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Unexplored Gravitational Waves on 10-100 Mpc scales

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based on T. Okumura & MS, *JCAP* 10 (2024) 060 : 2405.04210

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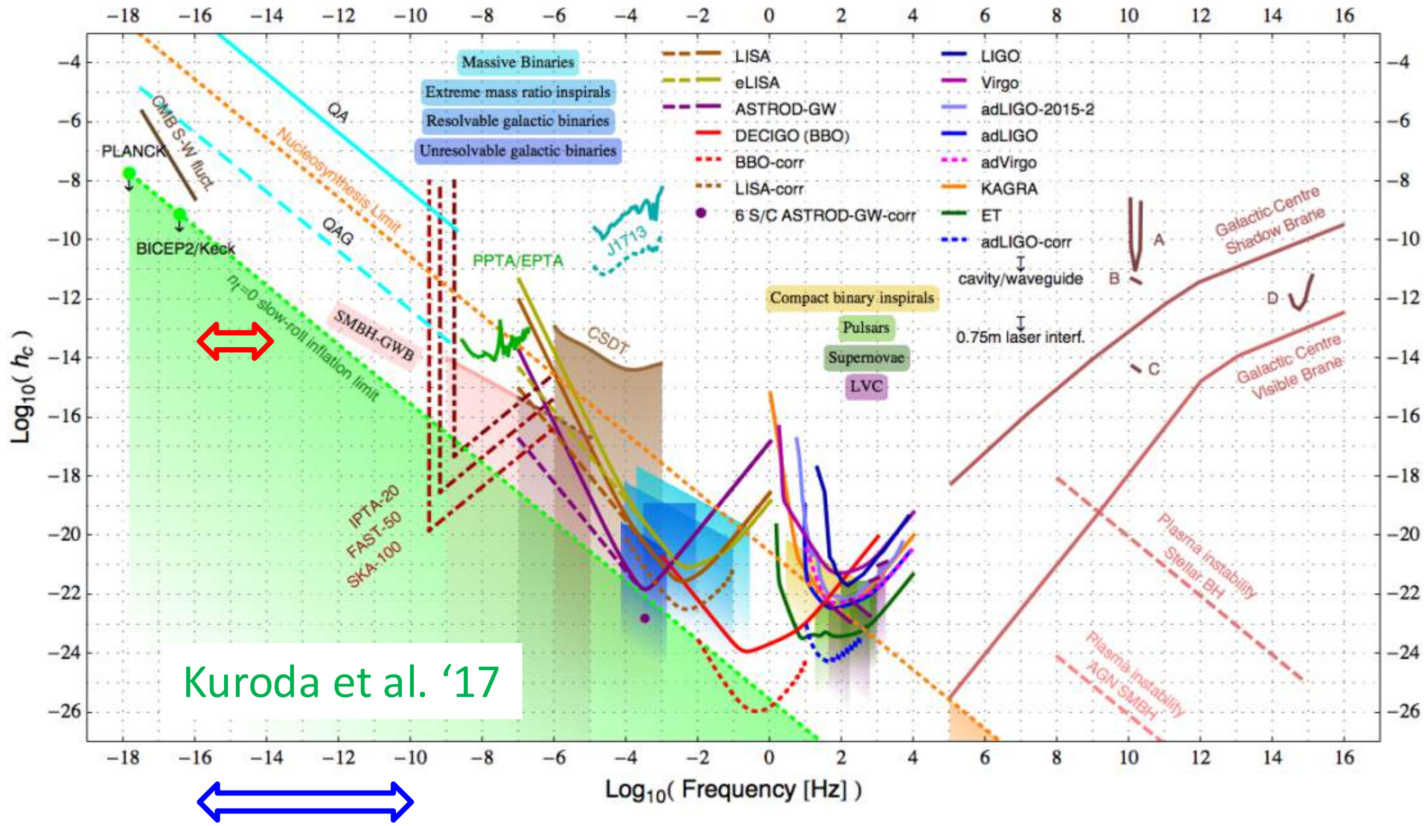
Multi frequency GW Cosmology



$10^{-16} – 10^{-10}$ Hz: empty window

(λ = 100 Mpc – 100 pc, P = 0.3Gyr – 300 yr)

We focus on λ = 1 – 100 Mpc ($f = 10^{-14}$–10^{-16}Hz)

any sources?

