Confinement and chiral symmetry breaking in a geometry with imaginary rotation

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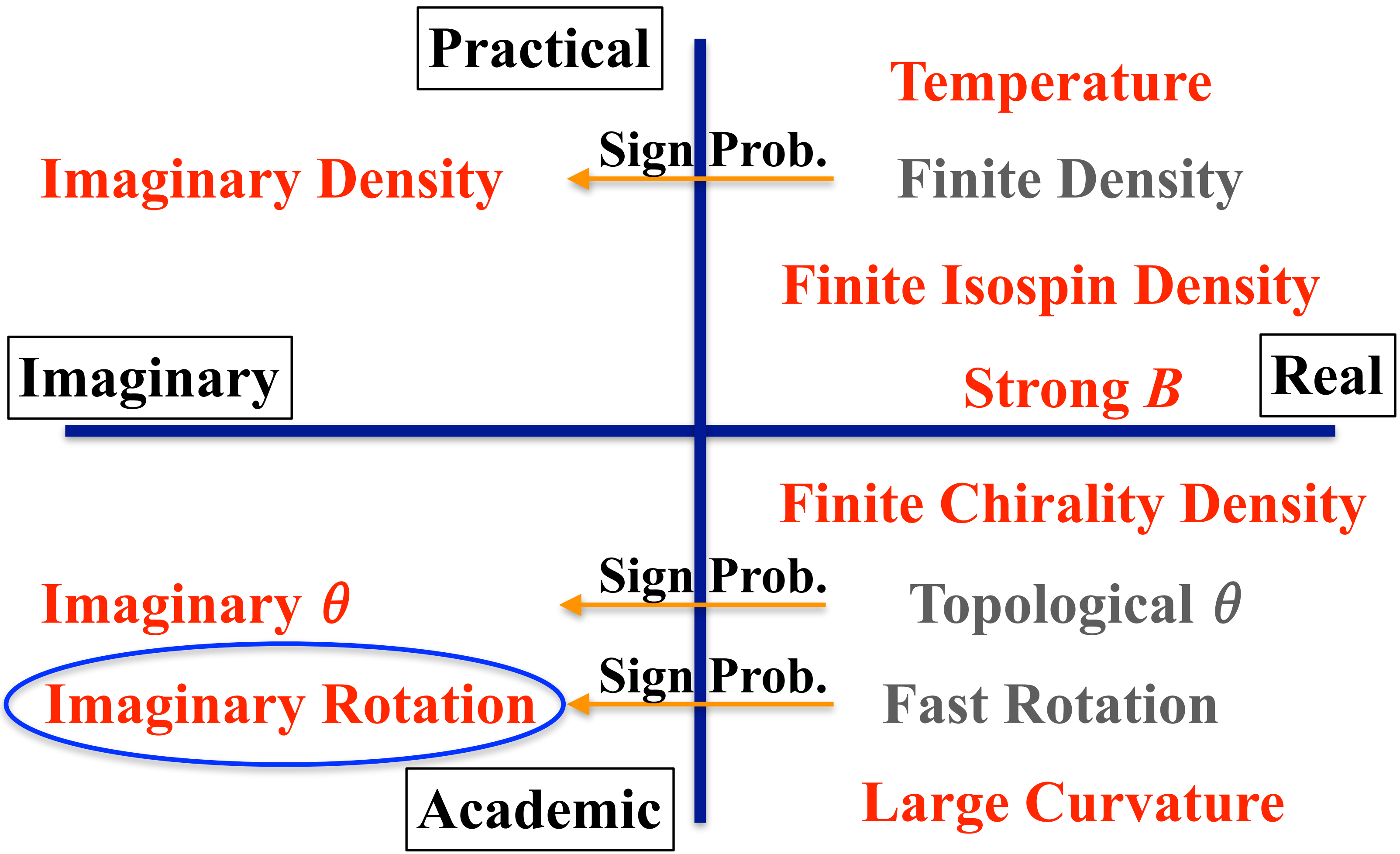
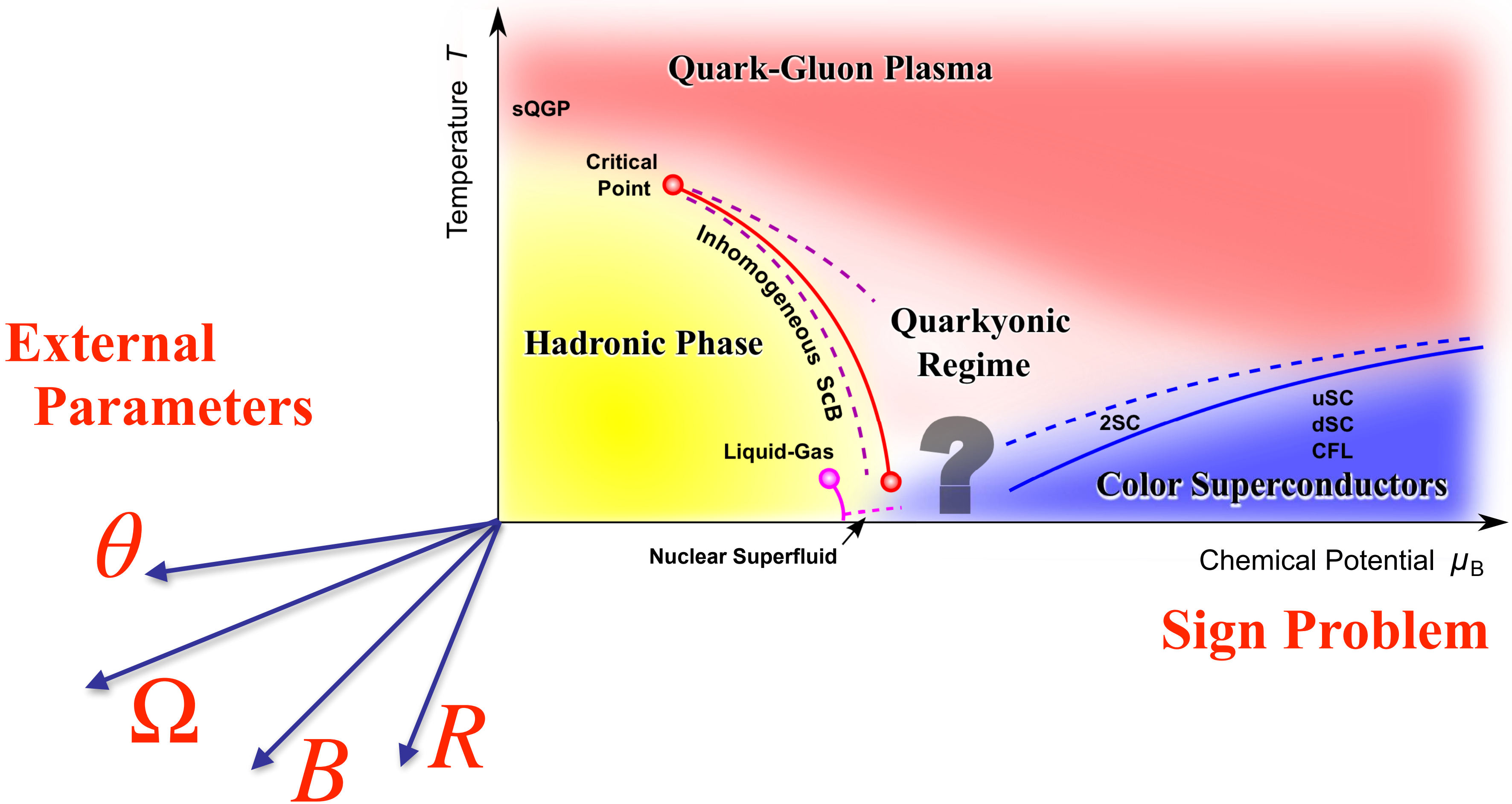
— Confinement and symmetry from vacuum to QCD phase diagram —

Various “Probes”



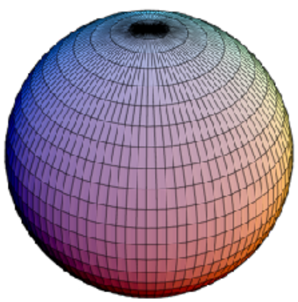
Various "Probes"



QCD with R

Determinant of the quark propagator with *R*

$$\mathrm{Tr}\mathrm{Log}G = \frac{1}{30}\Box R - \frac{1}{45}R_{\mu\nu}R^{\mu\nu} - \frac{7}{360}R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + 2m^4 + \frac{1}{3}m^2 R + \frac{1}{72}R^2$$



$$2(m^2 + R/12)^2$$

Flachi-Fukushima (2014)

Looks like a mass shift…
But QCD has chiral symmetry…?
Should be a spontaneous symmetry breaking.
Phase structure may be changed in curved spacetime.

QCD with Ω

In terms of the fluid language:

$$\beta_\mu p^\mu = \beta(p^0 - \underline{\boldsymbol{\Omega} \times \boldsymbol{x} \cdot \boldsymbol{p}})$$

$$= \boldsymbol{x} \times \boldsymbol{p} \cdot \boldsymbol{\Omega} = \boldsymbol{L} \cdot \boldsymbol{\Omega}$$

Can be fully relativistically generalized with $\Omega^\mu = \varepsilon^{\mu\nu\rho\sigma} u_\nu \partial_\rho u_\sigma$

Cranking Hamiltonian:

$$\hat{H} \;\to\; \hat{H} - \hat{\boldsymbol{J}} \cdot \boldsymbol{\Omega}$$

QCD with Ω

Metric in the rotating frame:

Euclidean Cylindrical + Imaginary Rotation

$$g_{\mu\nu} = \begin{pmatrix} 1-\Omega_I^2 r^2 & -i\Omega_I r^2 & 0 & 0 \\ -i\Omega_I r^2 & r^2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

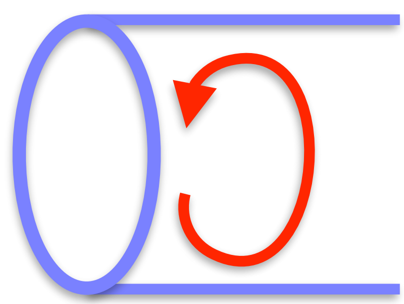
Analytical Continuation ➡ **Very singular due to causality**

