

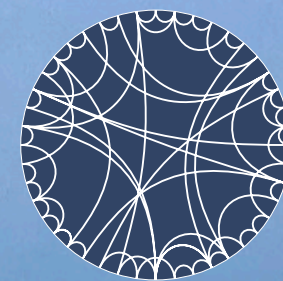
The QCD phase structure from first principles

Jan M. Pawłowski

Universität Heidelberg & ExtreMe Matter Institute

Benasque, February 13th 2025

for the fQCD collaboration



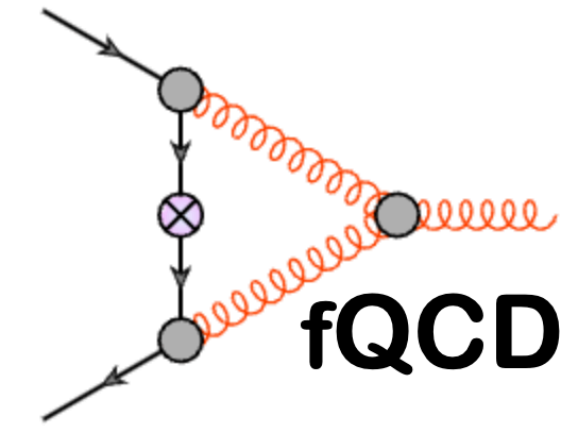
STRUCTURES
CLUSTER OF
EXCELLENCE



UNIVERSITÄT
HEIDELBERG
ZUKUNFT
SEIT 1386



fQCD collaboration



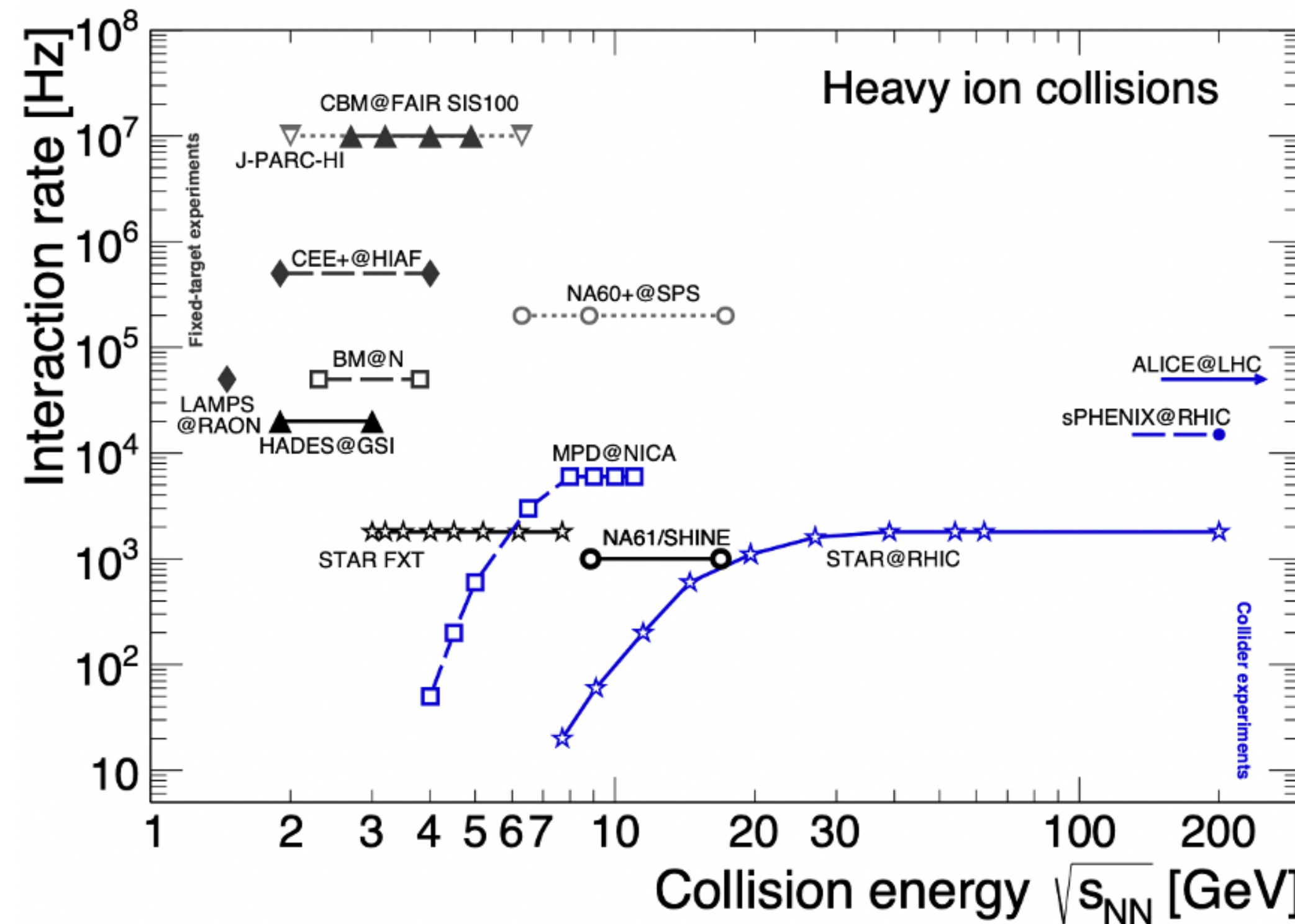
Dalian, Beijing, Darmstadt, Heidelberg, Gießen

**Braun, Chen, Fu, Gao, Geissel, Huang, Ihssen, Lu, Pawlowski, Rennecke,
Sattler, Schallmo, Tan, Töpfel, Wen, Wessely, Yin, Wang, Zorbach**

Experimental landscape

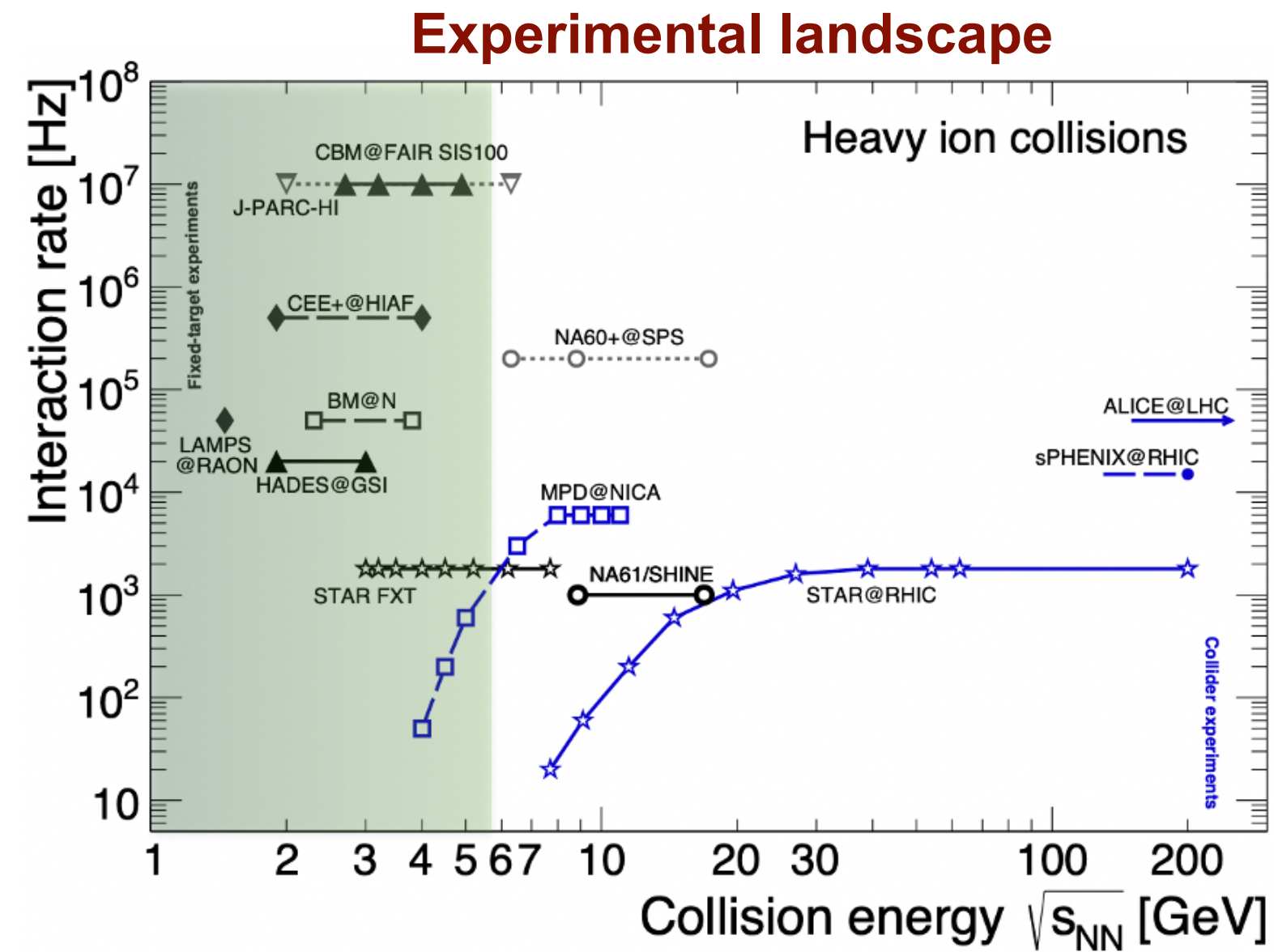
'The (experimental) future is bright'

Tetyana Galatyuk, Erice 2021



Galatyuk, A982 (2019) update 2021; CBM, EPJA 53 3 (2017) 60

How bright does it get?

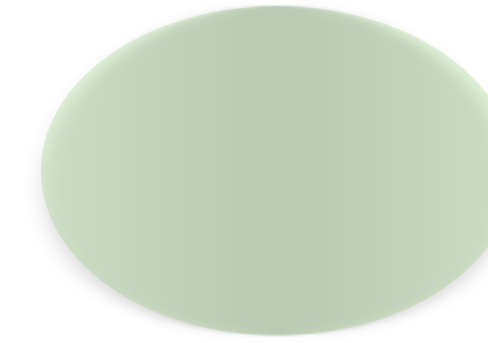


Galatyuk, A982 (2019) update 2021
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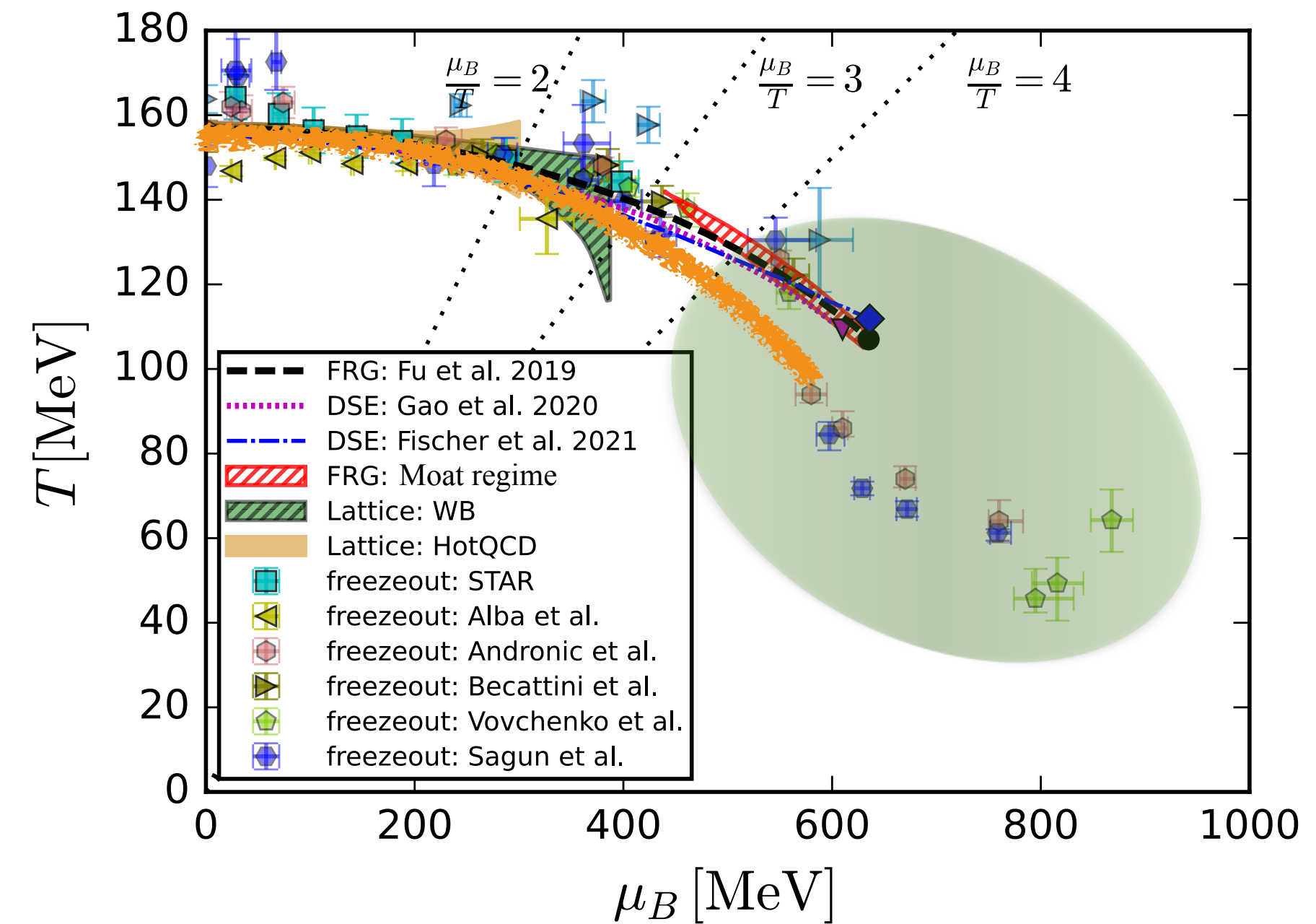
Freezeout curve

Fu, Luo, JMP, Rennecke, Wen, Yin, PRD 104 (2021) 9

CEP or other NEW physics/phases



Chiral phase structure (theory) & freeze out data (Exp. data+Pheno)

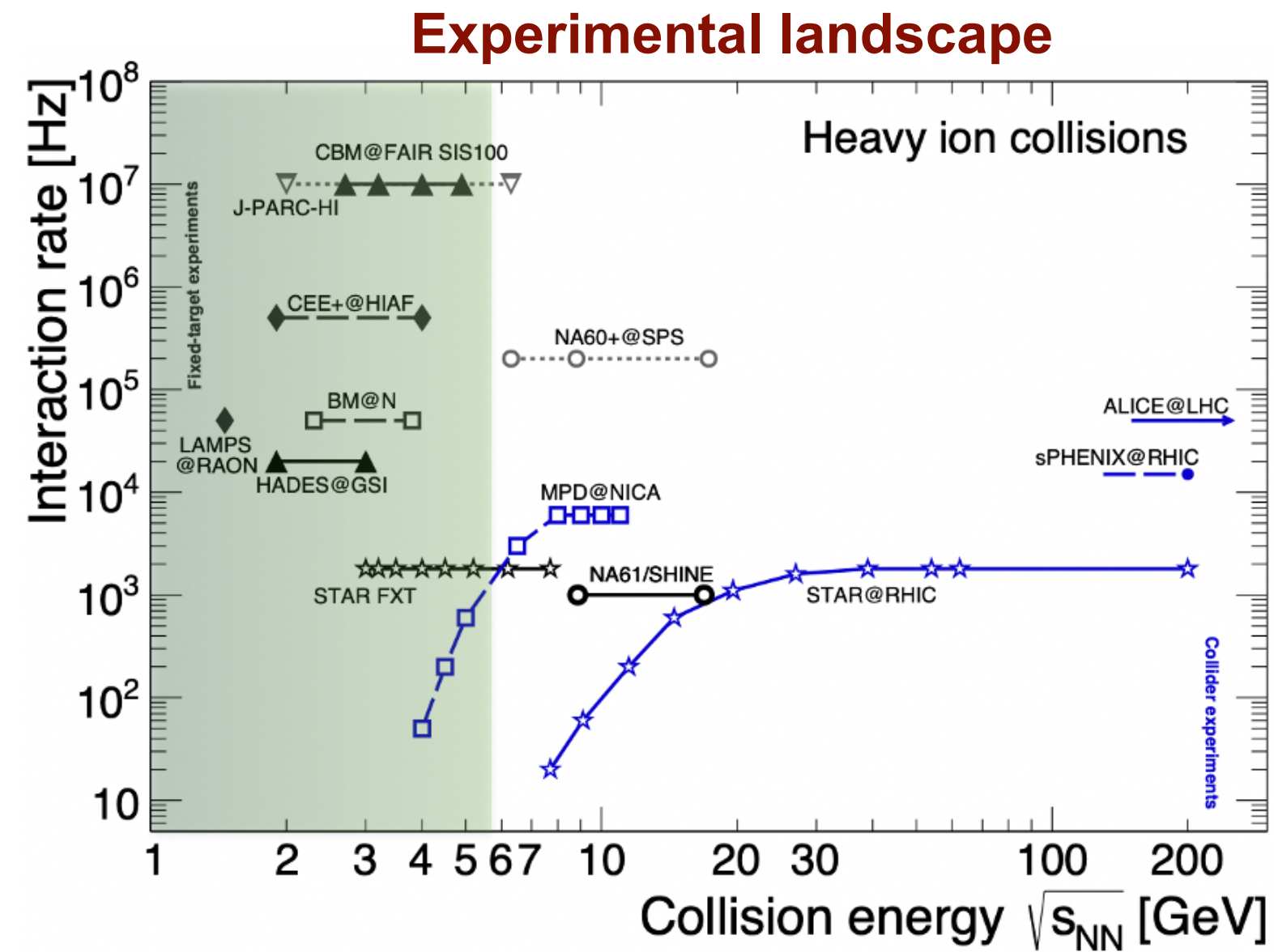


Fu, JMP, Rennecke, PRD 101 (2020) 054032

Gao, JMP, PLB 820 (2021) 136584

Gunkel, Fischer, PRD 104 (2021) 054022

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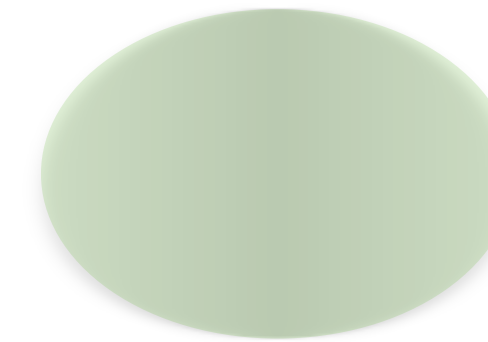
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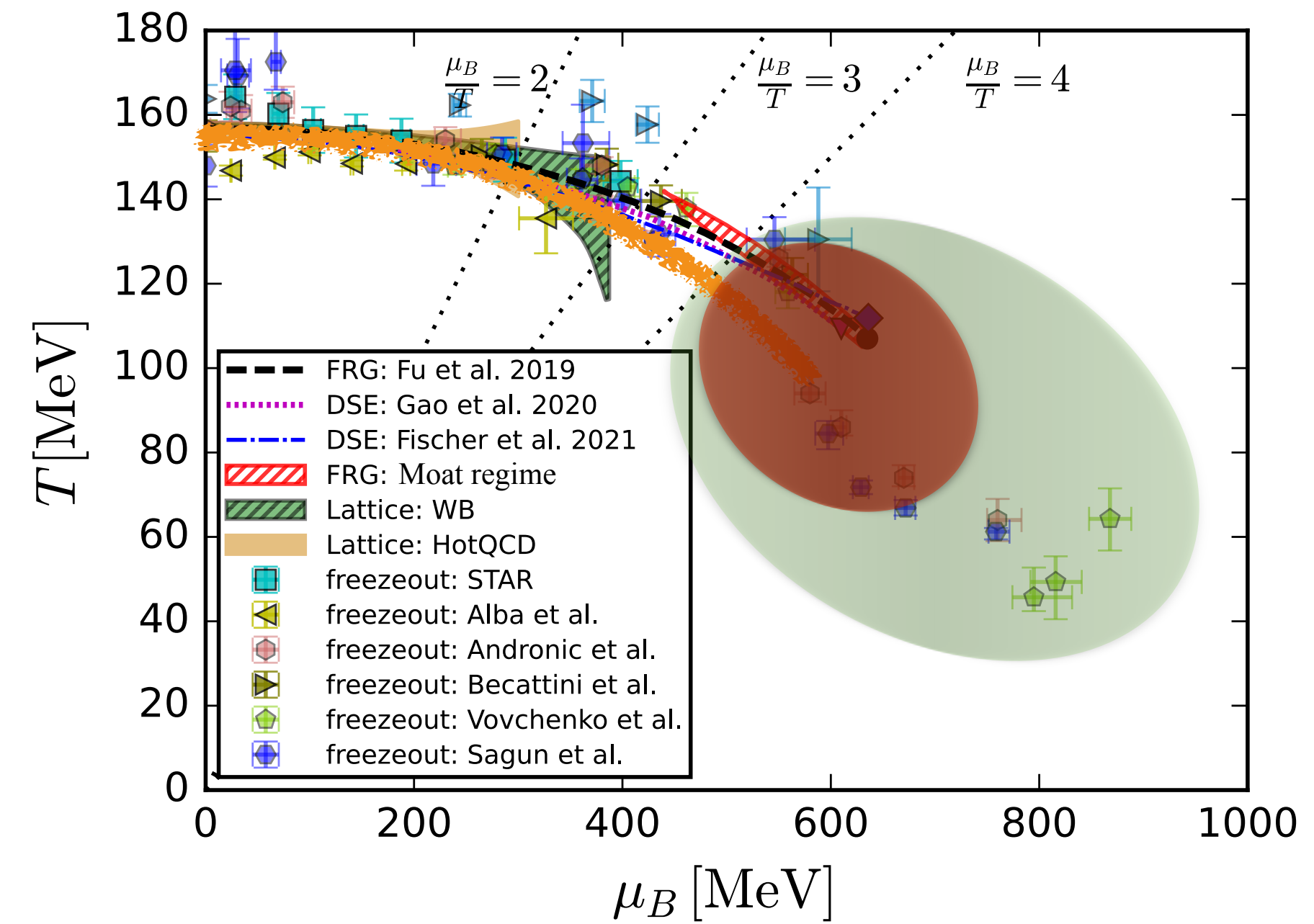
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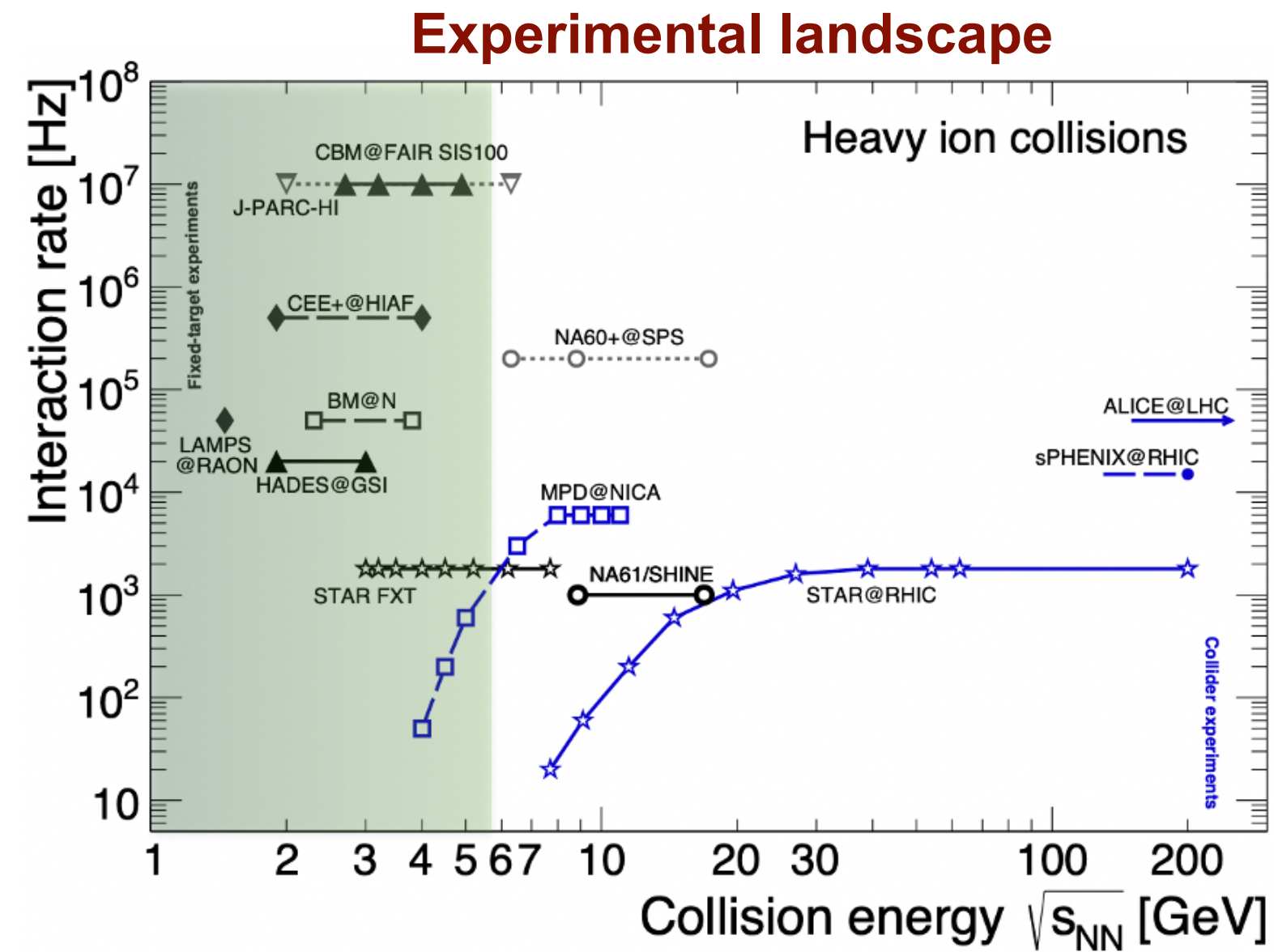


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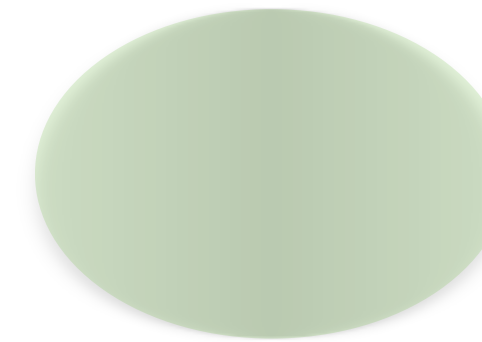
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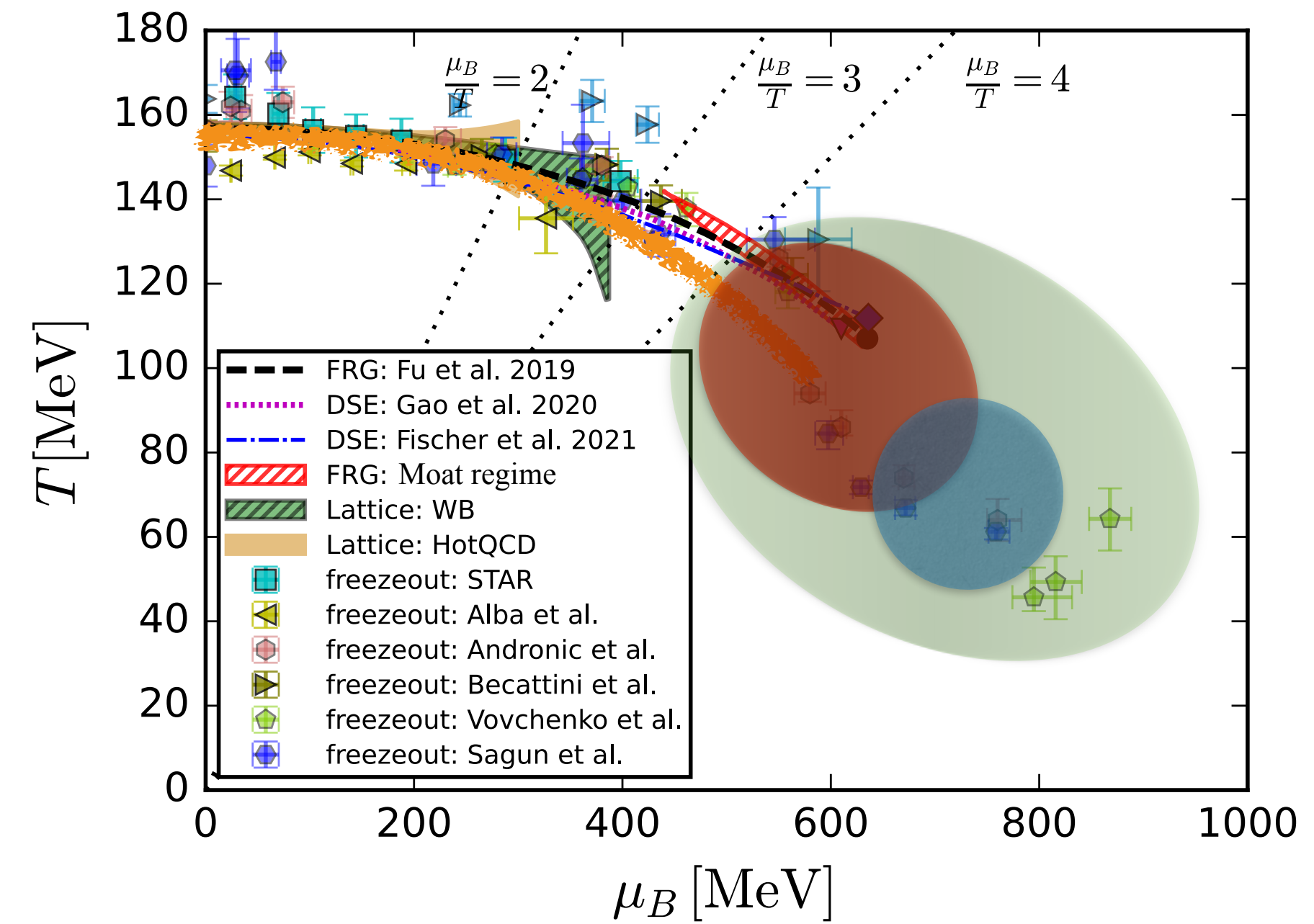
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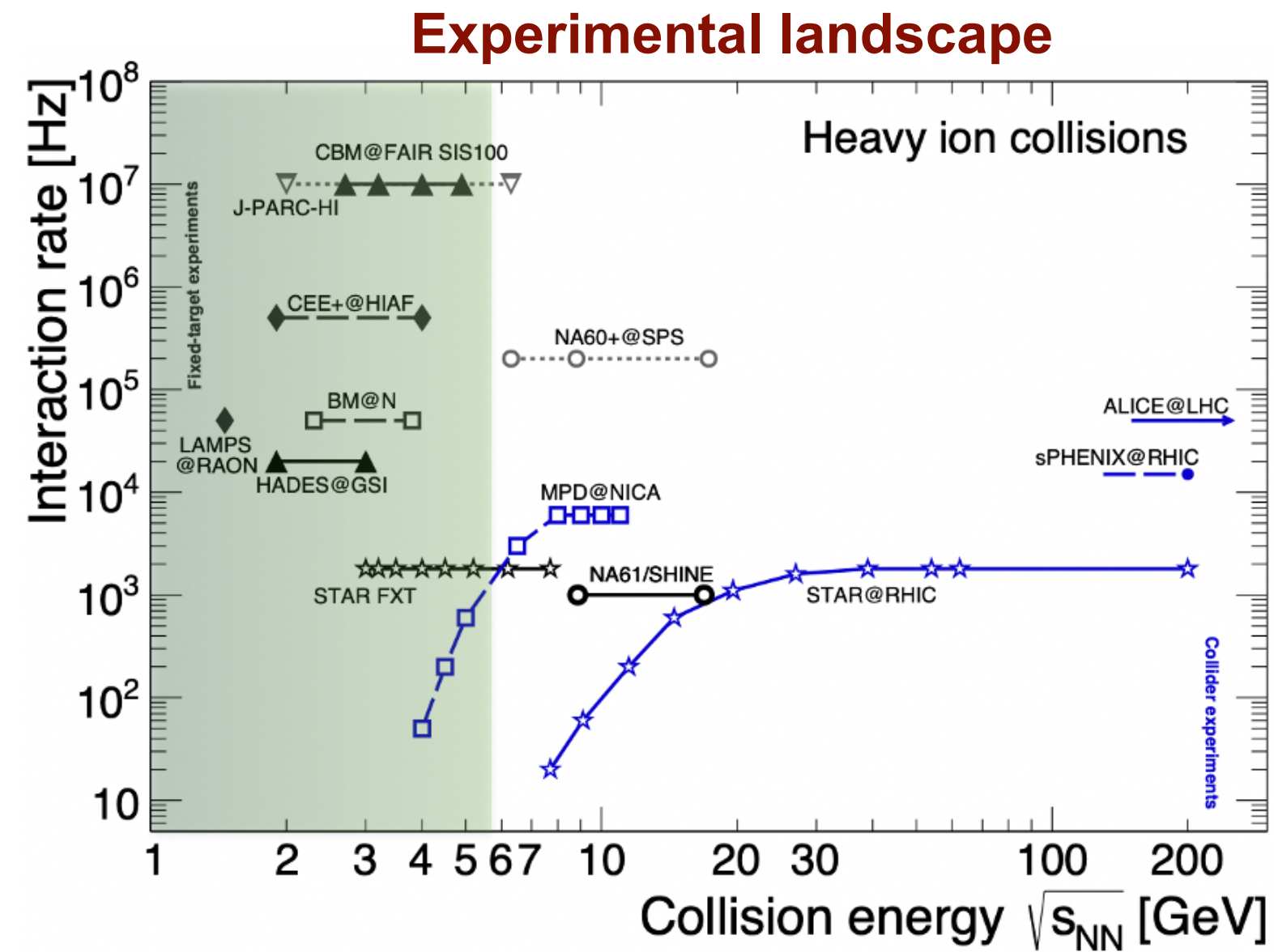


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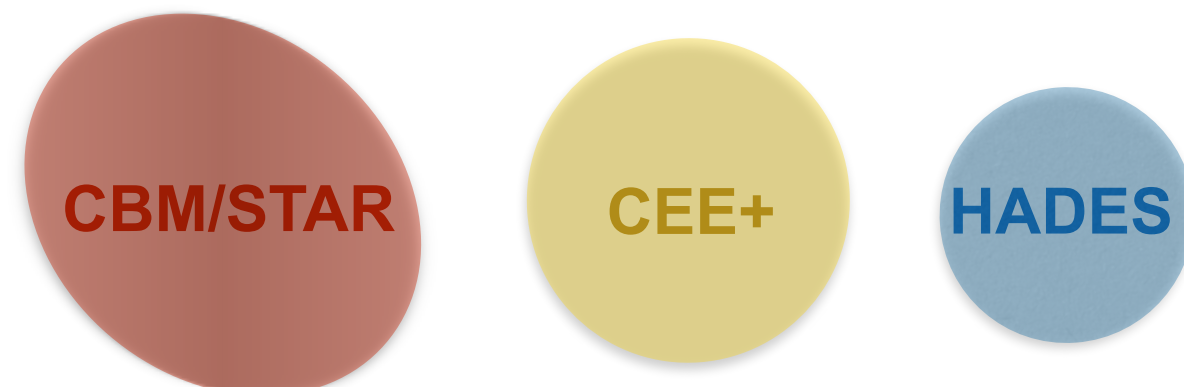
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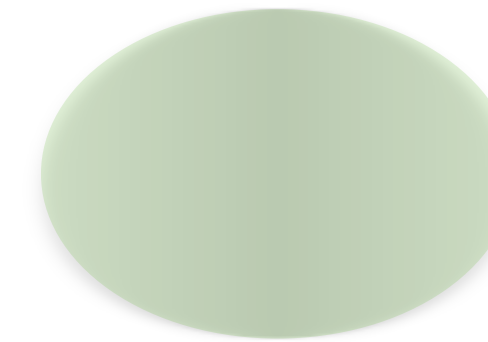
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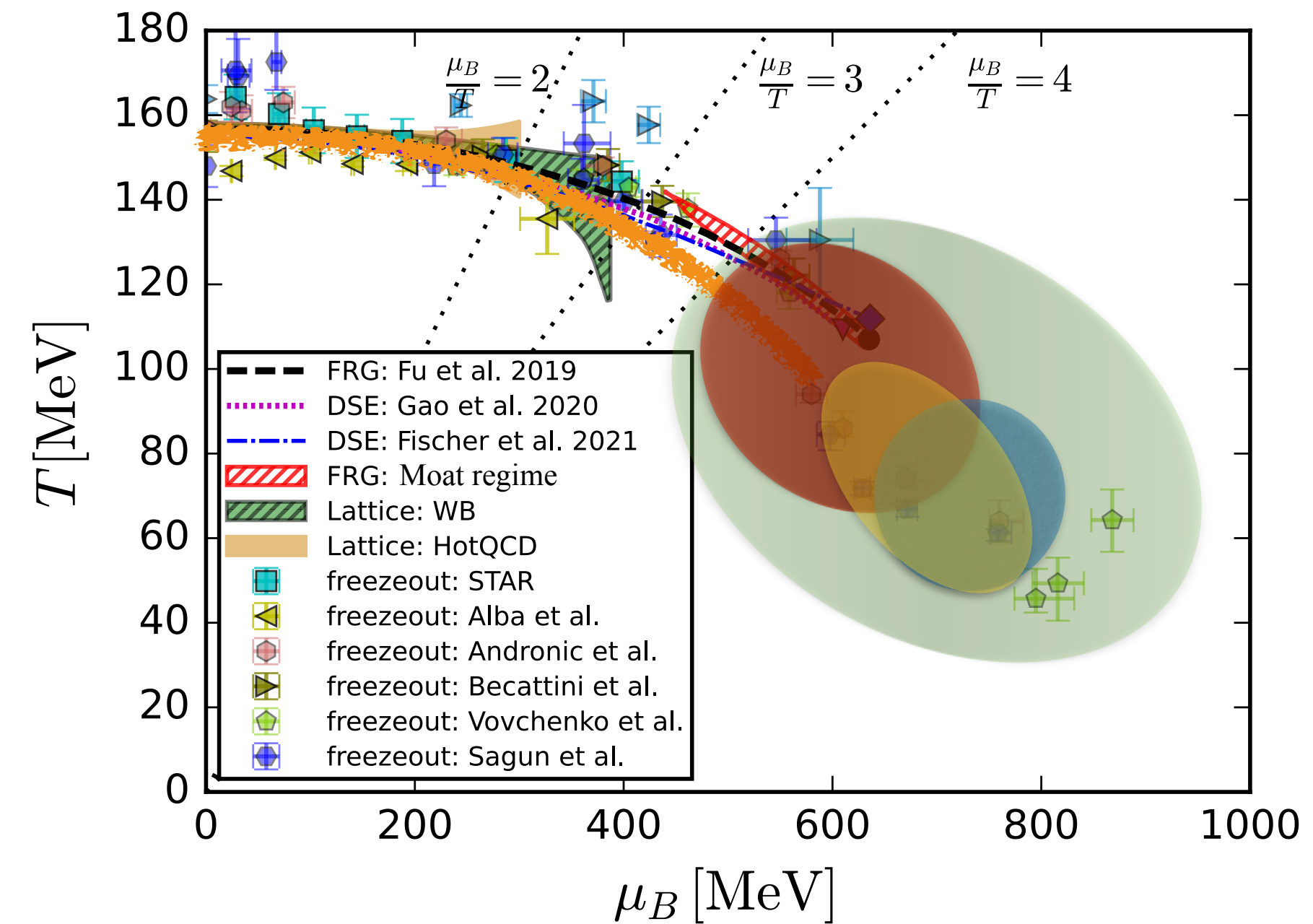
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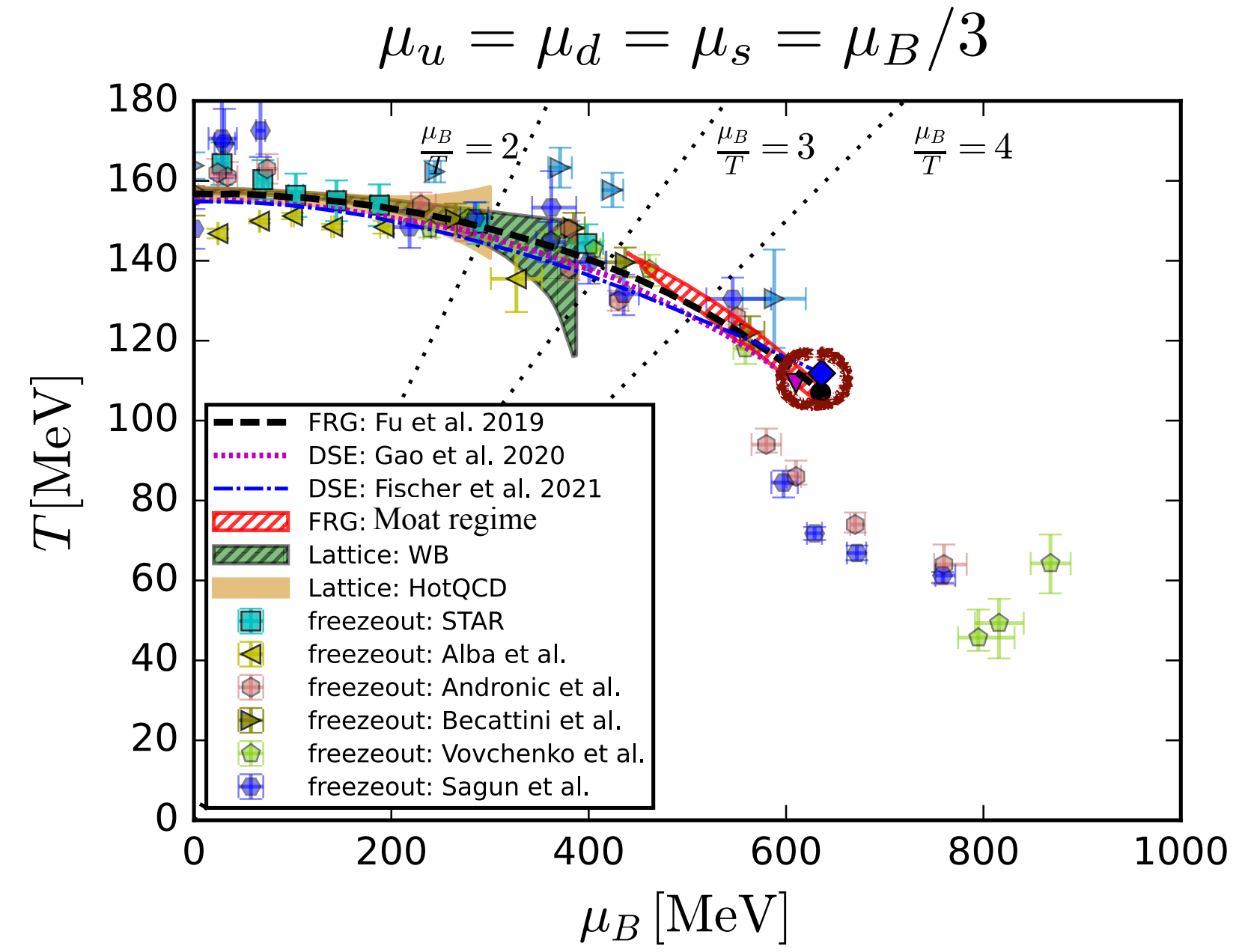


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Phase structure of QCD and the CEP



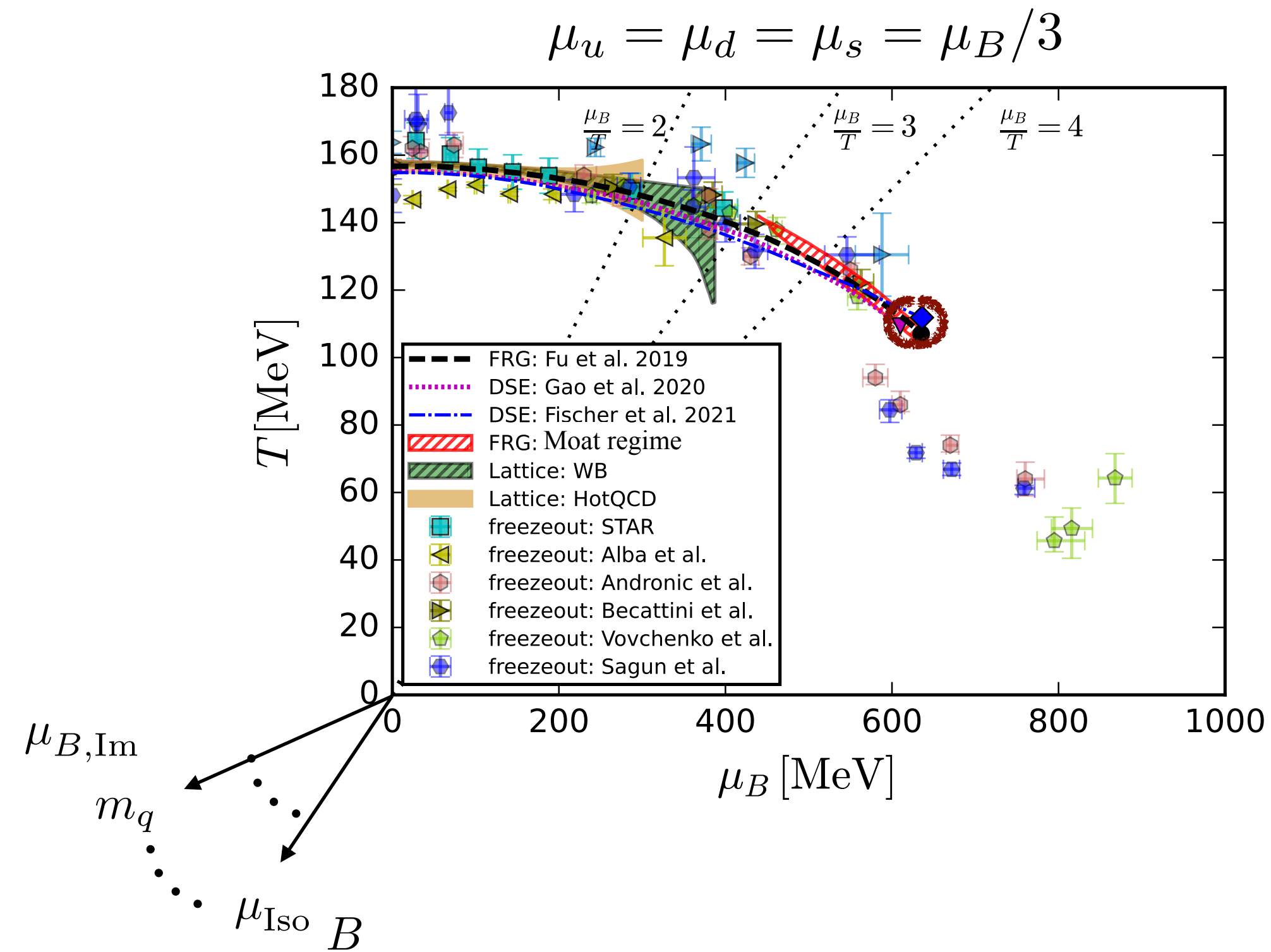
Functional QCD: CEP estimate

fRG: Fu, JMP, Rennecke, PRD 101 (2020) 054032

DSE: Gao, JMP, PLB 820 (2021) 136584
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$(\mu_B, T)_{\text{CEP}} \sim (600 - 650, 105 - 115)$ MeV

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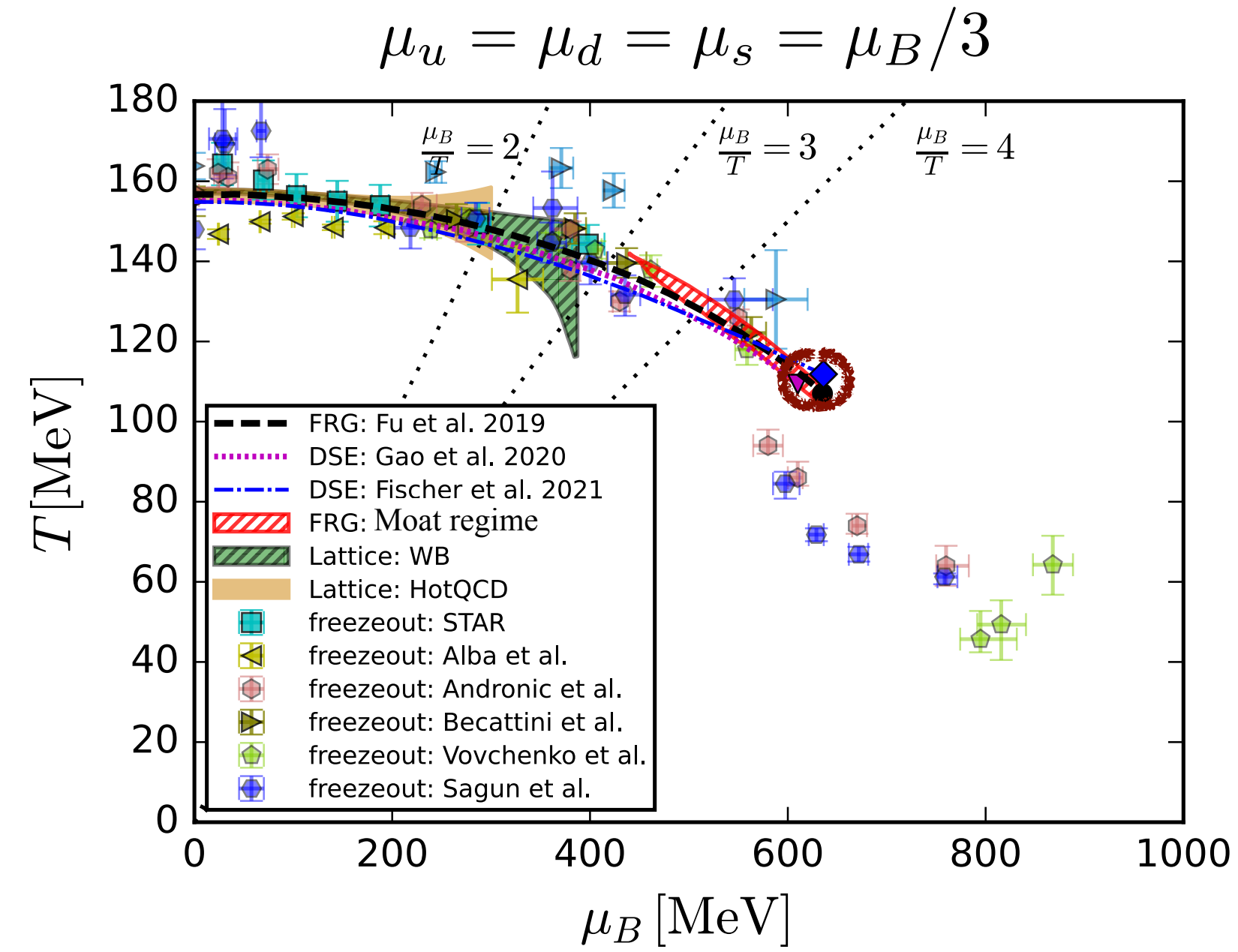
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$(\mu_B, T)_{CEP} \sim (600 - 650, 105 - 115) \text{ MeV}$

