Phase transition on superfluid vortices in Higgs-confinement crossover

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Tomoya Hayata (Keio Univ.)

based on arXiv: 2411.03676 accepted in JHEP

Summary

We focus on systems with $U(1)_{\text{global}} \times G_{\text{gauge}}$ symmetry

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Confined phase	Deconfined phase

$\beta_g = 1/g^2$

Strong coupling — Weak coupling

Adding fundamental charged matter and Higgsing it

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$U(1)$ superfluid

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We show something happened between strong and weak coupling.

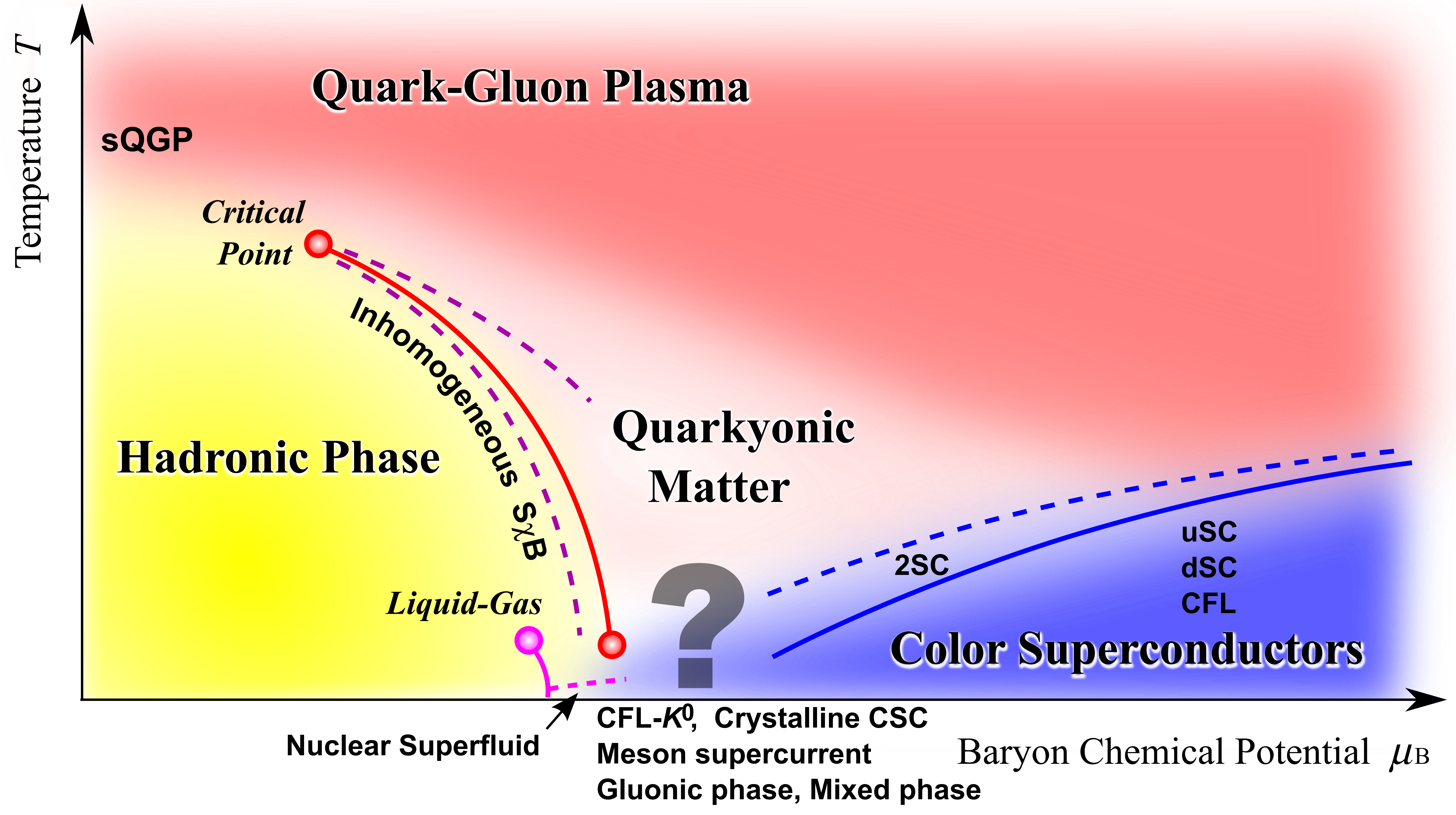
⇒Phase transition on vortex even in bulk has no phase transition.

Outline

- Motivation
- What we know about quark hadron continuity
- Phase transition on a vortices
- Summary and Outlook

Motivation: QCD phase diagram

Fukushima, Hatsuda, Rept. Prog. Phys. 74 (2011) 014001



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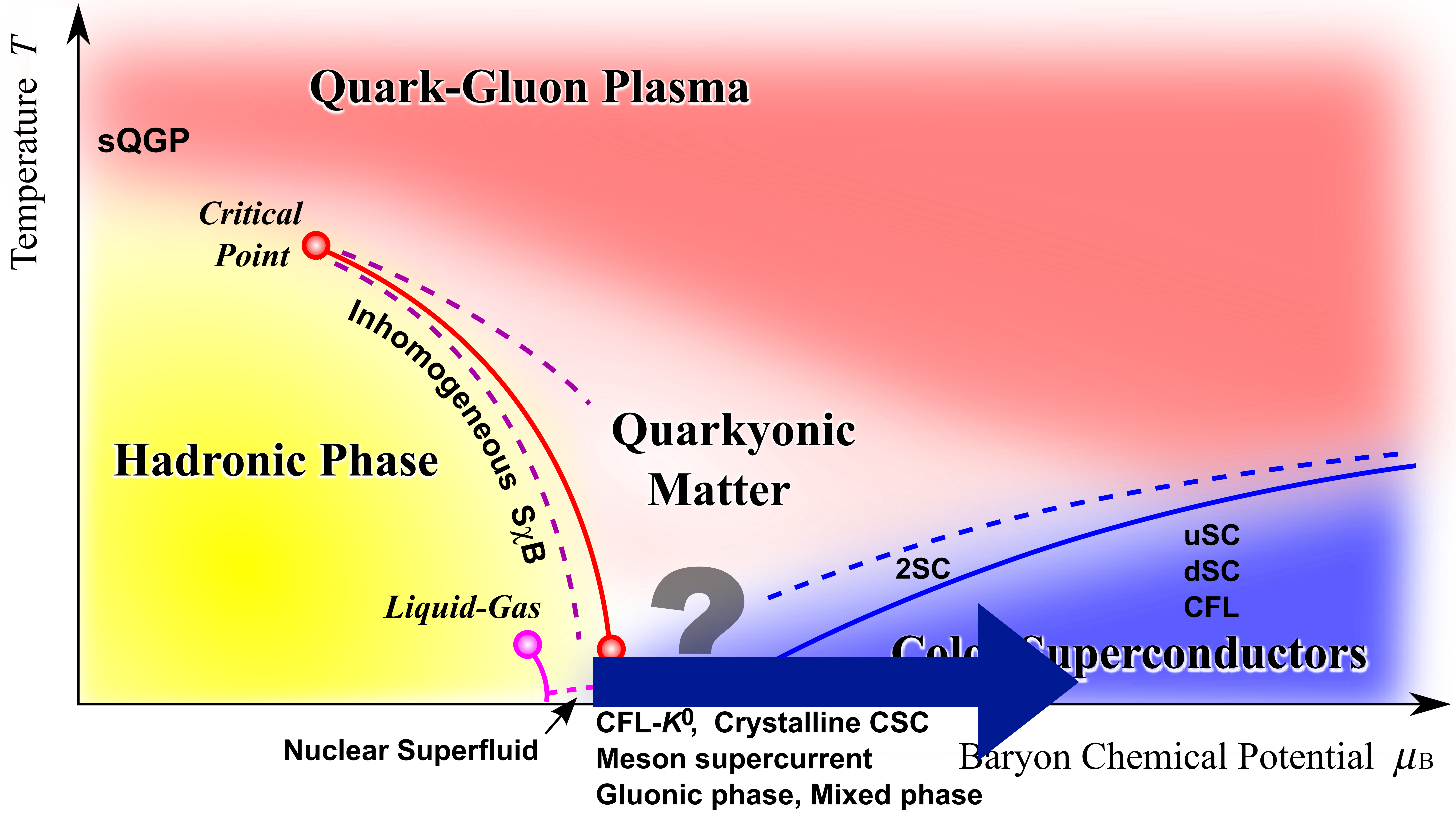

What we know

For 3-flavor QCD : $G = SU(3)_f \times U(1)_B$

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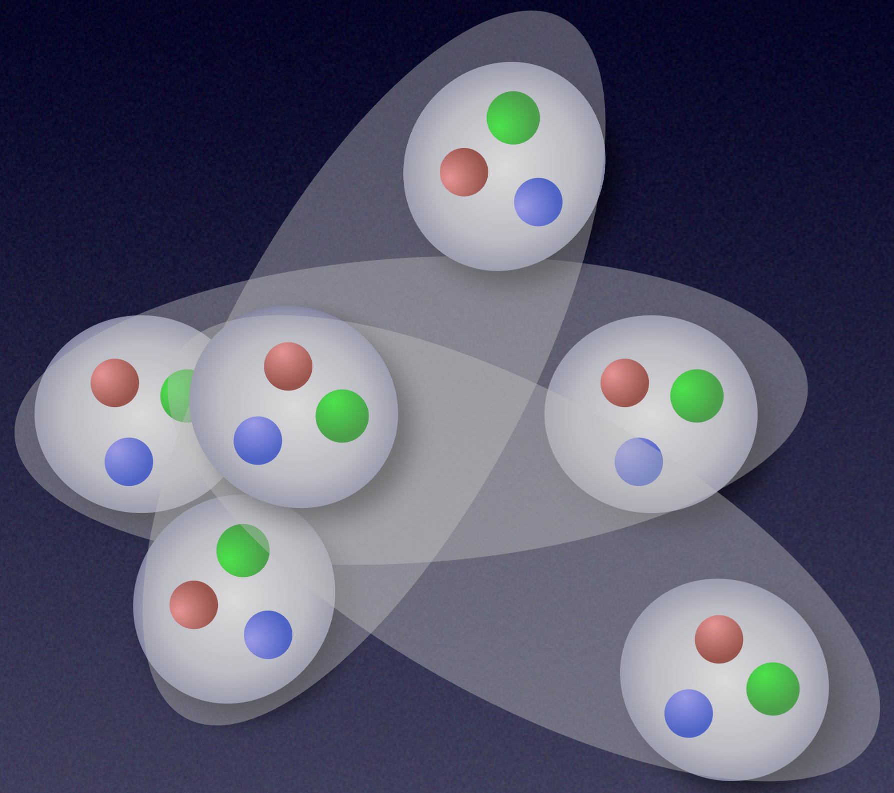

Baryon pair condensation

$\Delta = \langle \Lambda\Lambda \rangle \neq 0 \qquad \Lambda \sim uds$

$SU(3)_f \times U(1)_B \to SU(3)_f$

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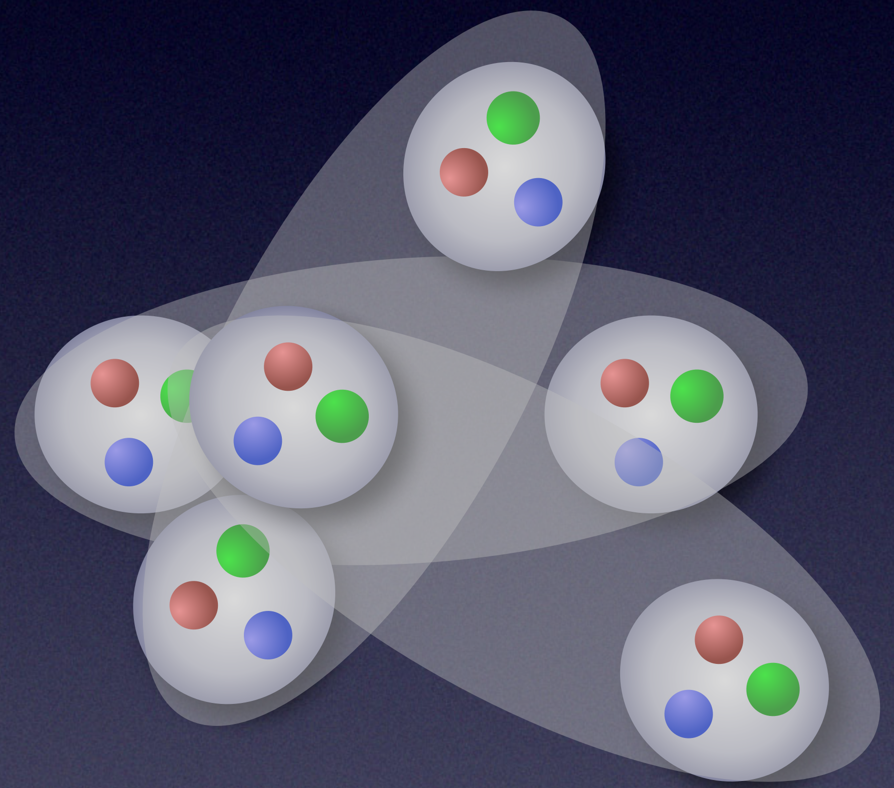