Geometrical condition:

$$(\tau, \theta, r, z) \;\sim\; (\tau+\beta, \theta \underline{-\beta\Omega_I}, r, z)$$

Imaginary time
× Imaginary angular velocity

Period $\beta = 1/T$

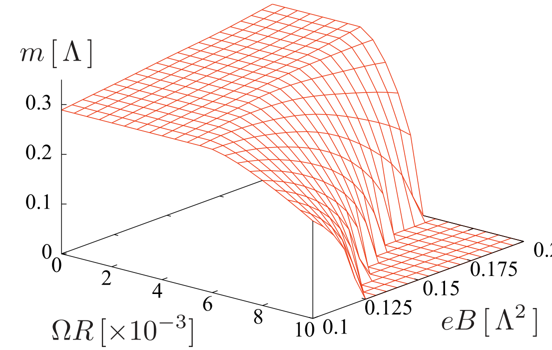


Rotation Controversies

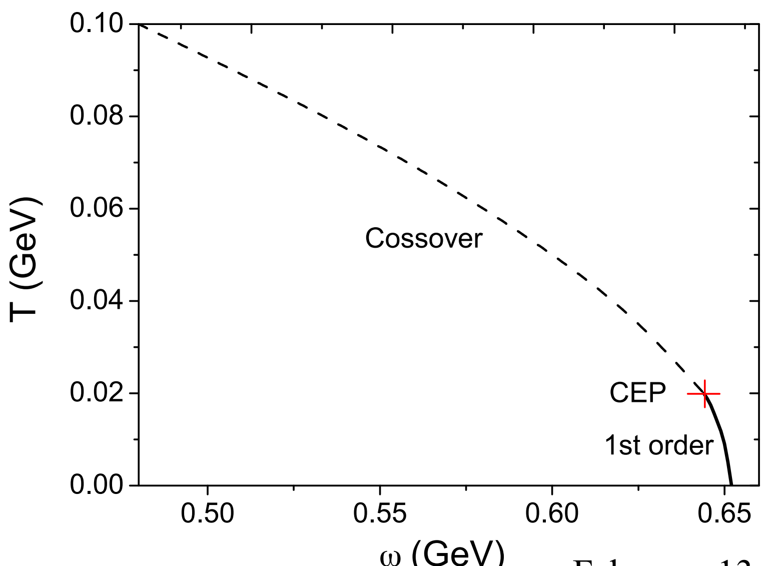
Angular Velocity ~ Finite Density

Chen-Fukushima-Huang-Mameda (2015)

$$H \rightarrow H - \boldsymbol{J}\cdot\boldsymbol{\Omega} \quad \Leftrightarrow \quad H - N\mu$$



Phase Diagram at Finite Angular Velocity



Jiang-Liao (2016)

This is a phase diagram at the rotation center in the quark model.
Chiral Symmetry

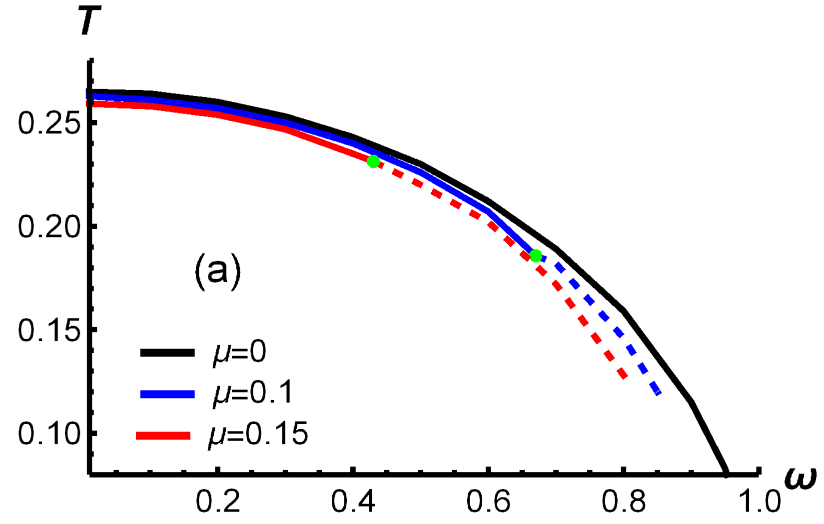
Rotation Controversies

Chen-Zhang-Li-Hou-Huang (2020)

They solved the 5D Einstein equations with

$$t \to \frac{1}{\sqrt{1-(\omega l)^2}}(t+\omega l^2\theta)$$

$$\phi \to \frac{1}{\sqrt{1-(\omega l)^2}}(\theta+\omega t)$$

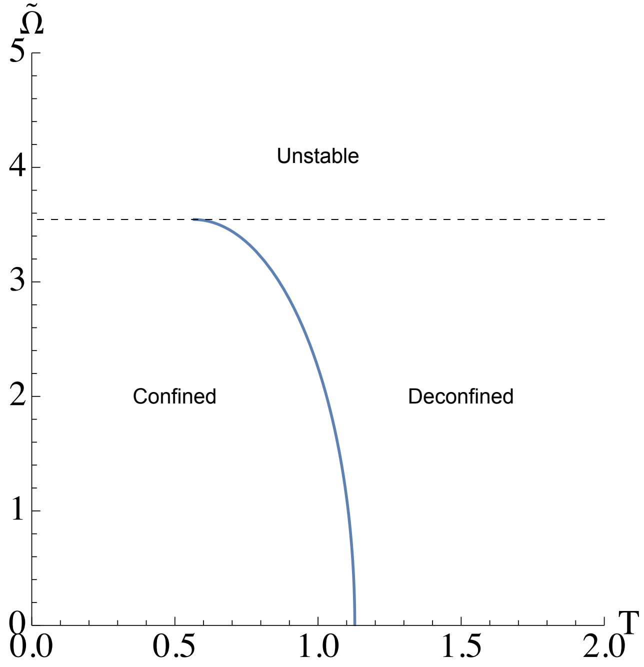


Hawking-Page phase transition modified by the orbital angular momentum. **Color Deconfinement**

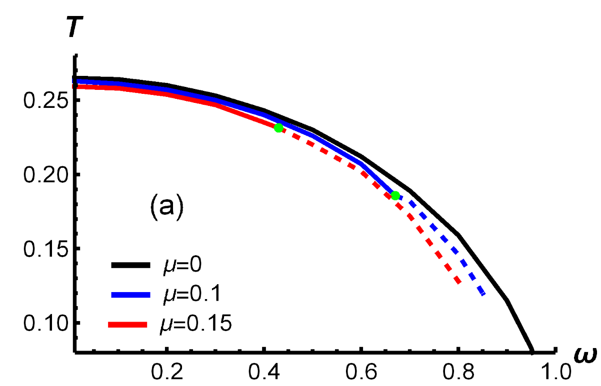
Rotation Controversies

Ahmed-Cong-Kubiznak-Mann-Visser (2023)

fixed $(J, \mathcal{V}, C)$:	$F \equiv E - TS$	**Canonical**
fixed $(\tilde{\Omega}, \mathcal{V}, C)$:	$W \equiv E - TS - \tilde{\Omega} J$	**Grand Canonical**
fixed $(J, \mathcal{V}, \mu)$:	$G \equiv E - TS - \mu C$	



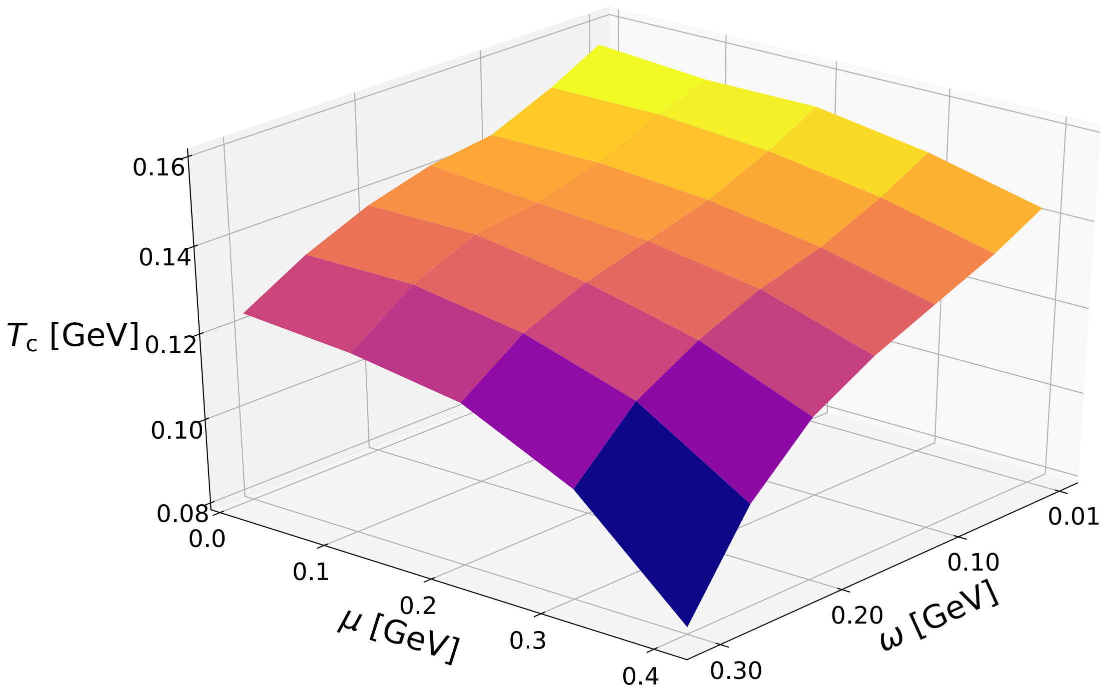
Consistent with Chen-Zhang-Li-Hou-Huang (2020)



Rotation Controversies

Fujimoto-Fukushima-Hidaka (2021)

Hadron Resonance Gas model predicts thermodynamics.
Pressure blows up around approximate T_c .



Full QCD ?