- Conventional inflationary GWs are perhaps too small:

$h \lesssim (L/H_o^{-1})^2 \text{x} 10^{-5} < 10^{-8} \iff$ scalar : $\Psi \sim 10^{-5}$

$\Updownarrow$

- h can be enhanced,

  e.g. by another tensor t coupled to h

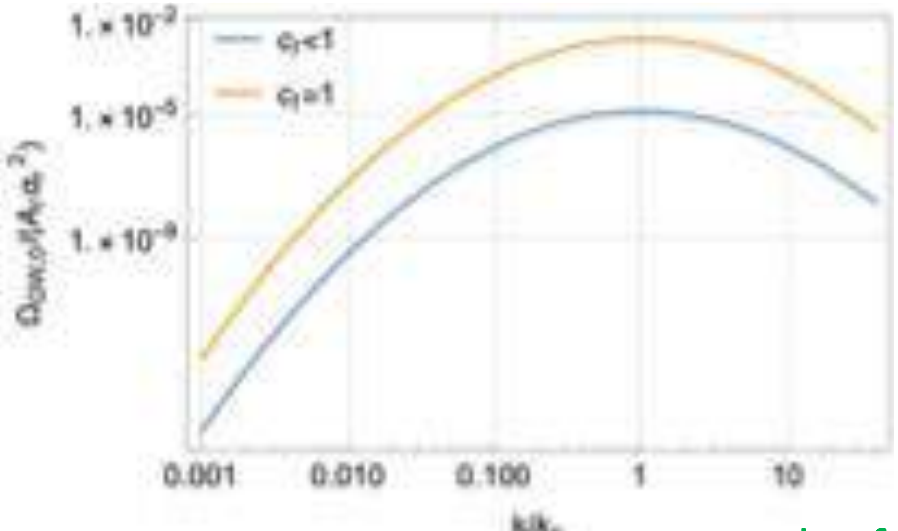
Gorji & MS, 2302.14080

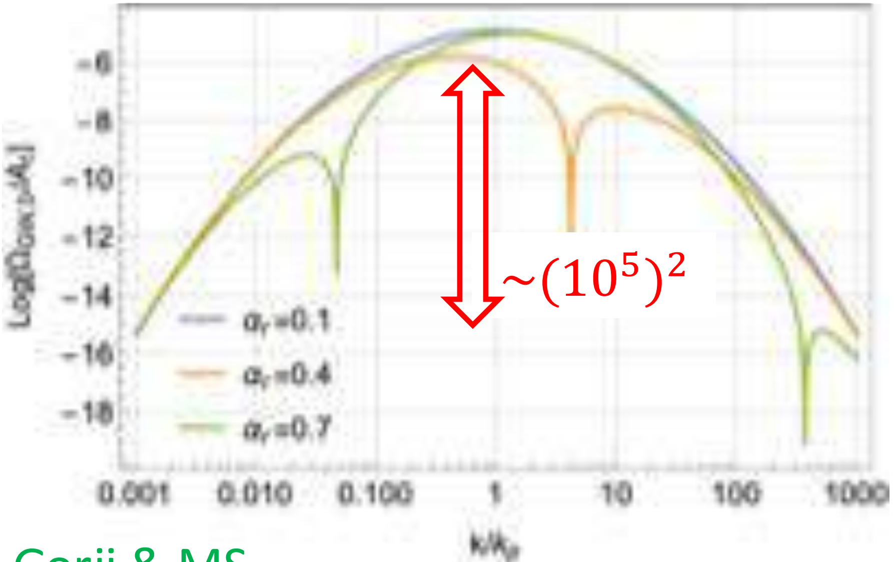
$L_{int} \sim \alpha_r H t^{ij} h'_{ij}$

