_l h_{ij} - 2a^3 \dot{h}_{ij} \partial_l h_{ij} \partial_l \left(\nabla^2\right)^{-1} \dot{\zeta} \right)$$

By writing the interactions with canonically normalized perturbation fields:

$$H_{INT1} = \frac{\sqrt{\varepsilon}}{2\sqrt{2} M_{pl}} \frac{H}{\eta} \theta_{ij} \theta_{ij} v$$

$$H_{INT2} = \frac{\sqrt{\varepsilon} H \eta}{4\sqrt{2} M_{pl}} v \partial_k \theta_{ij} \partial_k \theta_{ij}$$

$$H_{INT3} = \frac{\sqrt{\varepsilon} H \eta}{2\sqrt{2} M_{pl}} \theta''_{ij} \partial_k \theta_{ij} \partial_k \left(\nabla^2\right)^{-1} v$$