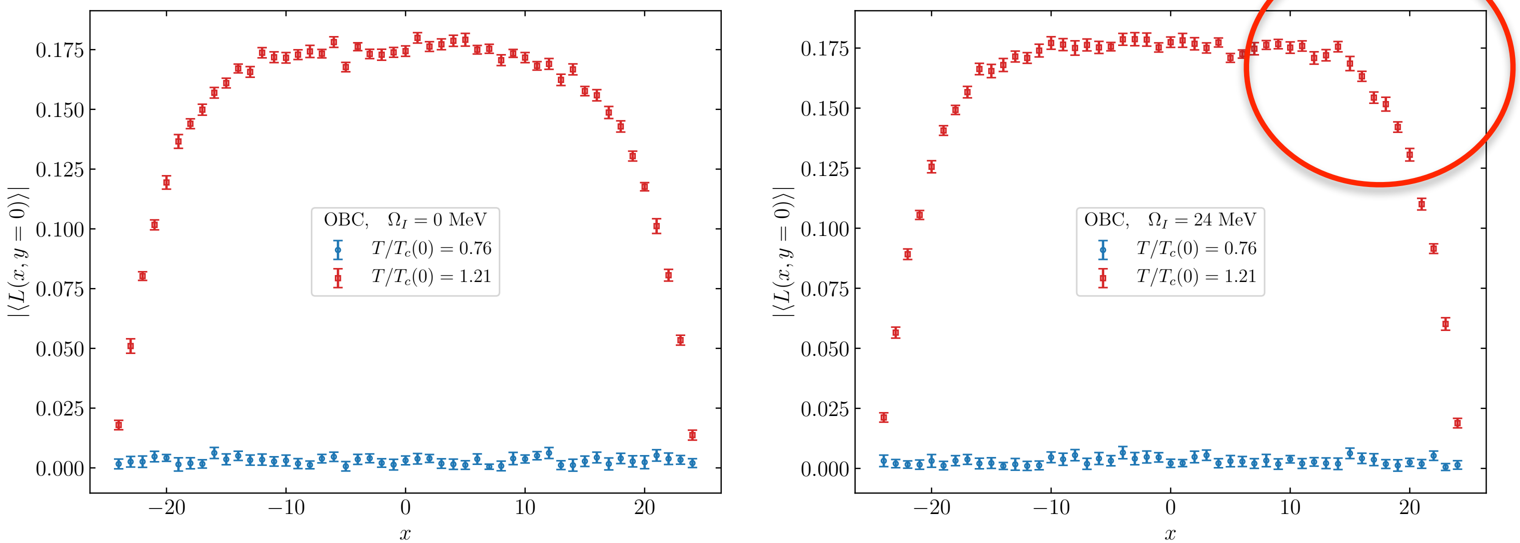
Lattice QCD ?

Rotation Controversies

First surprise — Braguta et al. (2021)

Small but sizable changes here

Pure gluonic theory on lattice

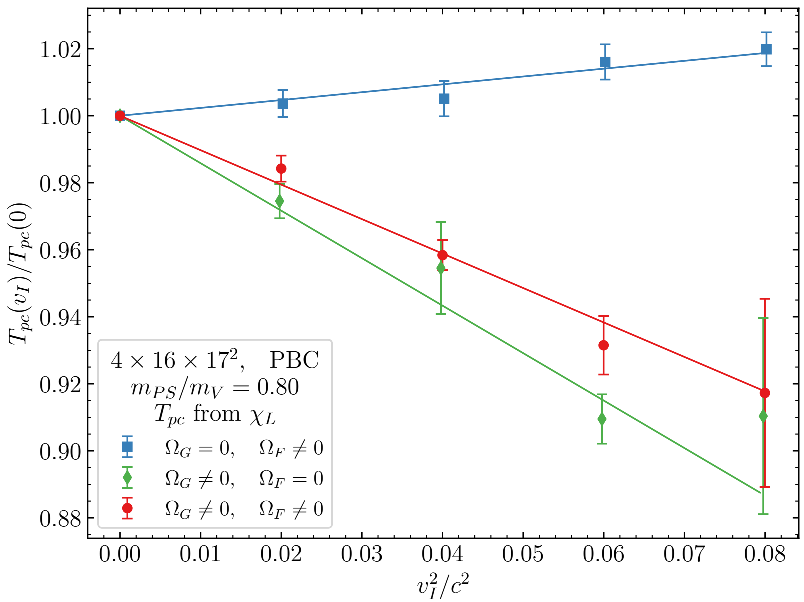


Imaginary rotation enhances the Polyakov loop
Real rotation induces more confinement!?

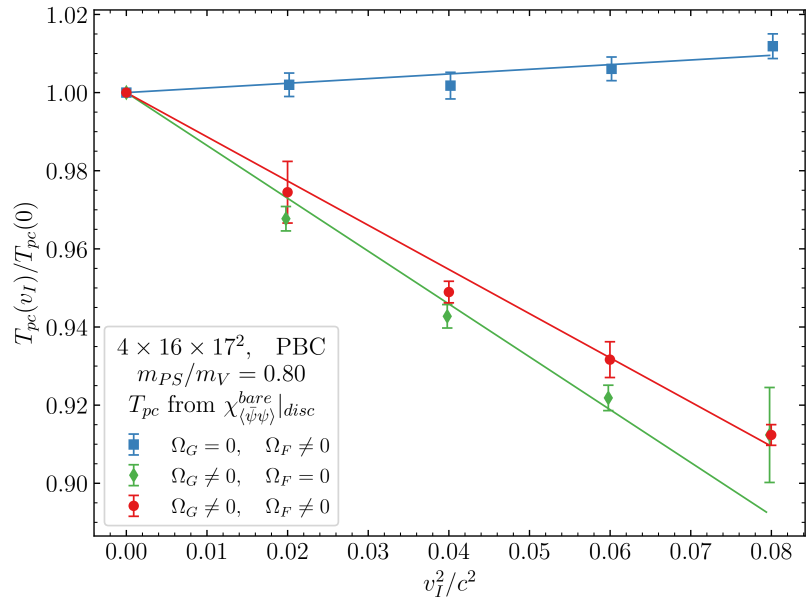
Rotation Controversies

Updates — Braguta-Kotov-Roenko-Sychev (2023)

[Deconfinement]



[Chiral Restoration]



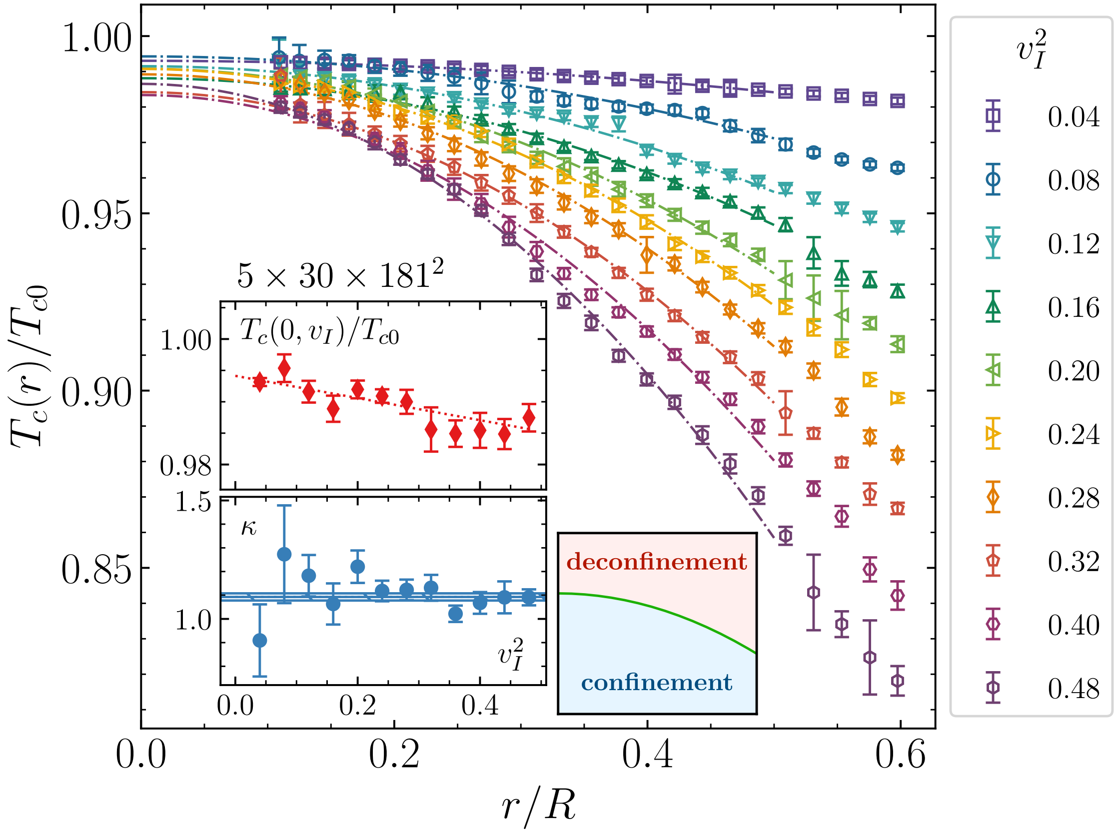
Imaginary Velocity →

Matter sector is consistent with other approaches.

Gluon sector is problematic !