**Collect all possible information/structure
for
physics understanding & extrapolations**

Phase structure of QCD and the CEP



Functional QCD: CEP estimate

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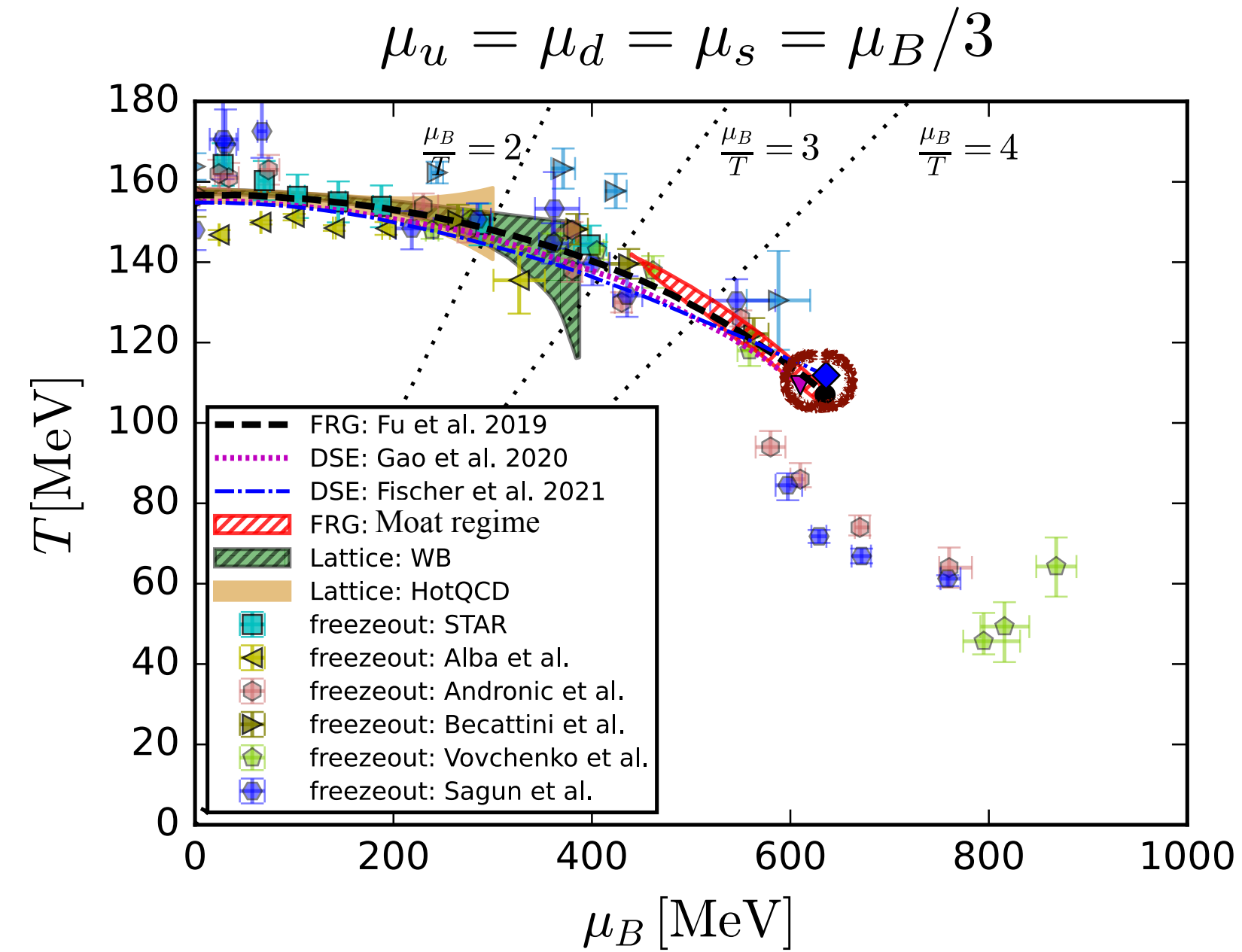
$$(\mu_B, T)_{\text{CEP}} \sim (600 - 650, 105 - 115) \text{ MeV}$$

Estimates & predictions

Requires computations in 1st principle QCD at

$$(\mu_B, T) \sim (\mu_B, T)_{\text{CEP}}$$

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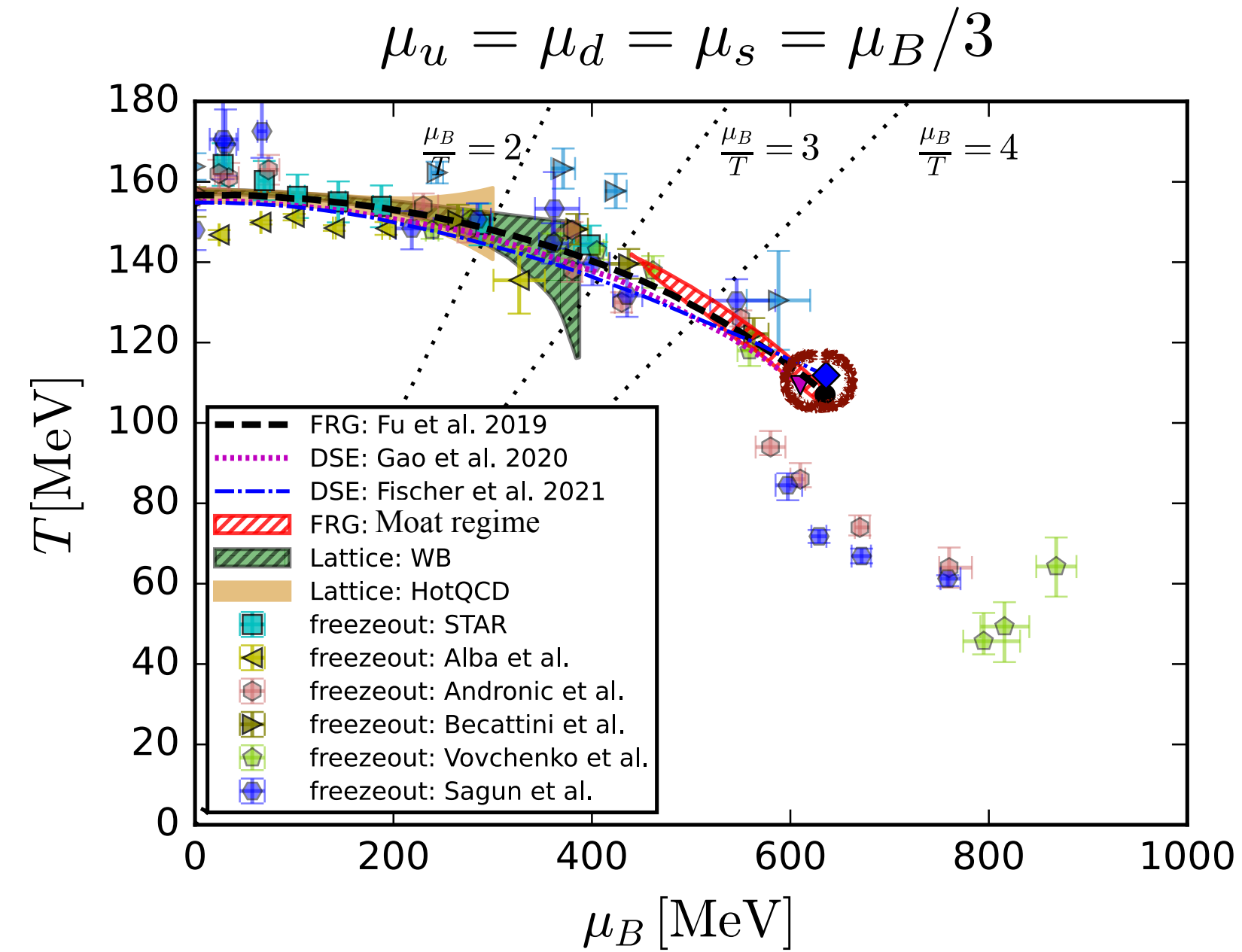
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Extrapolations for Pheno

Requires a discussion of the explicit & implicit assumptions

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Lattice extrapolations

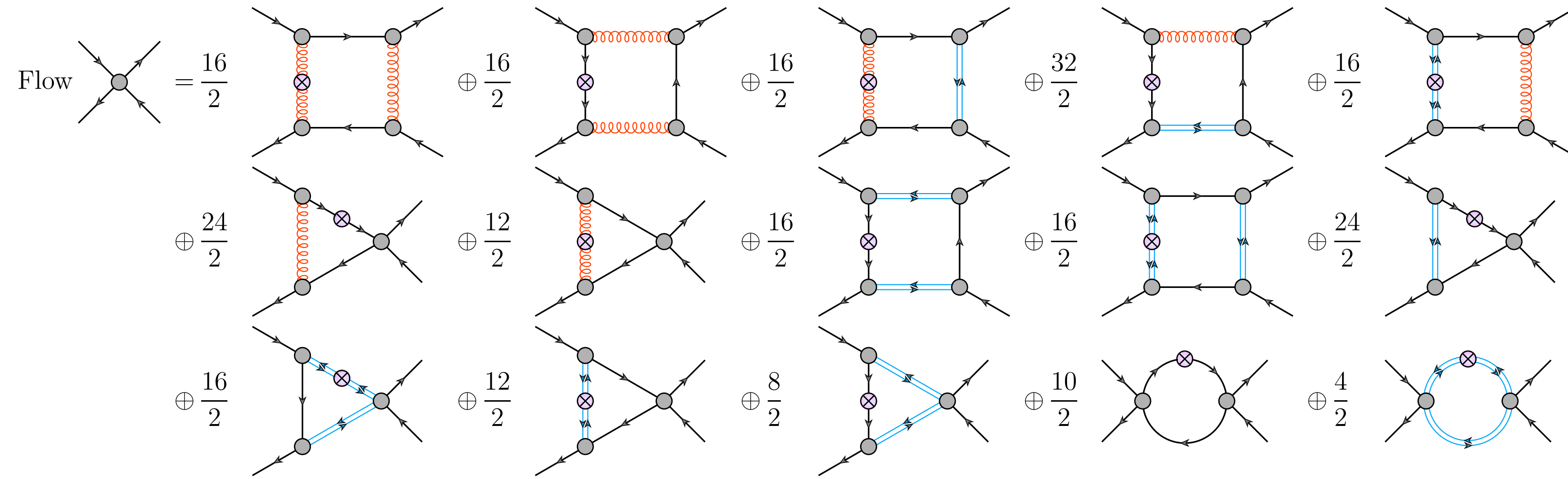
low energy effective theories:
QM, NJL, PQM, PNJL, ..., Holography

**Functional approaches are the only first principle approaches to date
that allow for *direct* computations at finite density with**

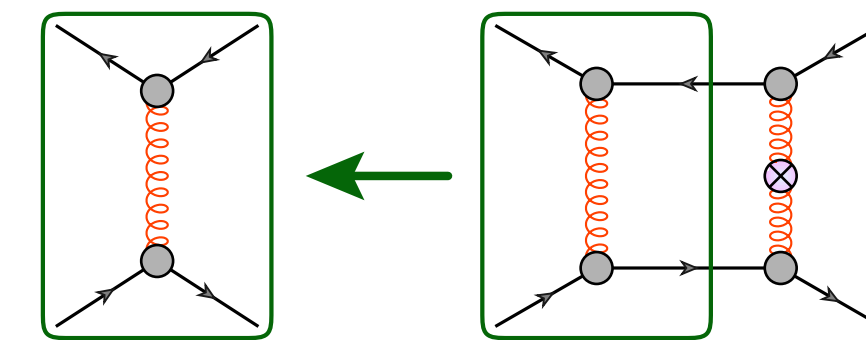
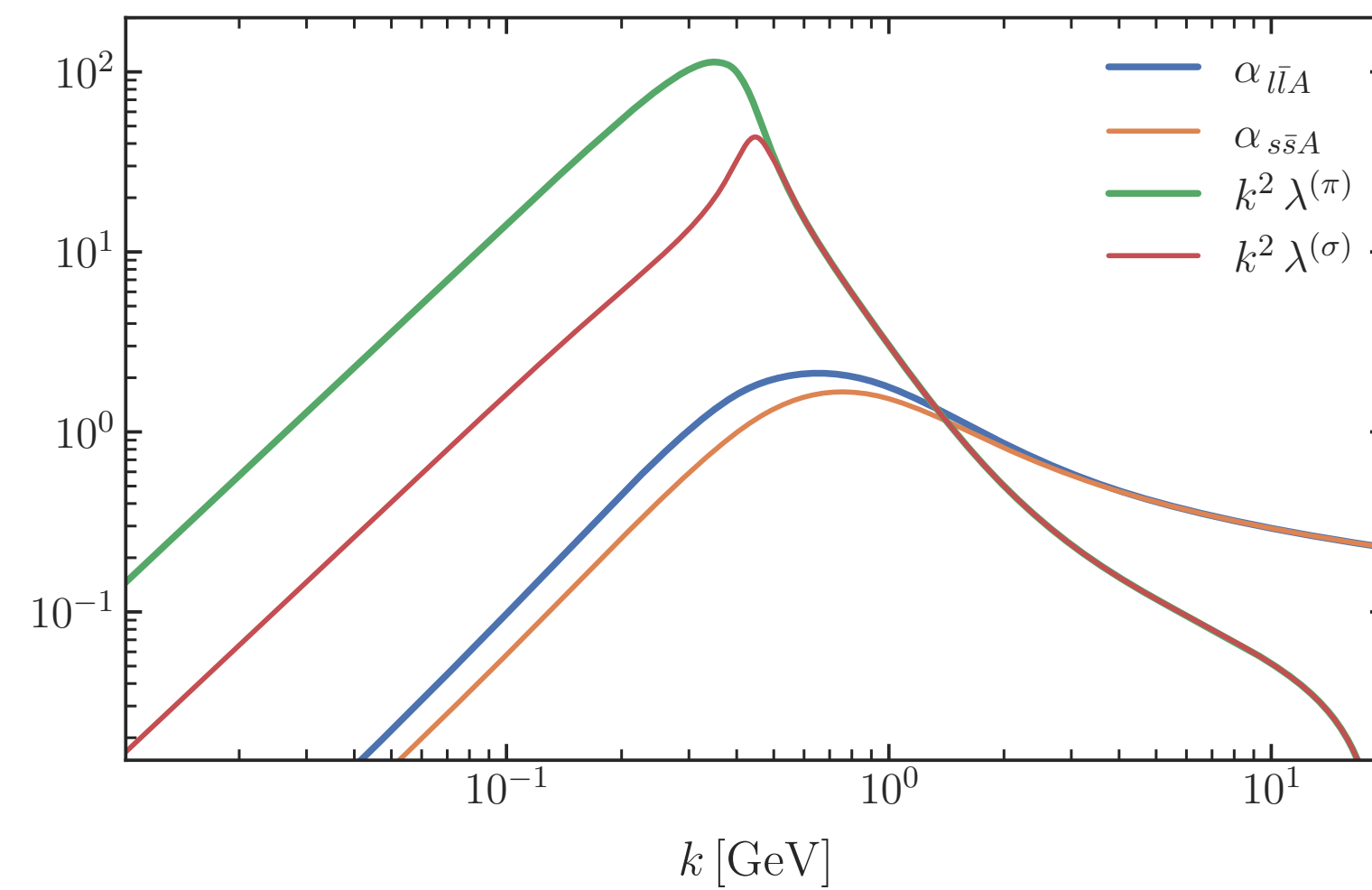
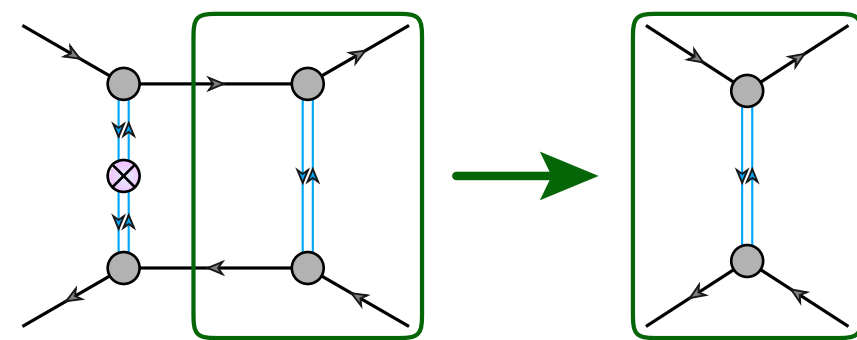
$$\frac{\mu_B}{T} \gtrsim 3$$

Phase structure from functional QCD: how to

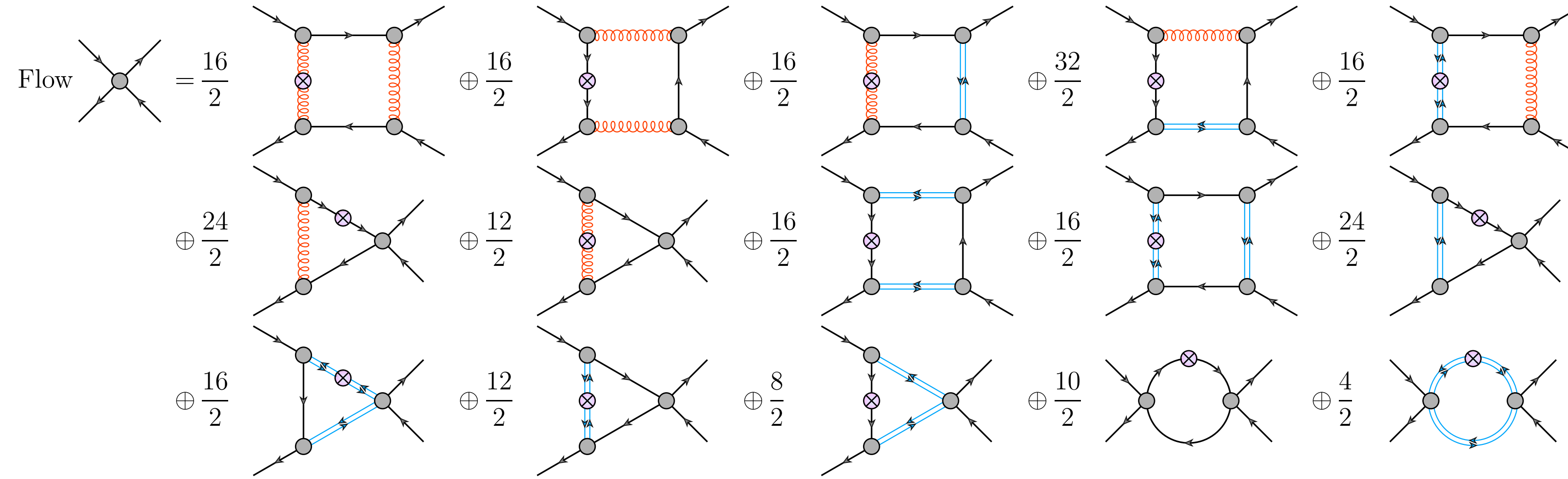
How to: systematic error estimates & the LEGO[®] principle



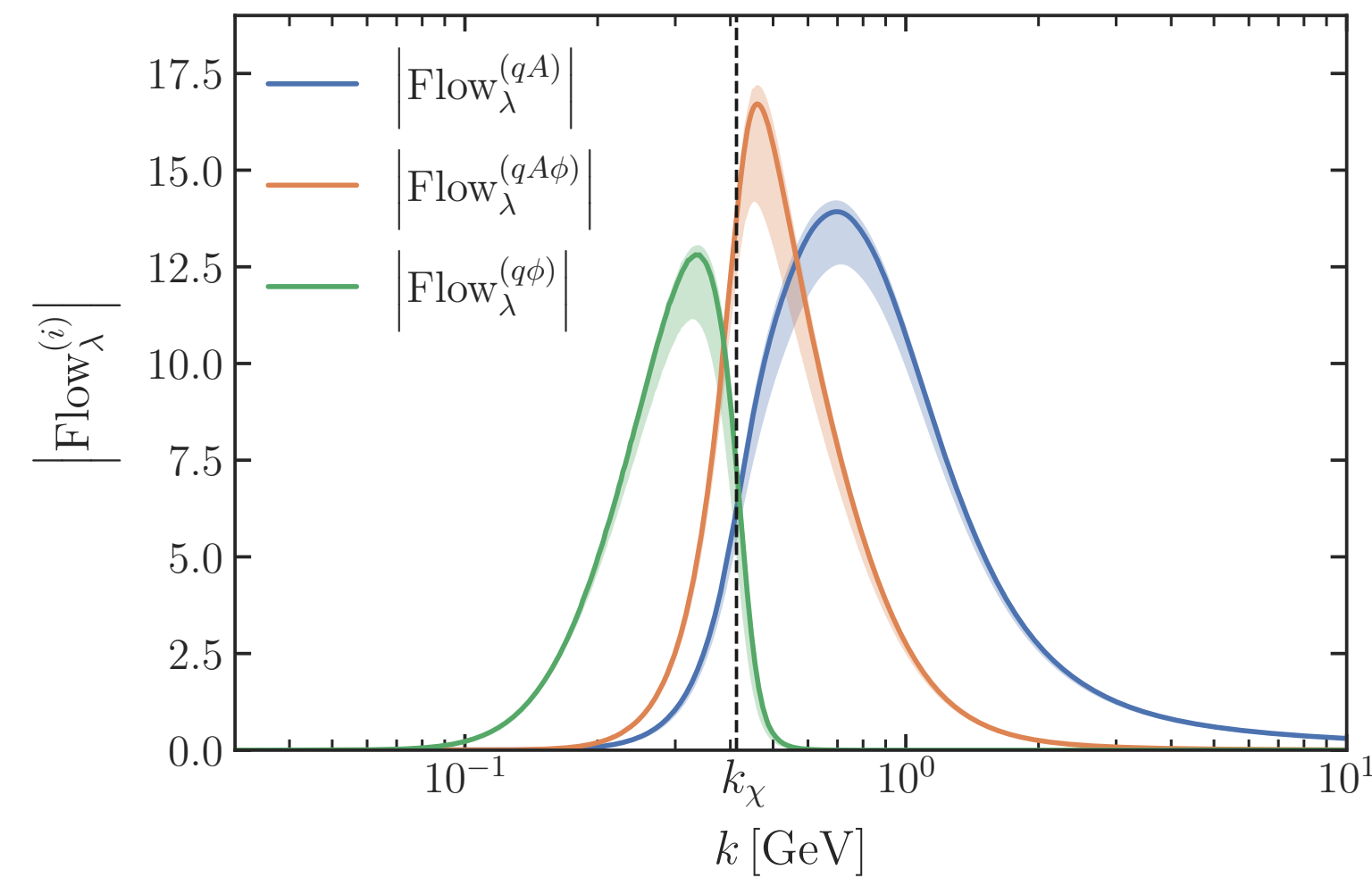
Example: 4-quark scattering vertex



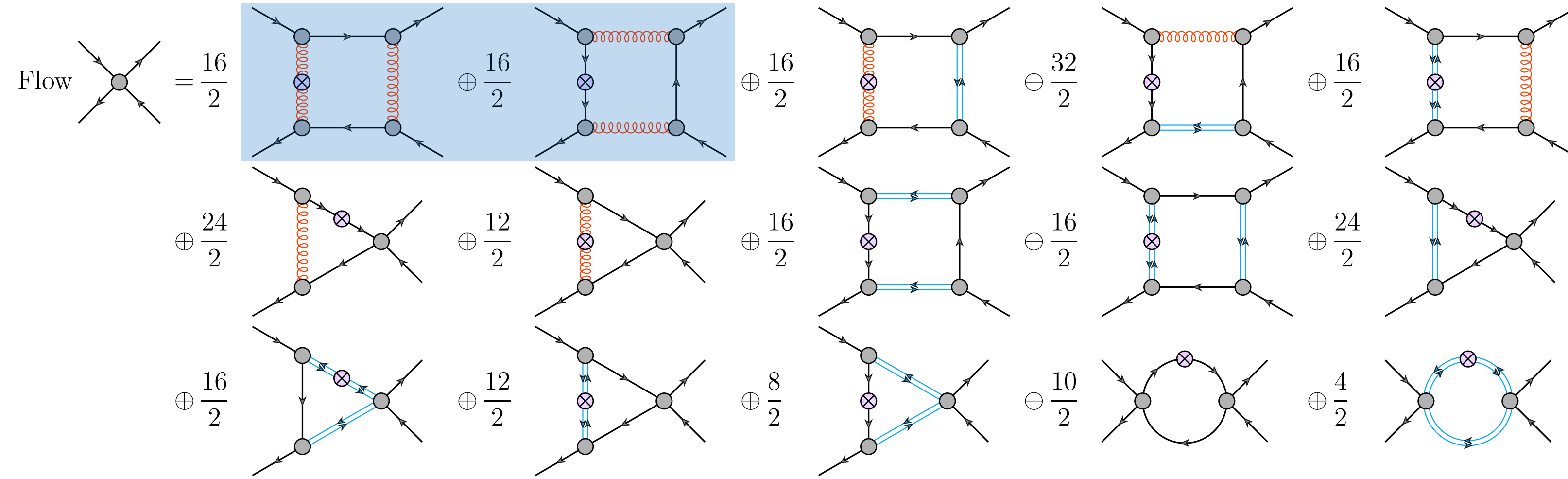
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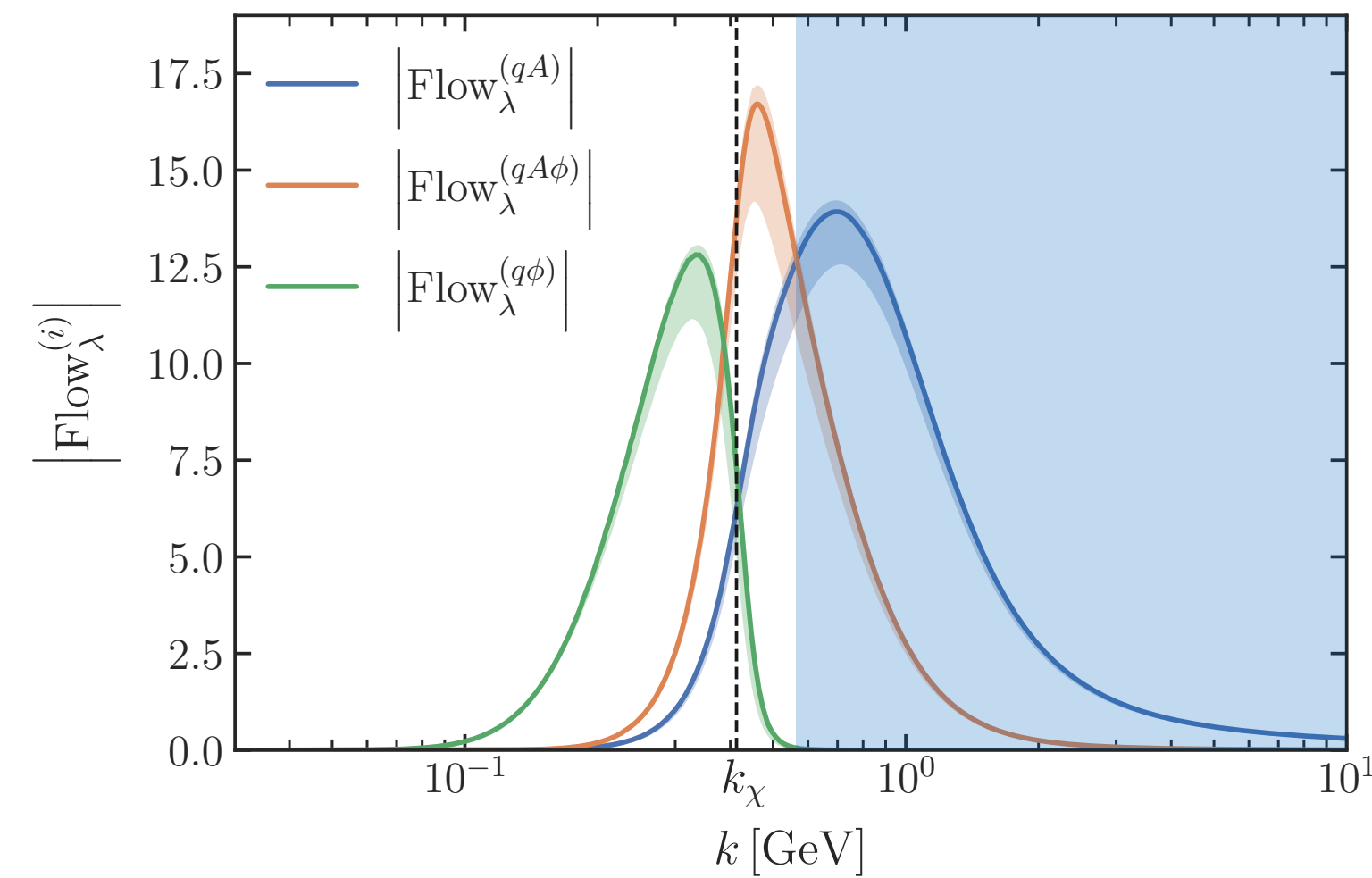
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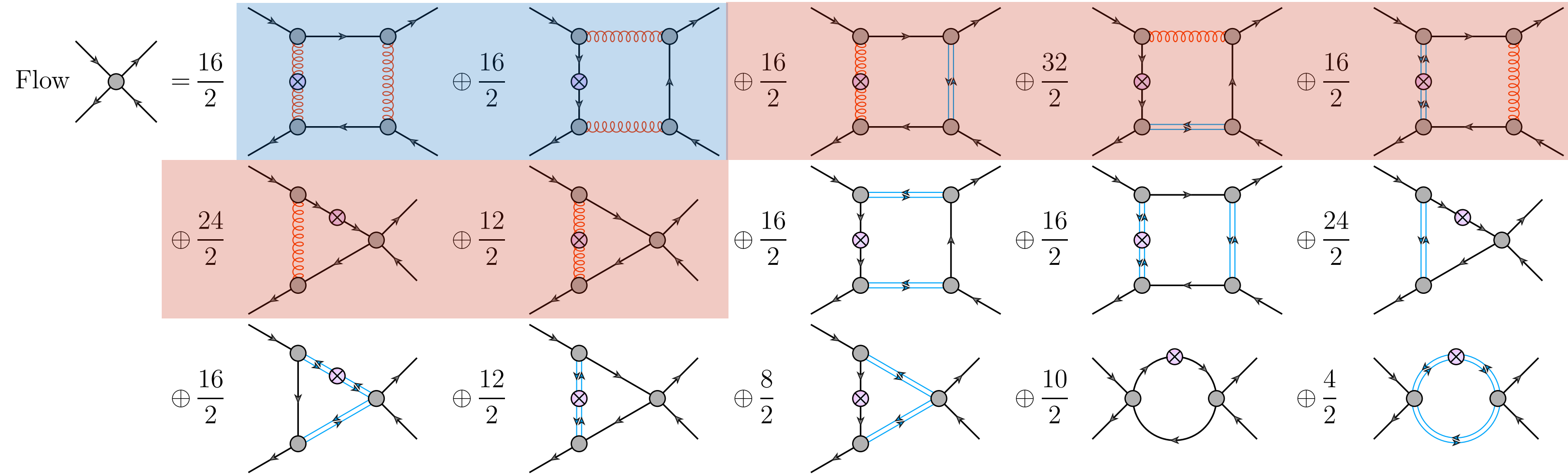
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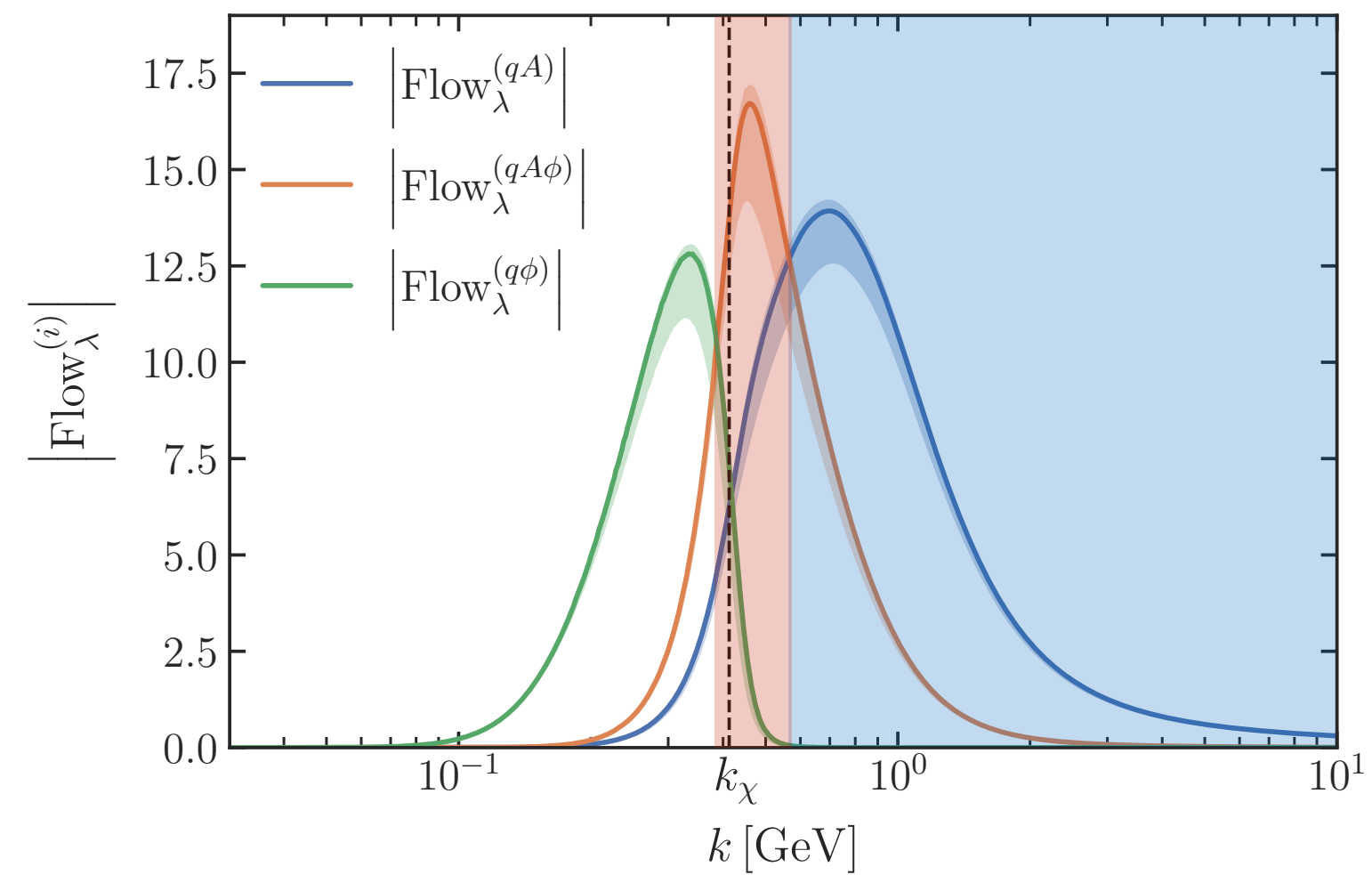
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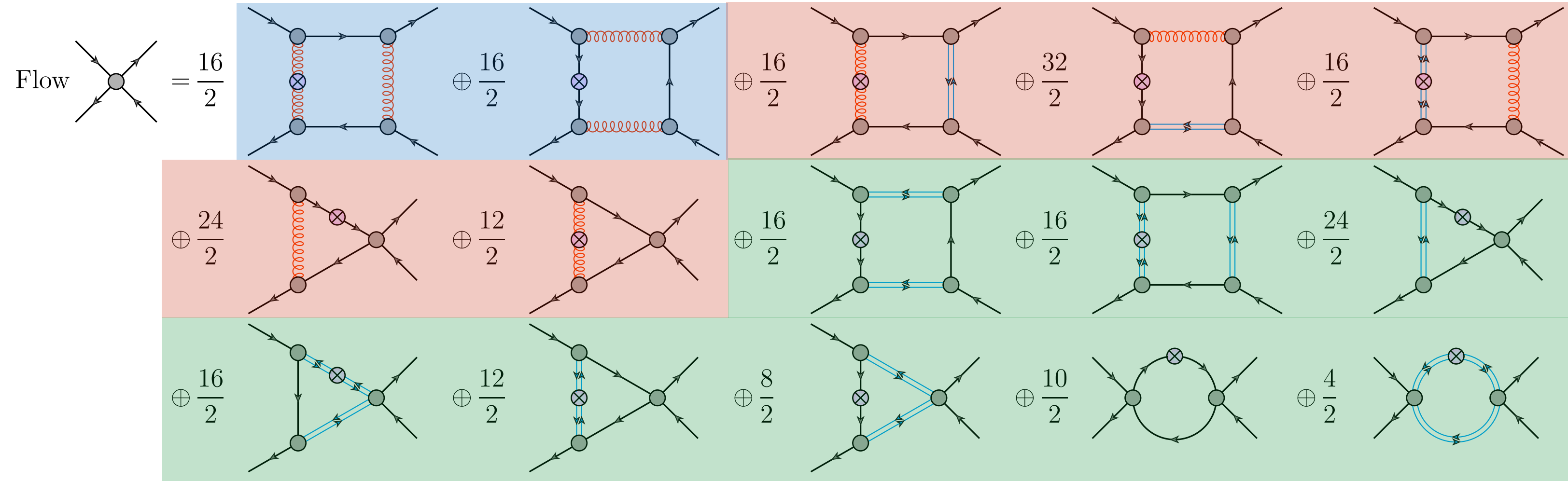
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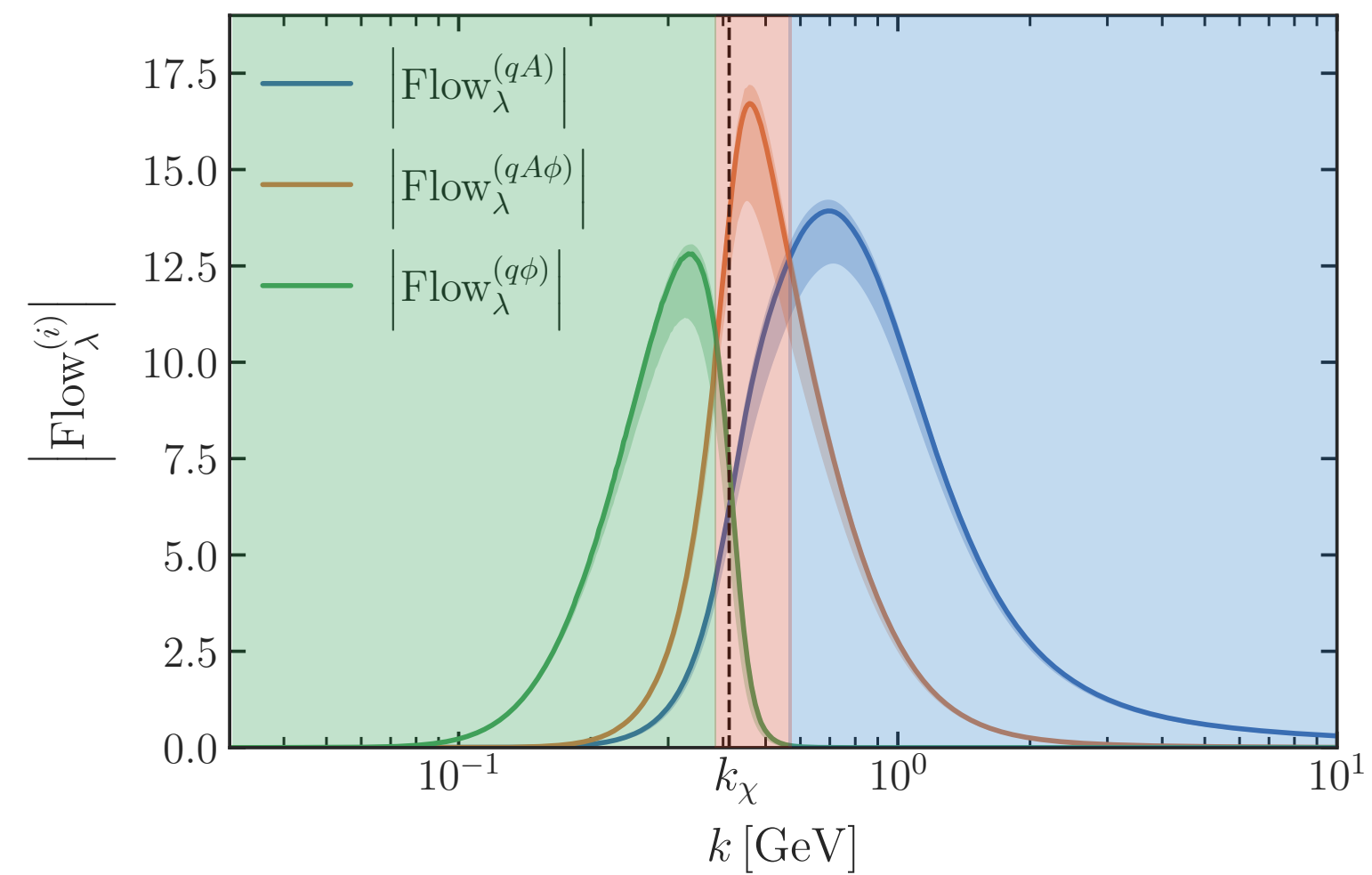
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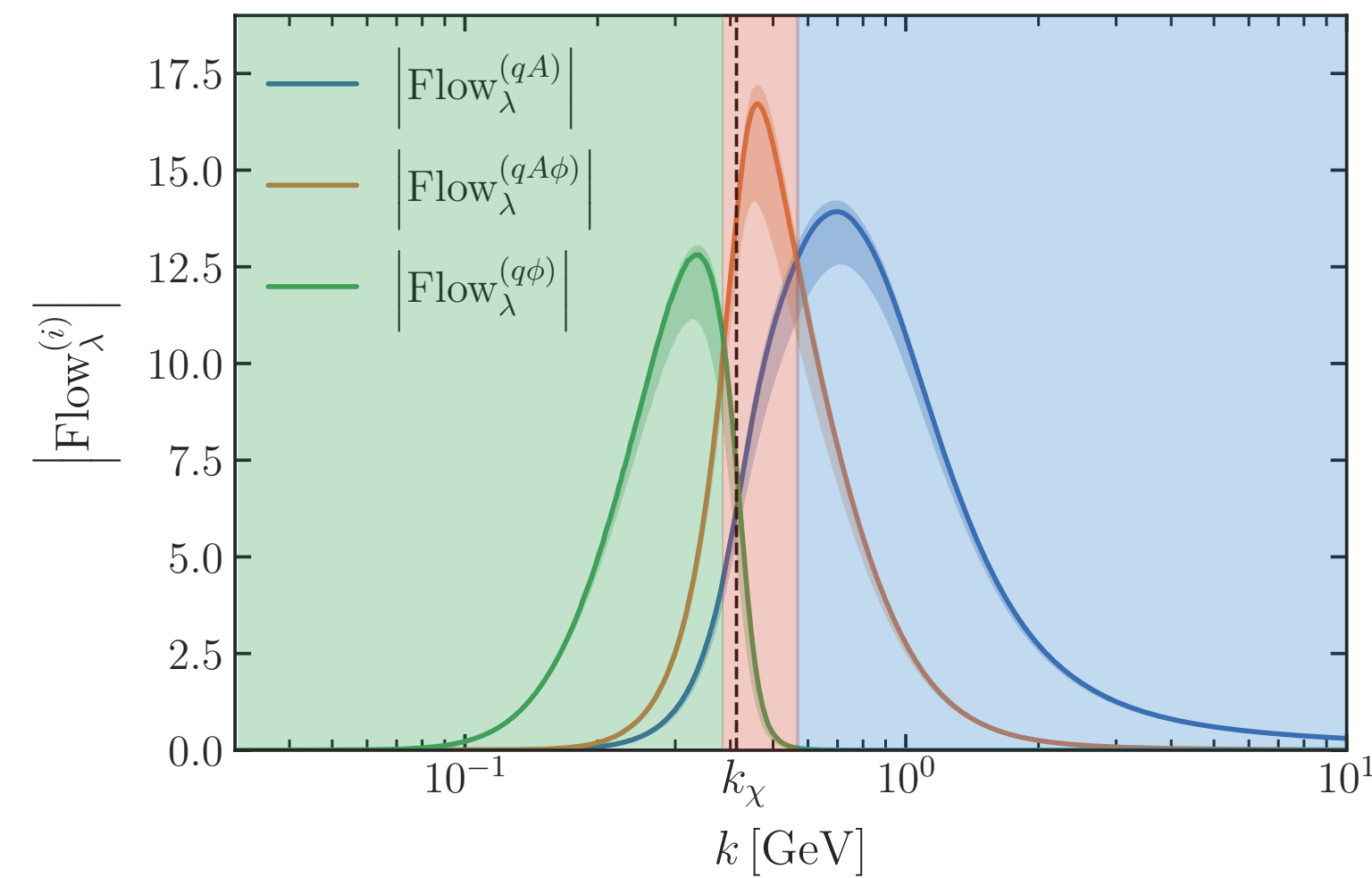
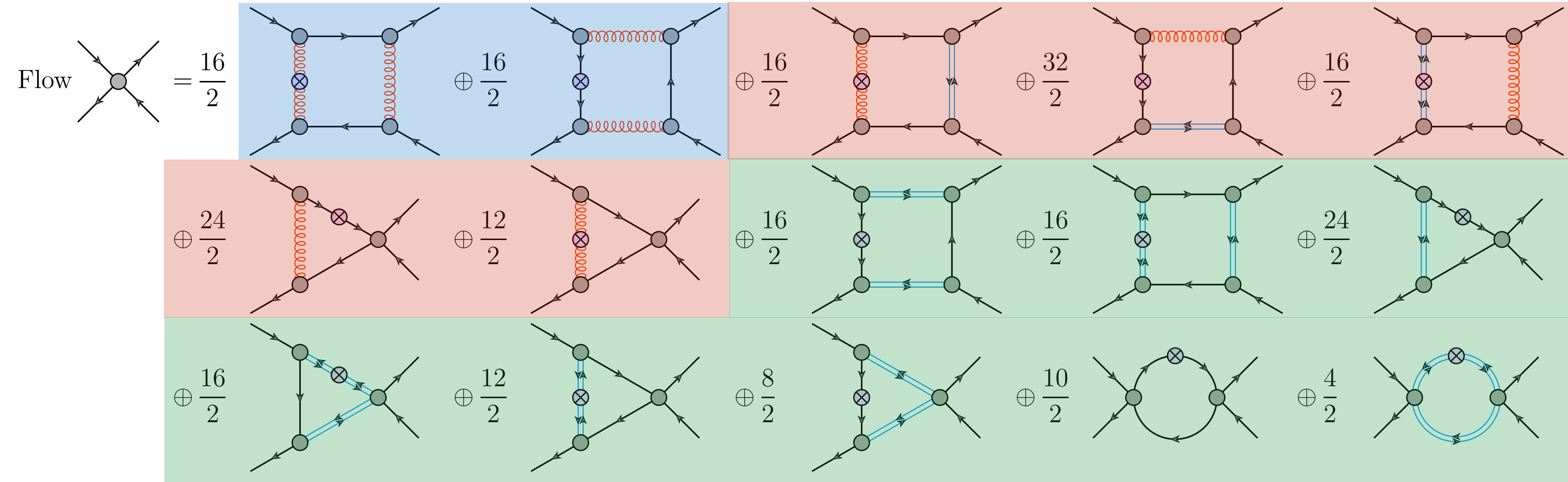
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How to: systematic error estimates & the LEGO[®] principle

