Duality in dense regime of QCD

Yuki Fujimoto
(UC Berkeley)

References:

[1] Y. Fujimoto, T. Kojo, L. McLerran, PRL132 (2024); arXiv:2410.22758.
[2] M. Bluhm, Y. Fujimoto, L. McLerran, M. Nahrgang, work in progress.
[3] Y. Fujimoto, K. Fukushima, W. Weise, PRD 101 (2020).
[4] Y. Fujimoto, arXiv:2502.01169 [cond-mat.supr-con].

14 February 2025 - Confinement and symmetry from vacuum to QCD phase diagram @ Benasque Science Center

Outline

1. Reinterpretation of Quarkyonic matter based on duality

2. Statistical mechanics of IdylliQ matter at finite temperature

3. Two-flavor color superconductor and its symmetry

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Emerging picture of neutron star EoS

Bedaque,Steiner (2015);
Tews,Carlson,Gandolfi,Reddy (2018);
Drischler,Han,Lattimer,
Prakash,Reddy,Zhao (2020);
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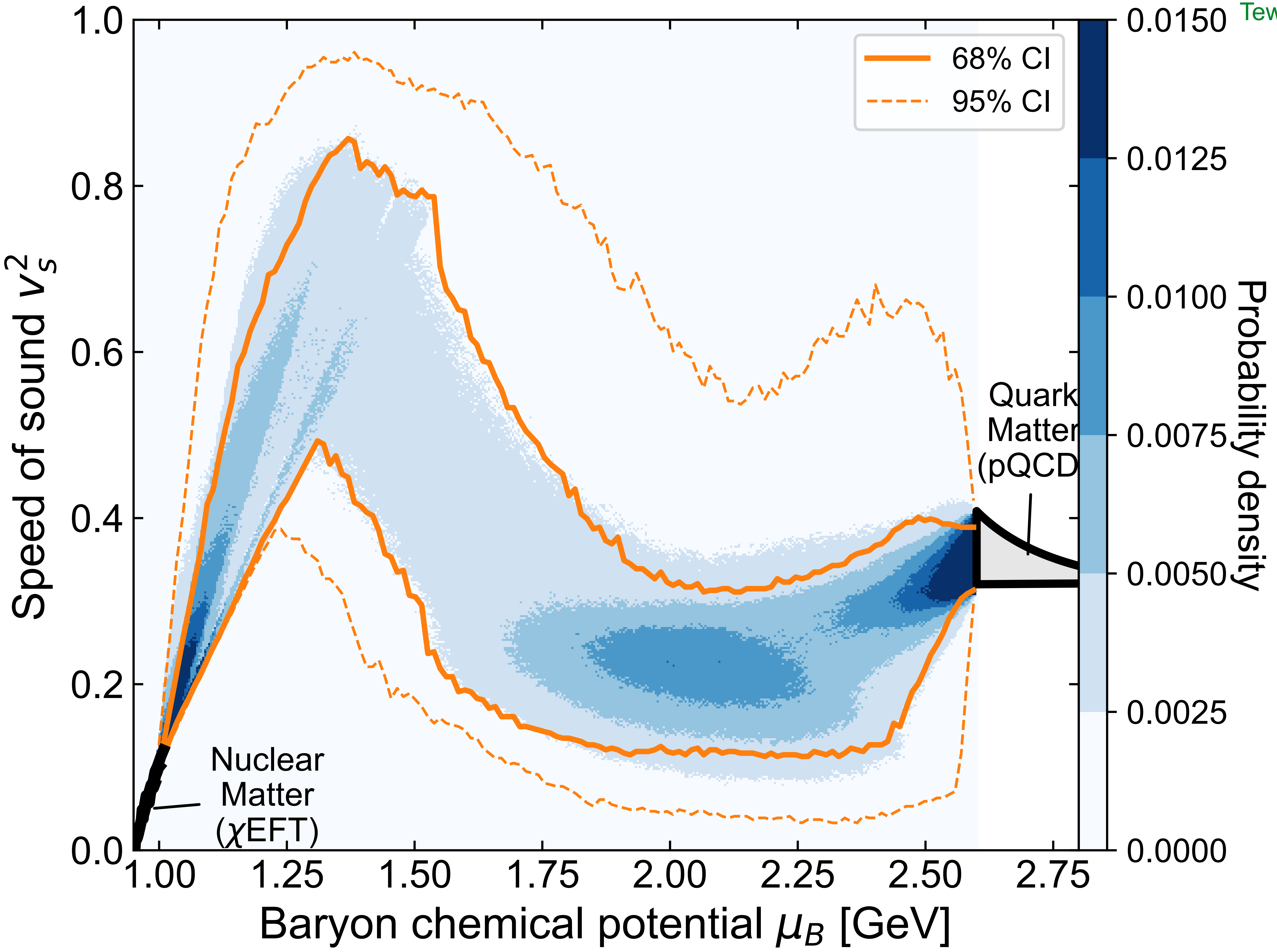
Brandes,Weise,Kaiser (2023)



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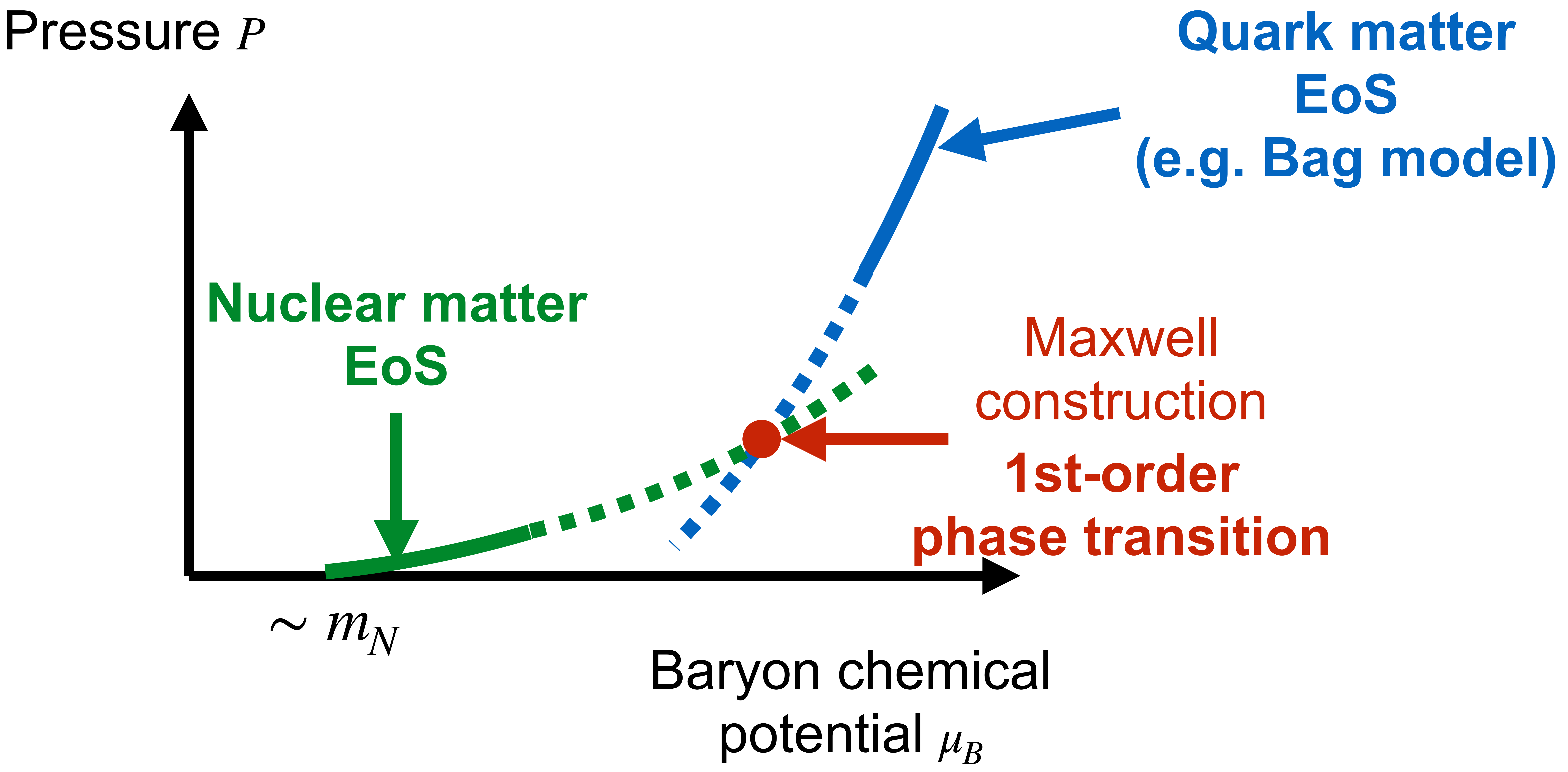
Naive quark deconfinement at high density

Baym,Chin (1976);
cf. Baym,Hatsuda,Kojo,Powell,Song,Takatsuka (2018)

EoS corresponding to the naive picture of deconfinement:



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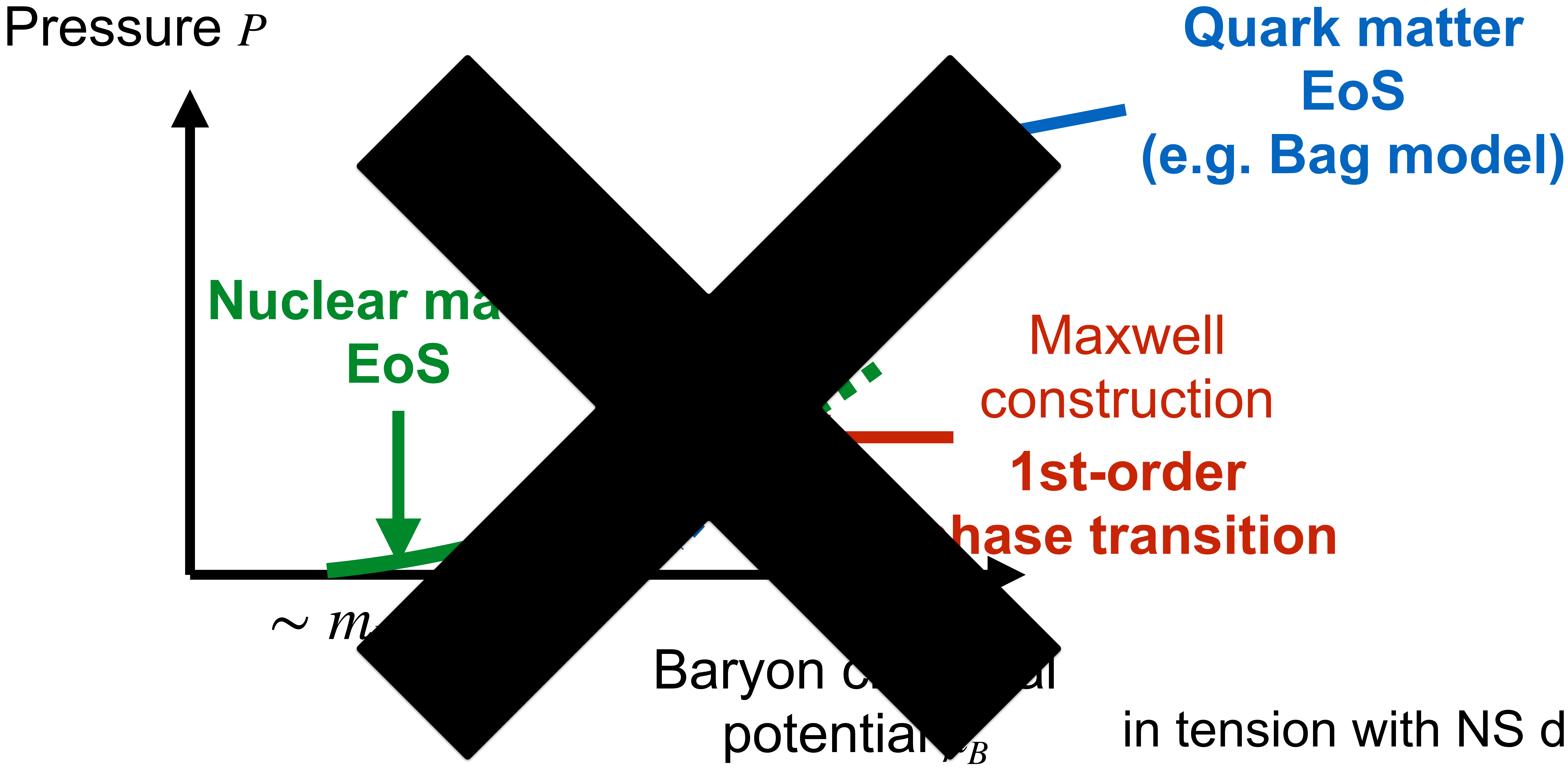


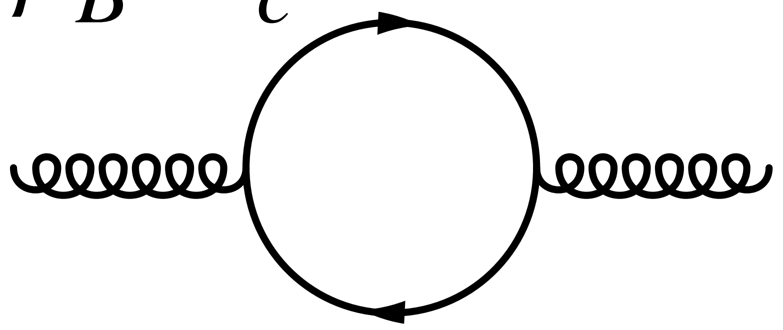
in tension with NS data

Central tenet of Quarkyonic matter: duality

Deconfinement at high density may not be that simple…

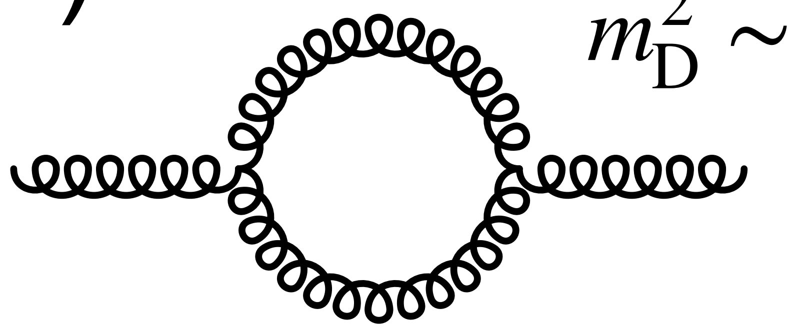
McLerran & Pisarski (2007): Quarks never deconfine in large-N_c QCD

$\mu = \mu_B / N_c$



$$m_D^2 \sim \frac{\lambda_{\text{'t Hooft}}\, \mu^2}{N_c} \to 0$$

cf) T



$m_D^2 \sim g^2 N_c T^2$

$\sim \lambda_{\text{'t Hooft}}\, T^2$

… (de)confinement is never affected by quark medium!

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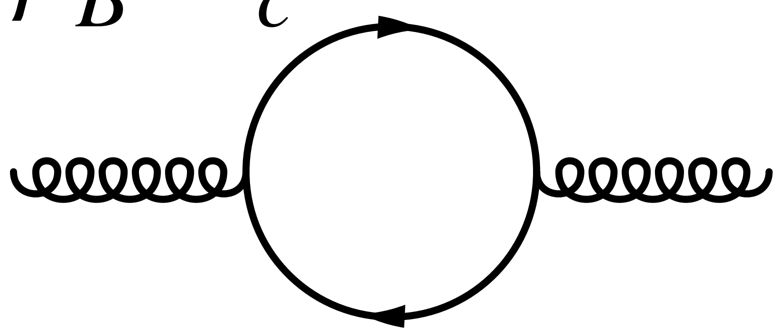

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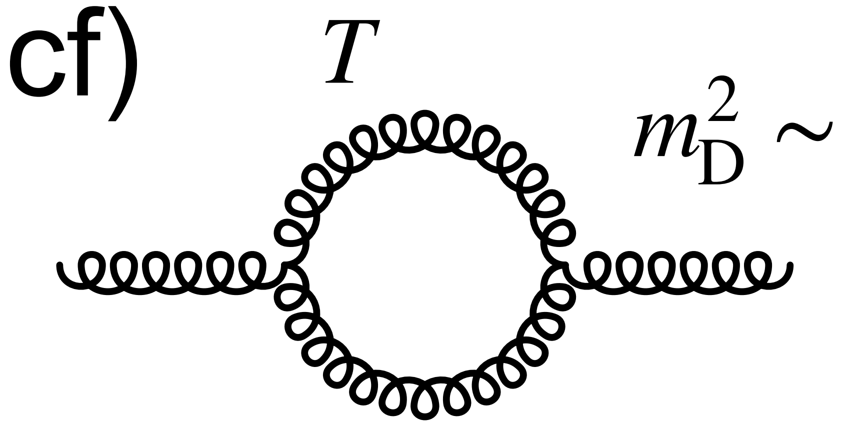
cf)



$m_{\rm D}^2 \sim g^2 N_c T^2$

$\sim \lambda_{\rm 't\ Hooft}\, T^2$

… (de)confinement is never affected by quark medium!

Dense large-Nc QCD matter can be described either as

- **Confined baryons** (because confining interaction is never screened)
- **Quarks** (at densities where weak-coupling QCD is valid)

→ implies **duality** between quark and confined baryonic matter

Quark yonic

Duality in Fermi gas: Idylliq model

Kojo (2021); Fujimoto,Kojo,McLerran, PRL 132 (2023)

Implement duality in Fermi gas model
(= simultaneous description in terms of baryons & quarks)

Fermi gas model w/ an explicit duality:

$$\varepsilon = \int_{\boldsymbol{k}} E_{\mathrm{B}}(k) f_{\mathrm{B}}(k) = \int_{\boldsymbol{q}} E_{\mathrm{Q}}(q) f_{\mathrm{Q}}(q)$$

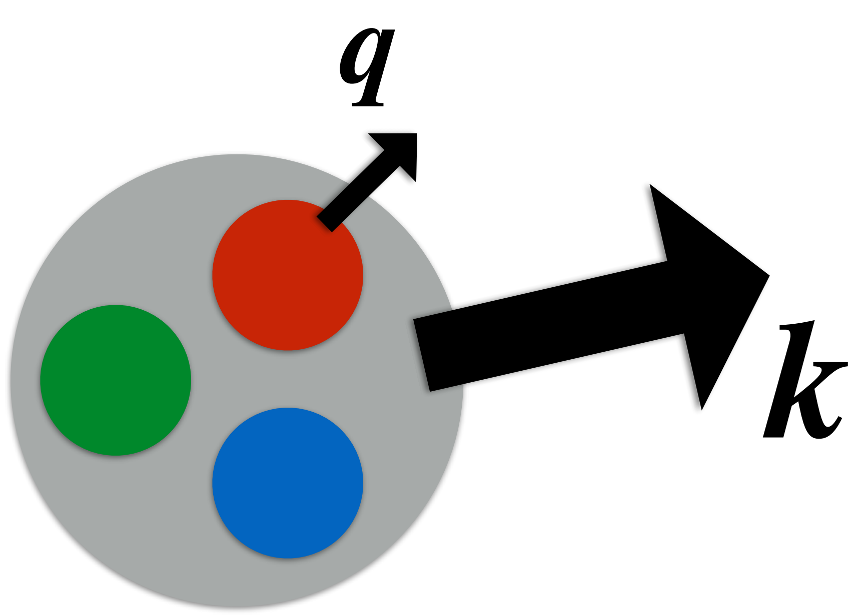
$$n_{\mathrm{B}} = \int_{\boldsymbol{k}} f_{\mathrm{B}}(k) = \int_{\boldsymbol{q}} f_{\mathrm{Q}}(q)$$

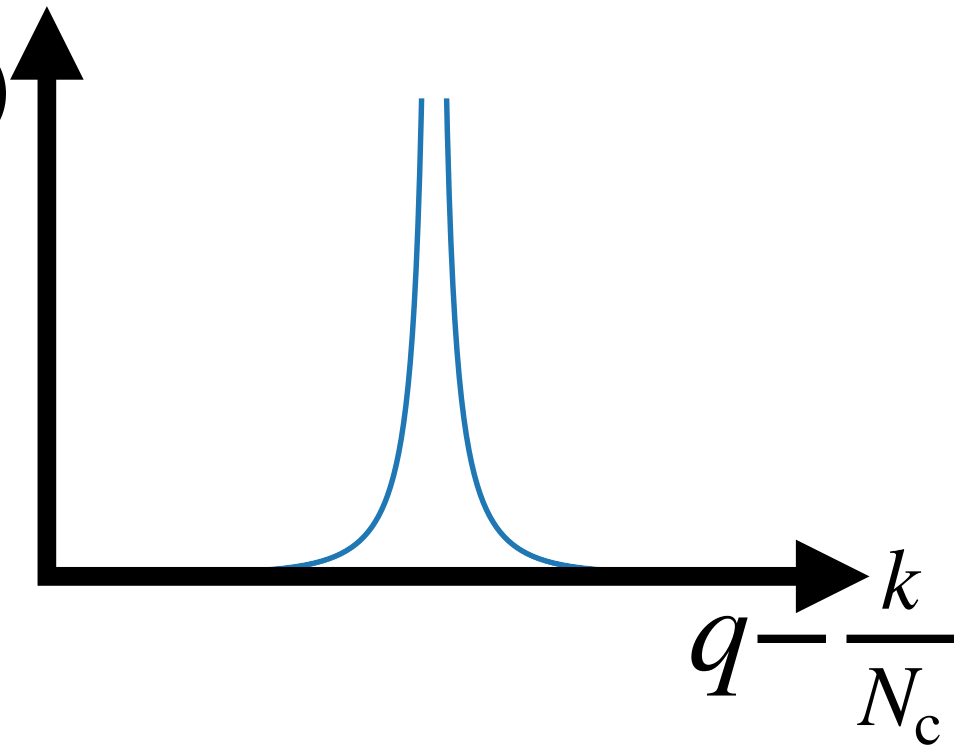
$0 \leq f_{\mathrm{B,Q}} \leq 1$: Pauli exclusion

$E_{\mathrm{B}}(k) = \sqrt{k^2 + M_N^2}$: ideal baryon dispersion relation

Modeling of confinement:

$$f_{\mathrm{Q}}(q) = \int_{\boldsymbol{k}} \varphi\left(\boldsymbol{q} - \frac{\boldsymbol{k}}{N_{\mathrm{c}}}\right) f_{\mathrm{B}}(k)$$



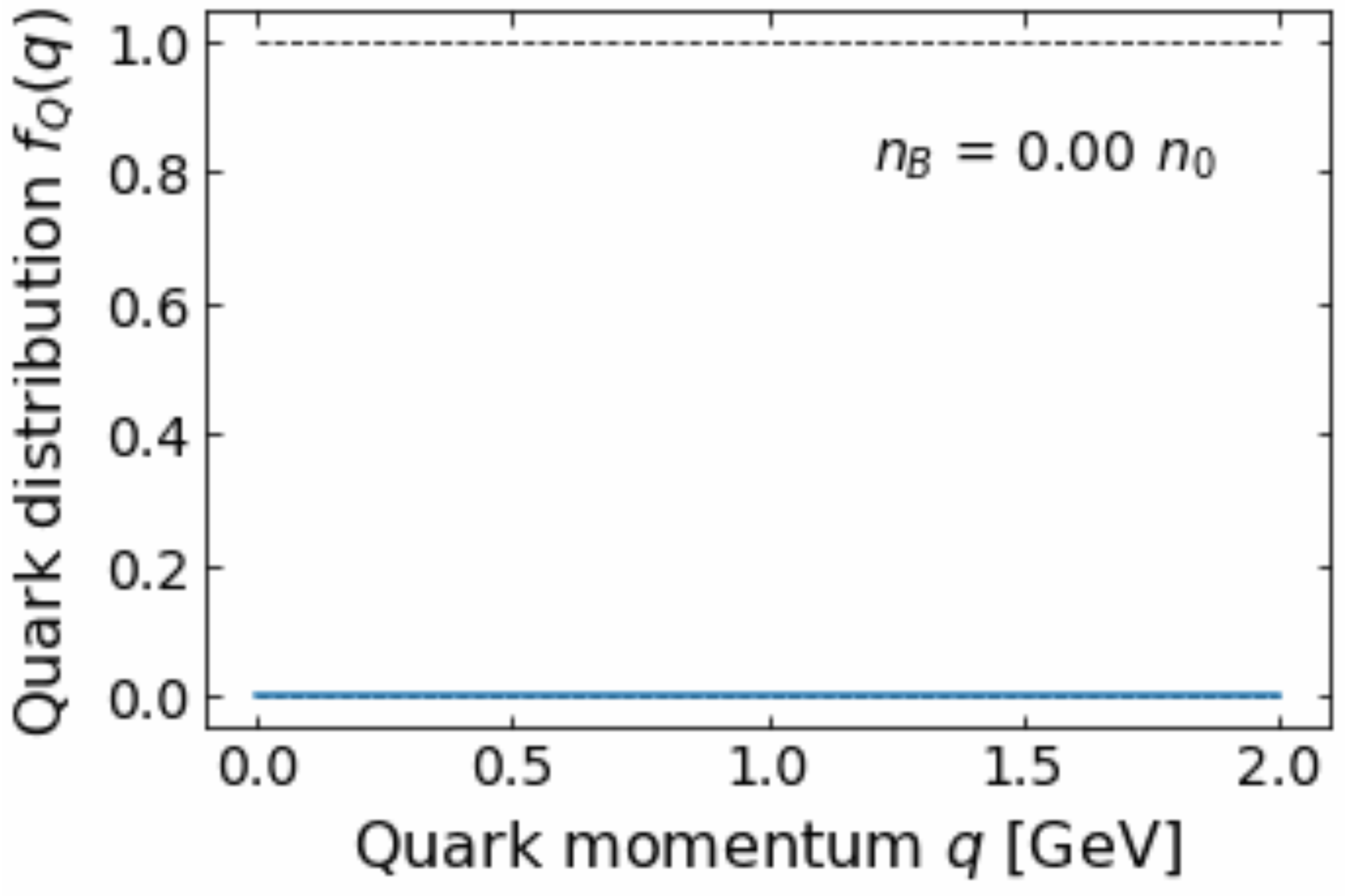


Ideal dual Quarkyonic model
→ **Find a solution for f_{B} and f_{Q} with minimum ε at a given n_B**

