

# String theory in a pinch

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Based on 2411.14998

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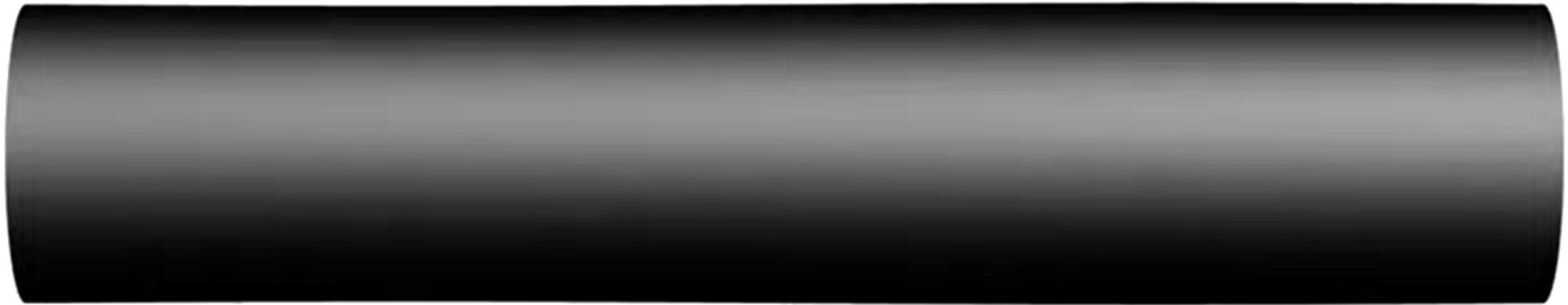


14/07/2025

# Motivation: naked singularities

- From smooth initial data to naked singularities
  - Critical Collapse Choptuik '93
  - Gregory-Laflamme (GL) instability Gregory and Laflame '93
- Learn how GR gets to its breaking point
- Modify CCC: assume short-living planckian object Empanan '20
- Before reaching Planck scales e.g. string theory

# Gregory-Laflamme (GL) instability



- Thermodynamic instability  $\rightarrow$  (hydro)dynamical instability

# Gregory-Laflamme (GL) instability

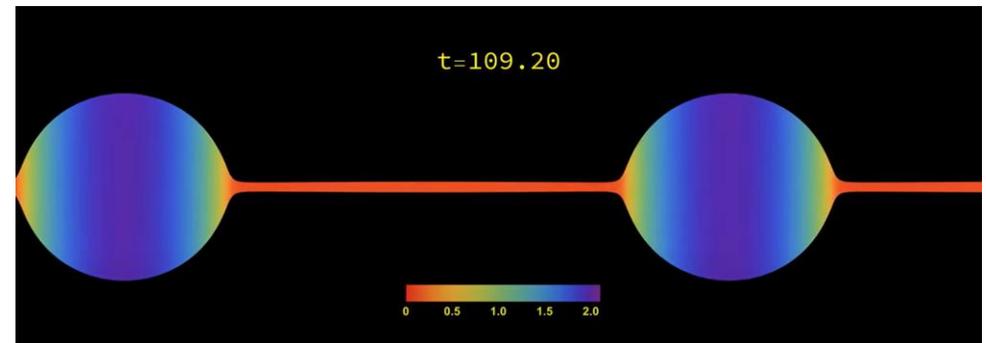
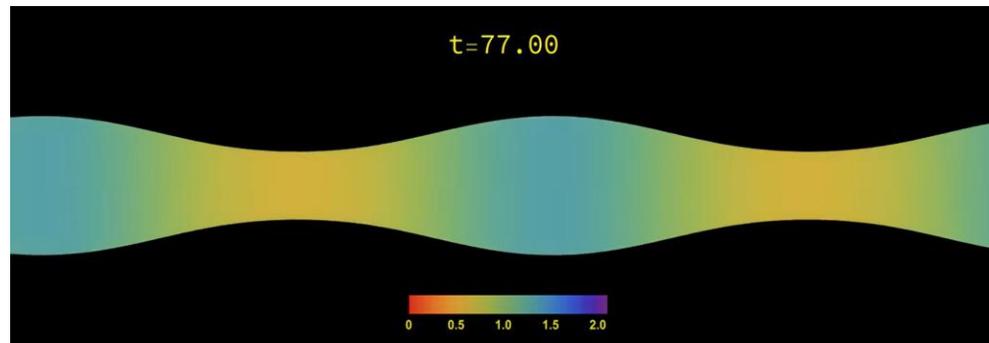
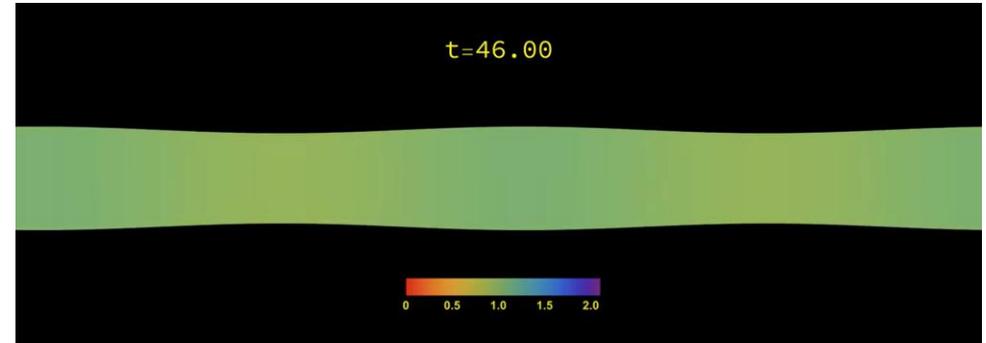
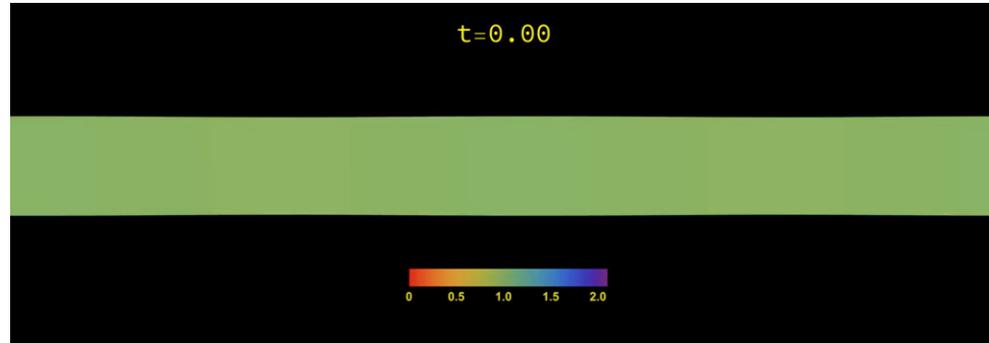


- Thermodynamic instability  $\rightarrow$  (hydro)dynamical instability
- $\delta g_{\mu\nu} \sim e^{\Omega t}$
- Non-uniform black strings are entropically disfavored

Wiseman '02

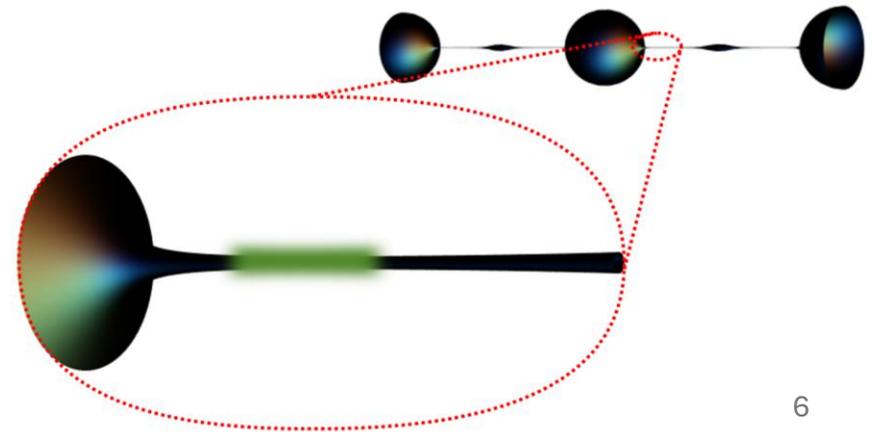
# Numerical evolution of the GL instability

Lehner and Pretorius '10    Figueras et al '23



# Approaching the singularity

- Singularity is reached in a finite time Lehner and Pretorius '10 Figueras et al '23
- Higher curvature corrections become important with time
- Eventually, the string scale  $l_s$  will be reached
- What happens then?



# Black hole/string correspondence

Susskind '93

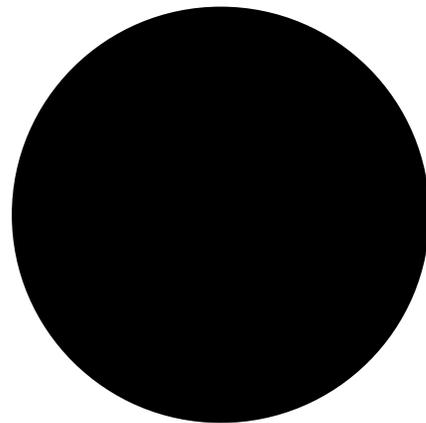
Howowitz, Polchinski '96

Ceplak, Emparan, Puhm, Tomasevic '23, '24

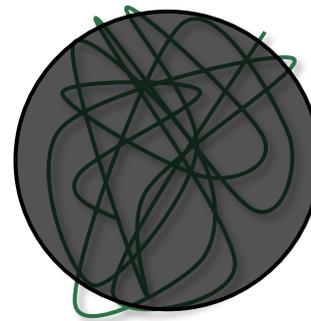
$$r_0 \sim GM \sim l_s \Rightarrow S_{BH} \sim S_{FS}$$

$$l_{FS} \gg l_s \sim r_0$$

$$l_{FS} \gg l_{IS} \sim l_s \sim r_0$$

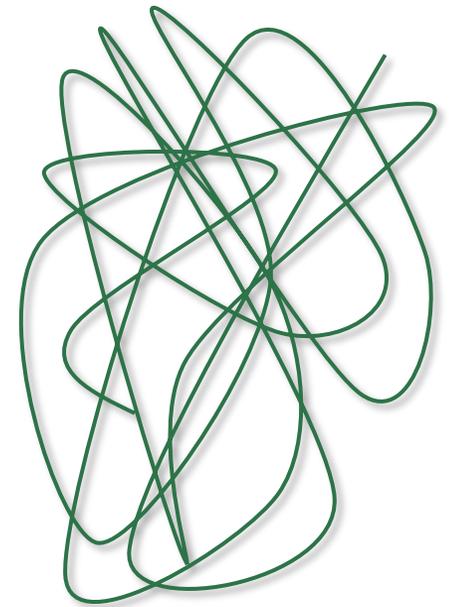


$g_s^2 S$



$g_s^2 S$

$$g_s^2 S \sim 1$$



# Self gravitating strings

- For  $T \lesssim T_H$ , highly excited string states can be collectively described
- Light mean-field  $\chi$  with  $m^2 \equiv \Delta_\beta \sim T_H - T$  Polchinski '85 Atick, Witten '88
- We need to add it at low energies