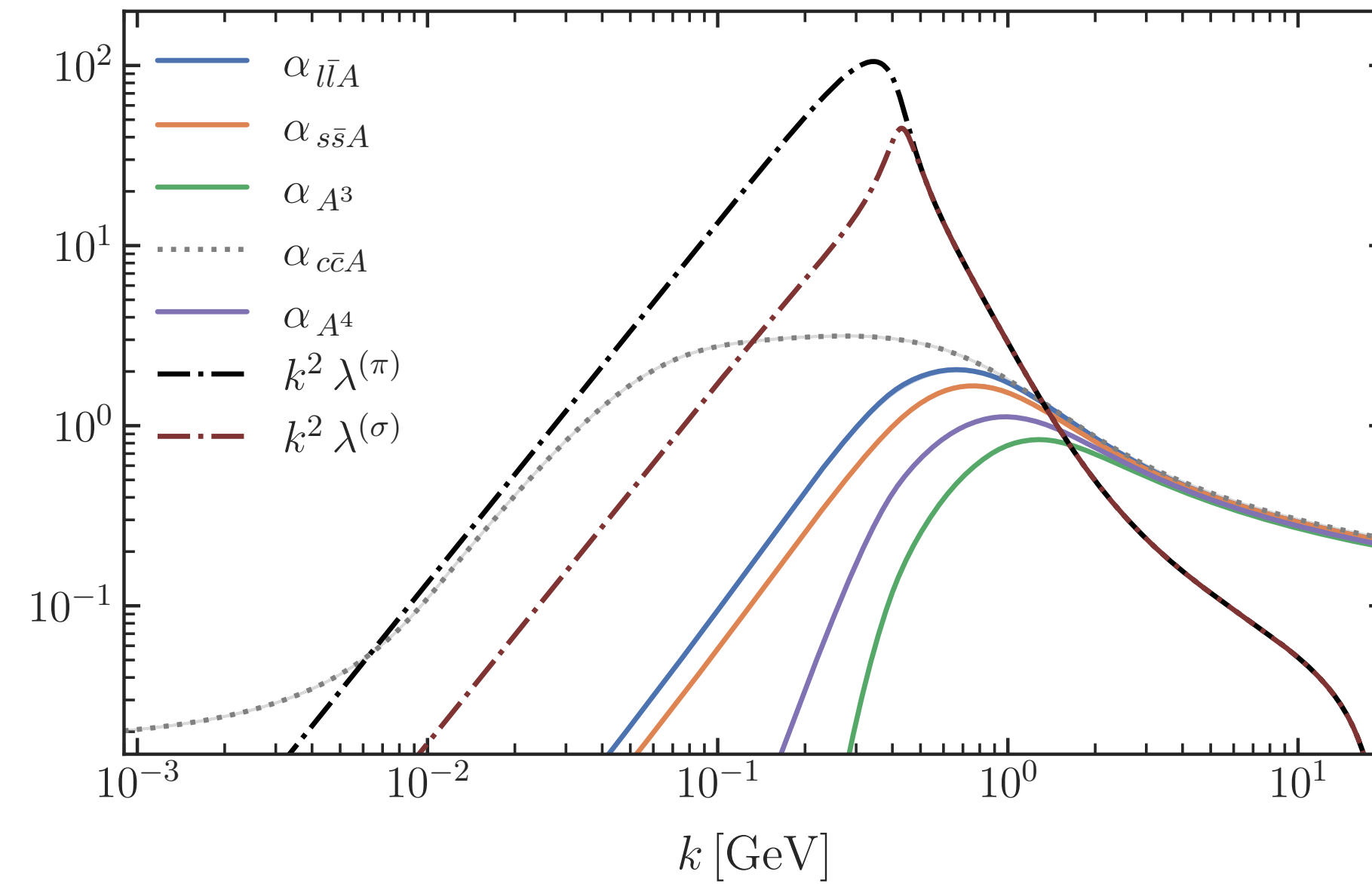


Access and combined use of
error estimates
from functional QCD & LEFTs

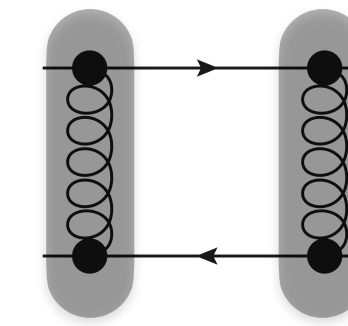
fQCD

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} \left(\text{orange loop} - \text{dashed loop} - \text{solid loop} + \frac{1}{2} \text{blue loop} \right)$$

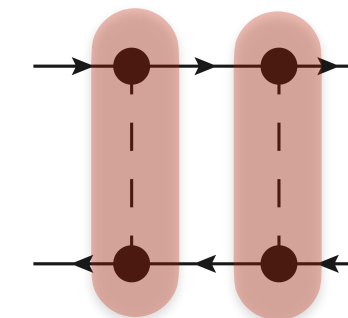
Sequential decoupling of gluon, quark, sigma, pion fluctuations



$$\frac{g_{lA}^2}{g_{sAs}^2}$$



$$k^2 \lambda(\pi, \sigma)$$



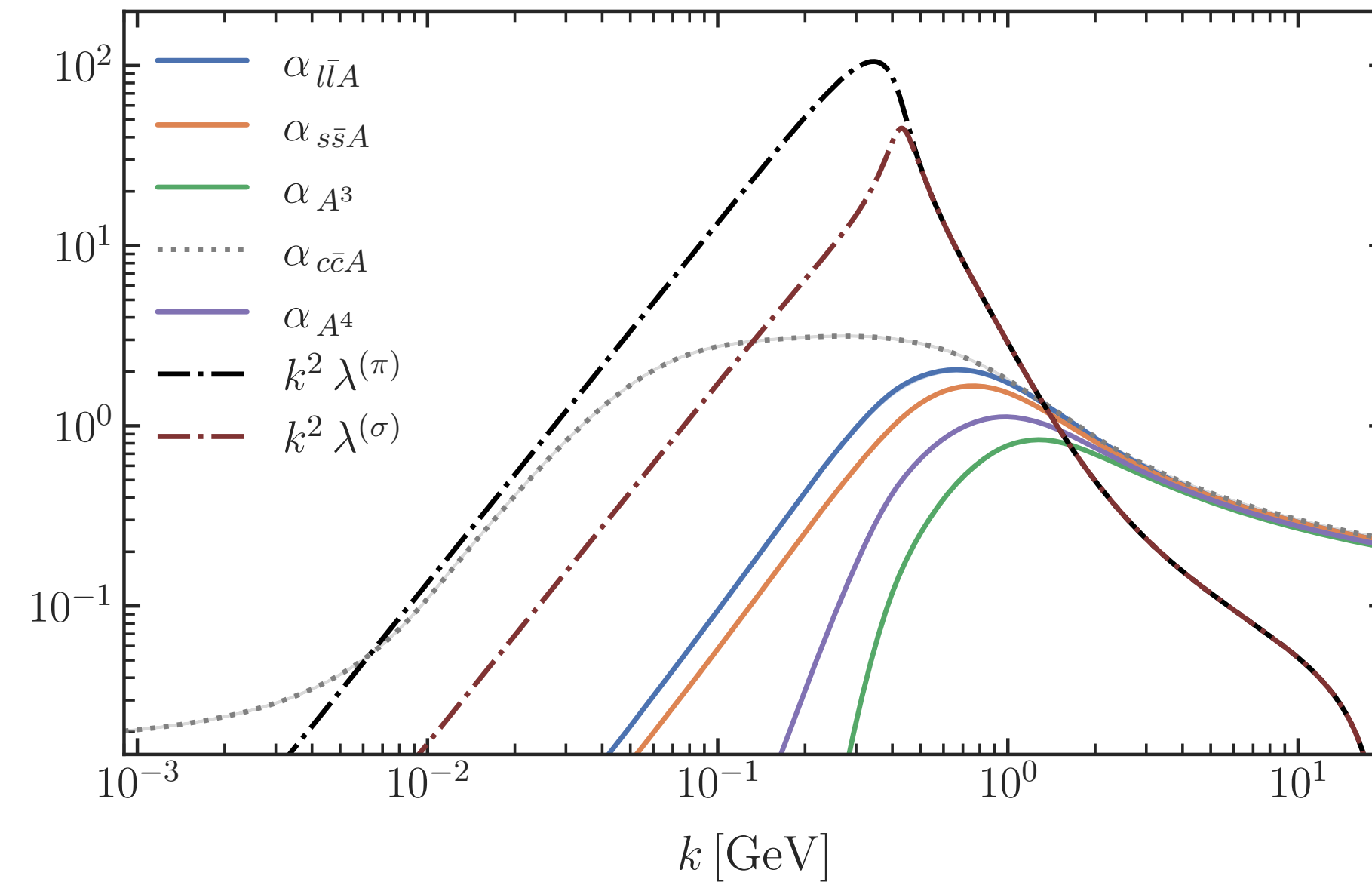
Vacuum: Ihssen, JMP, Sattler, Wink, arXiv:2408.08413

Phase structure: Fu, JMP, Rennecke, PRD 101, (2020) 054032

fQCD

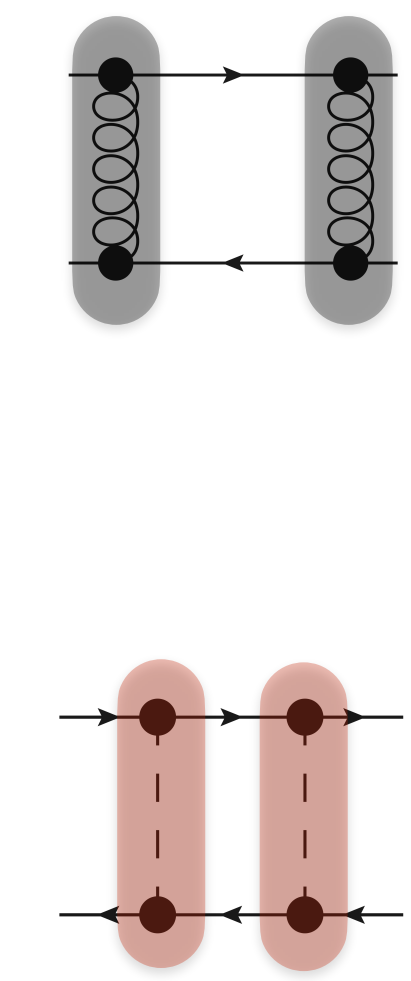
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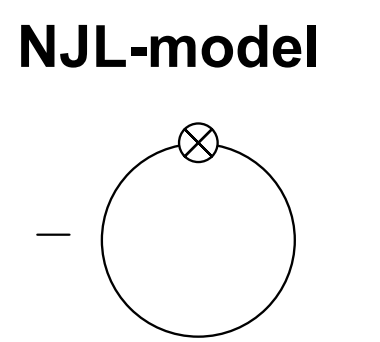
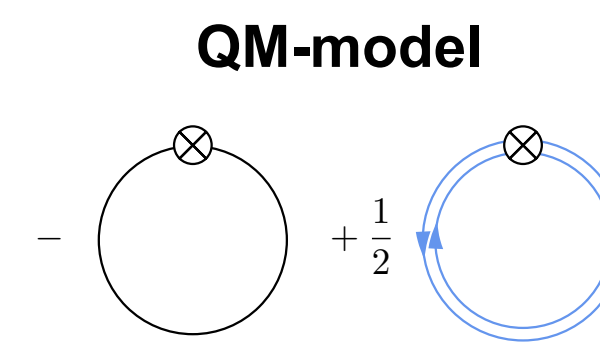
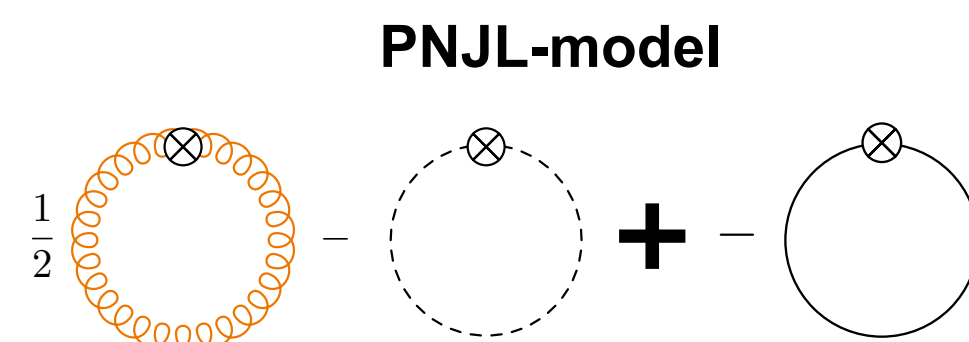
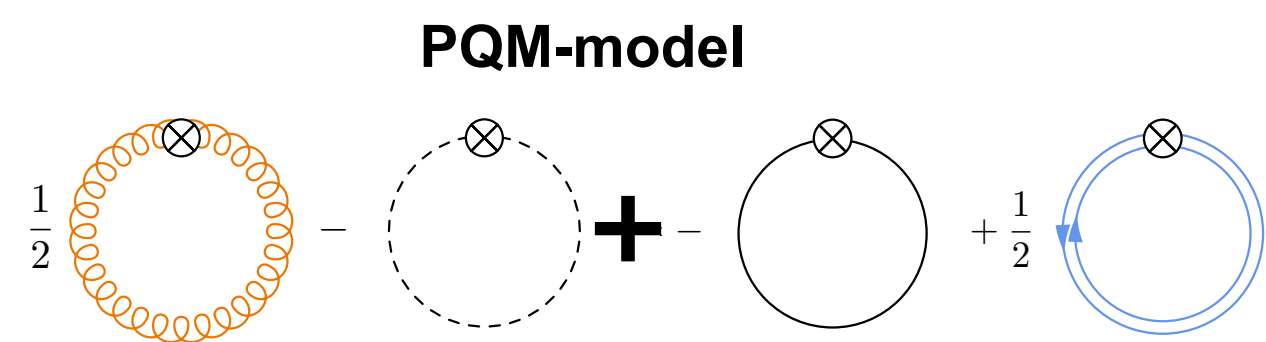
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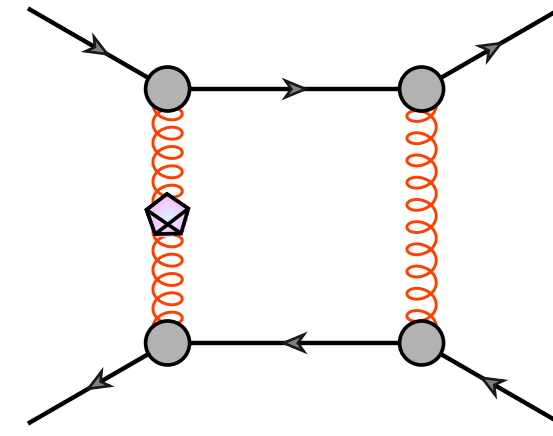


QCD-assisted low energy effective theories

How to: systematic error estimates & the LEGO® principle



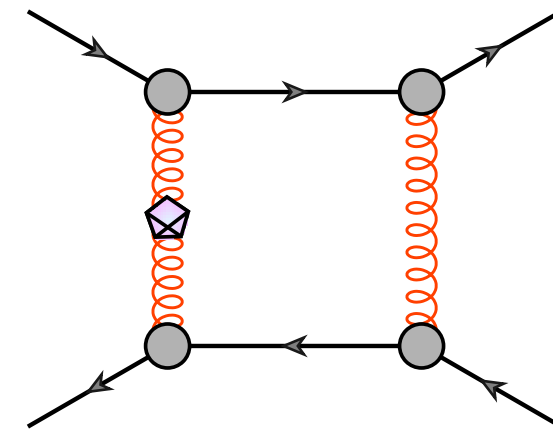
Example: Dissect quark-gluon box



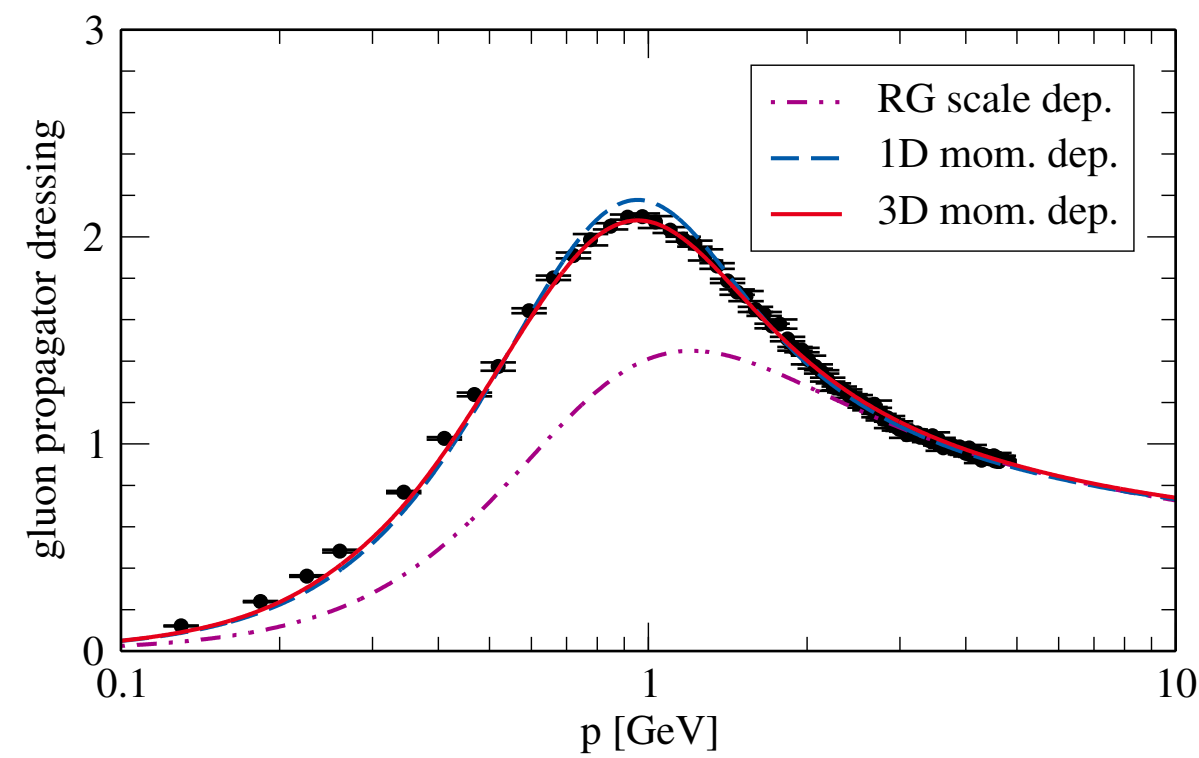
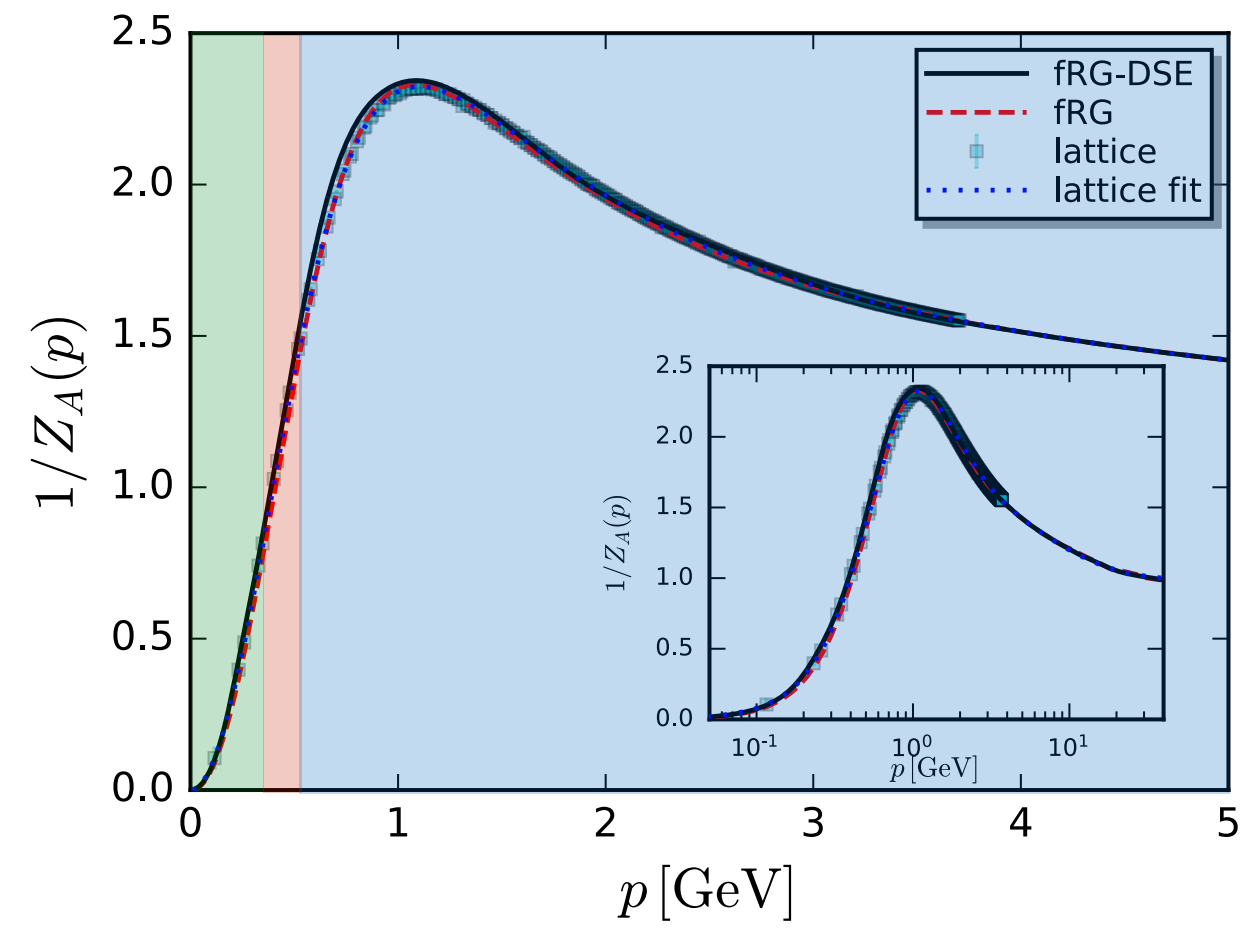
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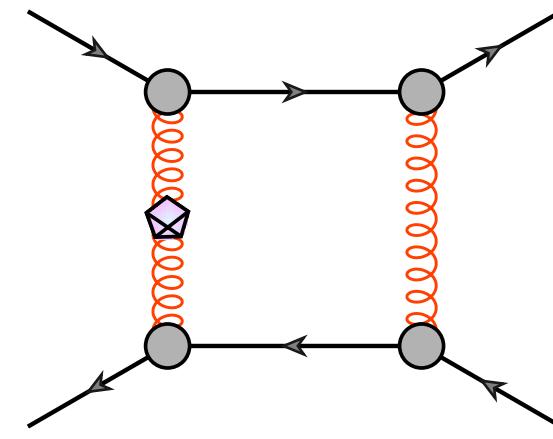
gluon two-point correlator 



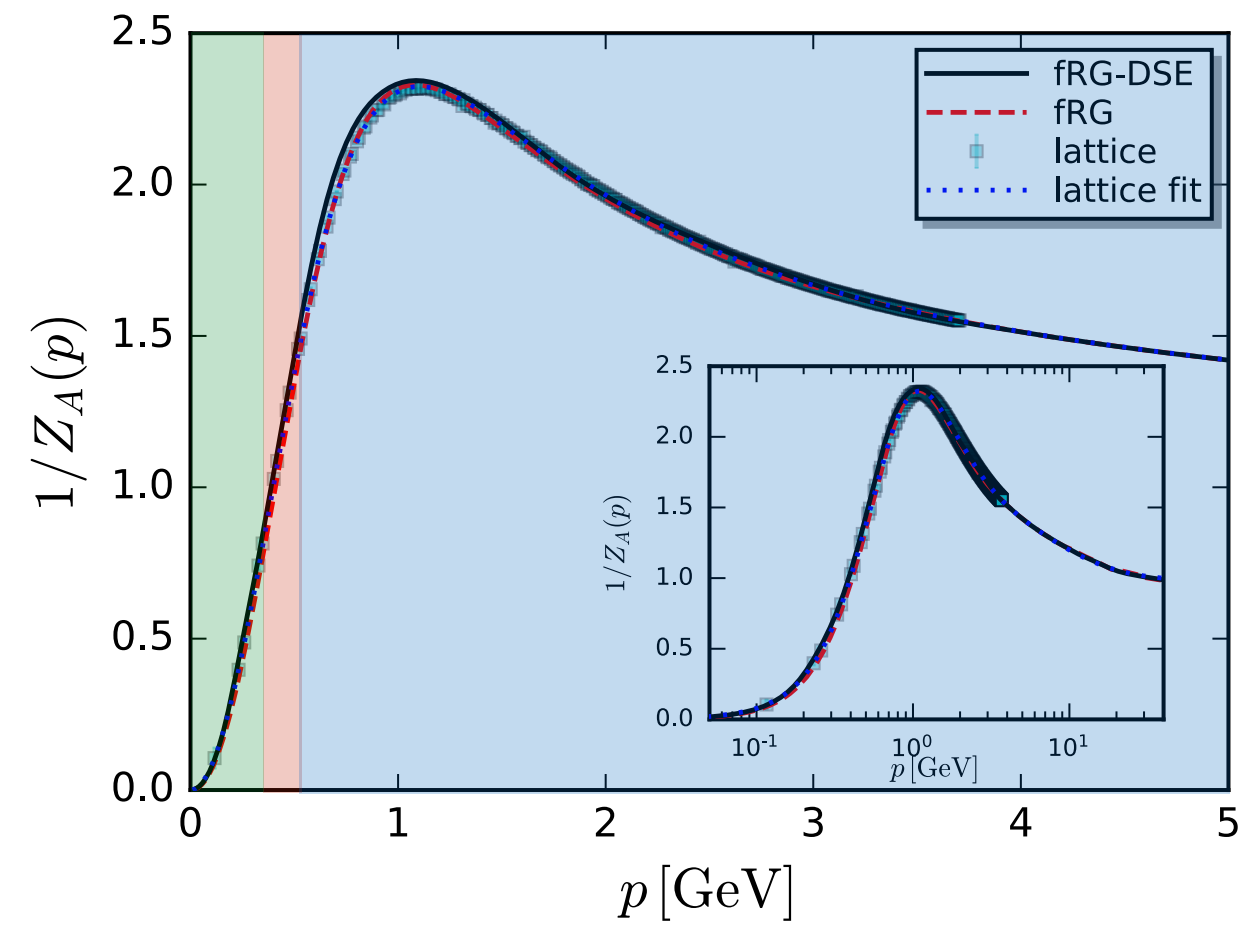
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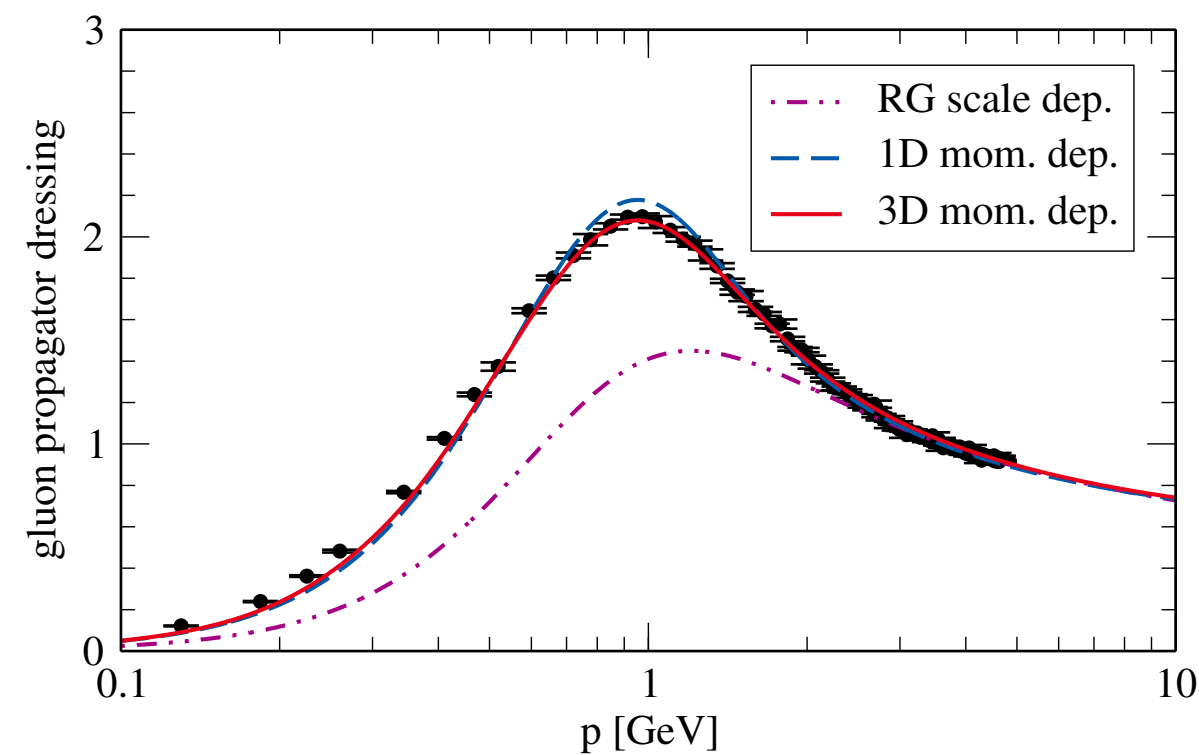
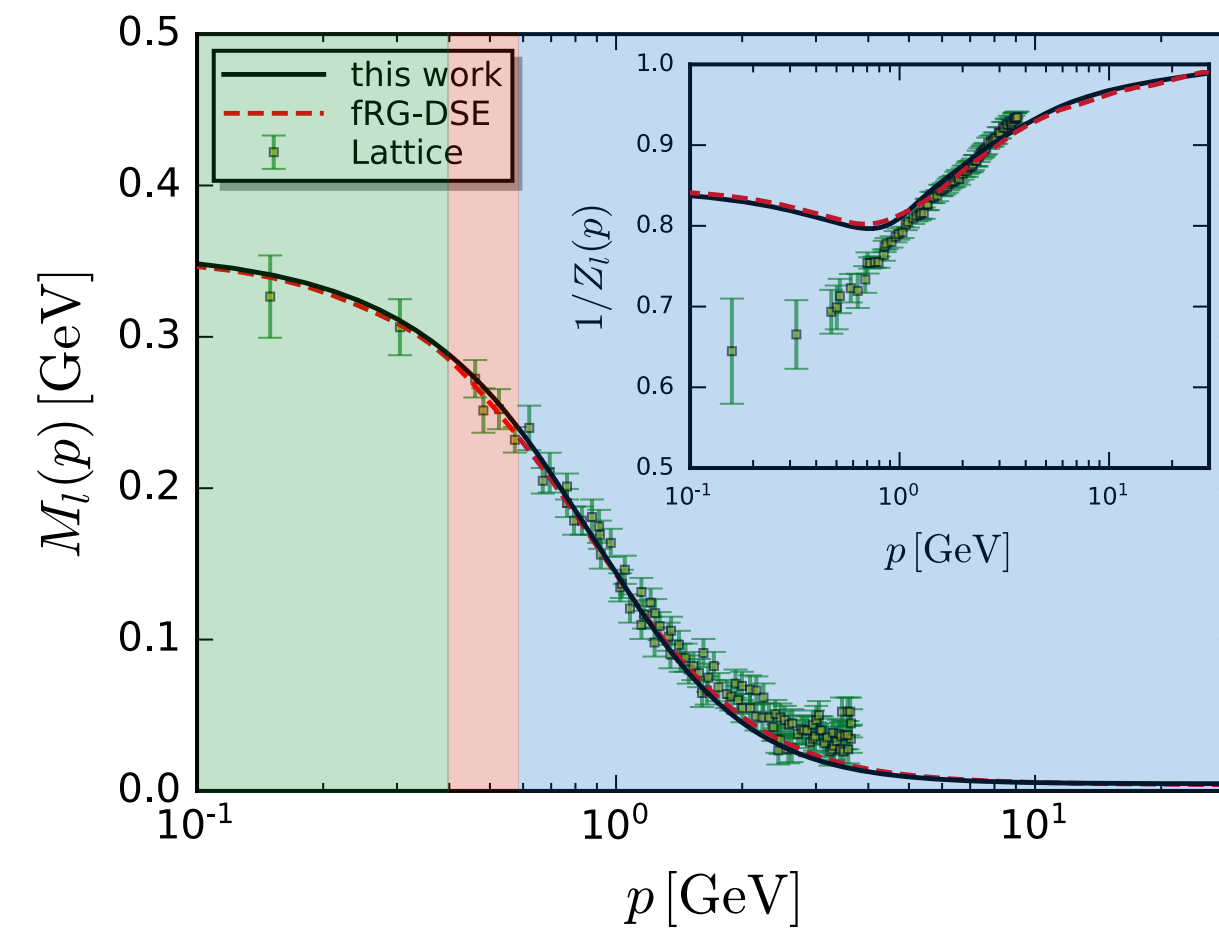
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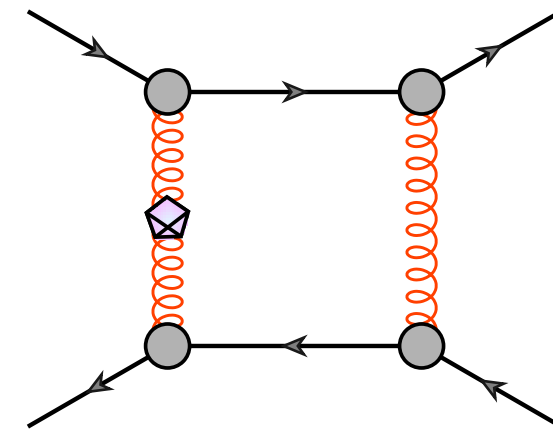
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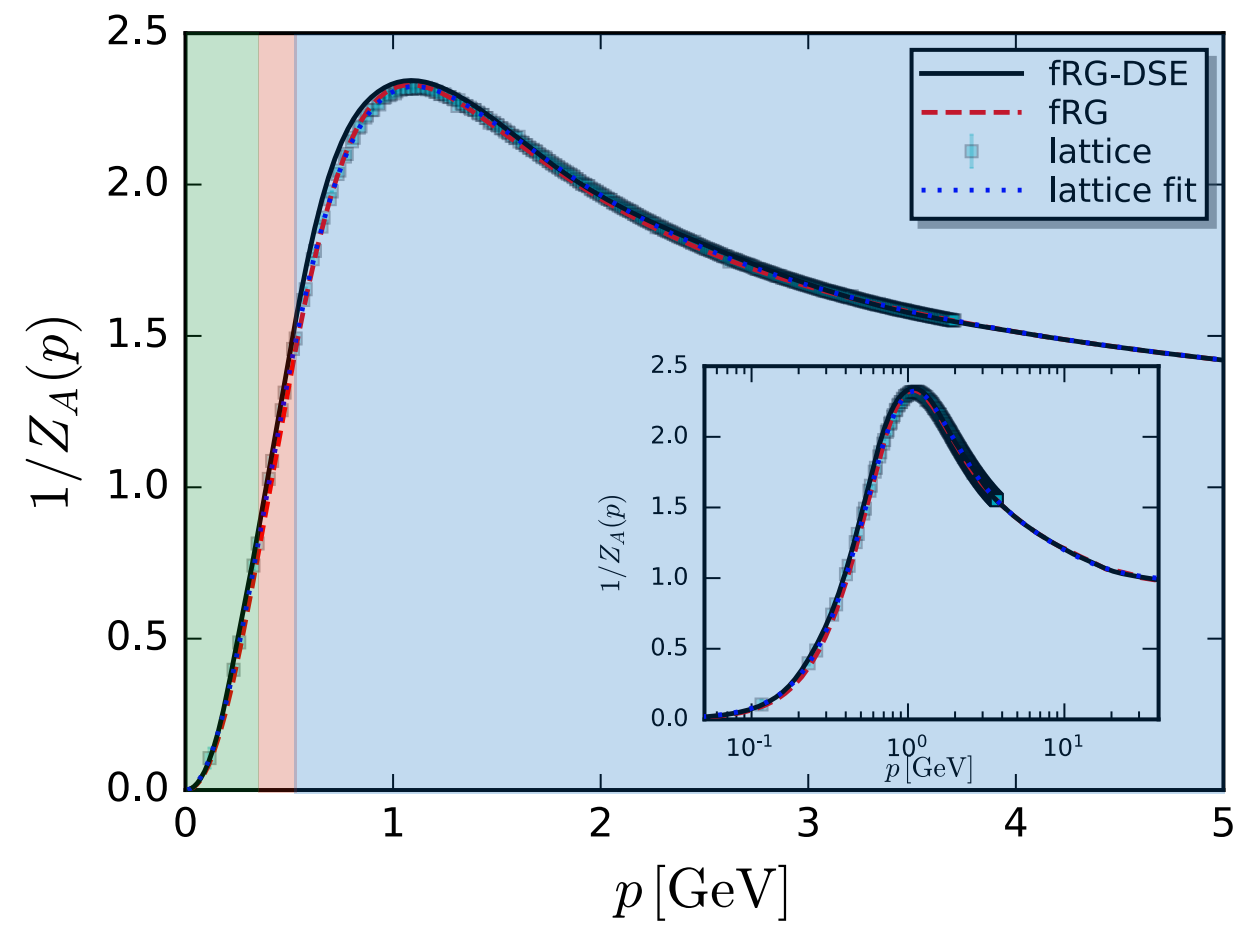


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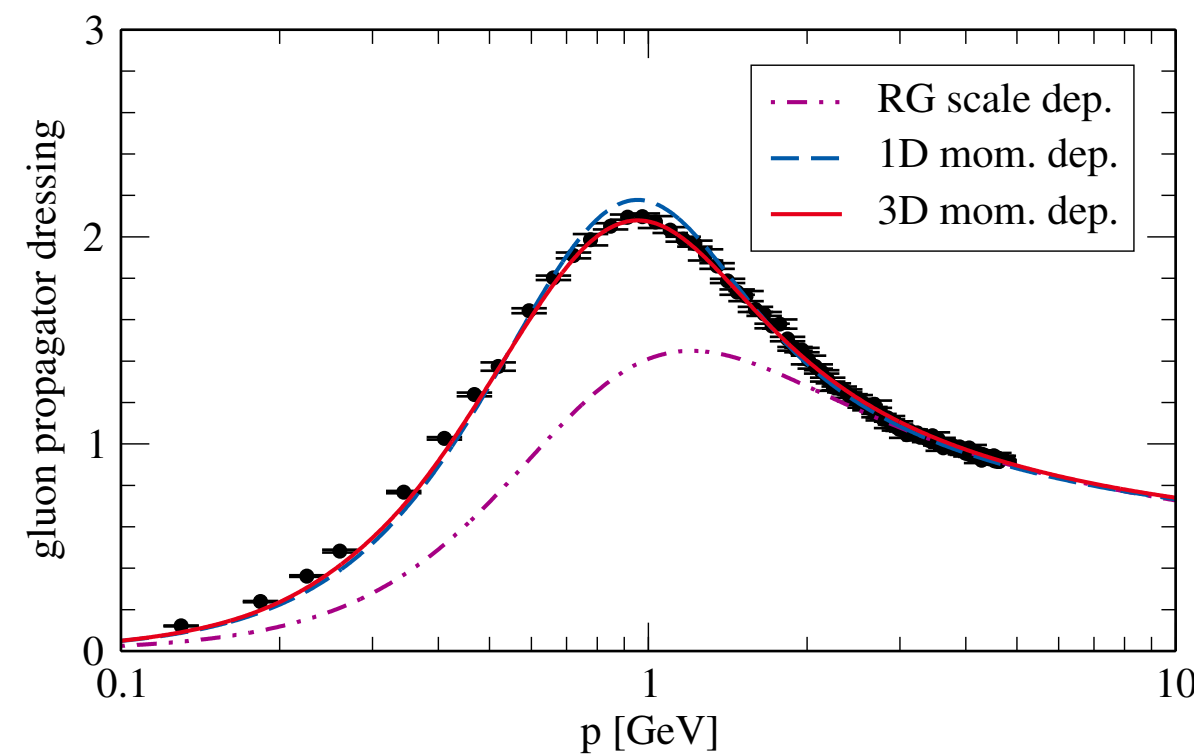
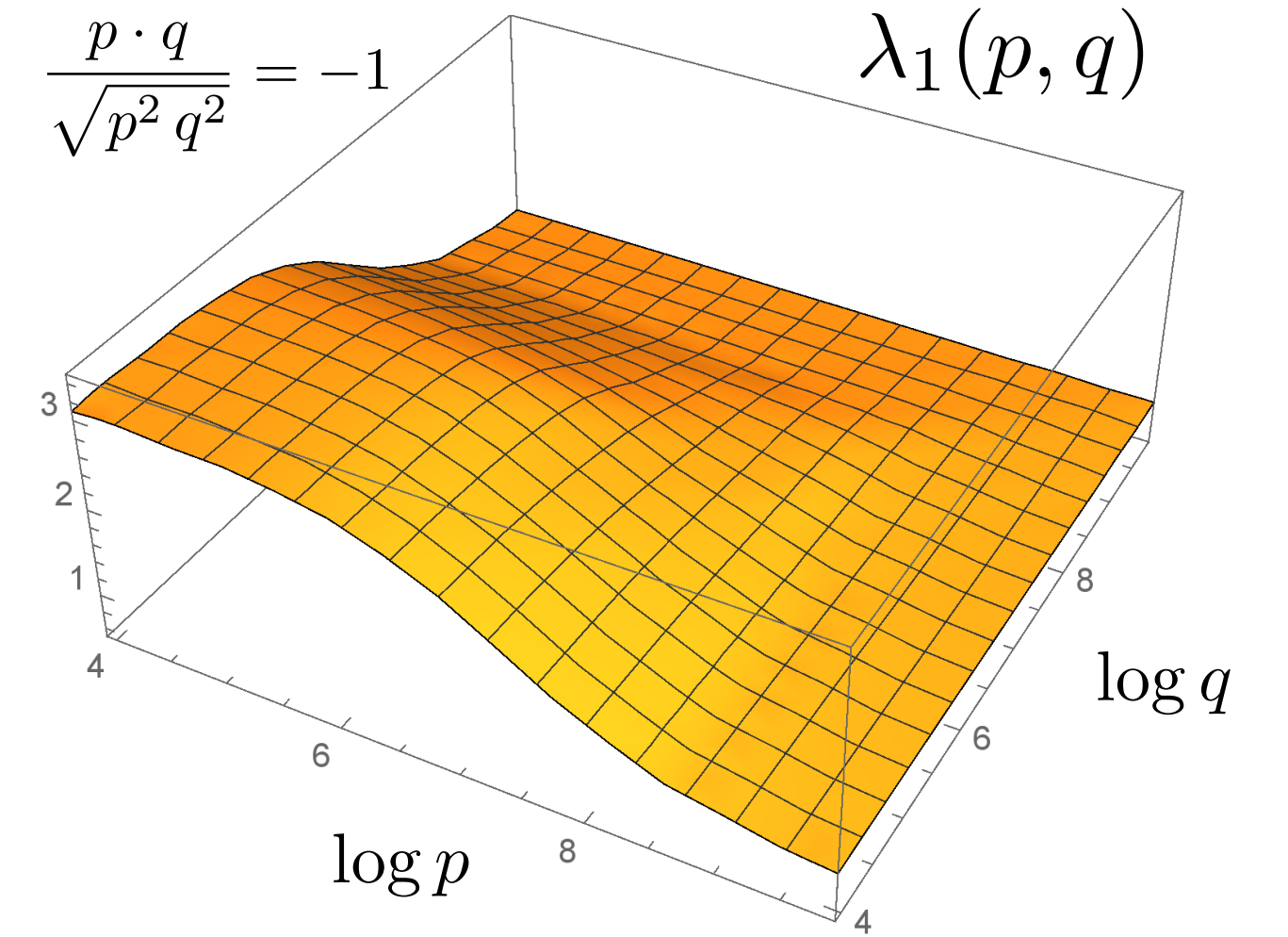
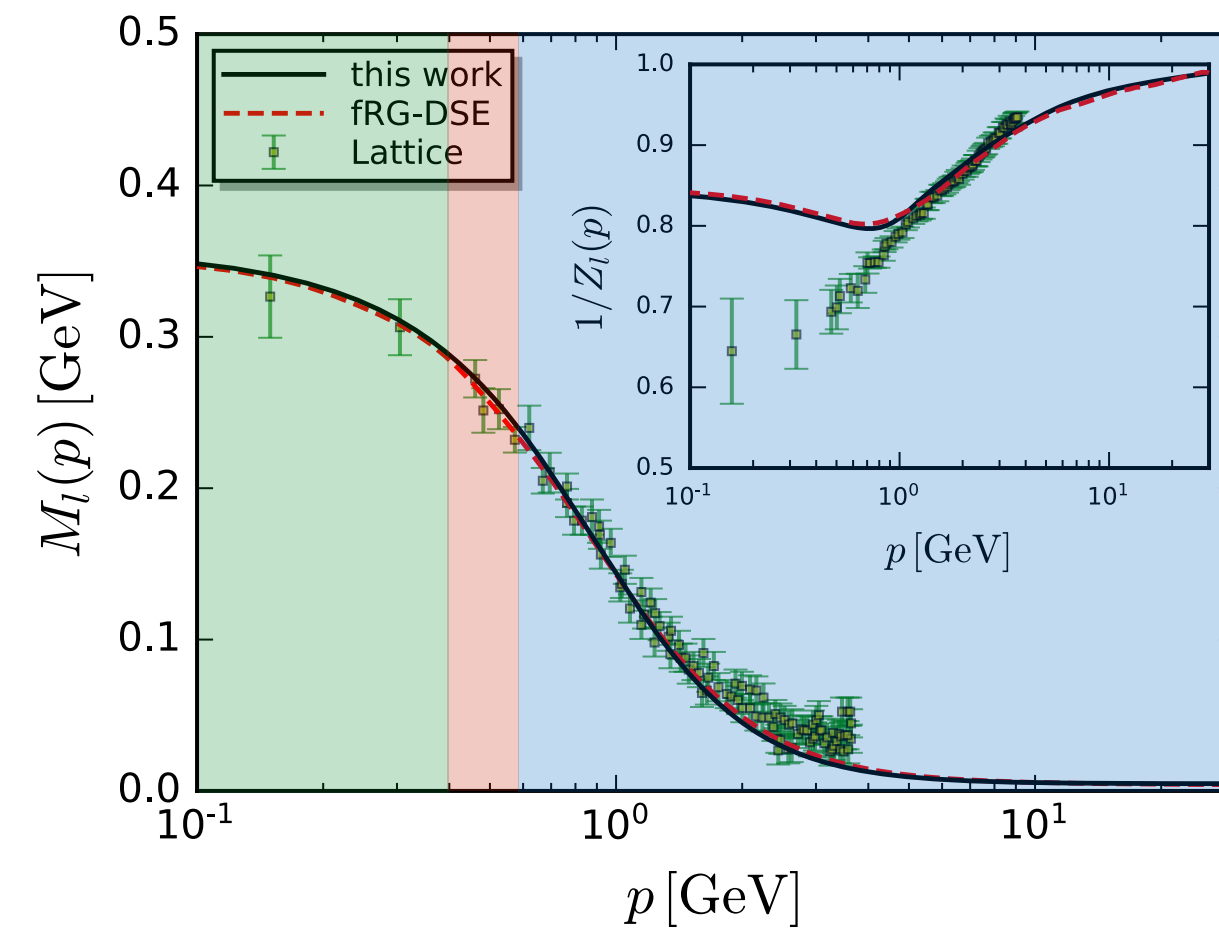


gluon two-point correlator 

quark-gluon scattering 



quark two-point correlator 



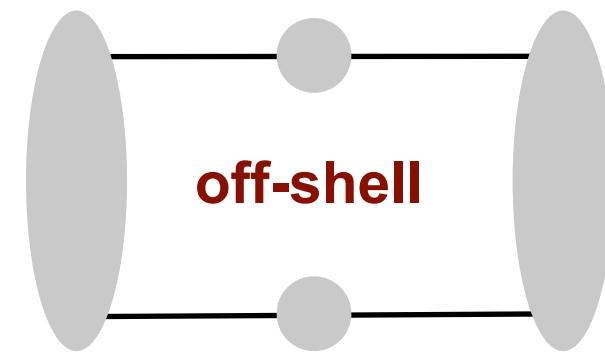
$\lambda_1(p, q), \dots, \lambda_8(p, q)$

Relevant: $\lambda_1, \lambda_4, \lambda_7$

Three remarks on Functional Methods for QCD

- off-shell representation of thermodynamic observables

e.g. $\text{Tr} \langle q(x) \bar{q}(x) \rangle$

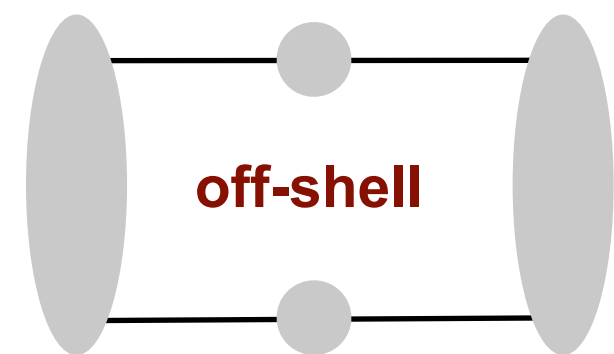


pressure, trace anomaly,
fluctuations, volume flucs., ...