talk by Tomo Fujita yesterday

excitation in $t^{ij} \Longrightarrow$ excitation in h_{ij}

Mohammad Ali (Iman) Gorji

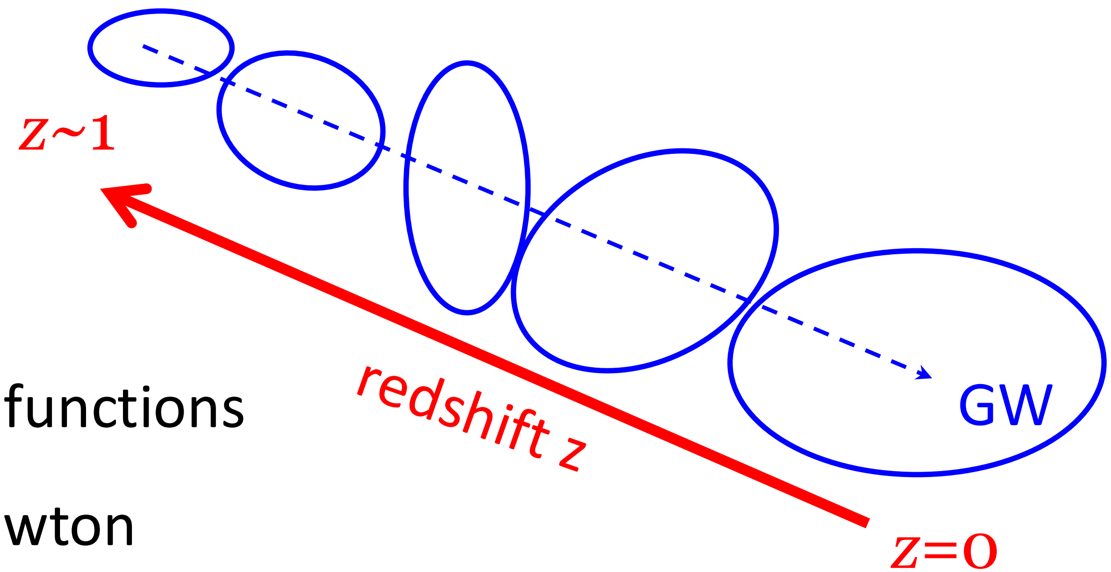




examples from Gorji & MS

GWs=tidal force=tidal deformation

- GWs are frozen (static in our timescale)
  = locally identical to scalar (Newtonian) tidal force
- The difference is its space-time dependence



- different correlation functions
  - in linear theory, Newton potential $\psi(t,x)$ is almost constant in time:

$\langle \Psi_k(t)\Psi_k(t')\rangle \sim$ time-indep $\sim P_\Psi(k)$

GWs propagate:

$\langle h_k(t)h_k(t')\rangle \sim$ time-dep $\sim P_h(t,t';k)$

Tidal force field

- metric perturbation

tensor = GWs

$$ds^2 = -a^2(\eta)\Big([1+2\Psi(\eta,\mathbf{x})]d\eta^2 + \{[1-2\Psi(\eta,\mathbf{x})]\delta_{ij} + h_{ij}^{\mathrm{TT}}(\eta,\mathbf{x})\}dx^i dx^j\Big)$$

scalar

- tidal force field

$$\delta R^k{}_{0j0}(\eta,\mathbf{x}) = \left[\Psi''(\eta,\mathbf{x}) + 2\mathcal{H}(\eta)\Psi'(\eta,\mathbf{x}) + \frac{1}{3}\nabla^2\Psi(\eta,\mathbf{x})\right]\delta^k_j + \delta^{ki}F_{ij}(\eta,\mathbf{x})$$

tidal force

- dimensionless tidal force field

$$f_{ij}(\eta,\mathbf{x}) \equiv \frac{F_{ij}(\eta,\mathbf{x})}{4\pi G\bar{\rho}(\eta)a^2} = s_{ij}(\eta,\mathbf{x}) + t_{ij}(\eta,\mathbf{x})$$

$$s_{ij}(\eta,\mathbf{x}) \equiv \frac{1}{4\pi G\bar{\rho}a^2}\left(\partial_i\partial_j - \frac{1}{3}\delta_{ij}\nabla^2\right)\Psi(\eta,\mathbf{x}) \quad \cdots \text{ scalar}$$

$$t_{ij}(\eta,\mathbf{x}) \equiv -\frac{1}{8\pi G\bar{\rho}a^2}\left[h''_{ij}(\eta,\mathbf{x}) + \mathcal{H}(\eta)h'_{ij}(\eta,\mathbf{x})\right] \quad \cdots \text{ tensor}$$

- tensor Fourier component

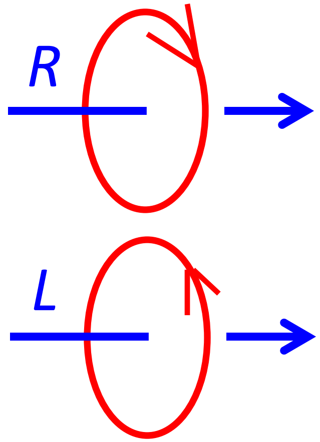
$$\boxed{t_{ij}(\eta,\mathbf{k}) = \frac{k^2}{3\mathcal{H}^2}h_{ij}(\eta,\mathbf{k})} \quad \text{for } k \gg \mathcal{H}$$

$$\left(\frac{k}{\mathcal{H}} = 40\sim 4000 \quad \text{for } k = 1\sim 100\ \mathrm{Mpc}^{-1}\right)$$

L-R helicity (polarization) decomposition

- define * operation,

$$X_{ij} \longrightarrow {}^{*}X_{ij}(\eta, \mathbf{x}) \equiv \frac{1}{2}\left(\epsilon_i{}^{mn}\partial_m X_{nj} + \epsilon_j{}^{mn}\partial_m X_{ni}\right)$$



then,

$$h_{ij}(\eta, \mathbf{k}) = \left[e_{ij}^{(R)}(\hat{\mathbf{k}})h_{(R)}(\eta, \mathbf{x}) + e_{ij}^{(L)}(\hat{\mathbf{k}})h_{(L)}(\eta, \mathbf{x})\right]$$