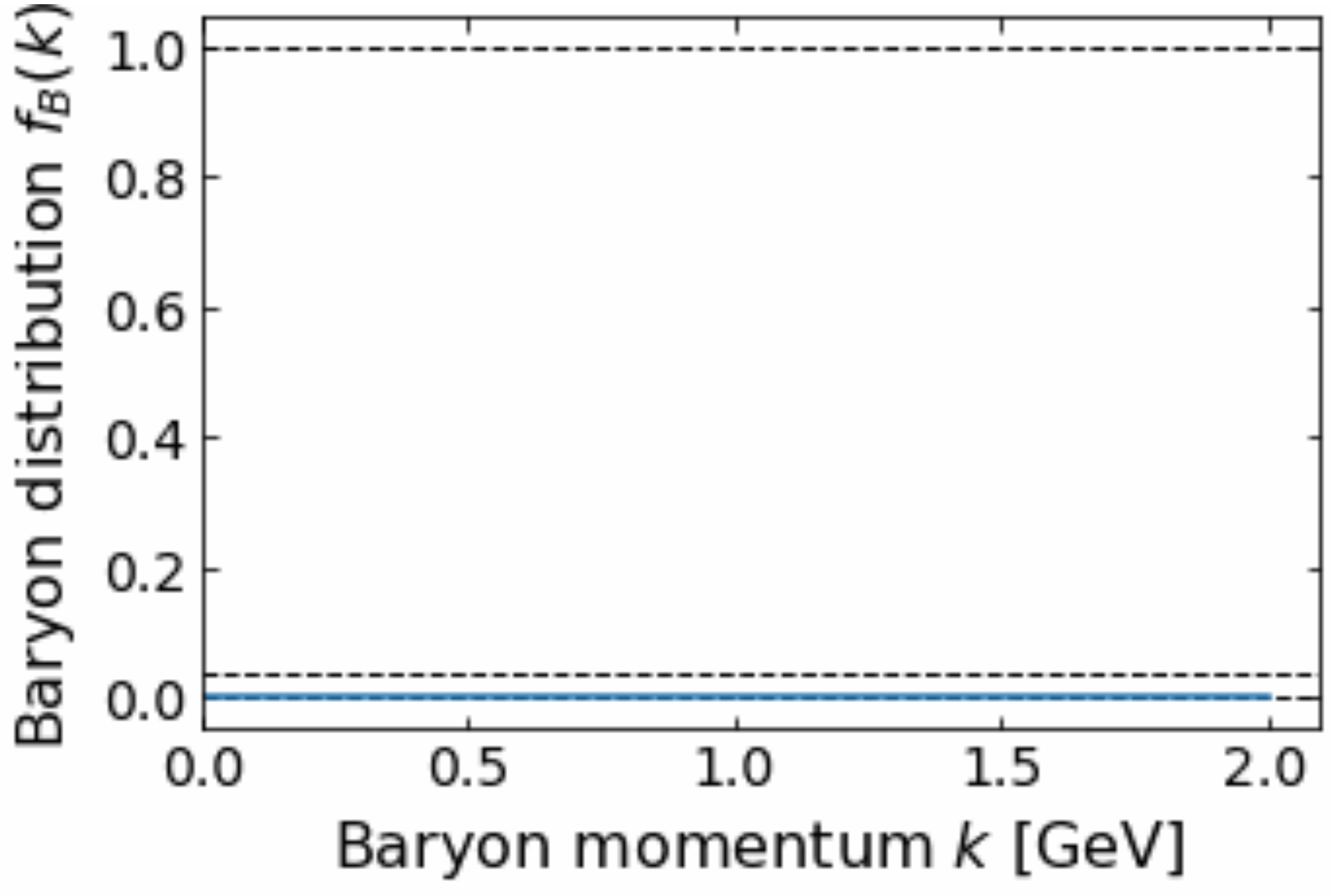
Overview on the solution of IdylliQ model

Fujimoto,Kojo,McLerran, PRL 132 (2023)

Quark distribution f_Q in momentum space



Baryon distribution f_B

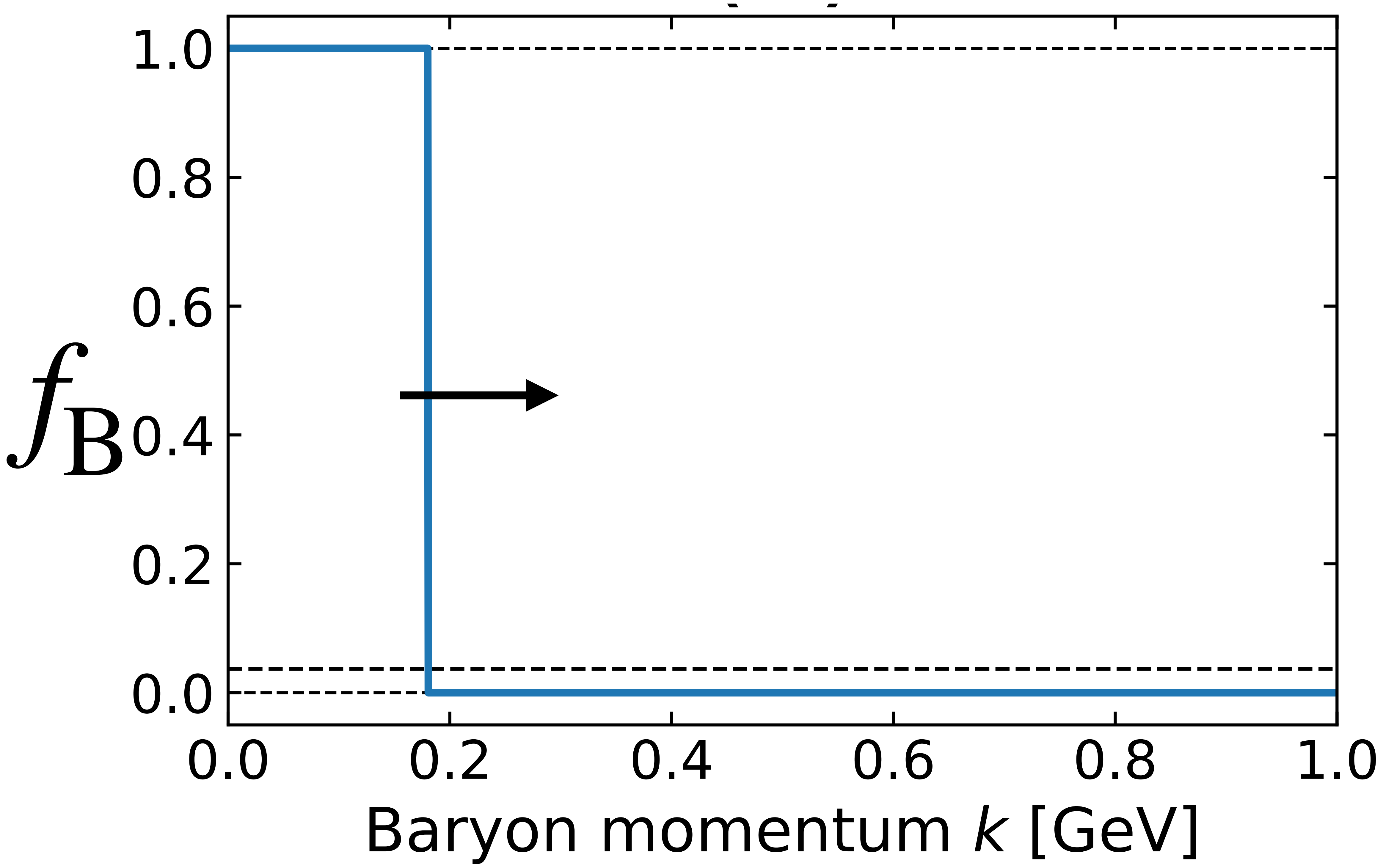


Solution of IdylliQ model

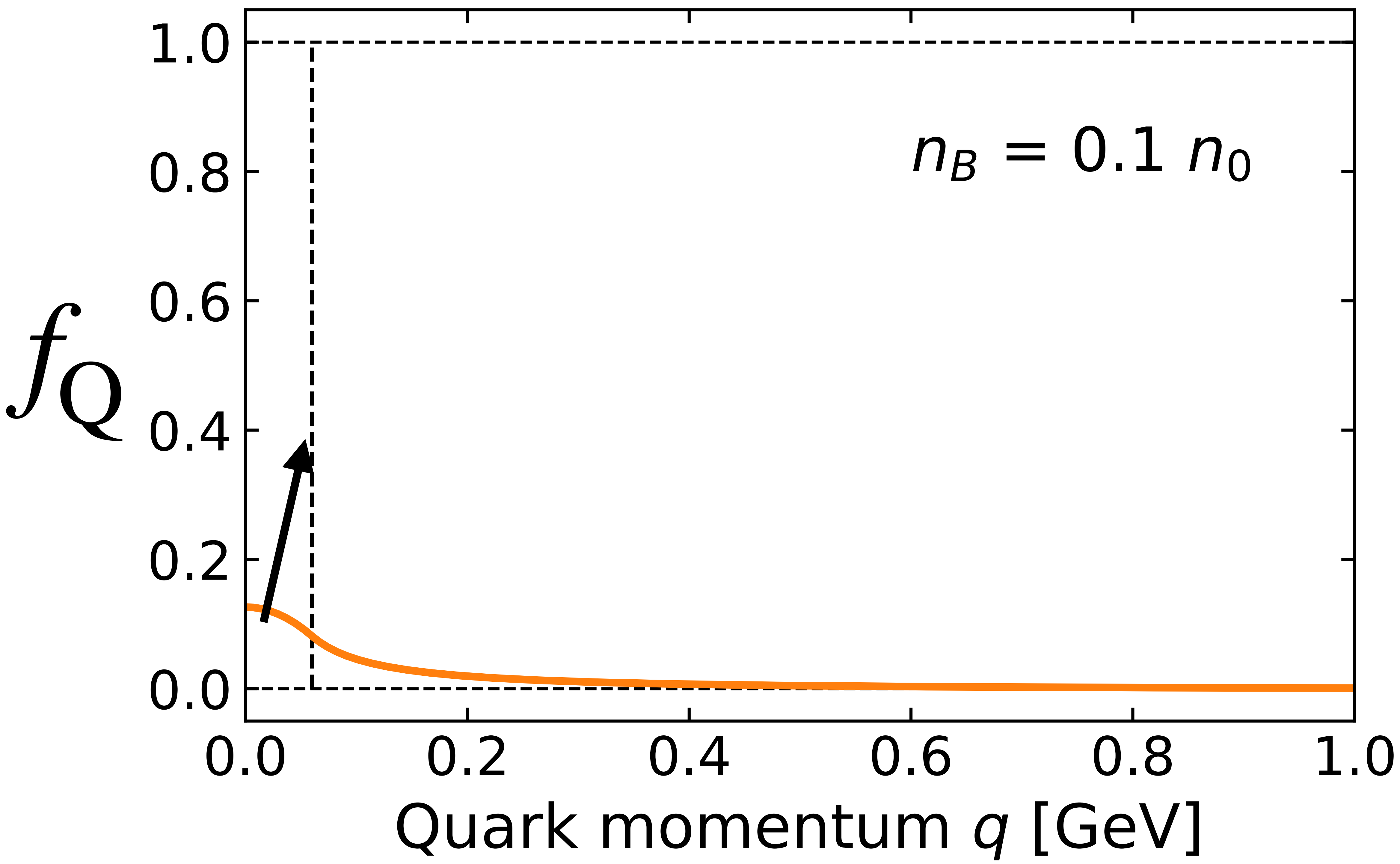
Kojo, PRD 104 (2021); Fujimoto,Kojo,McLerran, PRL 132 (2023)

At low density…

Fermi-Dirac distribution
for baryons



Quarks do not fill up
the Fermi sea yet

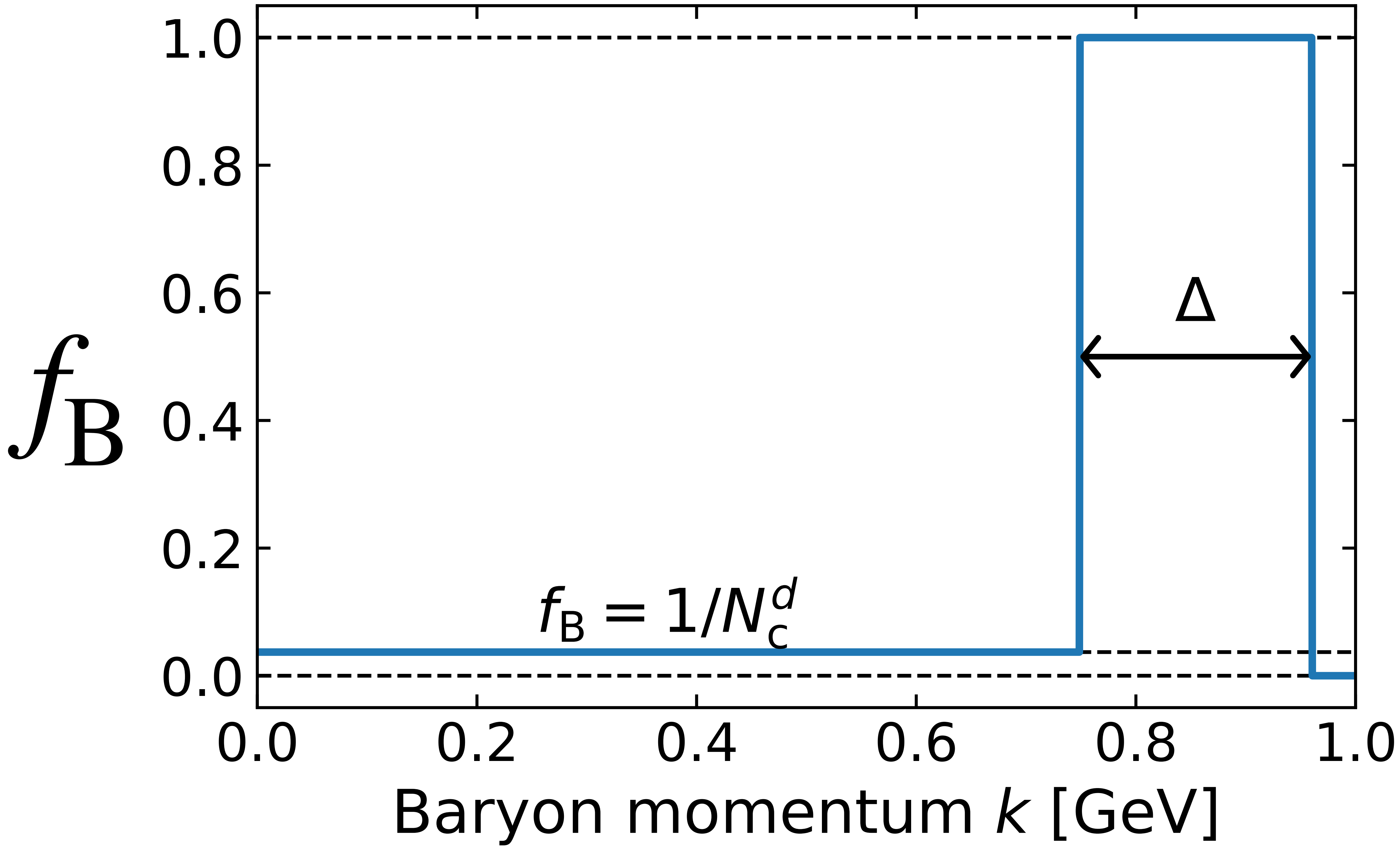


Solution of IdylliQ model

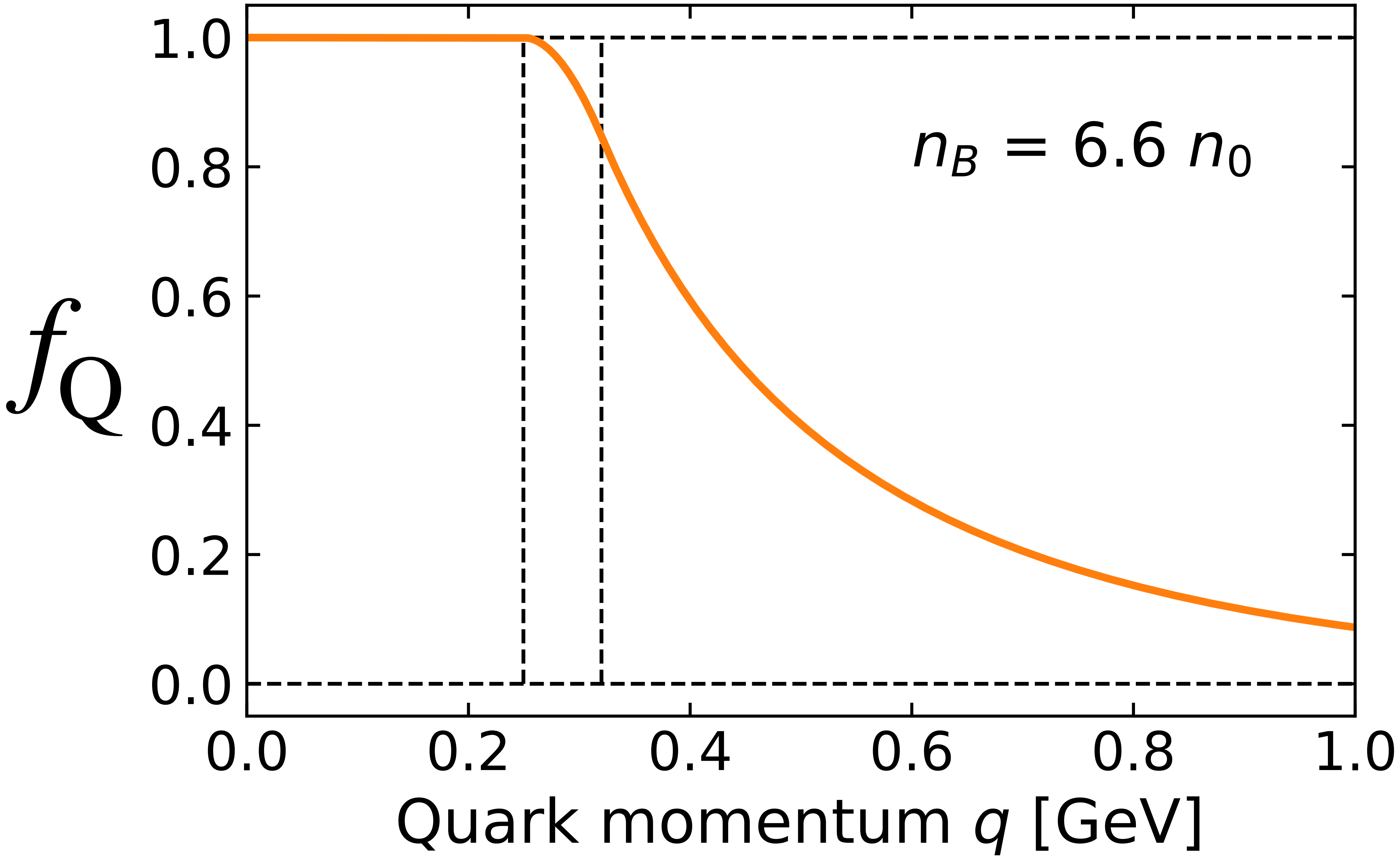
Fujimoto,Kojo,McLerran, PRL 132 (2023)

At sufficiently high density…

Fermi-Dirac distribution for baryons is modified



Quark obeys the FD distribution (with a tail from confinement)



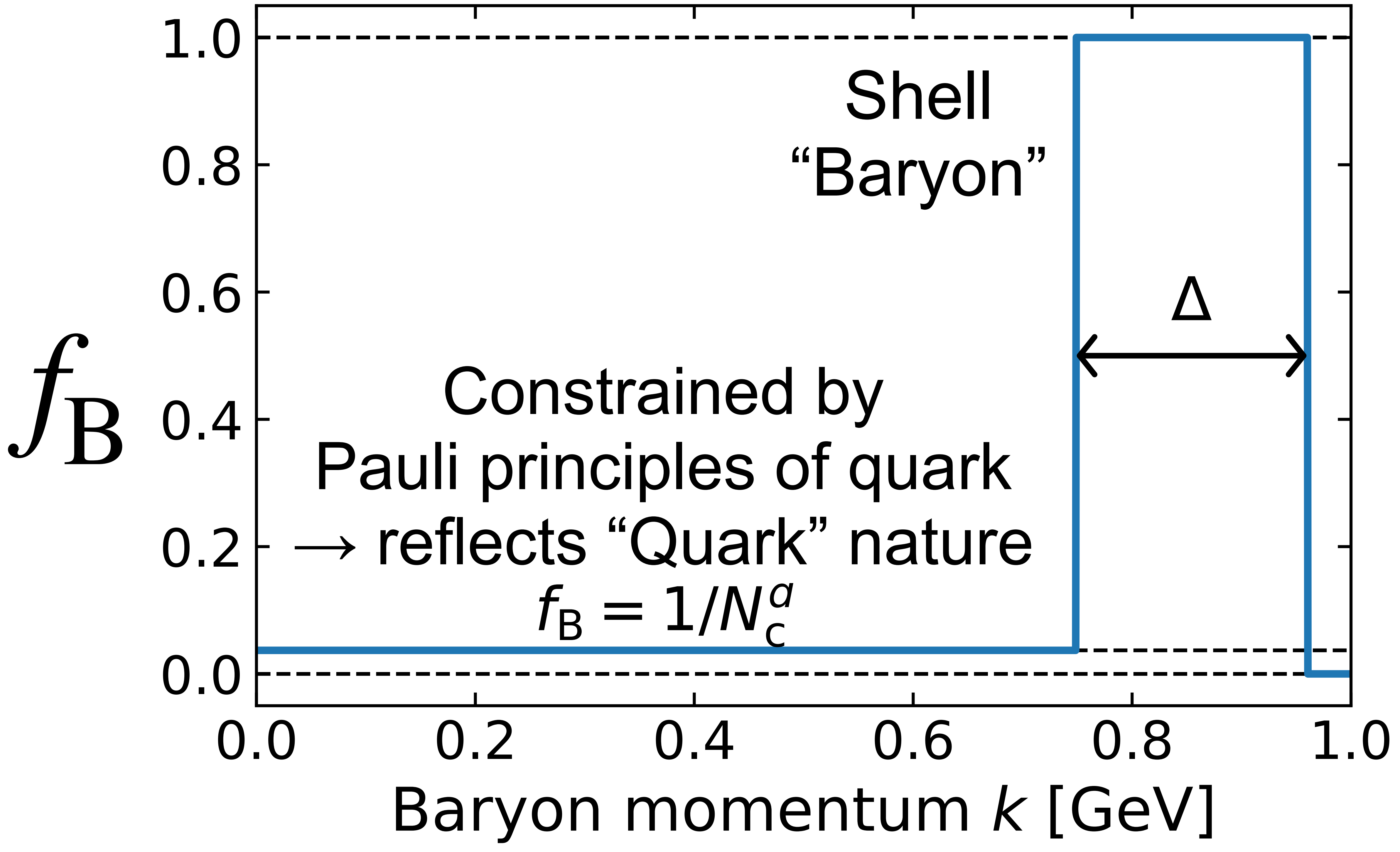
… characteristic feature of Quarkyonic matter