Baryon pair condensation

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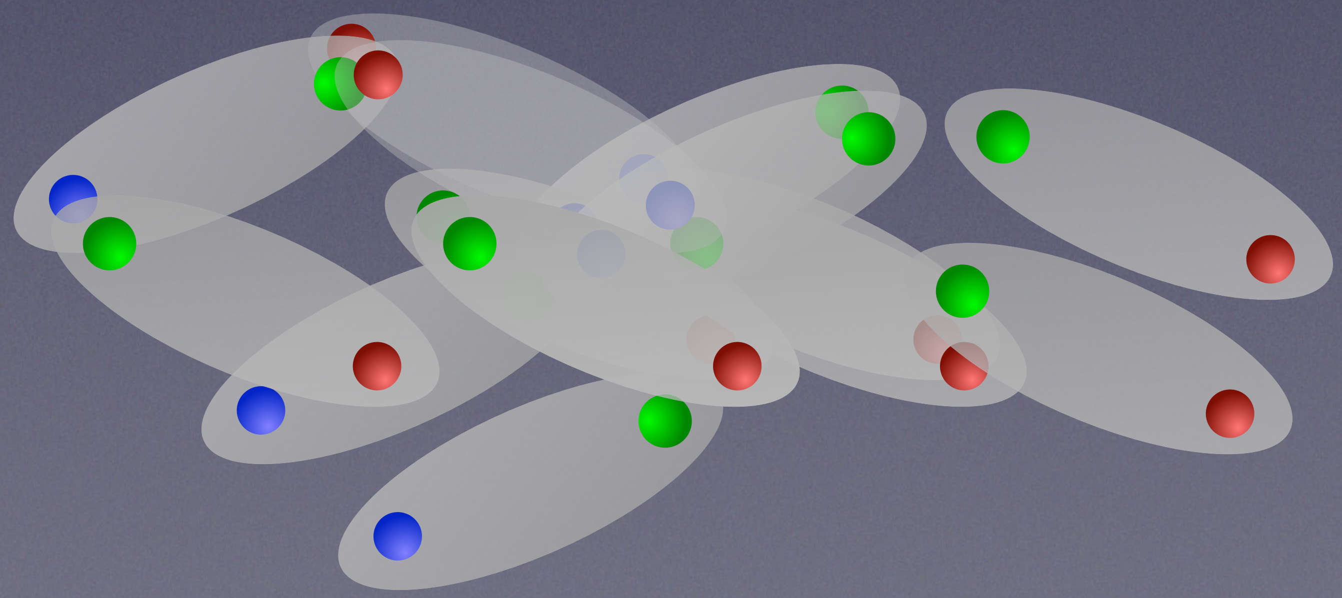
$$SU(3)_f \times U(1)_B \to SU(3)_f$$

- **Color super conductor (dense phase)**



"quark pair condensate"

$$(\Phi_L)^i_a = \epsilon^{ijk}\epsilon_{abc}\langle (q_L)^b_j (C q_L)^c_k \rangle = -\epsilon^{ijk}\epsilon_{abc}\langle (q_R)^b_j (C q_R)^c_k \rangle$$

$$SU(3)_f \times U(1)_B \to SU(3)_f$$

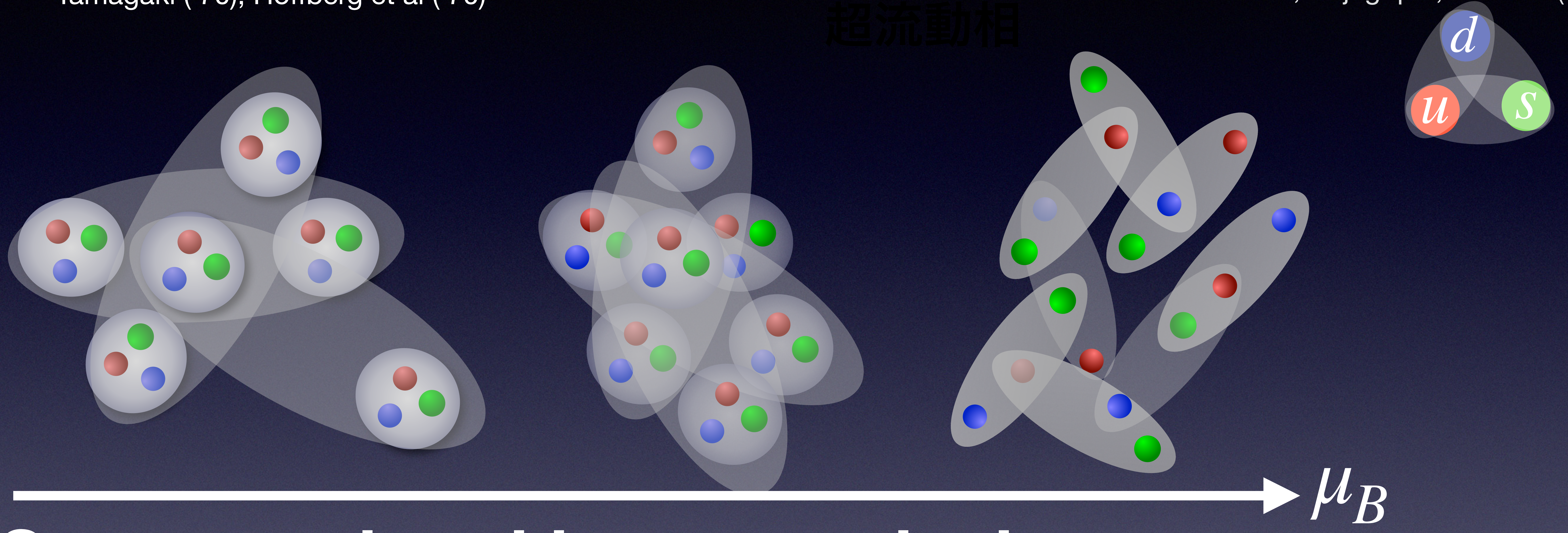
Quark hadron continuity

Hadronic superfluid

Tamagaki ('70), Hoffberg et al ('70)

Color flavor locked phase (CFL phase)

Alford, Rajagopal, Wilczek ('99)



Symmetry breaking patter is the same

⇒Quark hadron continuity

Excitations

Baryons ⇒ Quarks

Vector meson ⇒ Gluons

cf. Hatsuda, Tachibana, Yamamoto, Baym ('06)

Schäfer and Wilczek (1999)

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SPT phase, topological ordered phase, …

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SPT phase, topological ordered phase, …

Topological ordered phase

≈ Spontaneously broken generalized (discrete) global symmetries

Quantum electrodynamics

There are $U(1)_M^{[1]}$ magnetic 1-form symmetry

Vacuum	Superconductor
SSB $U(1)_M^{[1]}$	Unbroken $U(1)_M^{[1]}$

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Photons are Nambu-Goldstone modes

Superconductor

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Emergent symmetry (SSB) $\mathbb{Z}_2^{[1]} \times \mathbb{Z}_2^{[2]}$

Topological order

$\mathbb{Z}_2^{[1]}$: cooper pair has charge 2

$\mathbb{Z}_2^{[2]}$:π magnetic flux inside of vortex

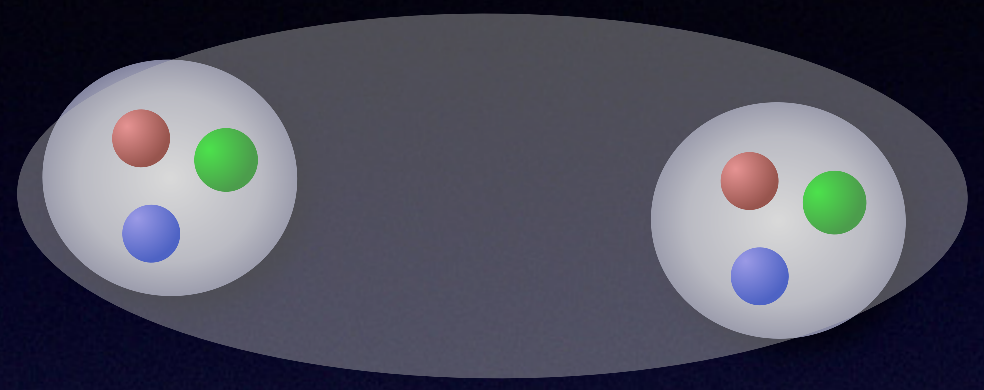
Thought experiment : rotating neutron stars

Quantum vortex

Consider continuity of vortices

- Circulation
- Emergent symmetry

Hadronic superfluid phase
di-baryons condense



$$\Delta = \langle \Lambda\Lambda \rangle \neq 0 \quad \Lambda \sim uds$$

Symmetry breaking pattern

$$SU(3)_f \times U(1)_B \to SU(3)_f$$

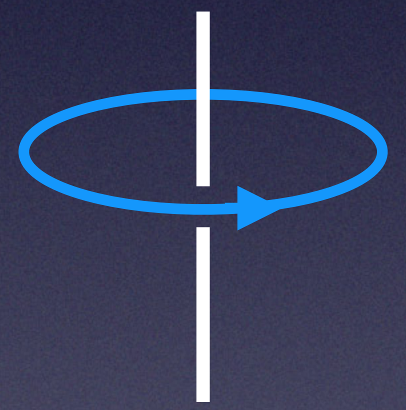
Topological excitation: U(1) vortex $\pi_1(U(1)_B) = \mathbb{Z}$

$$\phi = \Delta f(r) e^{i\theta} \quad \int \frac{d\theta}{2\pi} \in \mathbb{Z} \quad f \xrightarrow{r \to 0} 0 \quad f \xrightarrow{r \to \infty} 1$$