# Self gravitating strings

Howowitz, Polchinski '97  
Chen, Maldacena, Witten '21

$$I_d \sim \int d^d x \sqrt{g} e^{-2\phi_d} (-R - 4(\nabla\phi_d)^2 + (\nabla\varphi)^2 + |\nabla\chi|^2 + \Delta_\beta |\chi|^2 + \varphi |\chi|^2 + \dots)$$

- $D = d + 1$  spacetime dimensions
- $g$  spatial metric and  $\phi_d$  dilaton – fixed
- $\varphi$  is the gravitational potential
- For  $\Delta_\beta \ll 1$ ,  $\chi$  interaction with  $\varphi$  dominant

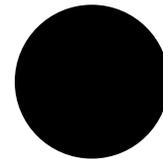
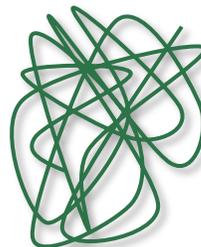
# Self gravitating strings

- Non relativistic boson star equations
  - Thermal configurations with  $S > 0$
  - Purely Euclidean

$$\nabla^2 \chi - (\Delta_\beta + \varphi) \chi = 0$$

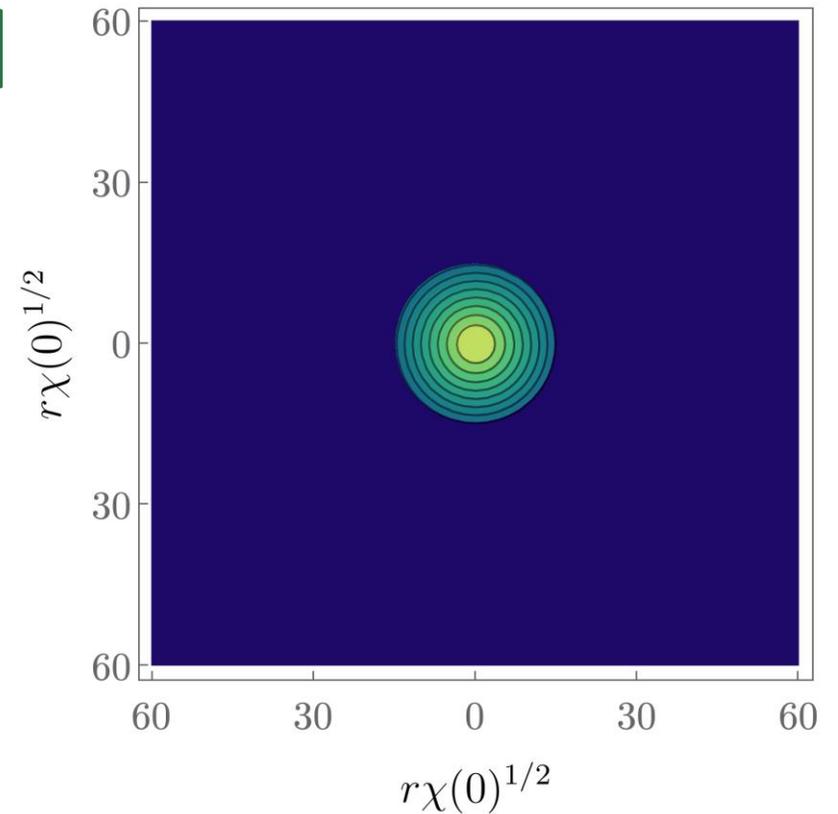
$$\nabla^2 \varphi - \frac{1}{2} \chi^2 = 0$$

- Valid when  $g_S^{\frac{4}{6-d}} \lesssim \Delta_\beta \ll 1$



# Horowitz-Polchinski (HP) ball

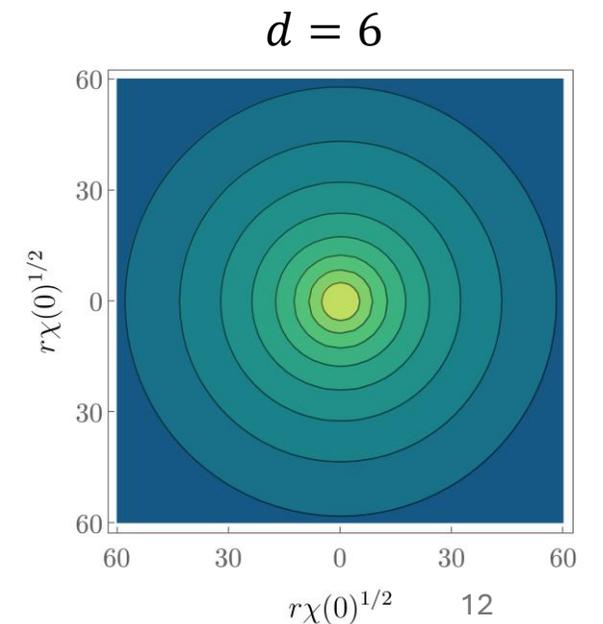
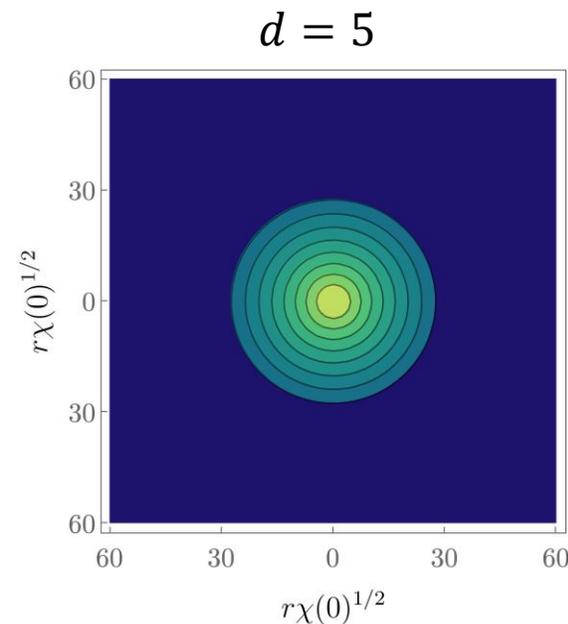
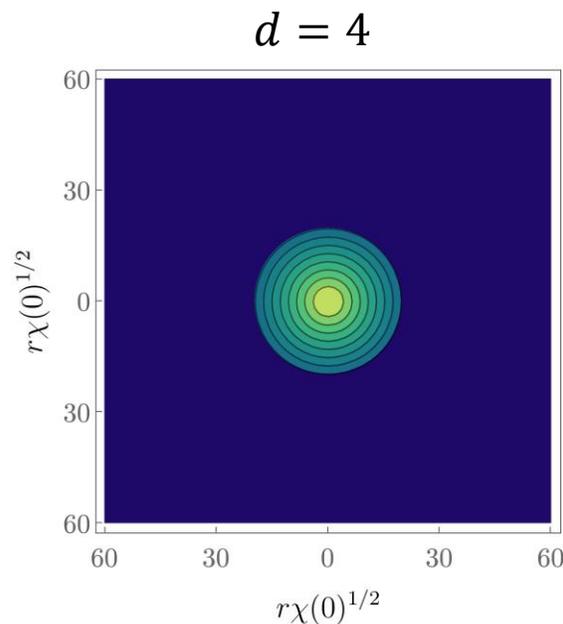
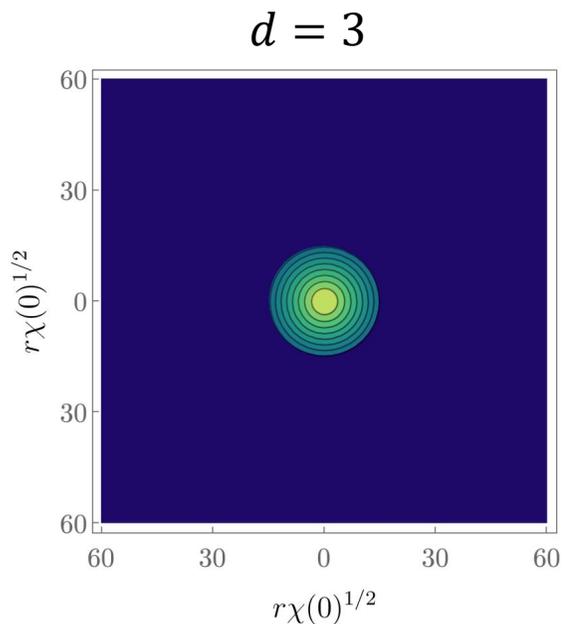
- Exponentially localized configurations
- Size grows with temperature
- In lower  $d$ , heavier stars = more compact
- When  $\Delta_\beta \sim 1$ ,  $r_b \sim l_s$



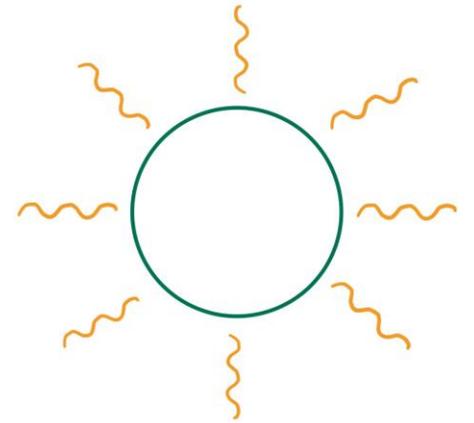
$$r_b \sim \frac{1}{\sqrt{\Delta_\beta}} \sim \sqrt{\frac{T}{T_H - T}}$$

# HP balls in higher dimensions

- As  $d \rightarrow 6$ ,  $\Delta_\beta \rightarrow 0$ . The star puffs up and out of the range of validity
- Corrections allow to construct stars below Hagedorn Balthazar, Chu, Kutasov '24
- We will focus on lower dimensions ( $d=4,5$ ) Chu '24

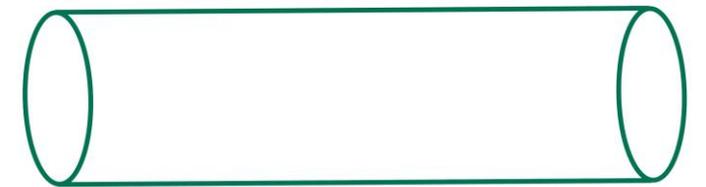


# Stringy GL

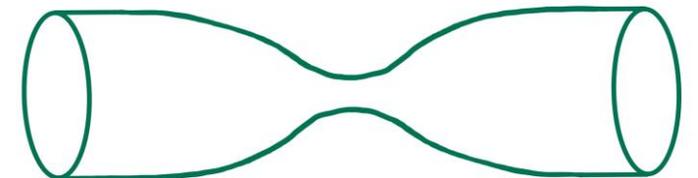


- HP balls have a negative Euclidean mode Chen, Maldacena, Witten '21

- Zero mode in  $d + 1$  dimensions (GL threshold)