Rotation Controversies

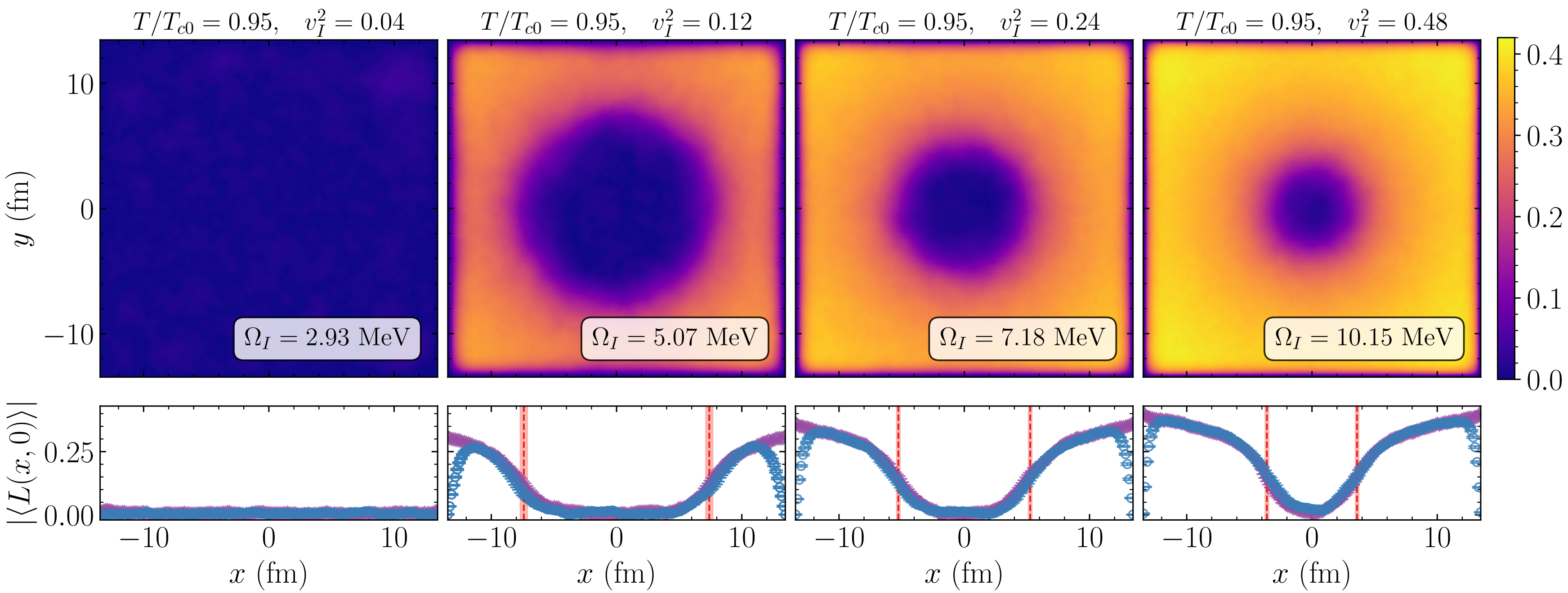
Further updates — Braguta-Chernodub-Roenko (2023)



Physics depends on the velocity $(\Omega_\mathrm{I} r)^2$ dominated by the orbital component?

Rotation Controversies

Further updates — Braguta-Chernodub-Roenko (2023)



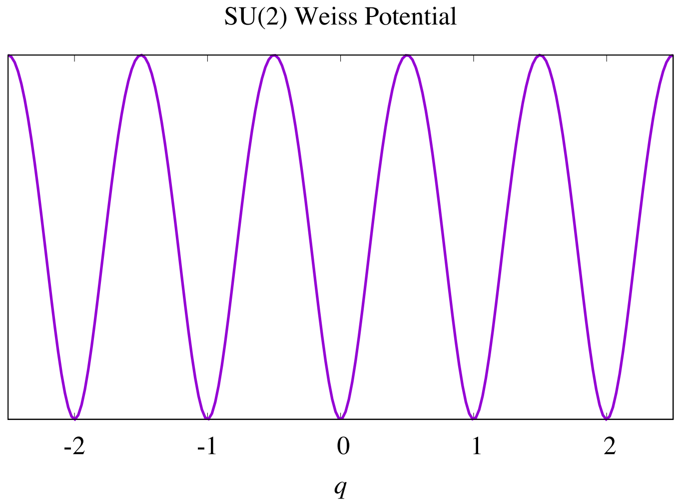
More "deconfined" for farer from the center

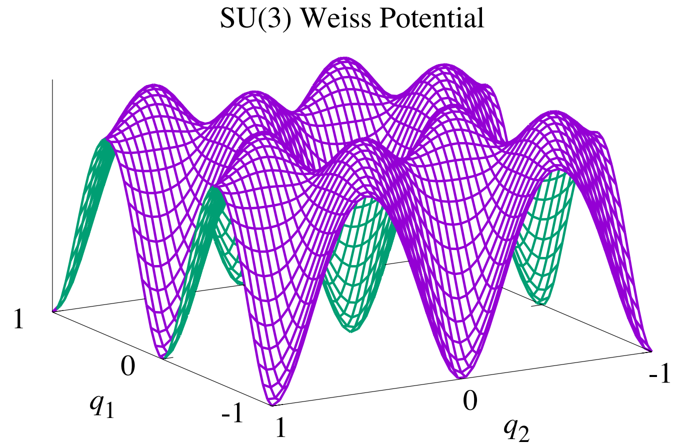
→ Real rotation would favor "confinement" ?

Weiss-GPY Potential

Polyakov loop ~ Color imaginary chemical potential

$$V_{\mathrm{glue}}[q] = 2V \int \frac{d^3p}{(2\pi)^3} \sum_{i>j} \left[\ln\left(1 - e^{-\beta|\boldsymbol{p}|+2\pi i q_{ij}}\right) + \ln\left(1 - e^{-\beta|\boldsymbol{p}|-2\pi i q_{ij}}\right) \right]$$





Center symmetry spontaneously broken at high *T*.

Weiss-GPY Potential

Rotating Weiss-GPY potential

Chen-Fukushima-Shimada (2022)

$$V = \frac{T}{4\pi^2}\sum_{\boldsymbol{\alpha}}\sum_{m\in\mathbb{Z}}\int_0^\infty k_\perp dk_\perp \int_{-\infty}^{\infty} dk_z \left[\underline{J^2_{m-1}(k_\perp r)} + \underline{J^2_{m+1}(k_\perp r)}\right]\mathrm{Re}\ln\left[1 - e^{-\left(\underline{|\vec{k}| - i\Omega_{\mathrm{I}} m}\right)/T + \underline{i\boldsymbol{\phi}\cdot\boldsymbol{\alpha}}}\right]$$

Helicity sum

$\boldsymbol{H - J\cdot\Omega}$

Polyakov loop background

Weiss-GPY Potential

Rotating Weiss-GPY potential

Chen-Fukushima-Shimada (2022)

$$V_g(\boldsymbol{\phi};\tilde{\Omega}_\mathrm{I}) = -\frac{2T^4}{\pi^2}\sum_{\alpha\in\Phi}\sum_{n=1}^{\infty}\frac{\cos(n\boldsymbol{\phi}\cdot\boldsymbol{\alpha})\cos\left(n\tilde{\Omega}_\mathrm{I}\right)}{\left\{n^2+2\tilde{r}^2\left[1-\cos\left(n\tilde{\Omega}_\mathrm{I}\right)\right]\right\}^2}$$

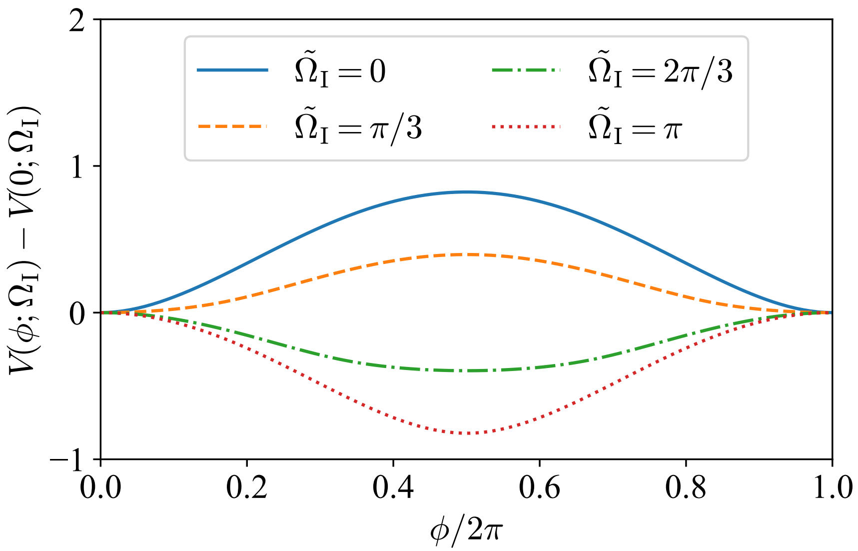
$$\tilde{\Omega}_\mathrm{I} = \Omega_\mathrm{I}/T\,,\quad \tilde{r} = rT$$

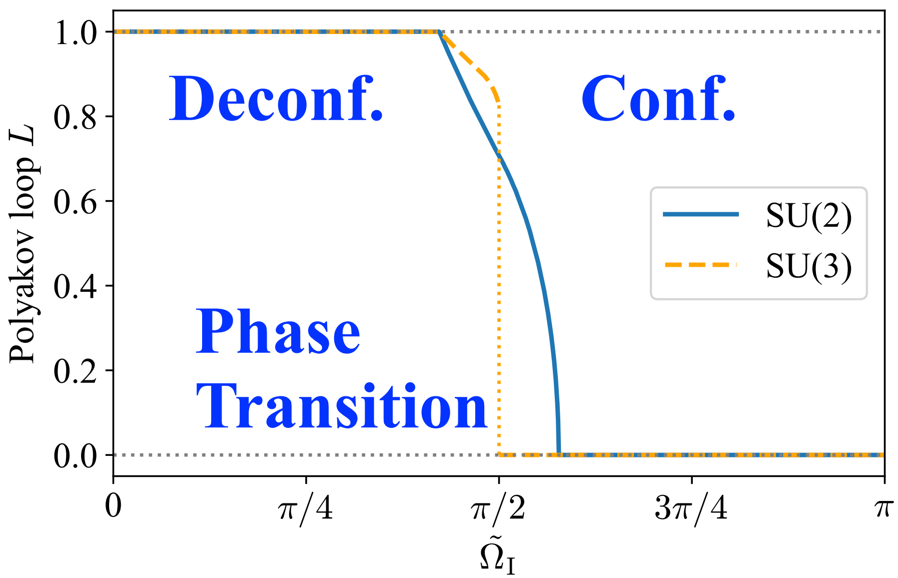
For any small *r*, the denominator can vanish for some complex angular velocity — Causality Singularity

Weiss-GPY Potential

Rotating Weiss-GPY potential

Chen-Fukushima-Shimada (2022)