Quantum number in Hadronic superfluid phase

Global $U(1)_B$ symmetry is broken

U(1) vortex: topological defect $\Delta e^{i\theta}$

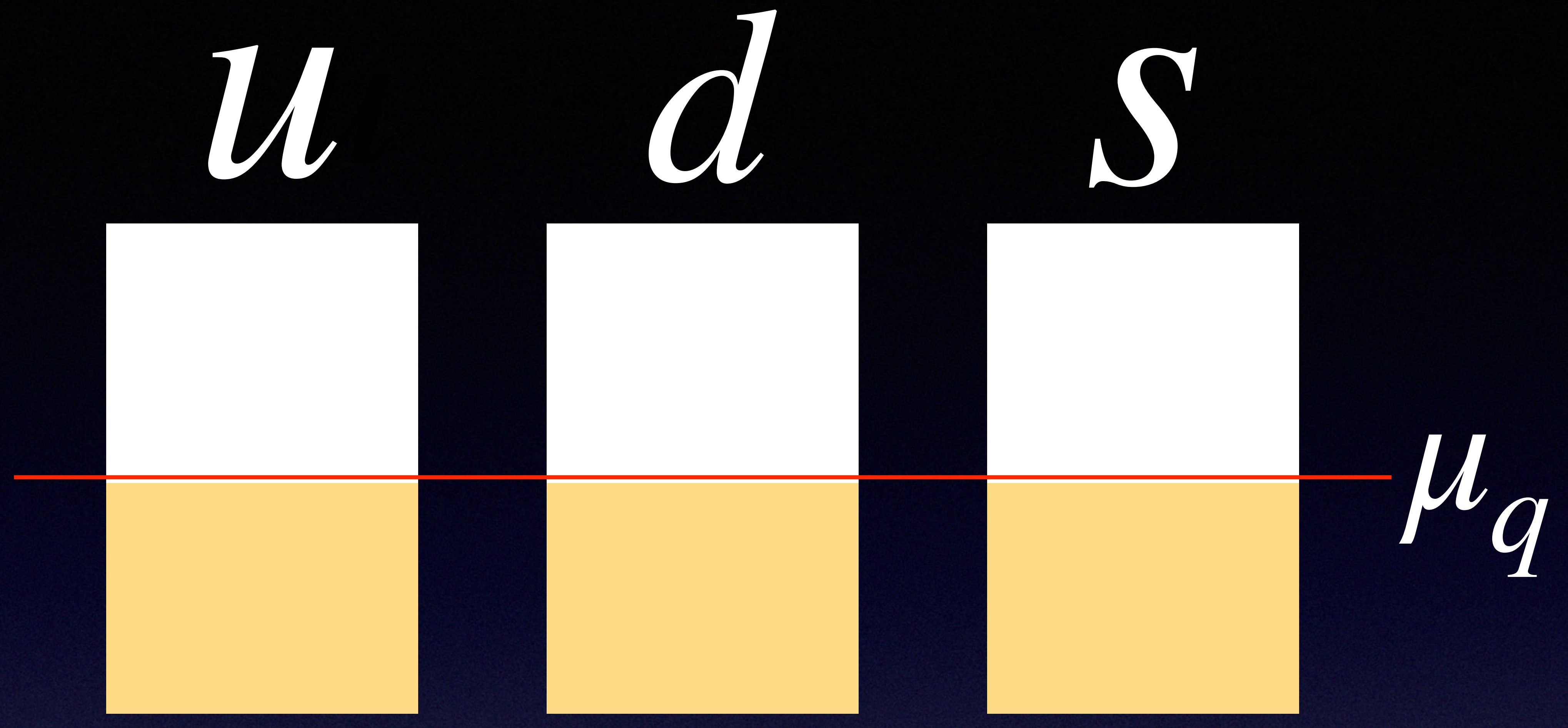


Circulation: $\int v = \int \frac{d\theta}{2\mu_B} = \frac{2\pi\nu_B}{2\mu_B}$

$\nu_B = \int \frac{d\theta}{2\pi}$**: Winding number**

$2\mu_B$**: Baryon chemical potential of order parameter**

Color-flavor locking phase



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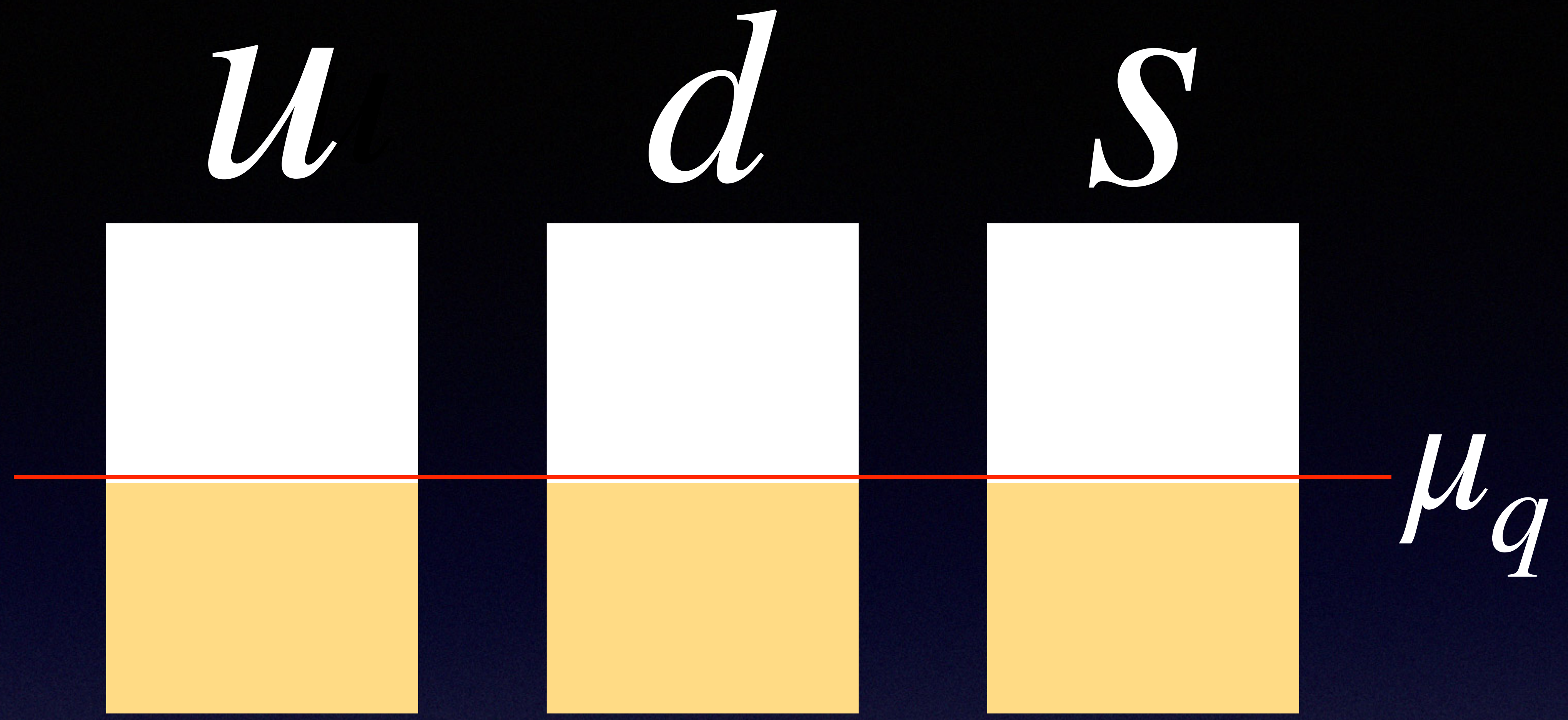
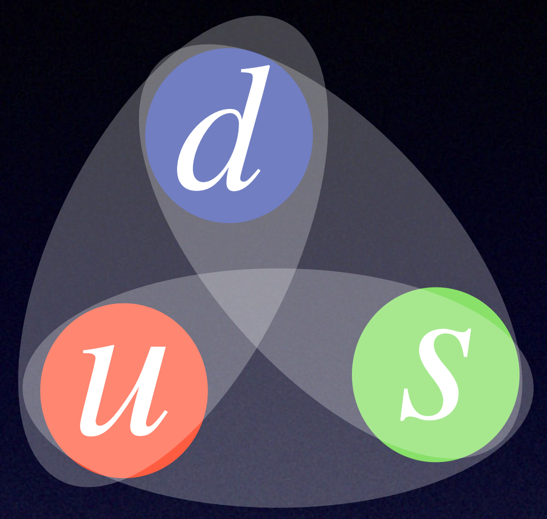


quark pair



$$(\Phi_L)^i_a = \epsilon^{ijk}\epsilon_{abc}\langle (q_L)^b_j (C q_L)^c_k \rangle \quad (\Phi_R)^i_a = \epsilon^{ijk}\epsilon_{abc}\langle (q_R)^b_j (C q_R)^c_k \rangle$$

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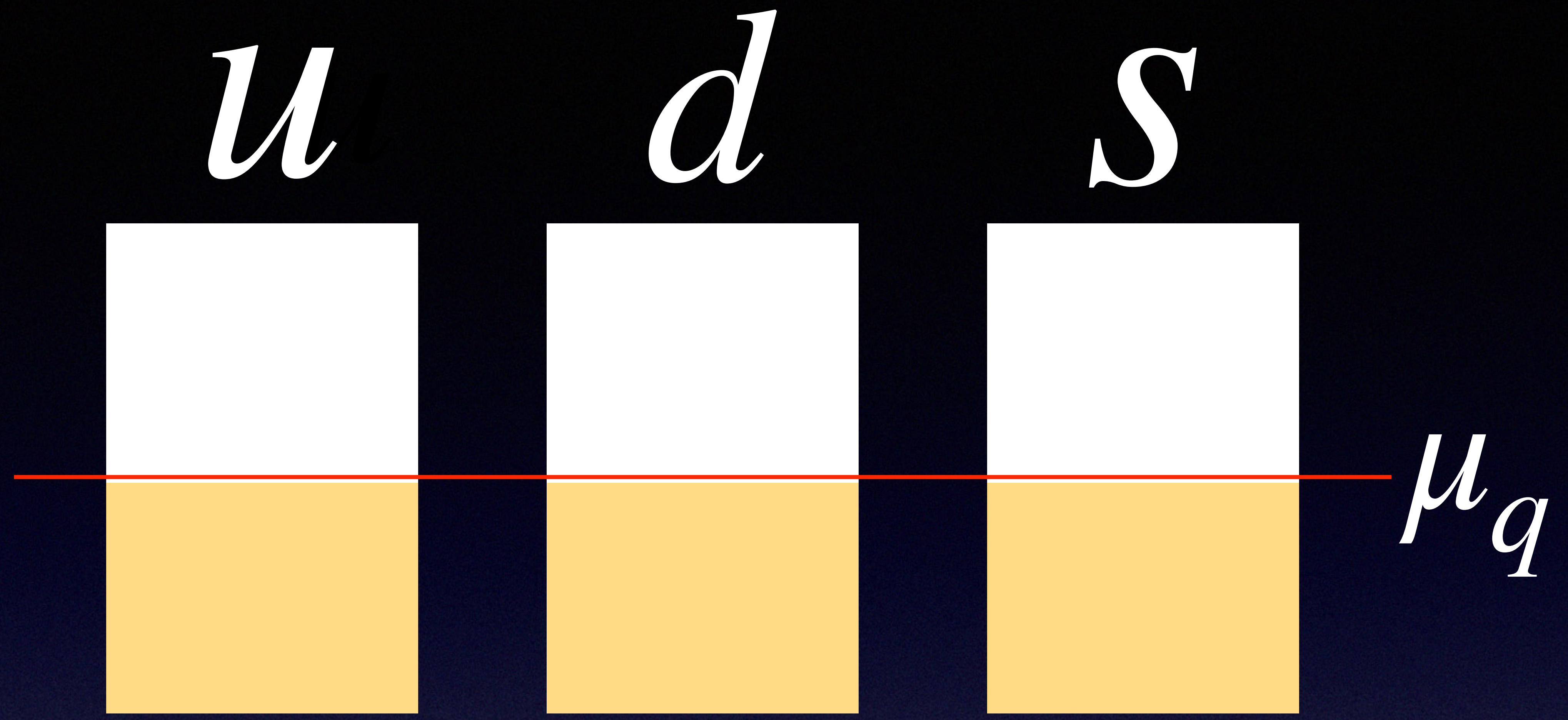

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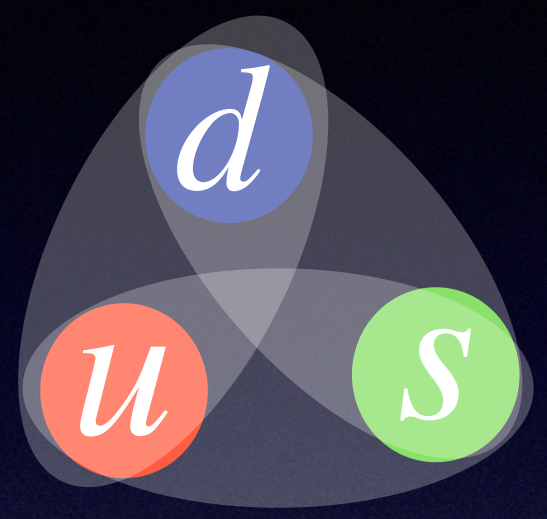