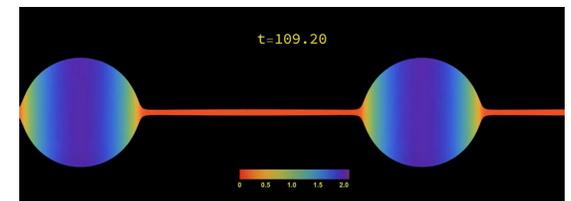
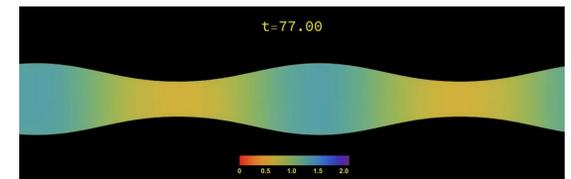
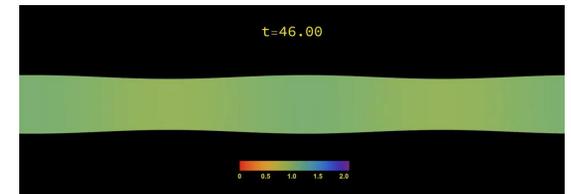
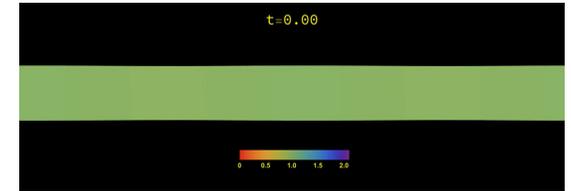
- Family of non-uniform stringy string configurations



# Fitting it into GL time evolution

- Dynamical stability?
- GL is not adiabatic
- Equilibrium is the first step
- Higher curvature corrections slow down the process

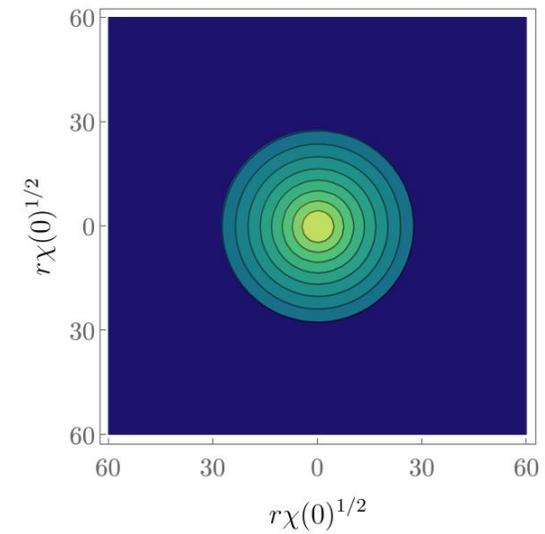
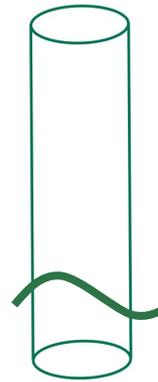
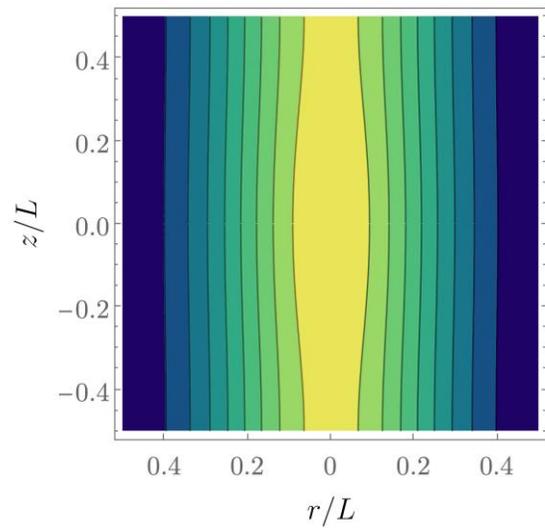
Figueras, Kovács, Yao



# Goal

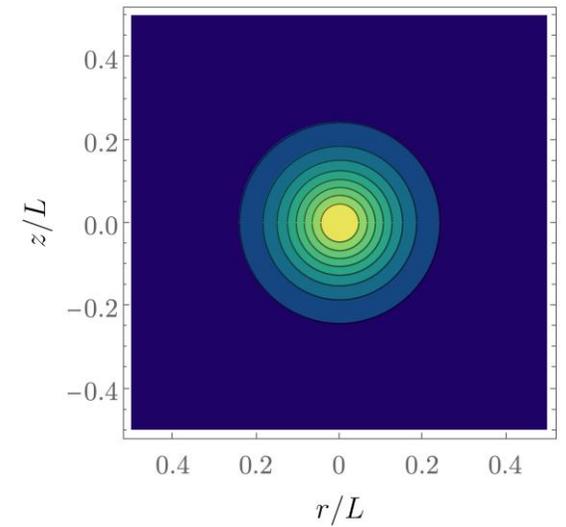
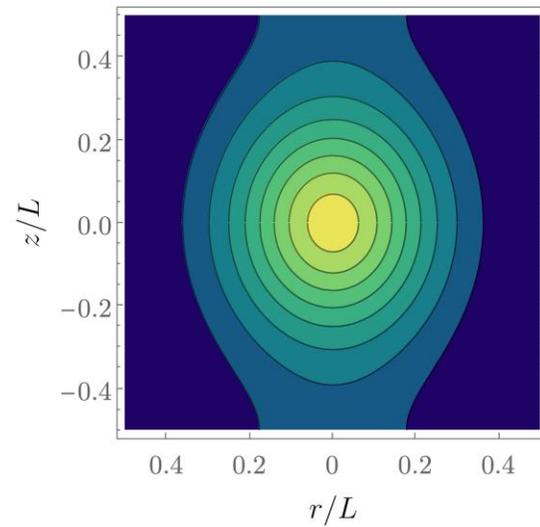
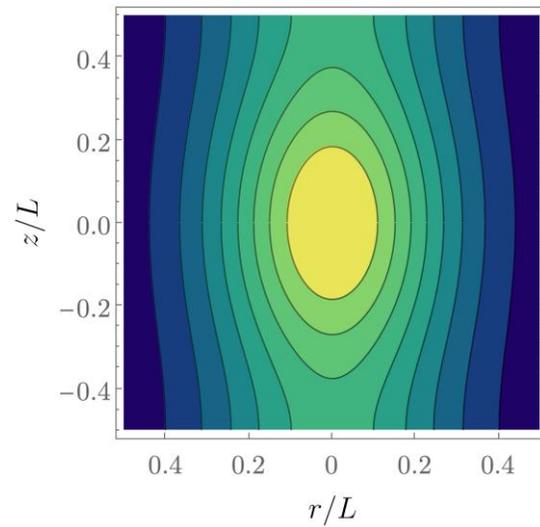
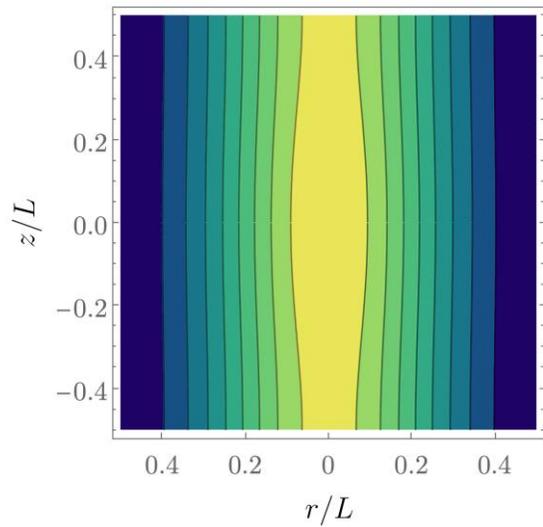
- Construct the full family of non-uniform HP strings (NUHPS)
- Which configuration is thermodynamically favored (UHPS or NUHPS)
- Does classical evolution towards pinching continue at  $l_S$

# Non-uniform HP configurations

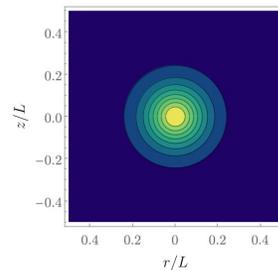
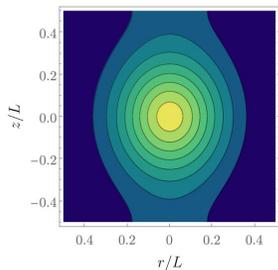
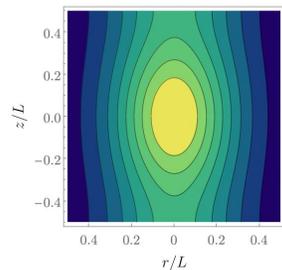
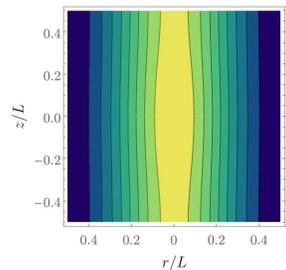