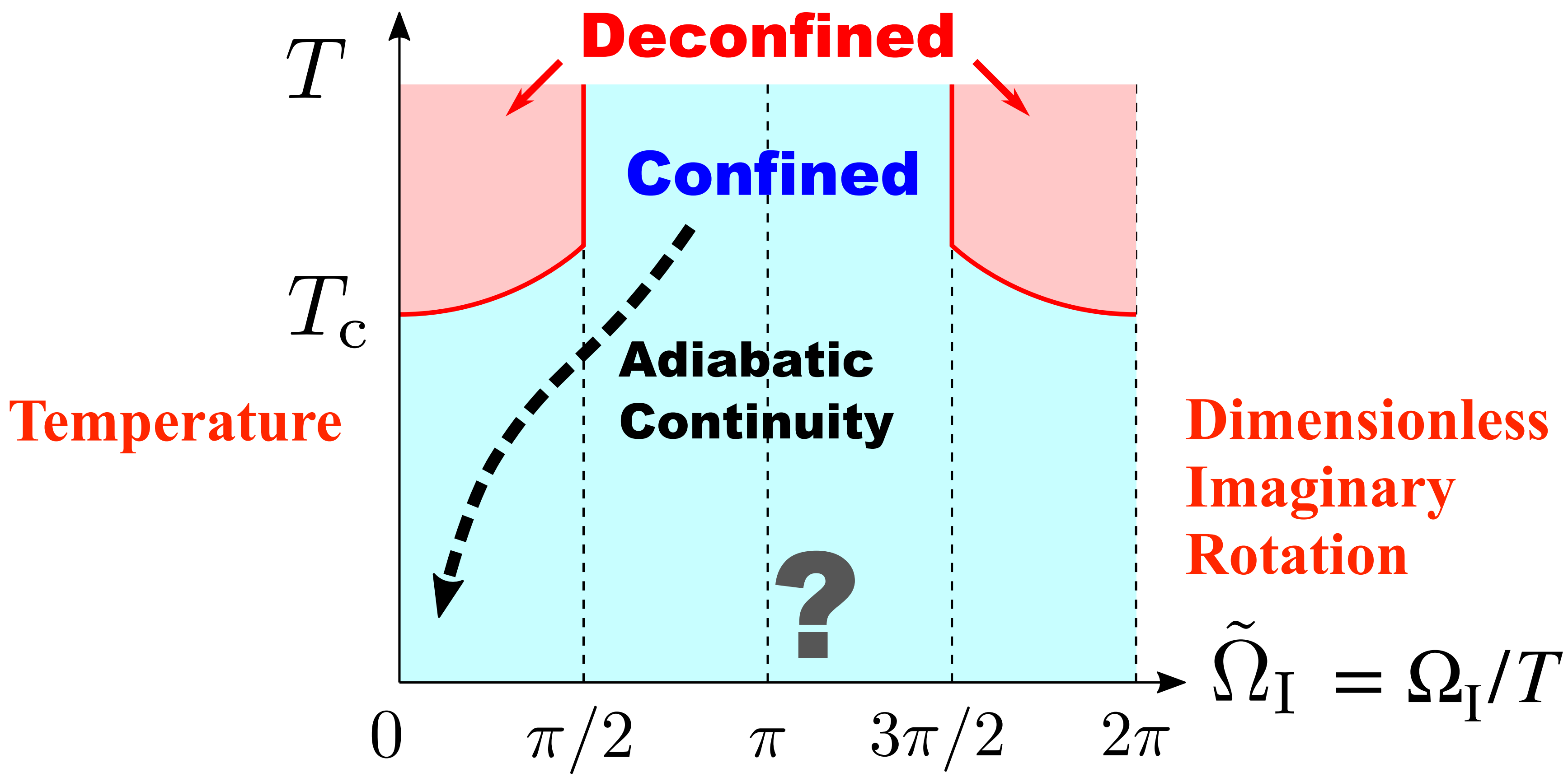




Gluons with $\tilde{\Omega}_I = \pi$ look like adjoint fermions:
$n_B(\varepsilon + i\pi T) = -n_F(\varepsilon)$ favoring confinement.

Weiss-GPY Potential

Phase diagram of rotating gluonic matter (no matter)



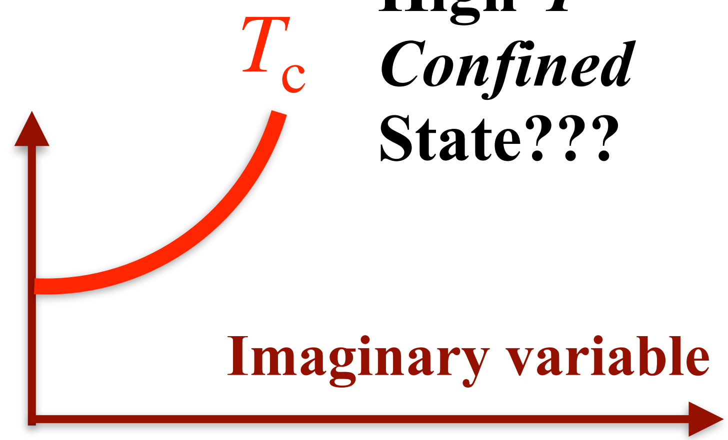
Chen-Fukushima-Shimada (2022)

Interpretation

Expected behavior of phase transition

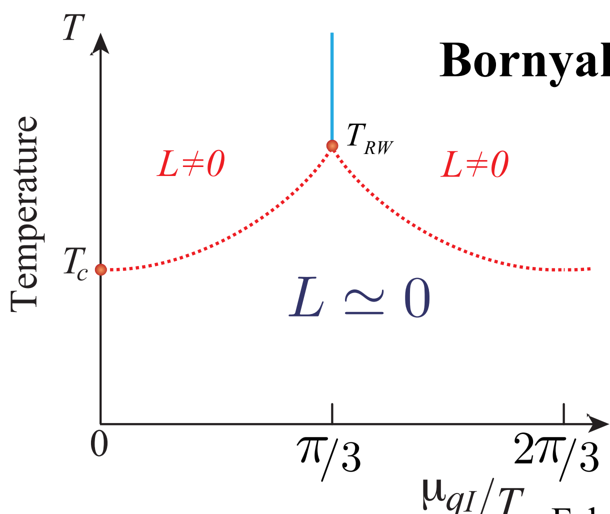
$$T_c(\Omega_I^2) \leftrightarrow T_c(-\Omega^2)$$

Increasing Function — Decreasing Function



High-*T* *Confined* State???

cf. Imaginary μ

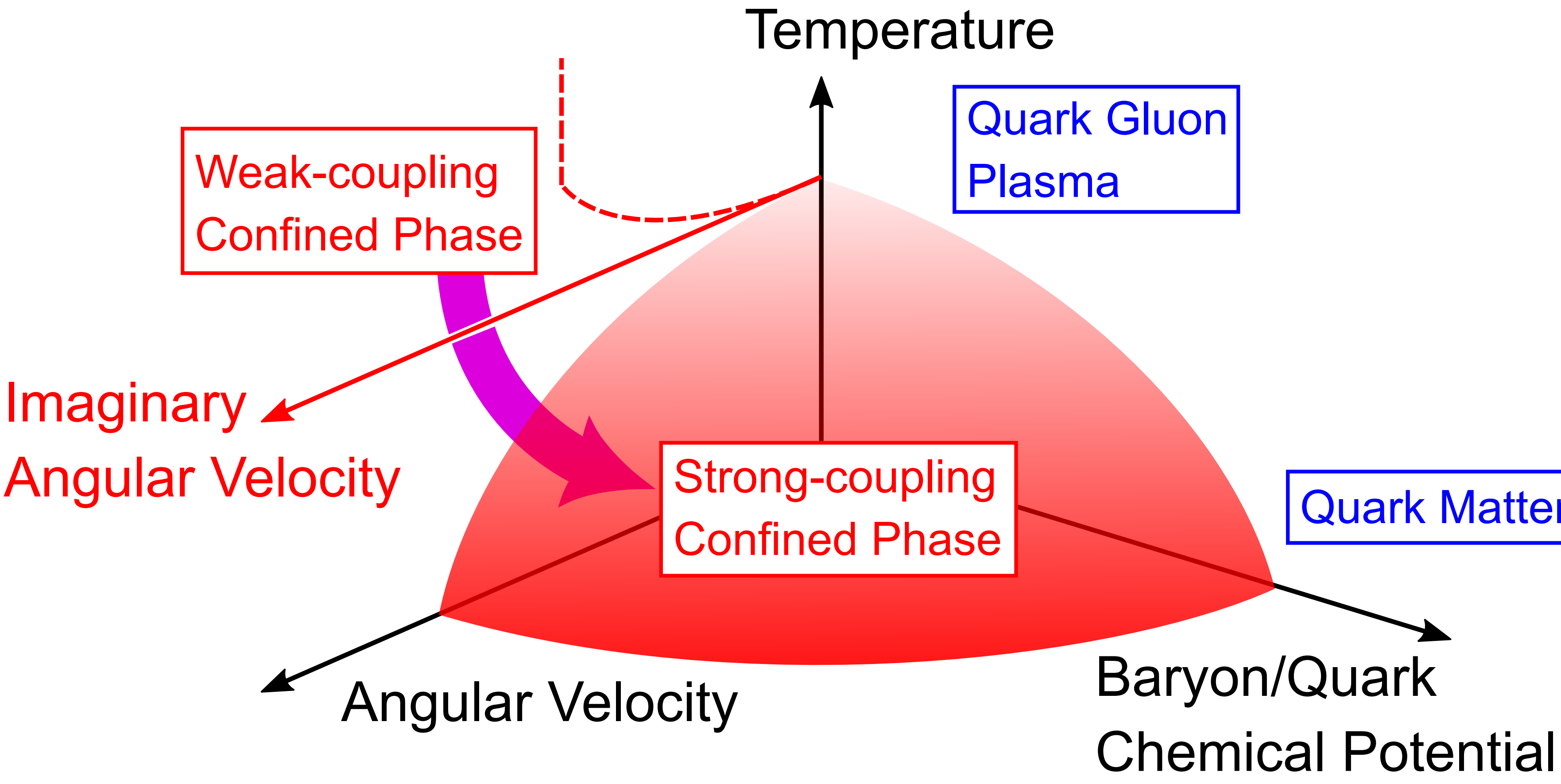


Bornyakov et al.

Before reaching high-T confinement, the periodicity by π/N_c prevents.