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$$\Phi := \Phi_L = -\Phi_R = \begin{pmatrix} \Delta_{\mathrm{CFL}} & 0 & 0 \\ 0 & \Delta_{\mathrm{CFL}} & 0 \\ 0 & 0 & \Delta_{\mathrm{CFL}} \end{pmatrix}$$

Topological excitations

cf. Eto, Hirono, Nitta & Yasui, PTEP 2014, 012D01 (2014)

order parameter space $G/H \simeq \frac{SU(3)_c \times U(1)_B}{\mathbb{Z}_3} \simeq U(3)$

U(1) vortex

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Non-abelian CFL vortex

Balachandran, Digal, Matsuura, PRD73, 074009 (2006)

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$$A_i = -\frac{\epsilon_{ij} x^j}{g_s^2 r^2}(1 - h(r))\mathrm{diag}\left(-\frac{2}{3}, \frac{1}{3}, \frac{1}{3}\right)$$

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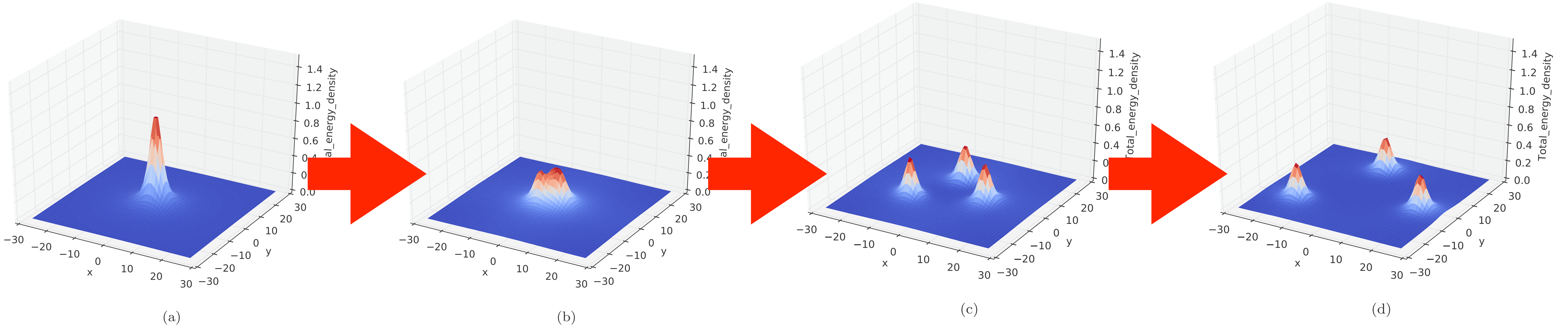
$$A_i = -\frac{\epsilon_{ij} x^j}{g_s^2 r^2}(1 - h(r))\mathrm{diag}\left(-\frac{2}{3}, \frac{1}{3}, \frac{1}{3}\right)$$ **both superfluidity and superconductivity**

Numerical Simulation

Alford, Mallavarapu, Vachaspati, Windisch, PRC 93, 045801 (2016)

U(1) vortex

non-abelian vortices



U(1) vortex decays into three non-abelian vortices

U(1) vortex in Hadronic phase

Alford, Baym, Fukushima, Hatsuda, Tachibana ('19)

Circulation $2\pi \frac{\nu_B}{2\mu_B}$

ν_B: **Winding number**

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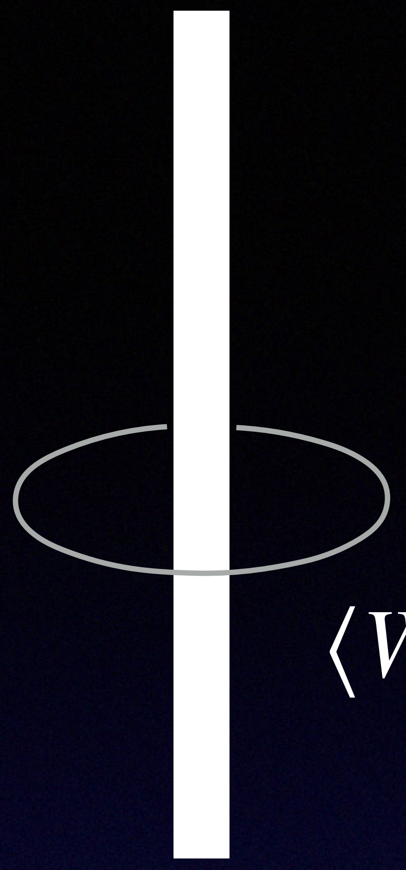

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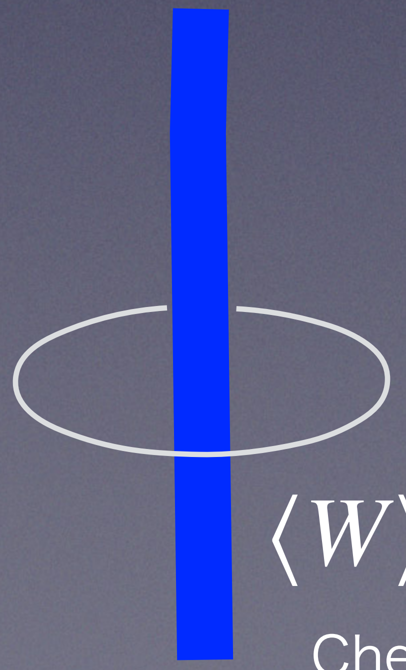

Non-abelian vortex in CFL

$\langle W \rangle = e^{i\frac{2\pi\nu_A}{3}} |\langle W \rangle|$

Circulation $\dfrac{2\pi\nu_A/3}{2\mu_q} = \boxed{2\pi \dfrac{\nu_A}{2\mu_B}}$

Cherman, Sen, Yaffe, PRD 100, 034015 (2019)

Topological ordered phase?

CFL vortex: emergent $\mathbb{Z}_3^{[2]}$ symmetry

However, it is not unbroken, i.e. not topological order

Hirono, Tanizaki ('19)

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⇒ This allow us to distinguish the phases.

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Cherman, Jacobson, Sen, Yaffe ('20), ('24)

Magnetic flux may penetrate through the vortices in the hadronic phase or dissipate during the transition.

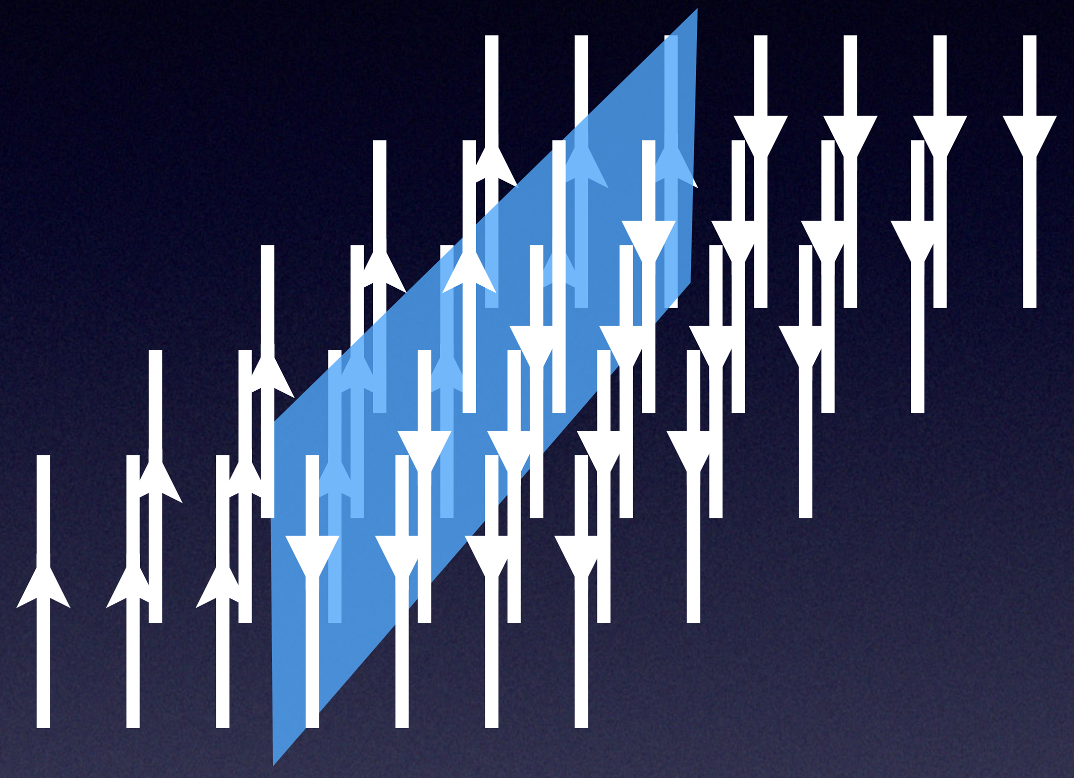
Hayashi ('23)

Outline

- Phase transition on a vortices
- Summary

Phase transition on a topological defect, while the bulk remains continuous?

Domain wall



vortex

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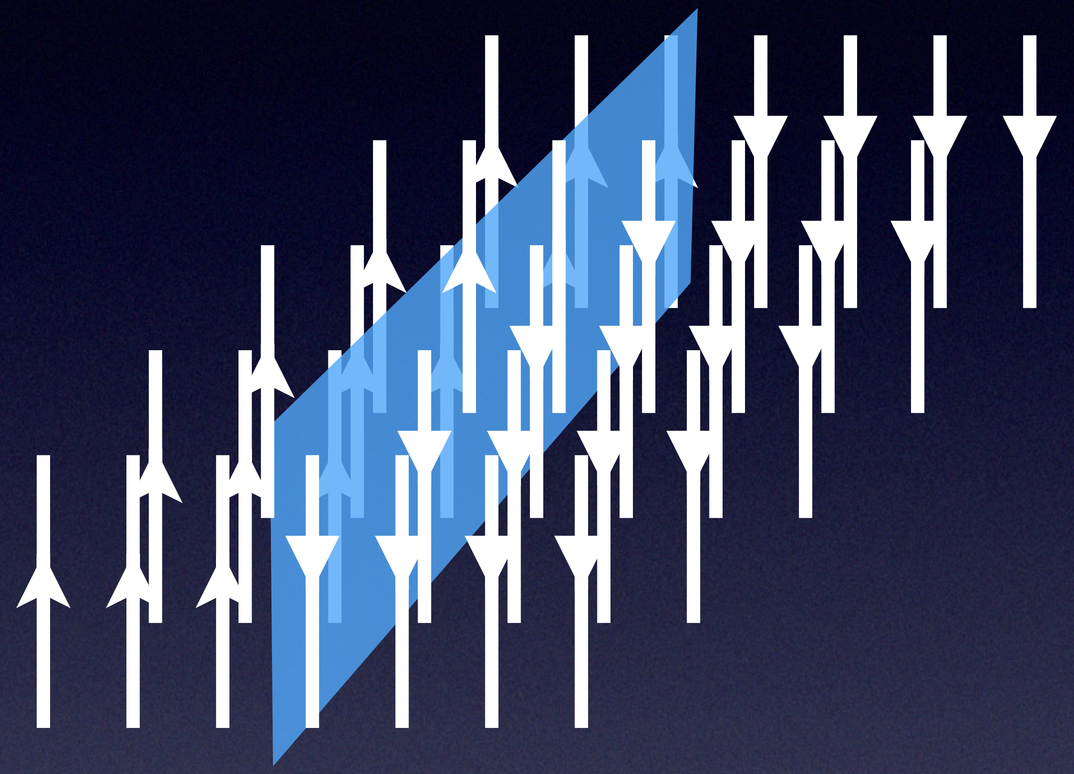

vortex

Our answer is YES!

Effective theory on a topological defect=
a lower-dimensional field theory may exhibit phase transition

Phase transition on a topological defect, while the bulk remains continuous?