# Non-uniform HP configurations

$$L/r_b$$

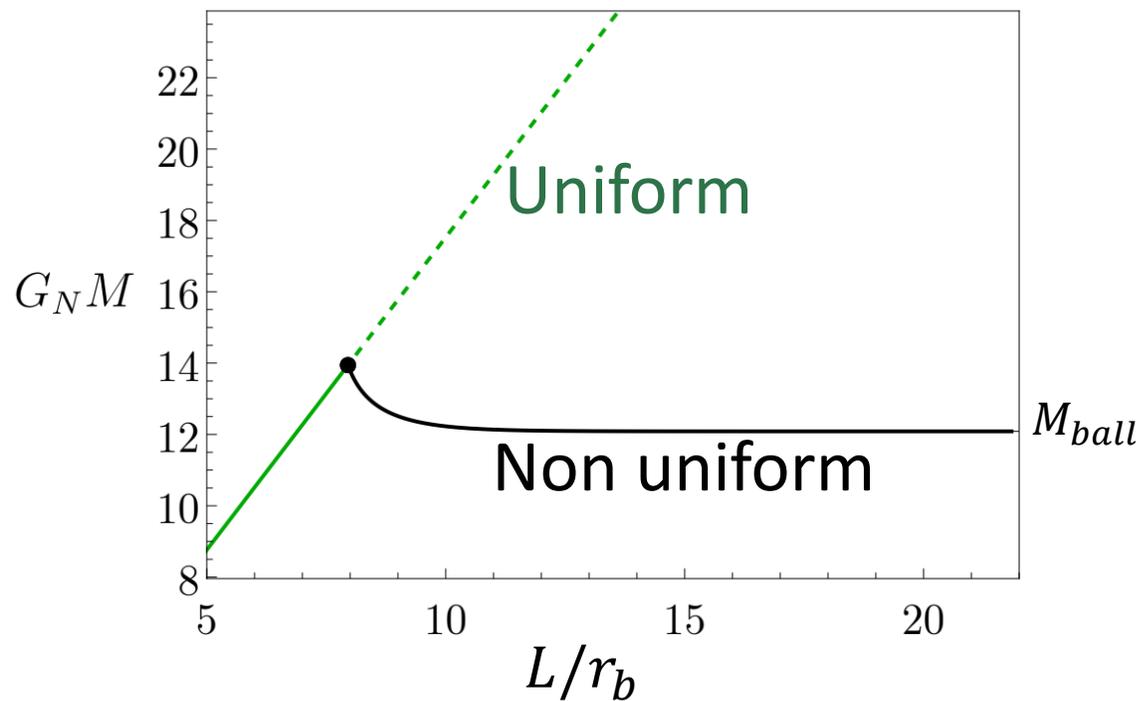


No topology change

$L/r_b$ 

$$\varphi \sim -\frac{G_N \rho(z)}{r^{d-3}}$$

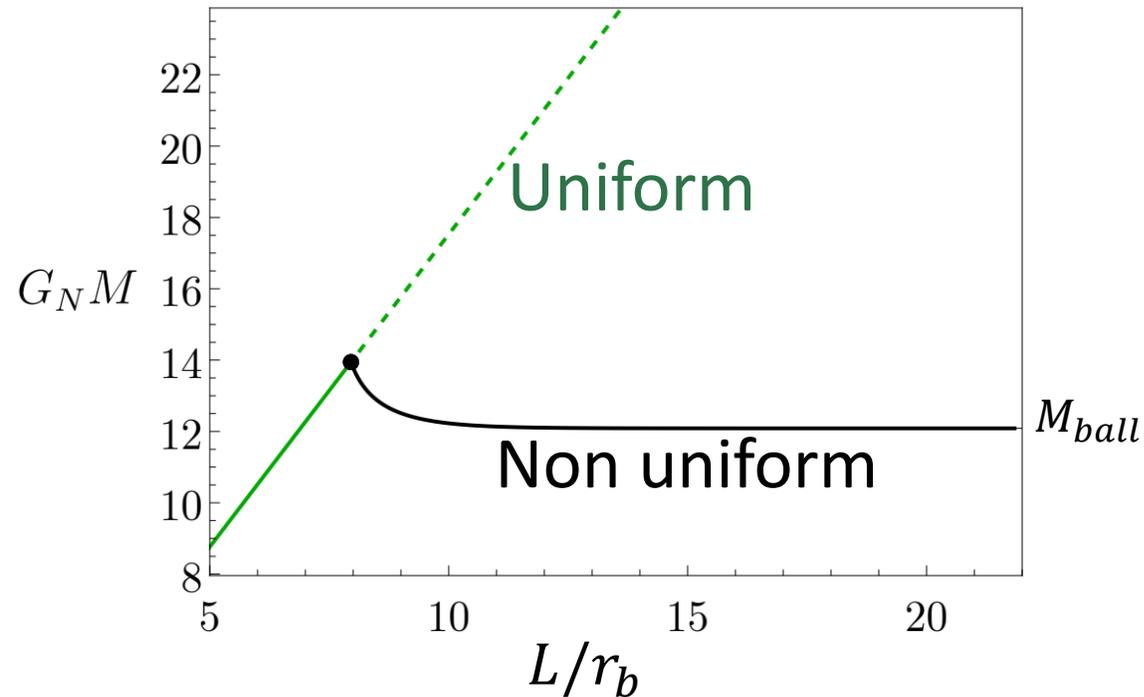
$$M \sim \int \rho(z) dz$$



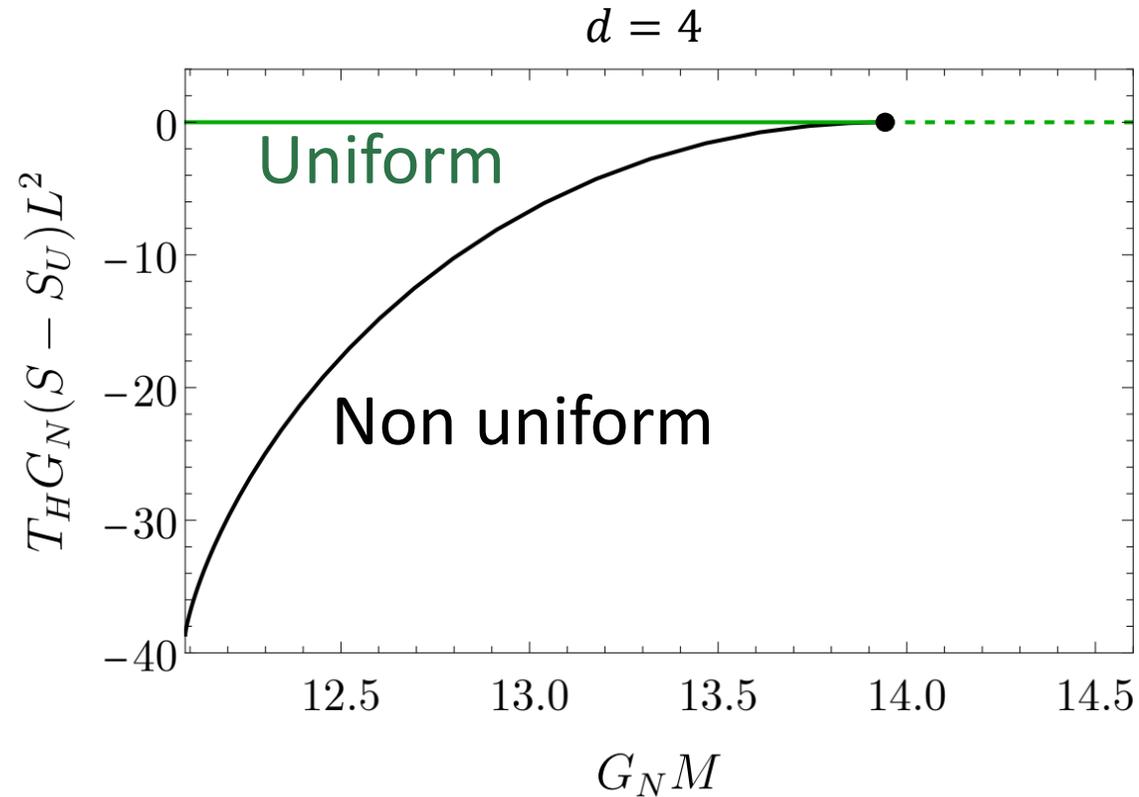
# Global stability: microcanonical

- $S = S_U + \int \beta(M, L) dM$  at fixed  $L$
- $\beta(M, L)$  from  $M(\beta, L)$