$$\longrightarrow {}^{*}h_{ij}(\eta, \mathbf{k}) = \underline{k}\left[e_{ij}^{(R)}(\hat{\mathbf{k}})h_{(R)}(\eta, \mathbf{k}) - e_{ij}^{(L)}(\hat{\mathbf{k}})h_{(L)}(\eta, \mathbf{k})\right]$$

except for the factor k, *op transforms R → R, L → −L

* can be used to detect parity violation (PV) in GWs

2-pt functions

$$\langle h_{(\lambda)}(\eta, \mathbf{k}) h_{(\lambda)}(\eta', \mathbf{k}') \rangle = (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}') P_{(\lambda)}(\eta, \eta', k) \qquad \lambda = R, L$$

$$\langle {}^* h(\eta, \mathbf{k}) h(\eta', \mathbf{k}') \rangle = (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}')\, k\, \chi(\eta, \eta', k) P_{(R+L)}(\eta, \eta', k)$$

$$\chi(\eta, \eta', k) \equiv \frac{P_{(R)}(\eta, \eta', k) - P_{(L)}(\eta, \eta', k)}{P_{(R+L)}(\eta, \eta', k)}$$: degree of PV

cosmological GW 2-pt fcns exhibit unique time-dependence

$$P_{(\lambda)}(\eta, \eta', k) = \frac{a_0^2}{a(\eta) a(\eta')} \boxed{\cos[k(\eta - \eta')]} P_{(\lambda)}(k) \qquad \text{for } k\eta \gg 1$$

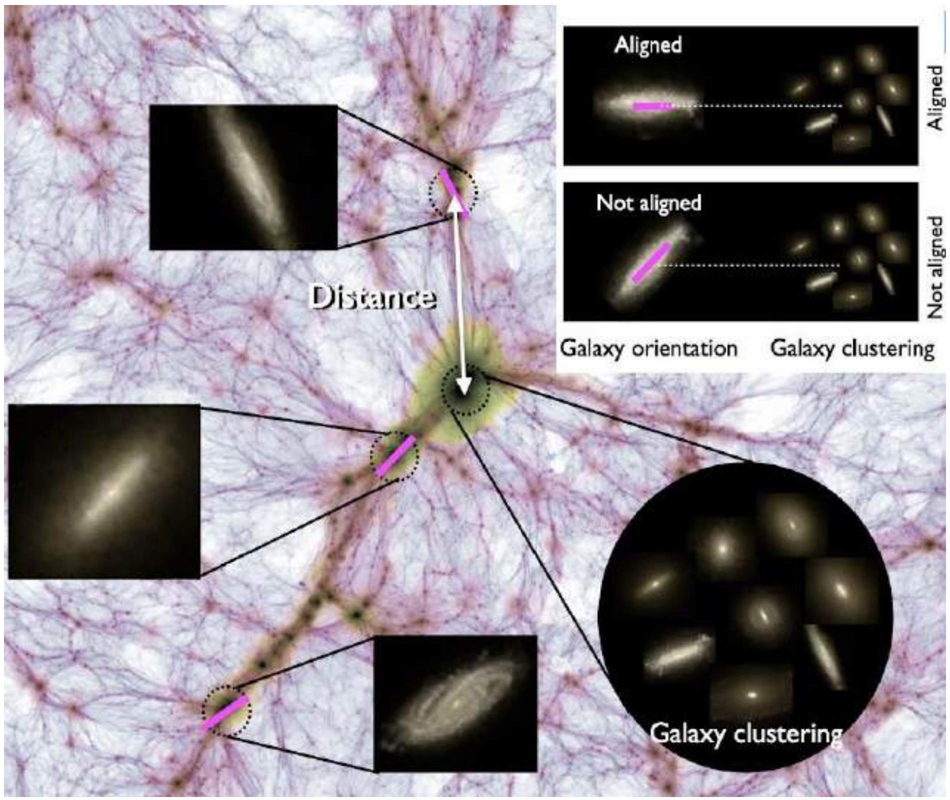
How can we probe cosmological GW background?

⇩

Intrinsic Alignment of galaxies

(not apparent alignment by gravitational lensing)

Intrinsic Alignment of galaxies



TNG Simulations image modified by Taruya-Okumura

- (scalar) tidal fields align galaxies

$$\gamma_{ij} = b_K^s s_{ij}$$

$$s_{ij}(\eta, \mathbf{x}) = \left(\frac{\partial_i \partial_j}{\nabla^2} - \frac{1}{3}\delta_{ij}\right)\delta_m(\eta, \mathbf{x})$$

Van Waerbeke et al. astro-ph/0002500v2, …

- GWs also align galaxies

$$\gamma_{ij}^{GW} = b_K^{GW} t_{ij}(\eta, \mathbf{x})$$

$$t_{ij}(\eta, \mathbf{k}) = \frac{k^2}{3\mathcal{H}^2} h_{ij}(\eta, \mathbf{k})$$