Interpretation

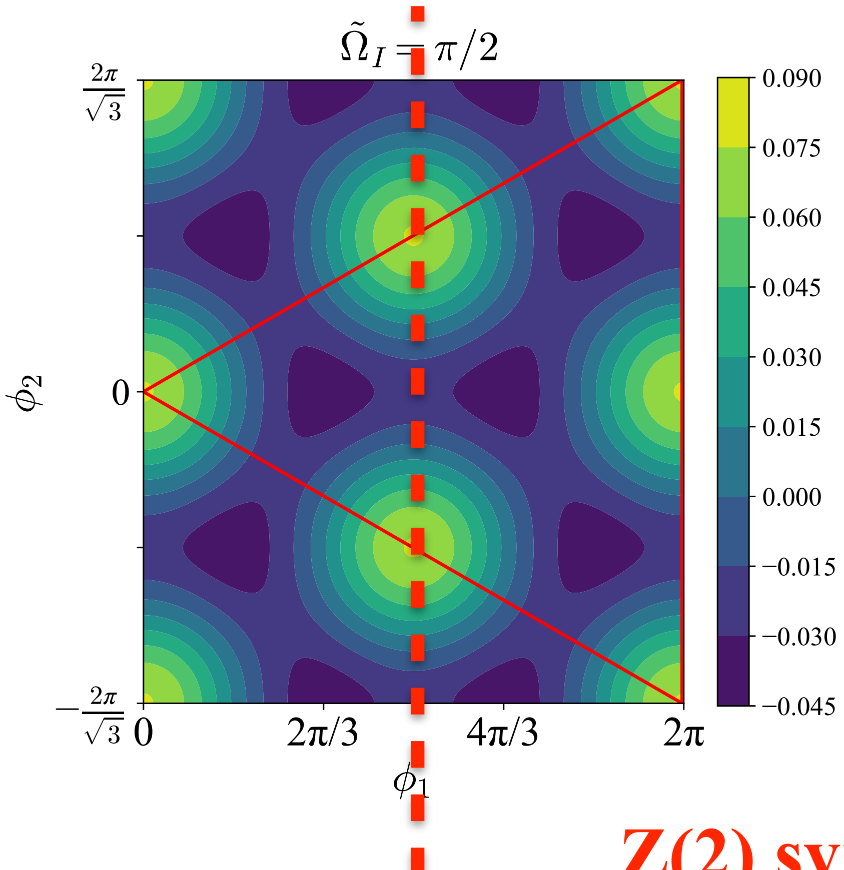
New approach to confinement physics

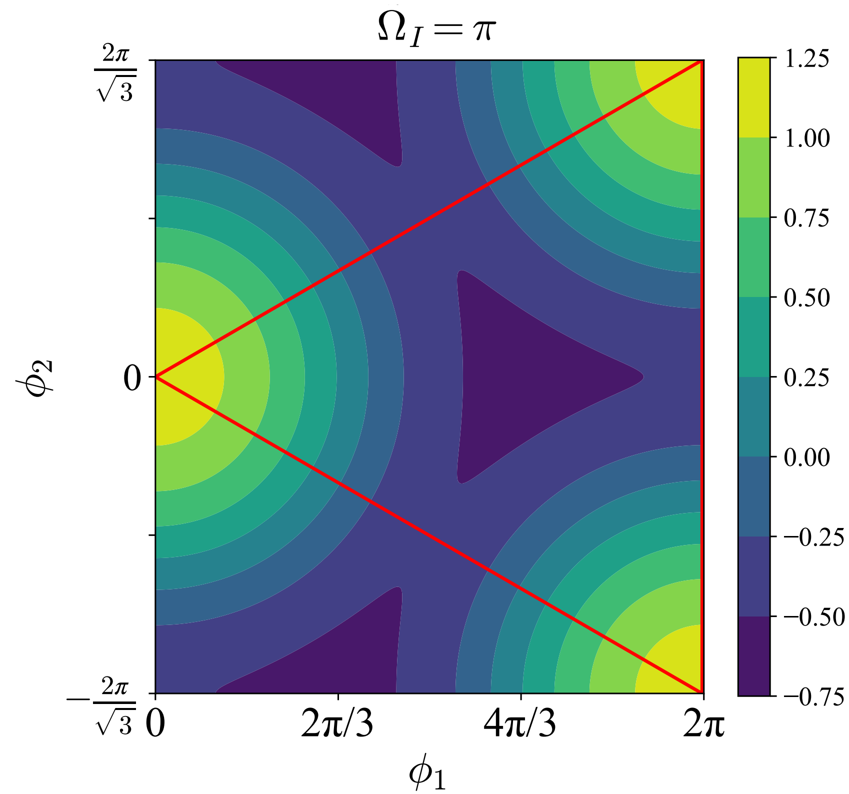


Interpretation

Accidental symmetry?

Chen-Fukushima-Shimada (2024)

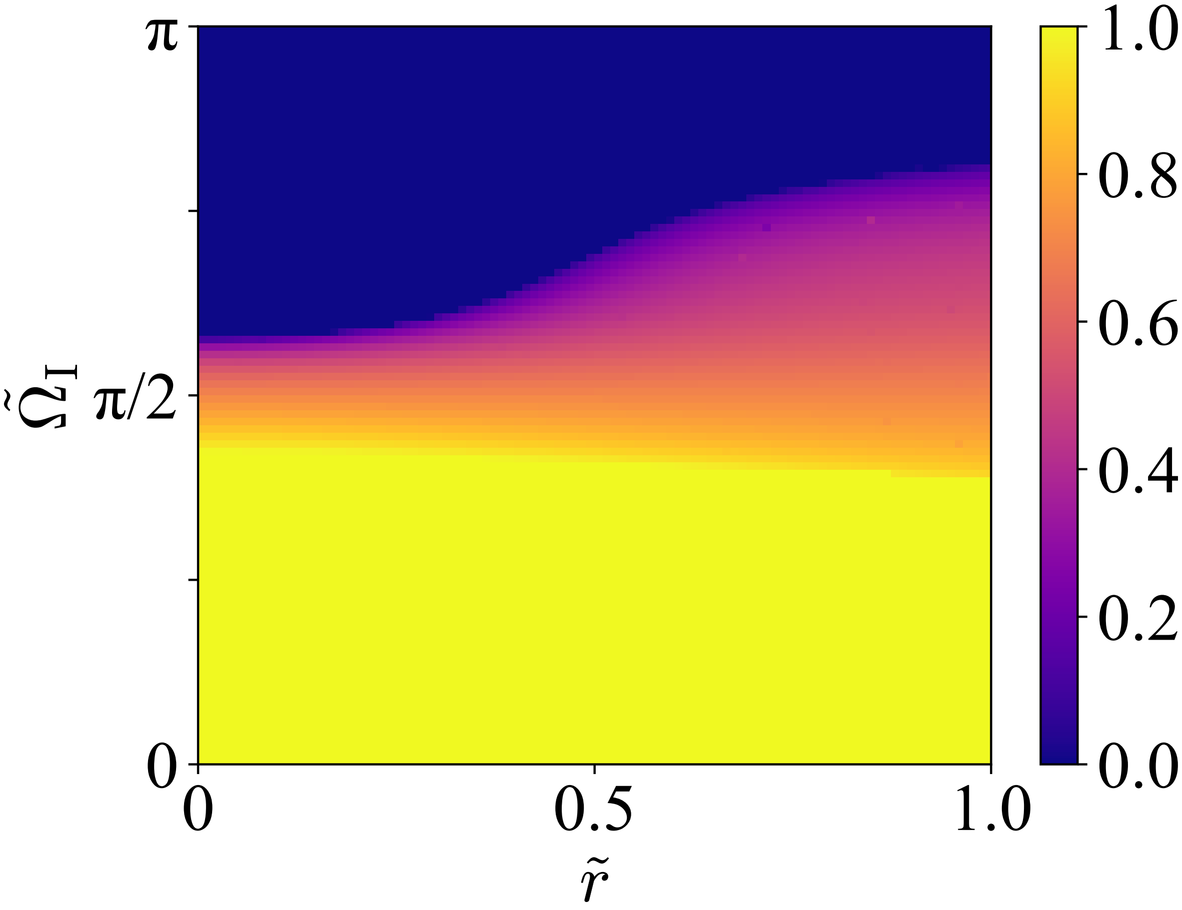




Z(2) symmetry that didn't exist…?

Inhomogeneity

Chen-Fukushima-Shimada (2024)



Dark color
~ Small PL
~ More Confined

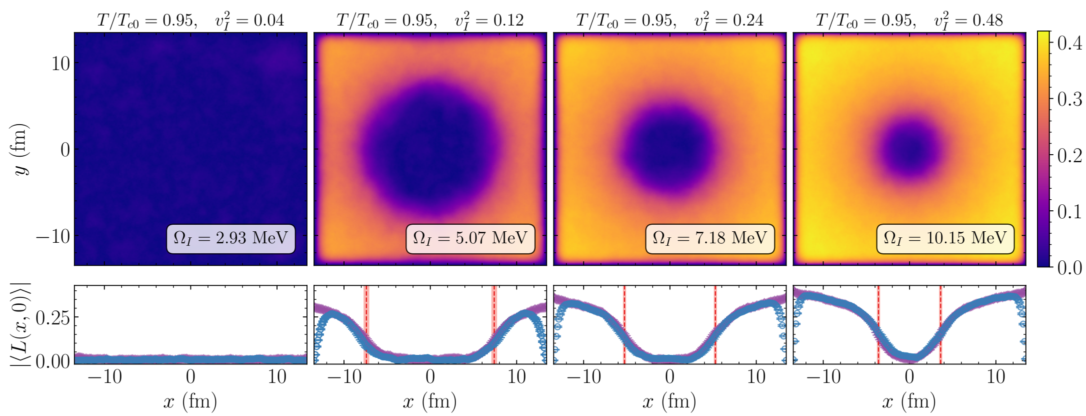
Bright color
~ Large PL
~ Deconfined

This is SU(2) and SU(3) is qualitatively the same.