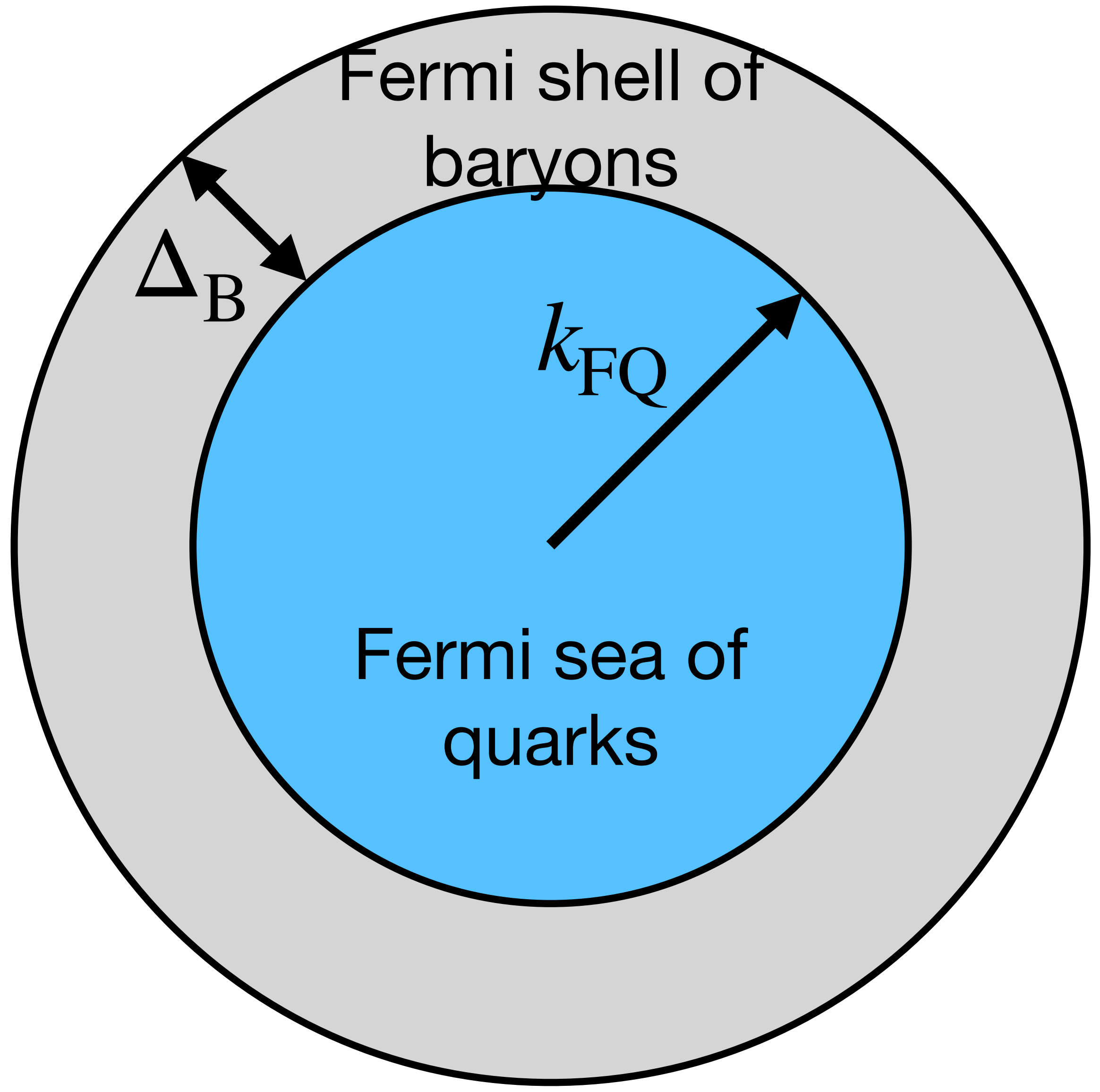
Reinterpretation of Quarkyonic shell structure

Fujimoto,Kojo,McLerran, PRL 132 (2023)

At sufficiently high density…

Fermi-Dirac distribution for baryons is modified





McLerran,Pisarski (2007);
McLerran,Reddy (2018);
Jeong,McLerran,Sen (2019);
many other works

Fermi shell structure emerges in f_B

Note: our picture is
purely baryonic description

This f_B leads to the same EoS based on the McLerran-Pisarski shell picture

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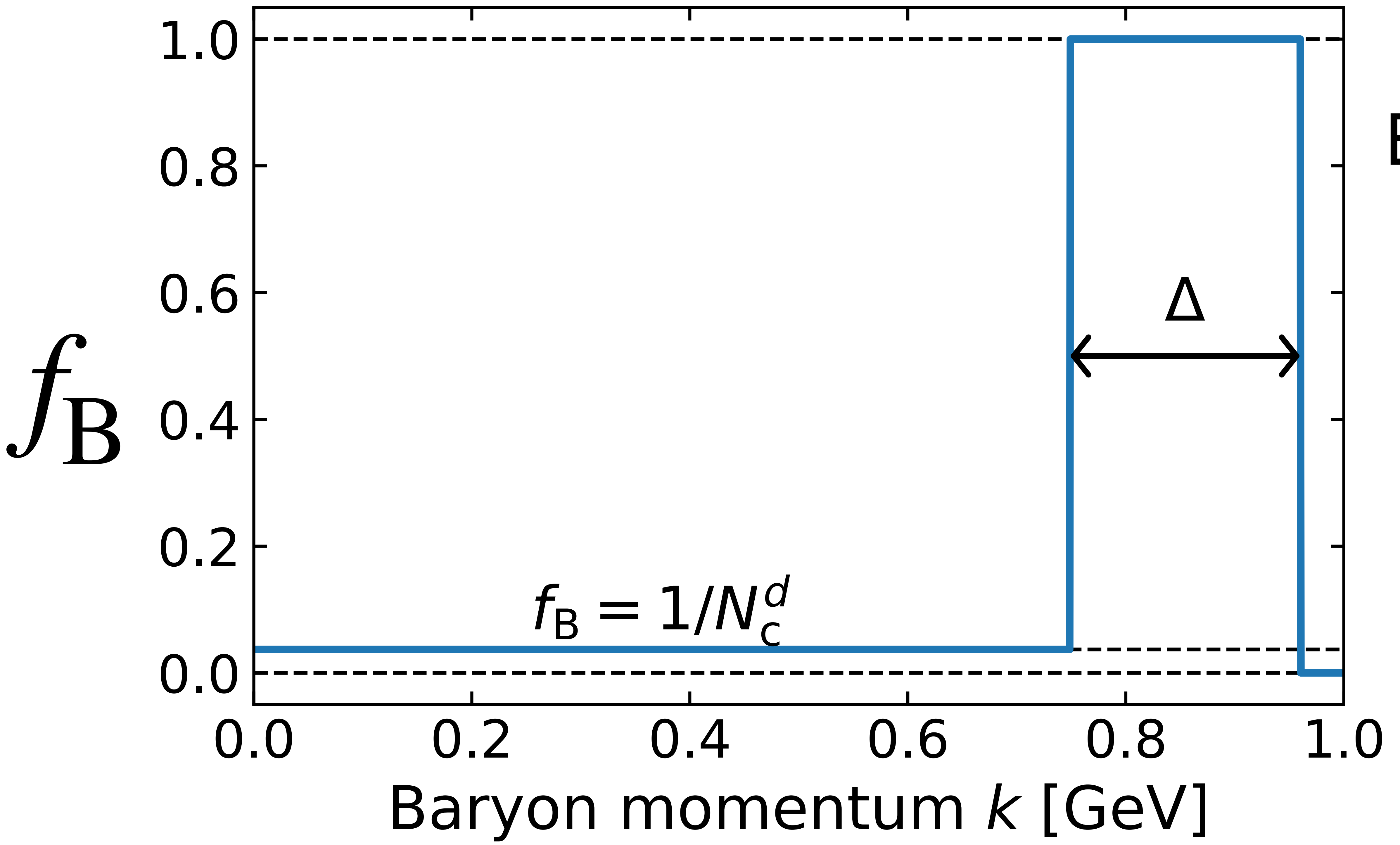
3. Two-flavor color superconductor and its symmetry

Problem of naive finite-T extension

Bluhm, Fujimoto, McLerran, Nahrgang, in preparation (2024?)



Entropy-density expression for ideal Fermi gas:

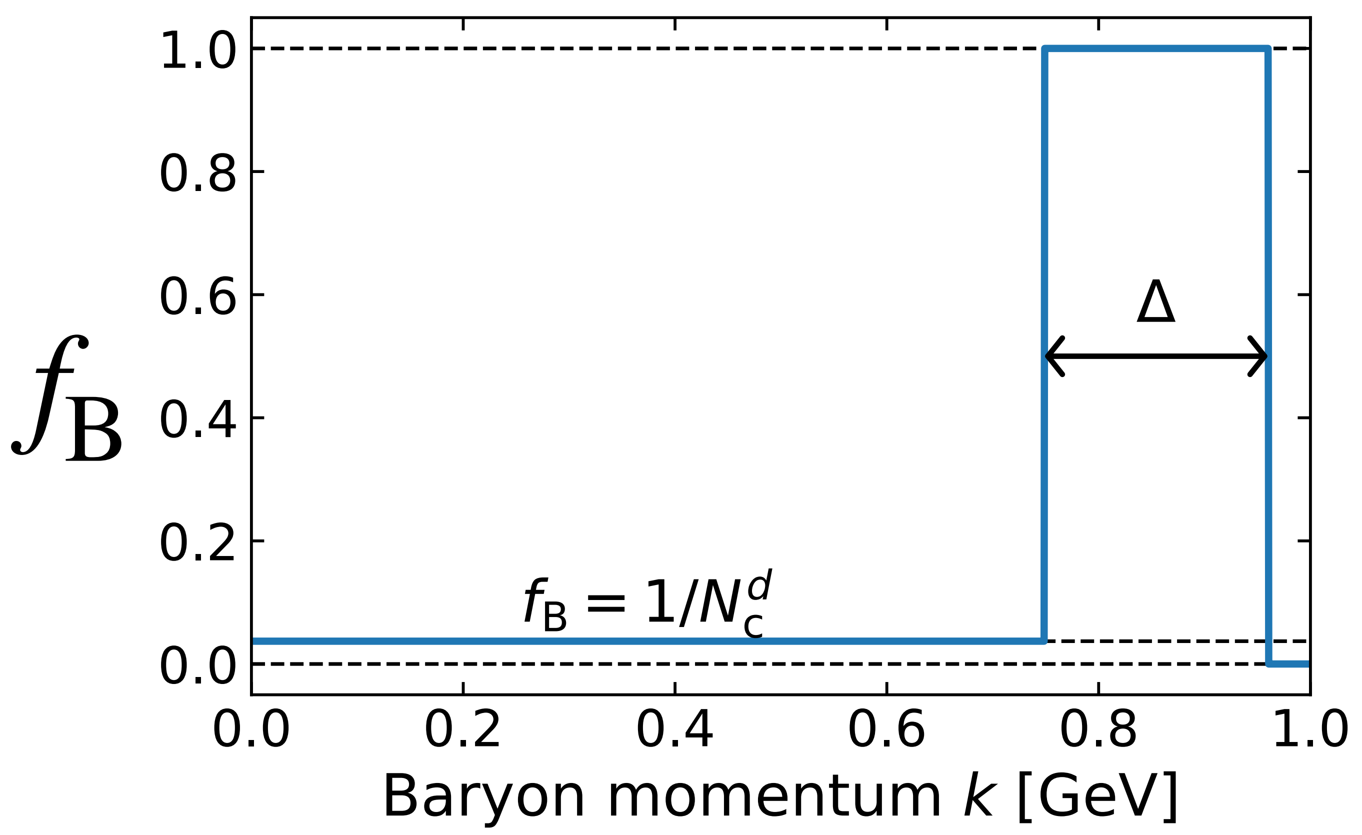
$$s = -\int_k \left[f \ln f + (1-f)\ln(1-f) \right]$$

→ for $f = f_B$, $s \neq 0$ even at $T = 0$!!

Problem: Entropy has to be zero at T = 0, but it is nonzero

Resolution: statistical mechanical approach

Bluhm, Fujimoto, McLerran, Nahrgang, in preparation (2024?)



Consider the following picture:

$$f_B(k) = g(k)\ n_{FD}(k)$$

Counts available states in $d^3\boldsymbol{k}$

$$\begin{cases} 1/N_c^3 & (k < k_{bu}) \\ 1 & (k > k_{bu}) \end{cases}$$

Fermi-Dirac distribution (step function)

Then, entropy density becomes

$$s = -\int_k g(k)\left[n_{FD} \ln n_{FD} + (1 - n_{FD})\ln(1 - n_{FD})\right]$$

$= 0$ at $T = 0$!!

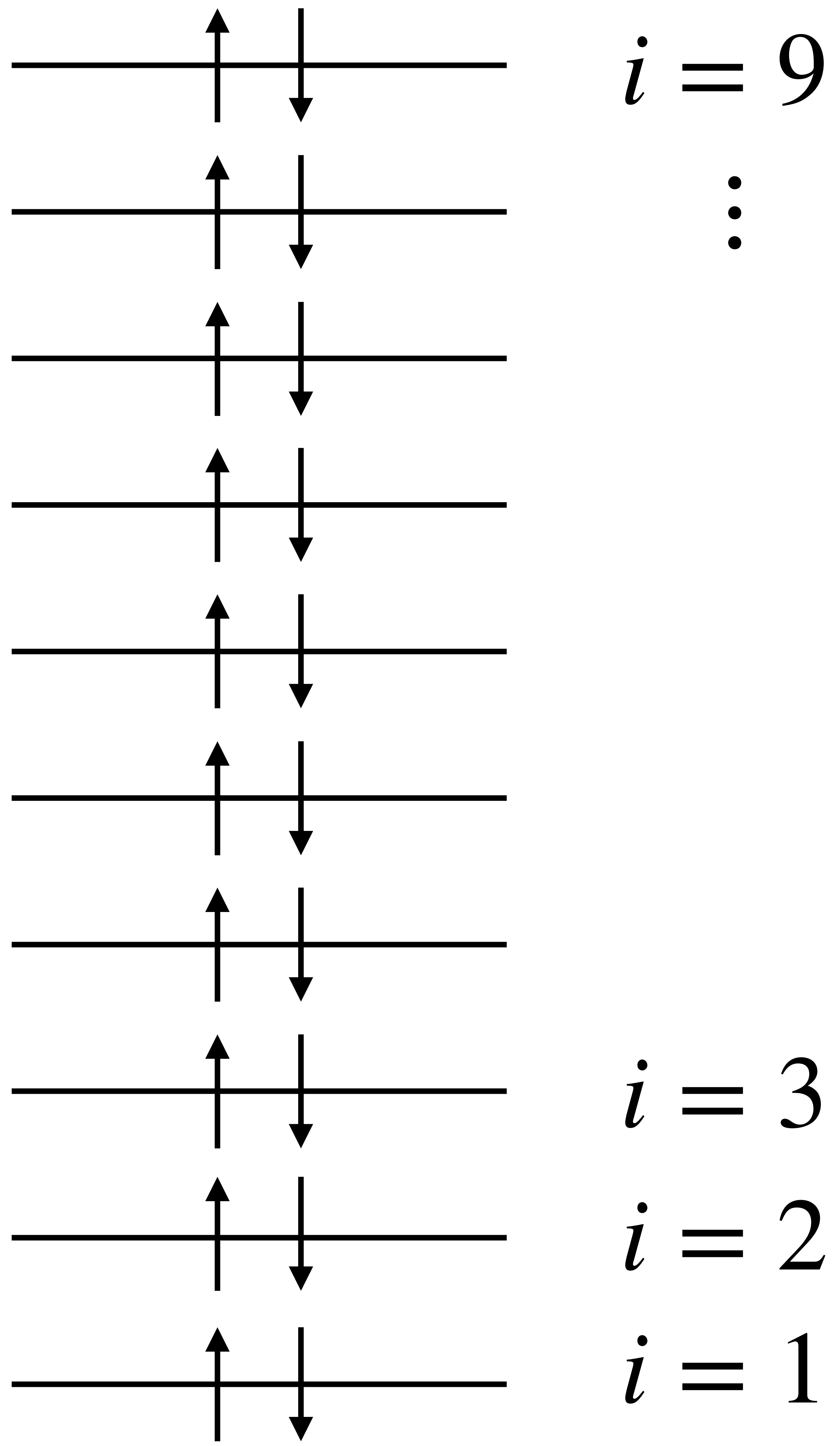
Resolution: statistical mechanical approach

Bluhm, Fujimoto, McLerran, Nahrgang, in preparation (2024?)

$$f_B(k) = g(k)\ n_{FD}(k)$$

…This expression naturally arises in statistical mechanics treatment

Consider baryon gas with
- quantum states $i = 1,2,\ldots$
- whose energy is $E_i = \sqrt{k_i^2 + M_N^2}$ and
- occupation number is $n_i = 0$ or 1



$$E = \sum_i E_i n_i$$

$$N = \sum_i n_i$$

Resolution: statistical mechanical approach

Bluhm, Fujimoto, McLerran, Nahrgang, in preparation (2024?)

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- quantum states $i = 1,2,\ldots$
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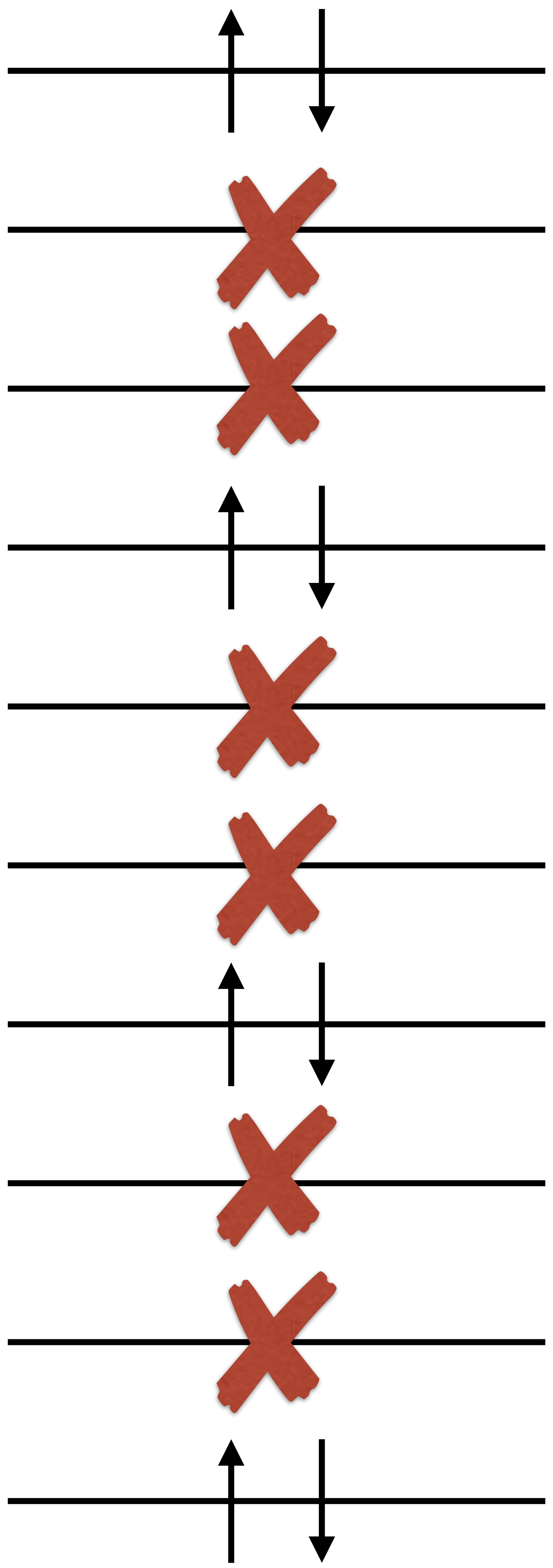
Suppose some states are forbidden by the external conditions:

in this case, the condition $f_Q \leq 1$ forbids some states to be occupied

$\rightarrow$ specify such information by factor g_i



$i = 9,\ g_9 = 1$

$\vdots$

$$E = \sum_i g_i E_i n_i$$

$$N = \sum_i g_i n_i$$

$i = 3,\ g_3 = 0$

$i = 2,\ g_2 = 0$

$i = 1,\ g_1 = 1$

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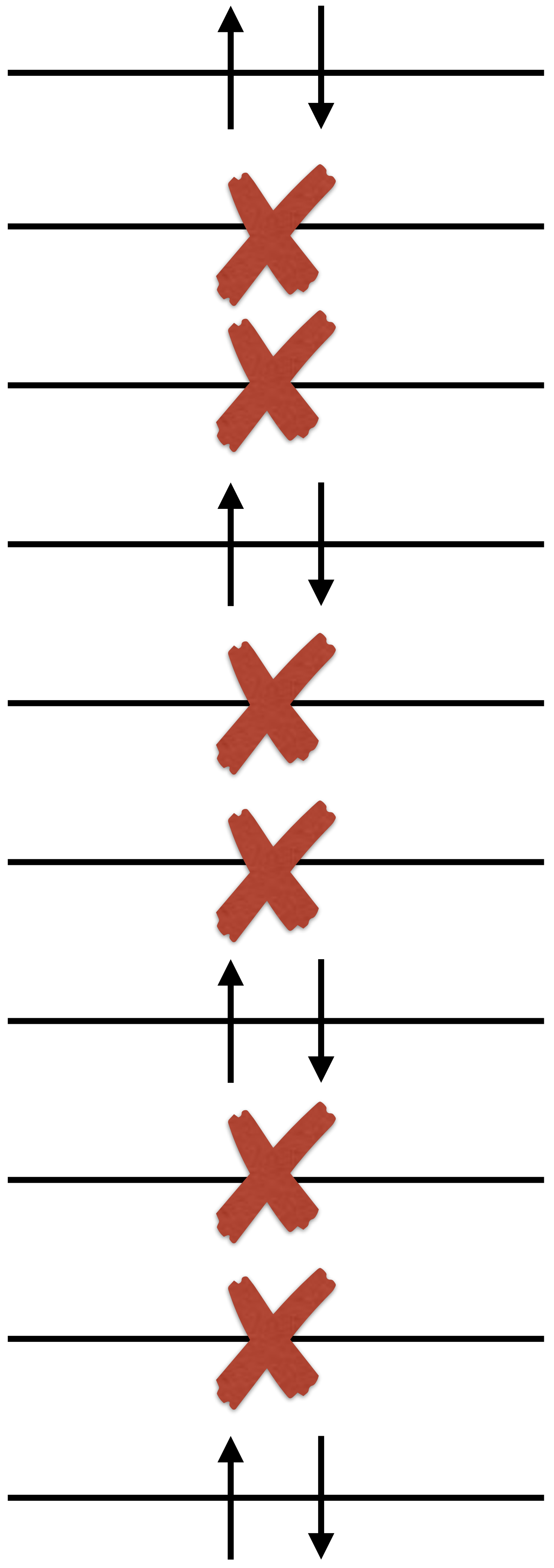
One can construct GC partition function:

$$\Xi = \sum_{N=0}^{\infty} e^{\hat{\beta}\hat{\mu}N} Z_N,$$

where $Z_N = \sum_{\{n_i\}}{}' \exp\left[-\hat{\beta}\sum_i g_i E_i n_i\right]$

Then, in thermodynamic limit $V \to \infty$,

$$\frac{k^2}{2\pi^2} g(k)\delta k = \sum_{k<k_i<k+\delta k} g_i\ , \qquad \frac{N}{V} = \int_k \underbrace{g(k) n_{\mathrm{FD}}(k)}_{f_{\mathrm{B}}(k)}$$



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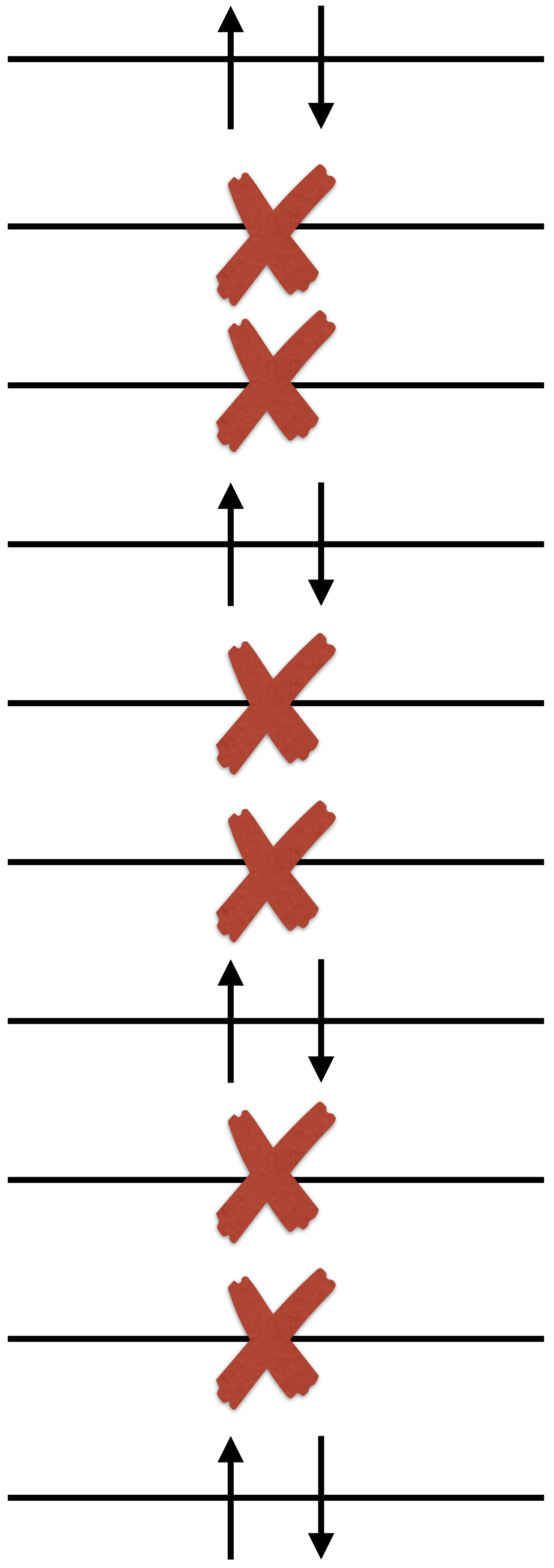

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Modification in physical T and μ

Bluhm,Fujimoto,McLerran,Nahrgang, in preparation

- **Interesting upshot:** physical temperature T and chemical potential μ becomes different from these appear in the partition function $\hat{T}$ and $\hat{\mu}$

$$\Xi = \sum_{N=0}^{\infty} e^{\hat{\beta}\hat{\mu}N} Z_N, \quad \text{where } Z_N = \sum_{\{n_i\}}{}' \exp\left[-\hat{\beta}\sum_i g_i E_i n_i\right]$$

- $\mu = \dfrac{\partial E}{\partial N} \neq \hat{\mu}, \quad \beta = \dfrac{\partial S}{\partial E} \neq \hat{\beta}$: This encodes the effect of confinement

… possible solution to hyperon puzzle?