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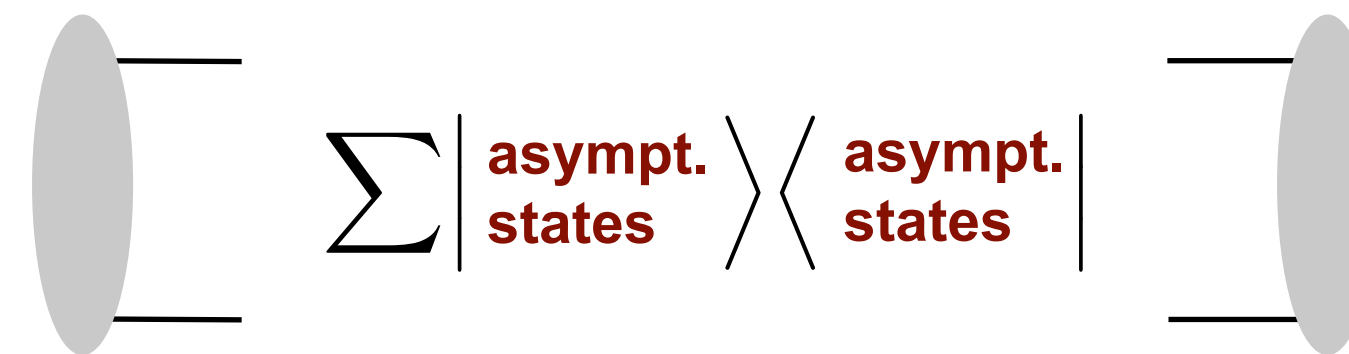
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on-shell

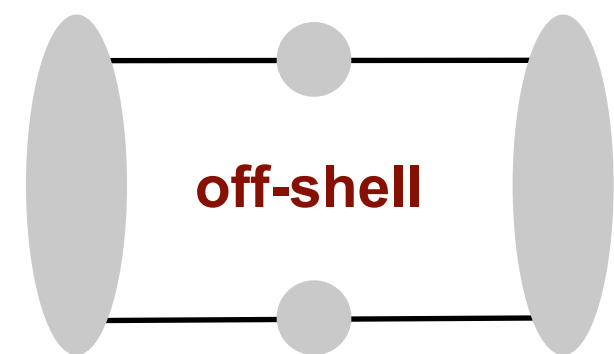


e.g. hadron resonances

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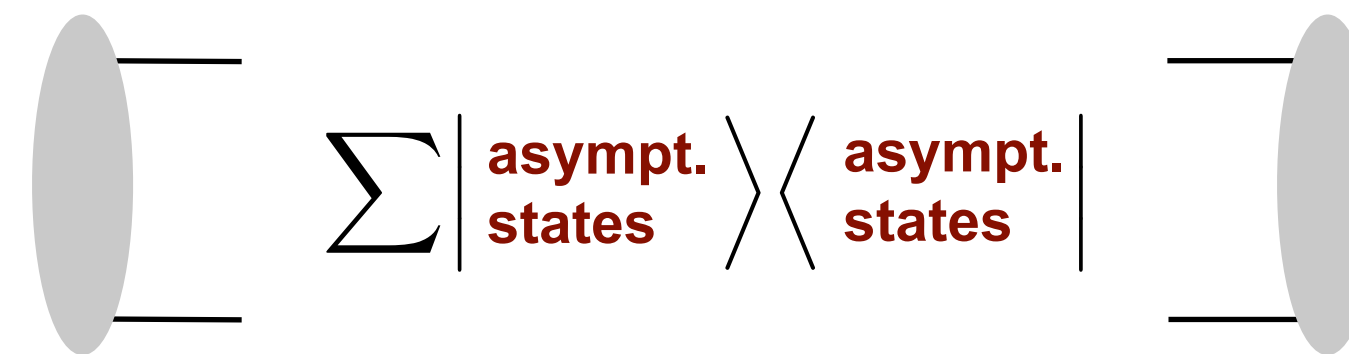
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on-shell



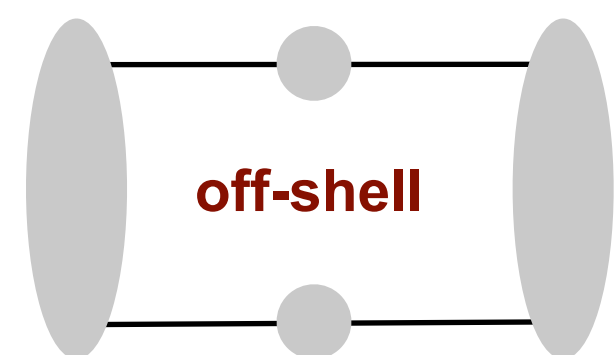
e.g. hadron resonances

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- off-shell representation of thermodynamic observables

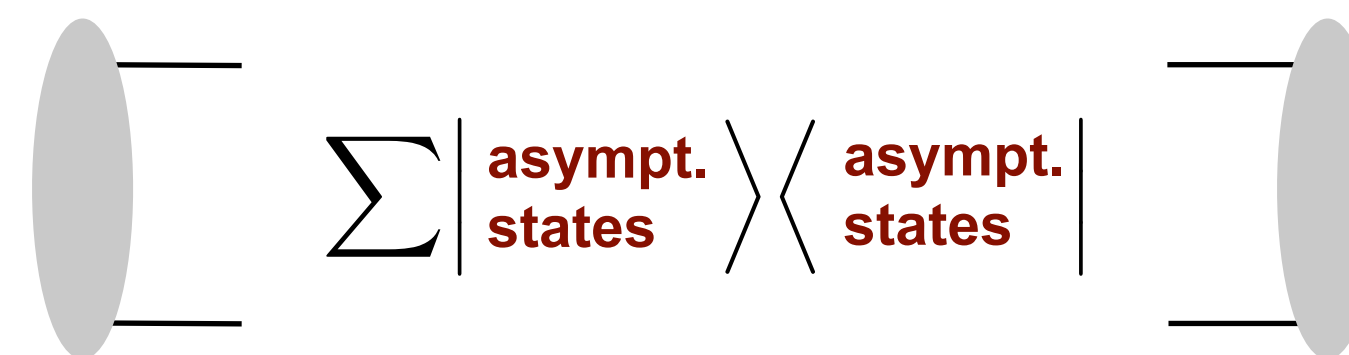
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on-shell



e.g. hadron resonances

- gauge fixing = parameterisation

$$\langle q(x_1) \cdots \bar{q}(x_{2n}) A_\mu(y_1) \cdots A_\mu(y_m) h(z_1) \cdots h(z_l) \rangle$$

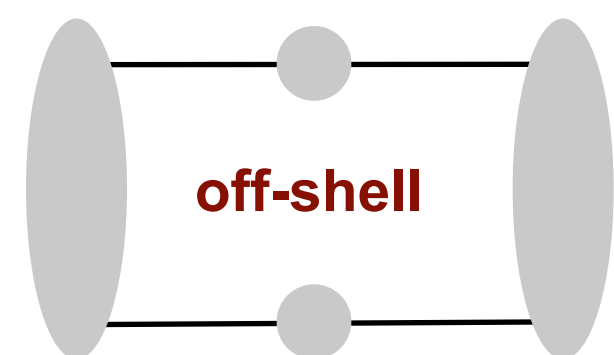
Consequences

I: simple correlations

Three remarks on Functional Methods for QCD

- off-shell representation of thermodynamic observables

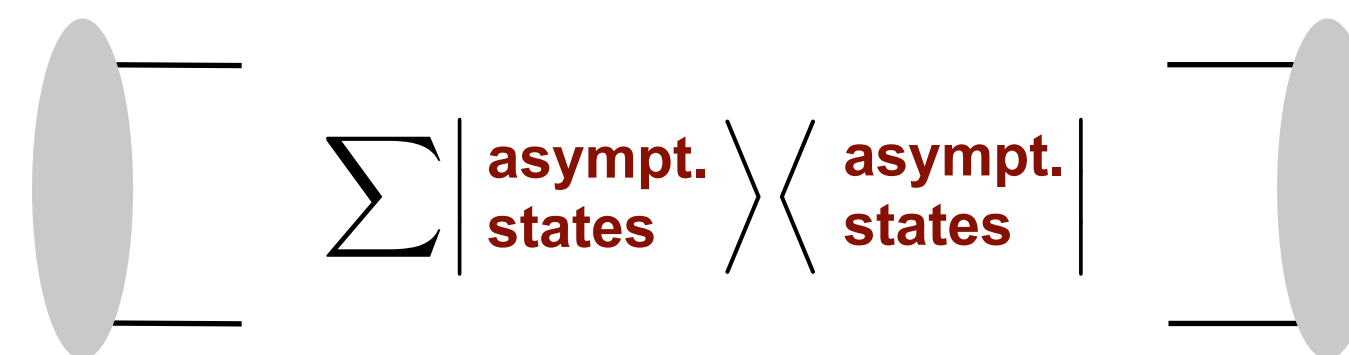
e.g. $\text{Tr} \langle q(x) \bar{q}(x) \rangle$



pressure, trace anomaly,
fluctuations, volume flucs., ...



on-shell



e.g. hadron resonances

- gauge fixing = parameterisation

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Consequences

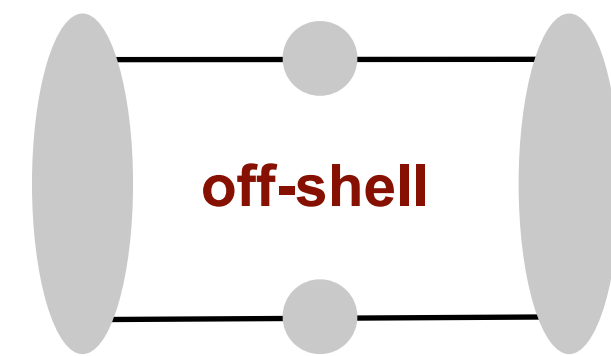
I: simple correlations

II: Difficult access to some observables
'No free lunch theorem'

Three remarks on Functional Methods for QCD

- off-shell representation of thermodynamic observables

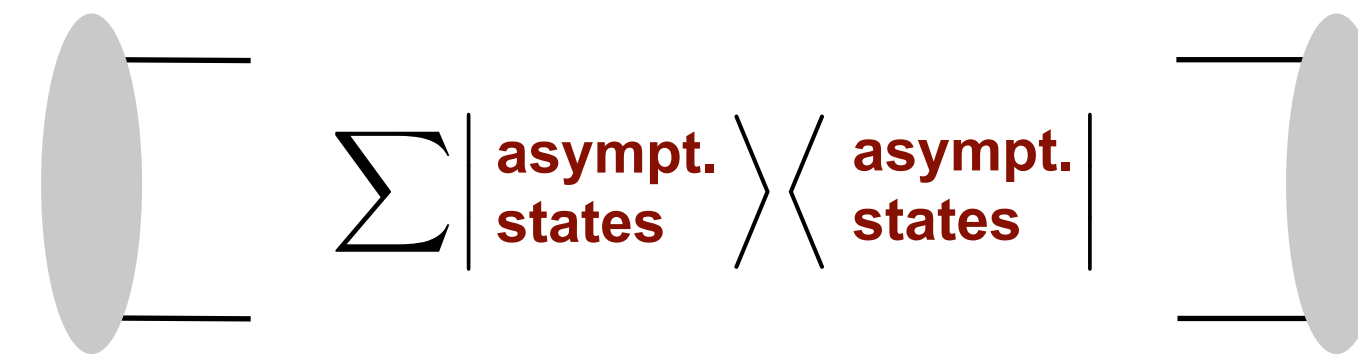
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on-shell



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Consequences

I: simple correlations

II: Difficult access to some observables
'No free lunch theorem'

- 'Your mean field is not my mean field'

$$\left. \frac{\delta S_{\text{cl}}[\phi]}{\delta \phi} \right|_{\phi=\bar{\phi}} = 0$$

$$\left. \frac{\delta \Gamma[\phi]}{\delta \phi} \right|_{\phi=\bar{\phi}_{\text{quant}}} = 0$$

How to: direct computations and the minimal point of view

Those are my interpretations,
and if you don't like them....
well, I have others

- Self-consistent truncations to functional relations define analytic functions in μ_B , eg:

$$\partial_t \left\langle q(x) \bar{q}(y) \right\rangle_{\mu_B} = \text{Loop} \left[\left\langle q(x) \bar{q}(y) \right\rangle_{\mu_B}, \left\langle q(x) A_\mu(y) \bar{q}(z) \right\rangle_{\mu_B}, \dots ; \mu_B \right]$$

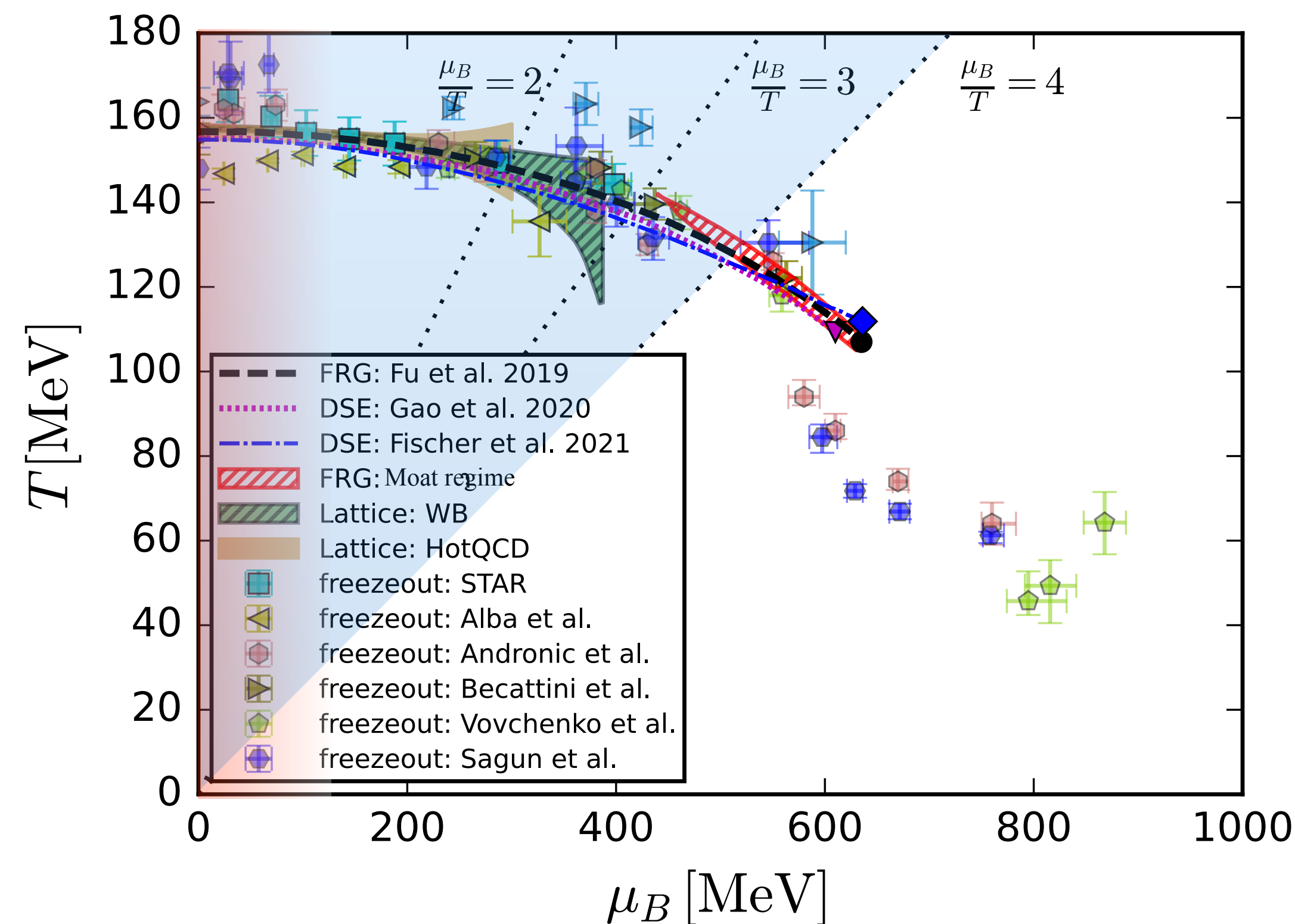
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- Consequences for functional QCD predictions at finite density



By now the best truncations to functional QCD pass lattice benchmark tests at vanishing and small chemical potential

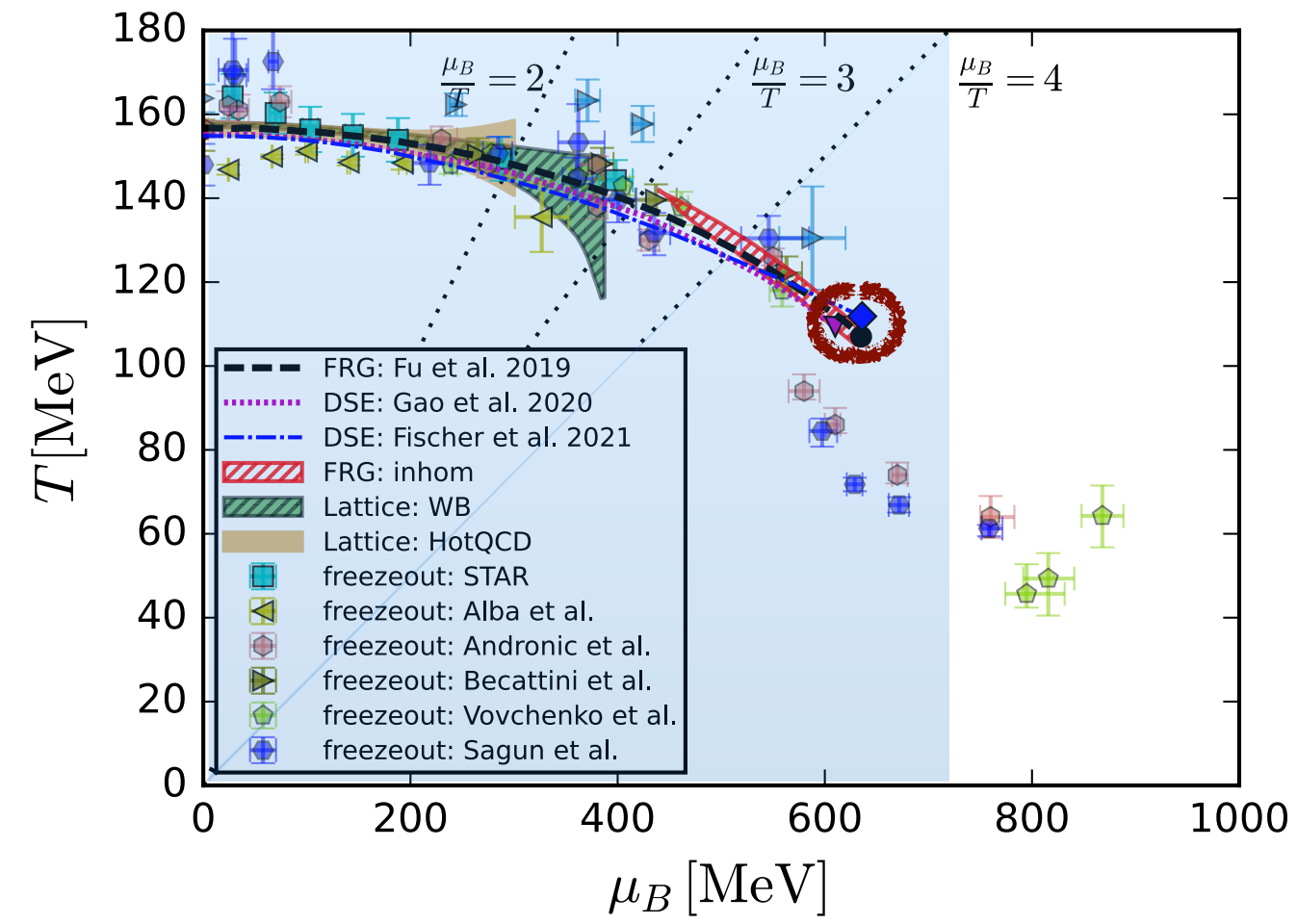
Regime of quantitative reliability of current best truncation

Unique: QCD-based analytic continuations that satisfy the lattice benchmarks at small chemical potential.

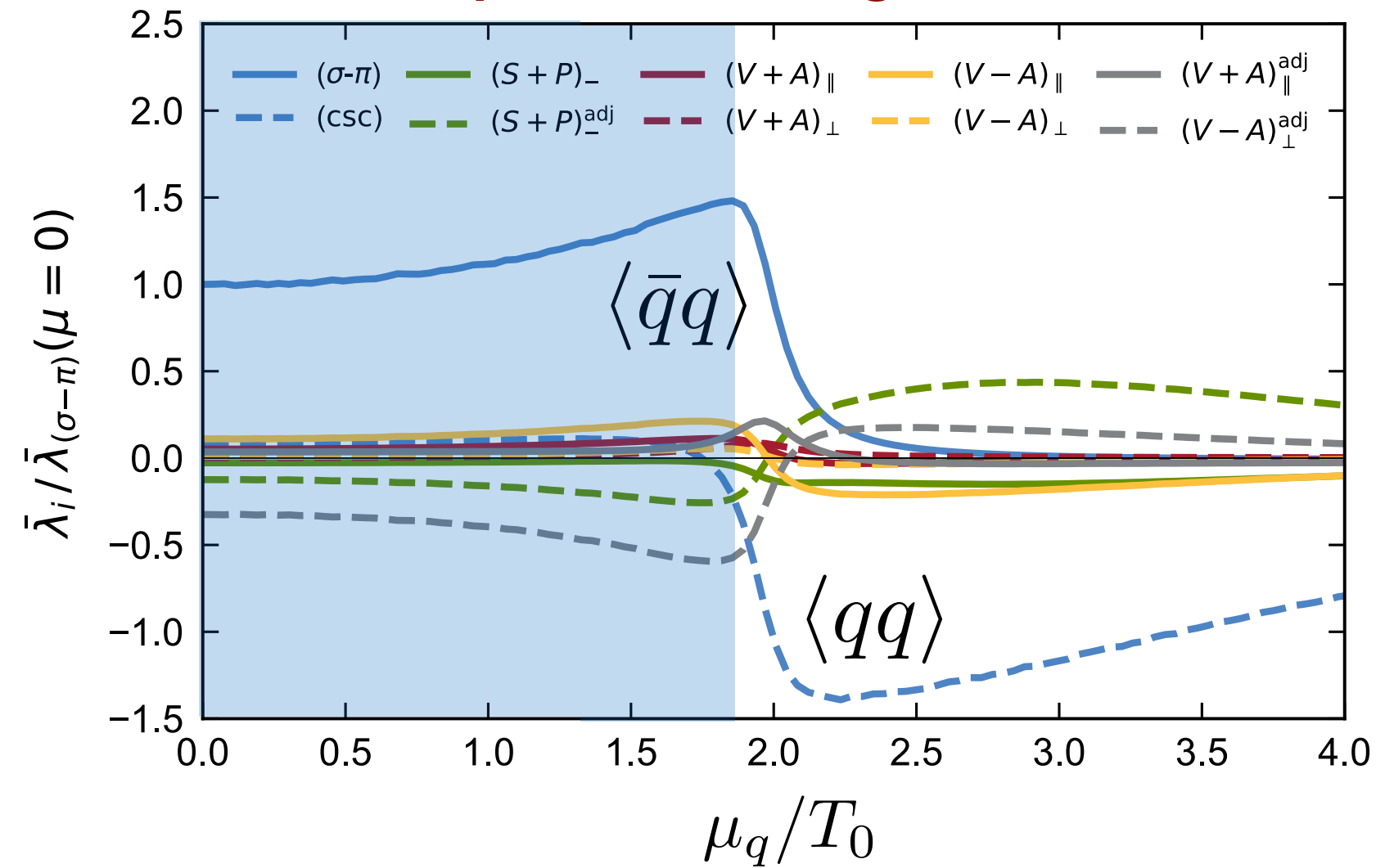
Great opportunity for a combined analysis of high density QCD (Exp. data + lattice QCD + functional QCD + LEFTs)

Phase structure from functional QCD: Predictions & estimates

Predictions & estimates



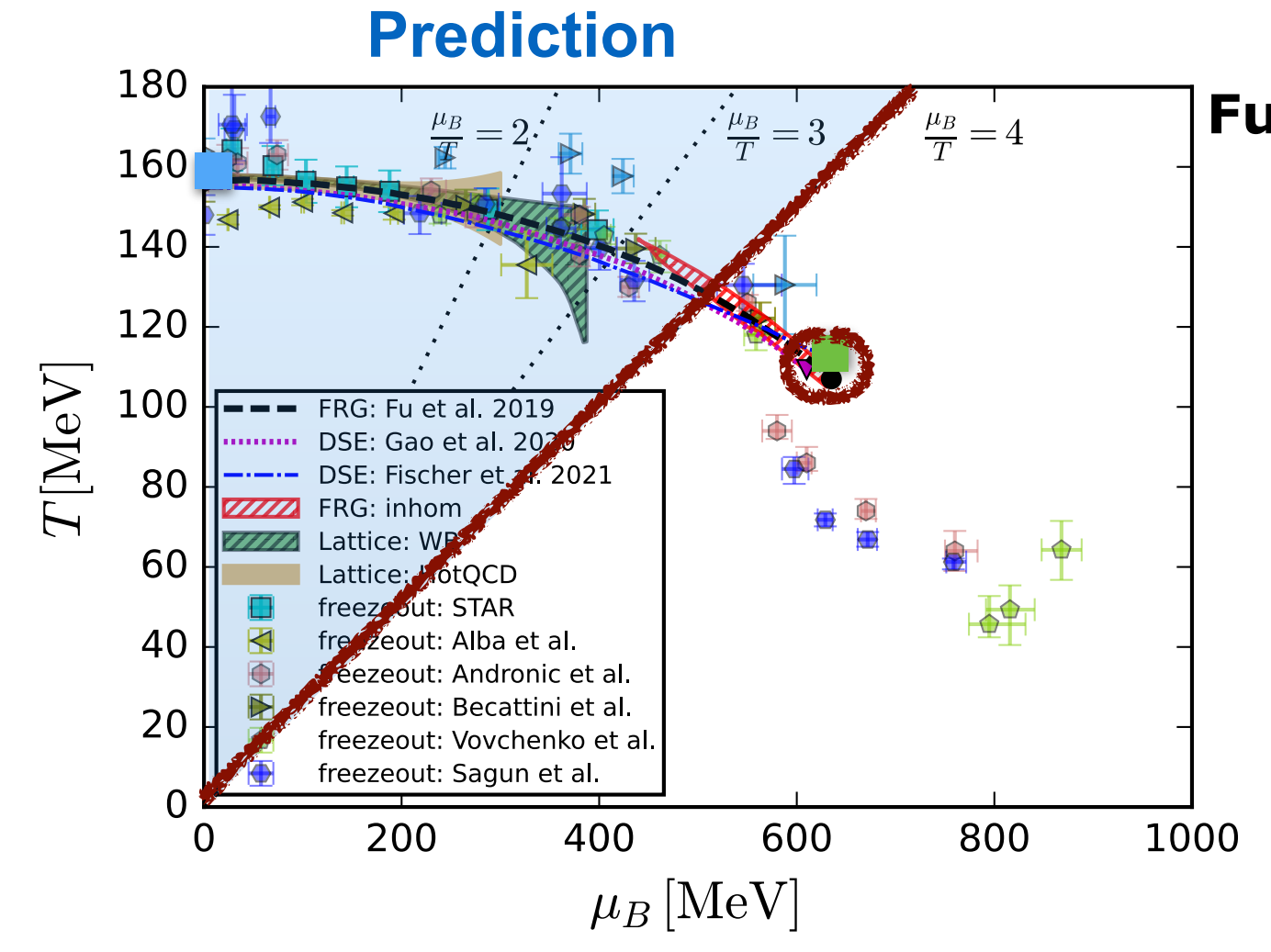
Four-quark scattering channels



Dominance of scalar-pseudoscalar fluctuations
Pions & sigma mode

Predictions & estimates

Moat regime
Pisarski, Rennecke, PRL 127 (2021) 152302



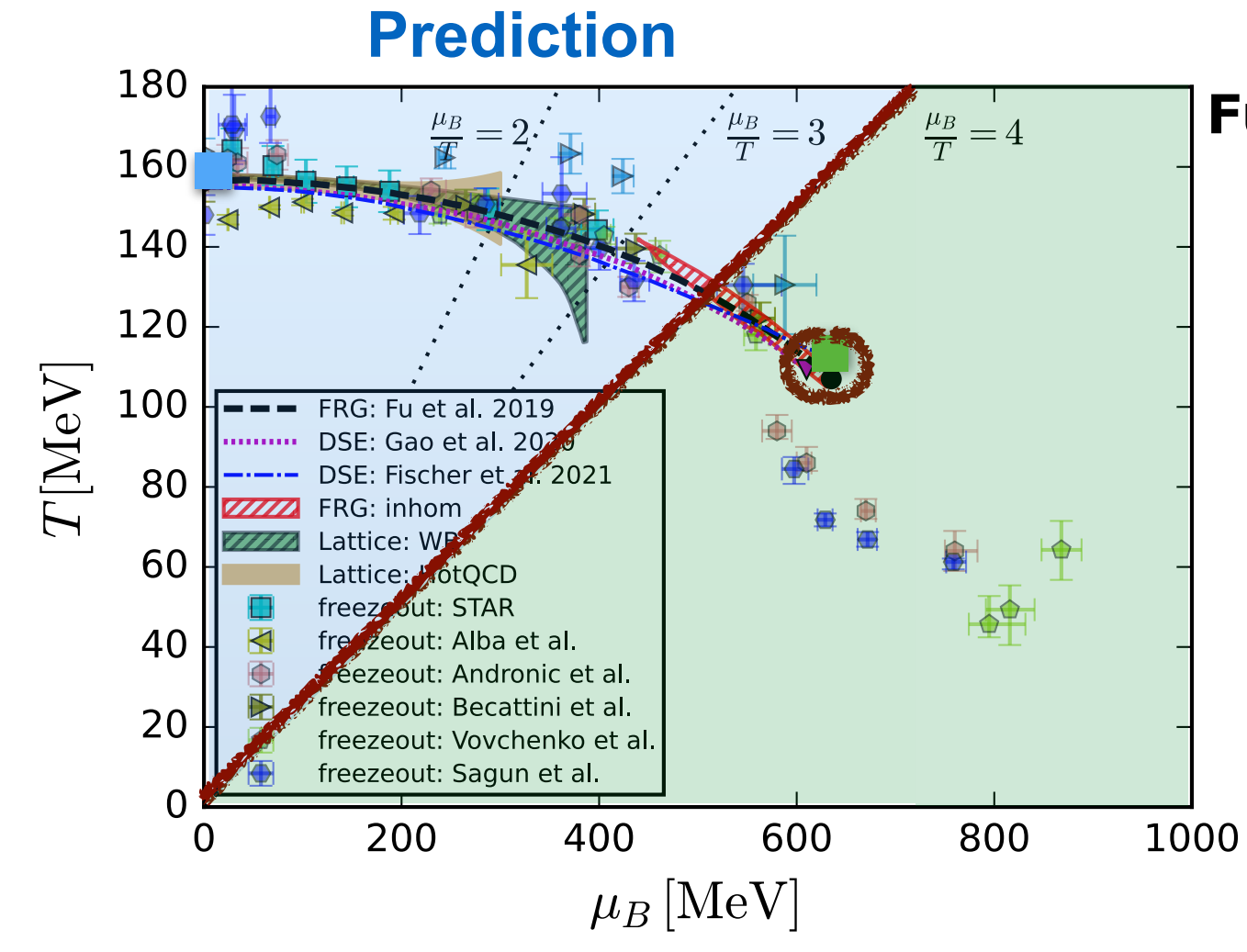
Fu, JMP, Rennecke, PRD 101 (2020) 054032

**Regime of quantitative reliability
of
current best truncation**

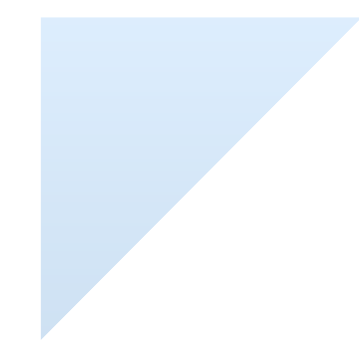
Predictions & estimates

Moat regime

Pisarski, Rennecke, PRL 127 (2021) 152302



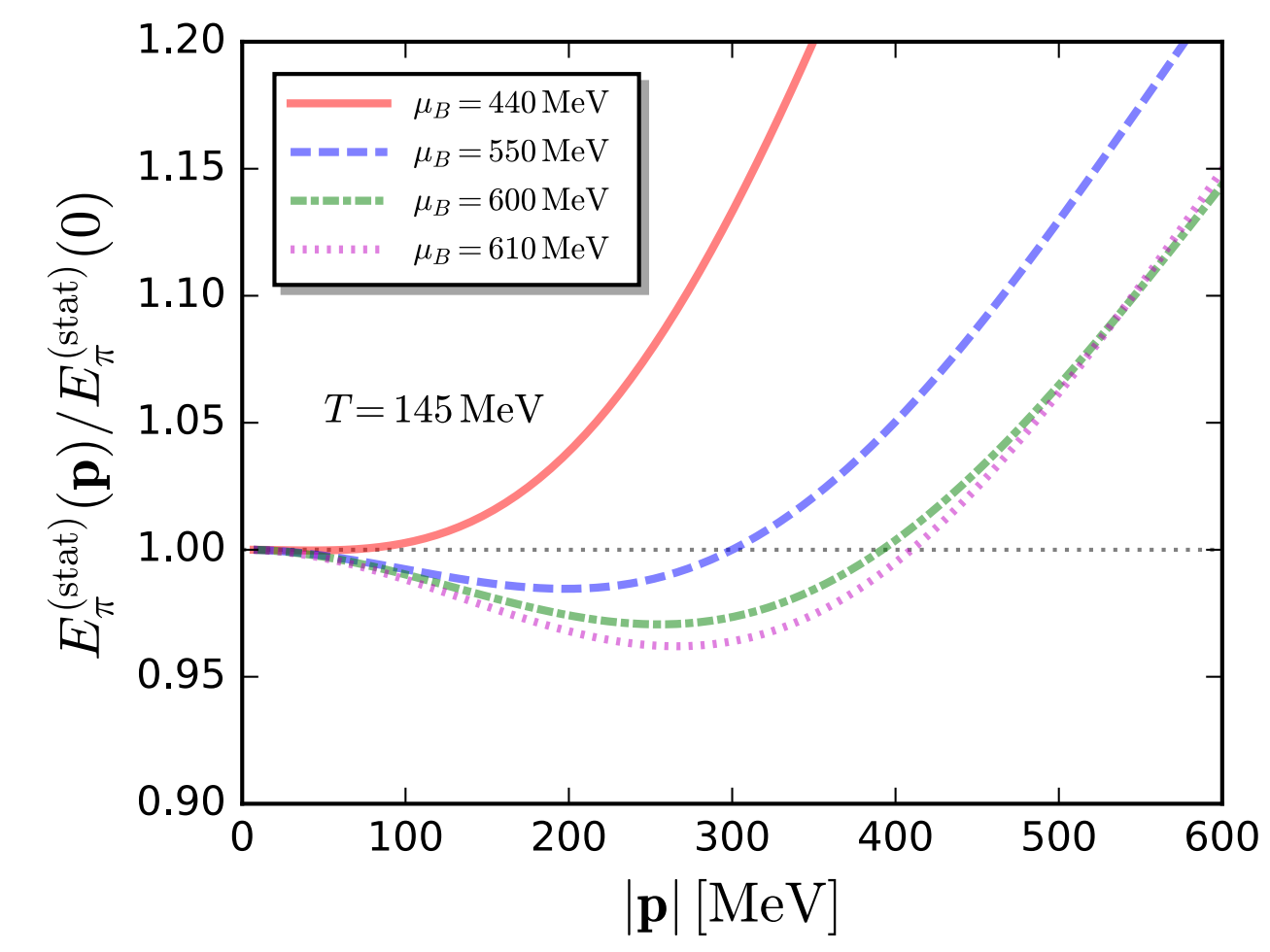
Fu, JMP, Rennecke, PRD 101 (2020) 054032



**Regime of quantitative reliability
of
current best truncation**

Estimate

Moat regime is not captured quantitatively



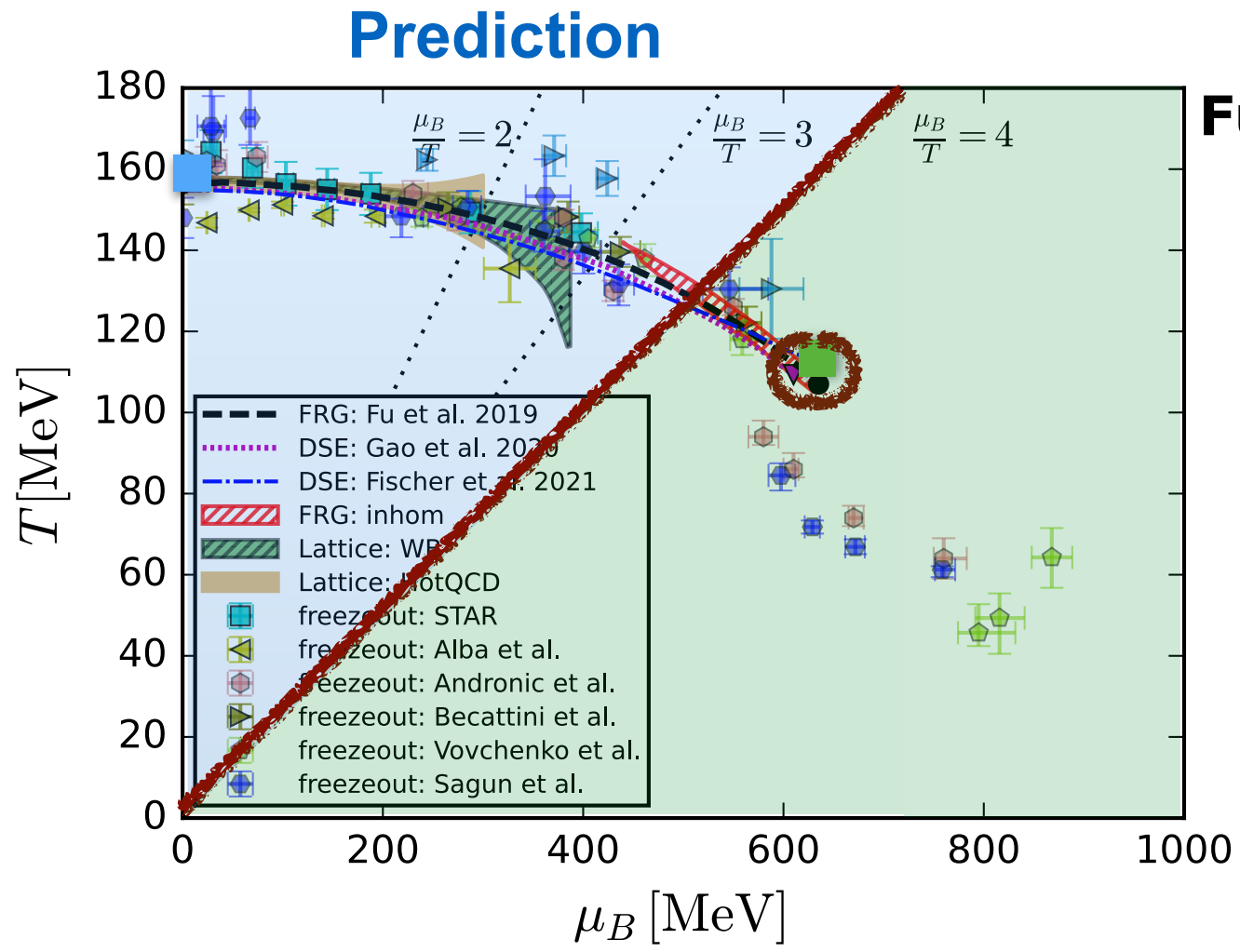
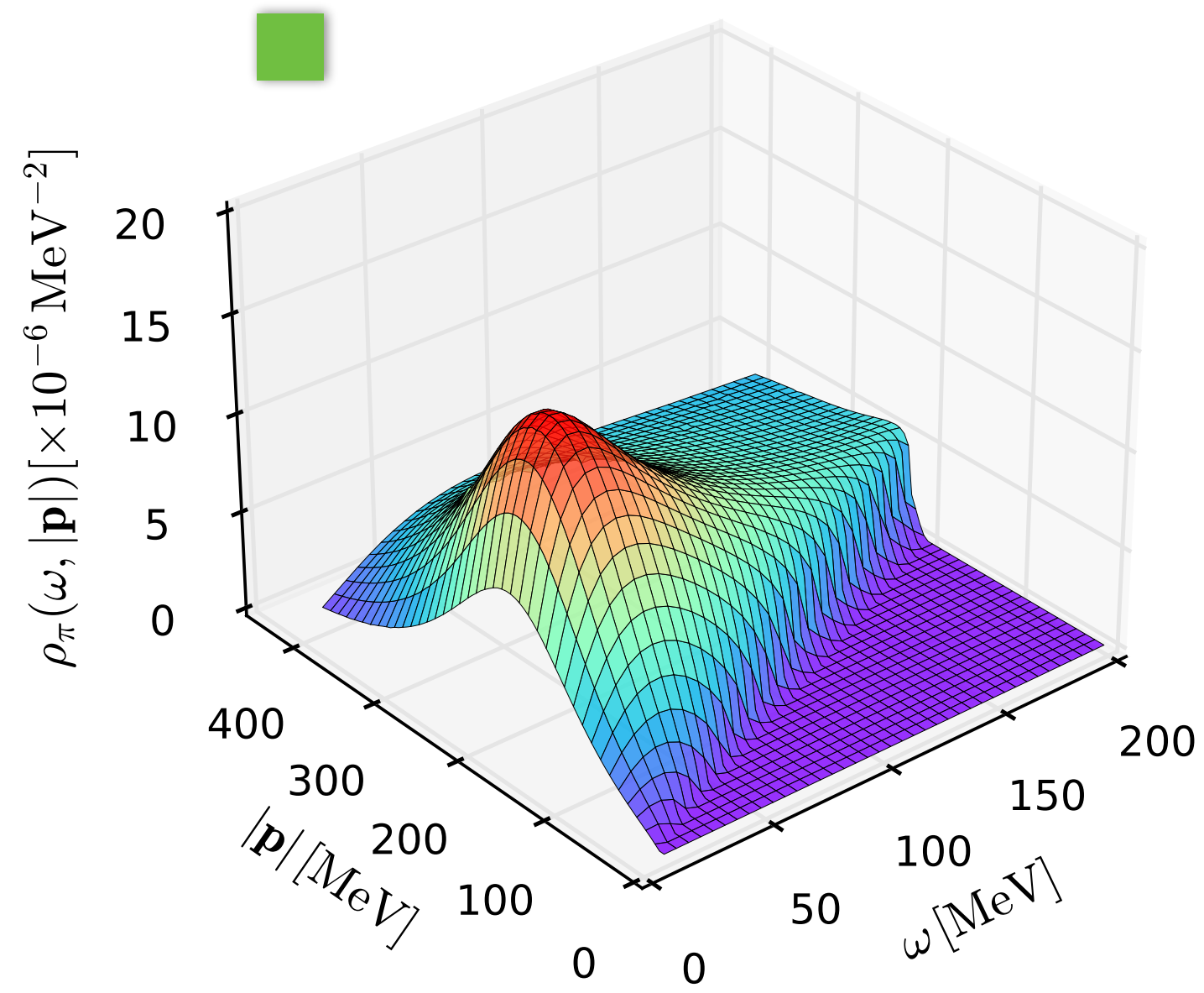
Fu, JMP, Pisarski, Rennecke, Wen, Shi Yin, 2412.15949

Predictions & estimates

Moat regime

Pisarski, Rennecke, PRL 127 (2021) 152302

T=114 MeV & $\mu_B=630$ MeV

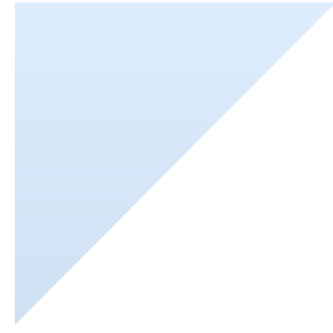


Fu, JMP, Rennecke, PRD 101 (2020) 054032

Estimate

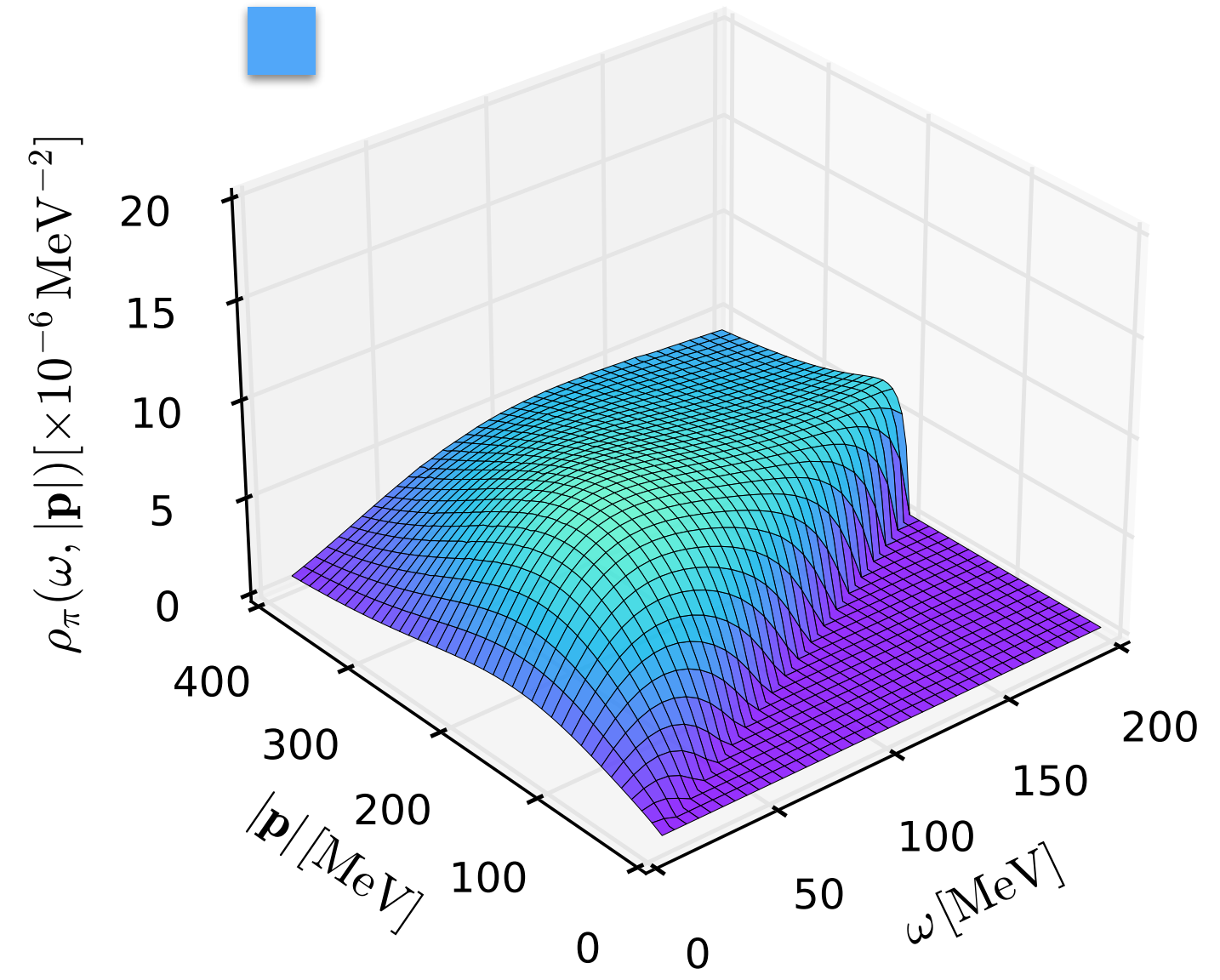
Moat regime is not captured quantitatively