Inhomogeneity

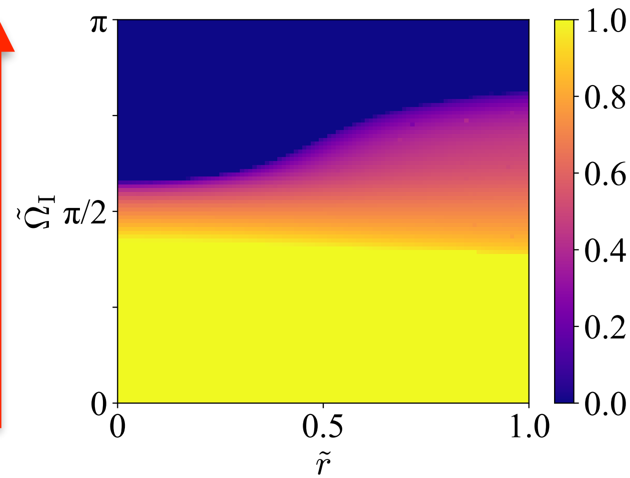
A tension of conflicting lattice…

Braguta-Chernodub-Roenko, PLB ('23) **Chen-Fukushima-Shimada, PLB ('24)**



Larger Imaginary Rotation

Darker ~ More Confined

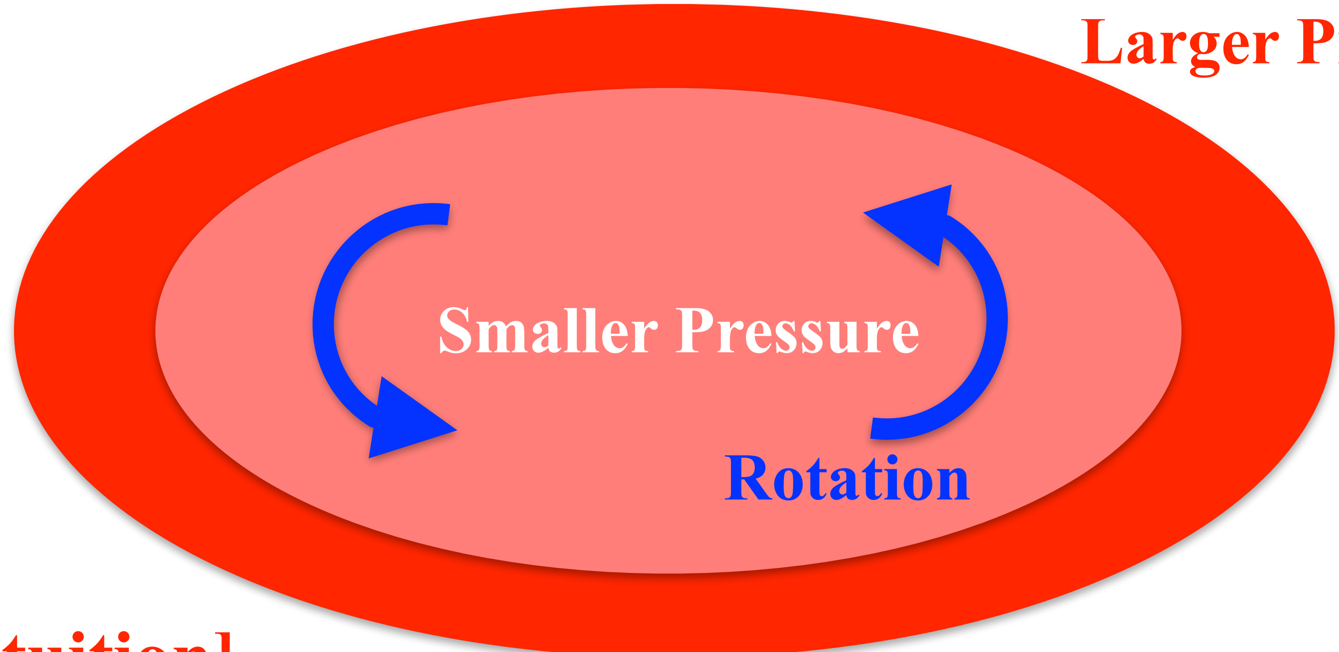


Non-perturbative Lattice
at $T \sim T_c$ and $\Omega_I \ll T$

Perturbative Calc
at $T \gg T_c$ and $\Omega_I > T$

Inhomogeneity

Both are very counter-intuitive!



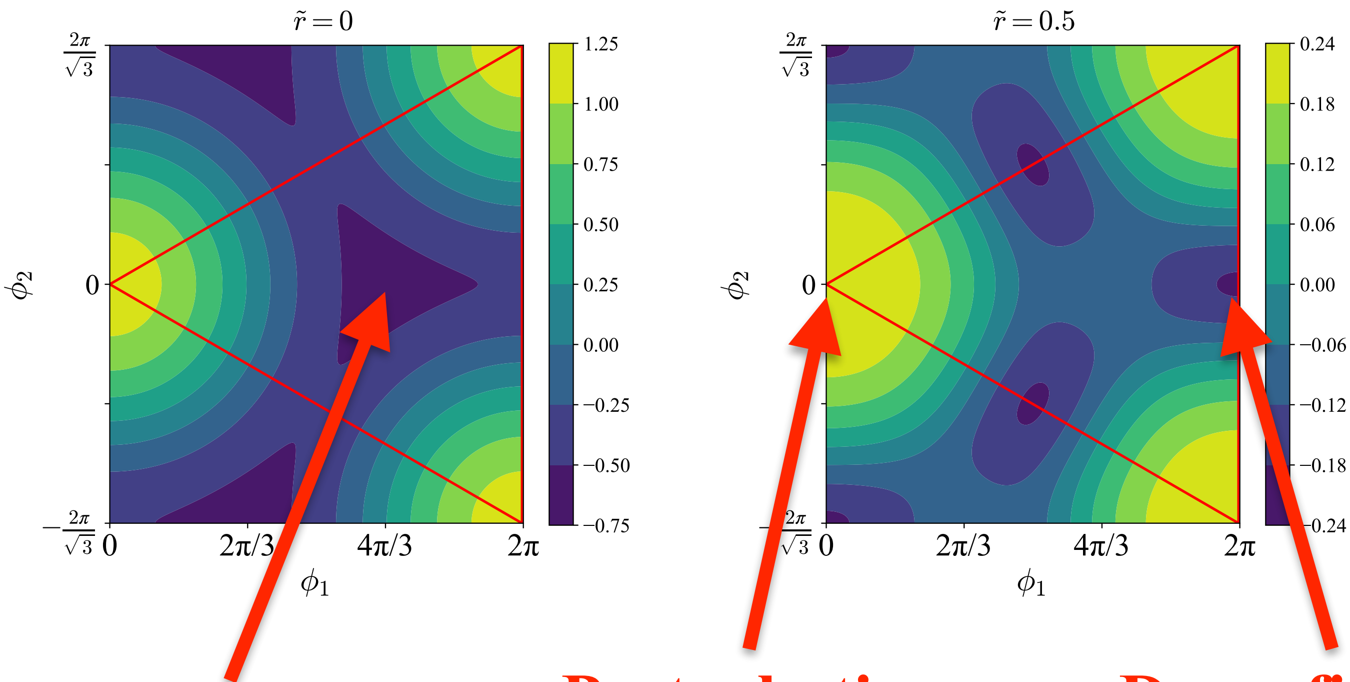
[Intuition]

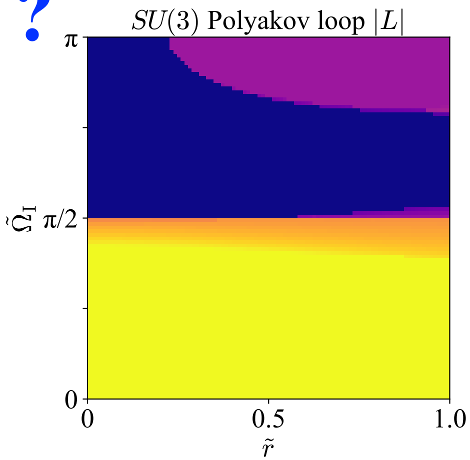
Larger *P* at outer ~ Higher *T* ~ More Deconfined

[Lattice/perturb.] **More Confined at outer…?**

Inhomogeneity

Unconventional "deconfined" phase ?





Confining Point

Perturbative vacuum still unstable

Deconfined but nontrivial PL background…?

Chiral Symmetry

Chen-Fukushima-Shimada (2024)

Adding “free” fermions with dynamical mass

$$\mathcal{Z}_{\mathrm{f}T,\omega} = \mathrm{Det}(\gamma^\mu G_{\mathrm{B}\,\mu} + m)$$

Search for the potential minimum of the Polyakov loop and the dynamical mass.

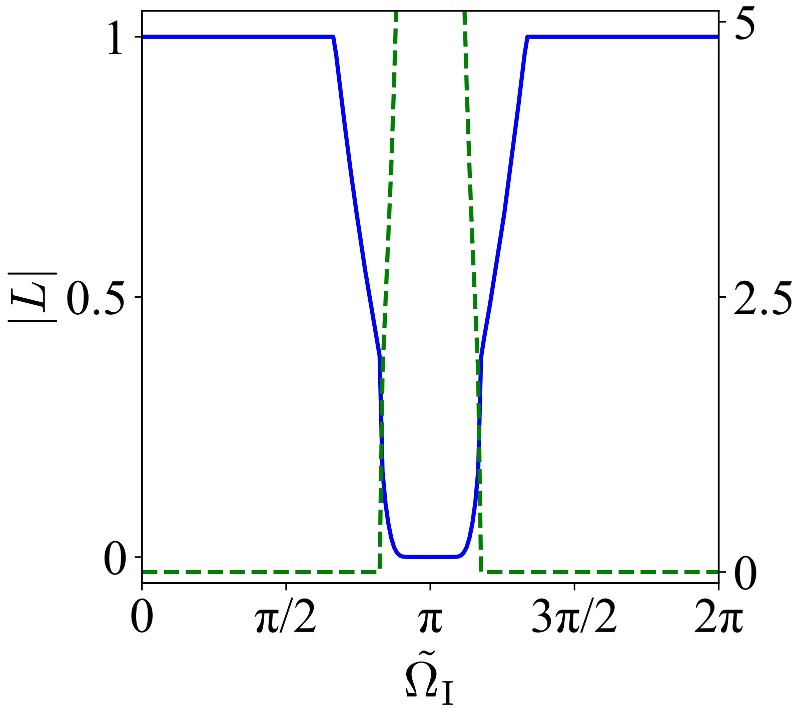
Once symmetry breaking is turned on, the mass blows up.

We may introduce a model such as NJL, but this is the most model-independent set-up.