Fujimoto,Kojo,McLerran (2024)

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Famous examples of duality in dense QCD

Essence of Quarkyonic duality:
confinement persists in the regime where quarks are natural d.o.f.

Rischke, Son & Stephanov (2001): **Two-flavor color superconductor (2SC)**

- Color superconductor "breaks" the gauge redundancy: $SU(3)_c \to SU(2)_c$
- Quarks are gapped by Δ
- No Debye/Meissner screening for $SU(2)$ gauge bosons at energy scale below Δ
  $\to$ only pure $SU(2)$ gluodynamics, which is confining!

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- **Additional examples:**
  * QCD at finite isospin density and zero baryon density [Son&Stephanov (2000)]
  * SU(2) QCD at finite baryon density [E.g. Iida,Itou,Murakami,Suenaga (2024)]

These examples shows the duality between confined hadrons and quarks
$\to$ Quarkyonic matter can persist even at $N_c = 2, 3$

Two-flavor color-superconducting (2SC) phase

- **Diquark condensate:** $\langle \hat{q}^{\top}_{\alpha A} C\gamma^5 \hat{q}_{\beta B} \rangle = \epsilon_{\alpha\beta\gamma}\epsilon_{AB} \langle \hat{\Phi}^{\gamma}_{\rm 2SC} \rangle$

- **2SC Ansatz** $\langle \hat{\Phi}^{\alpha}_{\rm 2SC} \rangle = \underline{\delta^{\alpha 3}\Delta_{\rm 2SC}}$

  (assuming unitary gauge fixing)

- **Symmetry breaking pattern**

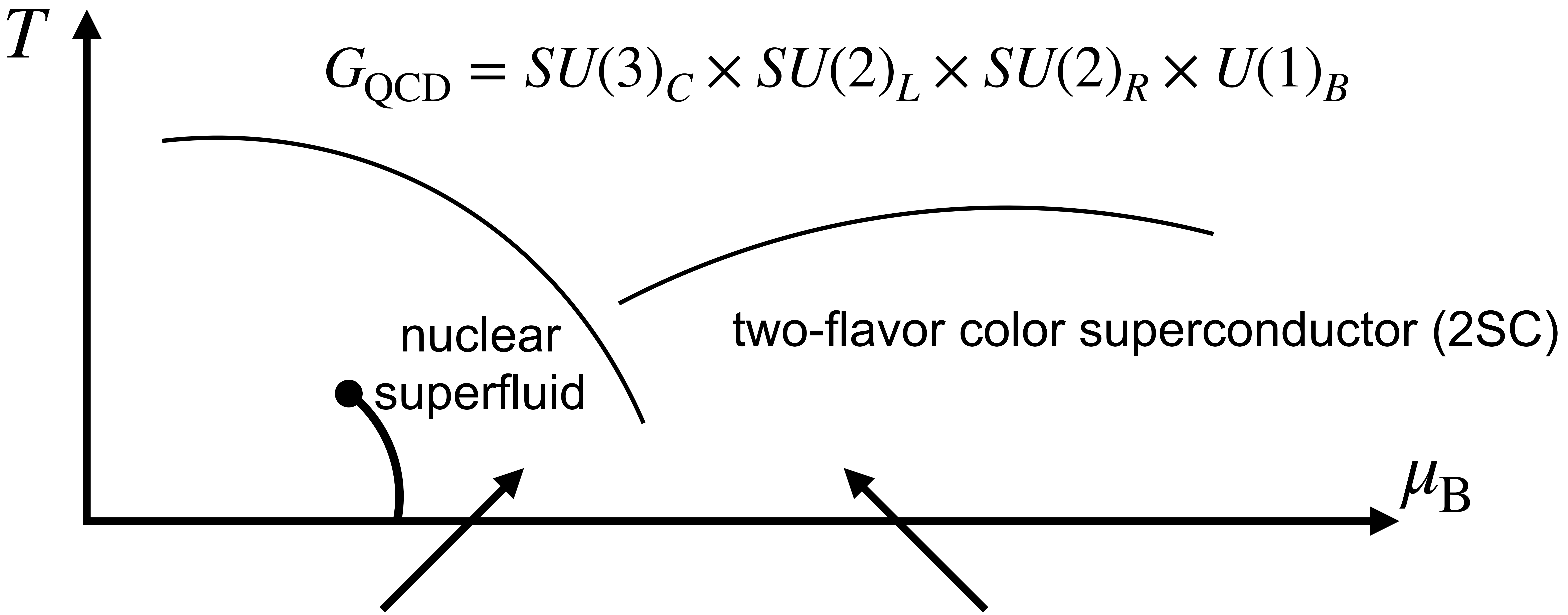
$$G_{\rm QCD} = {\rm SU}(3)_{\rm C} \times {\rm SU}(2)_{\rm L} \times {\rm SU}(2)_{\rm R} \times {\rm U}(1)_{\rm B}$$

$$\to\ G_{\rm 2SC} = {\rm SU}(2)_{\rm C} \times {\rm SU}(2)_{\rm L} \times {\rm SU}(2)_{\rm R} \times {\rm U}(1)_{\rm B}$$

… chiral symmetric & baryon number symmetry intact

Symmetries in QCD phase diagram

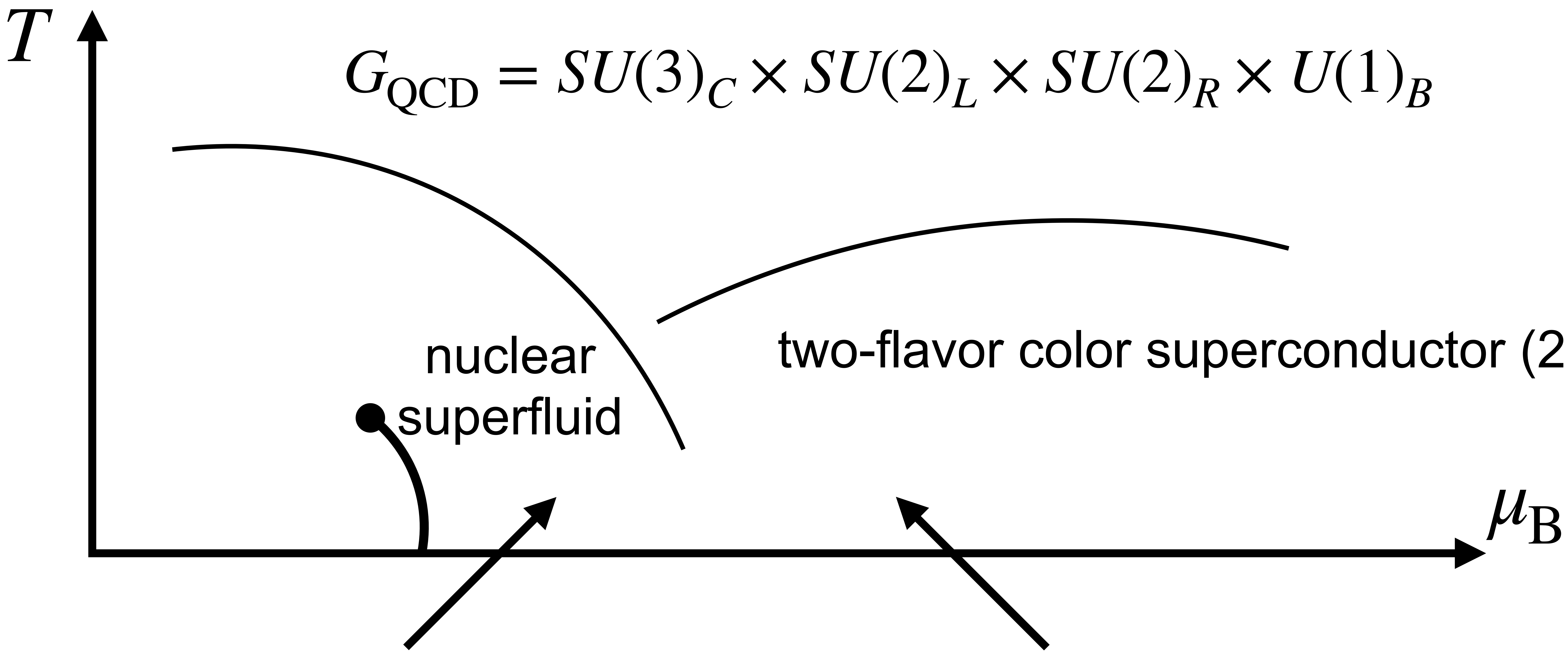
$$G_{\mathrm{QCD}} = SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_B$$



$$G_{\mathrm{nucl}} = SU(3)_C \times SU(2)_V$$

$$G_{\mathrm{2SC}} = SU(2)_C \times SU(2)_L \times SU(2)_R \times U(1)_B$$

Symmetries in QCD phase diagram



$G_{\mathrm{nucl}} = SU(3)_C \times SU(2)_V \longrightarrow G_{\mathrm{2SC}} = SU(2)_C \times SU(2)_L \times SU(2)_R \times U(1)_B$

chiral restoration

+ $U(1)_B$ restoration??

… additional complication in high density

Order parameters in color superconductor

Rajagopal, Wilczek (2000)

- Order parameters in color superconductor can be expressed in terms of gauge-invariant combination of diquark operators

- **Chiral order parameter for $SU(2)_L \times SU(2)_R$:**

$$\mathscr{M} = \delta^{\beta}_{\alpha}\delta^{\beta'}_{\alpha'}(\bar{q}^{\alpha}\bar{q}^{\alpha'})(q_{\beta}q_{\beta'}) \propto (\bar{q}^{\alpha}q_{\alpha})(\bar{q}^{\alpha'}q_{\alpha'})$$

- **Superfluid order parameter for $U(1)_B$:**

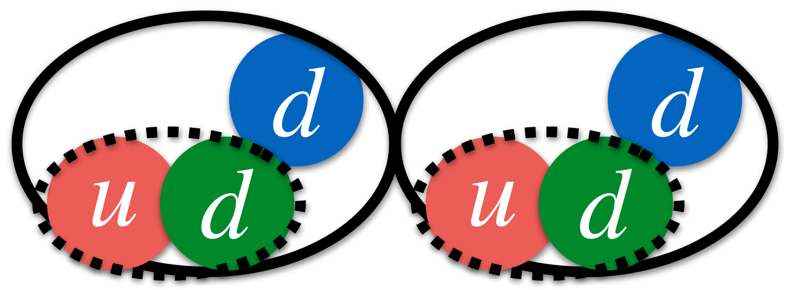
$$\Upsilon = \epsilon^{\alpha\beta\gamma}\epsilon^{\alpha'\beta'\gamma'}(q_{\alpha}q_{\alpha'})(q_{\beta}q_{\beta'})(q_{\gamma}q_{\gamma'}) \propto \underline{(\epsilon^{\alpha\beta\gamma}q_{\alpha}q_{\beta}q_{\gamma})(\epsilon^{\alpha'\beta'\gamma'}q_{\alpha'}q_{\beta'}q_{\gamma'})}$$

Can be matched with Cooper pairs in neutron matter

Additional pairing in 2SC phase

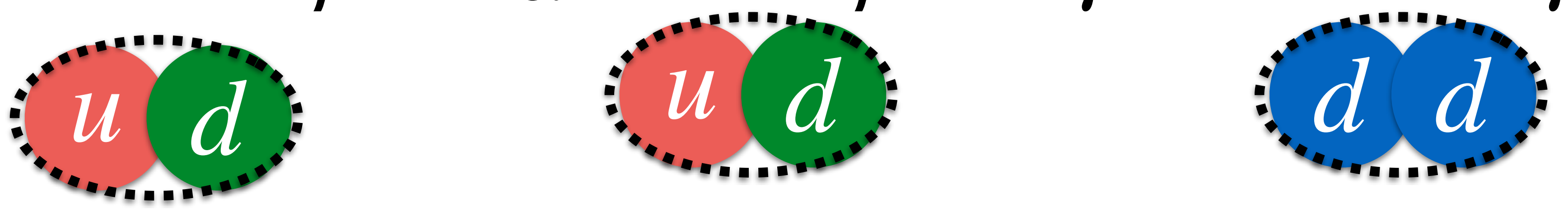
Fujimoto, Fukushima, Weise (2019)

- **Neutron superfluid OP:** $\Upsilon_{nn} \equiv n^\top C\gamma^i \nabla^j n$



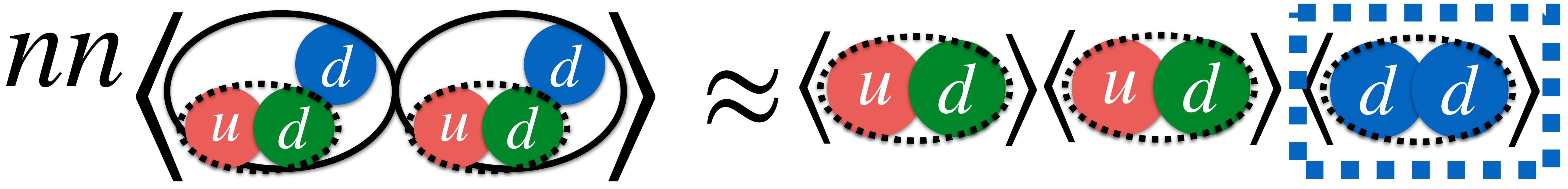
- **Rearranging quark fields:**

$$\Upsilon_{nn} \longrightarrow \epsilon^{\alpha\beta\gamma}\epsilon^{\alpha'\beta'\gamma'}(u_\alpha^\top C\gamma^5 d_\beta)(u_{\alpha'}^\top C\gamma^5 d_{\beta'})(d_\gamma^\top C\gamma^i \nabla^j d_{\gamma'})$$



- **Taking expectation value** (under mean-field approx.):

$$\langle \Upsilon_{nn} \rangle \approx \Phi_{2\mathrm{SC}}^{\alpha} \Phi_{2\mathrm{SC}}^{\alpha'} \langle d_\alpha^\top C\gamma^i \nabla^j d_{\alpha'} \rangle$$



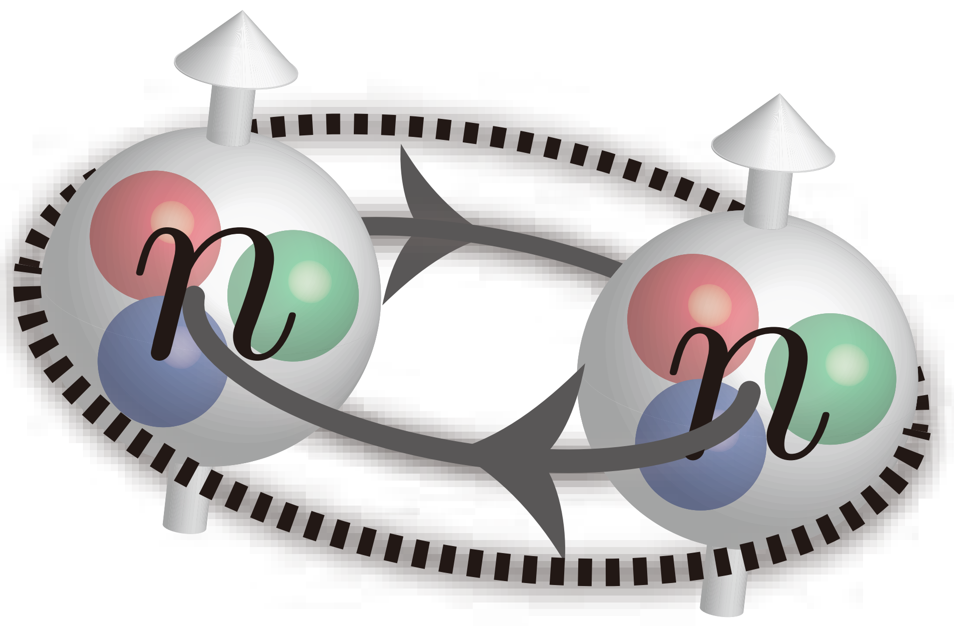
$2\mathrm{SC} + \langle dd \rangle$

Additional pairing breaks $U(1)_B$

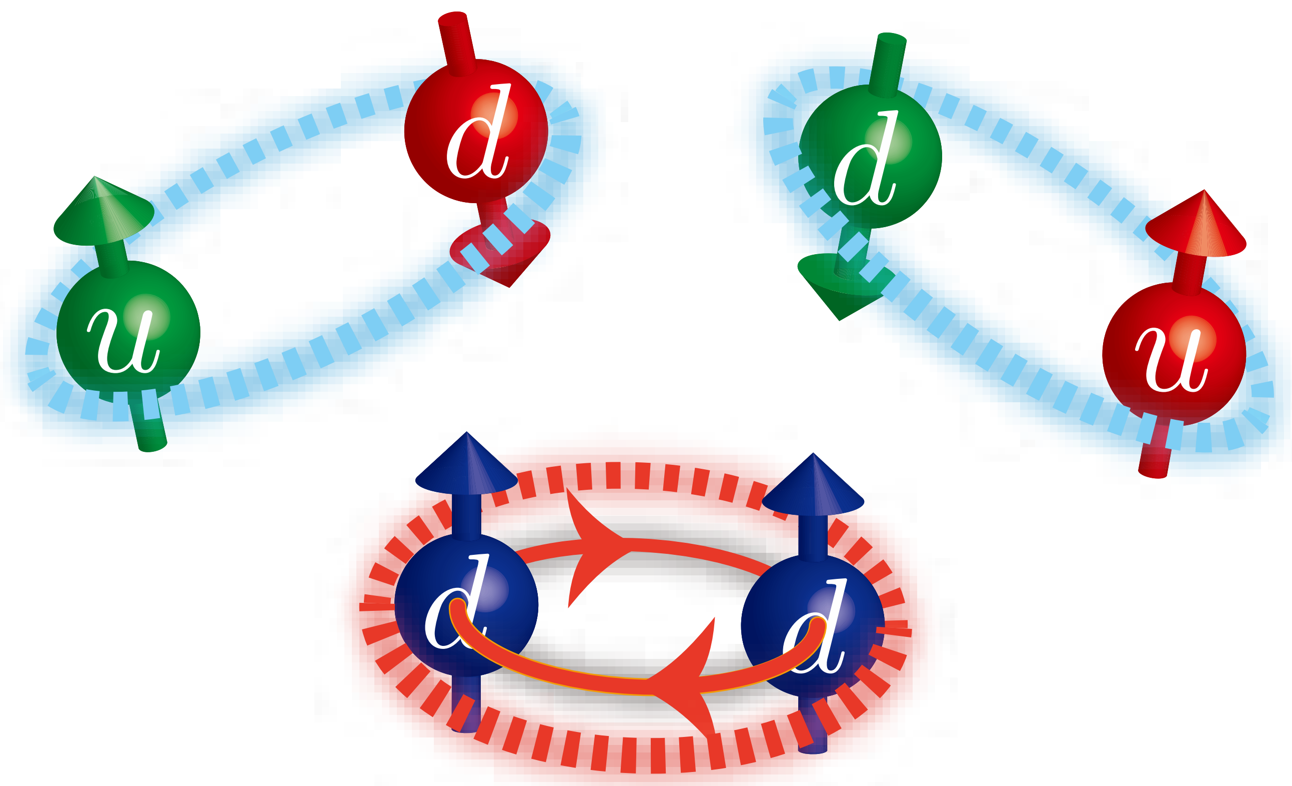
Additional pairing in 2SC phase

Fujimoto,Fukushima,Weise, arXiv:1908.09360:

Nuclear matter



2SC quark matter



$U(1)_B$ broken in both phases ⟷

$\sim n_0\,(=0.16\,\mathrm{fm}^{-3})$ $\sim 10\,n_0$ → n_B

2SC+<dd> phase

Fujimoto,Fukushima,Weise (2019)

- We have **2SC+<dd>**:

$$\langle ud \rangle \;\; + \;\; \langle \hat{d}^{\top} C \gamma^{i} \nabla^{j} \hat{d} \rangle$$

… <**dd**> breaks $\mathrm{U(1)_B}$

→ **matter is superfluid, the same as hadronic phase**

- $\langle \hat{d}_{\alpha}^{\top} C \gamma^{i} \nabla^{j} \hat{d}_{\beta} \rangle$ → Color should be **symmetric** (in $\mathbf{6}$ channel)

2SC+<dd> phase

Fujimoto,Fukushima,Weise (2019)

- We have **2SC+<dd>**:

$$\langle ud \rangle \;+\; \langle \hat{d}^{\top} C \gamma^{i} \nabla^{j} \hat{d} \rangle$$

… <**dd**> breaks $\mathrm{U}(1)_{\mathrm{B}}$

$\rightarrow$ **matter is superfluid, the same as hadronic phase**

- $\langle \hat{d}_{\alpha}^{\top} C \gamma^{i} \nabla^{j} \hat{d}_{\beta} \rangle$ $\rightarrow$ Color should be **symmetric** (in $\mathbf{6}$ channel)

OGE interaction between dd is **repulsive**

$\rightarrow$ **No Cooper pair formation?**

Kohn-Luttinger superconductivity

- Cooper pair formation: Attractive interaction necessary

- Kohn-Luttinger mechanism: Even when bare s-wave interaction is repulsive, induced interaction in higher partial wave l can be attractive

Kohn,Luttinger (1965)

- KL mechanism based on perturbation theory (1-loop):

Kohn,Luttinger (1965)

$$\Delta \sim \epsilon_{\mathrm{F}} \exp\left(-\#\, l^4\right)$$

- From the reanalysis using RG, it turns out:

Fujimoto (2025)

$$\Delta \sim \epsilon_{\mathrm{F}} \exp\left(-\#\, l\right)$$

KL effect at l = 1: perturbation theory vs RG

- Perturbation theory (1-loop):

  $$\Delta \sim \epsilon_{\rm F} \exp\left(-\#\, l^4\right)$$ Kohn,Luttinger (1965)