Pion spectral functions



Regime of quantitative reliability of current best truncation

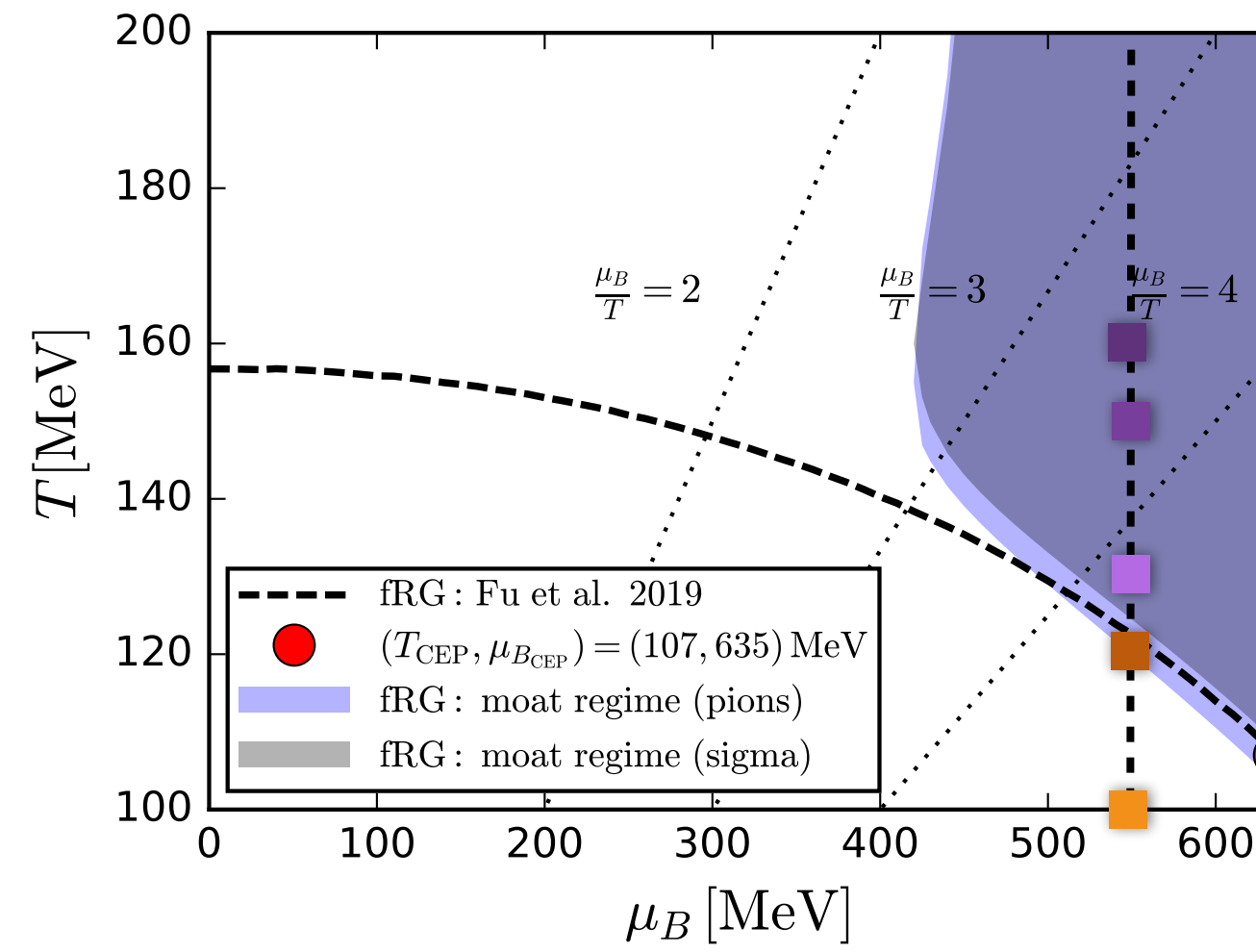
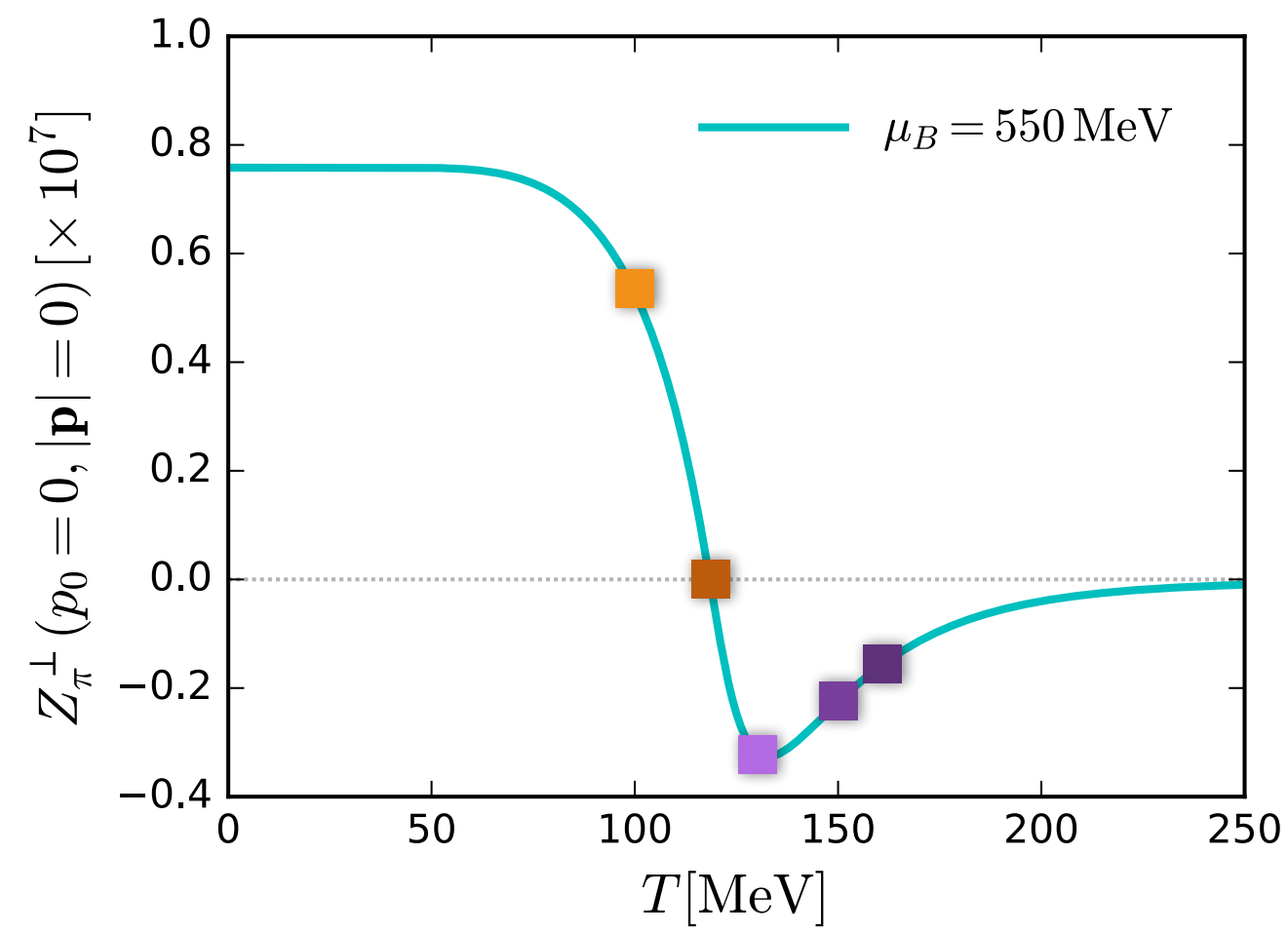
T=160 MeV & $\mu_B=0$ MeV



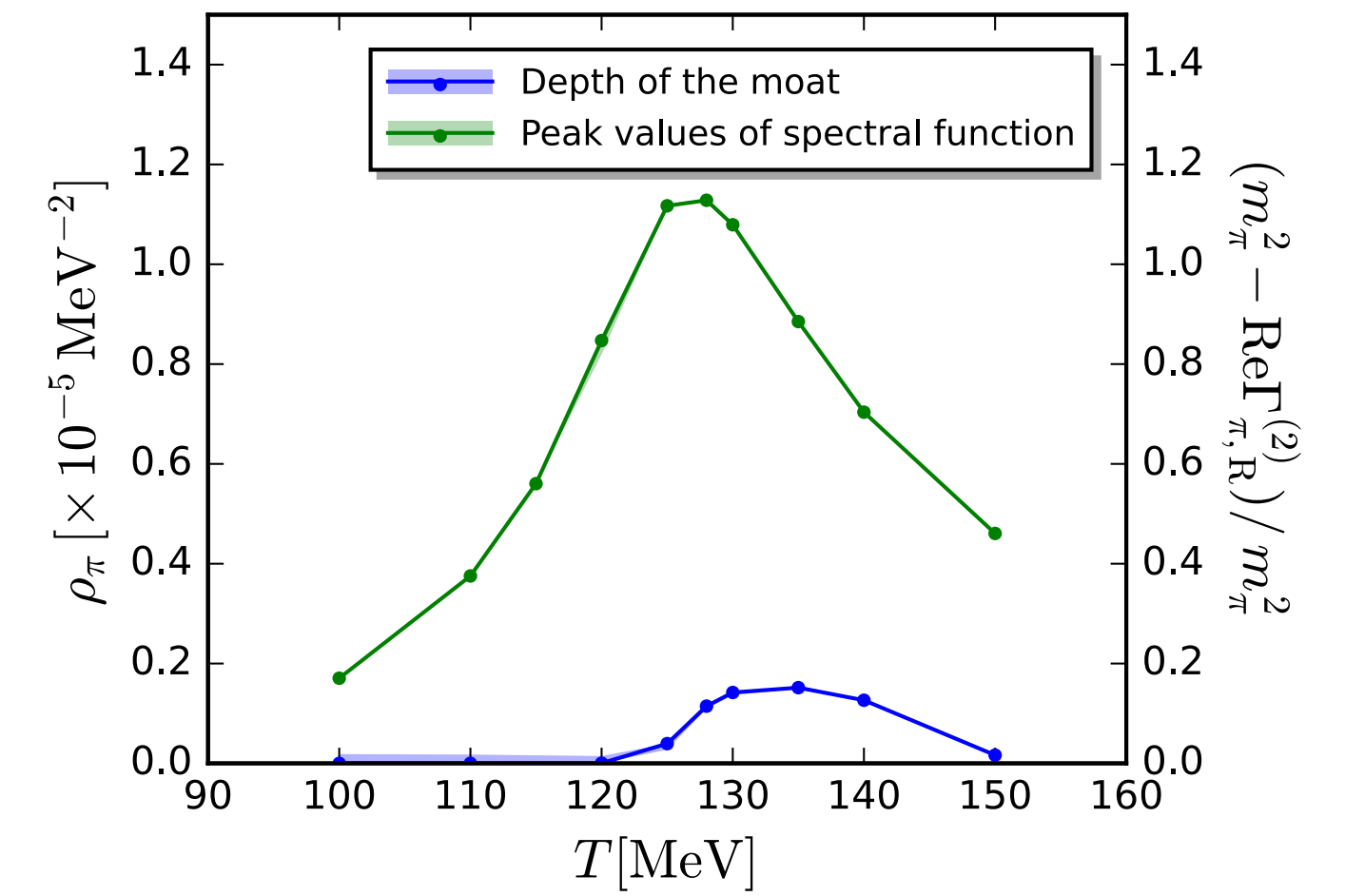
Fu, JMP, Pisarski, Rennecke, Wen, Shi Yin, 2412.15949

Predictions & estimates

Spatial wave function of the pion at $p=0$



Depth of the moat & spectral peak of the pion



Dissecting the moat & the moaton

$\mu_B = 650 \text{ MeV}$

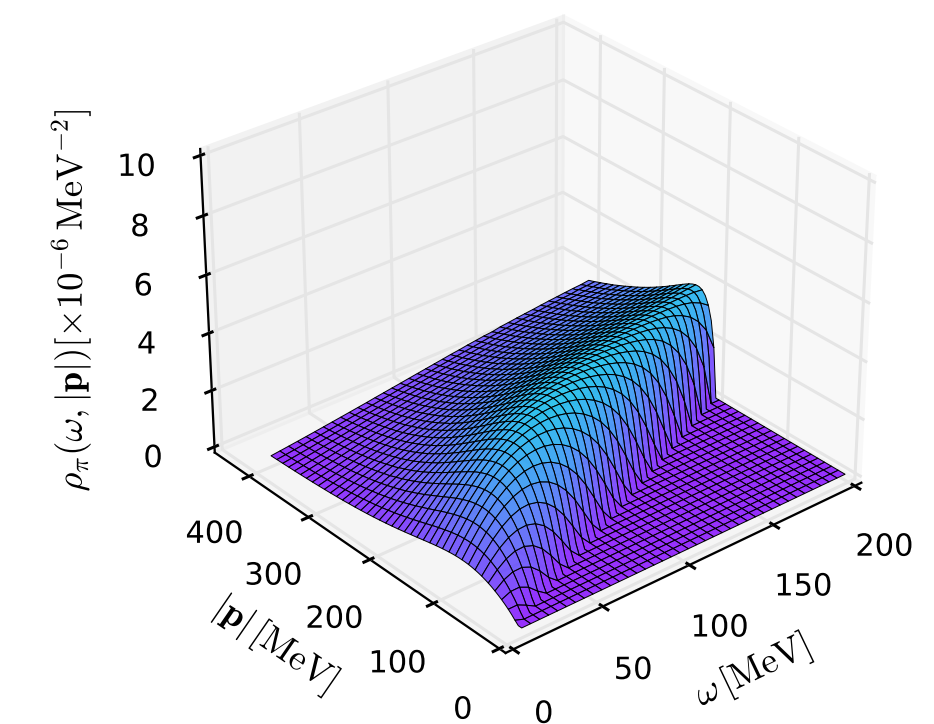
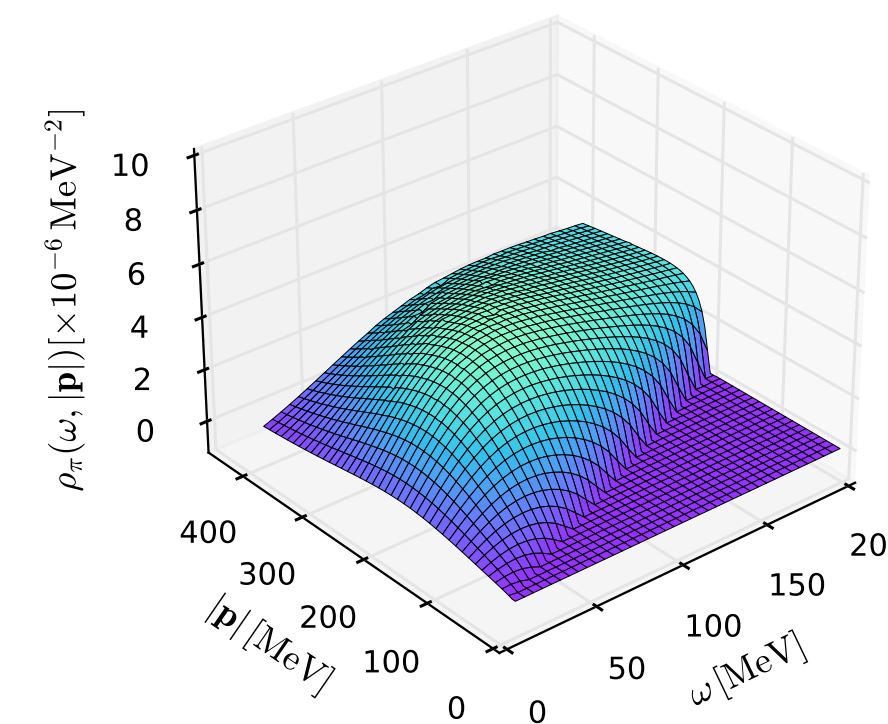
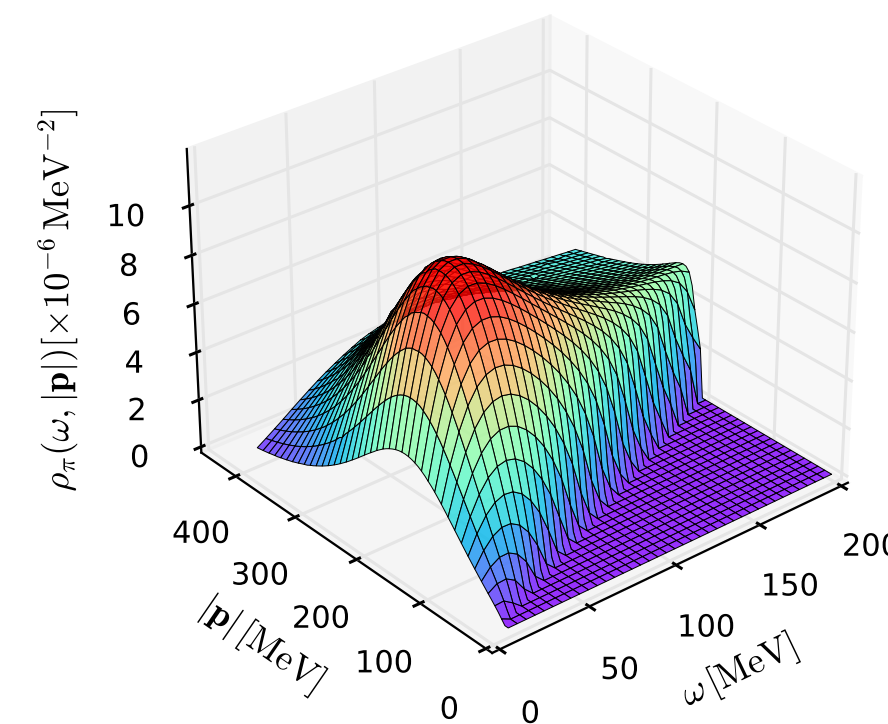
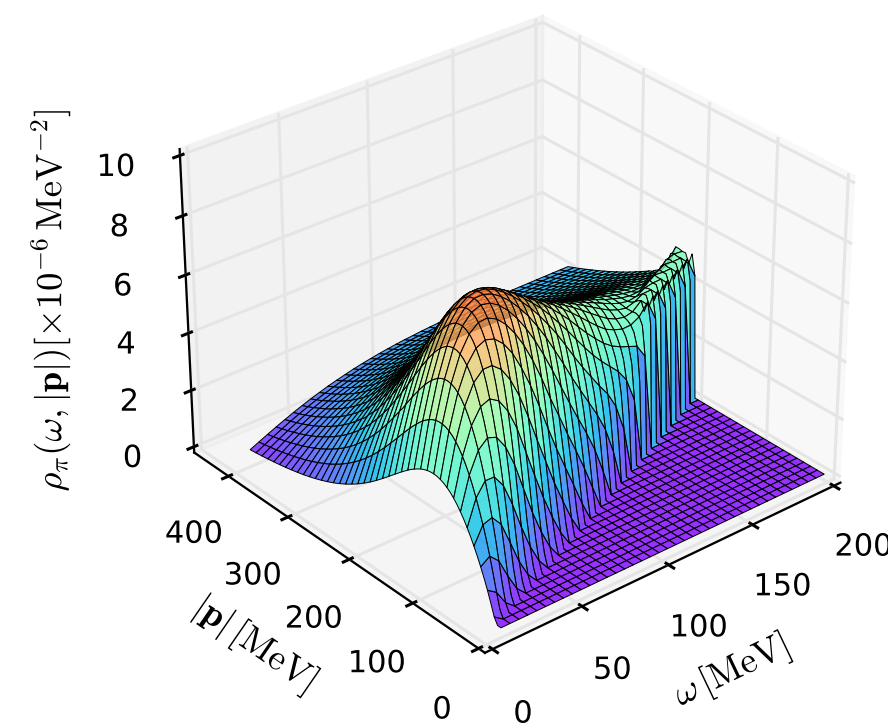
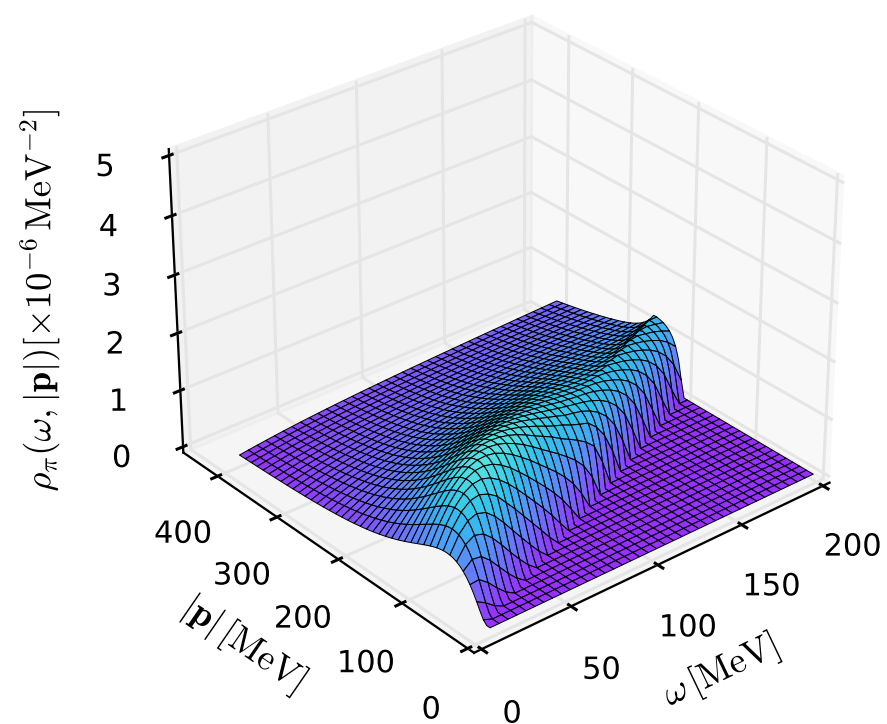
■ T=100 MeV

■ T=120 MeV

■ T=130 MeV

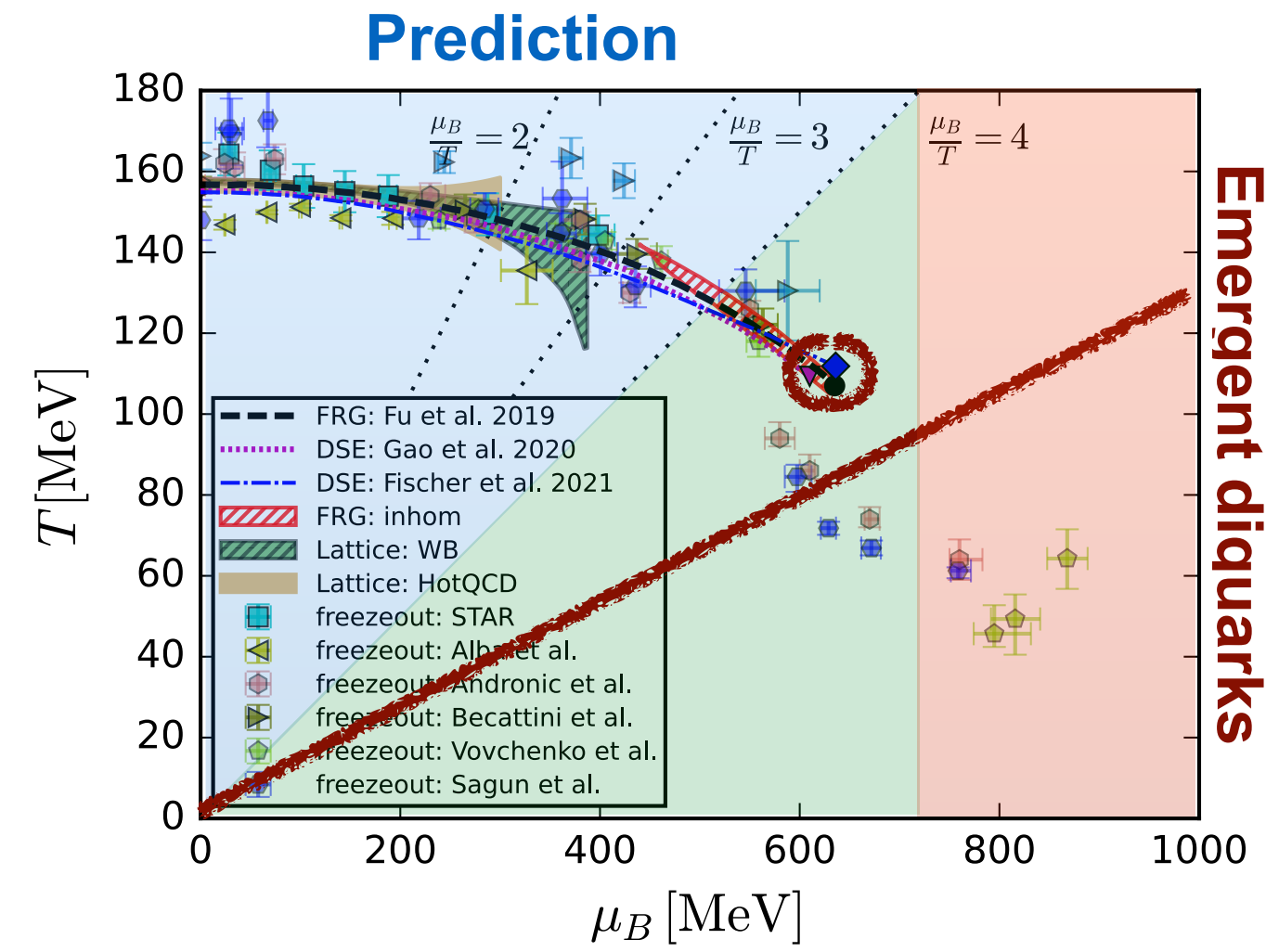
■ T=150 MeV

■ T=160 MeV

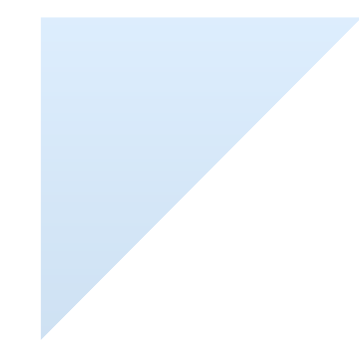


Predictions & estimates

Emergent diquarks

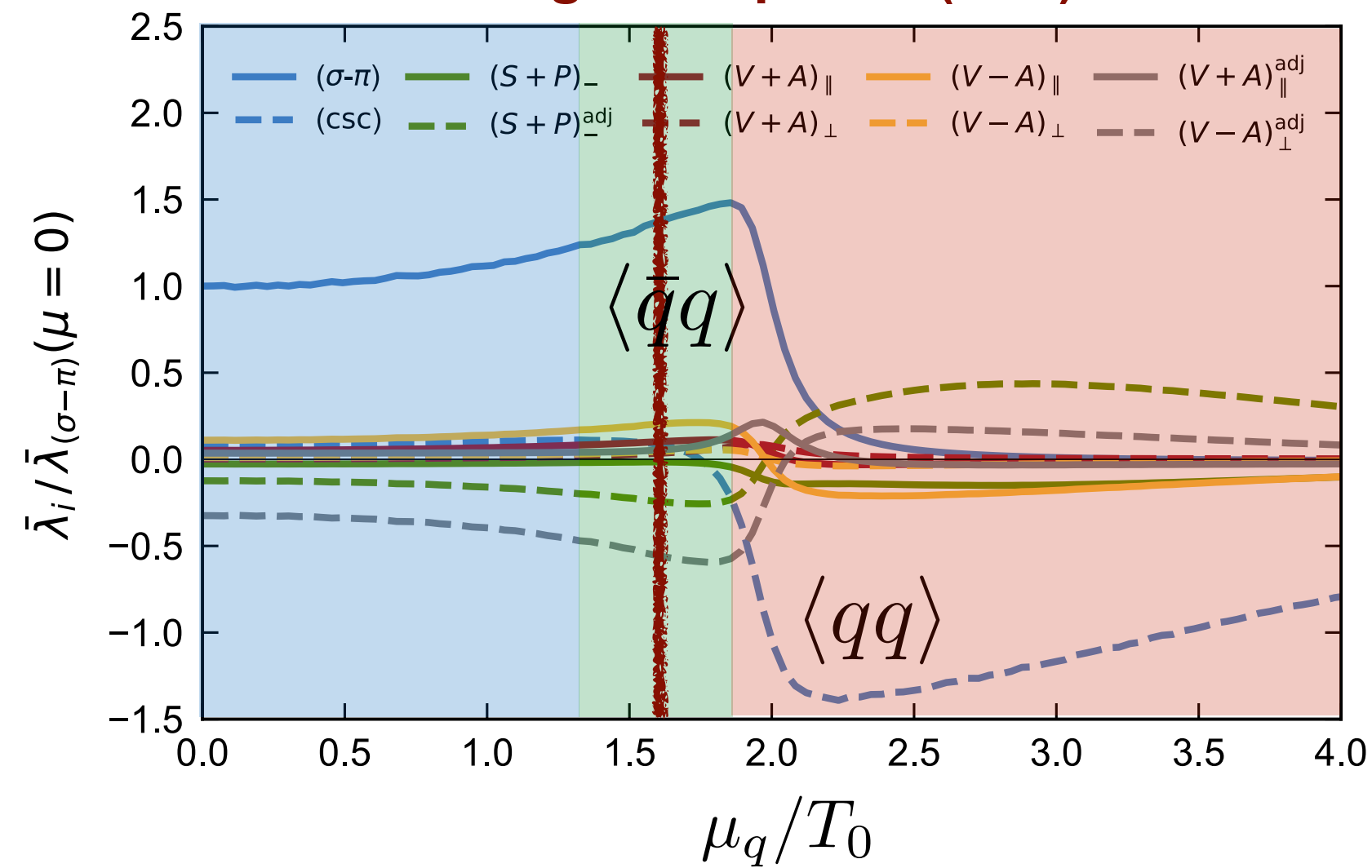


Estimate



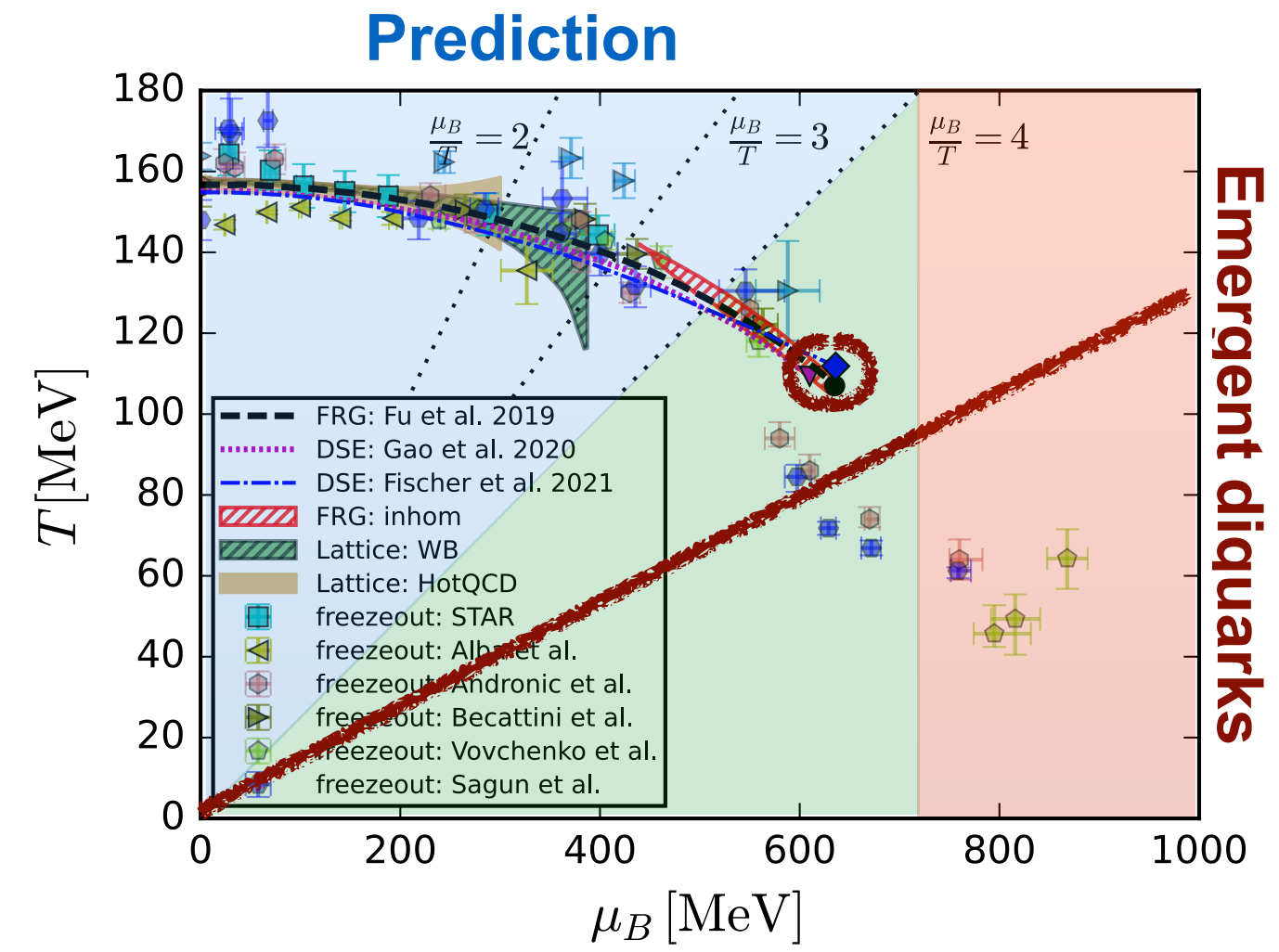
Regime of quantitative reliability of current best truncation

Emergent diquarks (fRG)

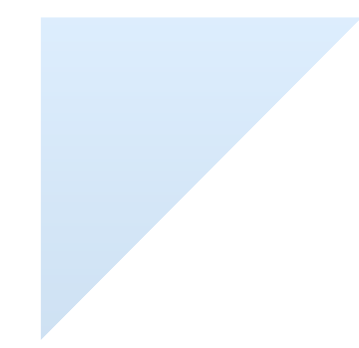


Predictions & estimates

Emergent diquarks

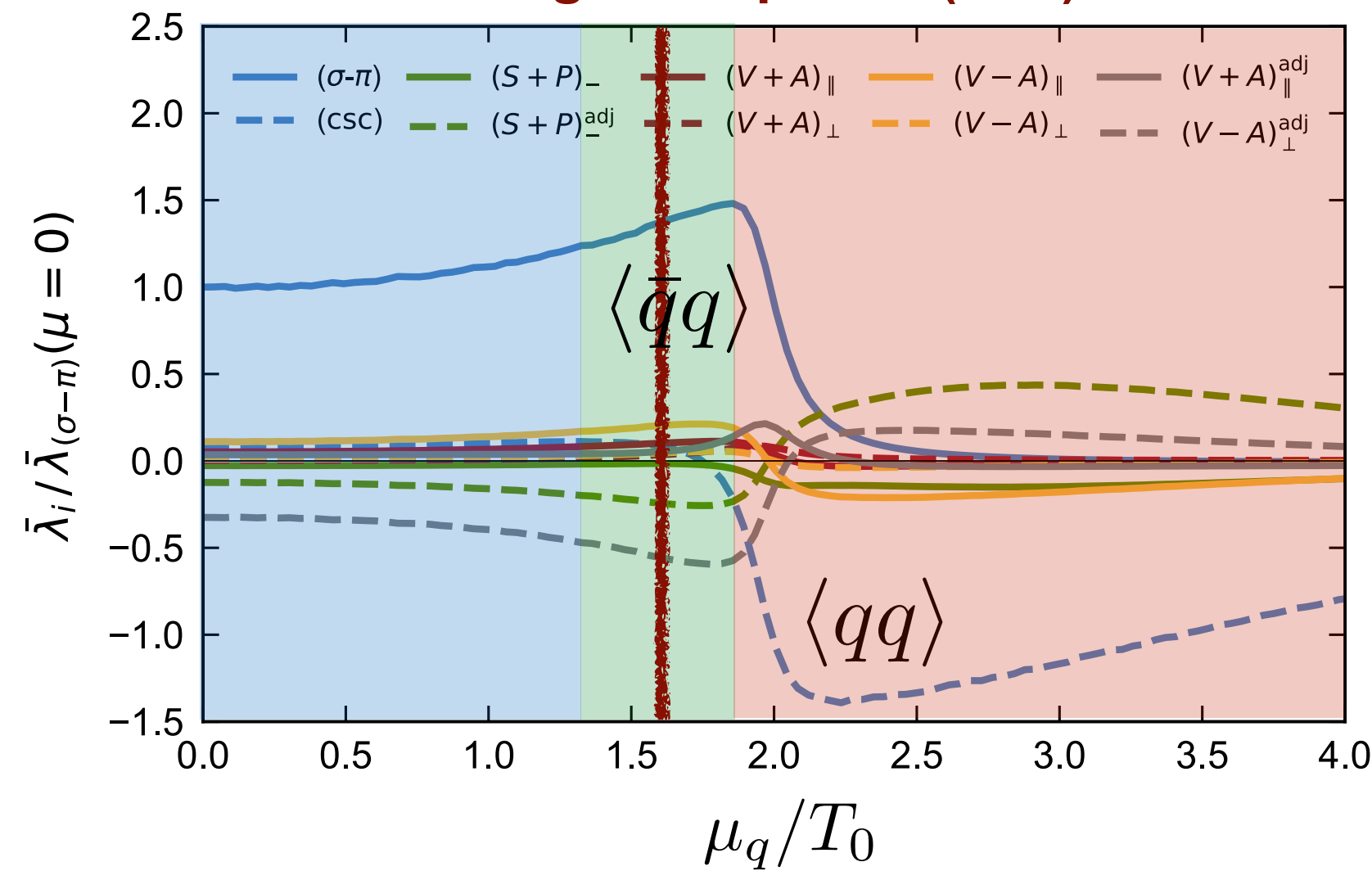


Estimate



Regime of quantitative reliability of current best truncation

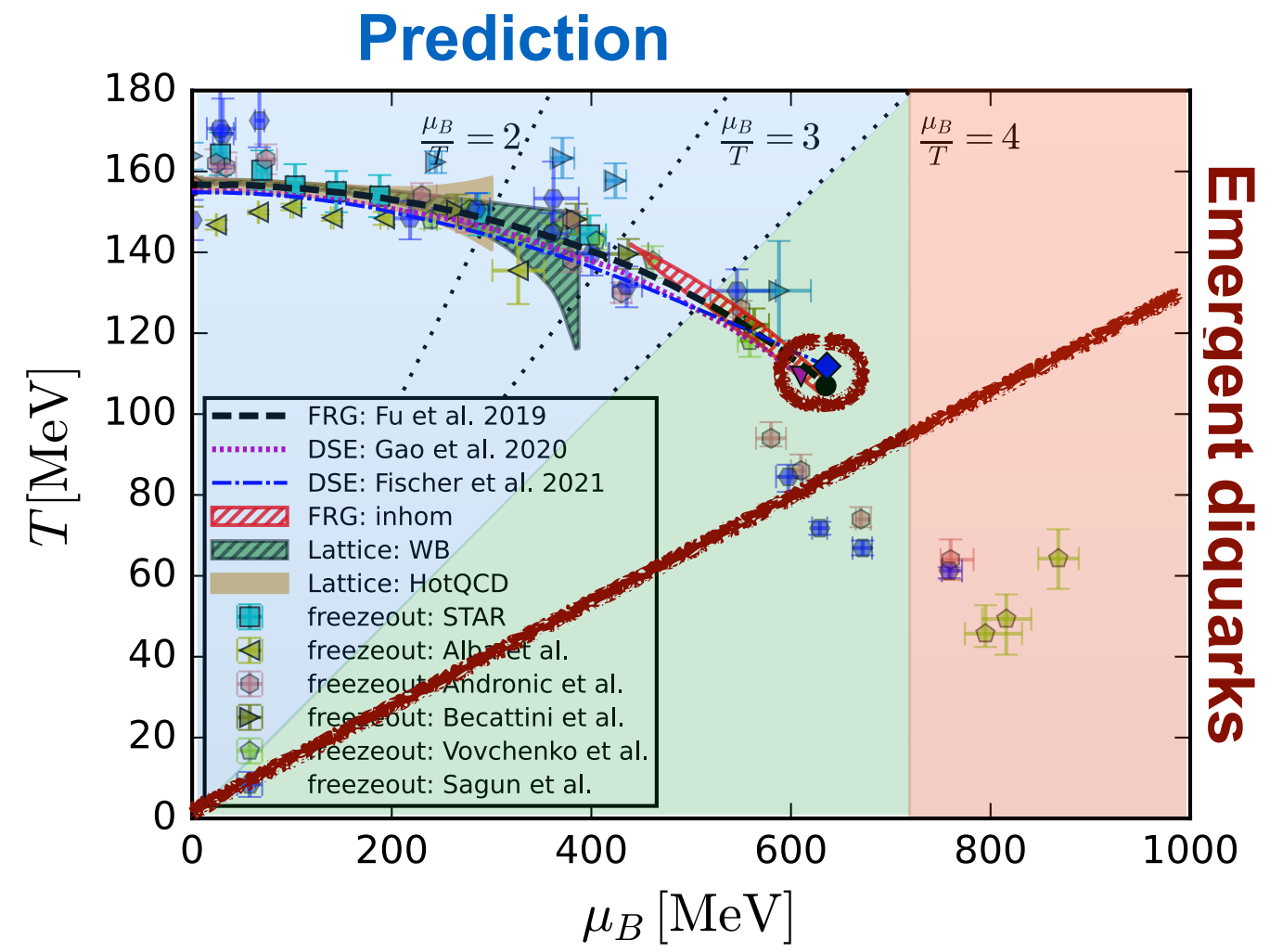
Emergent diquarks (fRG)



Emergent diquarks are not captured by extrapolations

Predictions & estimates

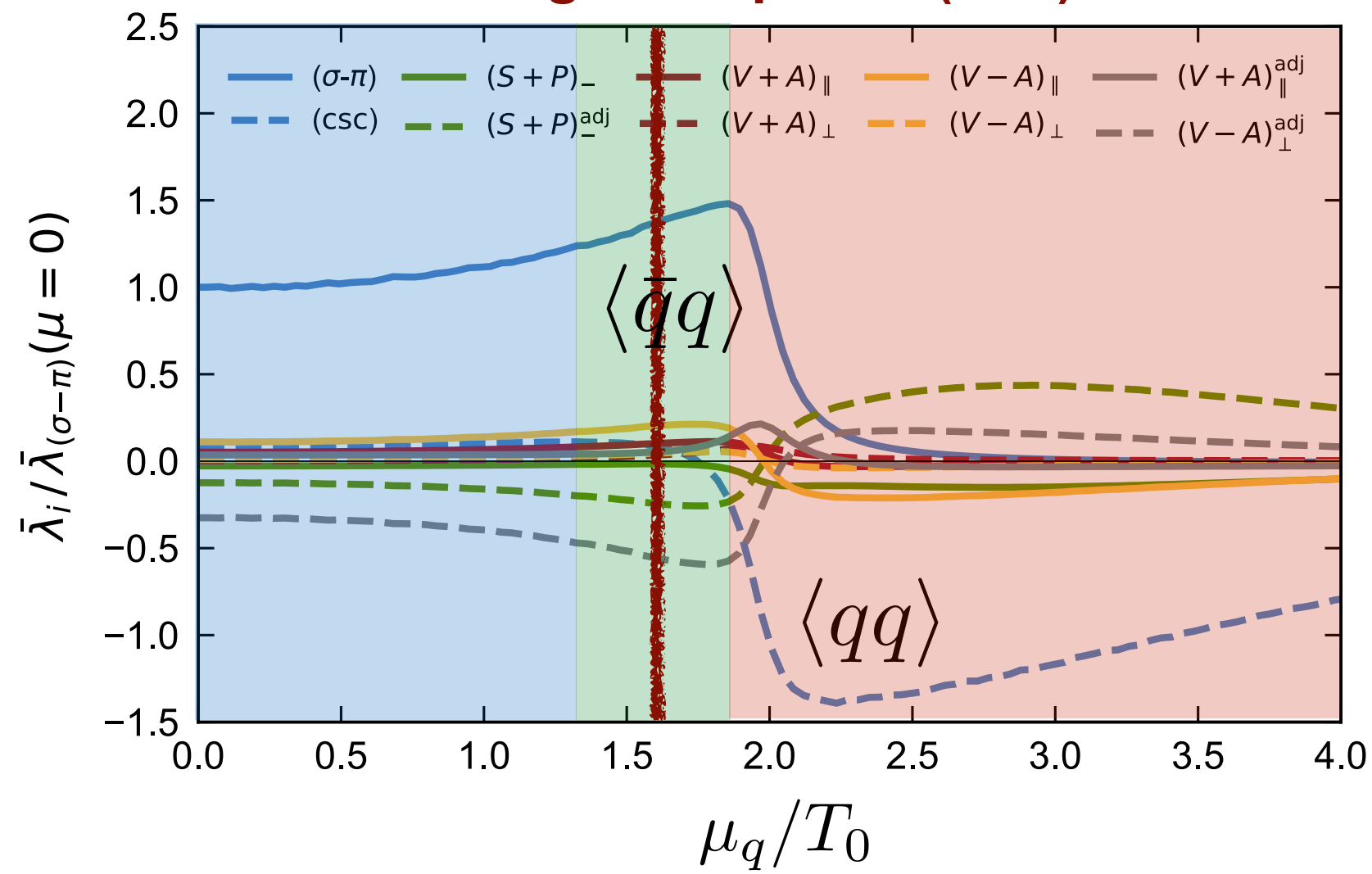
Emergent diquarks



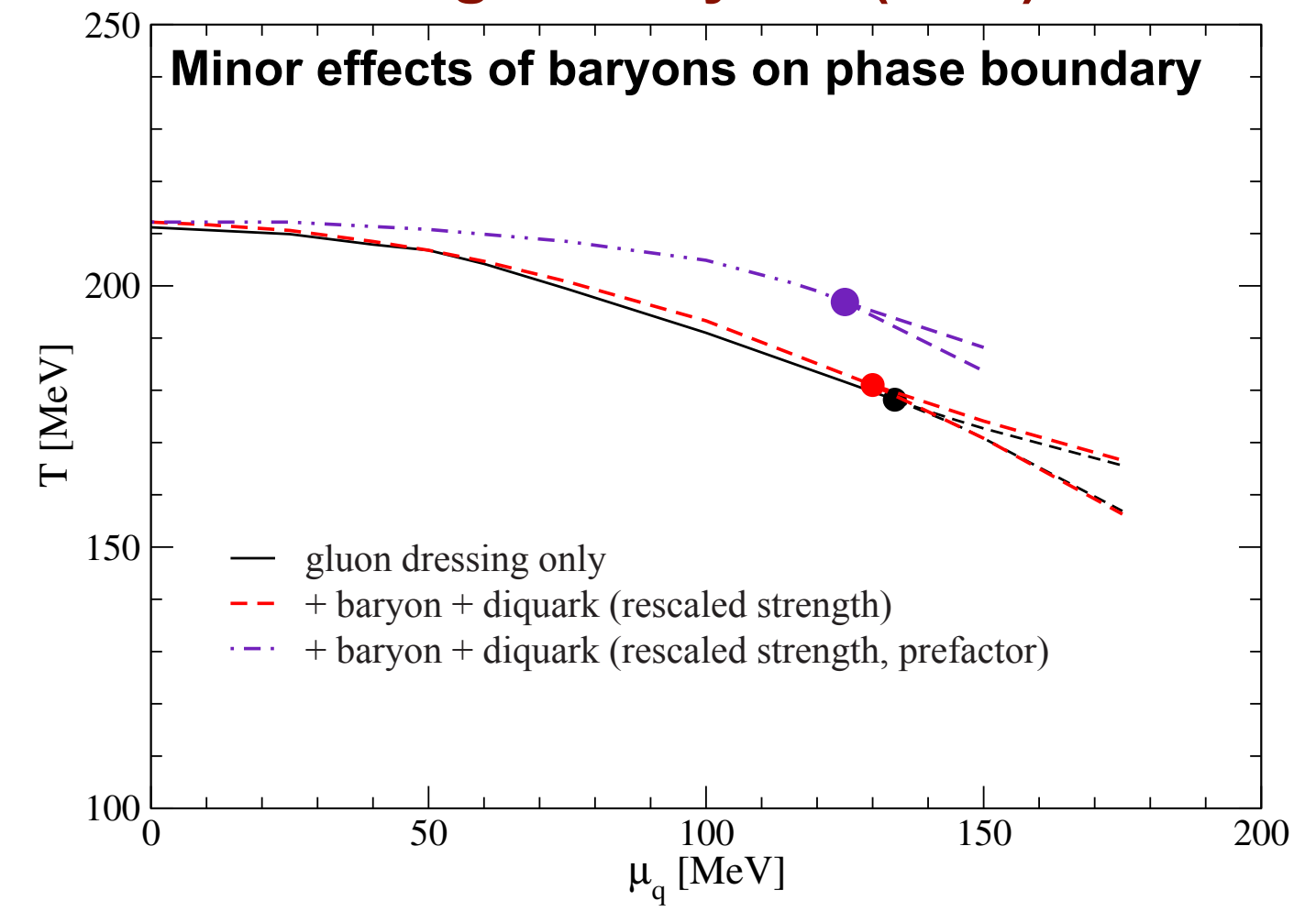
Regime of quantitative reliability of current best truncation

Estimate

Emergent diquarks (fRG)



Emergent baryons (DSE)

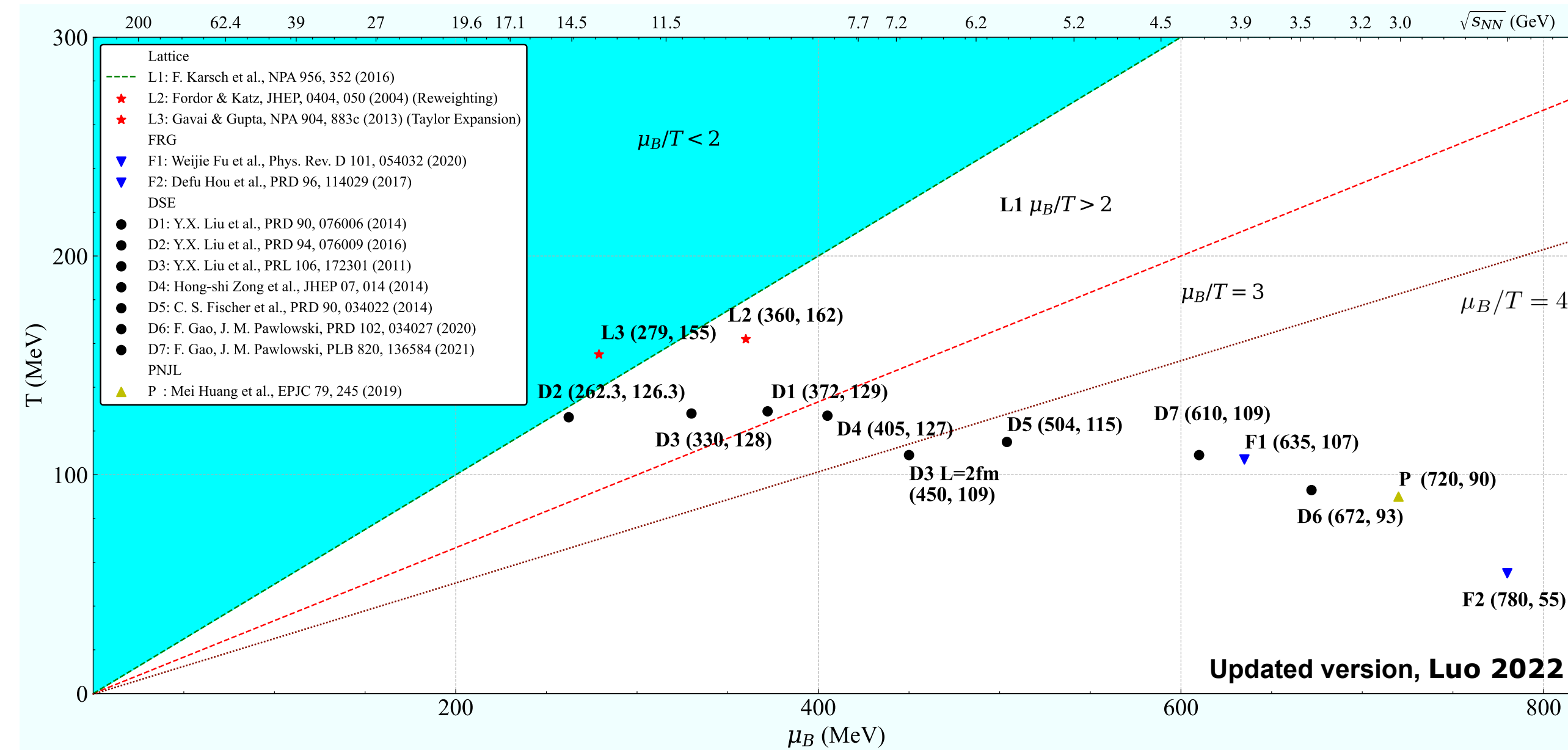


Predictions, estimates & extrapolations and how to judge them



Location of CP : Theoretical Prediction

Preliminary collection from Lattice, DSE, FRG and PNJL (2004-2020)



Large uncertainties for the estimation of CP location.