$$r_b \sim \sqrt{\frac{T}{T_H - T}}$$

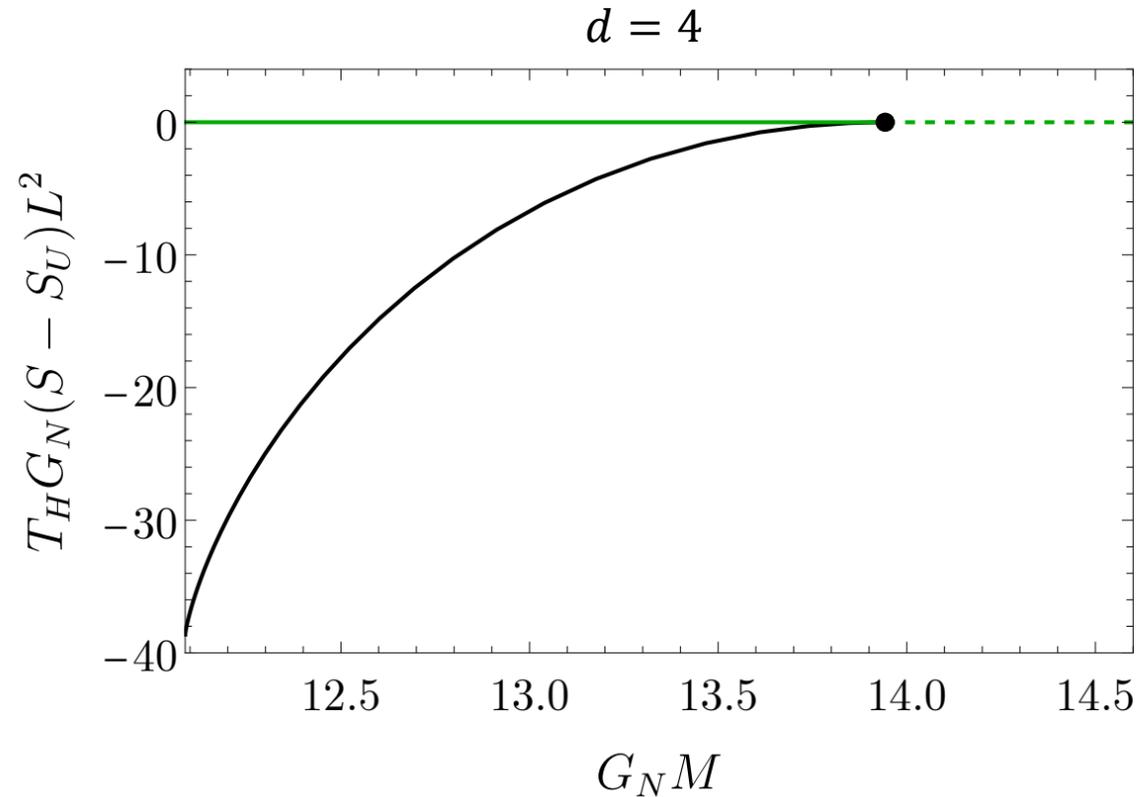


# Stability 1: Global



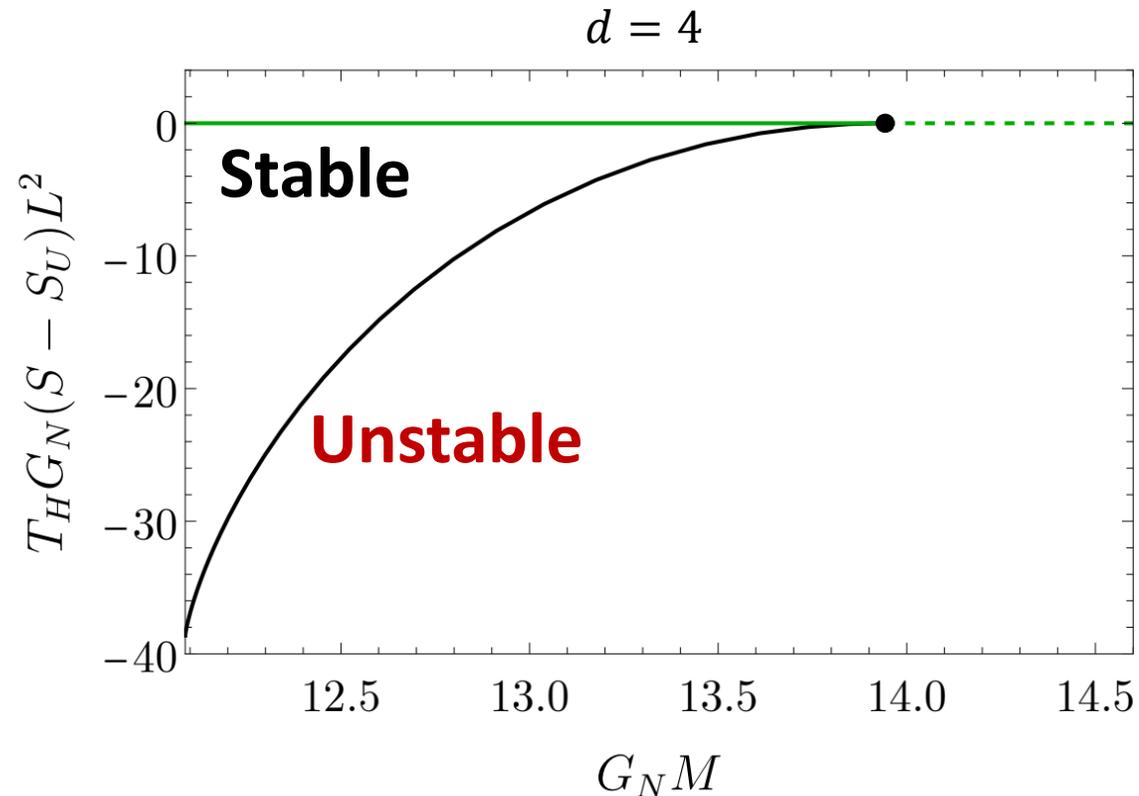
- Uniform strings are always preferred

# Stability 2: Morse theory



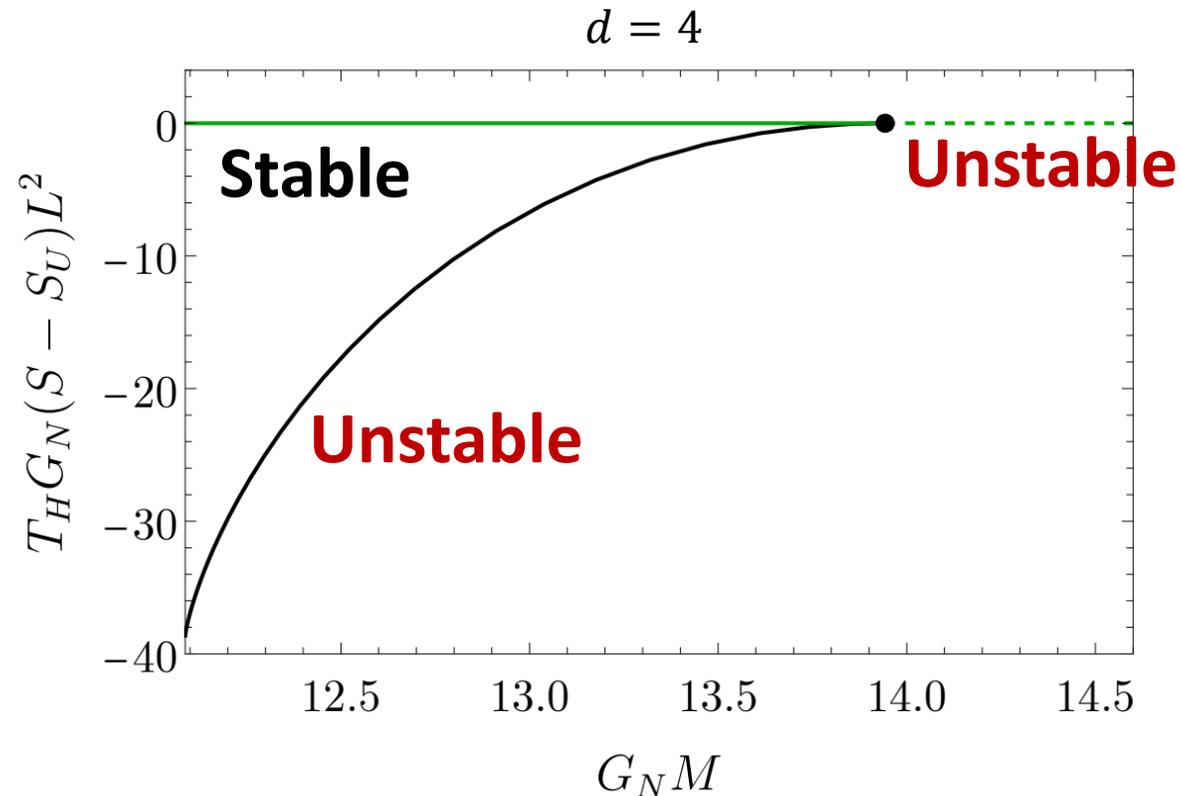
- When merging preserve # of unstable modes

# Stability 2: Morse theory



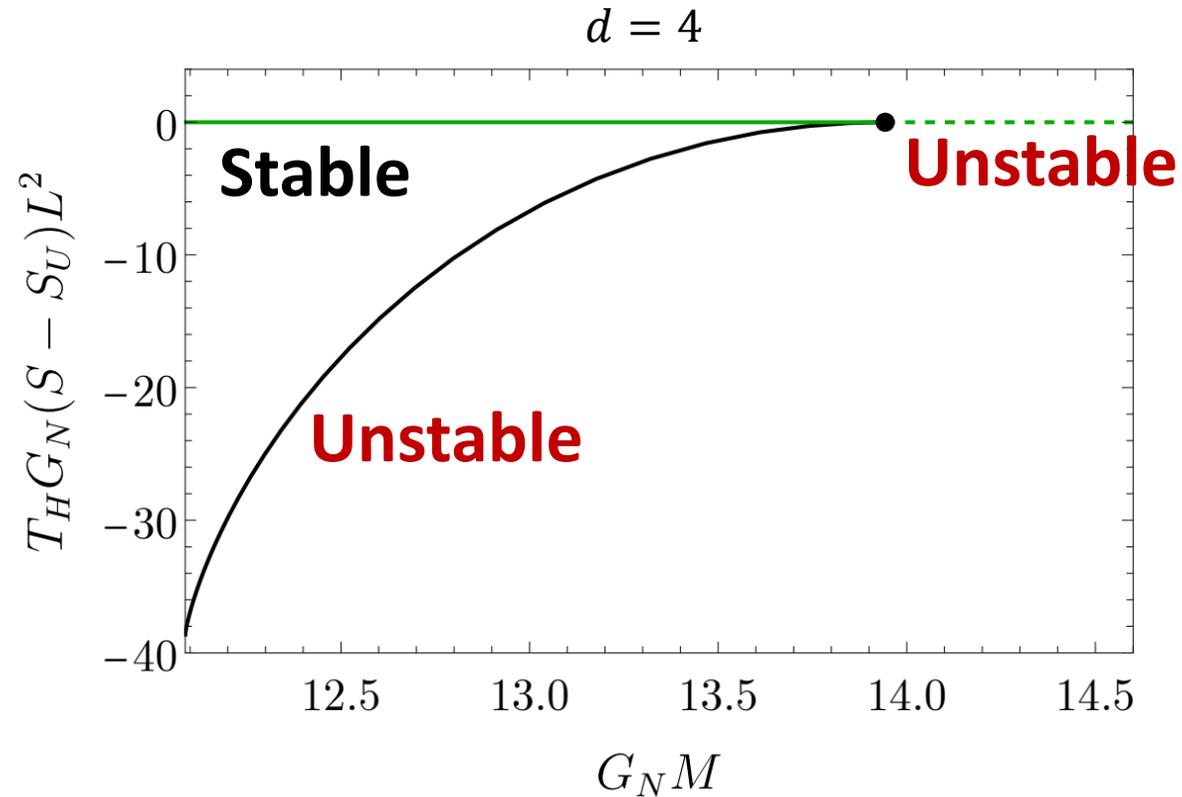
- When merging solutions preserve # of unstable modes

# Stability 2: Morse theory



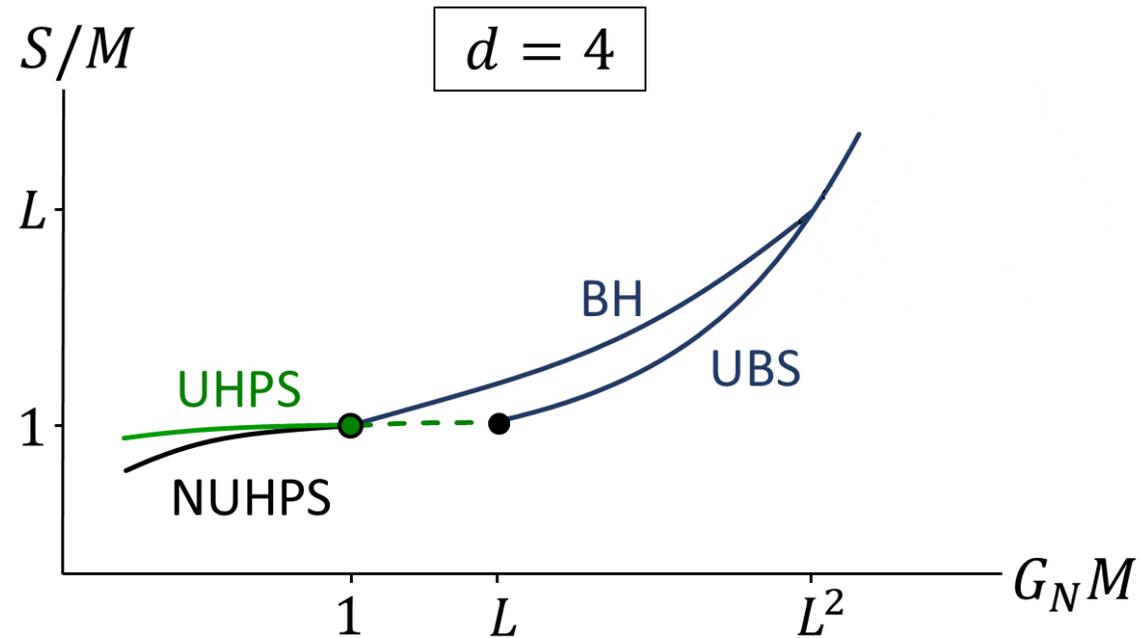
- When merging solutions preserve # of unstable modes

# Stability 2: Morse theory



- Lower  $M \rightarrow$  thicker string
- $C_U = \frac{dM}{dT} < 0 \rightarrow$  GL