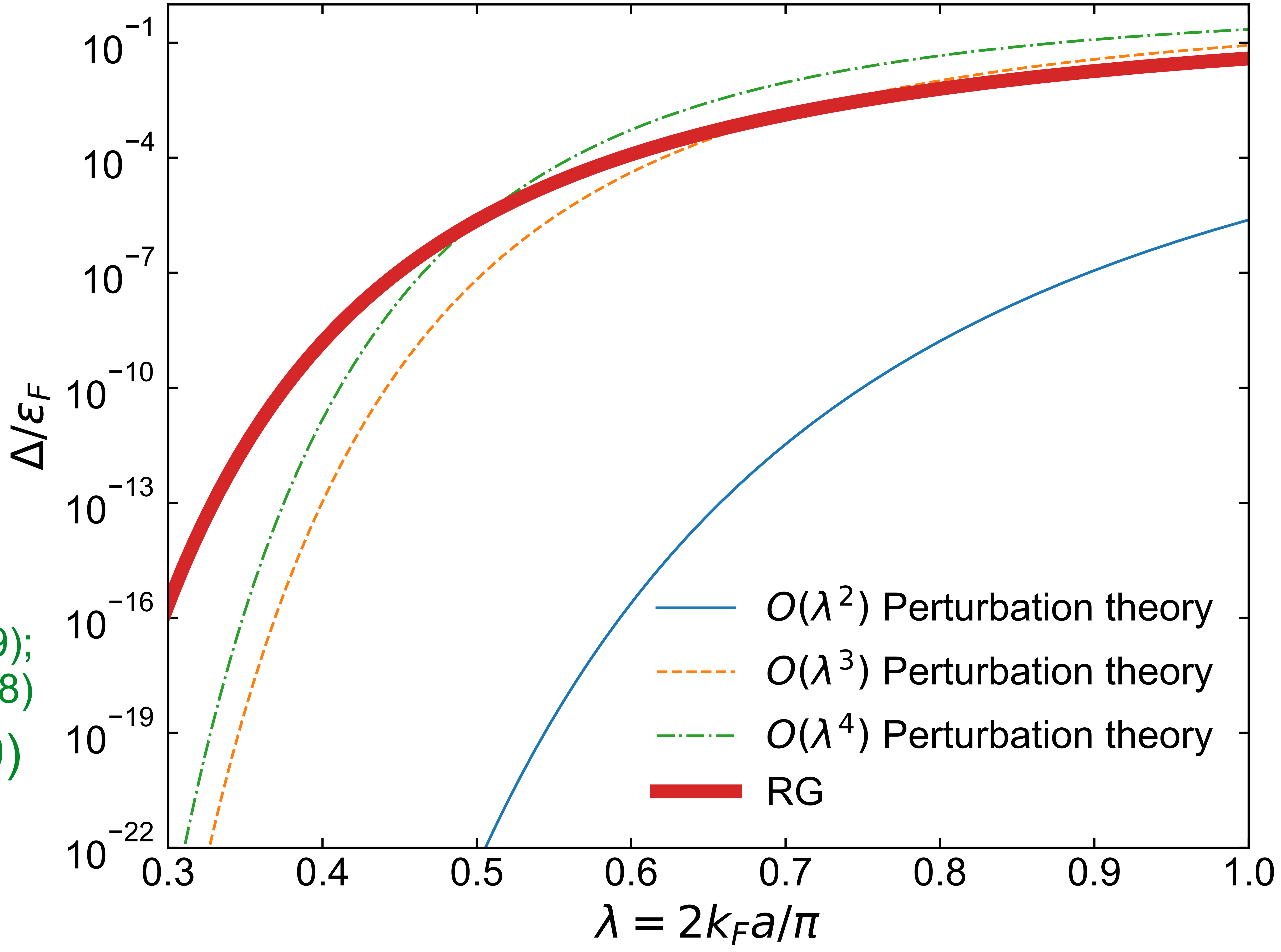
- RG:

  $$\Delta \sim \epsilon_{\rm F} \exp\left(-\#\, l\right)$$ Fujimoto (2025)

- Consider example at $l = 1$ perturbation theory: converges poorly, 1-loop deviates from 2 & 3-loop results

Fay,Layzer (1969); Kagan,Chubukov (1988)

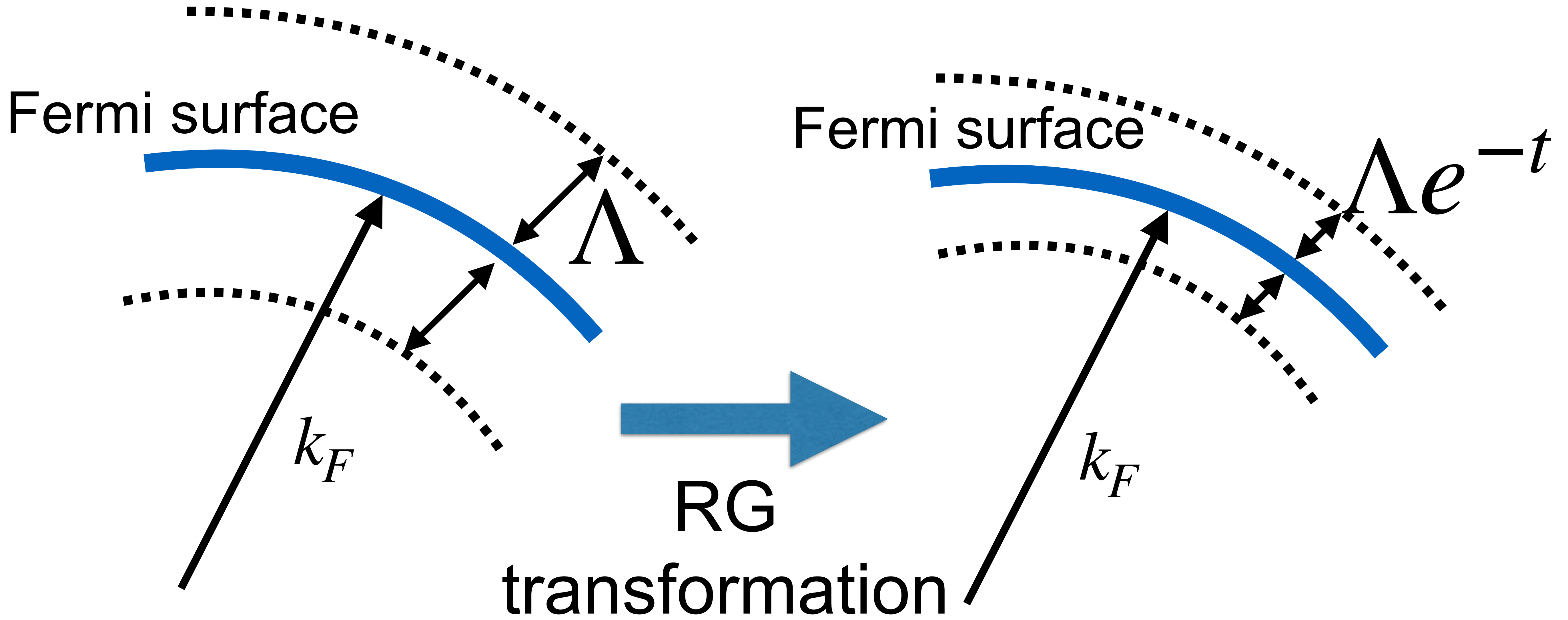
Efremov et al. (2000)



Subset of the subleading contributions has divergent integrand
→ it has to be resummed. Resummation done by RG

RG approach to BCS instability

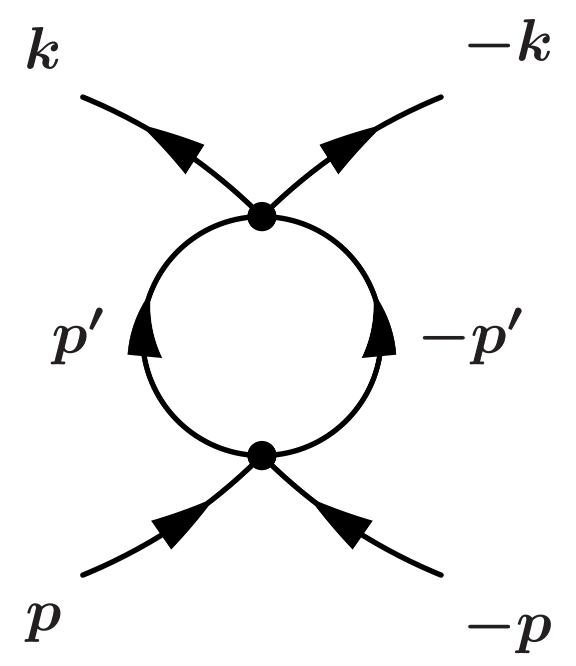
RG near the Fermi surface:



Benfatto,Gallavotti (1990); Polchinski (1992); Shankar (1993)…

RG equation:

$$\frac{dV_l(t)}{dt} = -V_l^2(t)$$



Solution: $V_l(t) = \dfrac{V_l(t=0)}{1 + V_l(t=0)\ln t}$

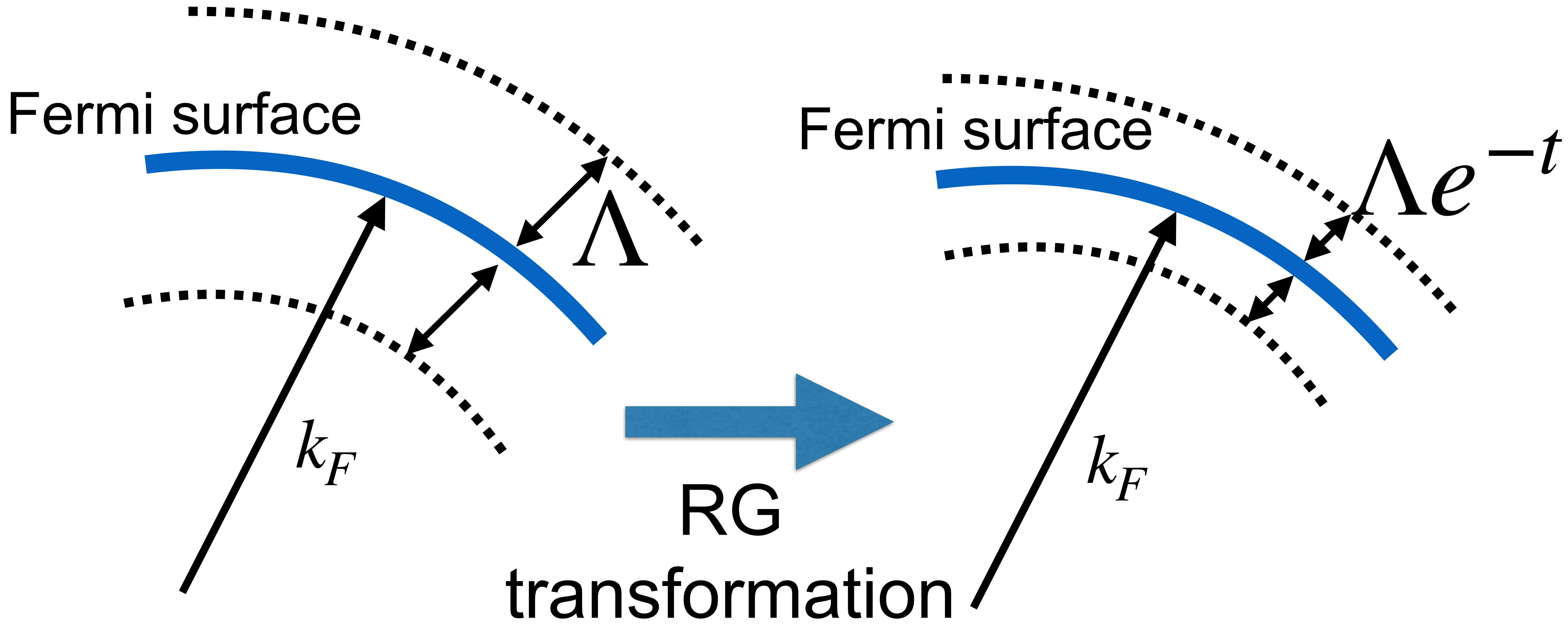
… singular at $\ln t = -1/V_l(0)$

when $V_l(0) < 0$ (BCS instability)

$\Delta \sim \Lambda$ at the singularity

RG approach to BCS instability

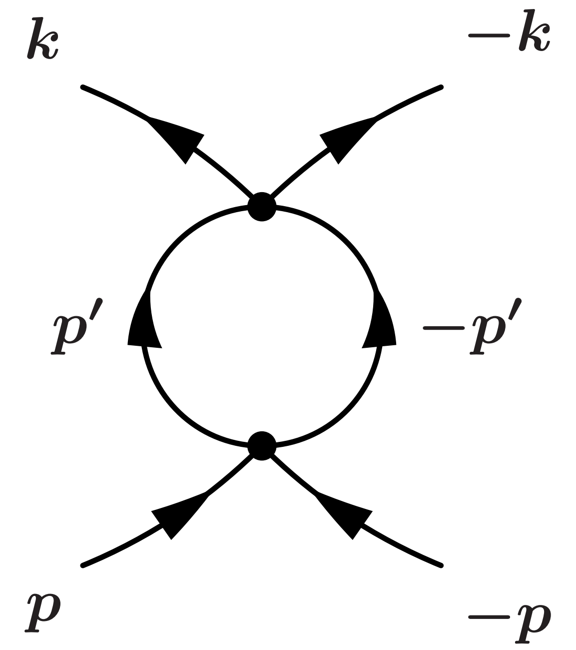
RG near the Fermi surface:



Benfatto,Gallavotti (1990); Polchinski (1992); Shankar (1993)…

RG equation:

$$\frac{dV_l(t)}{dt} = -V_l^2(t)$$



Solution: $V_l(t) = \dfrac{V_l(t=0)}{1 + V_l(t=0)\ln t}$

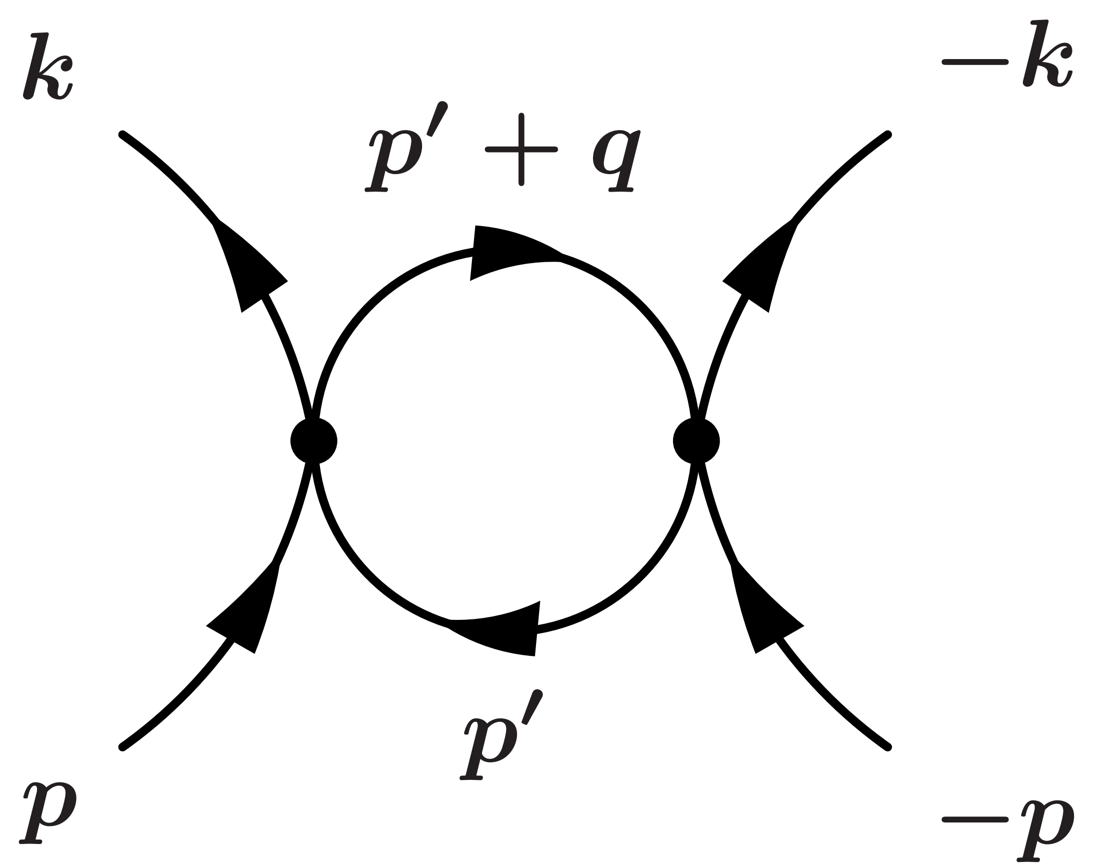
… singular at $\ln t = -1/V_l(0)$

when $V_l(0) < 0$ (BCS instability)

$\Delta \sim \Lambda$ at the singularity

Modified RG equation:

$$\frac{dV_l(t)}{dt} = -\frac{c}{2l+1}e^{-t} - V_l^2(t)$$



Inclusion of this term changes Δ as:

$$\Delta \sim \epsilon_{\mathrm{F}} \exp\left(-\# \, l\right)$$

Phenomenological relevance of this result

- Applies to Cooper pairing in quark matter

Fujimoto, work in progress

- **Kohn-Luttinger mechanism may give rise to the superconductivity in additional dd channel in the 2SC phase**

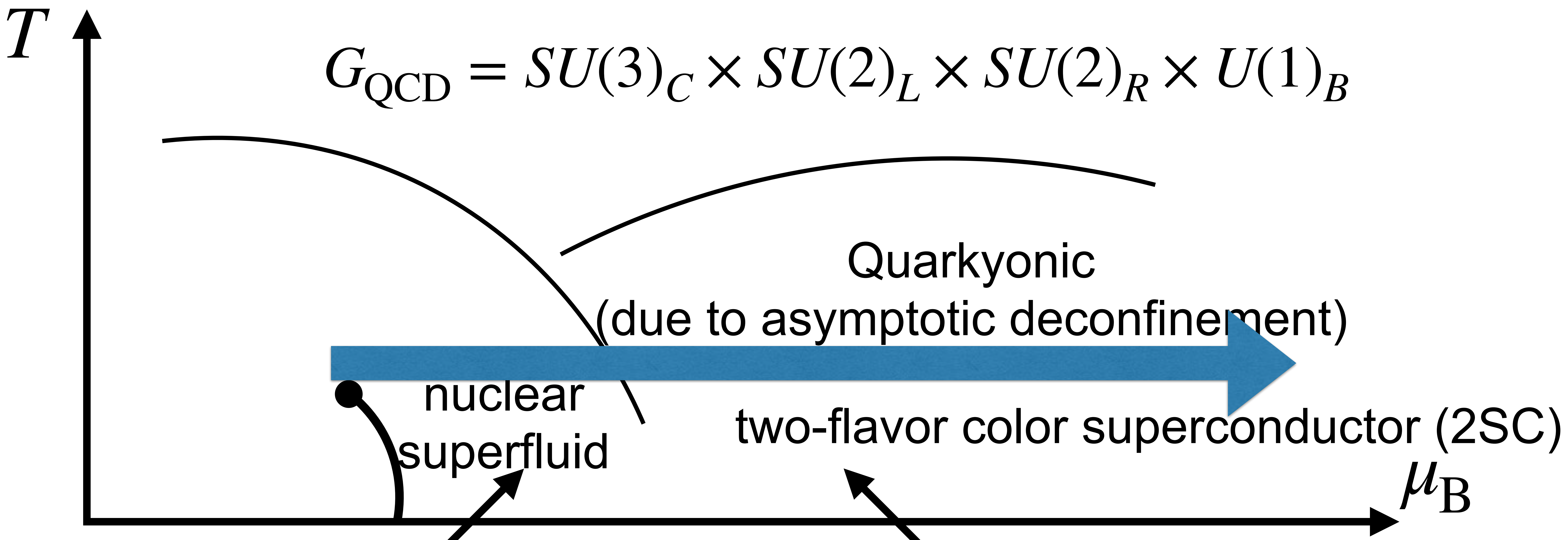
E.g. Baym,Hatsuda,Kojo,Powell,Song,Takatsuka (2017)

- The stiffening in the EoS (strong repulsive interaction) inevitably gives rise to large pairing gap in higher partial wave
  → **Bridges the EoS and transport property of neutron stars**
  Can be tested in multimessenger observation

See also: Kumamoto,Reddy (2024)

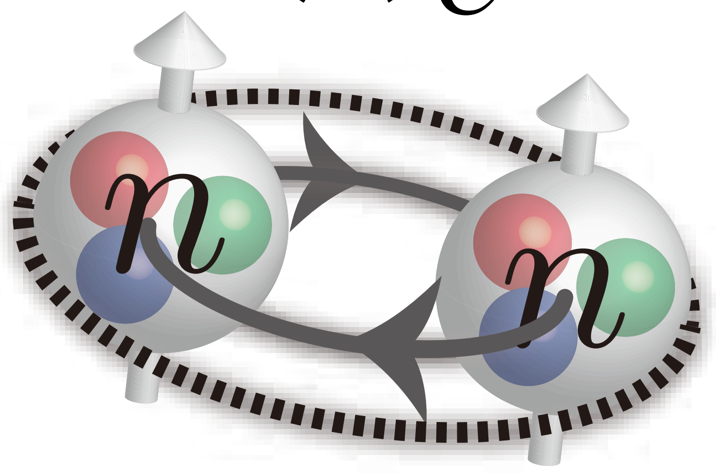
Summary

$$G_{\text{QCD}} = SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_B$$

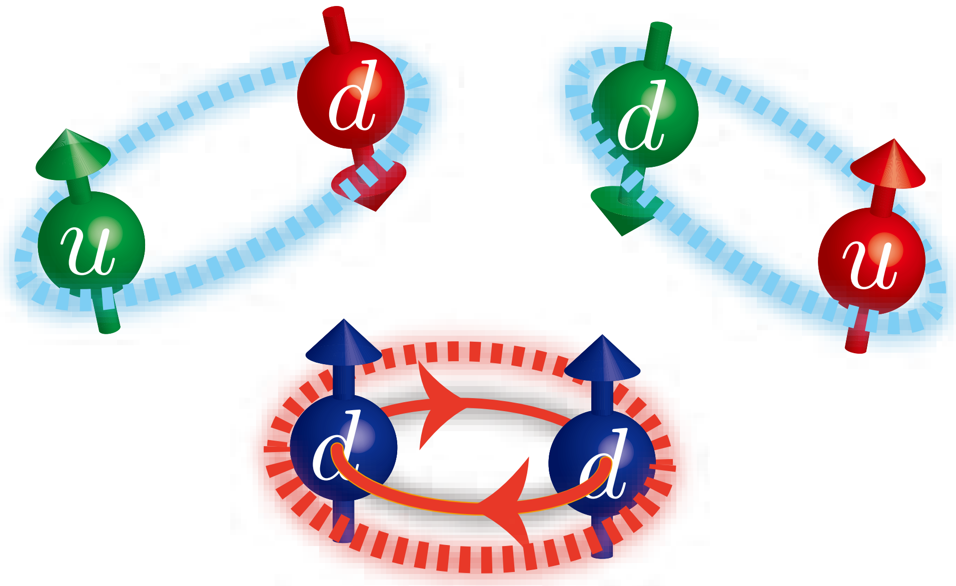


$$G_{\text{nucl}} = SU(3)_C \times SU(2)_V$$

$$G_{\text{2SC}} = SU(2)_C \times SU(2)_L \times SU(2)_R$$



superfluid neutrons



2SC + <dd> quark matter

Summary

- **Deconfinement at high baryon density:** may not be simple. Confinement persists up to high baryon density and the duality between baryons and quarks is implied **= Quarkyonic duality**

- **Quarkyonic matter:** As a result of the duality, the low-momentum part of baryon distribution is shown to be modified in a quite robust manner. I consider it as the defining property.

- **Statistical mechanics approach:** one can consider the gas of baryons and quark constraint as external conditions. Then, the quarkyonic property is recovered.