Schmidt, Pajer & Zaldarriaga, 1312.5616

Akitsu, Li & Okumura, 2209.06226

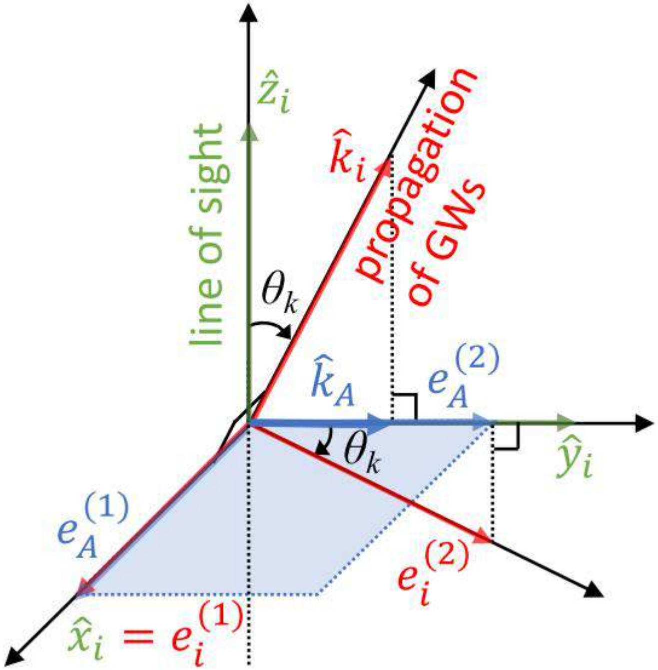
Projected tidal force field in 3D

- Observable = projected tidal field

$$\underbrace{e^{(1)} = \hat{x}\,,\ e^{(2)} = \hat{y}}_{\text{basis perpendicular to } n}\,,\ n = \hat{z}$$

line of sight

projection tensor: $P^i_m \equiv \delta^i_m - n^i n_m$



- Projected field in 3D

$$f_{AB} = P^i_A P^j_B f_{ij} \qquad (A,\ B\text{=1,2})$$

$$f^T_{AB} \equiv f_{AB} - \frac{1}{2} f^C{}_C$$

trace part is difficult to detect

- Projected (traceless) tidal force field

$$f^T_{AB} = s^T_{AB} + t^T_{AB}$$

scalar

tensor

How can we extract tensor part?

extracting PVGW components

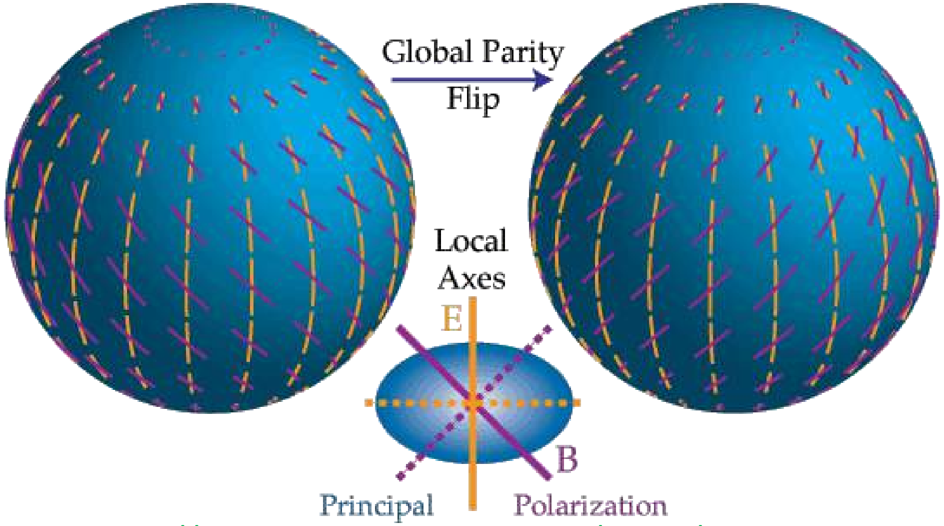
- conventional method: E/B decomposition

  E-mode = scalar + tensor

  B-mode = tensor

  $\Rightarrow$ non-zero $\langle B^2 \rangle$ = GW
  non-zero $\langle E\cdot B \rangle$ = PVGW

  E/B modes: defined globally



https://background.uchicago.edu/~whu/index.html

- new "local" method (complimentary to E/B method)

$$X_A^T(\eta, \mathbf{x}) \equiv \partial^B X_{BA}^T(\eta, \mathbf{x}), \qquad \boxed{{}^*X_A^T(\eta, \mathbf{x}) \equiv \epsilon^{BC} \partial_B X_{CA}^T(\eta, \mathbf{x})}$$

$$\boxed{{}^*X^T(\eta, \mathbf{x}) \equiv \partial^{A\,*} X_A^T(\eta, \mathbf{x}) = \epsilon^{BC} \partial^A \partial_B X_{CA}^T}$$