# Full picture of thermodynamic phases



- Below correspondence: stable UHPS
- Above correspondence: unstable UHPS to BH

# Main conclusion (d=4,5)

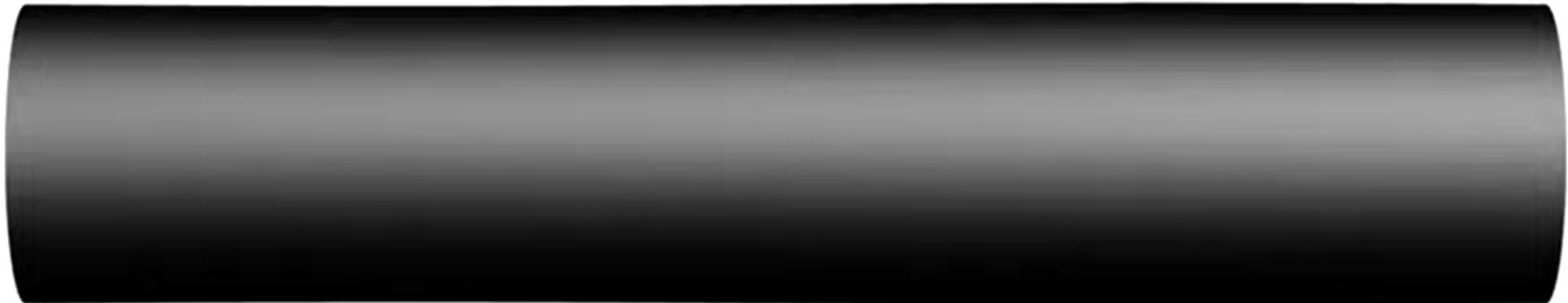
- Uniform stringy strings **can not** evolve into non uniform ones
- As the uniform strings are lighter, they become stable
- Opposite behavior to black strings