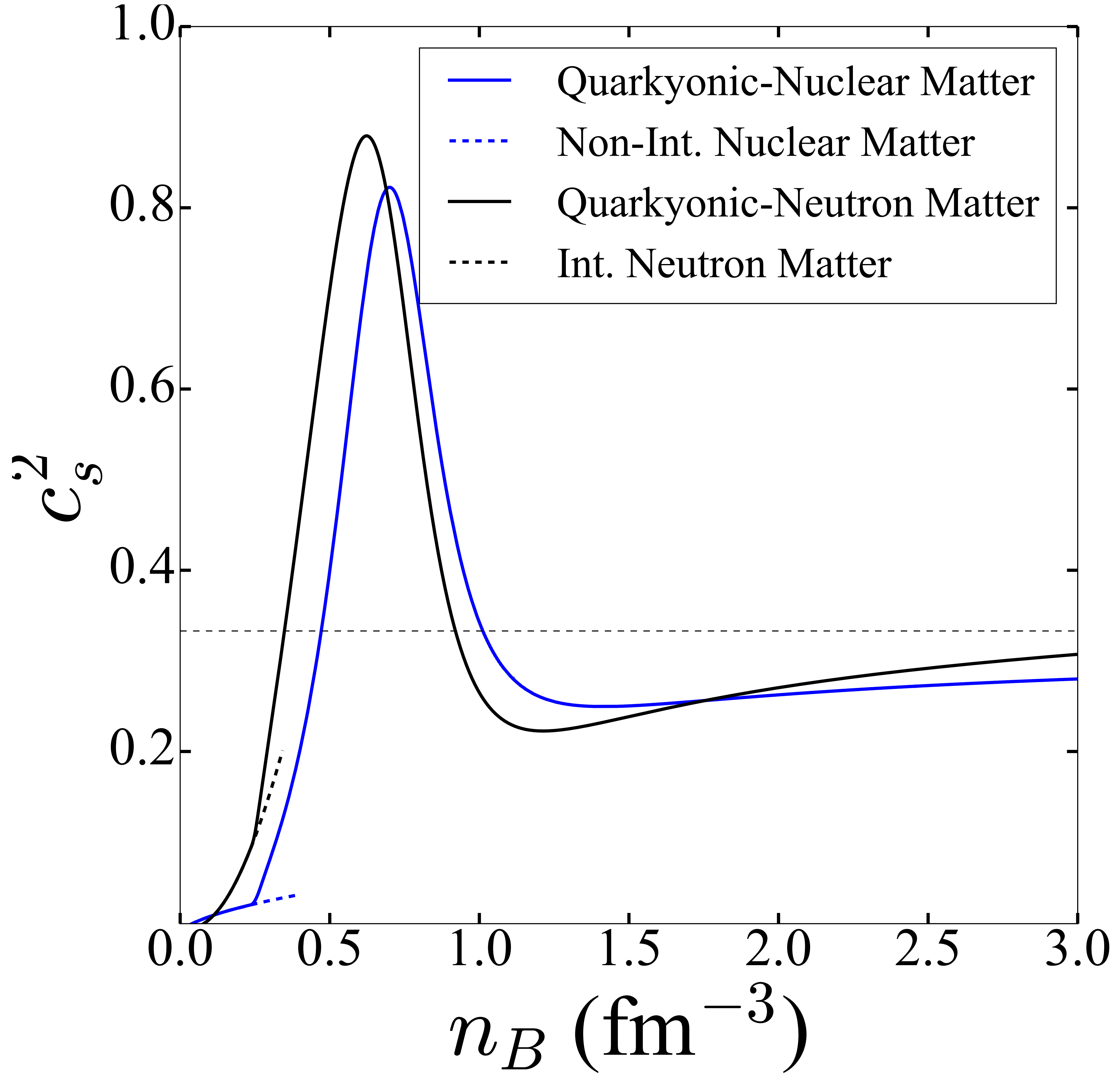
- **Two-flavor color superconductor:** Realizes quarkyonic duality, chirally symmetric matter

Bonus materials

EoS comparison: Quarkyonic model & Bayesian

Looks very similar…

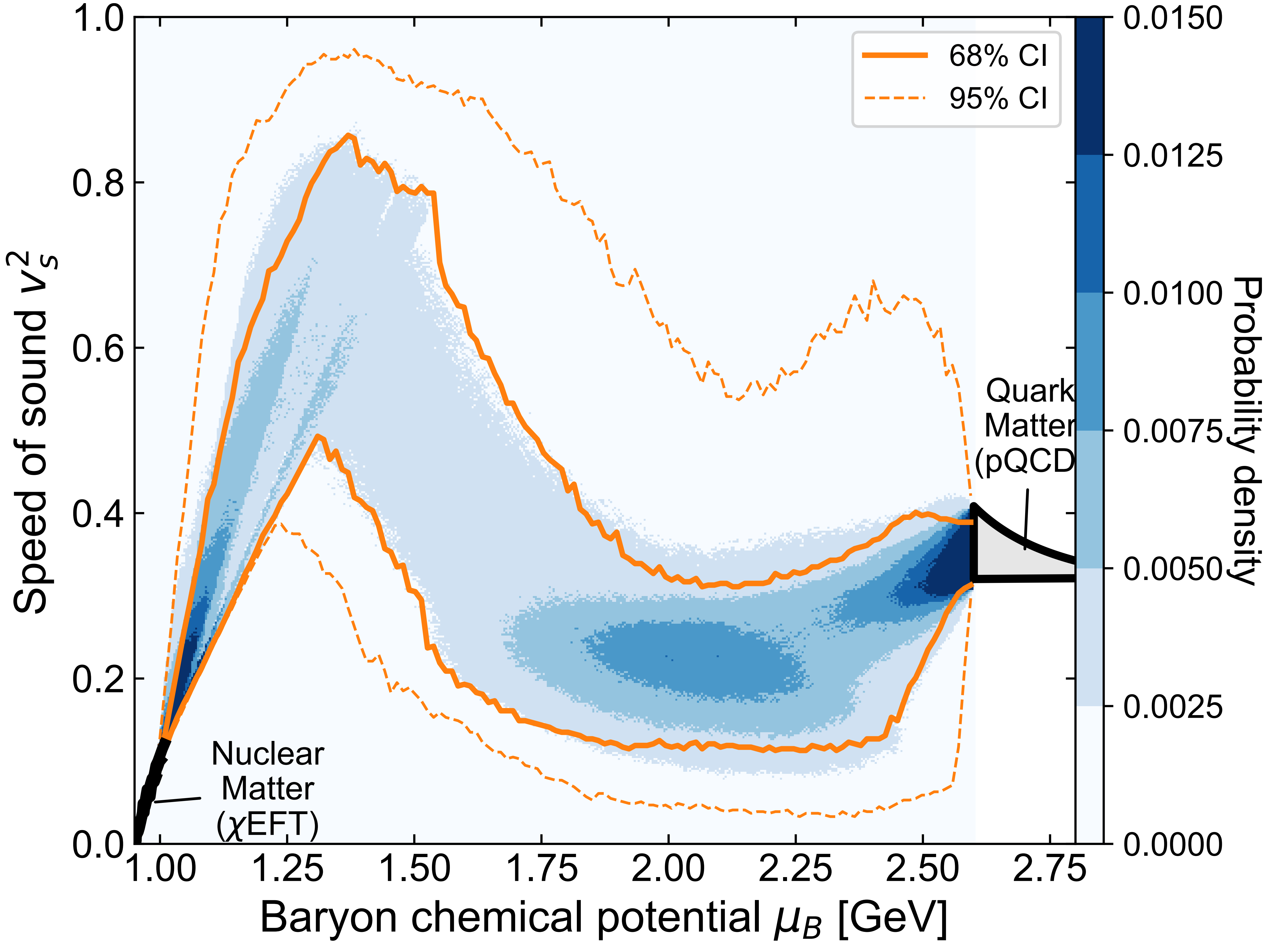
Model EoS of Quarkyonic matter



McLerran,Reddy (2018)

Bayesian

Fujimoto, 2408.12514 (2024)



EoS comparison: Quarkyonic model & Bayesian

Dense large-Nc QCD matter can be described either as

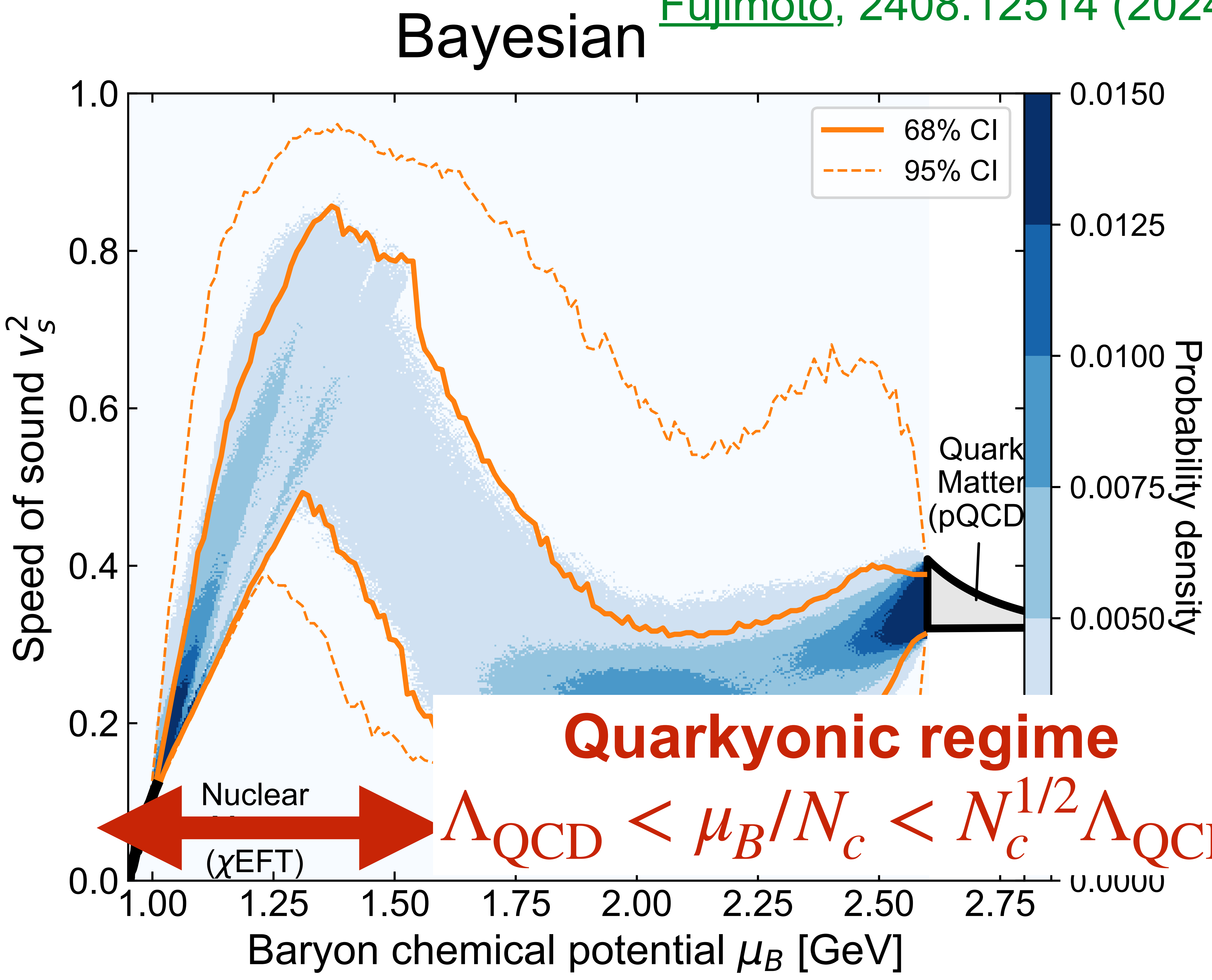
- **Confined baryons** (because confining interaction is never screened)

$$m_D^2 \ll \Lambda_{QCD}^2 \to \mu \ll \sqrt{N_c}\Lambda_{QCD}$$

- **Quarks** (at densities where weak-coupling QCD is valid)

$$\mu \gg \Lambda_{QCD}$$

Fujimoto, 2408.12514 (2024)



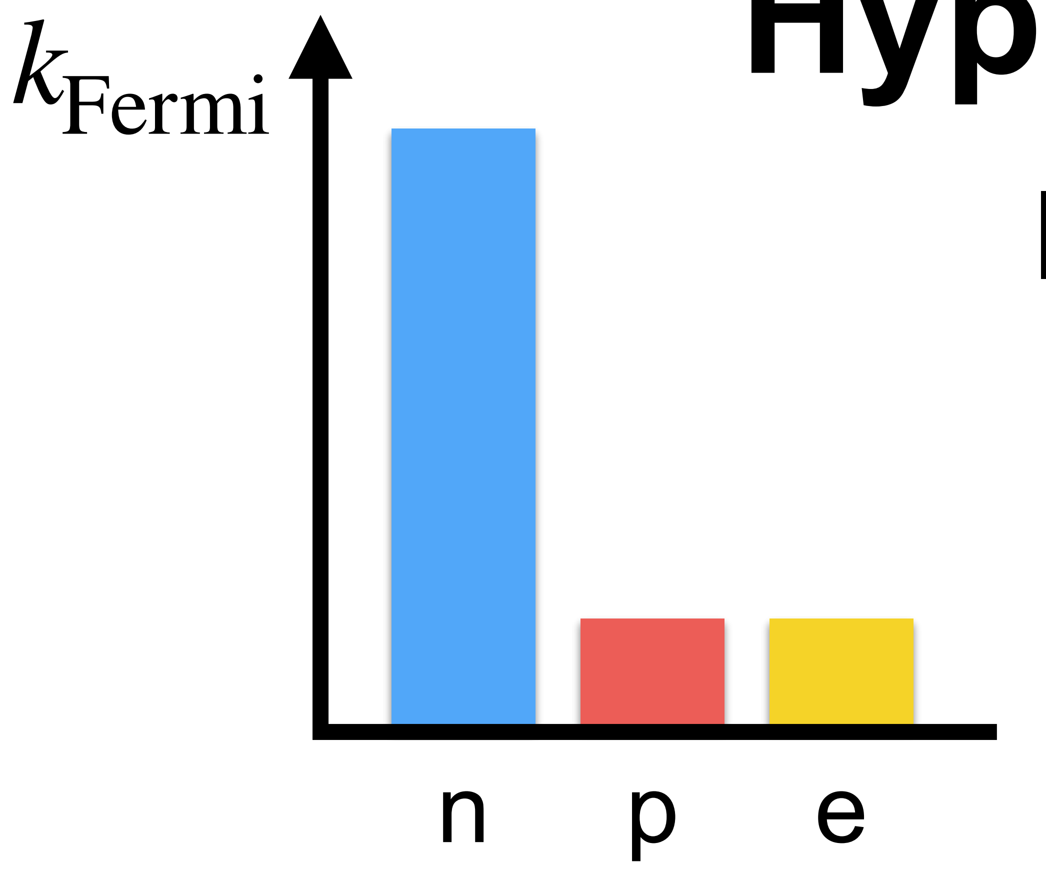
Quarkyonic regime

$$\Lambda_{QCD} < \mu_B/N_c < N_c^{1/2}\Lambda_{QCD}$$

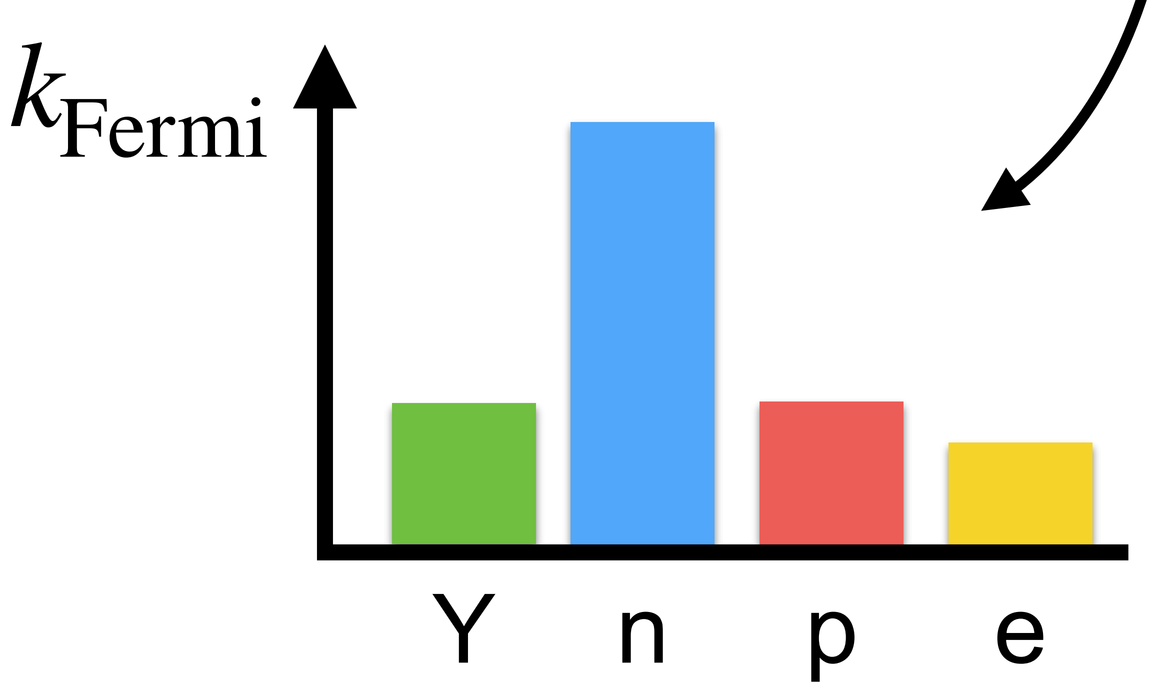
$$(\Lambda_{QCD} \simeq 0.3\,\text{GeV})$$