…free from scalar components

2-pt functions of projected tidal fields

- 2-pt fcns in terms of GW amplitude h

$$t_{ij}(\eta,\mathbf{k}) = \frac{k^2}{3\mathcal{H}^2} h_{ij}(\eta,\mathbf{k})$$

$$\Rightarrow \quad t_A = \frac{k^2}{3\mathcal{H}^2}\underbrace{h_A}_{\text{div } h}\,, \qquad {}^*t_A = \frac{k^2}{3\mathcal{H}^2}\underbrace{{}^*h_A}_{\text{rot } h}\,, \qquad \underbrace{\partial_A {}^*h^A = {}^*h^T}_{\text{div}\cdot\text{rot } h}$$

$$\langle {}^*h^T(\eta,\mathbf{x})\,{}^*h^T(\eta',\mathbf{x}')\rangle = \frac{a_0^2}{a(\eta)a(\eta')}\int \frac{k^6 dk}{2\pi^2} P_h(k)\Gamma_{{}^*h\,{}^*h}(kr, k\Delta\eta, \theta)$$

$$\langle {}^*h^{T,A}(\eta,\mathbf{x})\, h_A^T(\eta',\mathbf{x}')\rangle = \frac{a_0^2}{a(\eta)a(\eta')}\int \frac{k^4 dk}{2\pi^2} P_h(k)\underbrace{\chi(k)}_{\neq 0 \text{ if } \exists \text{PV}}\Gamma_{{}^*h\,h}(kr, k\Delta\eta, \theta)$$

$\Gamma_{XY}(kr, k\Delta\eta, \theta) \cdots$ "Overlap Reduction Function" for $<XY>$

$$r = |\mathbf{x}-\mathbf{x}'|\,,\ \ \Delta\eta = \eta - \eta'\,,\ \ \cos\theta = \frac{\mathbf{x}\cdot\mathbf{x}'}{|\mathbf{x}||\mathbf{x}'|}$$