Proposed GL picture

# Proposed picture for GL

**Trigger instability**

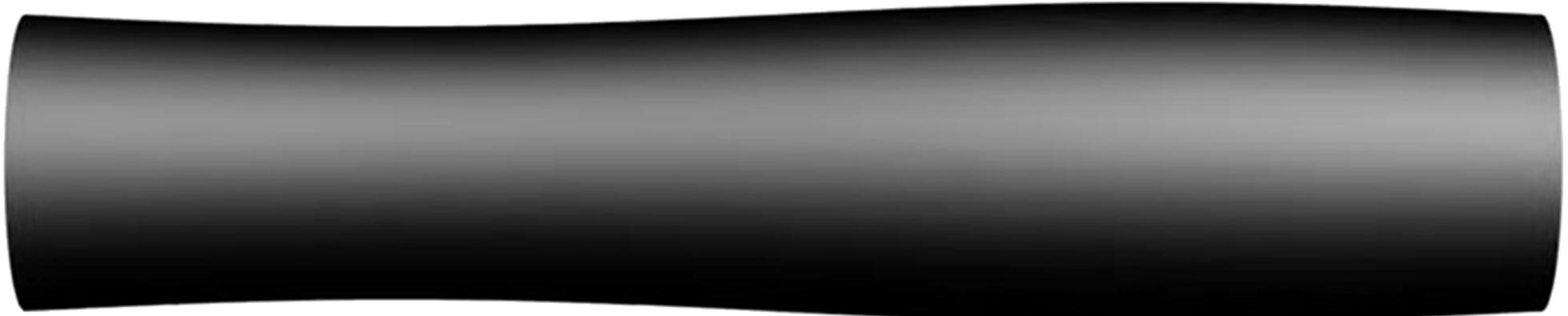
$$r_0 \gg l_s$$



# Proposed picture for GL

**Early stages within GR**

$$r_0 \gg l_s$$



# Proposed picture for GL

**Higher curvature corrections slow down the process**

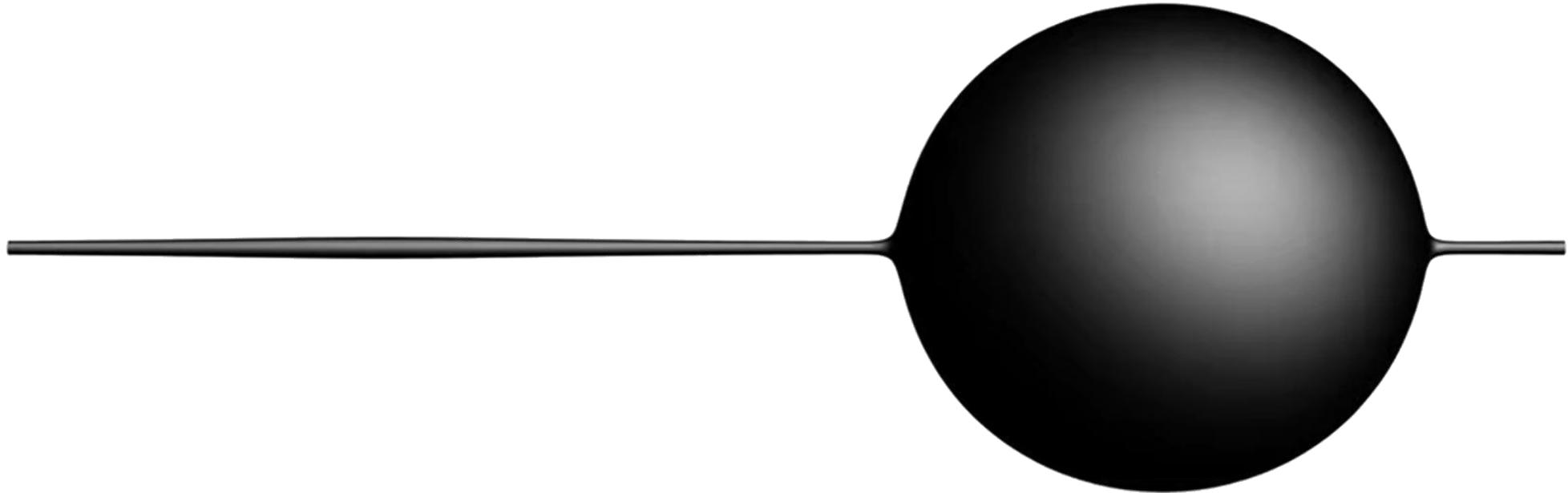
$$r_0 \gtrsim l_s$$



# Proposed picture for GL

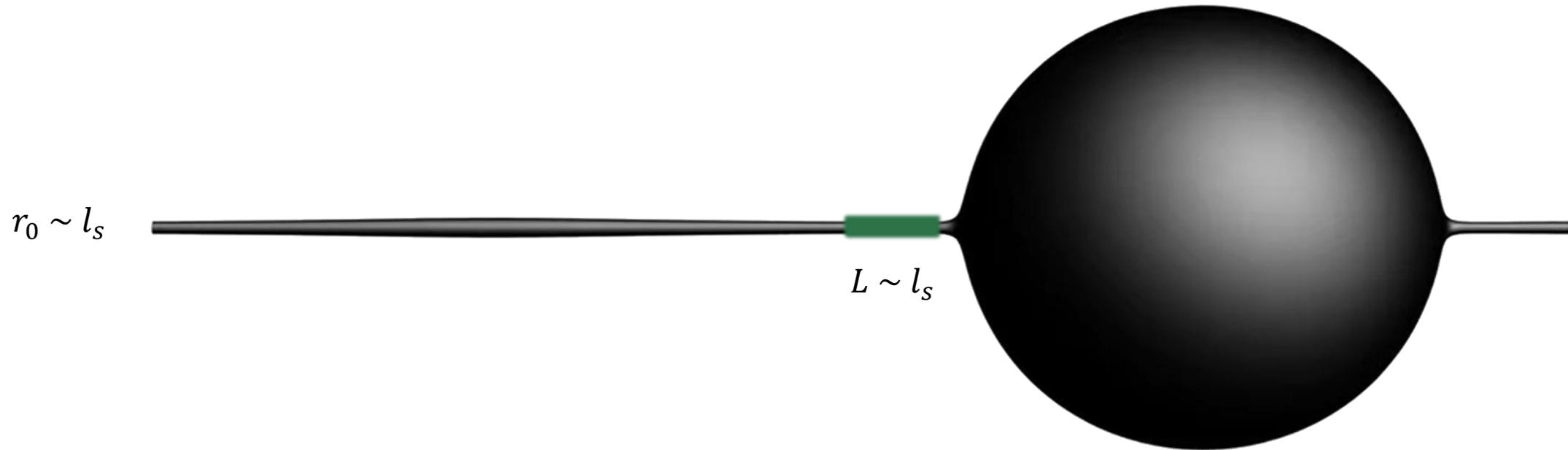
**Arrive to the string scale**

$r_0 \sim l_s$



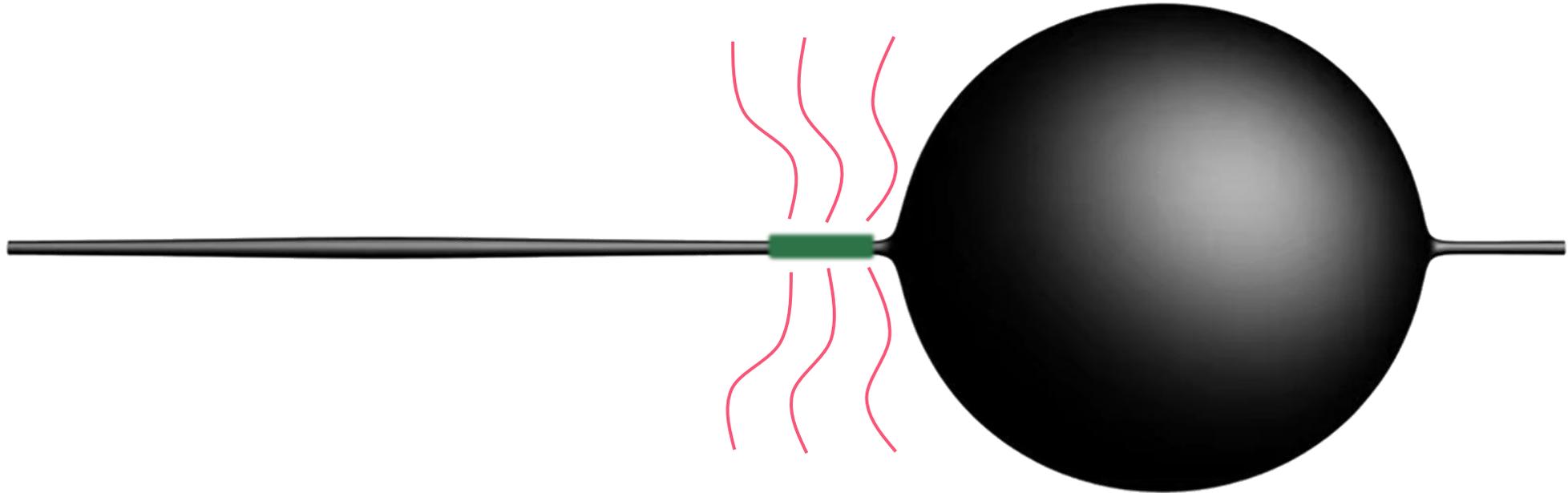
# Proposed picture for GL

**Classical evolution stops at a uniform stringy string**



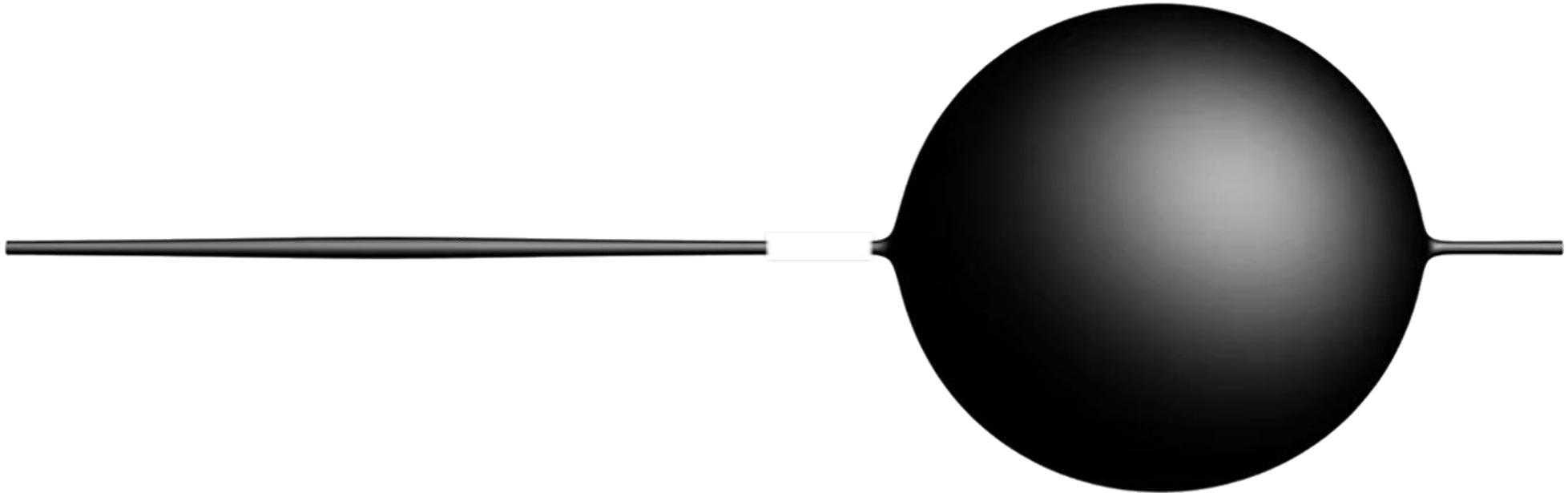
# Proposed picture for GL

**The neck slowly evaporates away**



# Proposed picture for GL

**Smooth pinch**



# Conclusions & Outlook

- Study of GL pinch with BH/string correspondence
- Uniform HP strings are preferred
- Evaporation leads to a slow pinch

Figueras, Kovács, Yao

- What about higher curvature effects in the picture (instability, non-uniform strings...)

Figueras, MSG

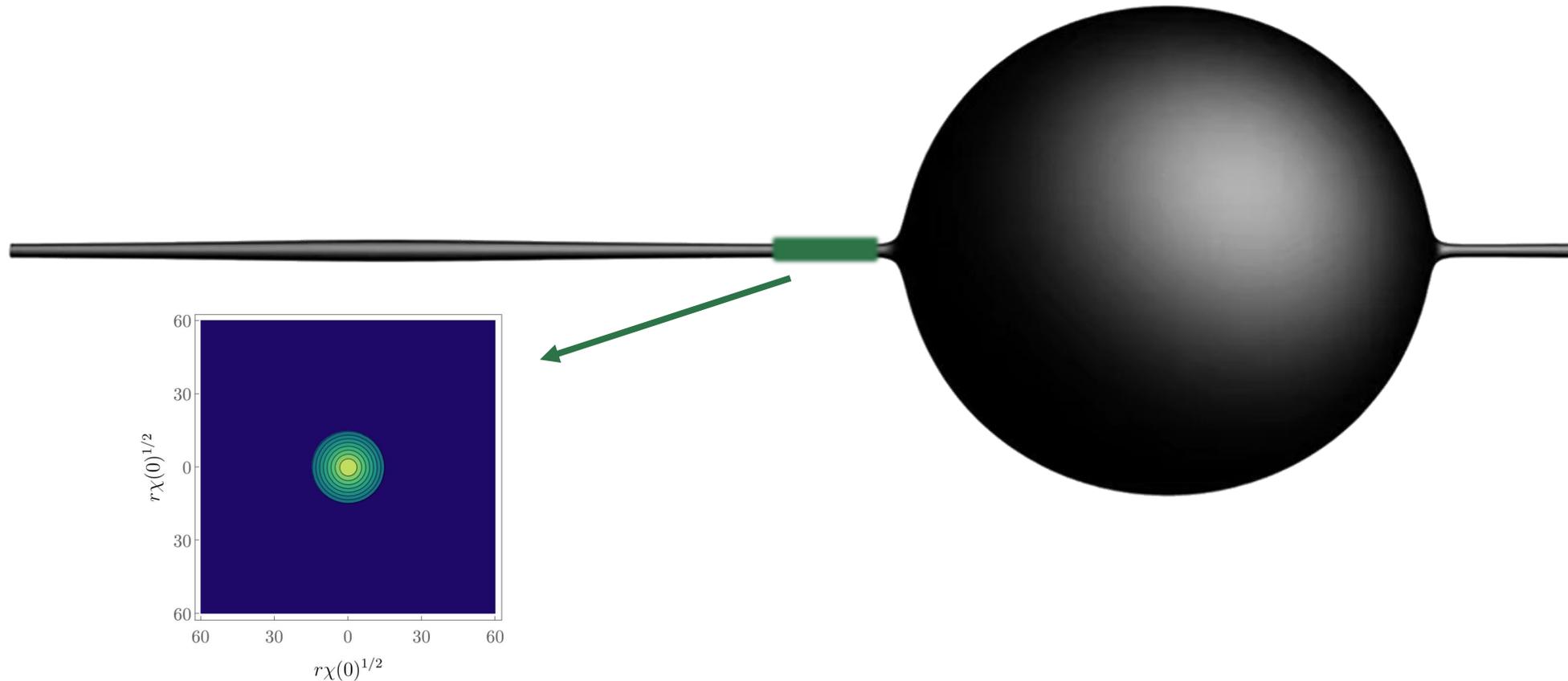
Thank you!

# Outlook

- Verify how higher curvature corrections alter the GL picture
  - Instability
  - Non-linear evolution
  - Family of non-uniform black strings
- Extension to charged versions

# Proposed picture for GL

**Classical evolution stops at a uniform stringy string**



# Limits of General Relativity

- General Relativity successful theory of gravity
- Leads to singularities where the theory breaks down
- Typically hidden: Cosmic Censorship Conjecture
- We have examples where a naked singularity is formed
  - Black Hole evaporation
  - Critical collapse
  - Gregory-Laflamme instability of black strings (this with pictures better)

# Gregory-Laflamme (GL) instability

- Thermodynamical instability implies a hydro instability
- Long horizons will be unstable: 5D black string
- Generic perturbations will trigger the instability
- Linear analysis predicts an exponential growth in the amplitude of perturbations
- Non-linear regime: numerical evolution of Einstein's equations

# Zero mode

- Find the (linear) deformation that is still a solution

- $d = 4,$        $\frac{L}{r_b} \approx 7.95$        $\frac{L}{r_0} \approx 7.17$

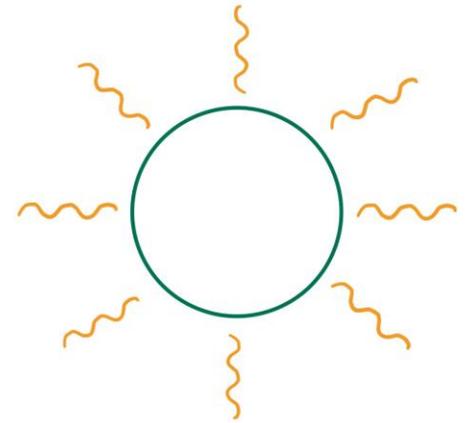
- $d = 5,$        $\frac{L}{r_b} \approx 4.84$        $\frac{L}{r_0} \approx 4.95$

- $d = 6,$        $\frac{L}{r_b} \approx 2.76$        $\frac{L}{r_0} \approx 3.97$

# Stringy GL

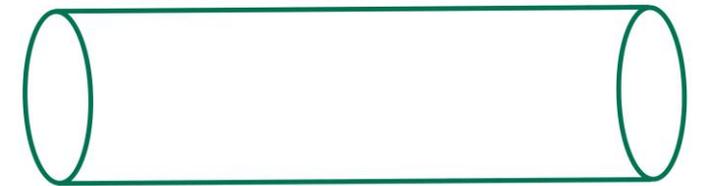
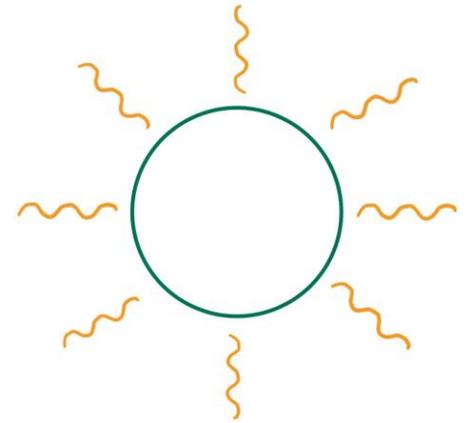
- HP balls have a negative Euclidean mode

Chen, Maldacena, Witten '21

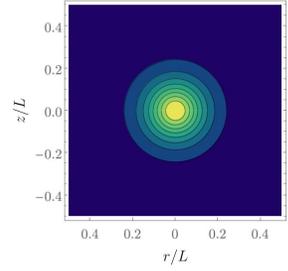
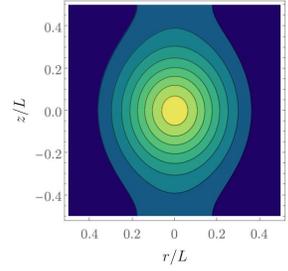
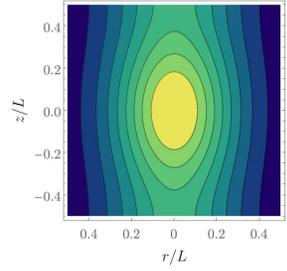
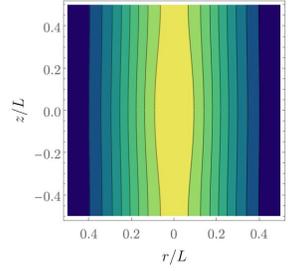


# Stringy GL

- HP balls have a negative Euclidean mode Chen, Maldacena, Witten '21
- Expect a GL instability in  $d + 1$  dimensions