Strangeness in neutron stars

Hyperon puzzle

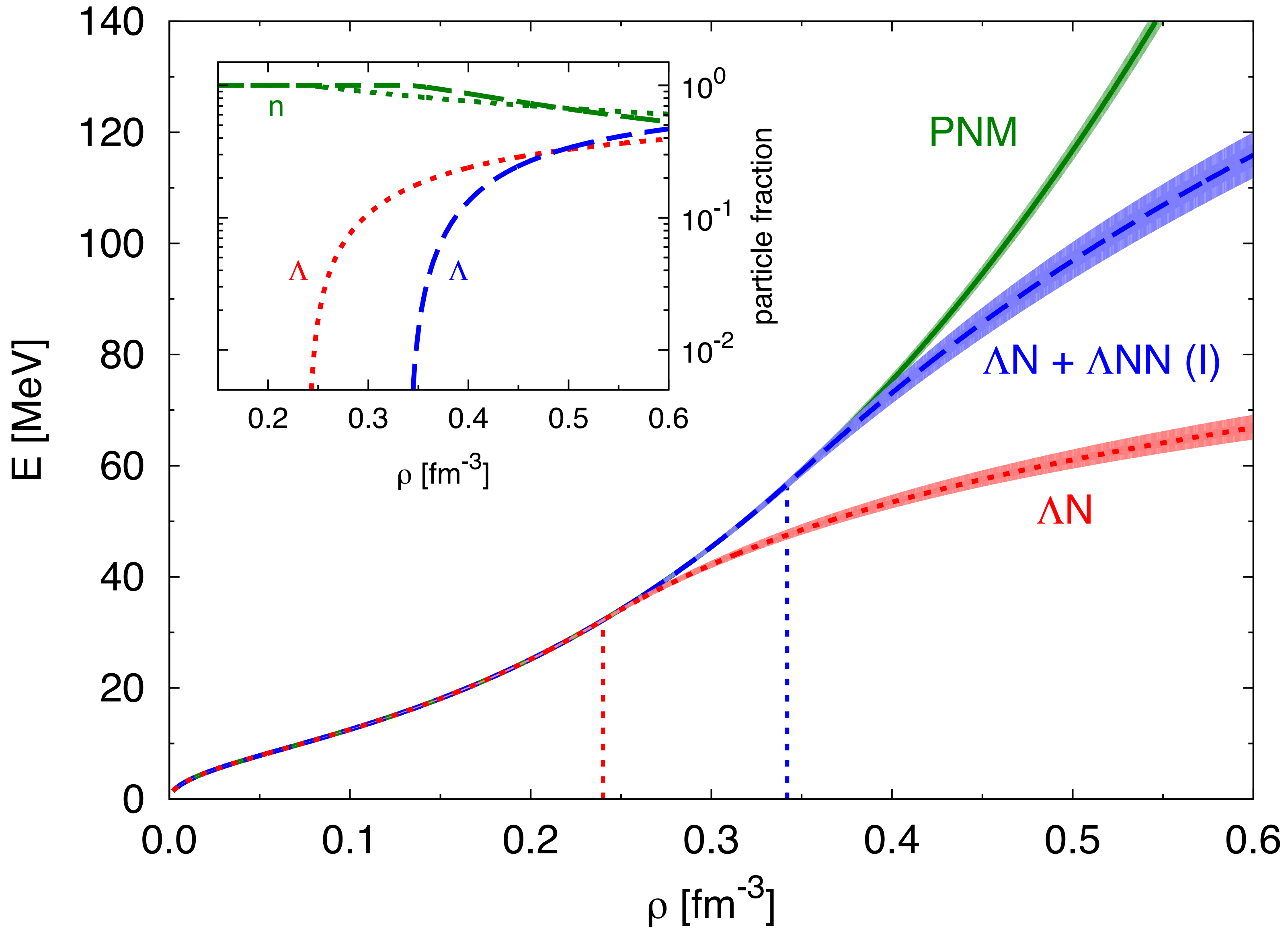


Inclusion of hyperons



Hyperons (Y) lower the energy density at a given baryon density

Hyperons soften the EoS drastically …



→ Cannot support heavy neutron stars

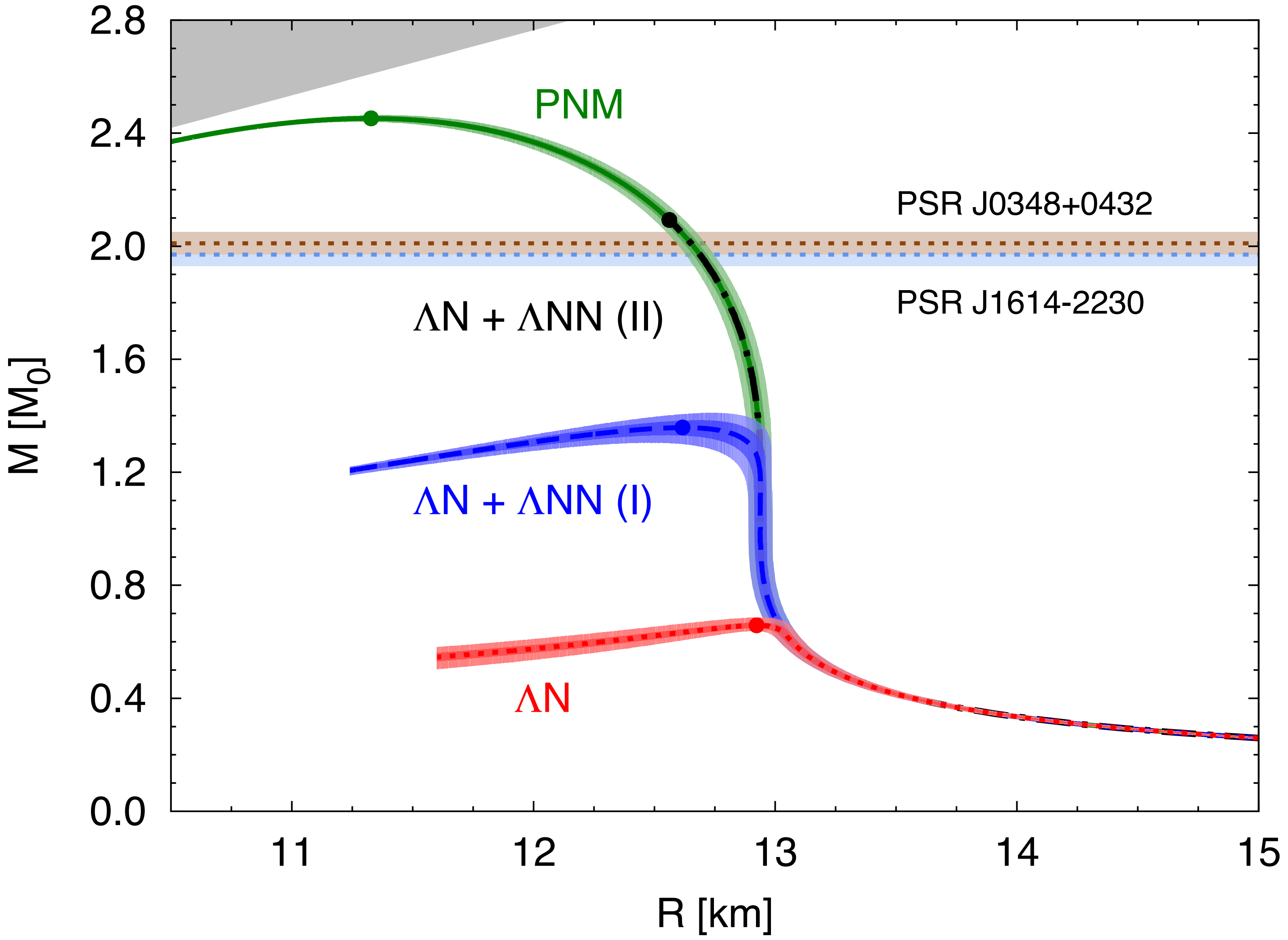


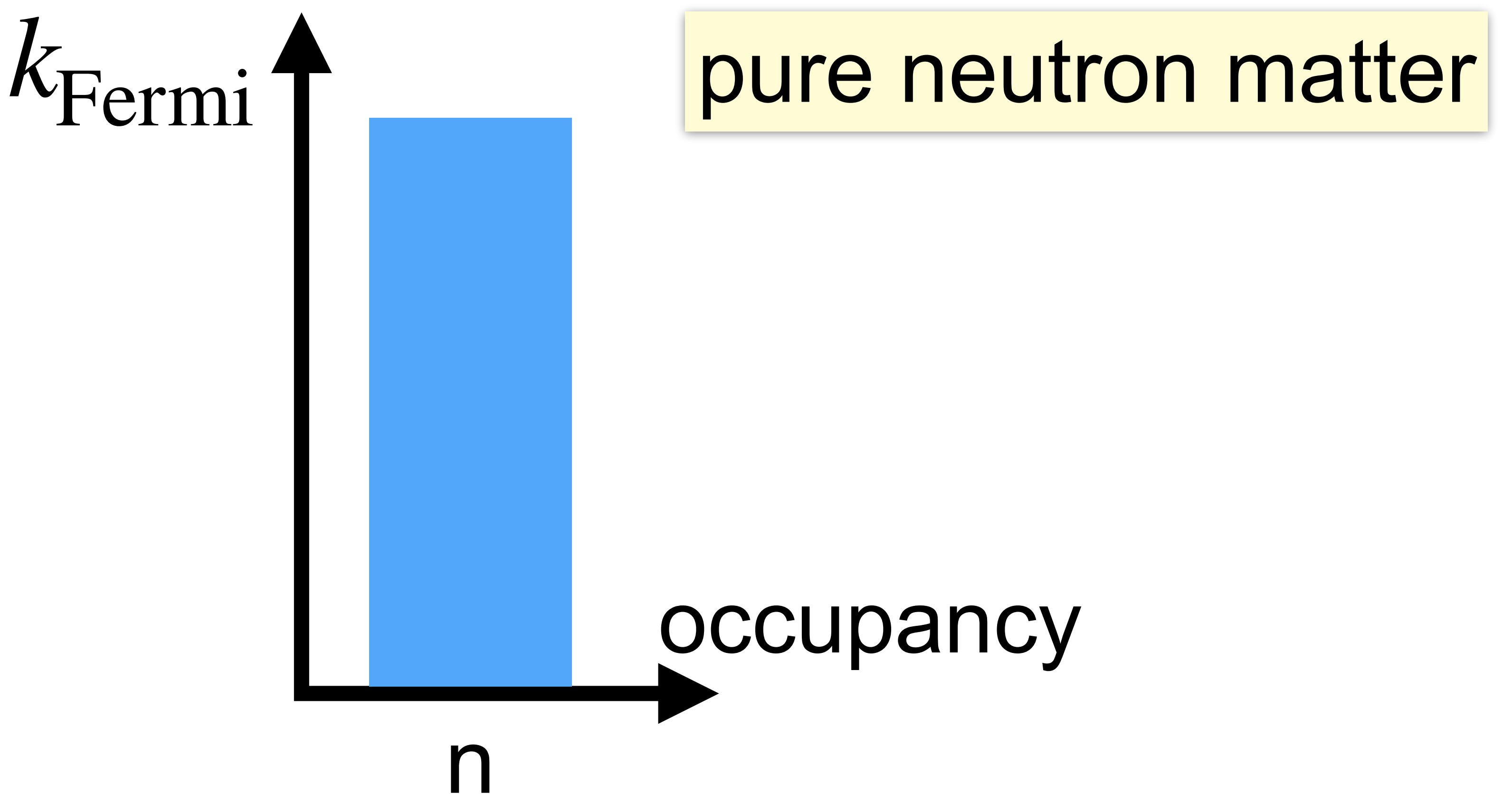
figure from Lonardoni et al. (2014)

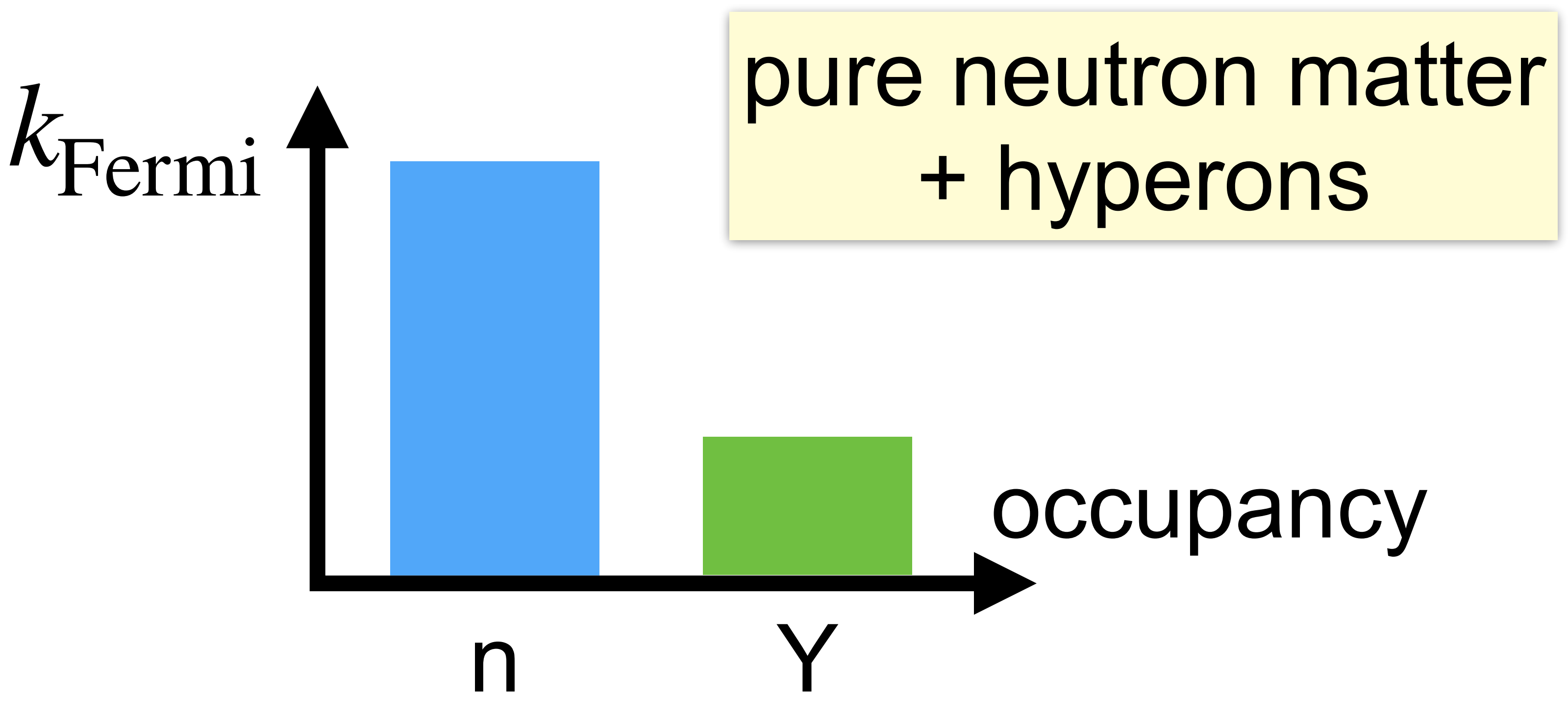
Hyperon puzzle

Quarkyonic solution to the hyperon puzzle

Fujimoto,Kojo,McLerran, 2410.22758 (2024)

Conventional picture:

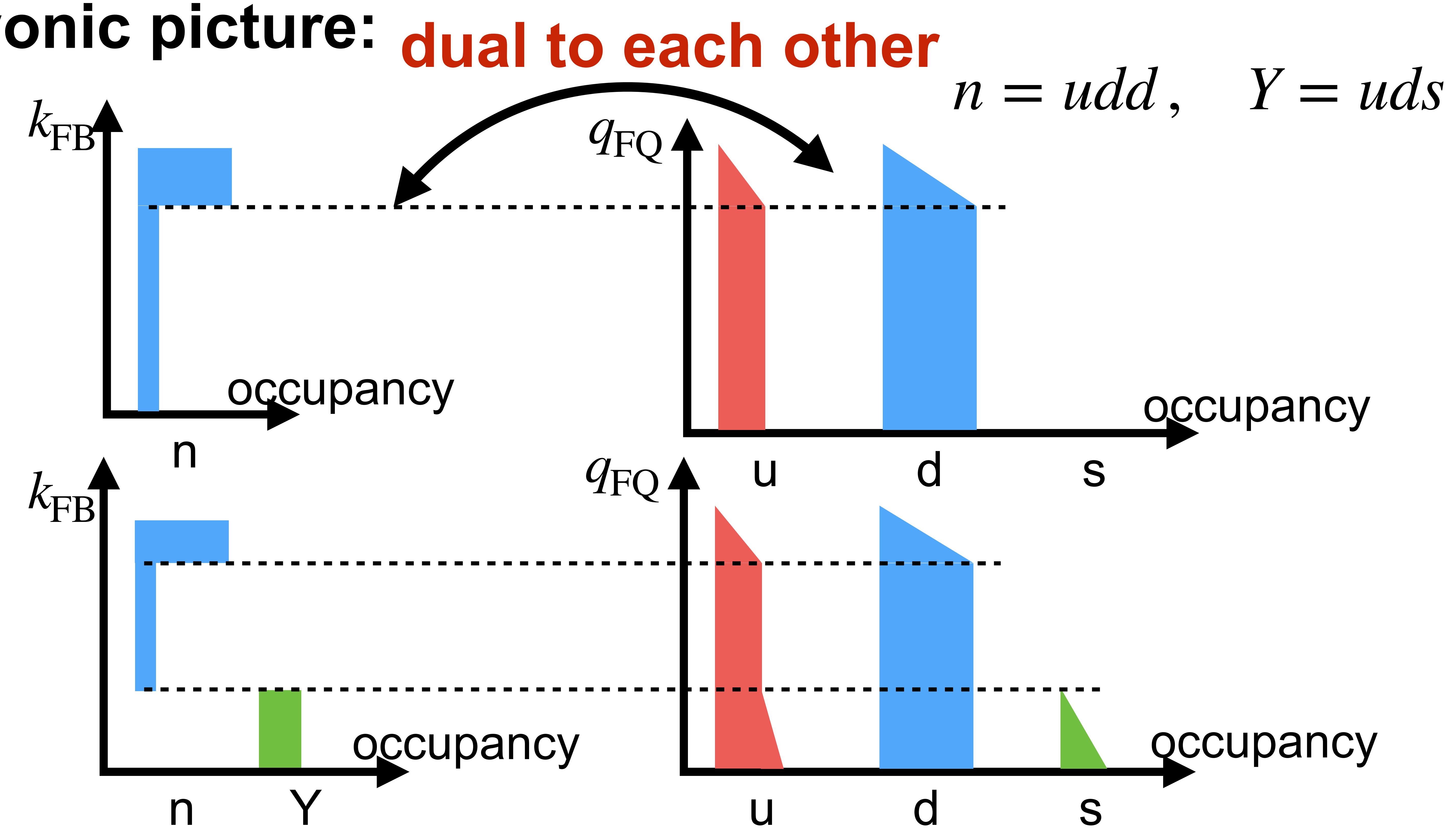




Hyperons (Y) lower the energy density at a given baryon density

Threshold: $\mu_B = M_Y$

Quarkyonic picture:



Y has to appear so that d-quark states are kept saturated:

n = udd → Y+Y = uuddss $\rightarrow \mu_Y = E_Y(k_Y) - \frac{1}{2}E_n(k_Y) + \frac{1}{2}\mu_n$

Threshold ($\mu_n = \mu_Y = \mu_B$ & $k_Y = 0$)

is shifted to: $\mu_B = 2M_Y - M_n = M_Y + (M_Y - M_n)$

Quarkyonic solution to the hyperon puzzle

Fujimoto,Kojo,McLerran, 2410.22758 (2024)

Equation-wise, one can understand the threshold shift as follows:

Hyperon chemical potential:

$$\mu_Y = \left(\frac{\partial \varepsilon}{\partial n_Y}\right)_{n_n} = E_Y(k_{\mathrm{F},Y}) - \frac{1}{2}E_N(k_{\mathrm{F},Y}) + \frac{1}{2}\mu_n$$

Beta equilibrium condition:

$$\mu_S = 0 \Rightarrow \mu_i = B_i\mu_B + Q_i\mu_Q$$

i.e. $\mu_n = \mu_B$, $\mu_Y = \mu_B$ (now we limit ourselves to $\mu_Q = 0$)

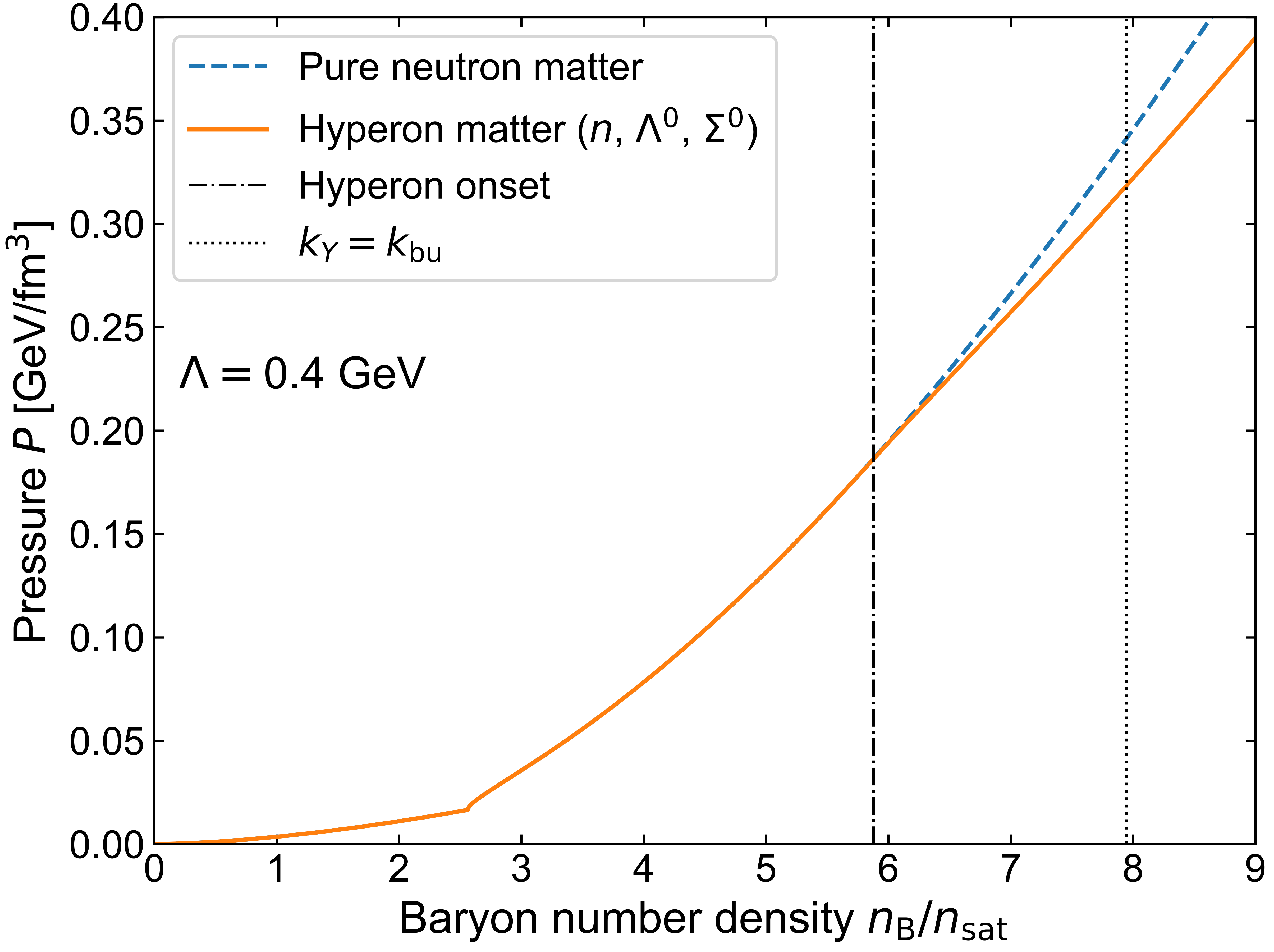
Hyperon threshold:

when the Fermi momentum of hyperons is $k_{\mathrm{F},Y} = 0$

$$\mu_B^{\mathrm{thres}} = M_Y + (M_Y - M_N)$$

Quarkyonic solution to the hyperon puzzle

Fujimoto,Kojo,McLerran, 2410.22758 (2024)



Due to the saturation of d-quark states, softening in the hyperon EoS is mild

This is purely the effect of FD statistics!
No interaction except for the one implicitly in the confining relation.

Usual solutions of the hyperon puzzle requires very strong repulsive interaction

QCD inequality: derivation

Cohen (2003); Fujimoto,Reddy (2023);
see also: Moore,Gorda (2023)

- **Dirac operator**: $\mathscr{D}(\mu) \equiv \gamma^\mu D_\mu + m - \mu\gamma^0$, **property**: $\det \mathscr{D}(-\mu) = [\det \mathscr{D}(\mu)]^*$

- **QCD$_I$**: $Z_I(\mu_I) = \int [dA]\ \det \mathscr{D}(\frac{\mu_I}{2}) \det \mathscr{D}(-\frac{\mu_I}{2}) e^{-S_G} = \int [dA]\ \left|\det \mathscr{D}(\frac{\mu_I}{2})\right|^2 e^{-S_G}$

u quark   d quark

charge conjugation symmetry $\mu_B \to -\mu_B$

- **QCD$_B$**: $Z_B(\mu_B) = \int [dA]\ \det \mathscr{D}(\frac{\mu_B}{N_c}) \det \mathscr{D}(\frac{\mu_B}{N_c})\ e^{-S_G} = \int [dA] \operatorname{Re} \left[\det \mathscr{D}(\frac{\mu_B}{N_c})\right]^2 e^{-S_G}$

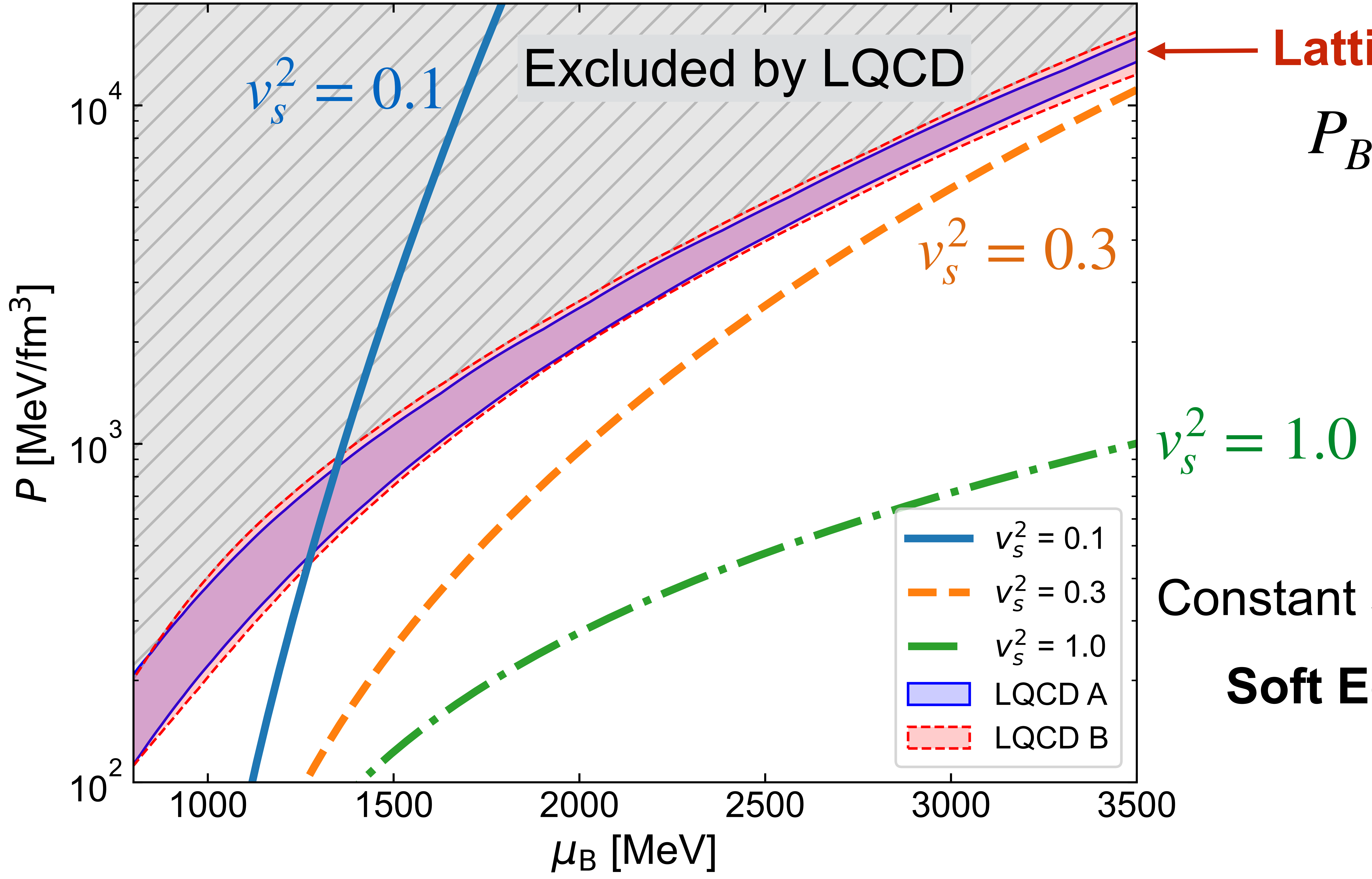
Note: this is **isospin symmetric** because there is no isospin imbalance

- From the relation $\operatorname{Re} z^2 \leq |z^2| = |z|^2$:

$$Z_B(\mu_B) \leq \int [dA]\ \left|\det \mathscr{D}(\frac{\mu_B}{N_c})\right|^2 e^{-S_G} = Z_I\left(\mu_I = \frac{2}{N_c}\mu_B\right)$$

Direct use of QCD inequality

Lattice data: Abbott et al. (2023); Fujimoto,Reddy (2023)



Lattice data: upper bound

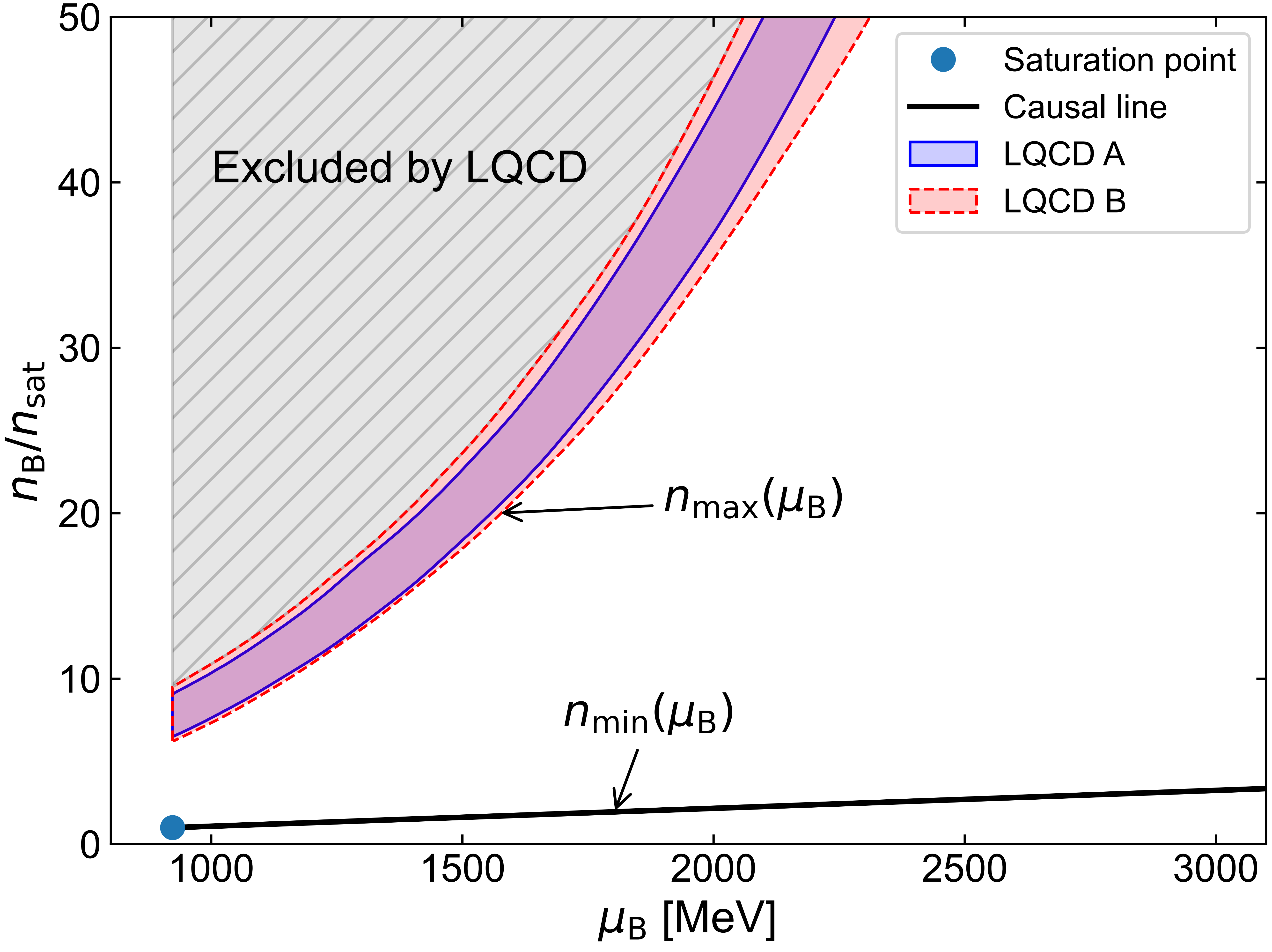
$$P_B(\mu_B) \le P_I\left(\mu_I = \frac{2}{N_c}\mu_B\right)$$