The above 2-pt functions can be obtained by local operations

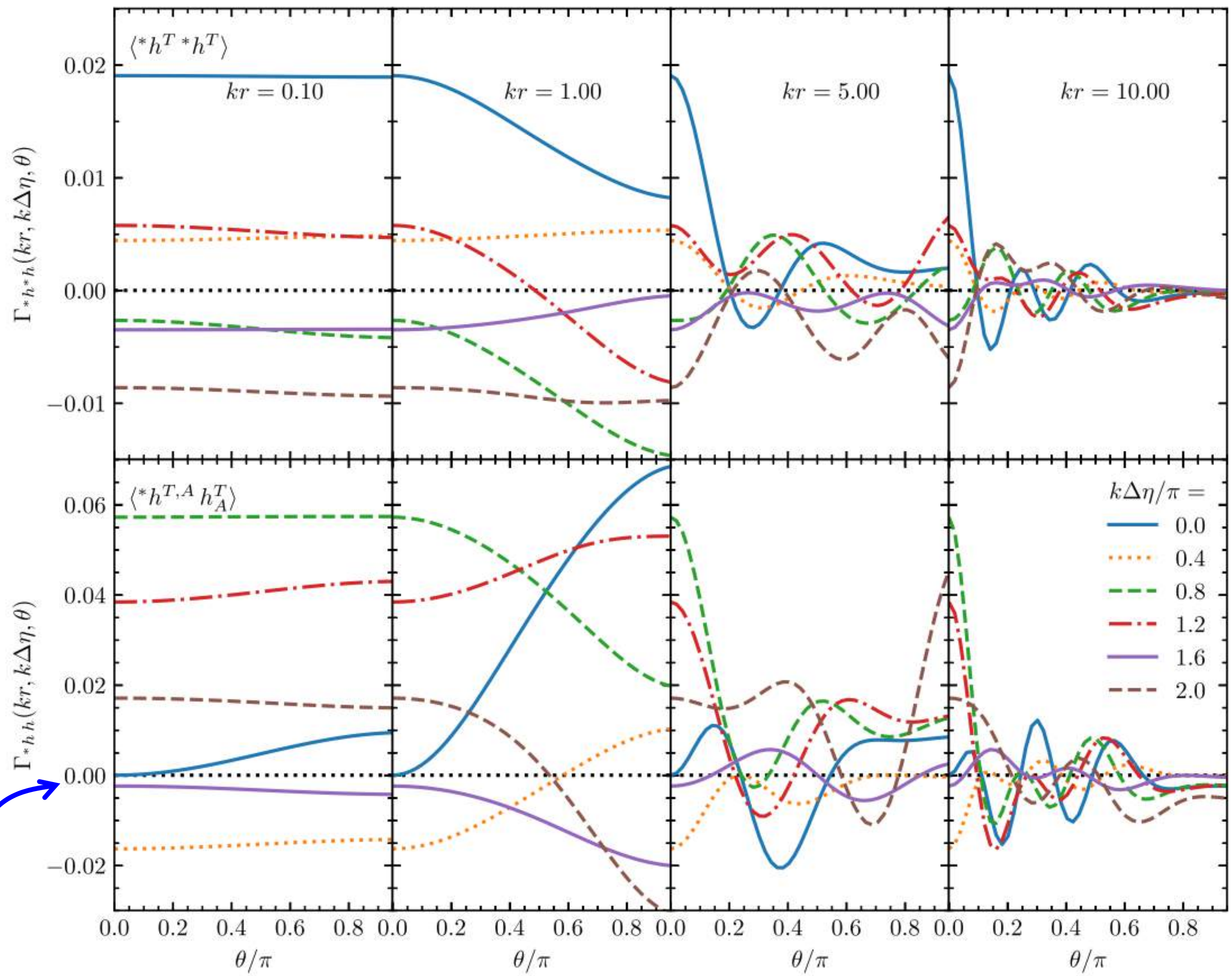
ORFs for <$^*h^*h$> and <$^*h\ h$>

<$^*h^T\,{}^*h^T$>

GW signal

<$^*h^{T,A}\ h^T{}_A$>

PVGW signal



PV signal vanishes in the limit (ϑ, $k\Delta\eta$) → (0, 0)