Disclaimer

Most functional computations (LEFT or QCD) have not been set-up for CEP-predictions!

Lack of predictive power for CEP-predictions is no quality measure!

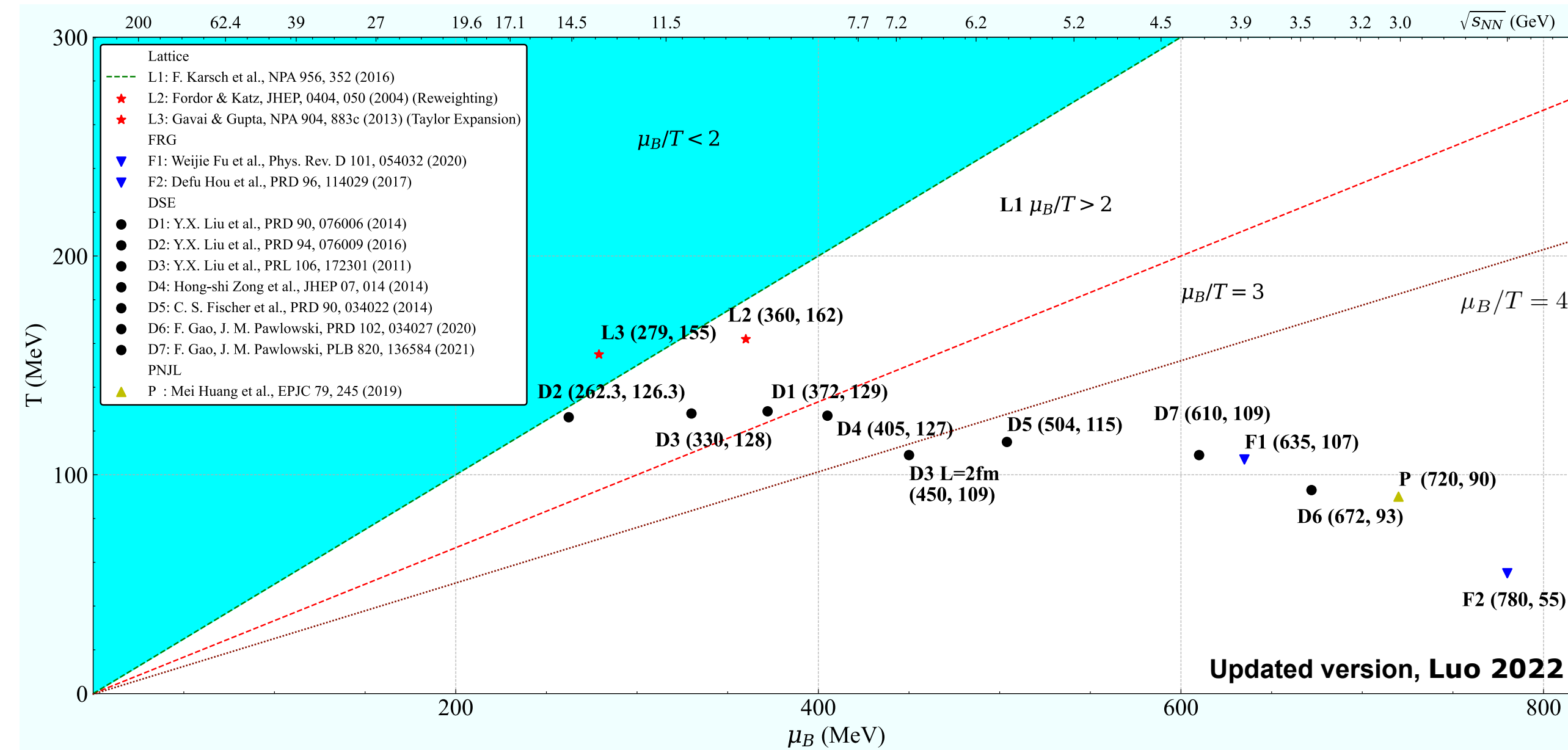
CEP is standing for 'regime with new physics'

Predictions, estimates & extrapolations and how to judge them



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Common folklore since ~2004



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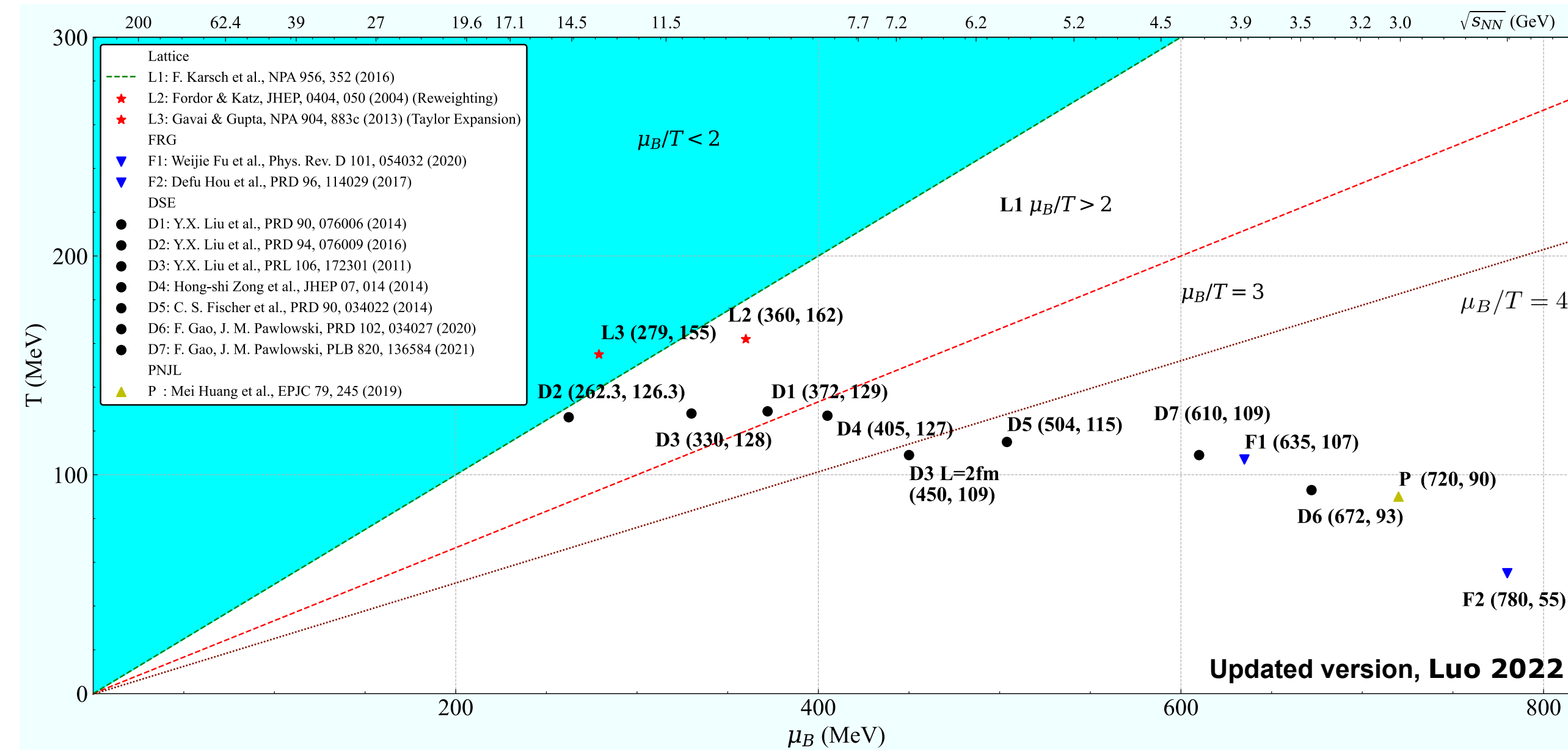
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Predictions, estimates & extrapolations and how to judge them



Location of CP : Theoretical Prediction

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Remove CEP-predictions

RHIC-BES Seminar Oct. 6th 2020, Xiaofeng Luo

(i) 'old' CEPs: lattice, Functional QCD approaches, LEFTS (updated computations available)

(ii) LEFTs & Functional Results (qualitative approximations) that miss lattice benchmarks at $\mu_B = 0$

(iii) LEFTs with CEPs at large density (missing quark-gluon back reaction)

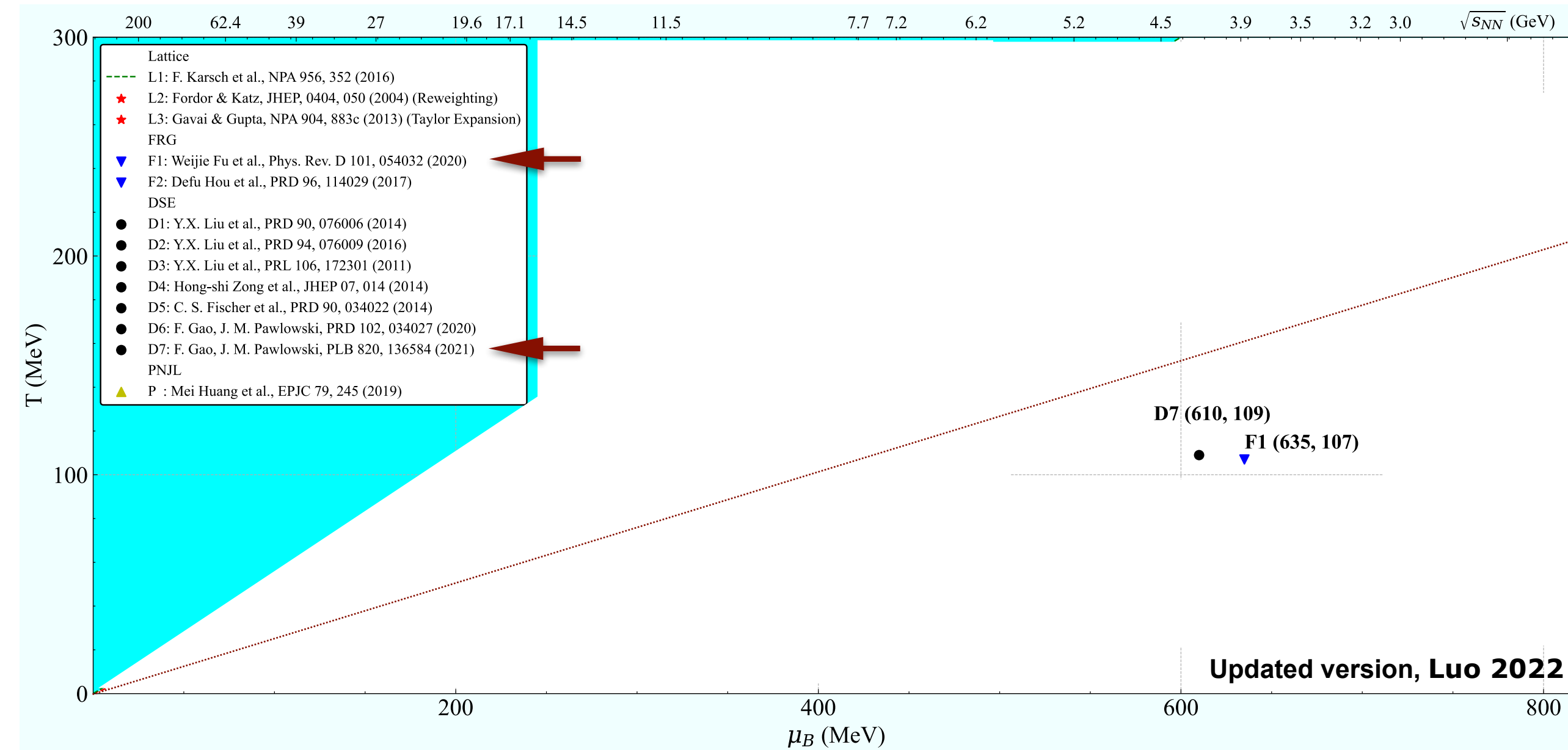
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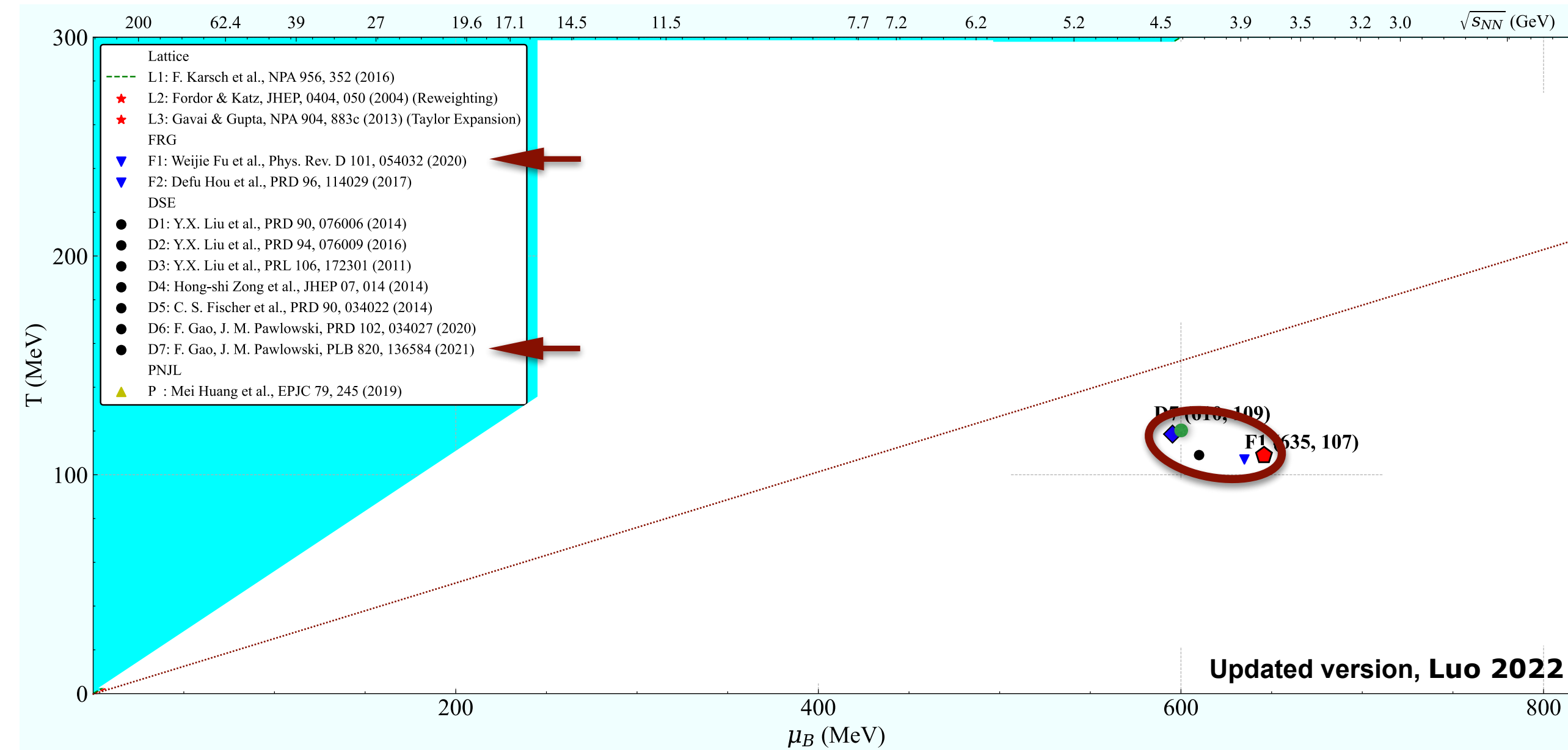


Location of CP : Theoretical Prediction

Preliminary collection from Lattice, DSE, FRG and PNJL (2004-2023)

Functional QCD

- ◆ Gao, Lu, JMP, Schneider, in prep (DSE)
- ◆ Fu, JMP, Rennecke, Wen, Yin, in prep (fRG)
- Gunkel, Fischer, PRD 104 (2021) 054022 (DSE)



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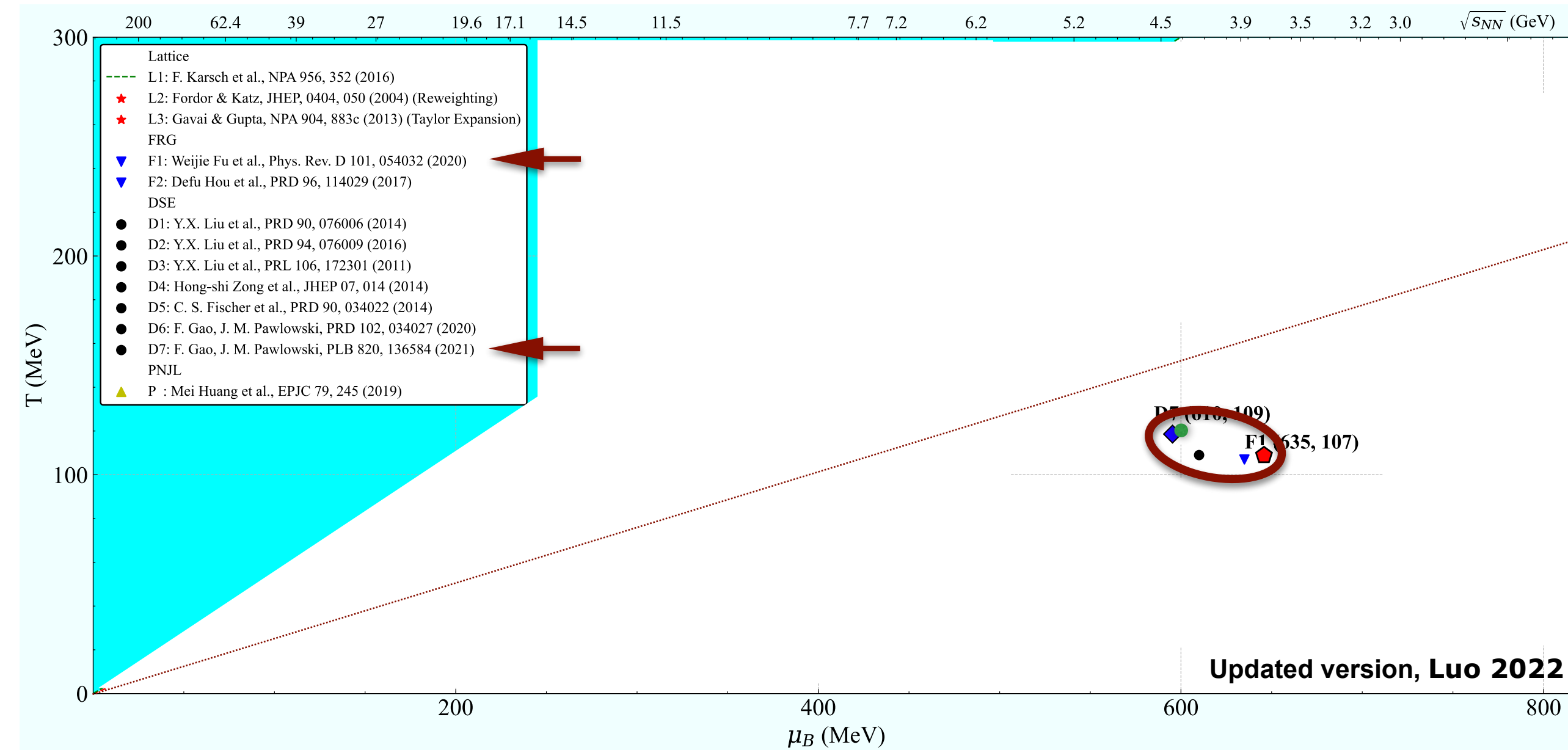


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Still small uncertainties for the estimation of **CP location / Onset of new phases**

Remove CEP-predictions

RHIC-BES Seminar Oct. 6th 2020, Xiaofeng Luo

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Predictions, estimates & extrapolations and how to judge them



Location of CP : Theoretical Prediction

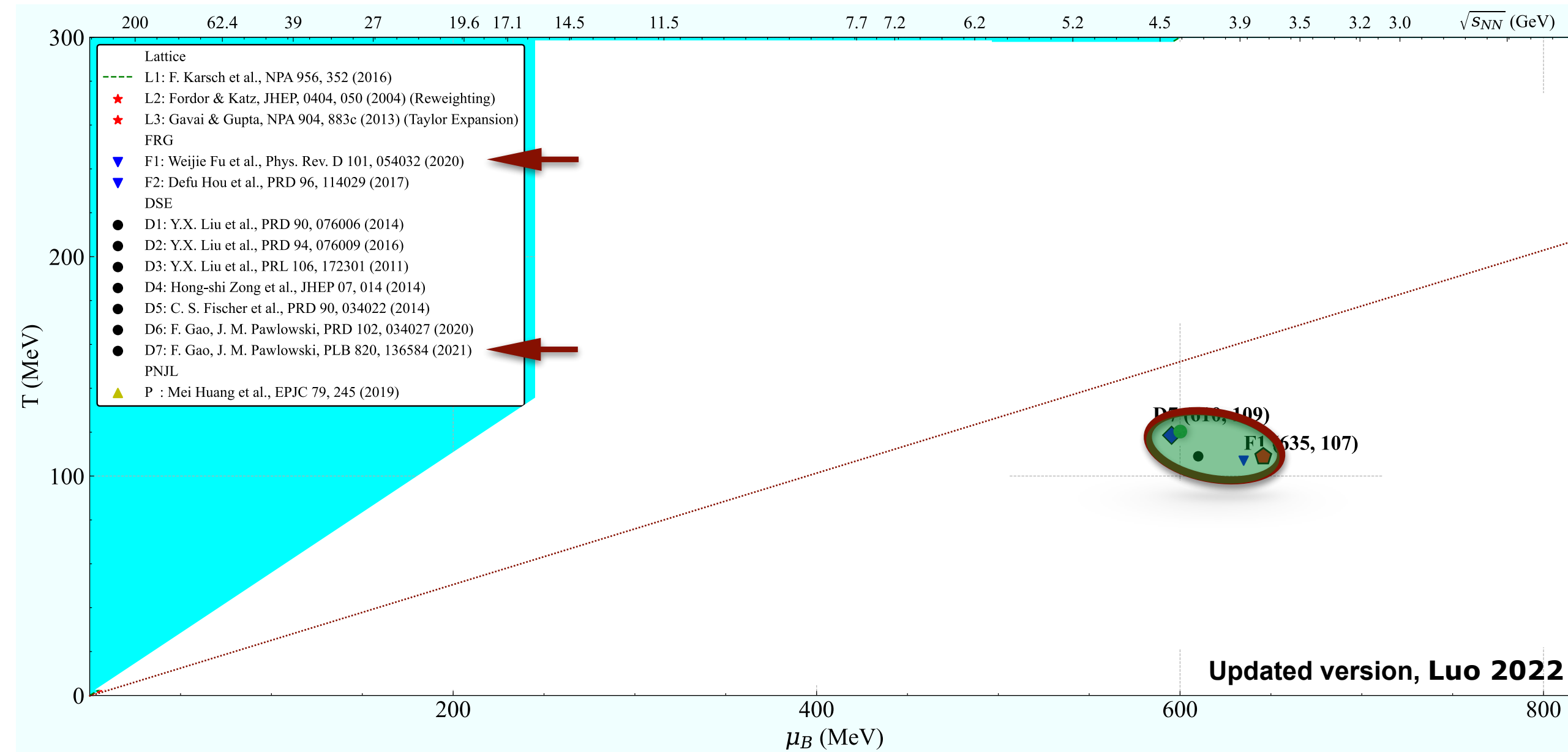
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Functional QCD

- ◆ Gao, Lu, JMP, Schneider, in prep (DSE)
- ◆ Fu, JMP, Rennecke, Wen, Yin, in prep (fRG)
- Gunkel, Fischer, PRD 104 (2021) 054022 (DSE)

Extrapolations

- Lattice extrapolations:
- Basar, PRC 110 (2024) 015203
 - Bielefeld-Parma, arXiv:2405.10196
 - ⋮
- Holographic models:
- Hippert, Grefa, Manning, Noronha,
 - Noronha-Hostler, Portillo Vazquez, Ratti,
 - Rougemont, Trujillo, arXiv: 2309.00579
 - ⋮



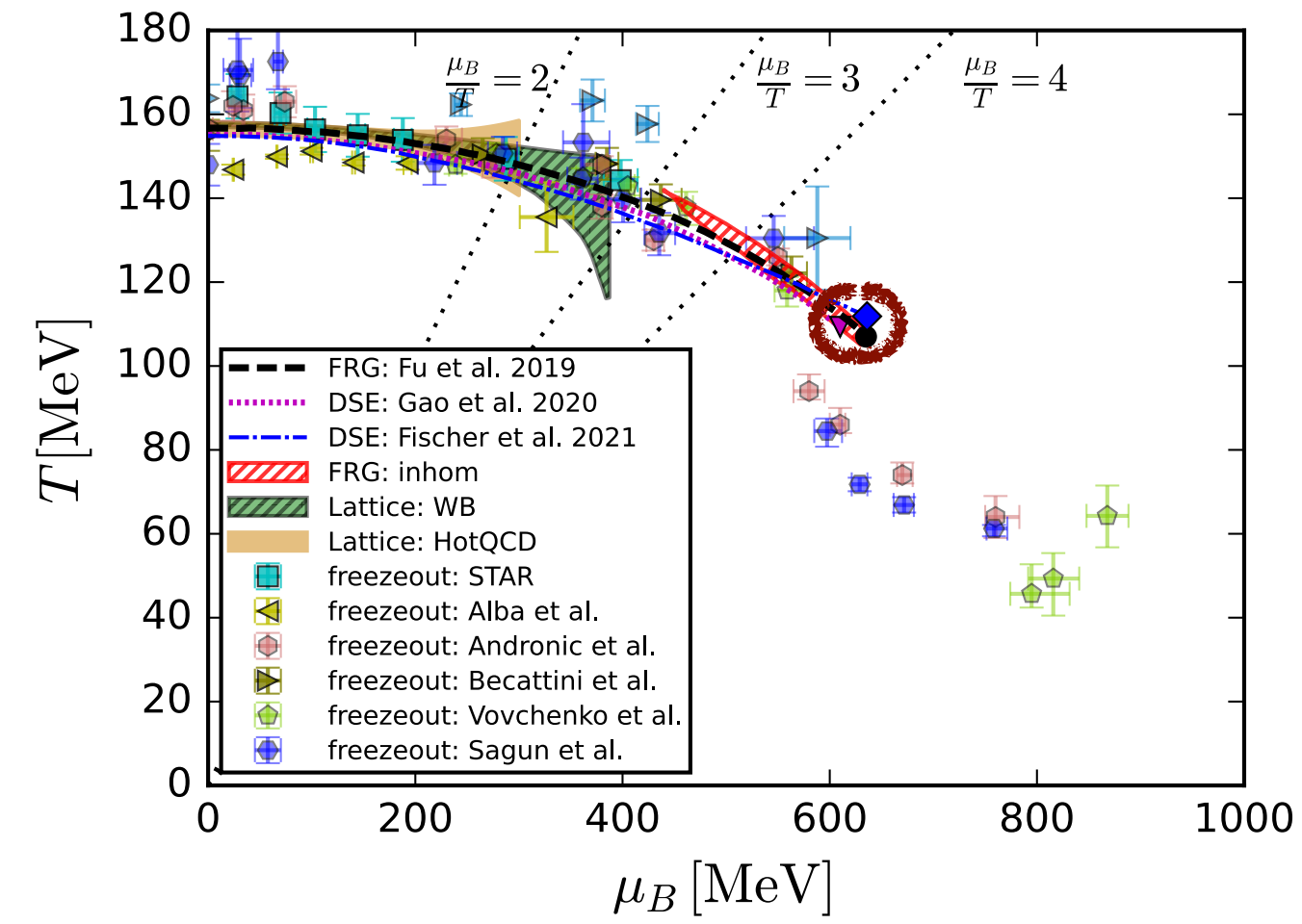
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Remove CEP-predictions

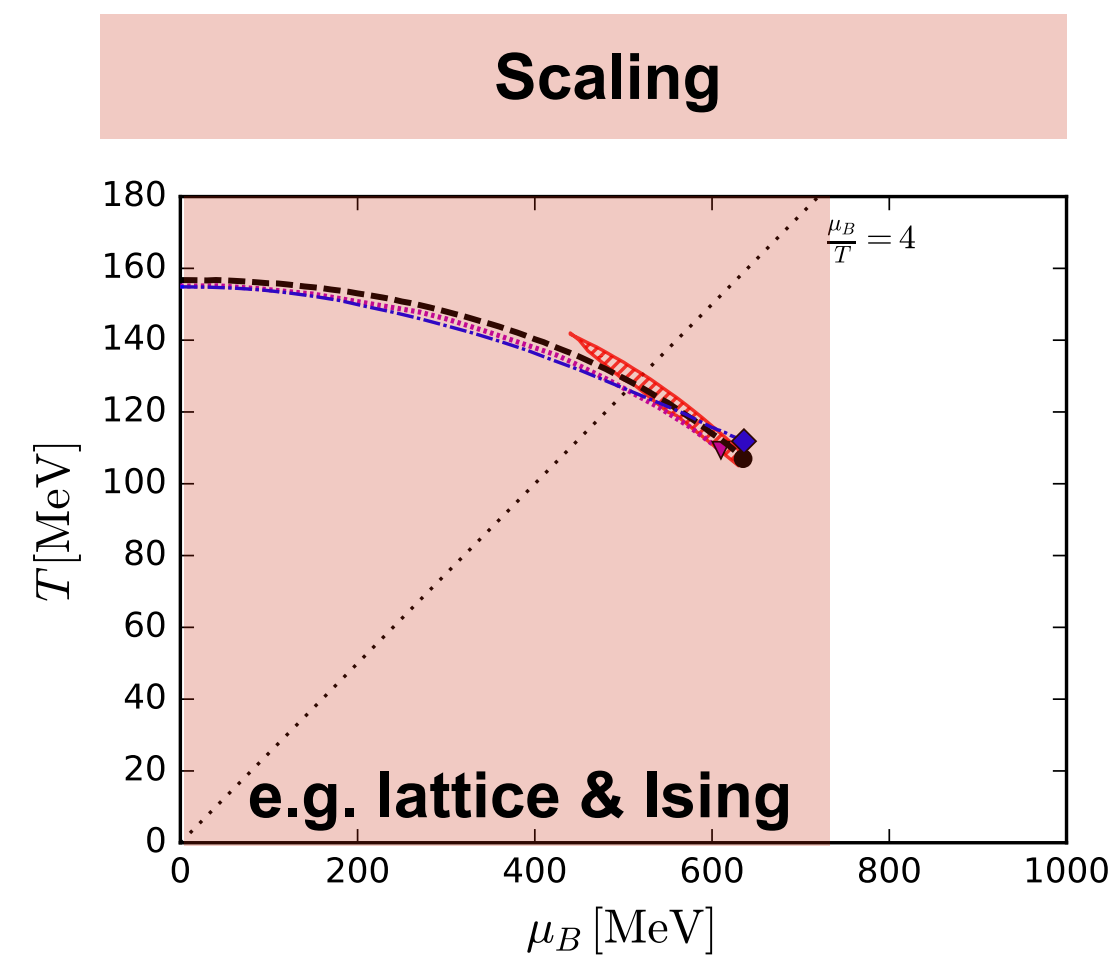
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Predictions, estimates & extrapolations and how to use them

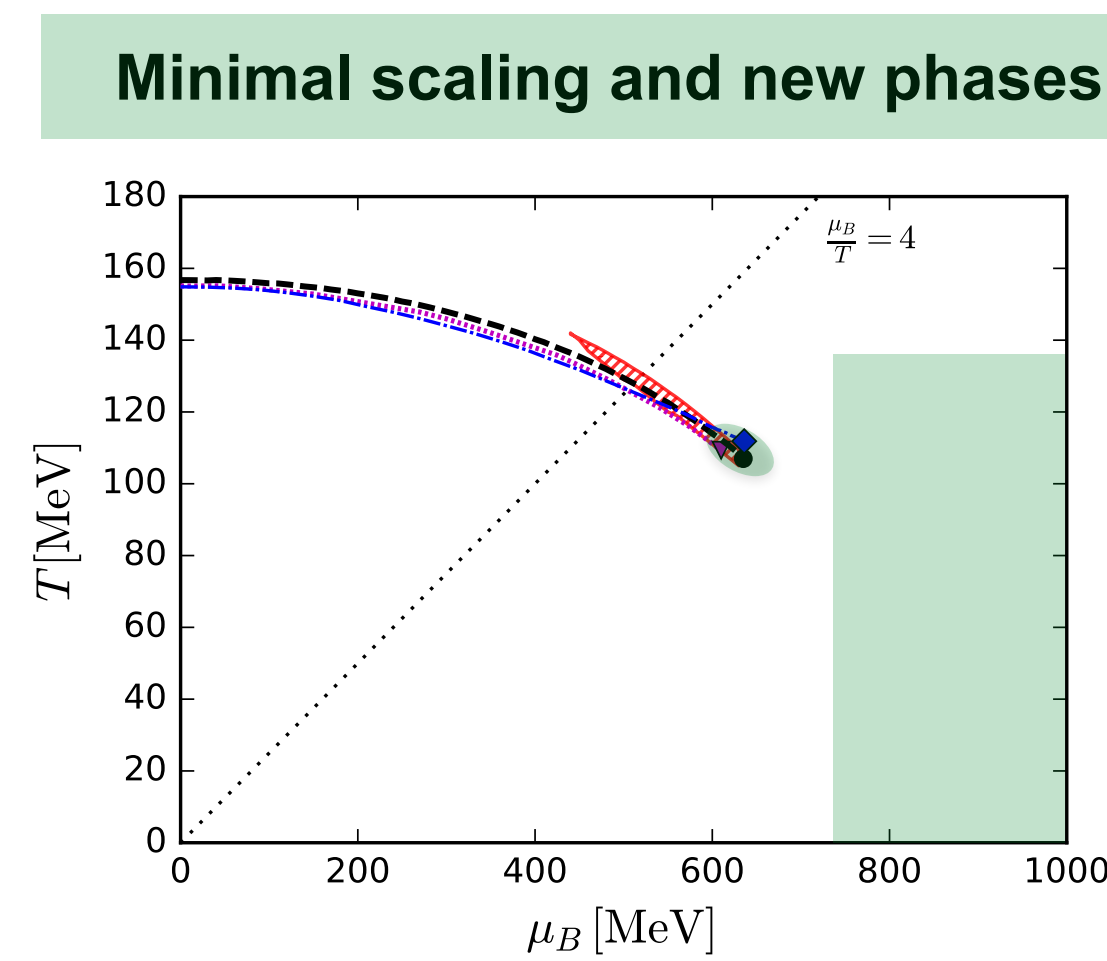
Predictions, estimates & extrapolations and how to use them



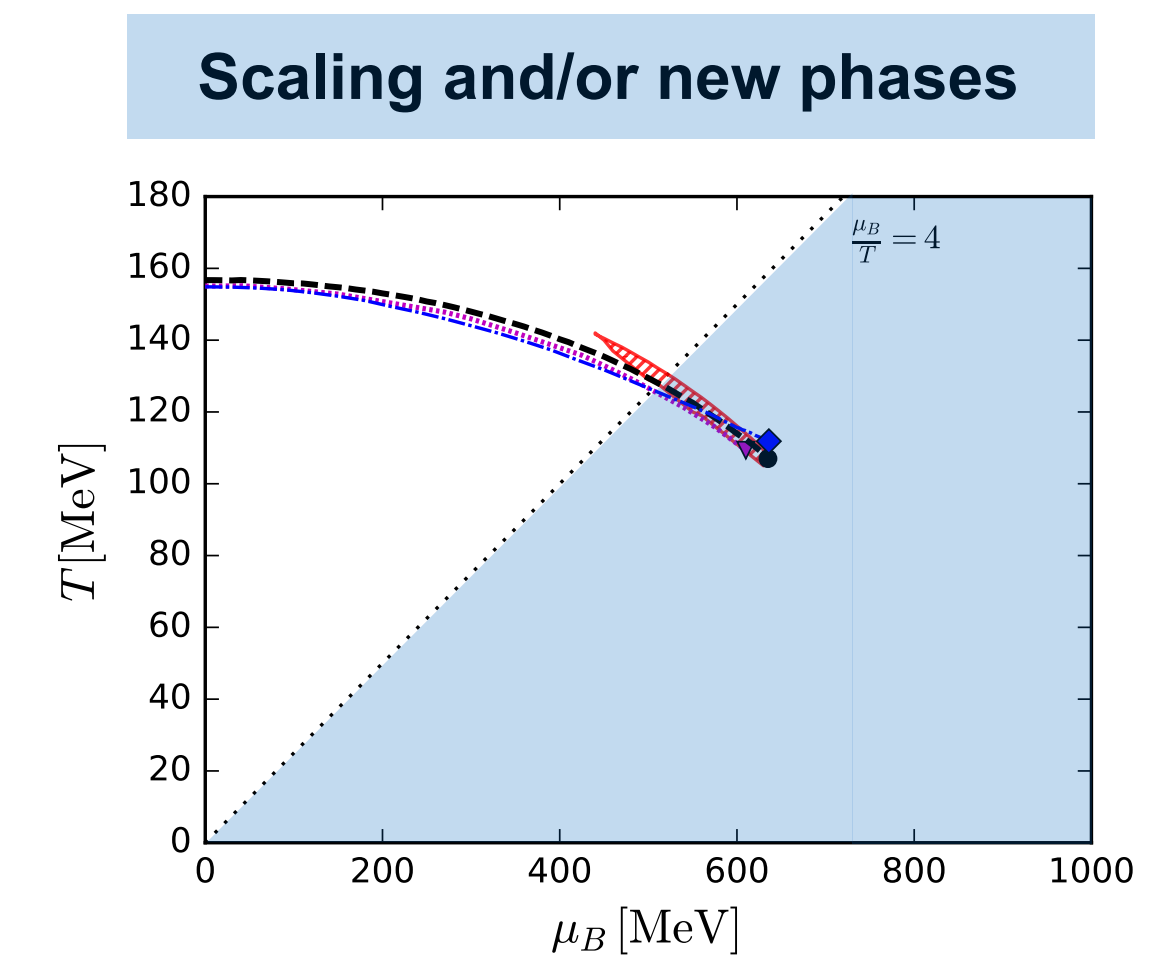
Scenario I



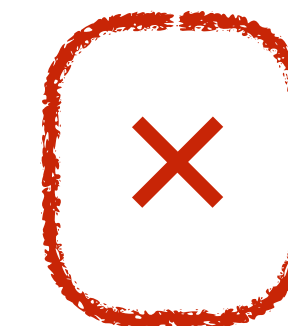
Scenario II



Scenario III

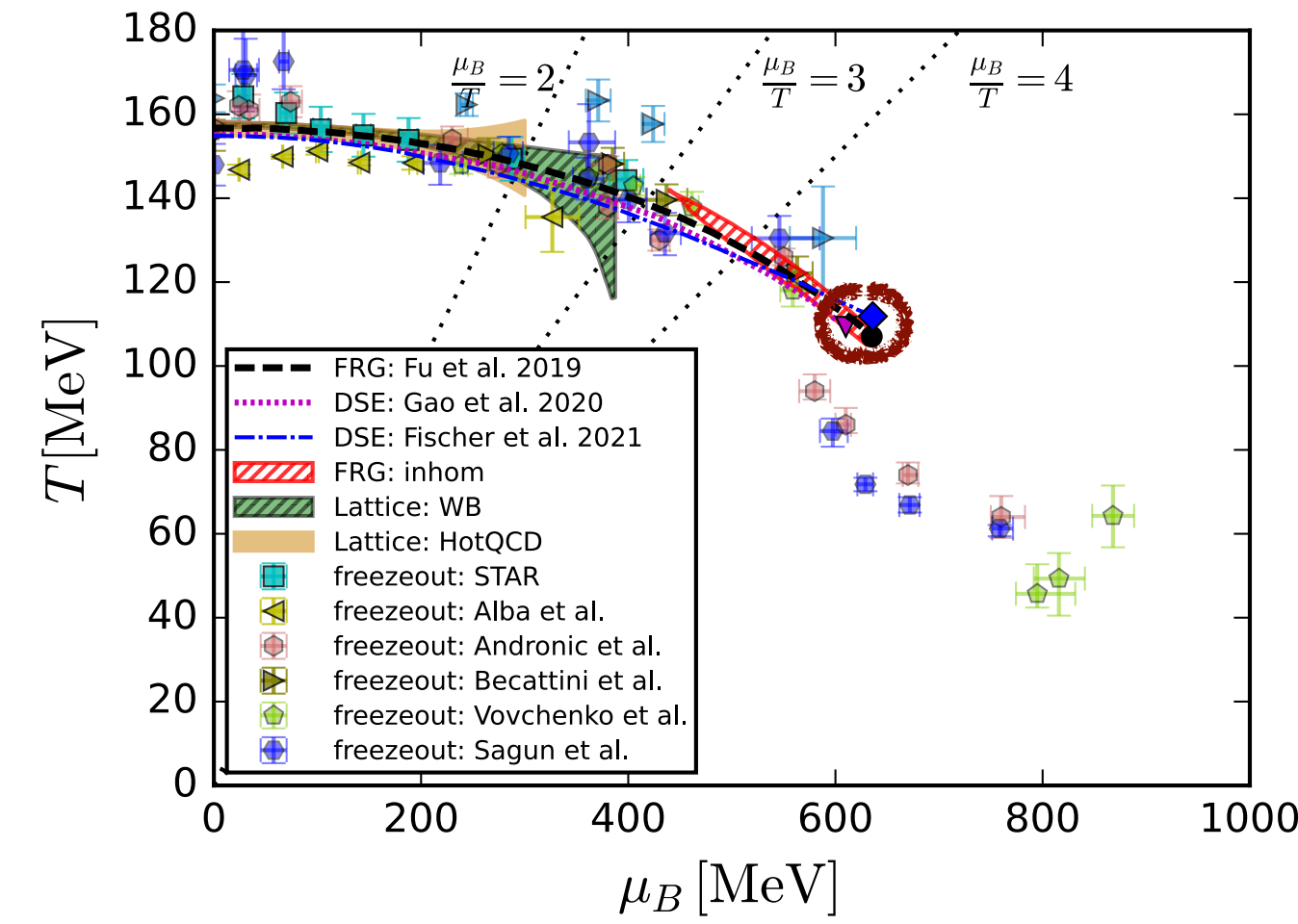


**Extrapolations
for
Pheno**



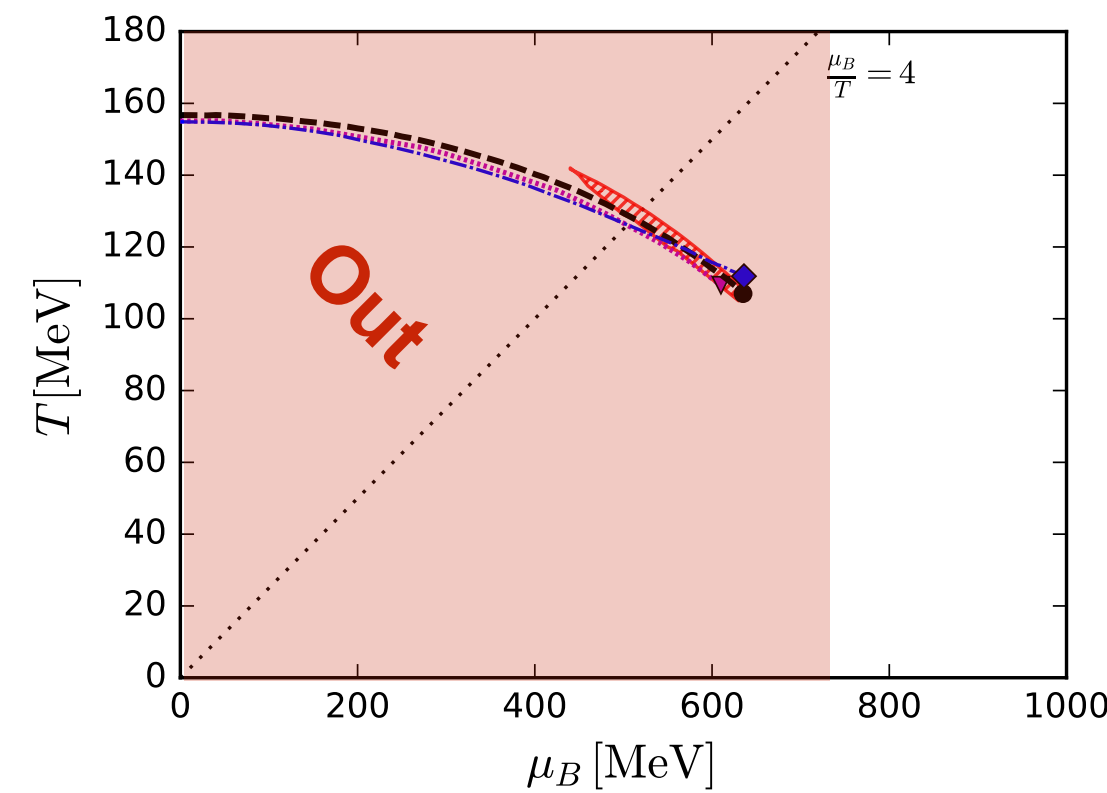
Predictions, estimates & extrapolations and how to use them

Out by the LEGO[®] principle
 Fu, JMP, Rennecke, PRD 101 (2020) 054032
 +
 Size of scaling regime in LEFTs
 Schaefer, Wambach, PRD 75 (2007) 085015
 Braun, Klein, Piasecki, EPJC 71 (2011) 1576
 ⋮