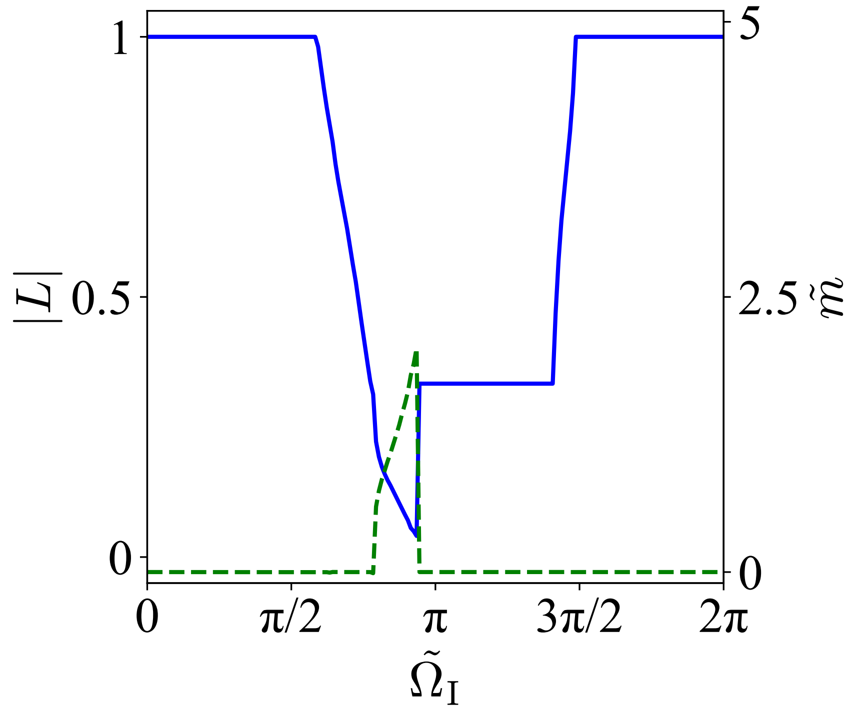
Chiral Symmetry

Chen-Fukushima-Shimada (2024)

SU(2) full (2 flavor)



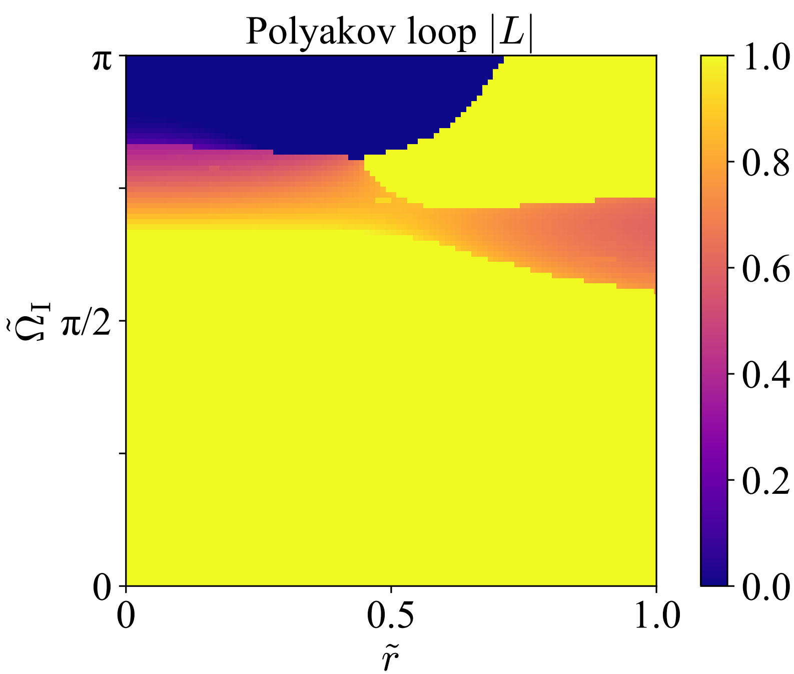
SU(3) full (2 flavor)



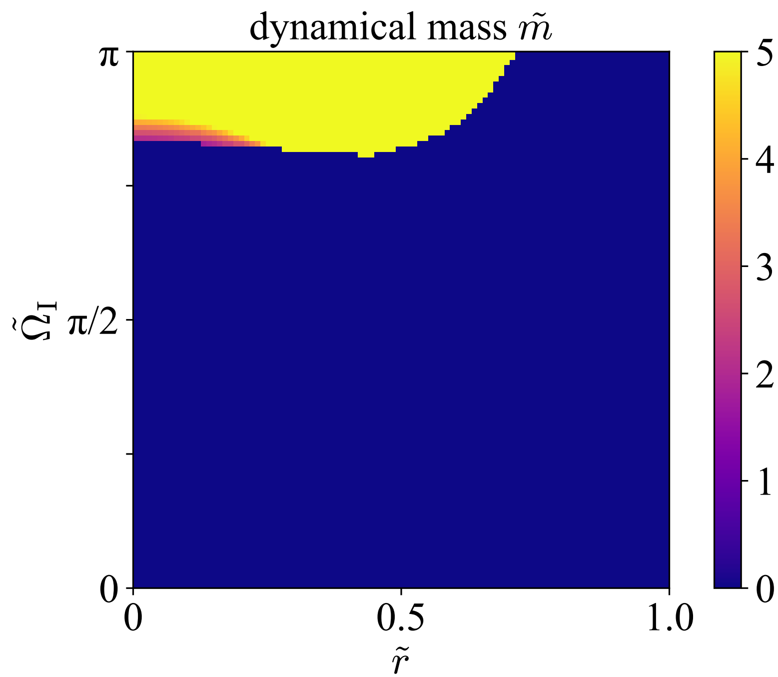
Almost correlated… but SU(3) looks horrible…

Chiral Symmetry

Polyakov loop



Chiral condensate

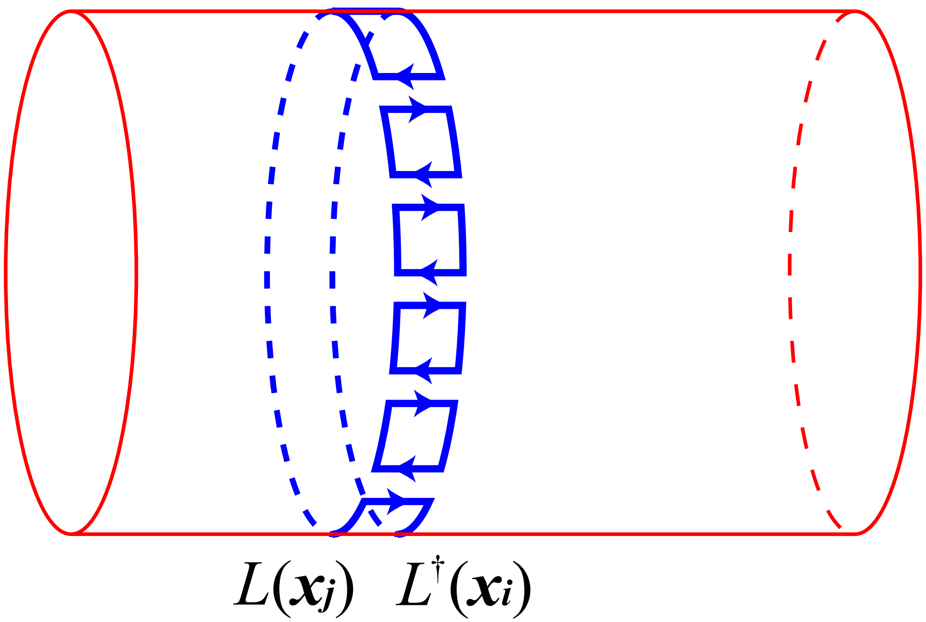


Chen-Fukushima-Shimada (2024)

Fermion mass and the PL are strongly correlated but something funny in the intermediate regions…

Strong Coupling Analysis

Fukushima-Shimada (soon)



Disordered Phase
~ Smaller *J*
~ More Confined

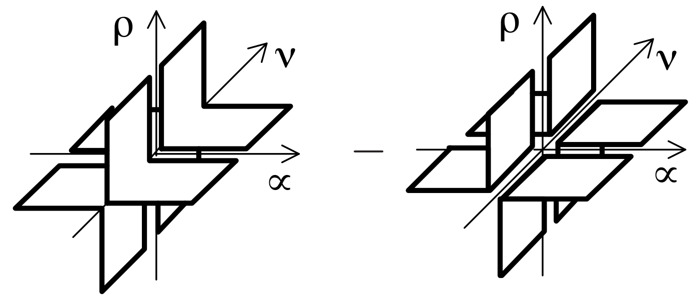
$$S \sim J \sum_{\text{n.n.}} \mathrm{tr} L_n \mathrm{tr} L_{n'}$$

Rotation correction?

Strong Coupling Analysis

Hirono-Yamamoto (2013) **Fukushima-Shimada (soon)**

$$
\begin{aligned}
S_G = \sum_x \beta \Big[& (1+r^2\Omega^2)\left(1-\frac{1}{N_c}\mathrm{Re\,tr}\bar{U}_{xy}\right) \\
& + (1+y^2\Omega^2)\left(1-\frac{1}{N_c}\mathrm{Re\,tr}\bar{U}_{xz}\right) \\
& + (1+x^2\Omega^2)\left(1-\frac{1}{N_c}\mathrm{Re\,tr}\bar{U}_{yz}\right) \\
& + 3 - \frac{1}{N_c}\mathrm{Re\,tr}\left(\bar{U}_{x\tau}+\bar{U}_{y\tau}+\bar{U}_{z\tau}\right) \\
& - \frac{1}{N_c}\mathrm{Re\,tr}\Big(y\Omega\bar{V}_{xy\tau} - x\Omega\bar{V}_{yx\tau} \\
& + y\Omega\bar{V}_{xz\tau} - x\Omega\bar{V}_{yz\tau} + xy\Omega^2\bar{V}_{xzy}\Big)\Big].
\end{aligned}
$$



Leading correction to *J* is NEGATIVE.

Imaginary rotation more confinement!

Summary (Real Rotation)

- **Controversies: Real rotation causes…**
  - □ Decreasing T_c in the matter sector. ← **Okay**
  - □ Increasing T_c in the gluonic sector. ← **Conflicting**
  - □ Increasing T_c in the full theory. ← **Subtle**

- **Controversies: Inhomogeneity patter shows…**
  - □ More confinement at outer regions. ← **Theory**
  - □ Which is more natural? More deconfinement?

- **Controversies: Chiral condensate exhibits…**
  - □ Something too complicated to be true? ← **No lattice yet**