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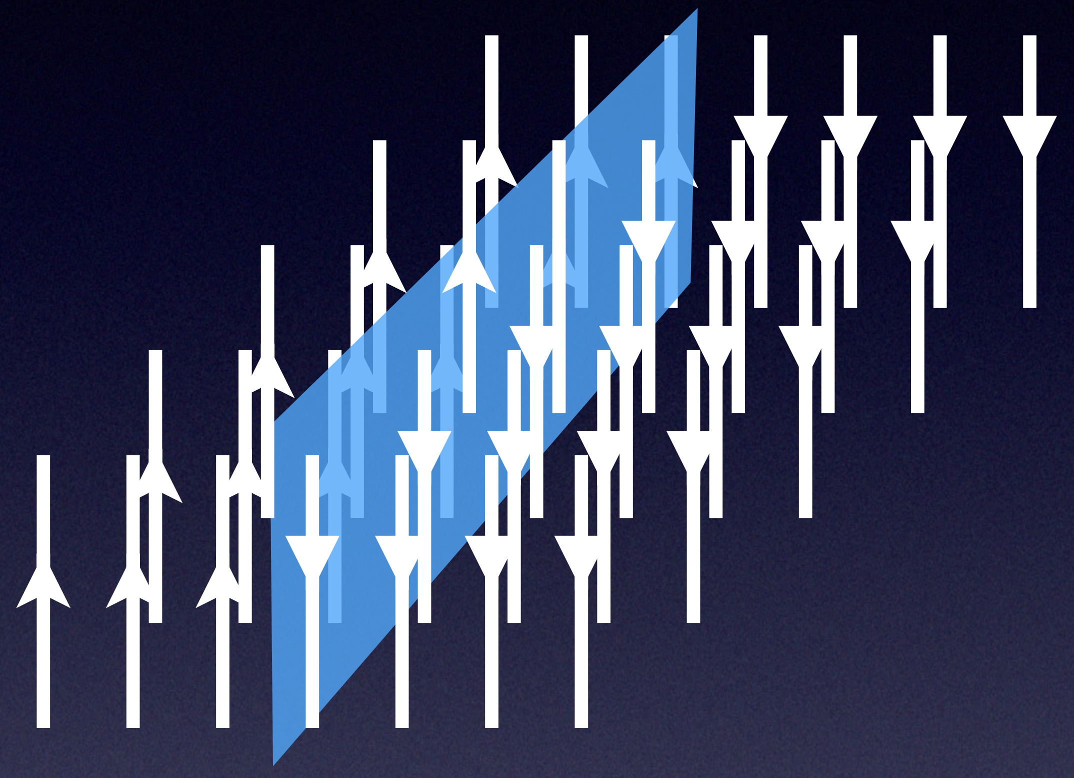

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Phase transitions may occur in quantum vortices.

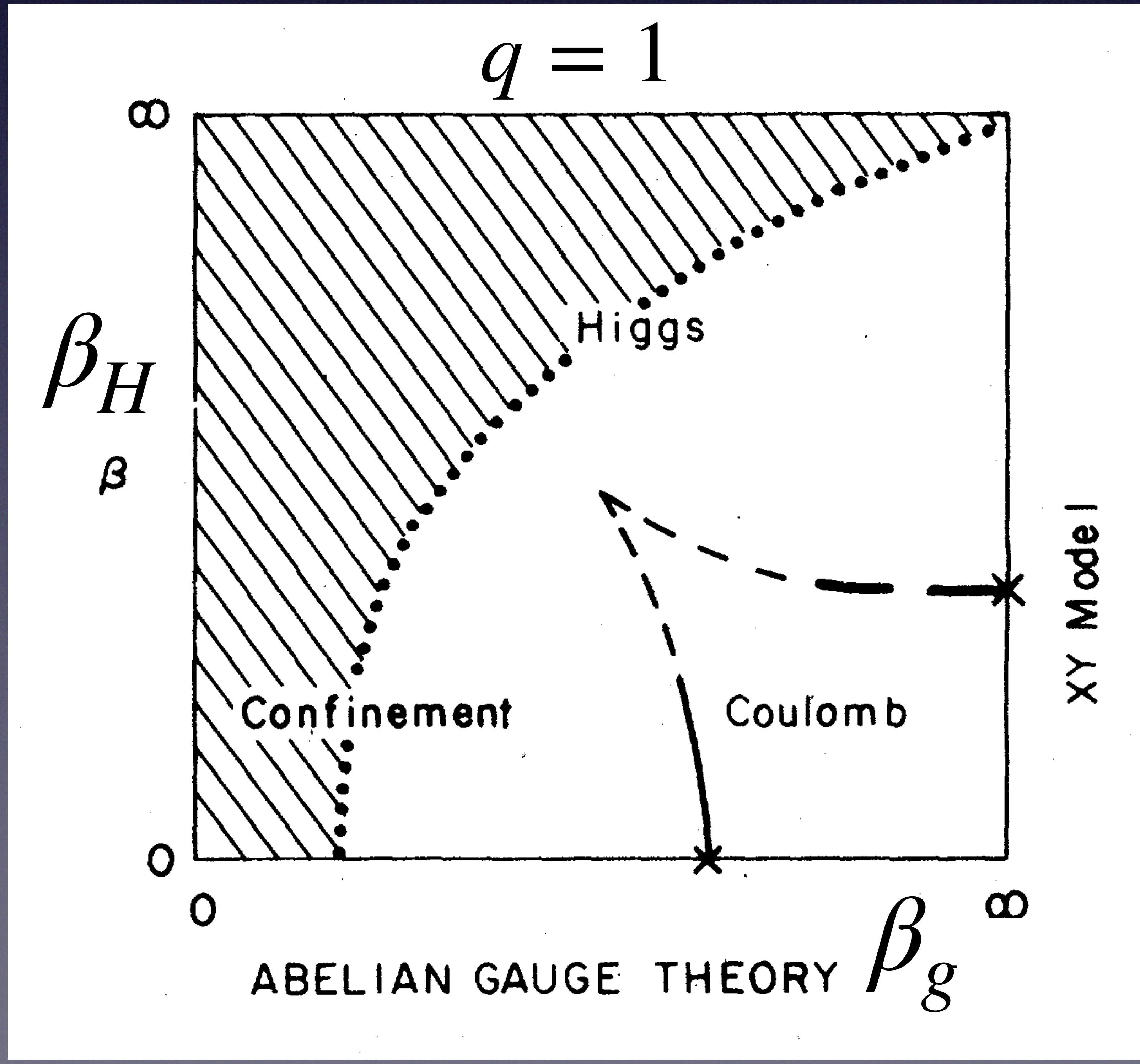
Abelian Higgs model in (3+1) dimensions

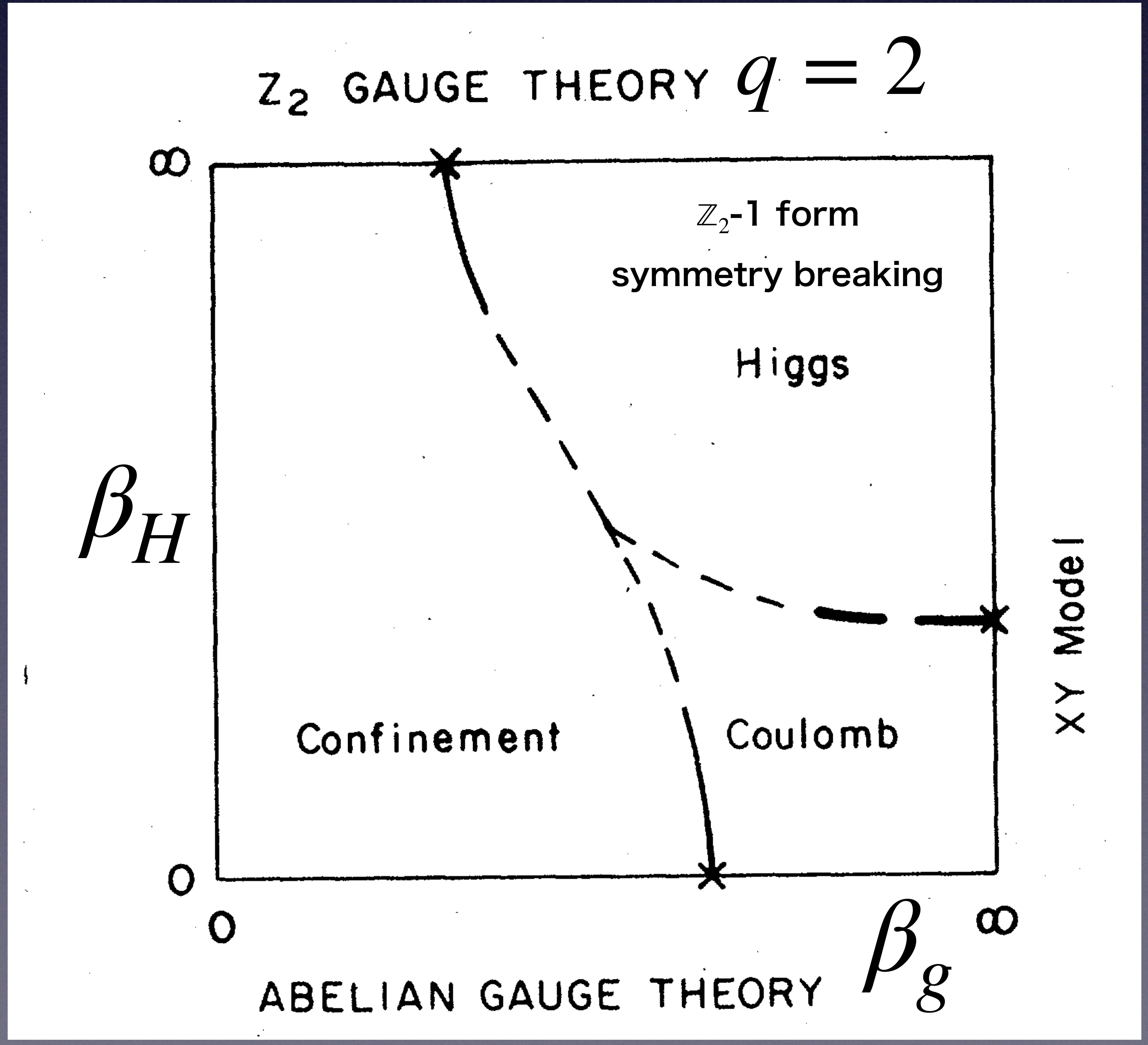
$$S = -\beta_g \sum_{x,\mu<\nu} \cos\left(F_{\mu\nu}(x)\right) - \beta_H \sum_{x,\mu} \cos\left(\Delta_\mu \varphi(x) - qA_\mu(x)\right)$$

Field strength **Scalar field (phase dof)** **Gauge field**

$$\Delta_\mu \varphi_a(x) = \varphi_a(x+\hat{\mu}) - \varphi_a(x)$$

Fradkin-Schenker Phys. Rev. D 19, 3682 ('79)





$\mathrm{U(1)_{gauge}} \times \mathrm{U(1)_{global}}$ **lattice model**

cf. Motrunich, Senthil ('05)

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Field strength

Scalar field
(phase dof)

Gauge field

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Symmetry

$$\mathrm{U}(1)_{\mathrm{gauge}} : \begin{array}{l} \varphi_1 \to \varphi_1 - \lambda \\ \varphi_2 \to \varphi_2 - \lambda \\ A_\mu \to A_\mu + \Delta_\mu \lambda \end{array}$$

$$\mathrm{U}(1)_{\mathrm{global}} : \begin{array}{l} \varphi_1 \to \varphi_1 + \theta \\ \varphi_2 \to \varphi_2 - \theta \end{array}$$

$$\mathbb{Z}_{2F} : \begin{array}{l} \varphi_1 \to \varphi_2 \\ \varphi_2 \to \varphi_1 \end{array}$$