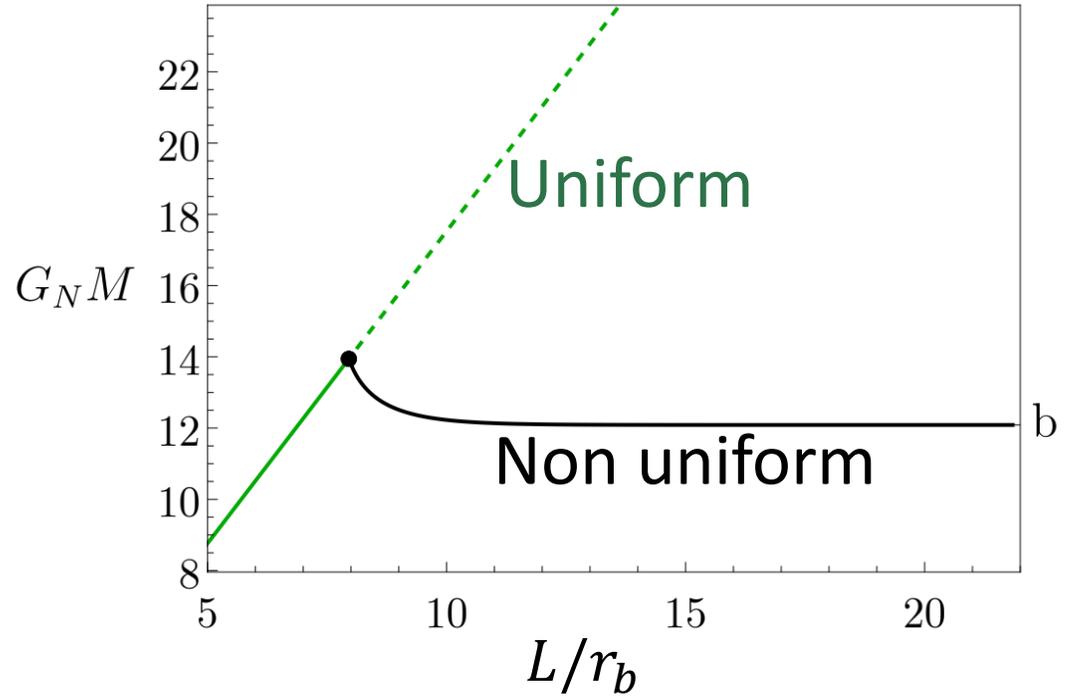


$L/r_b$

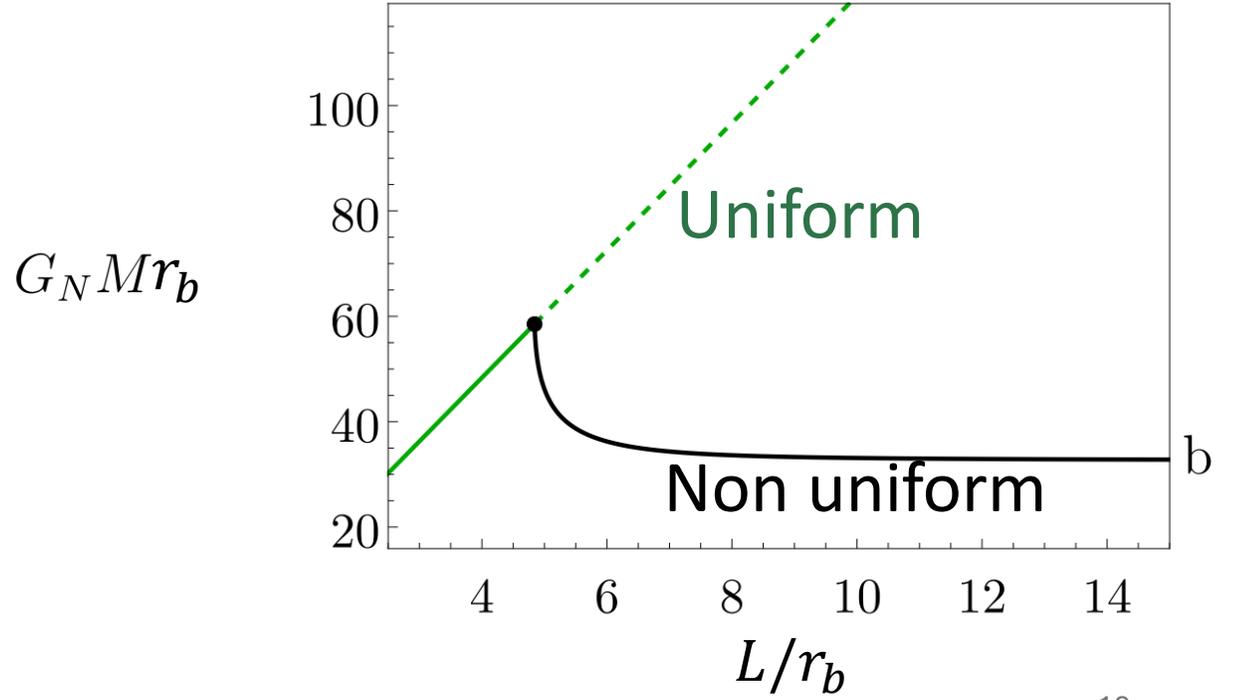


$$\varphi \sim -\frac{G_N \rho(z)}{r^{d-3}}$$

$d = 4$

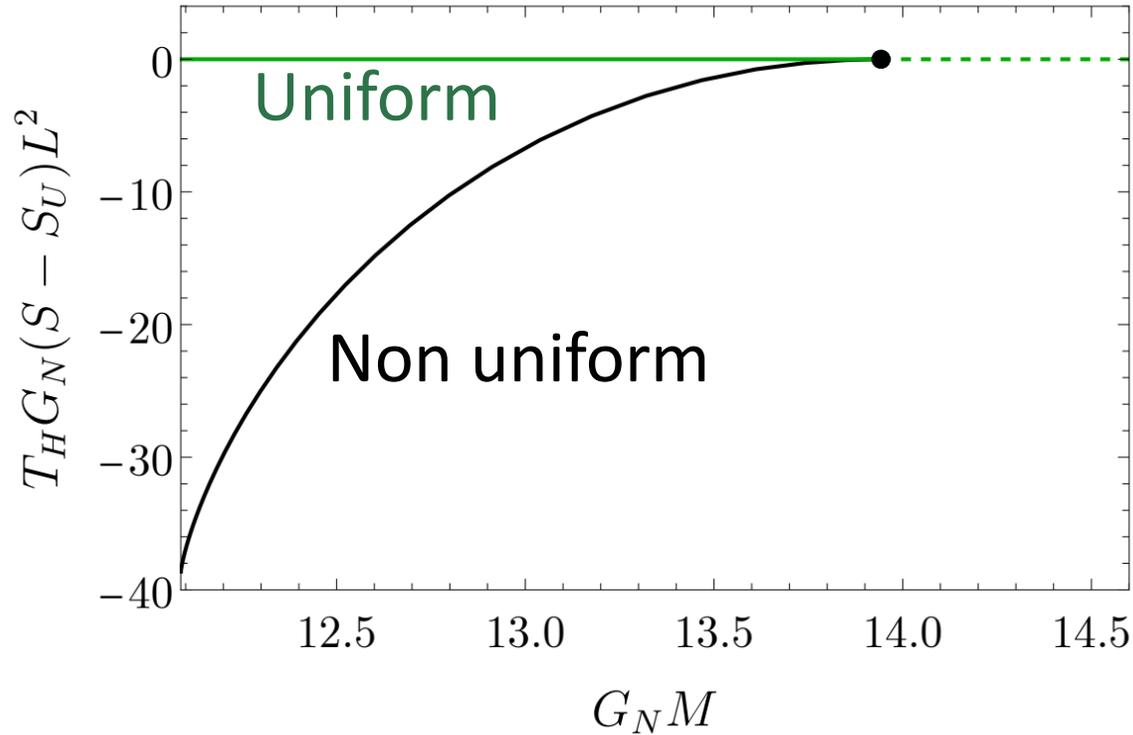


$d = 5$

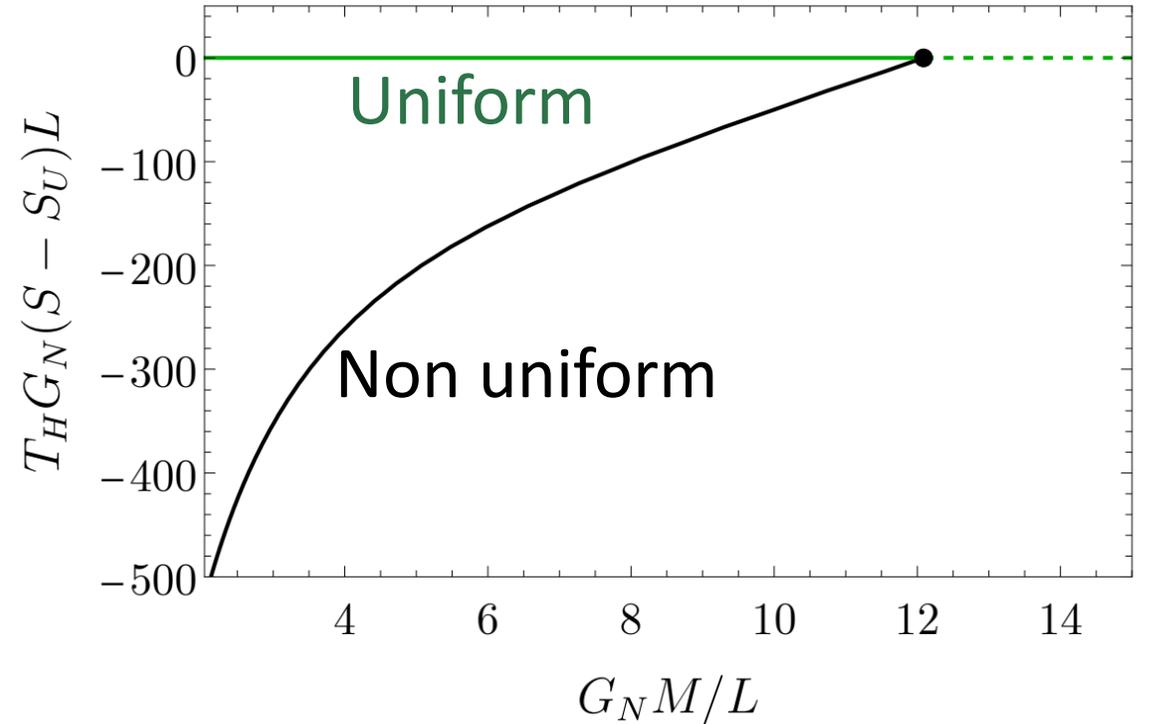


# Stability 1: Global

$d = 4$



$d = 5$



- Uniform strings are always preferred