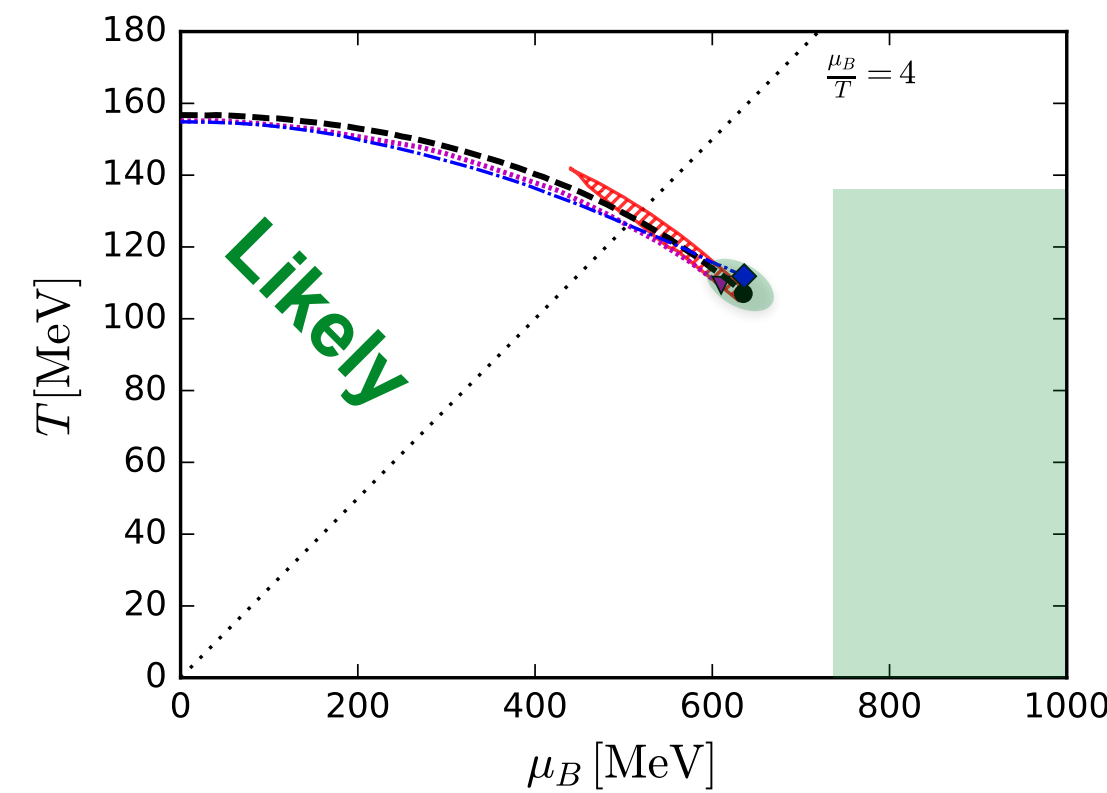
Scenario I

Scaling



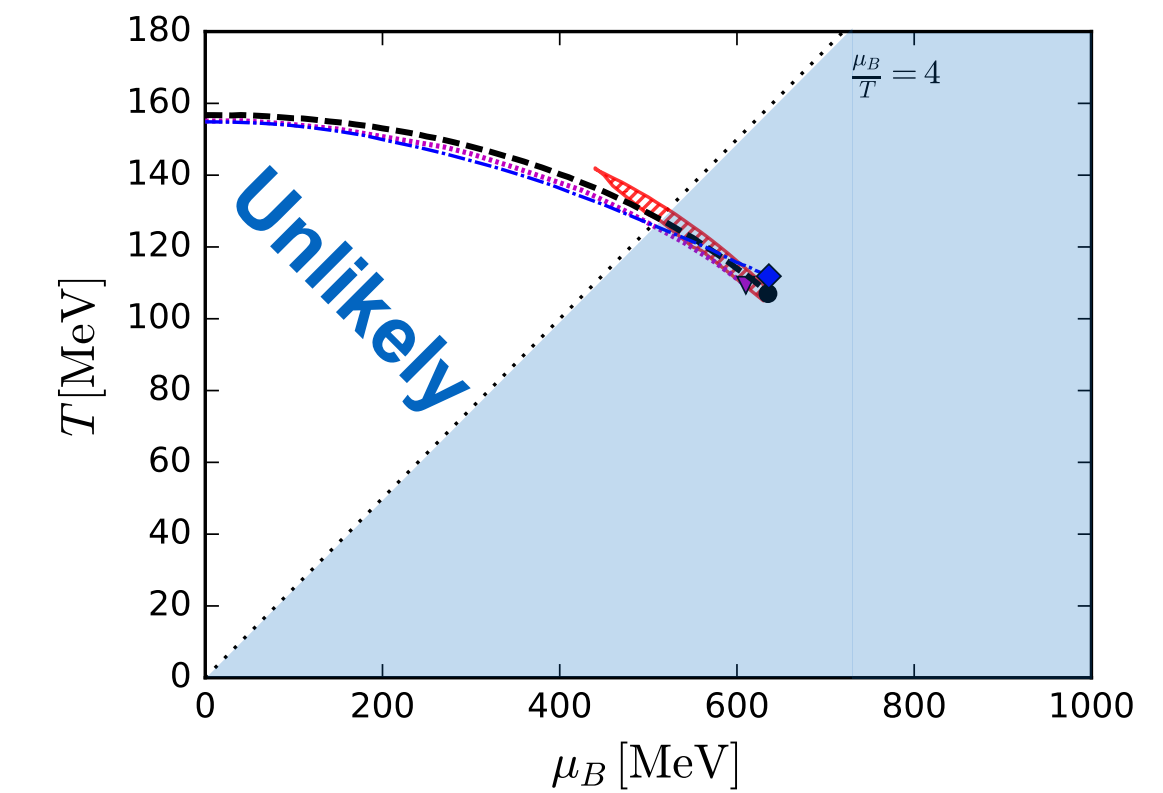
Scenario II

Minimal scaling and new phases



Scenario III

Scaling and/or new phases



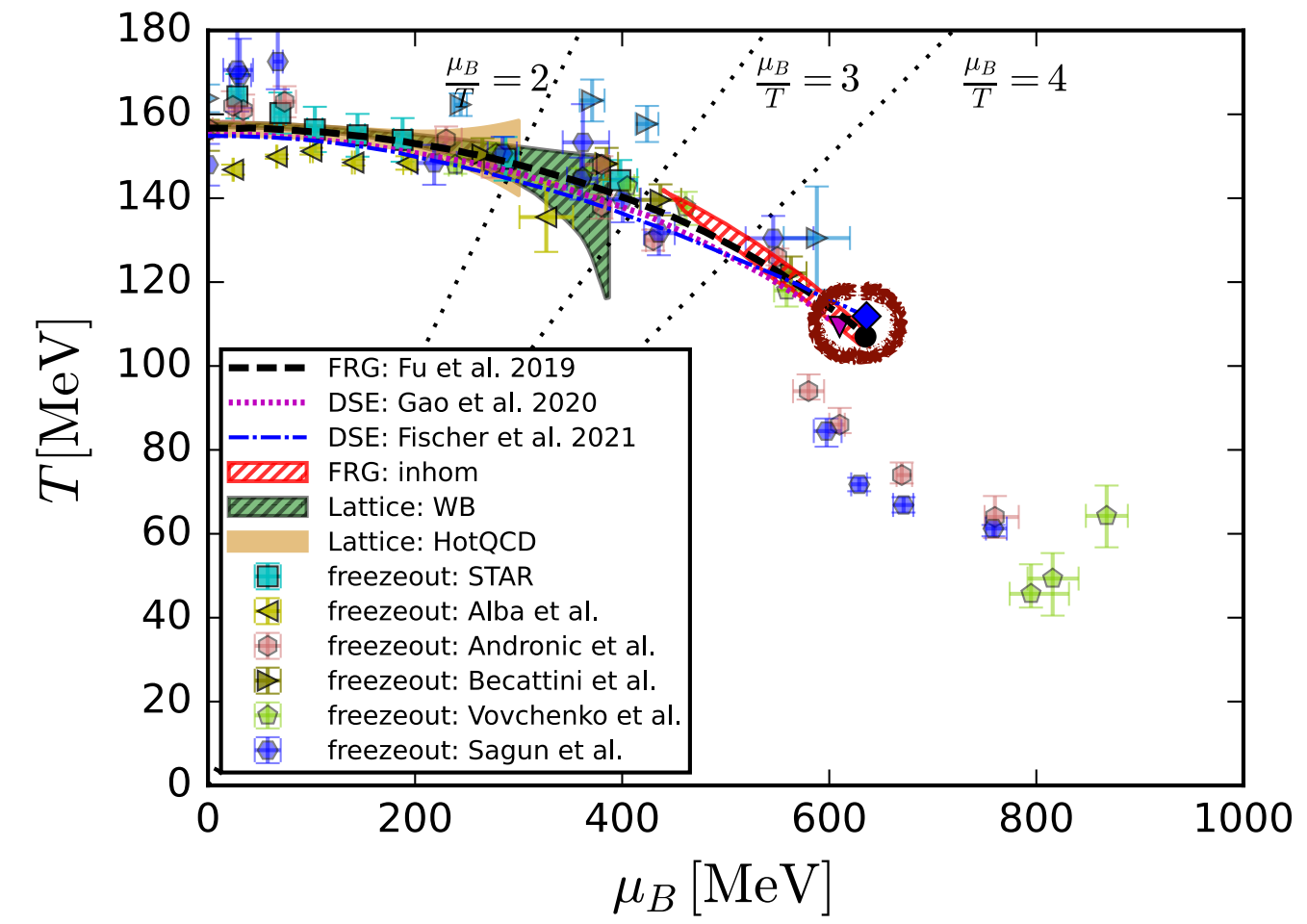
Braun, Fu, JMP, Rennecke, Rosenblüh, Yin, PRD 102 (2020) 056010

Gao, JMP, PRD 105 (2022) 094020

Soft modes in hot QCD matter: Braun, Chen, Fu, Gao, Huang, Ihssen, JMP, Rennecke, Sattler, Tan, Wen, Yin, arXiv:2310.19853

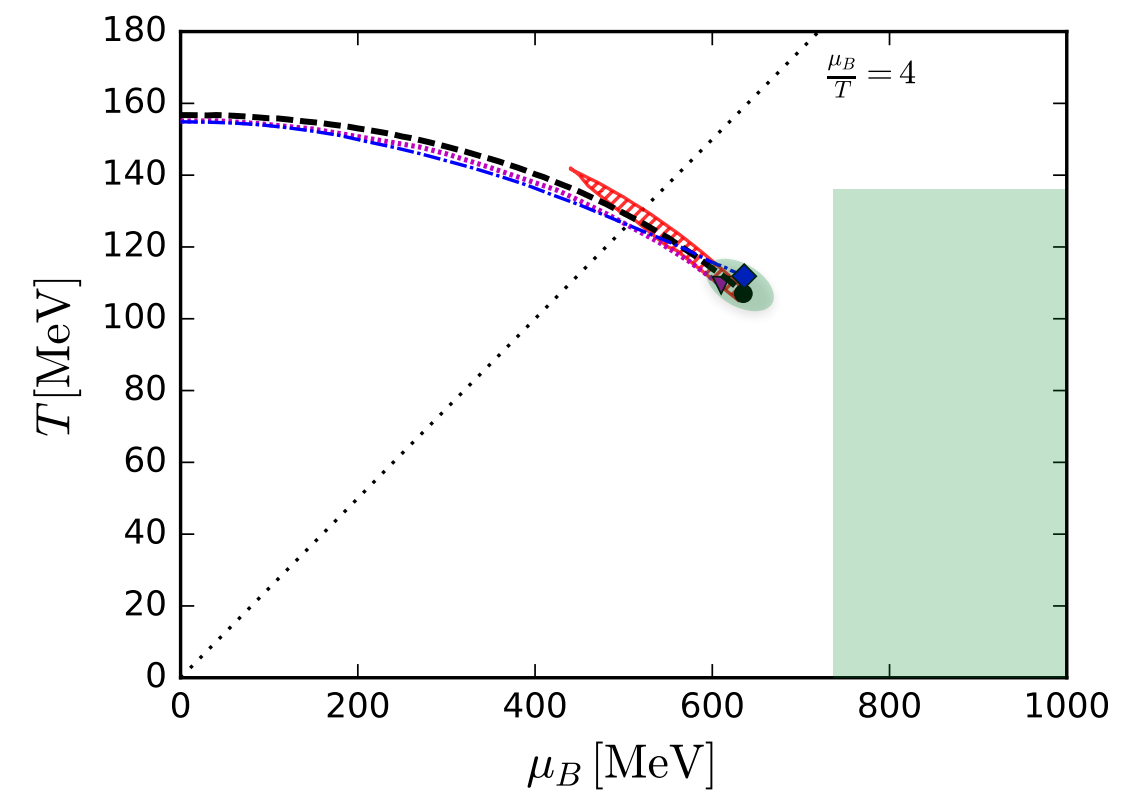
+ many results in dynamical low energy effective theories
LEGO[®] principle

Predictions, estimates & extrapolations and how to use them



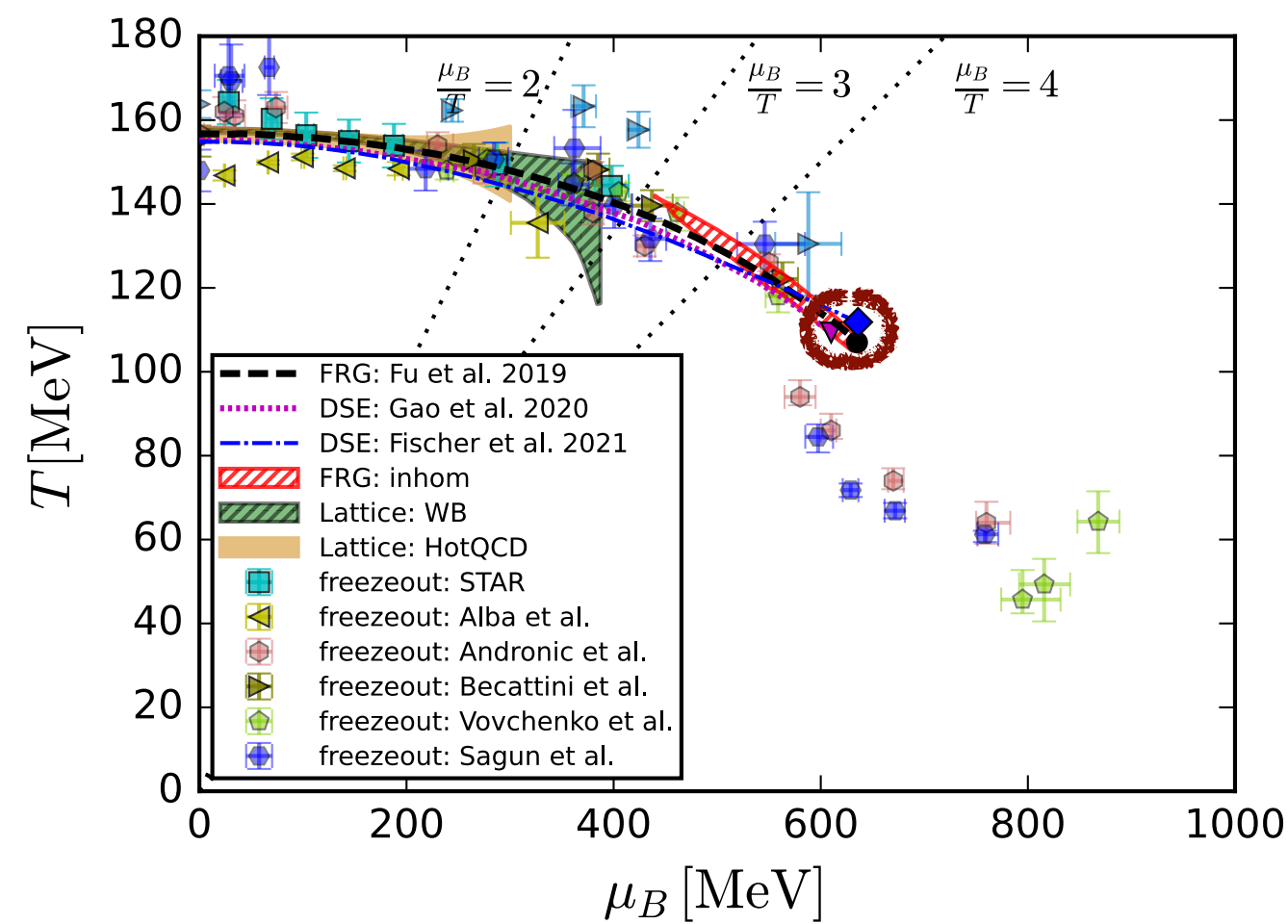
Scenario II

Minimal scaling and new phases



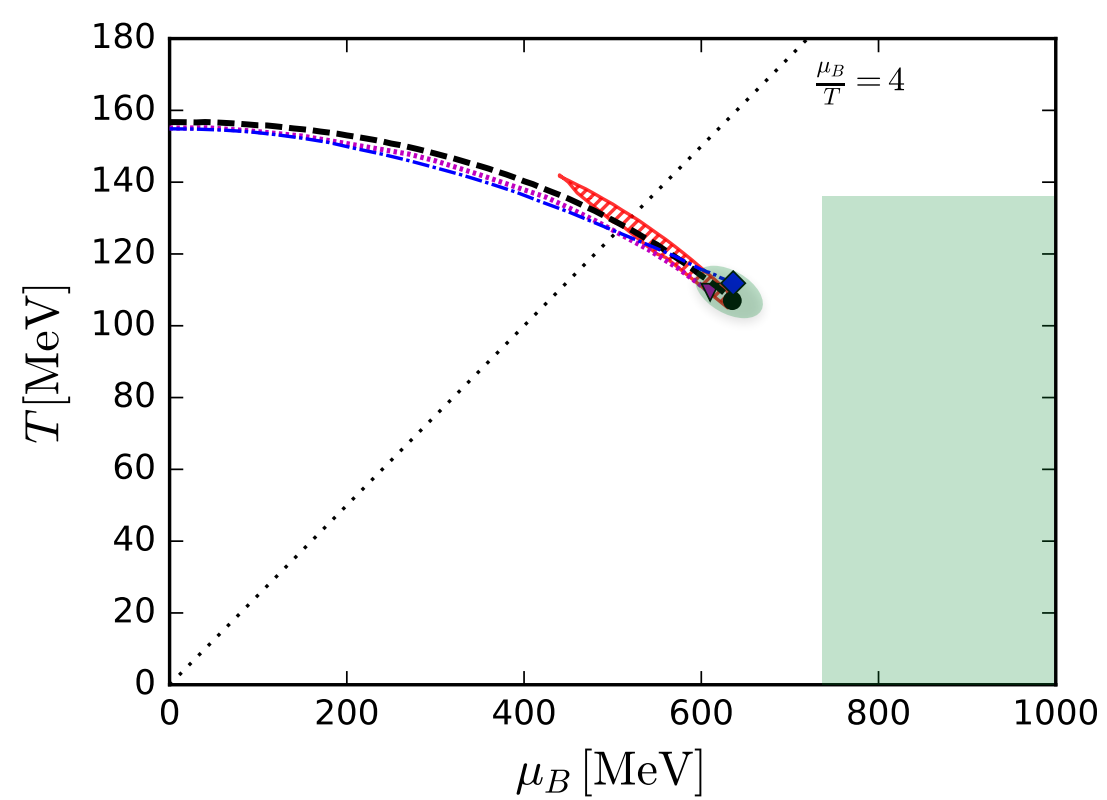
Extrapolations
for
Pheno

Predictions, estimates & extrapolations and how to use them



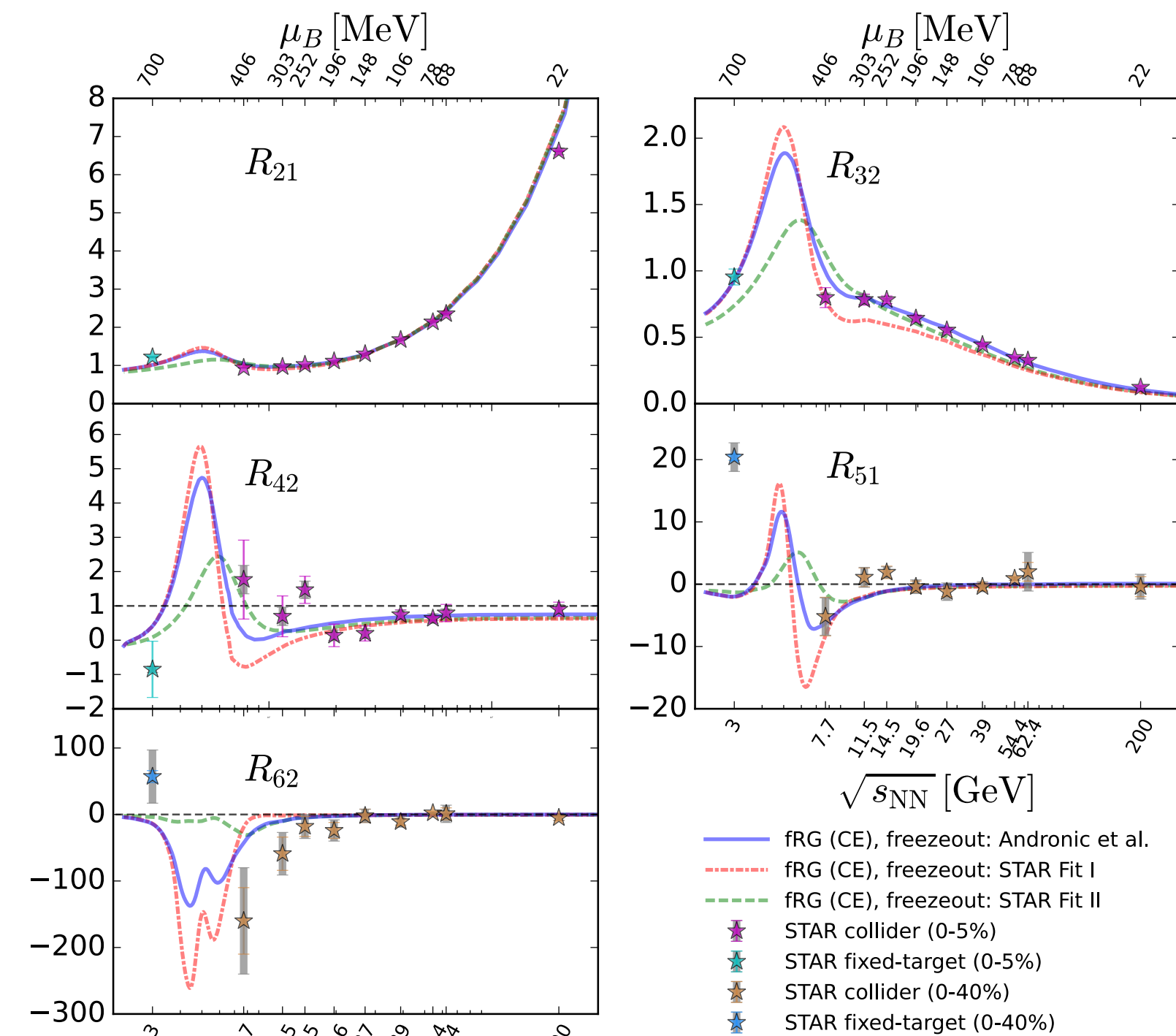
Scenario II

Minimal scaling and new phases



Ripples of the critical end point

baryon & proton number fluctuations



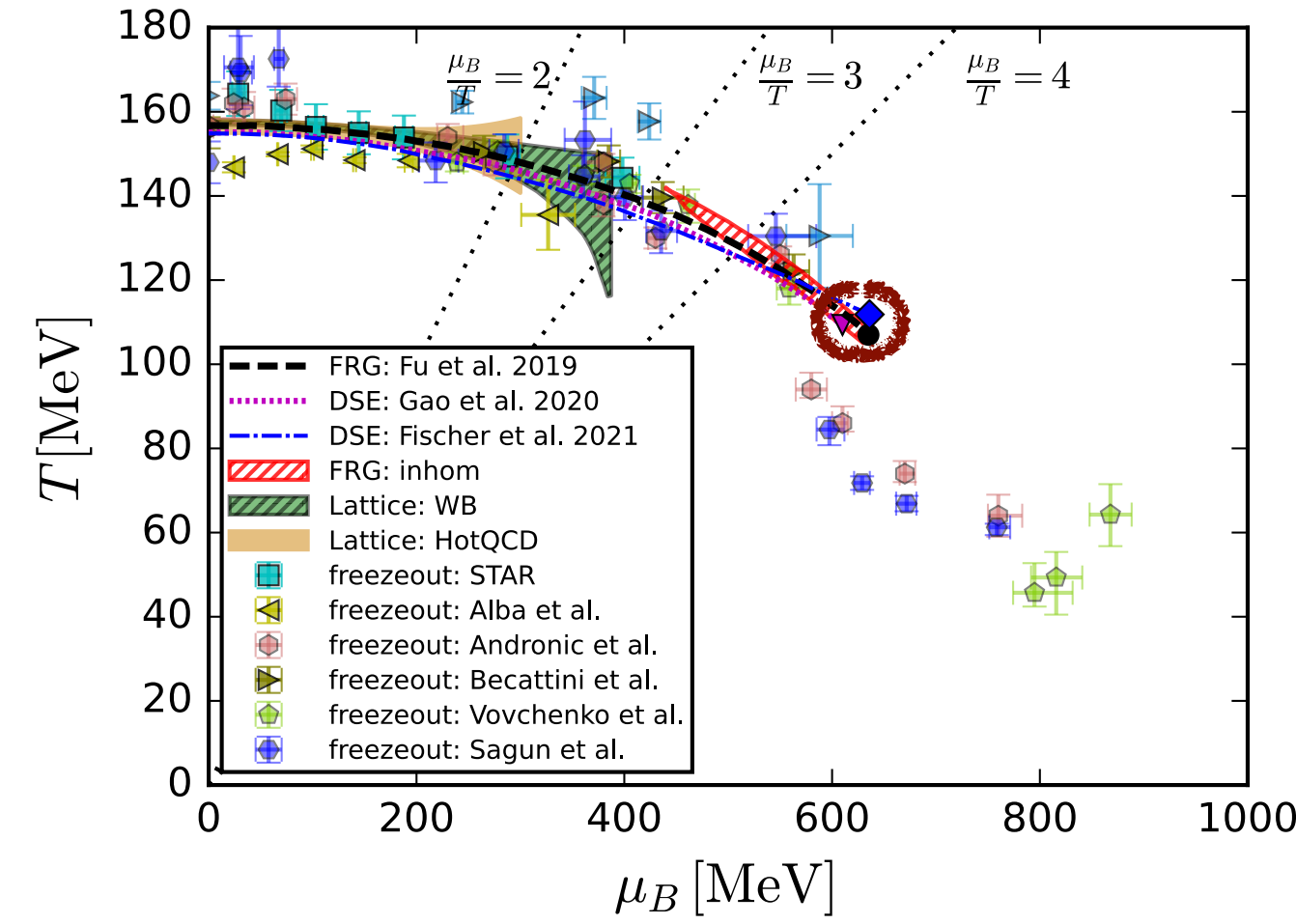
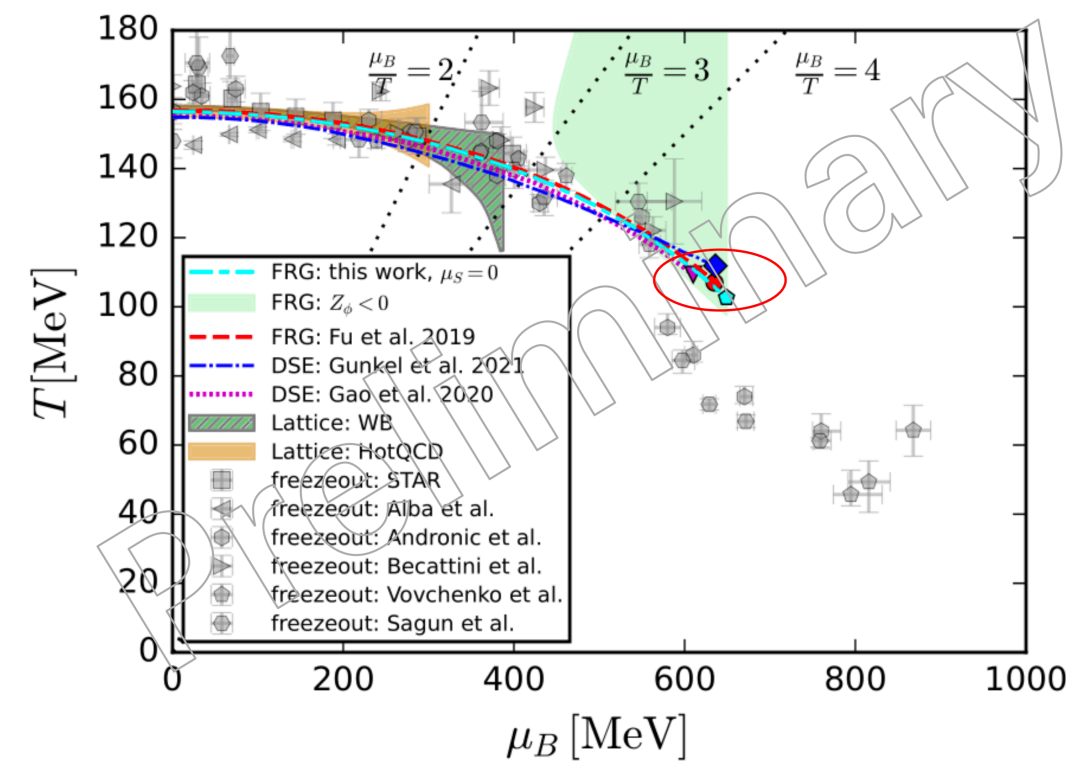
Extrapolations
for
Pheno



Predictions, estimates & extrapolations and how to use them

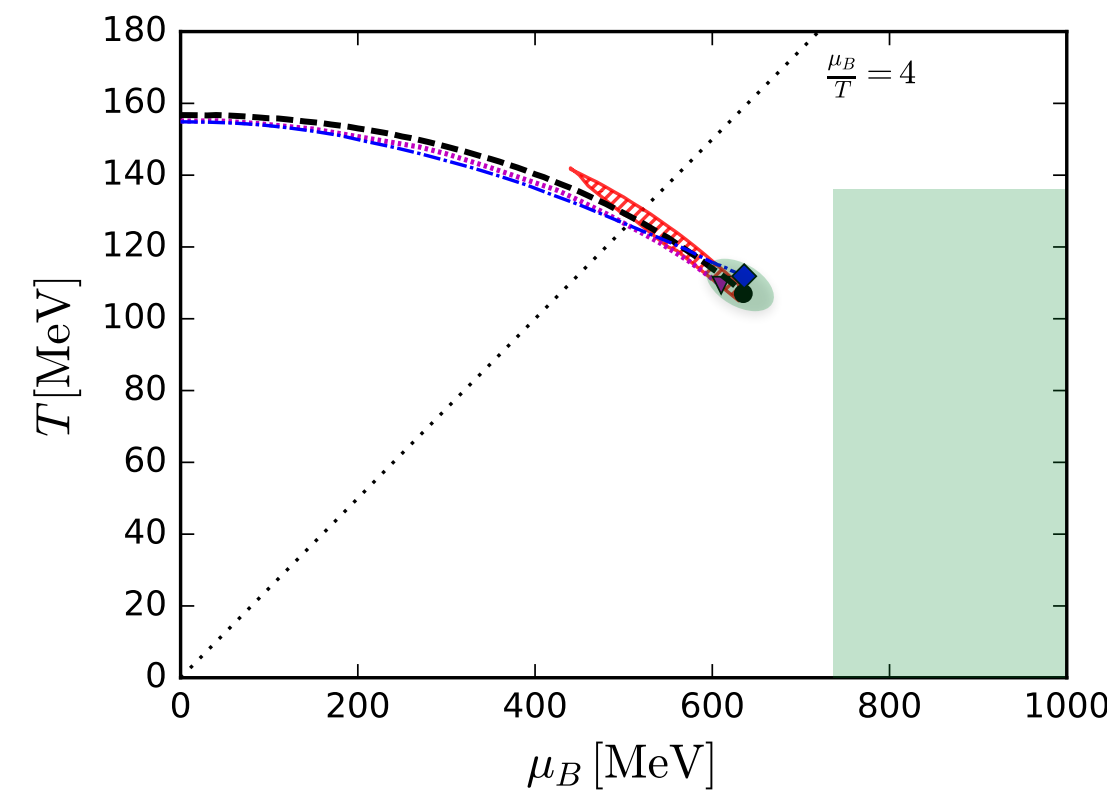
Strangeness neutrality

baryon & proton number fluctuations



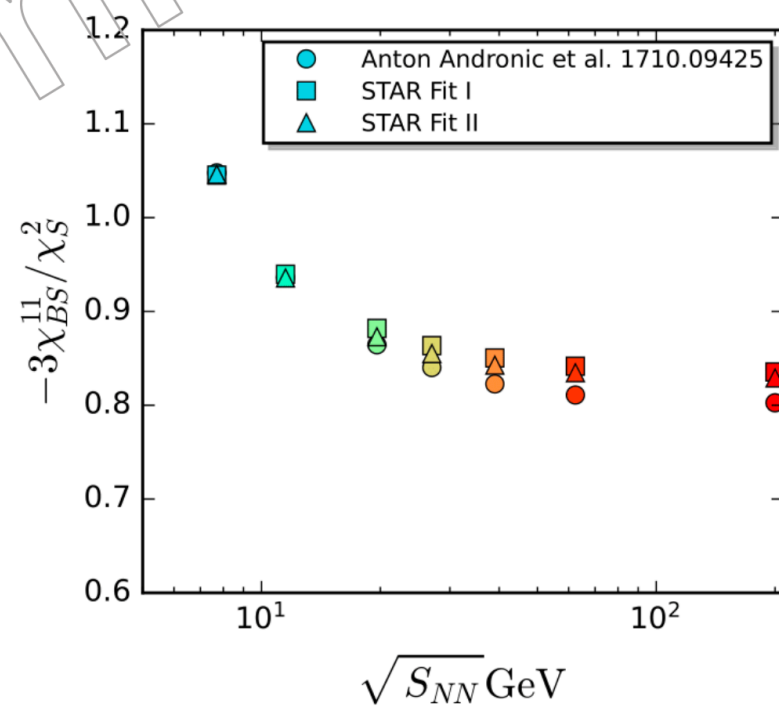
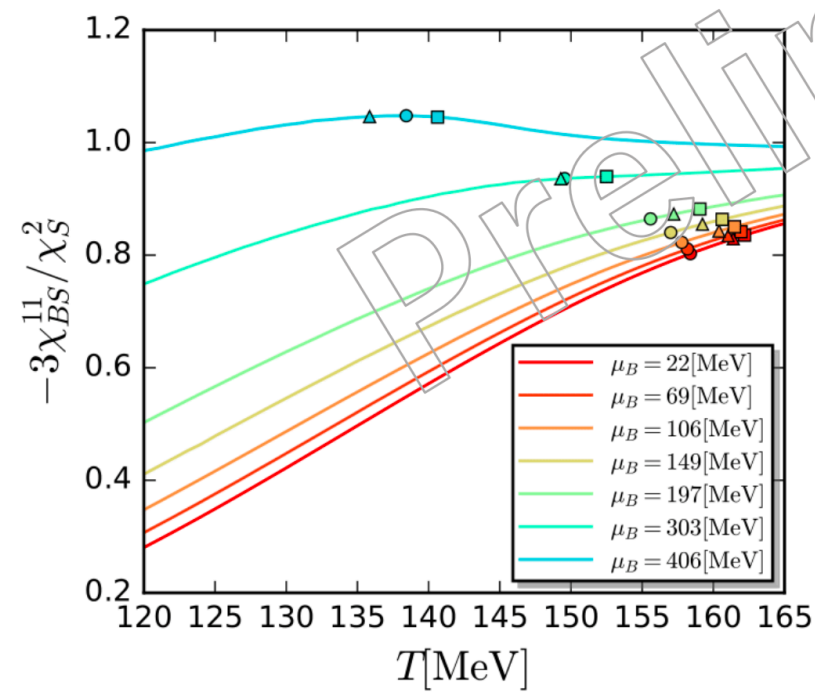
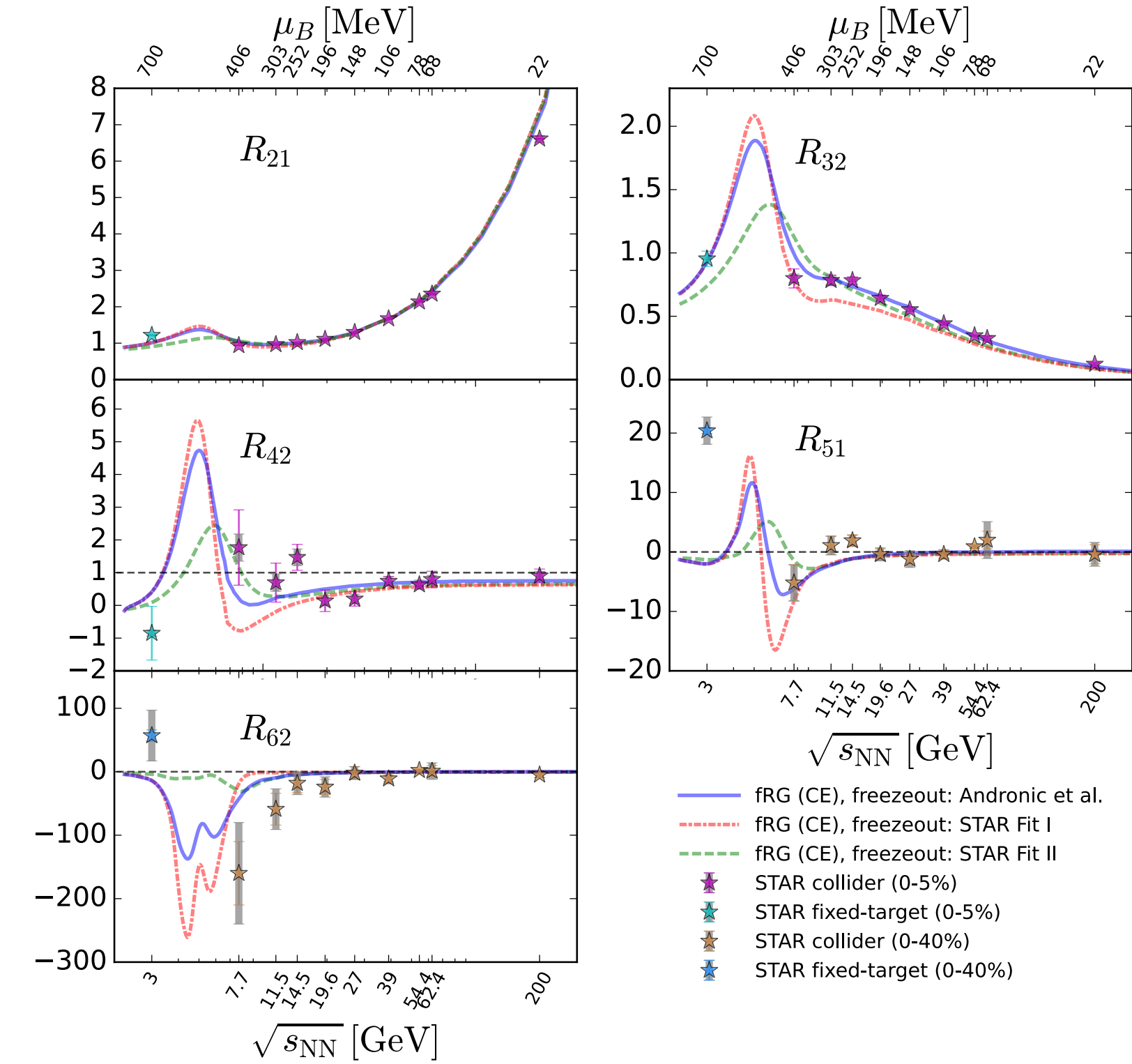
Scenario II

Minimal scaling and new phases



Ripples of the critical end point

baryon & proton number fluctuations

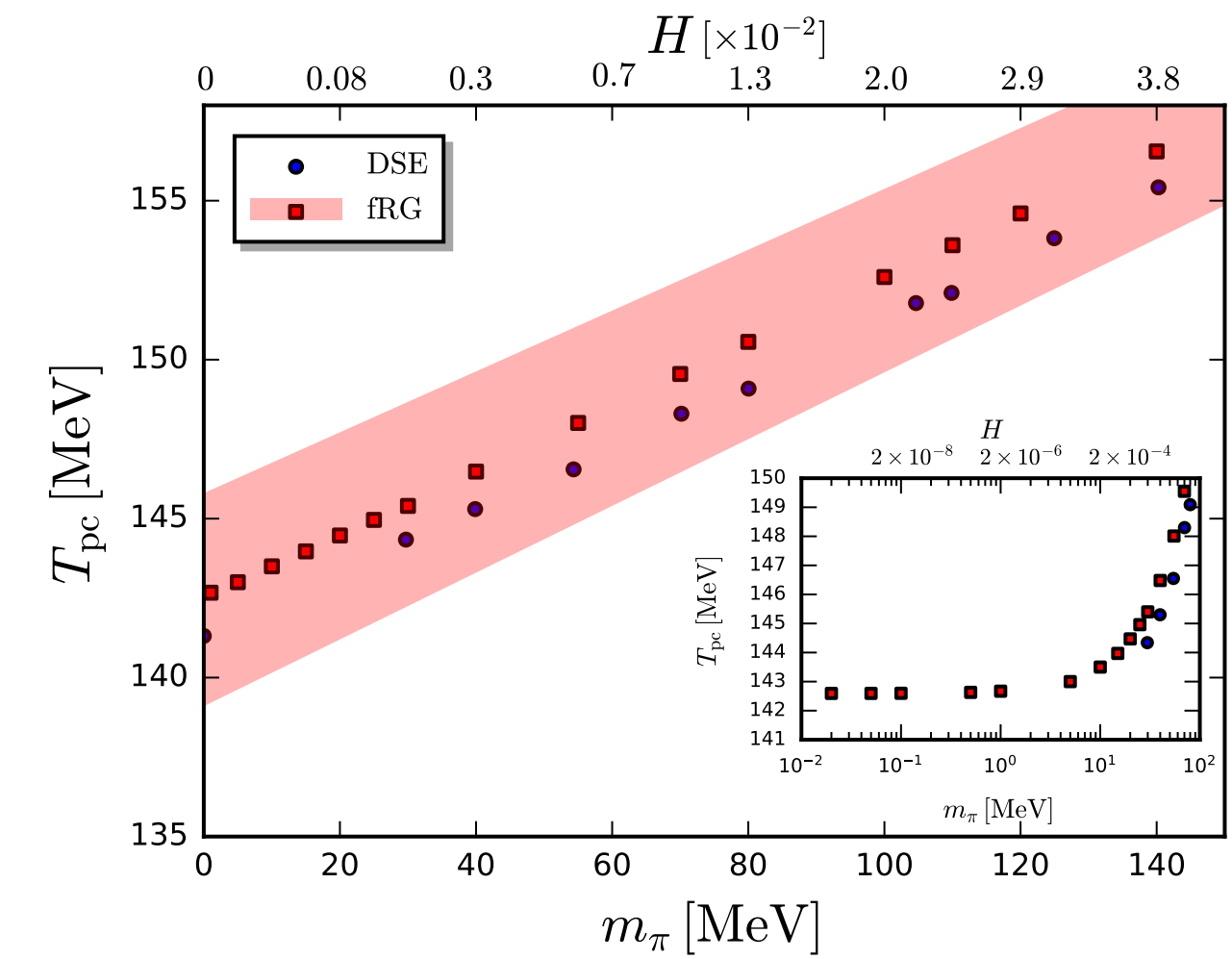
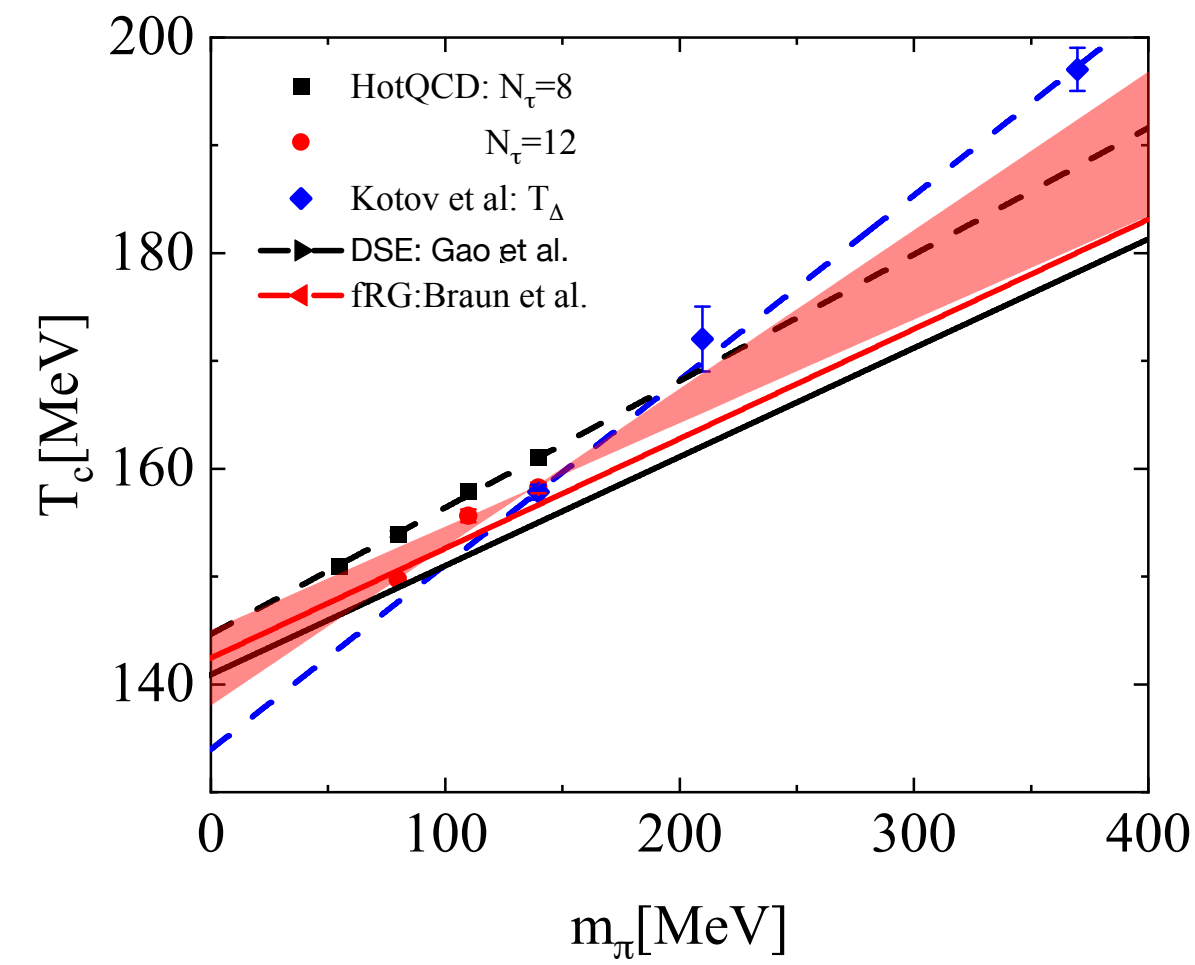
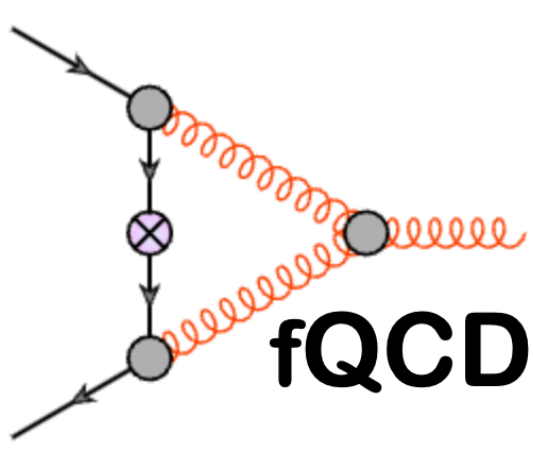


Chiral dynamics & soft modes

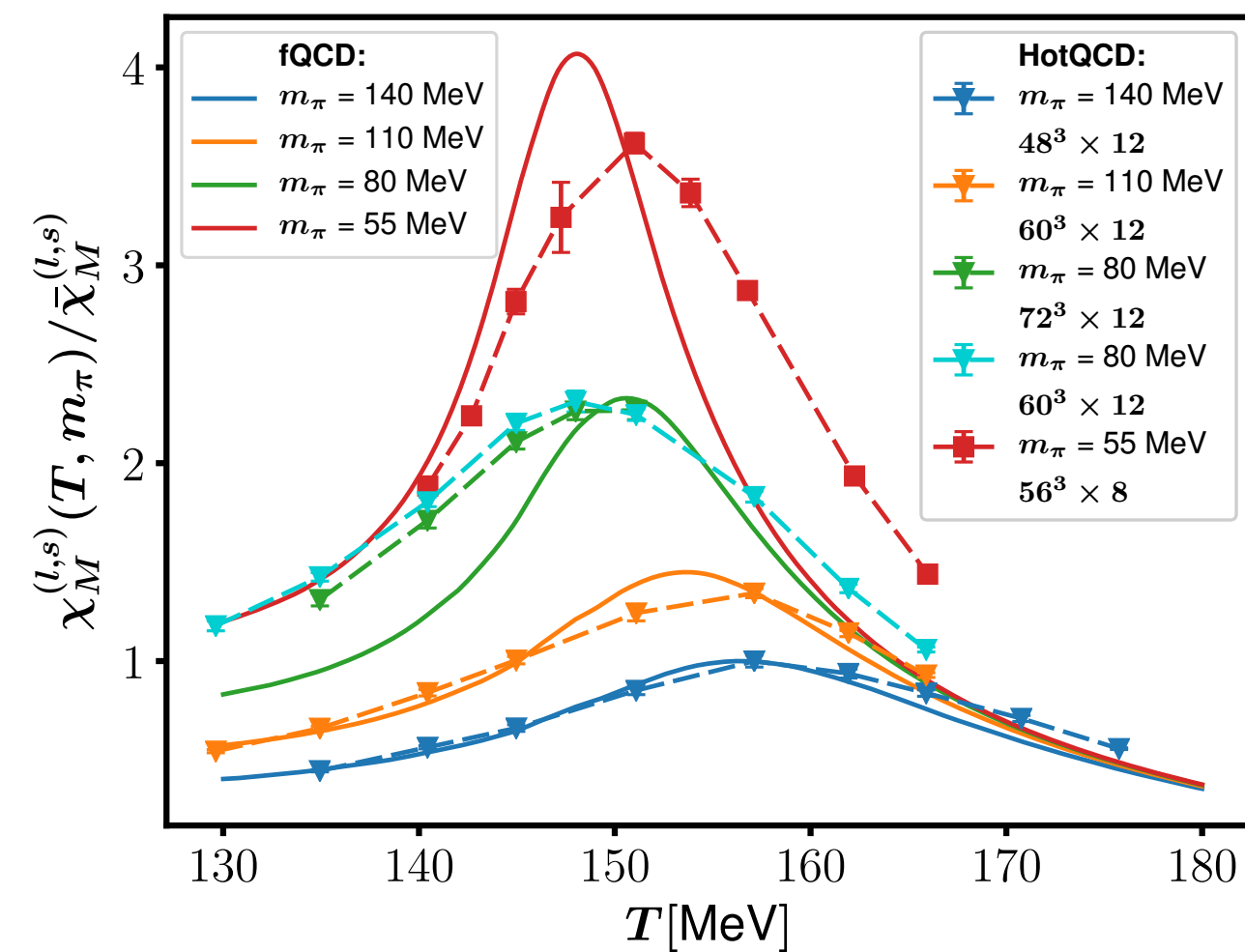
To be (critical) or not (to be)