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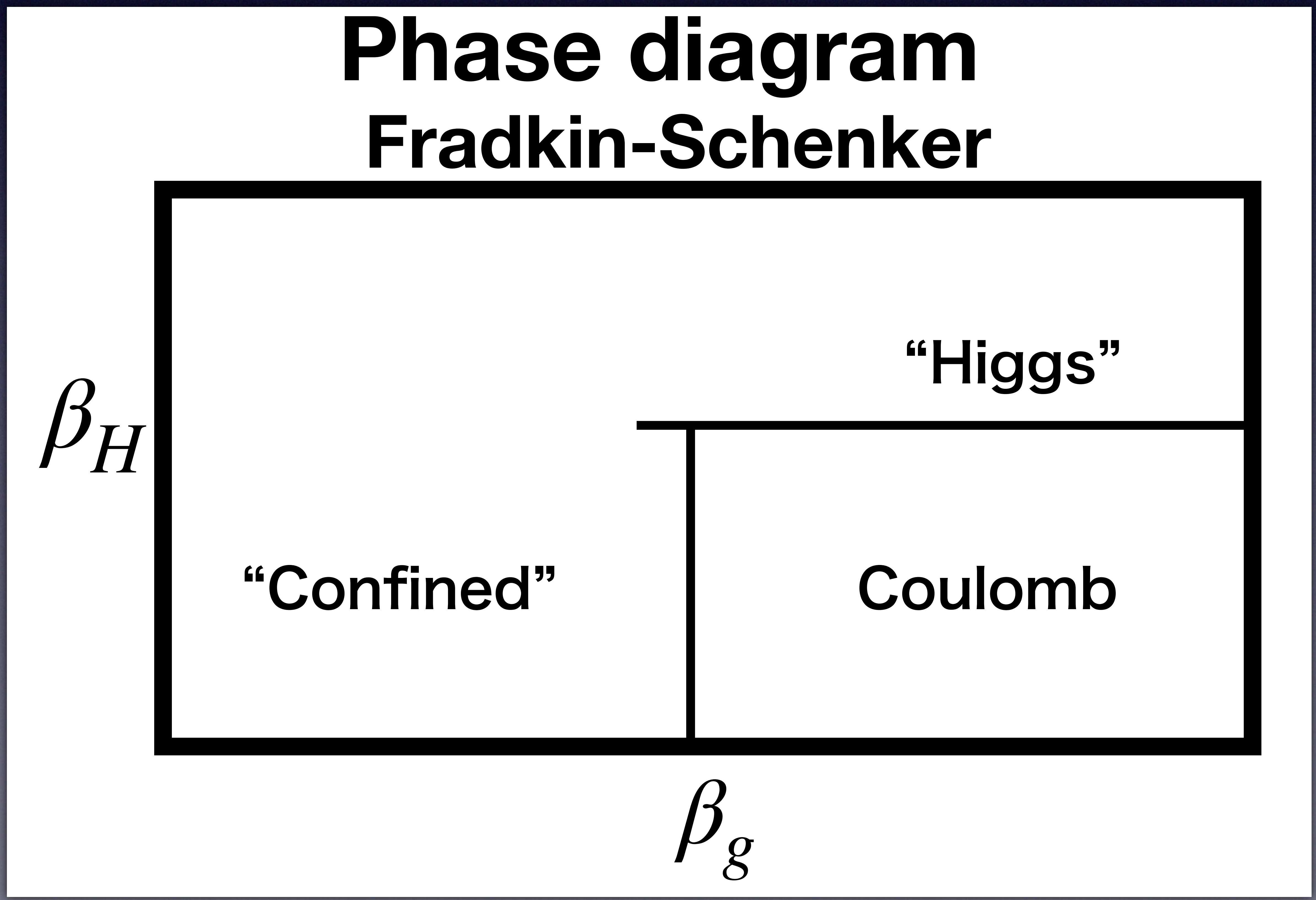

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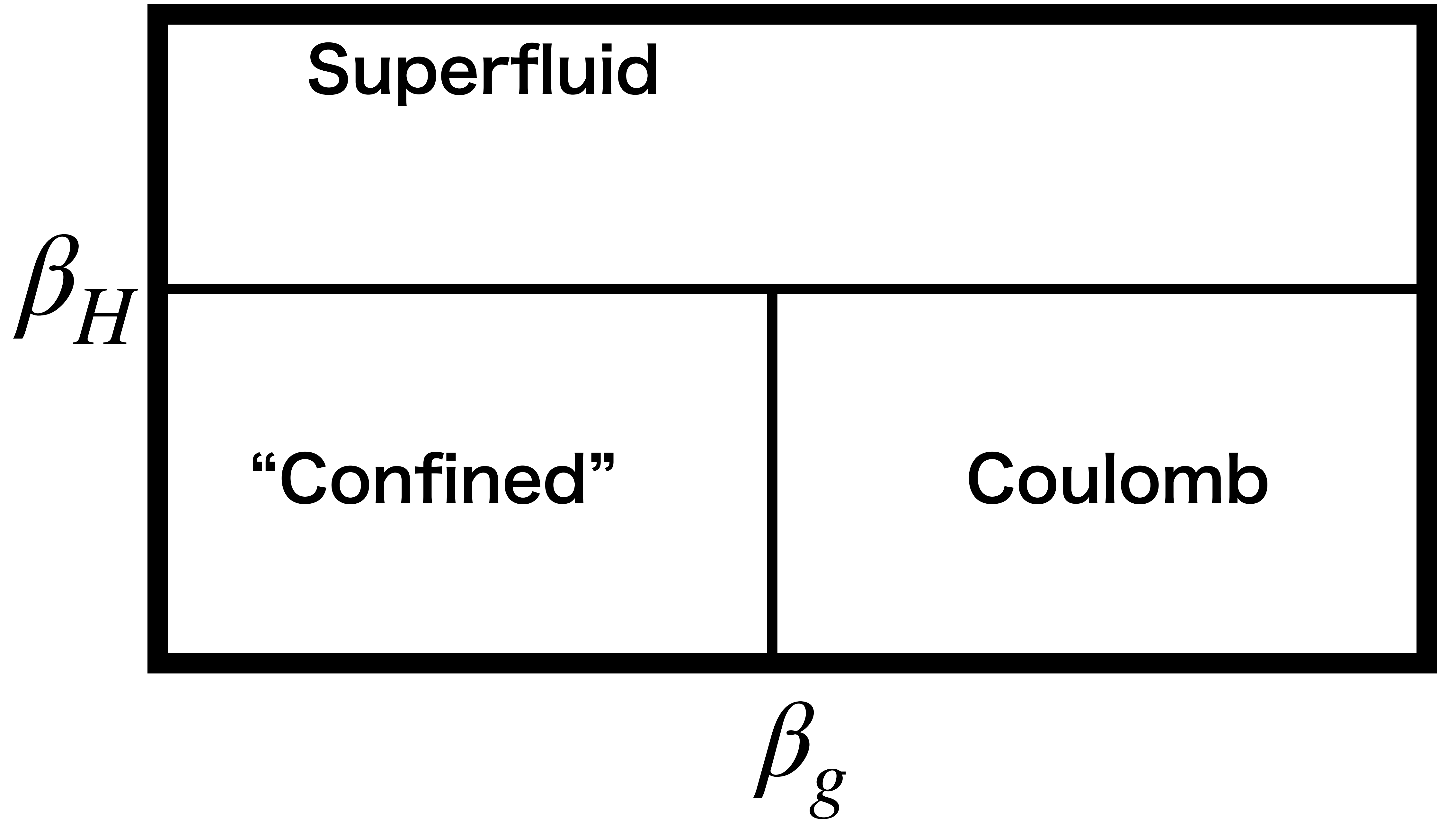
Phase diagram



$\mathrm{U}(1)_{\mathrm{gauge}} \times \mathrm{U}(1)_{\mathrm{global}}$ **lattice model**

cf. Motrunich, Senthil ('05)

$$S = -\beta_g \sum_{x,\mu<\nu} \cos\left(F_{\mu\nu}(x)\right) - \beta_H \sum_{x,\mu} \sum_{a=1,2} \cos\left(\Delta_\mu \varphi_a(x) + A_\mu(x)\right)$$

Field strength

Scalar field (phase dof)

Gauge field

$\Delta_\mu \varphi_a(x) = \varphi_a(x+\hat{\mu}) - \varphi_a(x)$

Symmetry

$$\mathrm{U}(1)_{\mathrm{gauge}} : \begin{array}{l} \varphi_1 \to \varphi_1 - \lambda \\ \varphi_2 \to \varphi_2 - \lambda \\ A_\mu \to A_\mu + \Delta_\mu \lambda \end{array}$$

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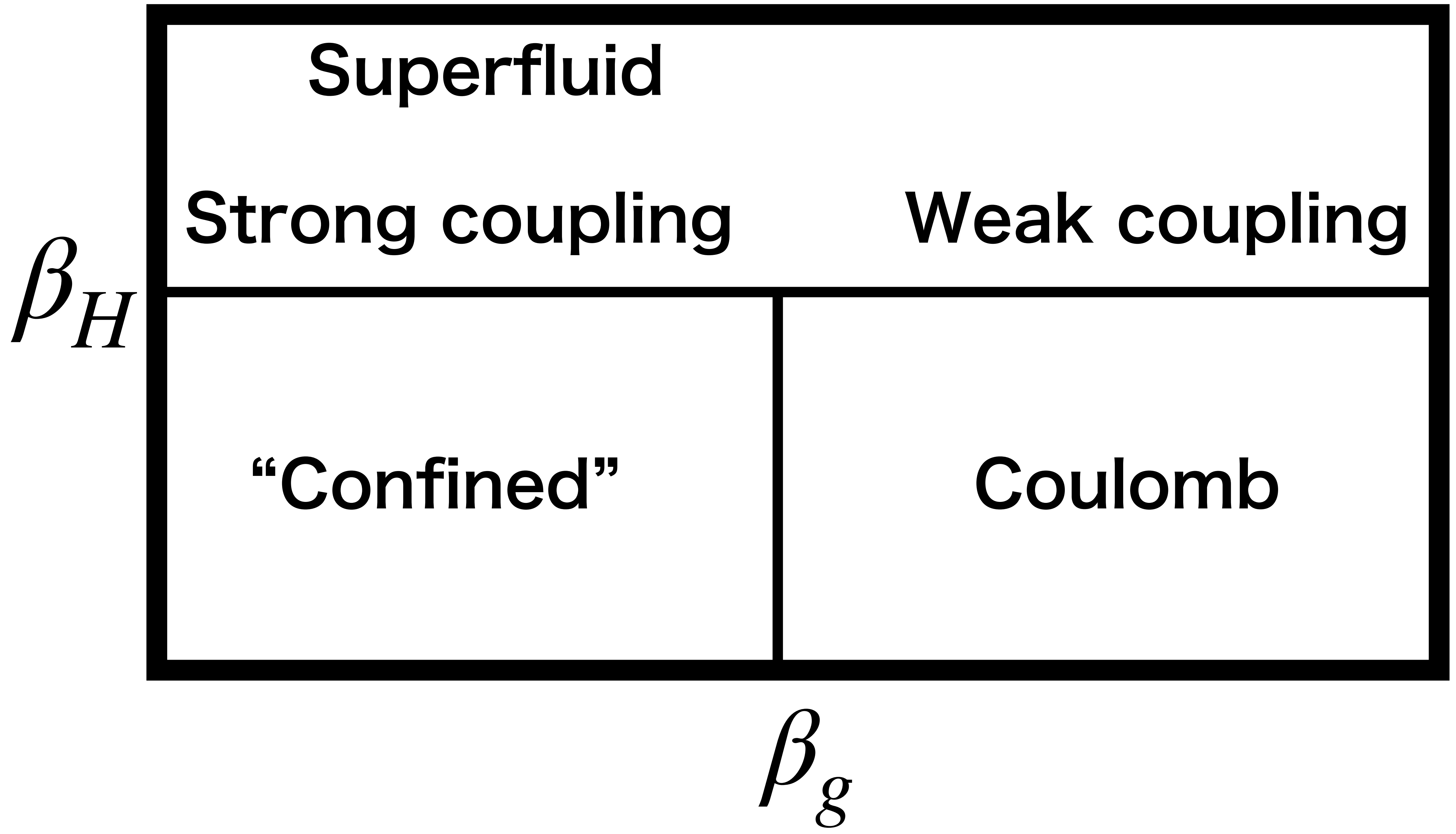