Constant sound speed EoS: $P(\varepsilon) \propto v_s^2 \varepsilon$

Soft EoS (smaller P at a given ε) is excluded

Bounds on $n_B(\mu_B)$

Komoltsev,Kurkela (2021); Fujimoto,Reddy (2023)



Properties $n_B(\mu_B)$ must satisfy:

① Stability:

$$\frac{d^2P}{d\mu_B^2} \geq 0 \Rightarrow \frac{dn_B}{d\mu_B} \geq 0$$

② Causality $v_s^2 \leq 1$:

$$v_s^2 = \frac{n_B}{\mu_B}\frac{d\mu_B}{dn_B} \leq 1 \Rightarrow \frac{dn_B}{d\mu_B} \geq \frac{n_B}{\mu_B}$$

③ QCD inequality on the integral:

$$\int_{\mu_{sat}}^{\mu_B} d\mu' \, n_B(\mu') \leq P_I\left(\mu_I = \frac{2}{N_c}\mu_B\right)$$

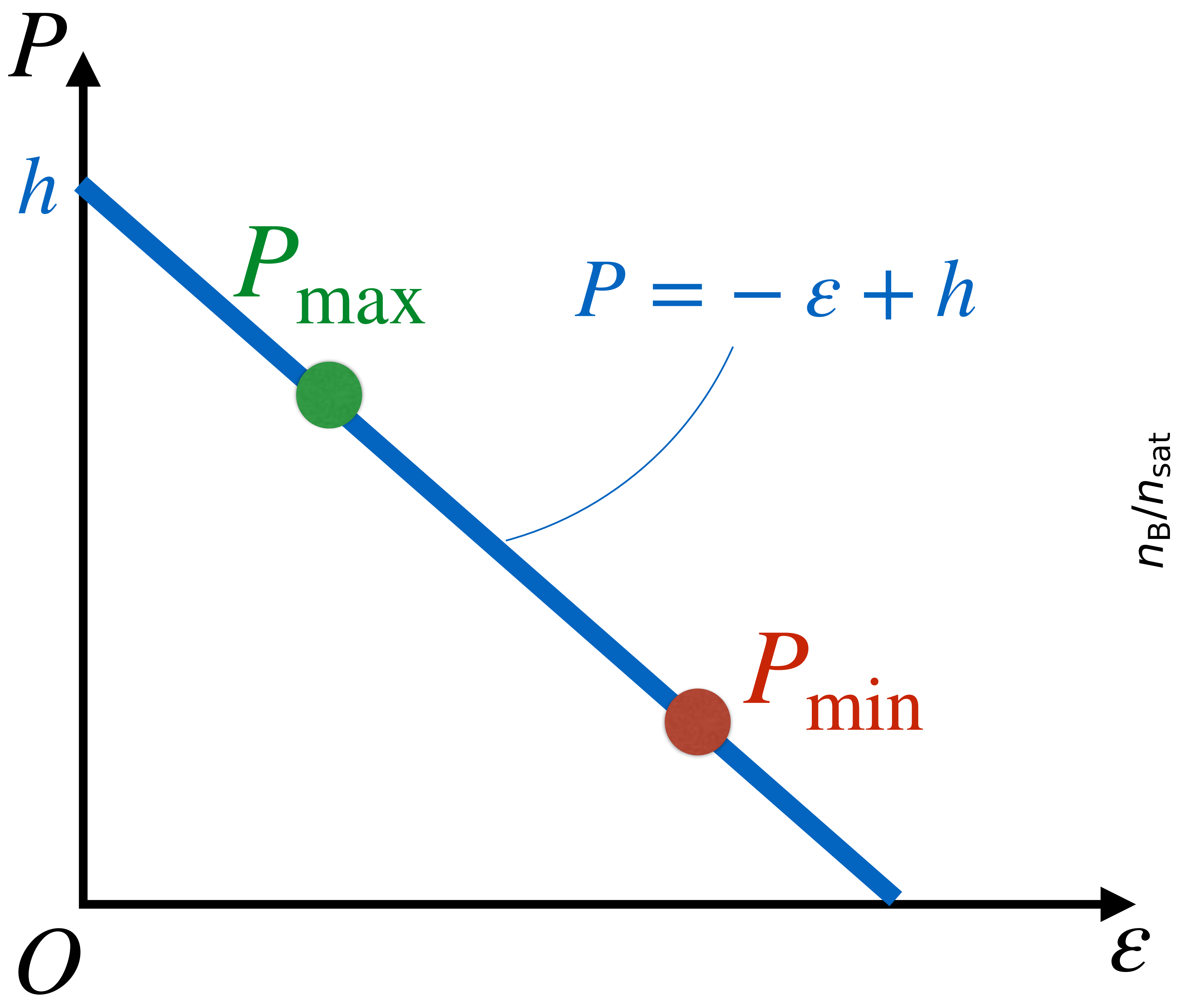
Lower bound of the integral must be specified

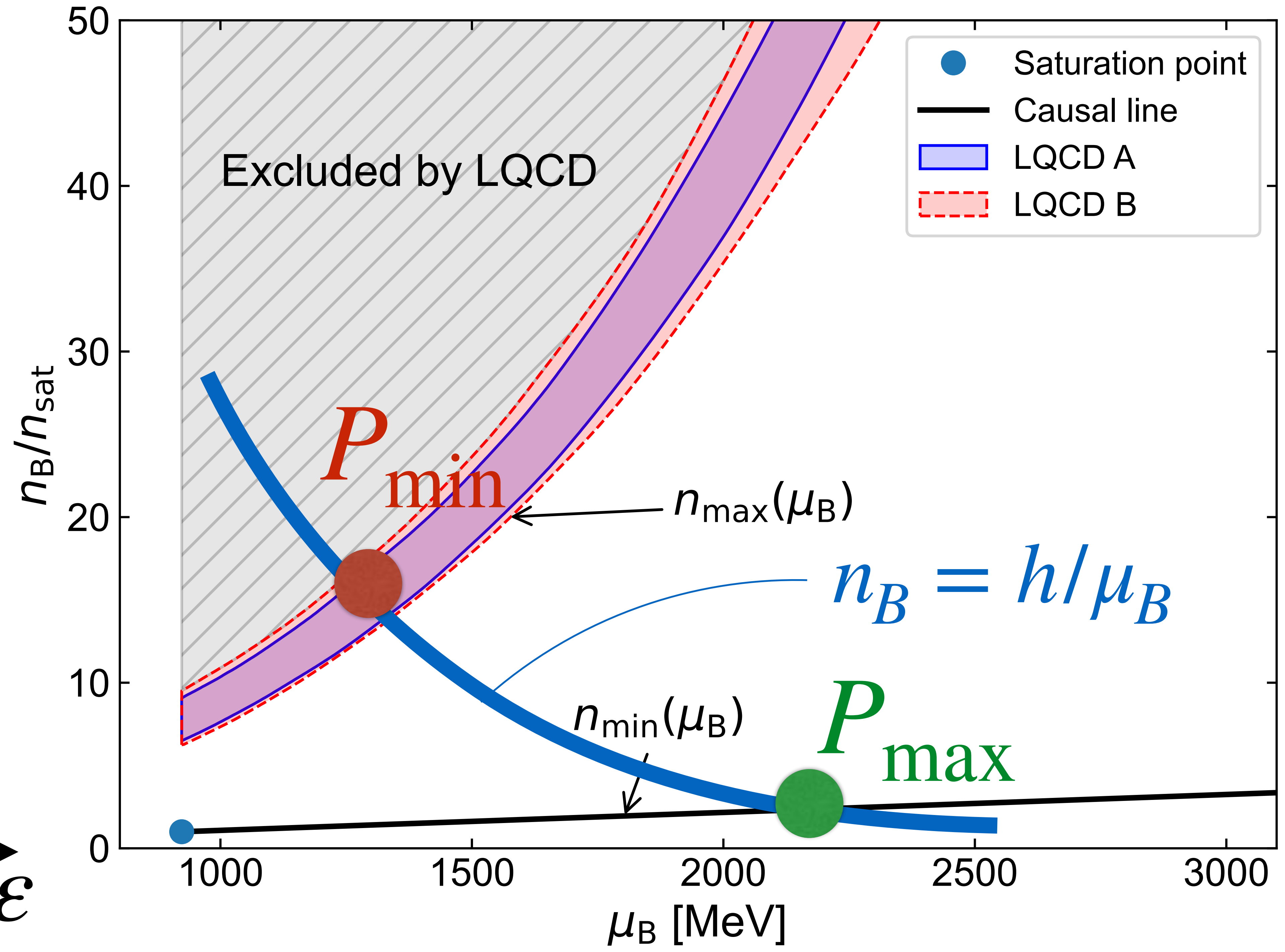
fix it to the **empirical saturation property**

Bounds on $P(\varepsilon)$

Isenthalpic line: $h = \mu_B n_B = \varepsilon + P = \text{const}$

Komoltsev,Kurkela (2021); Fujimoto,Reddy (2023)



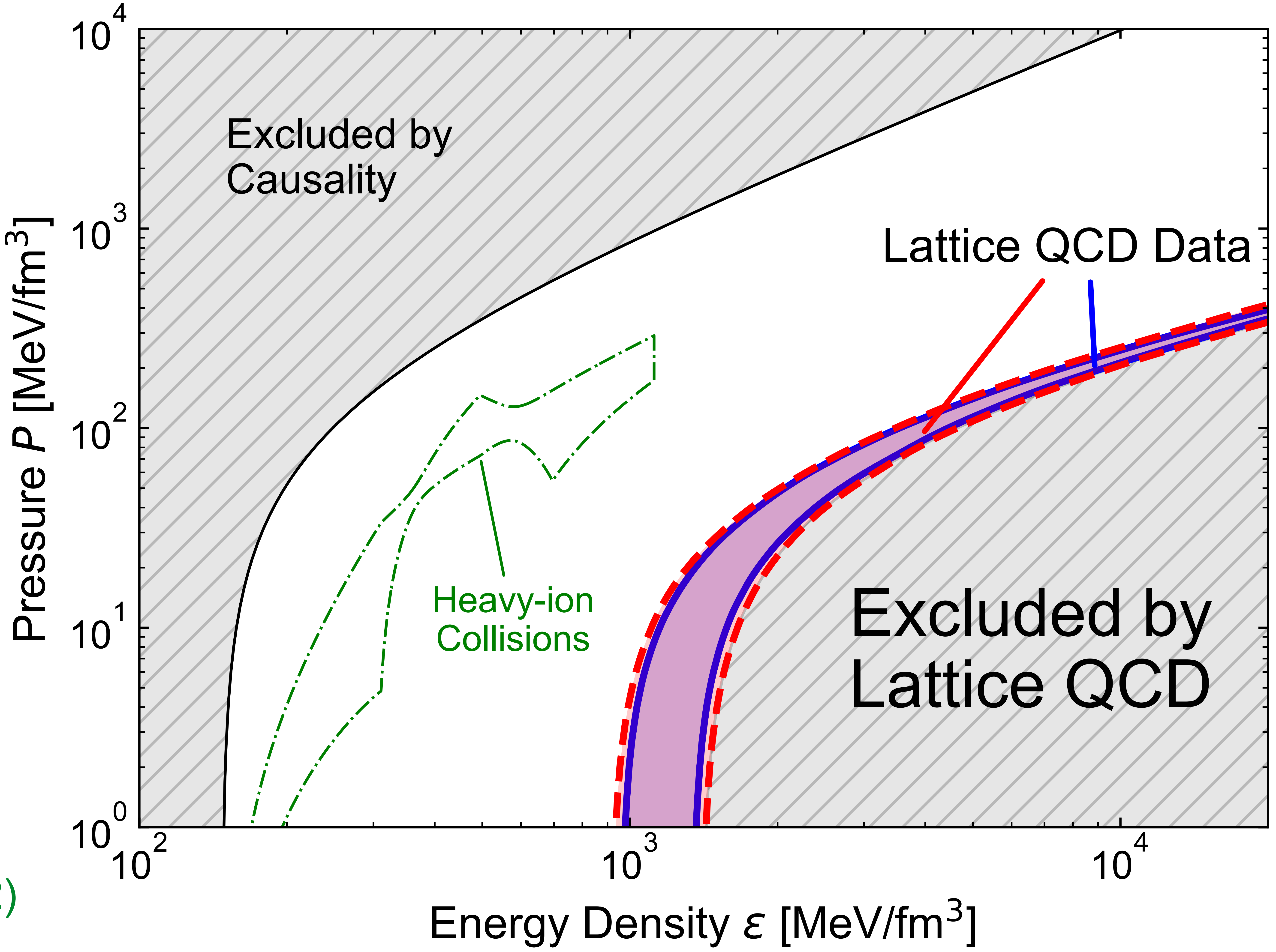


by changing value of h, the trajectories of $P_{\min}$ ($P_{\max}$)
gives the lower (upper) bound for $P(\varepsilon)$

Robust bounds on $P(\varepsilon)$

Fujimoto, Reddy (2023)

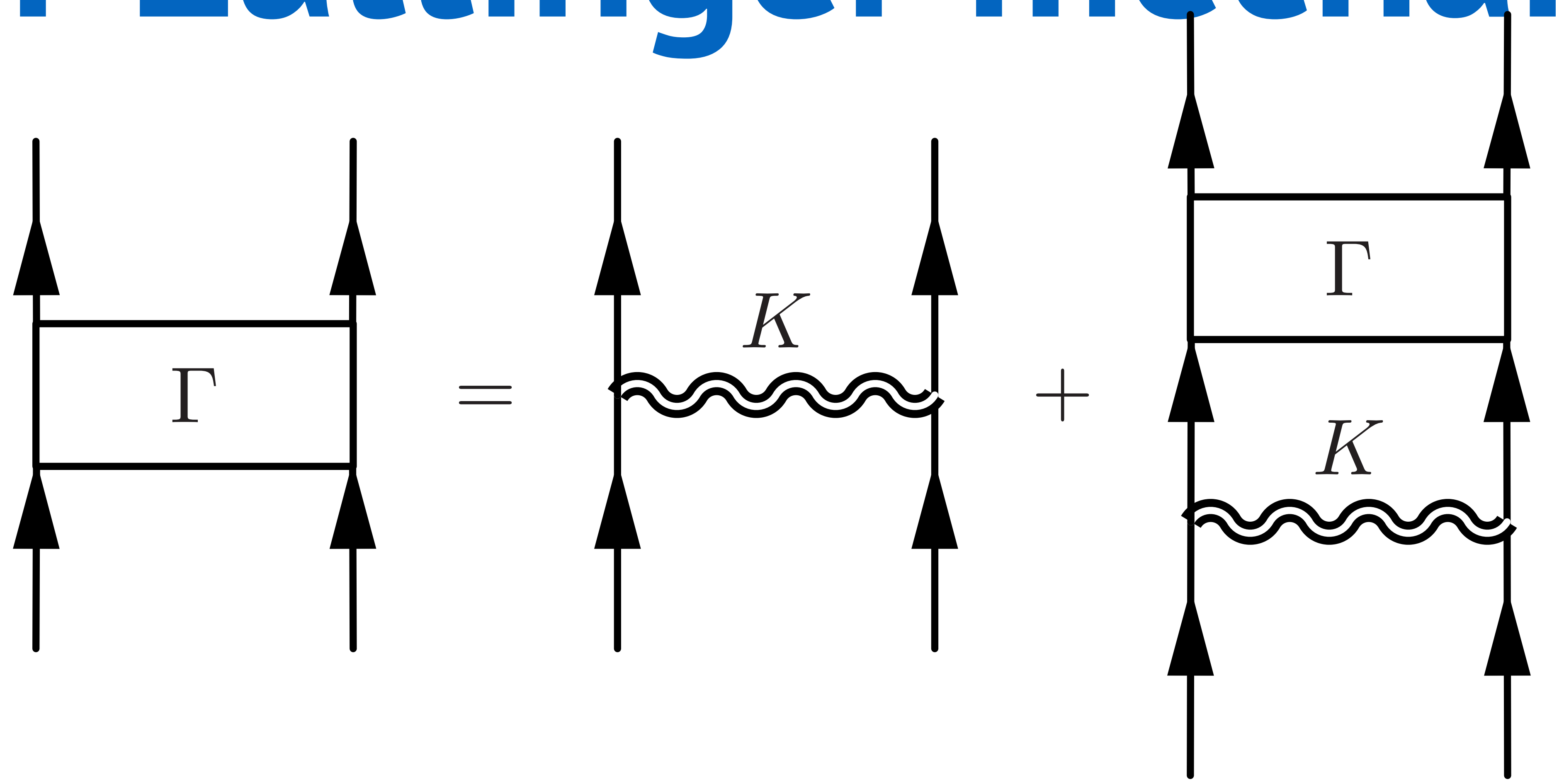
From the relation $\varepsilon = -P + \mu_B n_B$:

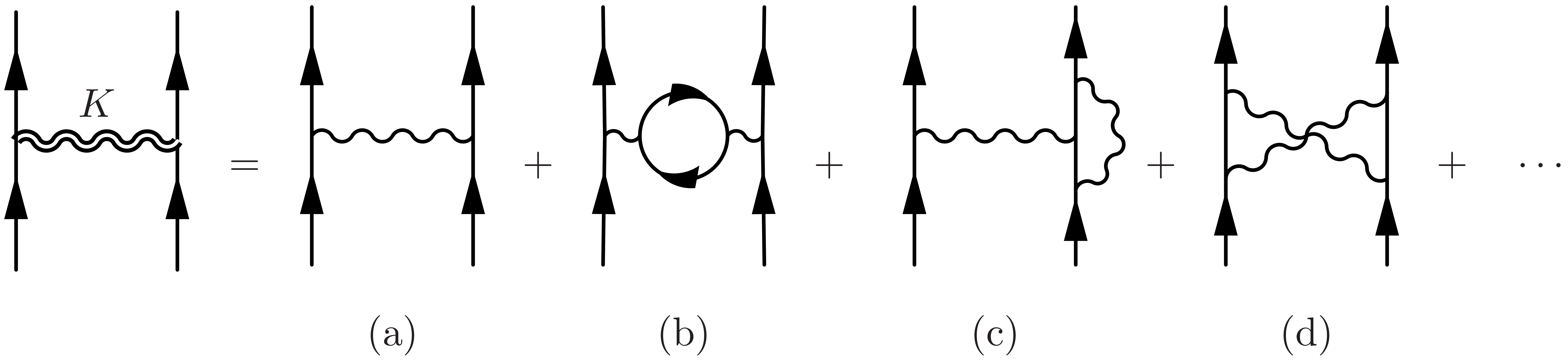


Soft EoS at large ε is excluded

Heavy-ion:
Oliinychenko et al.(2022)

Kohn-Luttinger mechanism



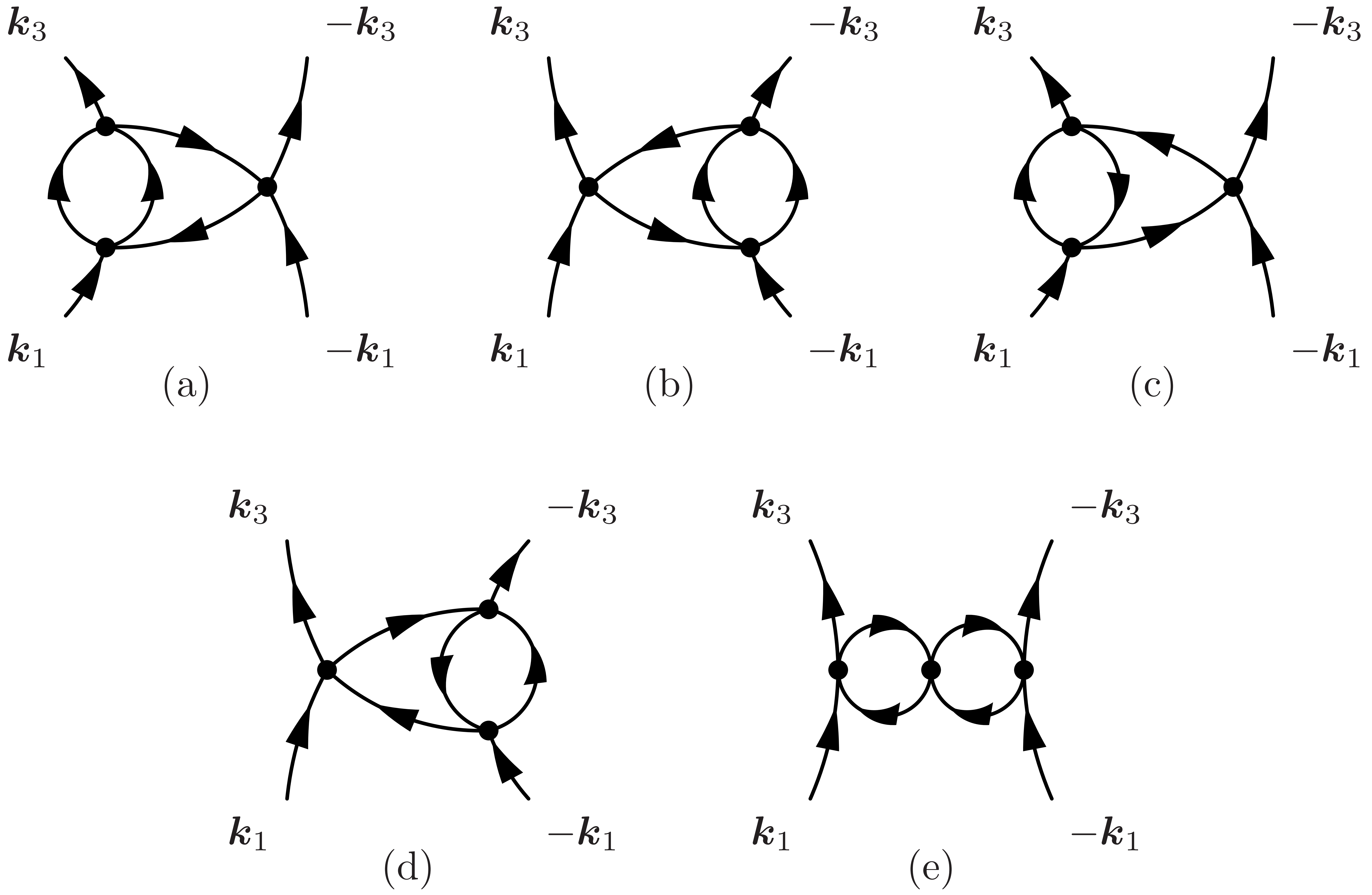


Partial wave expansion: $K(\theta) = \sum_l (2l+1) K_l P_l(\cos\theta)$

$K_l^{(a)} \sim e^{-l} \sim 0$

$K_l^{(b,c,d)} \sim \dfrac{(-1)^l}{l^4}$

Higher-order diagrams in perturbation theory



Emerging picture of neutron star EoS

Bedaque,Steiner (2015);
Tews,Reddy,