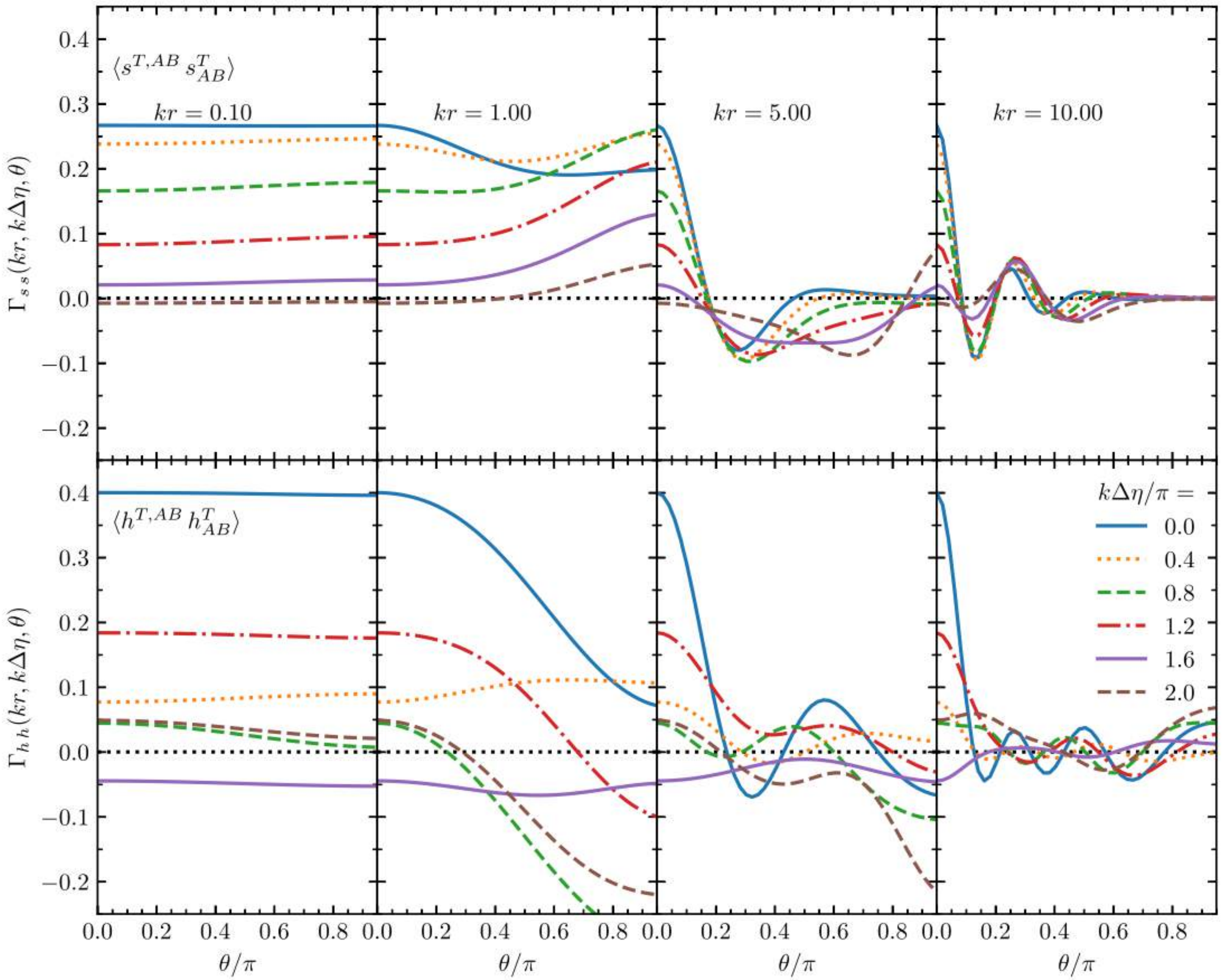
ORFs for scalar and tensor

$<s^{T,AB}\, s^{T}{}_{AB}>$

scalar tidal field

$<h^{T,AB}\, h^{T}{}_{AB}>$

tensor field



- In principle it's possible to distinguish tensor from scalar contributions by template matching applied to the data.

Summary

- Taking the divergence and the curl of the projected tidal field $\boldsymbol{f}$: $\nabla \cdot \nabla \times \boldsymbol{f}$, allows us to extract GW signals locally, free from scalar (gravitational potential) tidal force field.

- Taking the cross-correlation between the divergence and the curl of $\boldsymbol{f}$: $\langle \nabla \boldsymbol{f} \cdot \nabla \times \boldsymbol{f} \rangle$, we can extract PVGW signals.

- Our method is complimentary to the E/B-method, as it can extract GWs locally, in principle.

- Projection of the signal-to-noise ratio is left for future work.

Appendix: Relation to E/B-modes

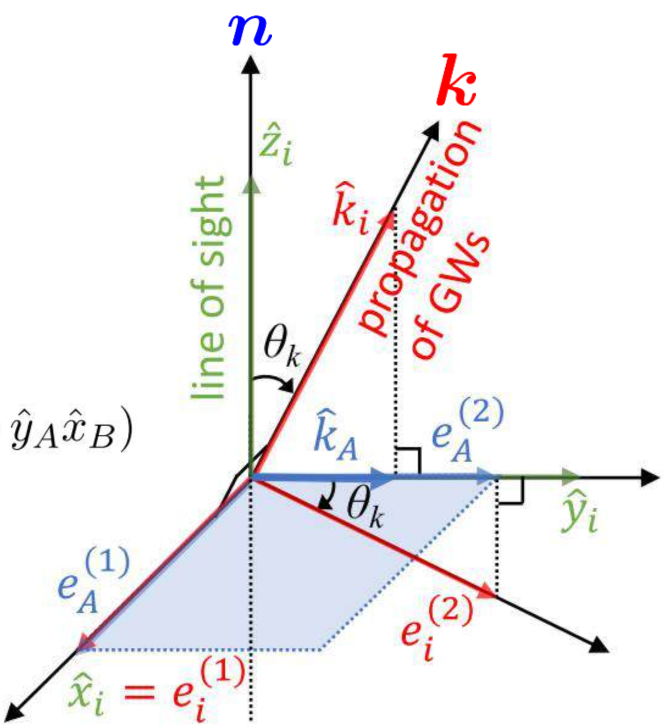
- Fourier modes

$$h_{(E)} = h_{AB}e^{AB}_{(E)} = \frac{1}{\sqrt{2}}\frac{1+\mu_k^2}{2}(h_{(R)} + h_{(L)}),$$

$$h_{(B)} = h_{AB}e^{AB}_{(B)} = \frac{i}{\sqrt{2}}\mu_k(h_{(R)} - h_{(L)}).$$

$$e^{(E)}_{AB} = \frac{1}{\sqrt{2}}\left(\hat{x}_A\hat{x}_B - \hat{y}_A\hat{y}_B\right), \quad e^{(B)}_{AB} = \frac{1}{\sqrt{2}}\left(\hat{x}_A\hat{y}_B + \hat{y}_A\hat{x}_B\right)$$

$$\mu_k \equiv \frac{\boldsymbol{k}\cdot\boldsymbol{n}}{k}$$



- Power spectra

$$P_{EE}(\eta,\eta',\mathbf{k}) = \frac{1}{8}(1+\mu_k^2)^2(P_{(R)} + P_{(L)}) = \frac{1}{8}(1+\mu_k^2)^2 P_h(\eta,\eta',k),$$

$$P_{BB}(\eta,\eta',\mathbf{k}) = \frac{1}{4}\mu_k^2(P_{(R)} + P_{(L)}) = \frac{1}{4}\mu_k^2 P_h(\eta,\eta',k),$$

$$P_{EB}(\eta,\eta',\mathbf{k}) = \frac{i}{4}\mu_k(1+\mu_k^2)(P_{(R)} - P_{(L)}) = \frac{i}{4}\mu_k(1+\mu_k^2)\chi(k)P_h(\eta,\eta',k)$$