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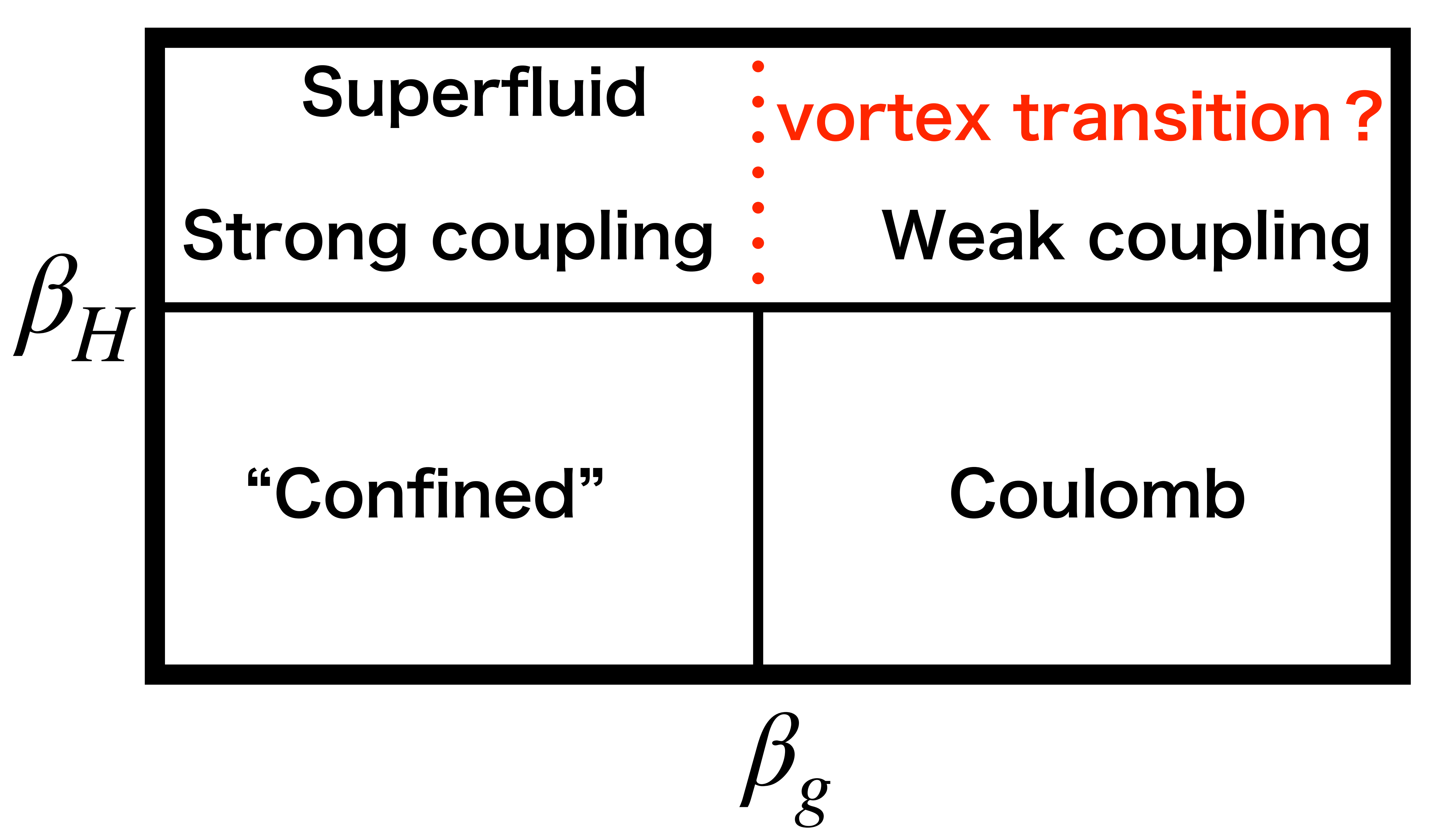

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Field strength

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Gauge field

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Emergent symmetry at large β_H (SSB of $U(1)_{\mathrm{global}}$)

YH, Kondo ('22)

Emergent	$U(1)^{[2]}$	$\mathbb{Z}_2^{[2]}$
Symmetry operator	$e^{i\frac{\theta}{2\pi}\int_C (d\varphi_1 - d\varphi_2)}$	$e^{i\frac{1}{2}\int_C (d\varphi_1 + d\varphi_2)}$

Example: $\mathrm{U}(1)_{\mathrm{gauge}} \times \mathrm{U}(1)_{\mathrm{global}}$ lattice model

Strong coupling $\beta_g \ll 1$

Weak coupling $\beta_g \gg 1$

Example: $\mathrm{U}(1)_{\mathrm{gauge}} \times \mathrm{U}(1)_{\mathrm{global}}$ lattice model

Strong coupling $\beta_g \ll 1$

Integrating over gauge fields

$$S_{\mathrm{eff}} = -\sum_{x,\mu} \ln I_0 \left[2\beta_H \cos\left(\frac{\Delta_\mu \varphi_1(x) - \Delta_\mu \varphi_2(x)}{2} \right) \right]$$

$I_0(z)$:**Modified Bessel**

Weak coupling $\beta_g \gg 1$

$$S = -\beta_g \sum_{x,\mu<\nu} \cos\left(F_{\mu\nu}(x)\right)$$

$$-\beta_H \sum_{x,\mu} \sum_{a=1,2} \cos\left(\Delta_\mu \varphi_a(x) + A_\mu(x)\right)$$

Example: $\mathrm{U}(1)_{\mathrm{gauge}} \times \mathrm{U}(1)_{\mathrm{global}}$ lattice model

Strong coupling $\beta_g \ll 1$

Integrating over gauge fields

⬇

$$S_{\mathrm{eff}} = -\sum_{x,\mu} \ln I_0 \left[2\beta_H \cos\left(\frac{\Delta_\mu \varphi_1(x) - \Delta_\mu \varphi_2(x)}{2} \right) \right]$$

$I_0(z)$:Modified Bessel

Essential d.o.f. is $\varphi_1 - \varphi_2$

i.e., one d.o.f.

Weak coupling $\beta_g \gg 1$

$$S = -\beta_g \sum_{x,\mu<\nu} \cos\left(F_{\mu\nu}(x)\right)$$
$$-\beta_H \sum_{x,\mu} \sum_{a=1,2} \cos\left(\Delta_\mu \varphi_a(x) + A_\mu(x)\right)$$

Distinguishable φ_1 and φ_2

$\mathbb{Z}_{2F}$ is spontaneously broken on the vortices

Criterion of symmetry breaking:

When discrete symmetry is broken: twisting the boundary conditions by the symmetry causes the formation of domain walls

Example: Ising model

$\mathbb{Z}_2$ broken phase

$$\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow | \downarrow\downarrow\downarrow\downarrow\downarrow\downarrow$$

domain wall

$\mathbb{Z}_2$ unbroken phase

random configuration

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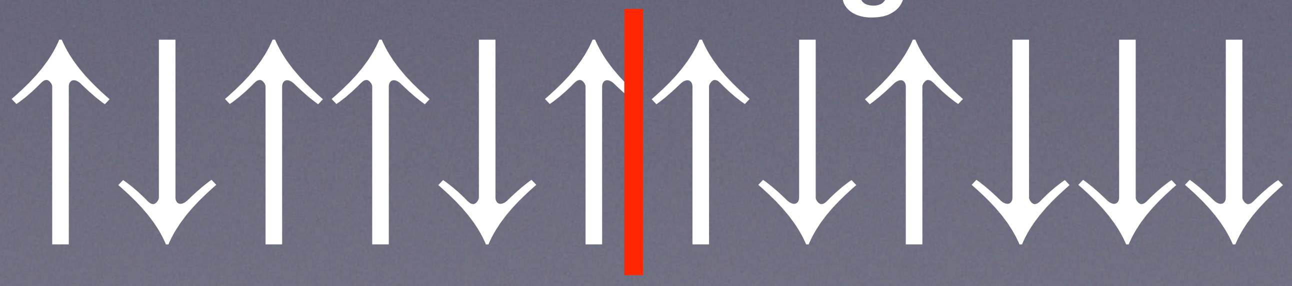
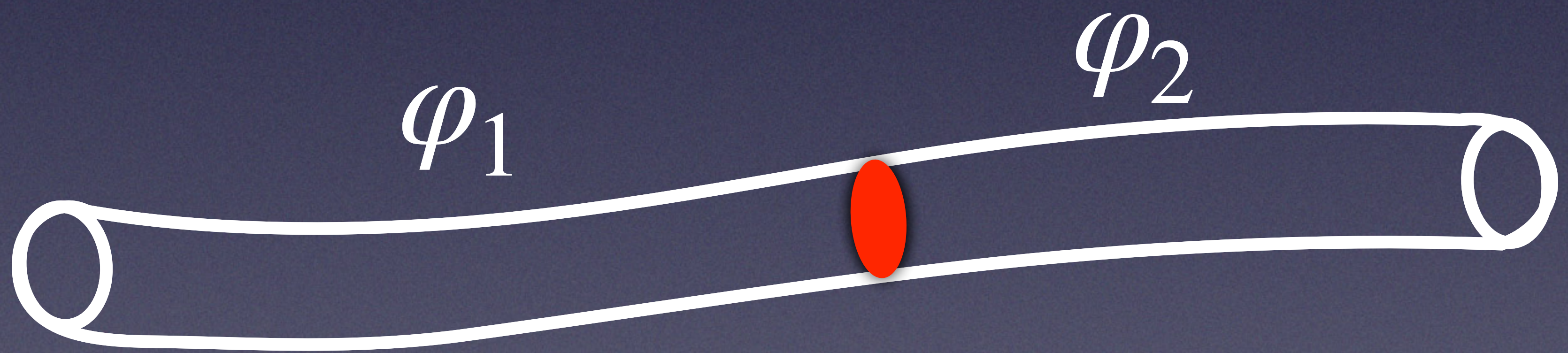


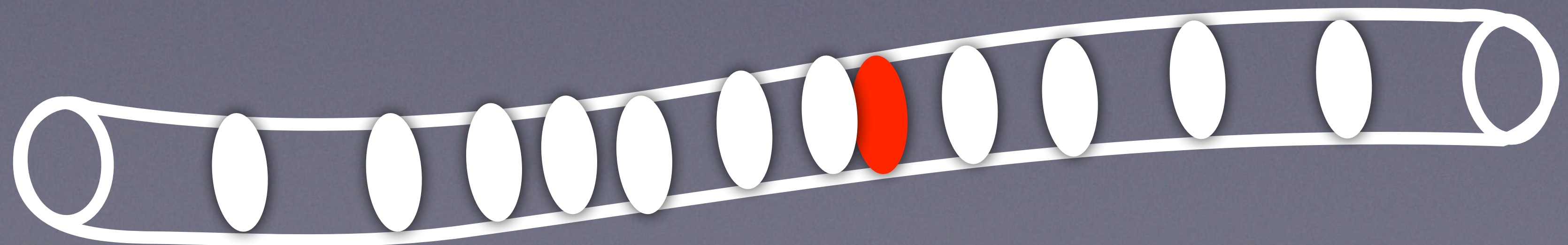
$\mathrm{U}(1)_{\mathrm{gauge}} \times \mathrm{U}(1)_{\mathrm{global}}$ model

Weak coupling ($\mathbb{Z}_{2F}$ broken)



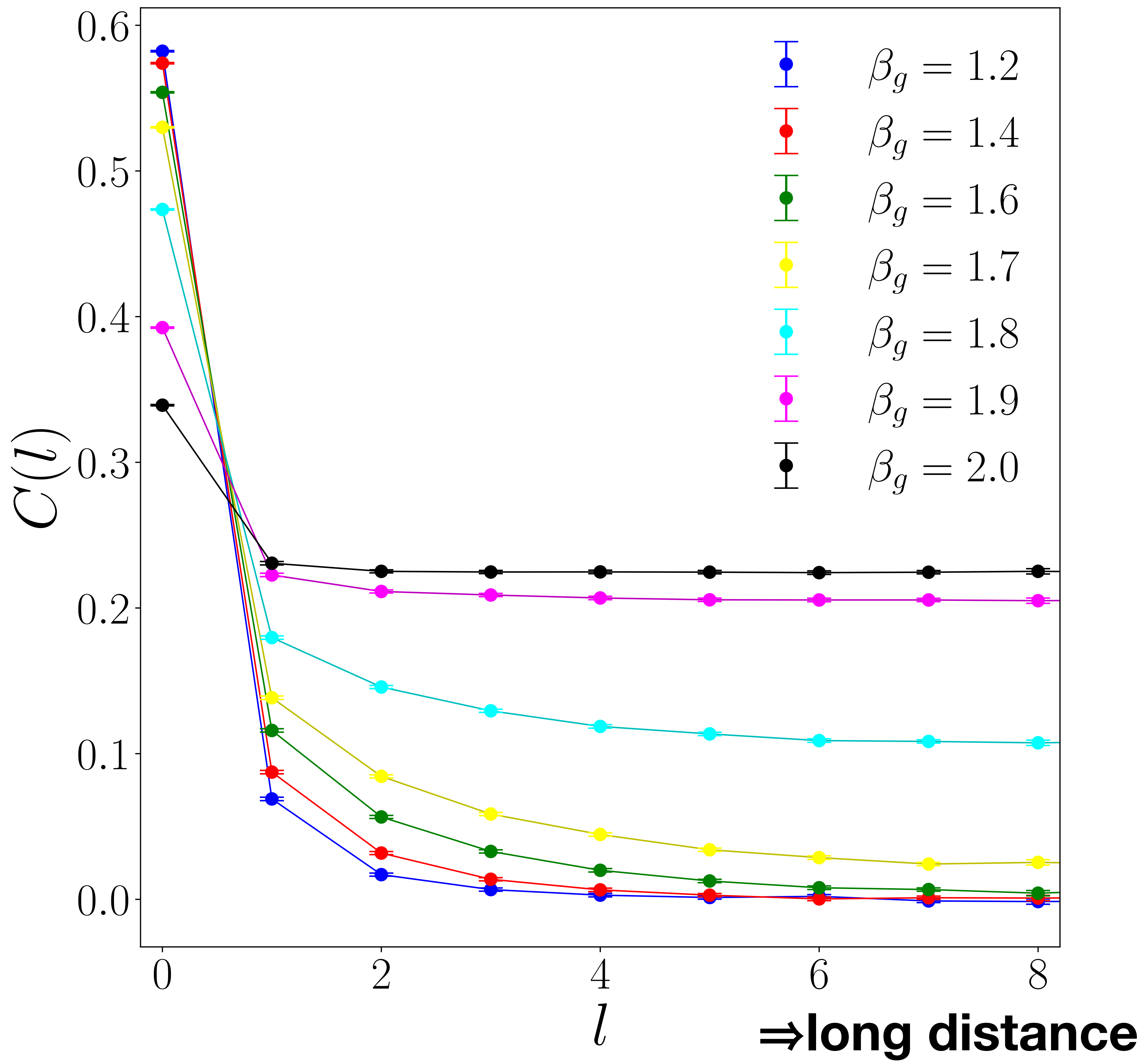
Strong coupling ($\mathbb{Z}_{2F}$ unbroken)

randomized junctions



Numerical simulation

Correlation function of magnetic flux



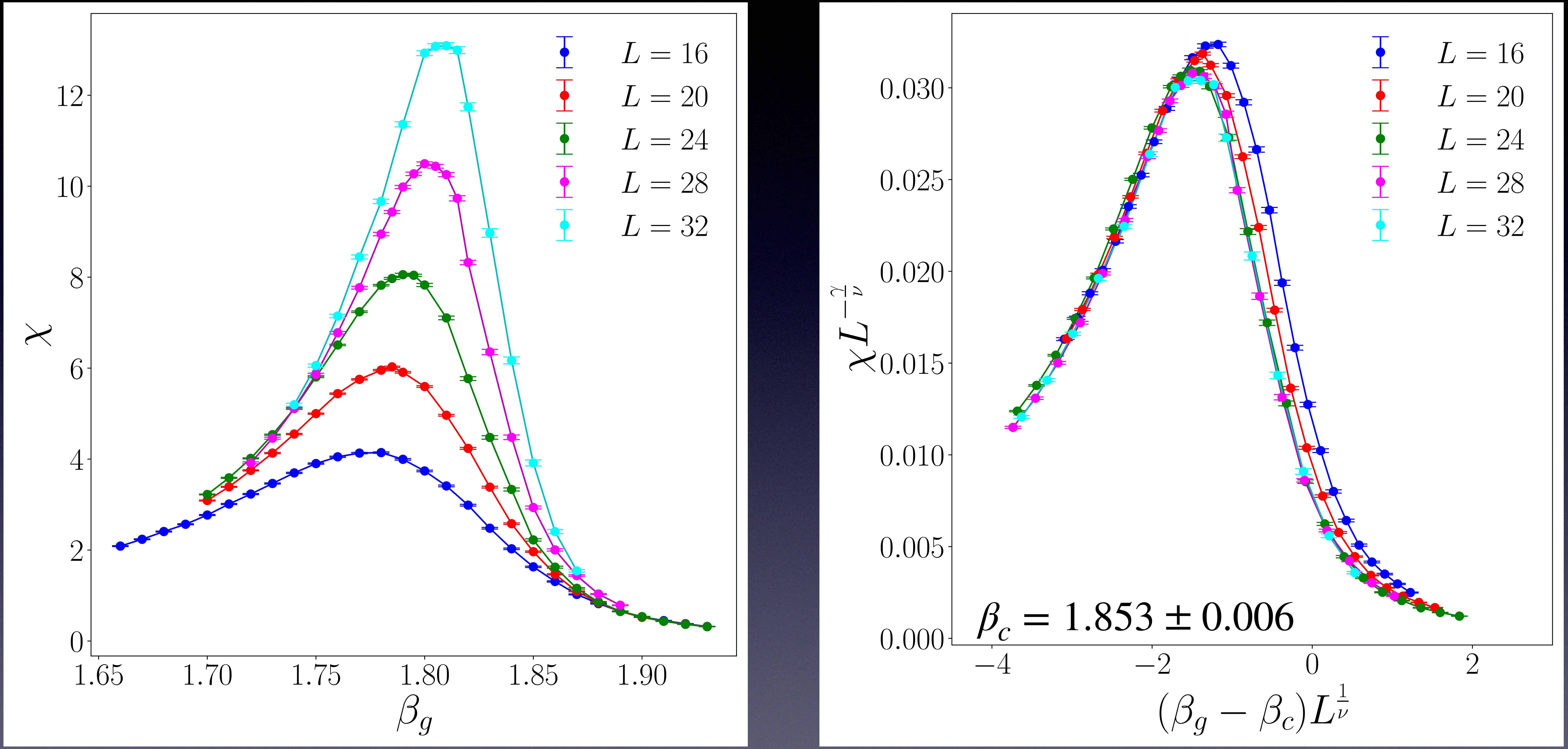
vortex

At weak coupling long-range correlation

Spontaneous symmetry breaking

⇒ Phase transition on a vortex

Critical point



Ising universality class $\nu = 1, \gamma = 7/4$

predicted in Motrunich, Senthil ('05)

Summary

We found the phase transition on a vortex between strong and weak gauge couplings in superfluid phase

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More generally, there can be phase transitions of various defects

Codimension 1: transition on a domain wall
Codimension 2: transition on a vortex
Codimension 3: Level crossing

Phase transitions on domain wall junctions are also possible

Example: Level crossing

Consider hadronic and superconducting regime

Muto, Hayashi, YH ('25)

Example: Level crossing

Consider hadronic and superconducting regime

Muto, Hayashi, YH ('25)

Test charge

Example: Level crossing

Consider hadronic and superconducting regime

Muto, Hayashi, YH ('25)

hadronic regime

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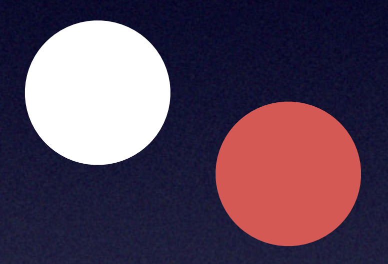

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Screened by antiquark

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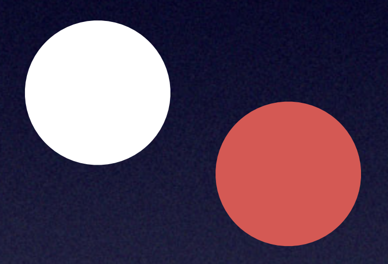

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superconducting regime

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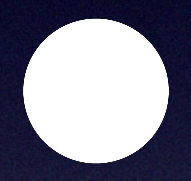

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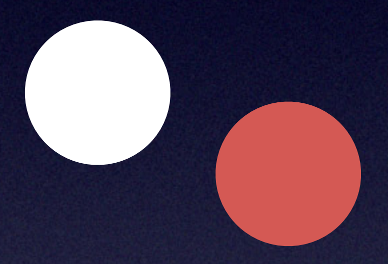

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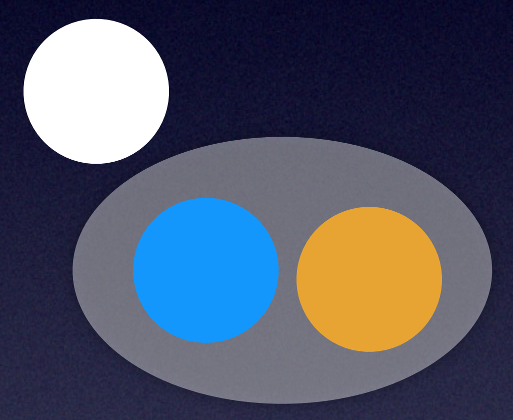

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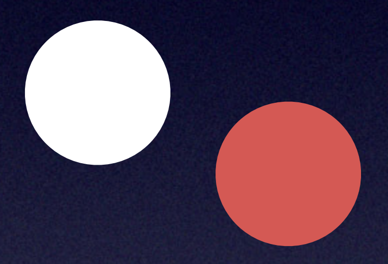

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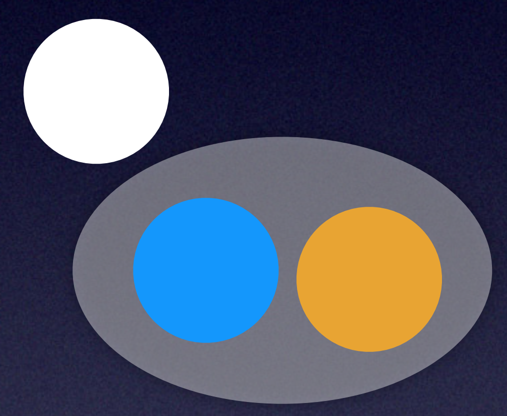
Fermion parity odd

superconducting regime

Test charge



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Fermion parity even

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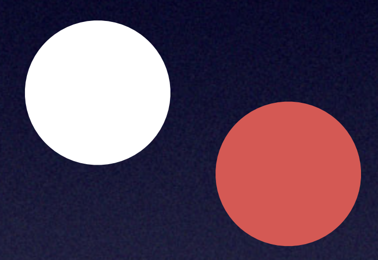

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Fermion parity odd

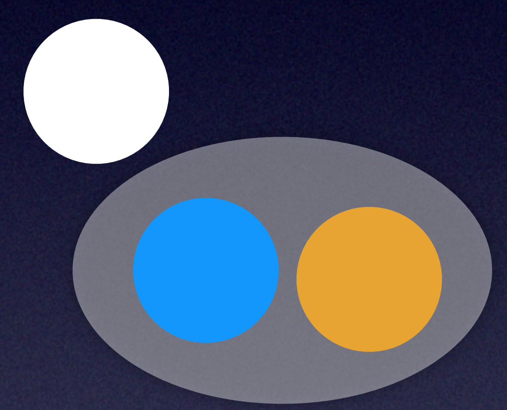
Order parameter $$\frac{\langle FP \rangle}{\langle P \rangle} = -1$$

P: Polyakov loop

F: Fermion parity operator

superconducting regime

Test charge



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Fermion parity even

$$= +1$$

Outlook

EFT on $U(1) \times U(1)$ model〜Ising model

EFT of CFL phase $\sim CP(2)$ model

Ground state of $CP(2)$ model
Gapped phase, no flavor breaking
⇒ continuously connects to the hadronic phase？

What happens if fermion d.o.f. is included？