Chiral transition temperature

$$H = \frac{m_l}{m_s}$$



Magnetic susceptibility

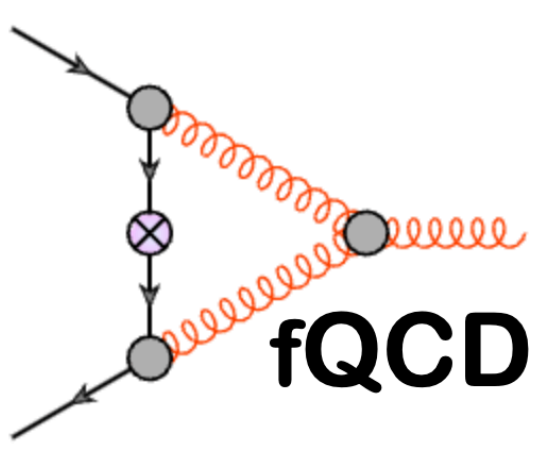


Braun, Fu, JMP, Rennecke, Rosenblüh, Yin, PRD 102 (2020) 056010

Gao, JMP, PRD 105 (2022) 094020

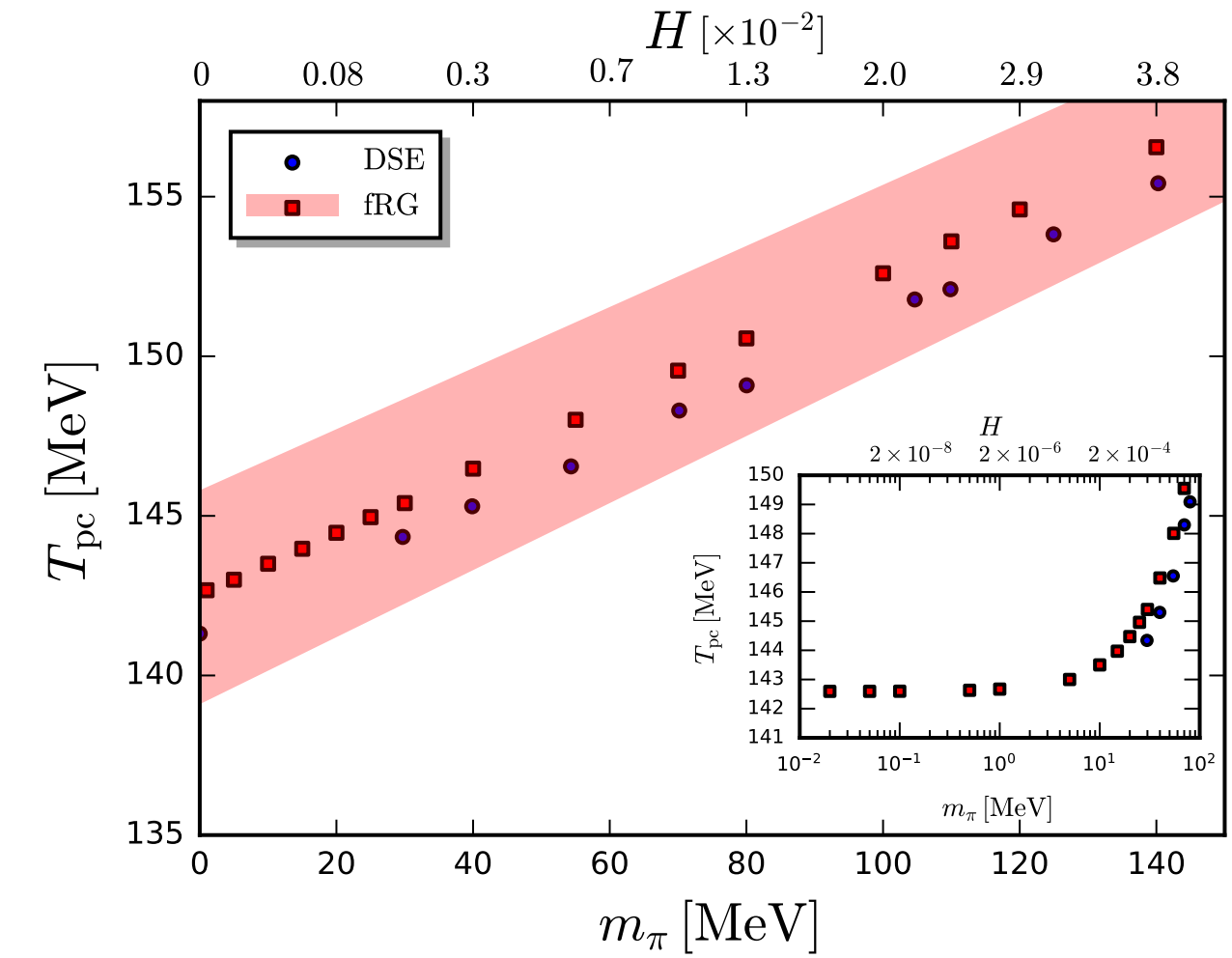
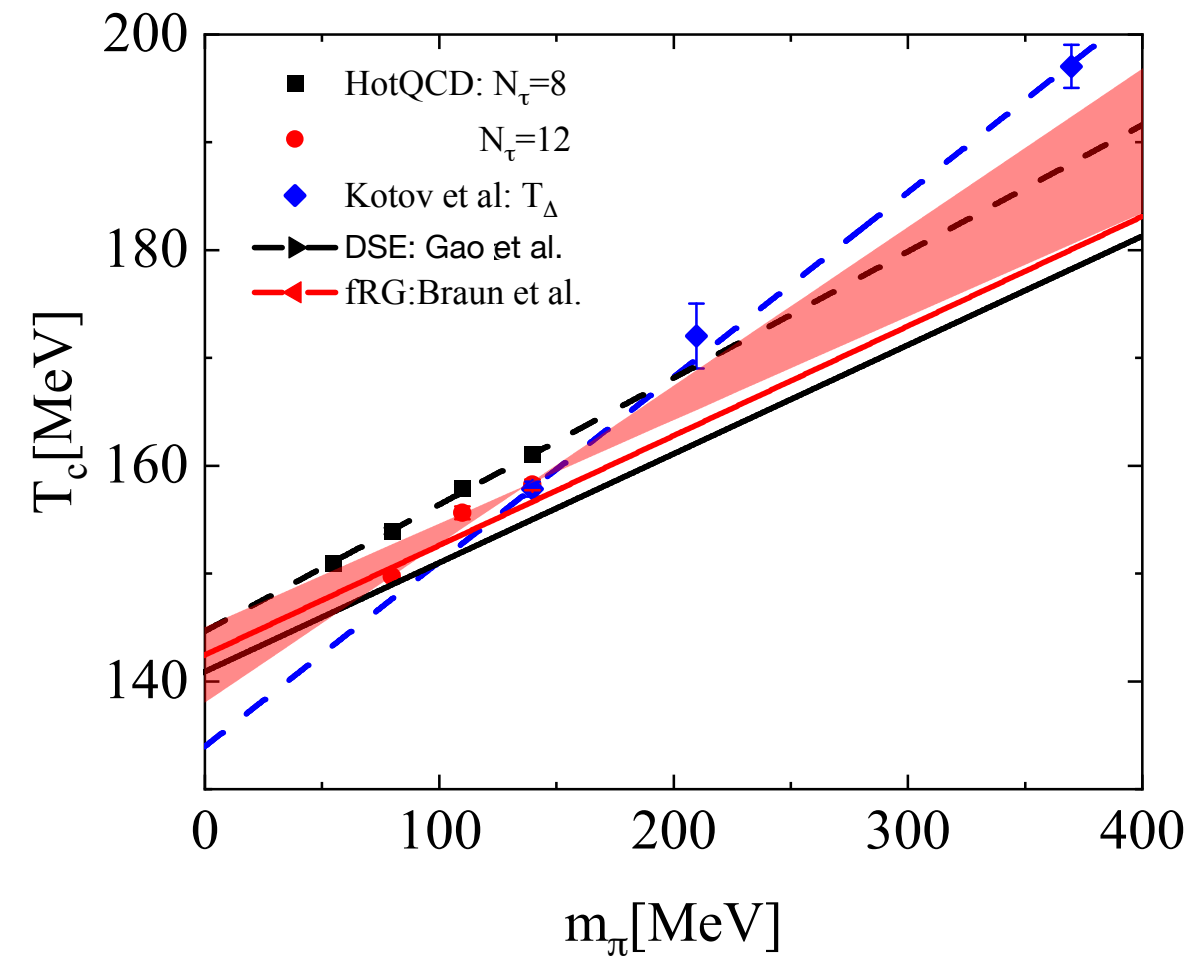
Braun, Chen, Fu, Gao, Huang, Ihssen, JMP, Rennecke, Sattler, Tan, Wen, Yin, 2310.19853

To be (critical) or not (to be)



Chiral transition temperature

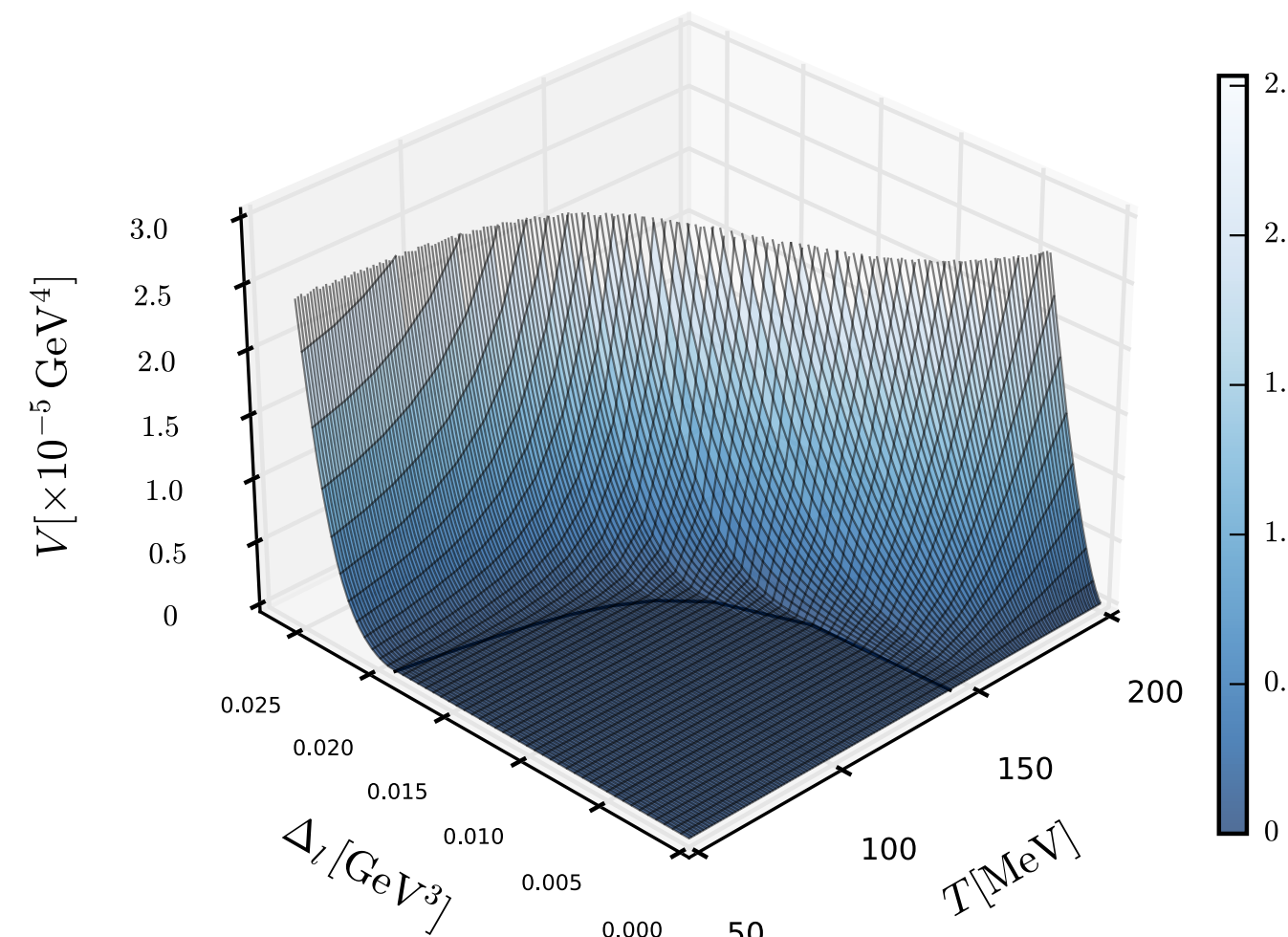
$$H = \frac{m_l}{m_s}$$



Order parameter potential & scaling

$$V_\chi \approx \Delta_l^n \longleftrightarrow \Delta_l(H) \propto H^{\frac{1}{n-1}}$$

(Critical) exponent: $\frac{1}{\delta} = \frac{1}{n-1}$



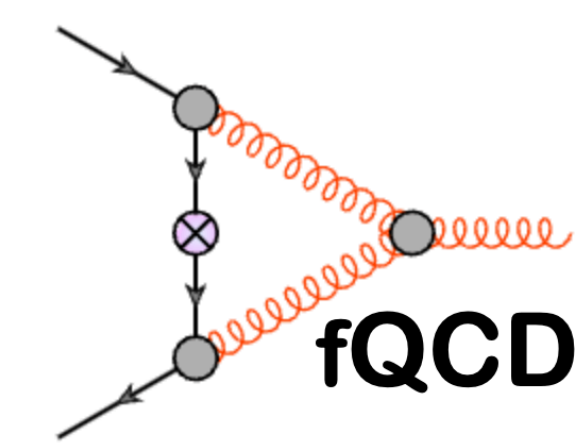
$$V_\chi^{(\text{fRG})} \approx V_\chi^{(\text{DSE})}$$

Braun, Fu, JMP, Rennecke, Rosenblüh, Yin, PRD 102 (2020) 056010

Gao, JMP, PRD 105 (2022) 094020

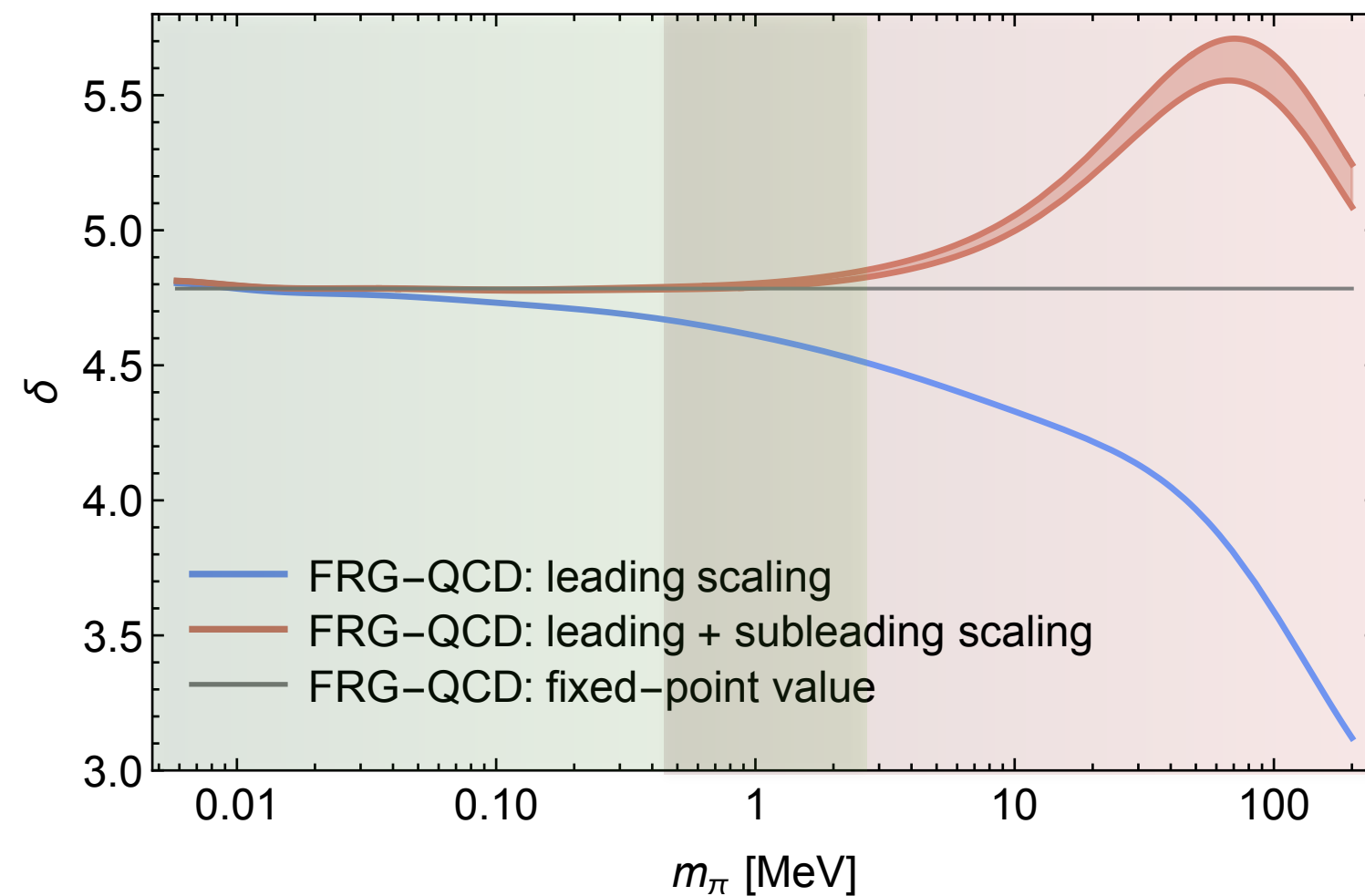
Braun, Chen, Fu, Gao, Huang, Ihssen, JMP, Rennecke, Sattler, Tan, Wen, Yin, 2310.19853

Chiral dynamics & quasi-massless modes



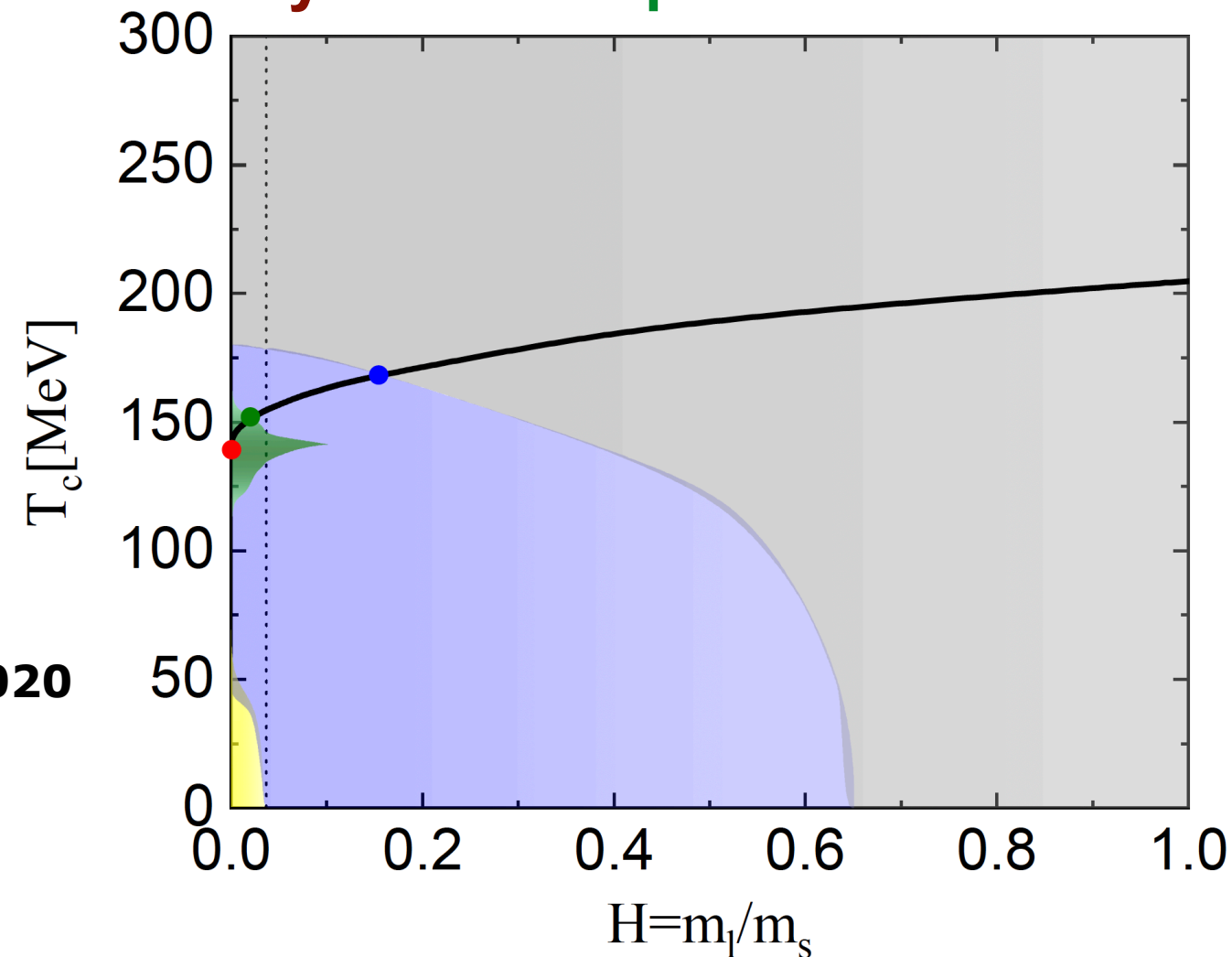
Critical O(4) scaling

Scaling coefficient as function of the pion mass



Trivial $\Delta_l^{1+\delta}$ scaling

Chiral dynamics & quasi-massless modes



Critical scaling



Gao, JMP, PRD 105 (2022) 094020

Far away from the critical regime for $m_\pi \gtrsim 1$ MeV

$$\Delta_l(T, H) \approx \Delta_{l,\chi}(0) \left(c_0 + c_{\frac{1}{5}} H^{\frac{1}{5}} + c_{\frac{1}{3}} H^{\frac{1}{3}} + c_1 H \right)$$



$$V_\chi(\Delta_l) \propto$$

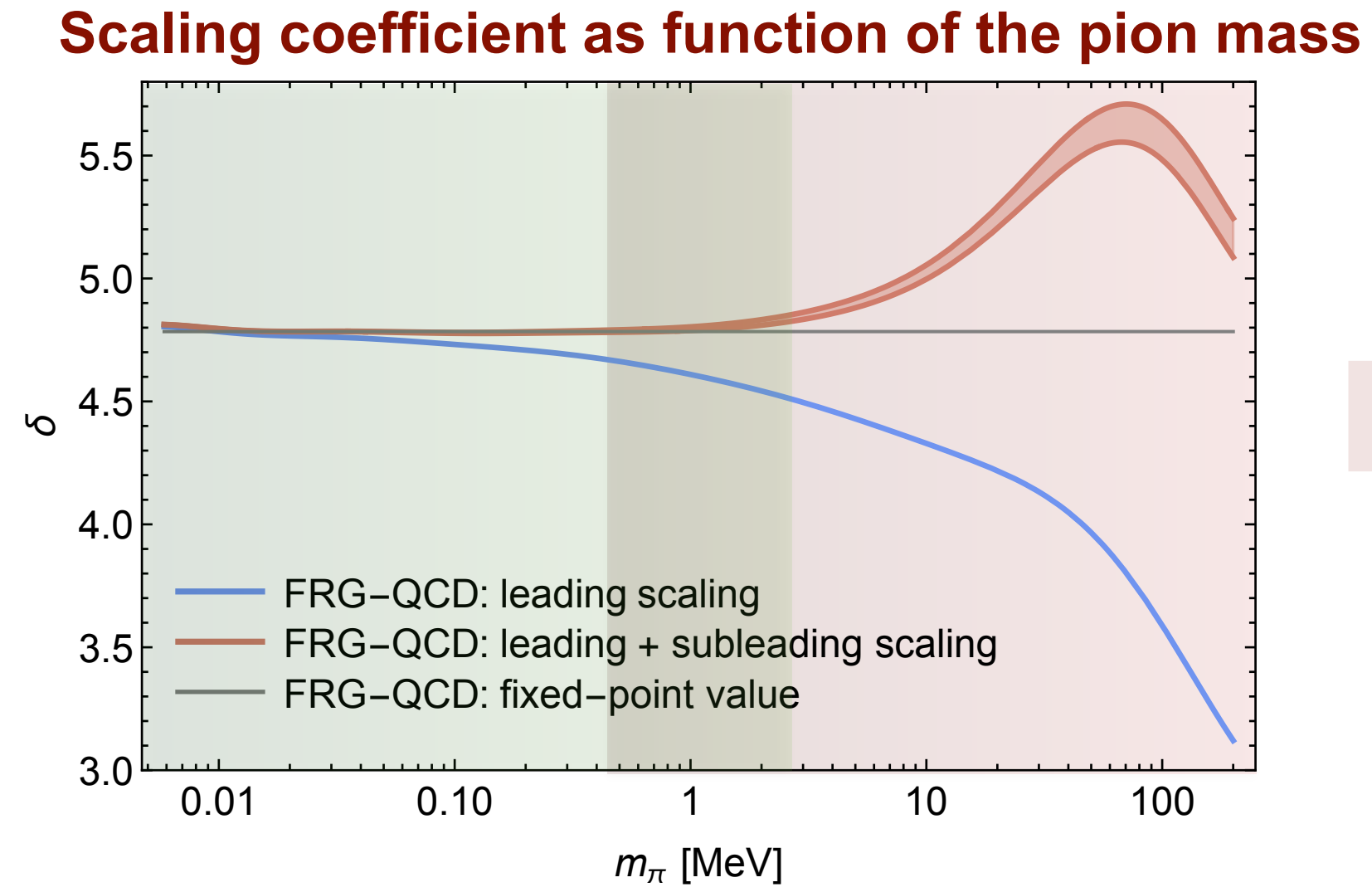
$$\Delta_l^6$$

$$\Delta_l^4$$

$$\Delta_l^2$$

Chiral dynamics & quasi-massless modes

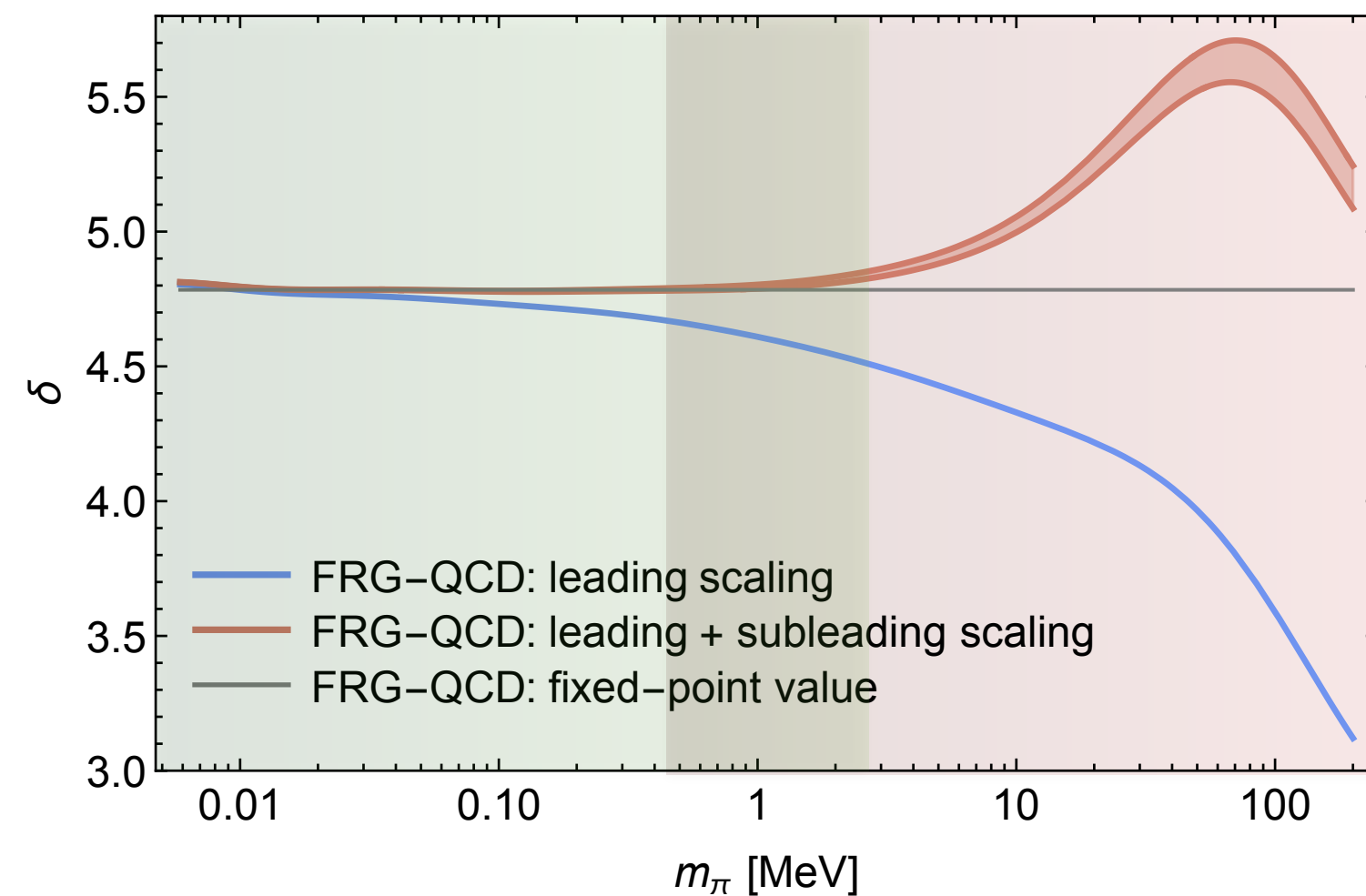
$$\Delta_l(m_\pi) \propto m_\pi^{2/\delta} [1 + a_m m_\pi^{2\theta_H} + \dots]$$



Trivial $\Delta_l^{1+\delta}$ scaling

Chiral dynamics & quasi-massless modes

Scaling coefficient as function of the pion mass



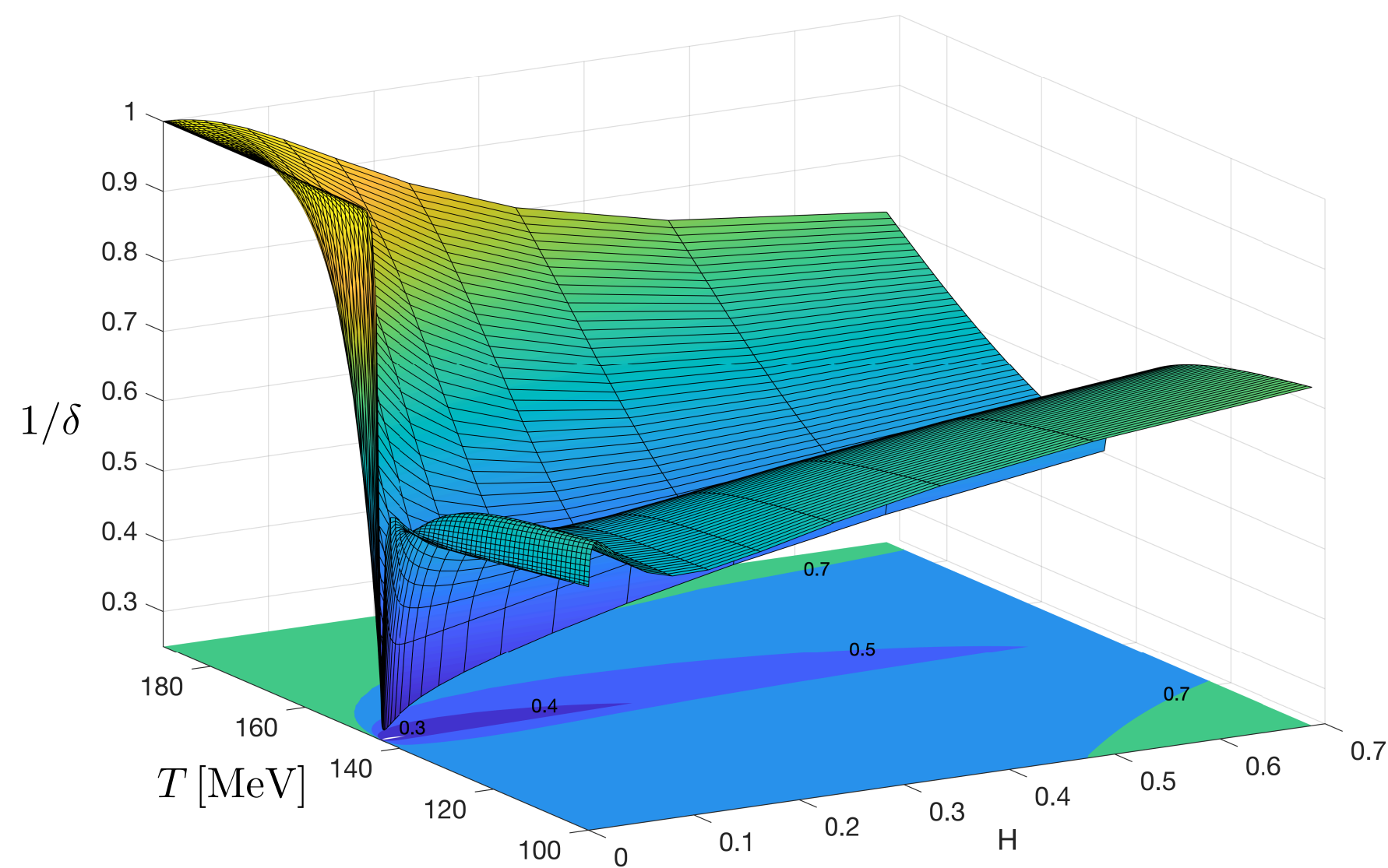
Trivial $\Delta_l^{1+\delta}$ scaling

$$\Delta_l(m_\pi) \propto m_\pi^{2/\delta} [1 + a_m m_\pi^{2\theta_H} + \dots]$$

Small chiral scaling regime

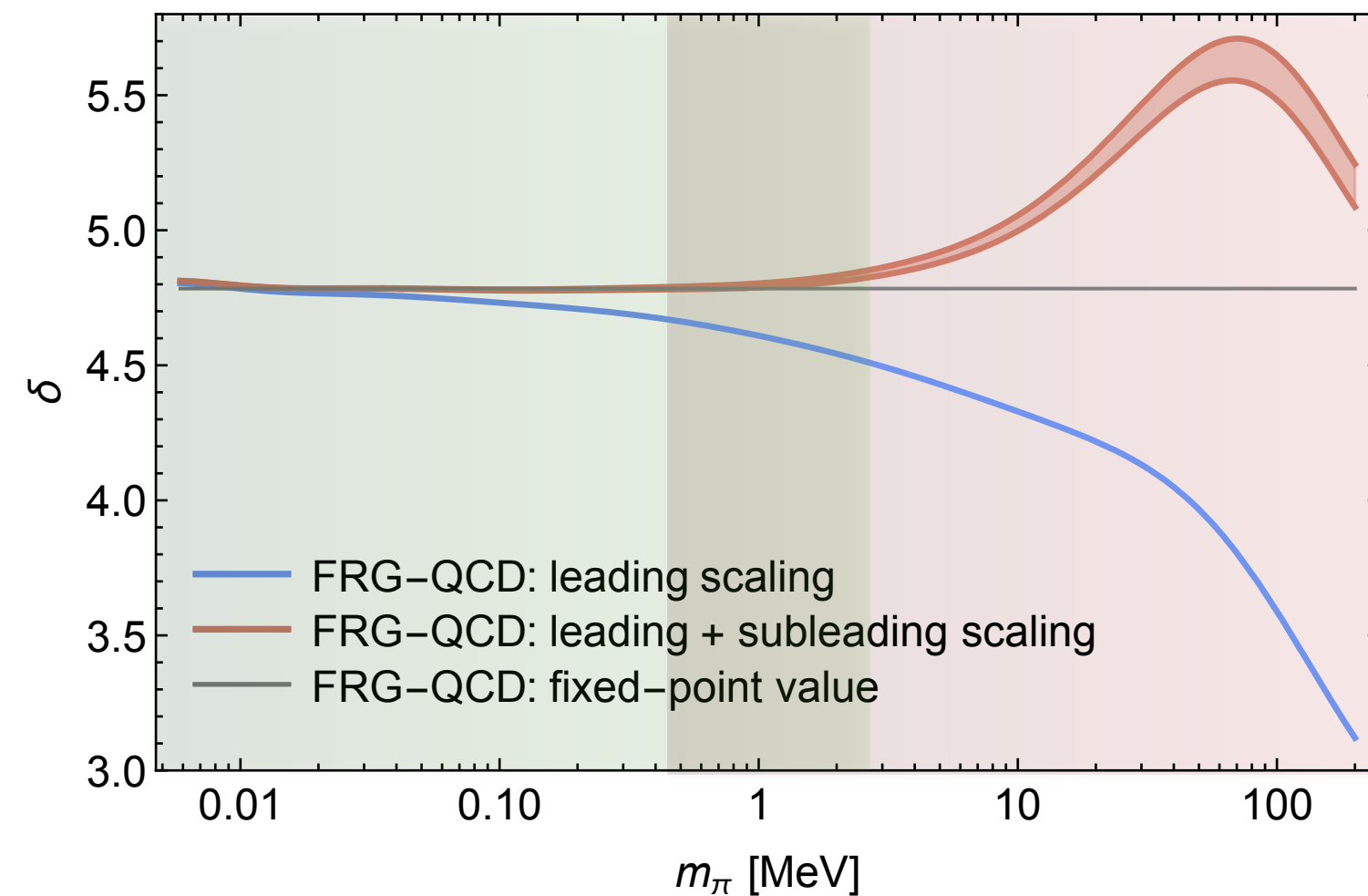


Small critical regime around pot. CEP



Chiral dynamics & quasi-massless modes

Scaling coefficient as function of the pion mass



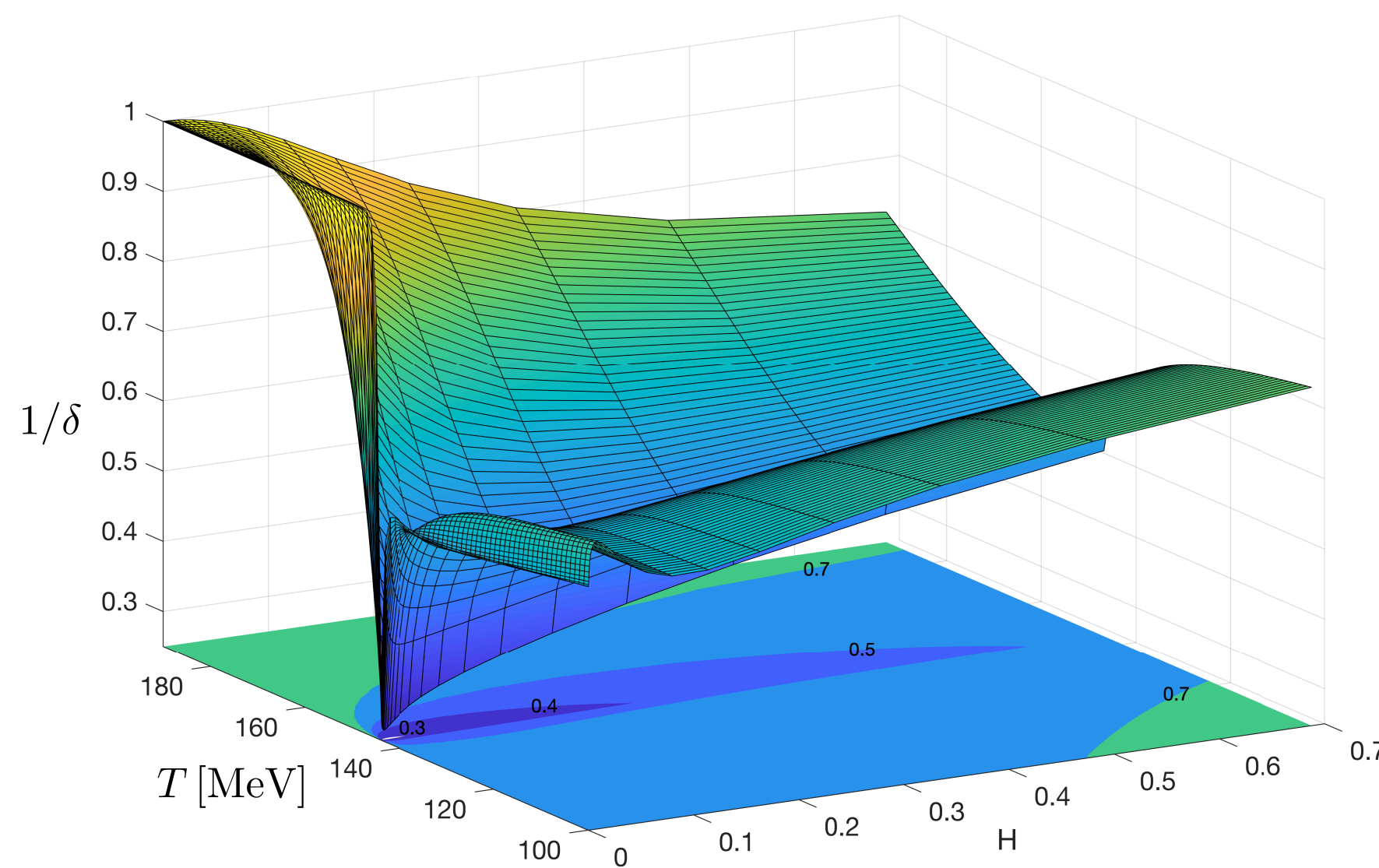
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Small chiral scaling regime



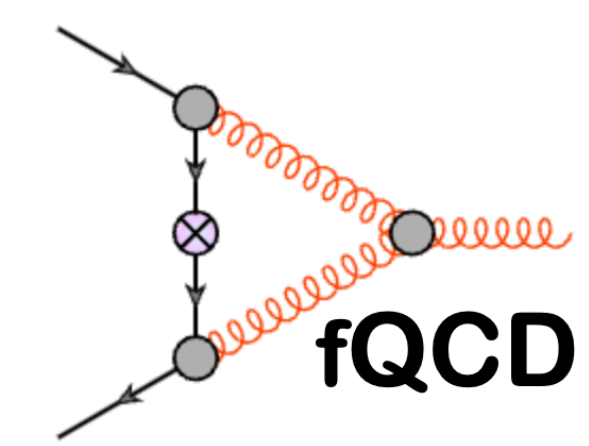
Small critical regime around pot. CEP



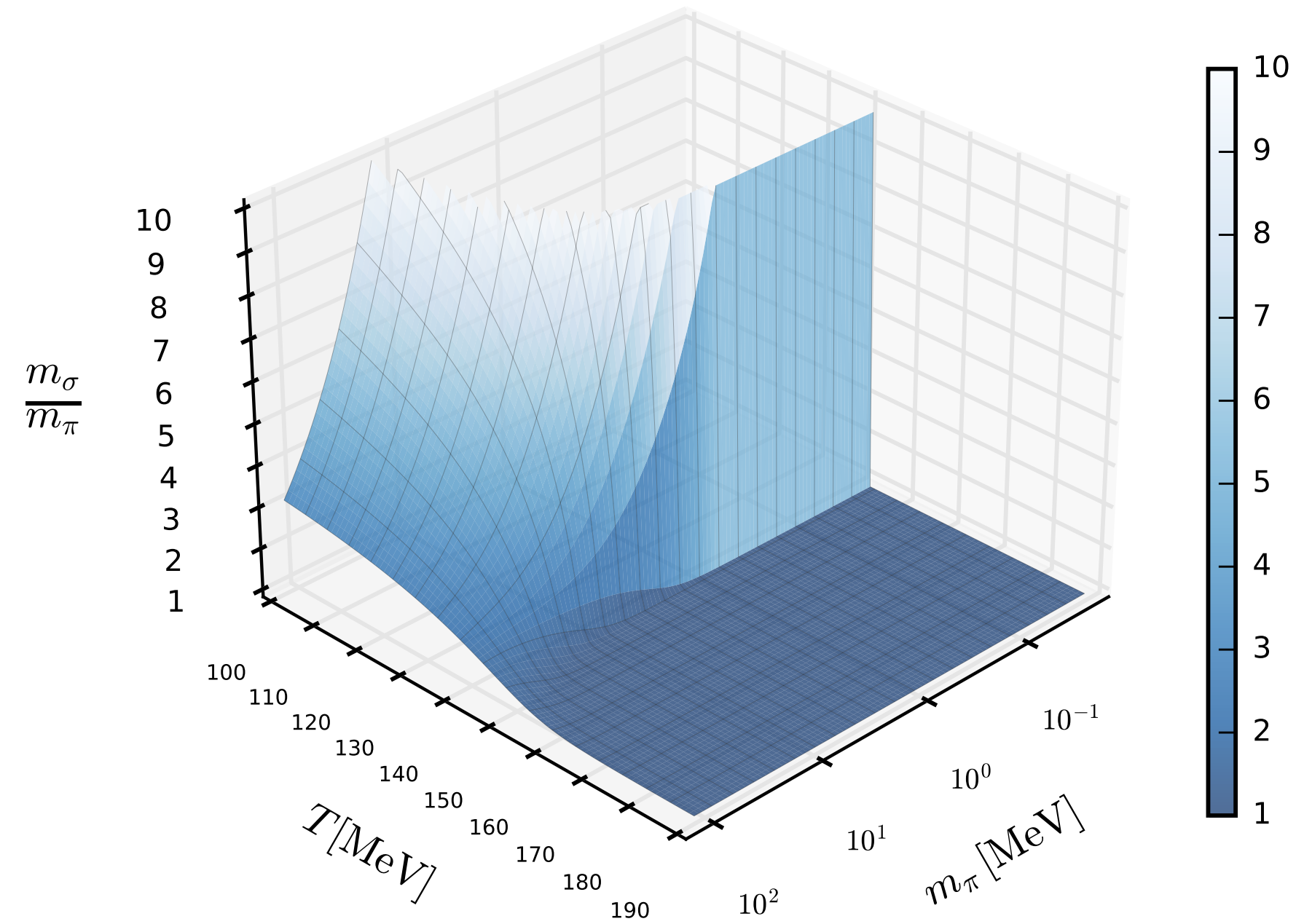
!!Great News!!

Location of CEP/New phase accessible via combination of precision measurements & computations

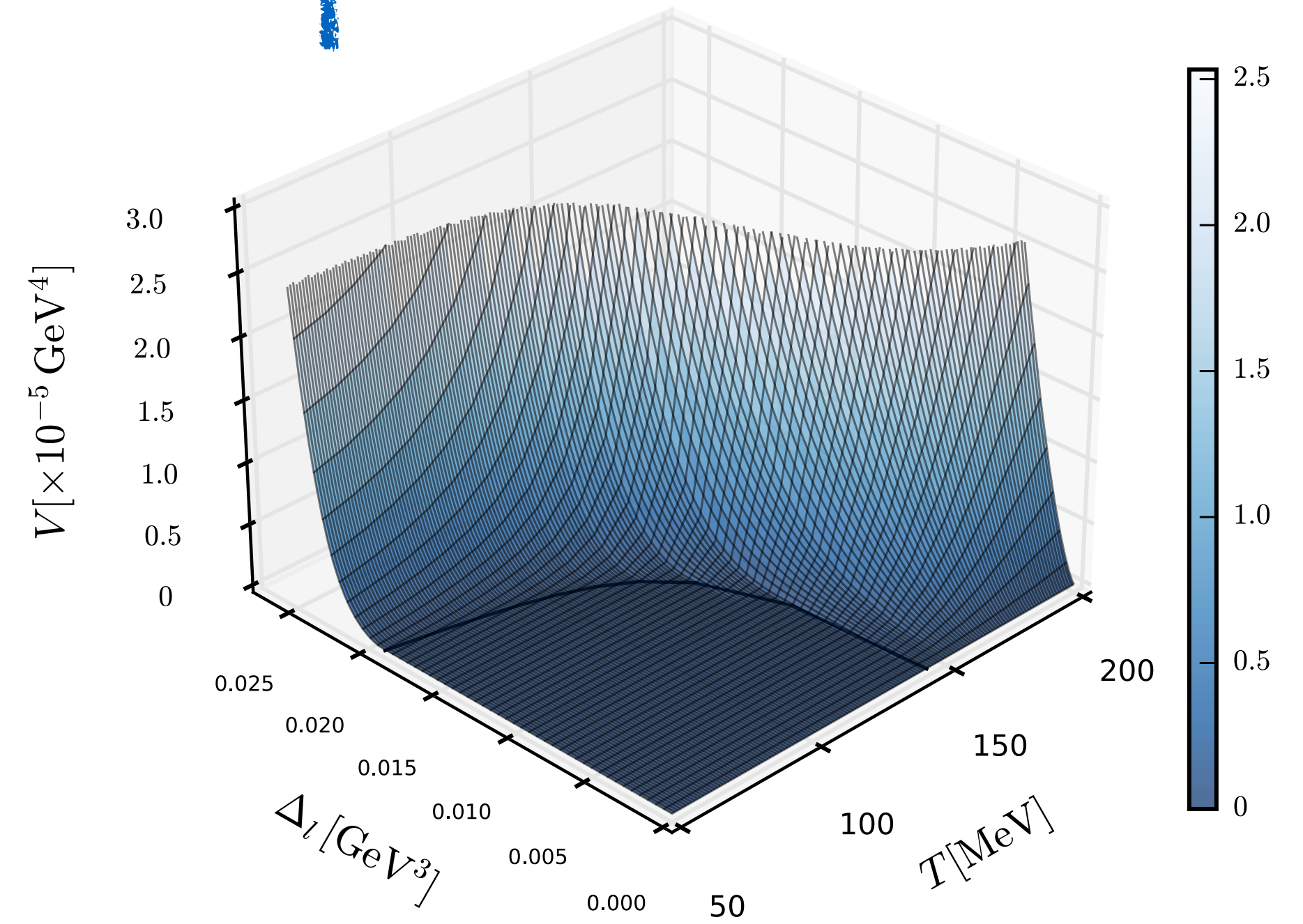
Full order parameter potential



Measure: correlation length



Use for chiral dynamics in heavy ion collisions

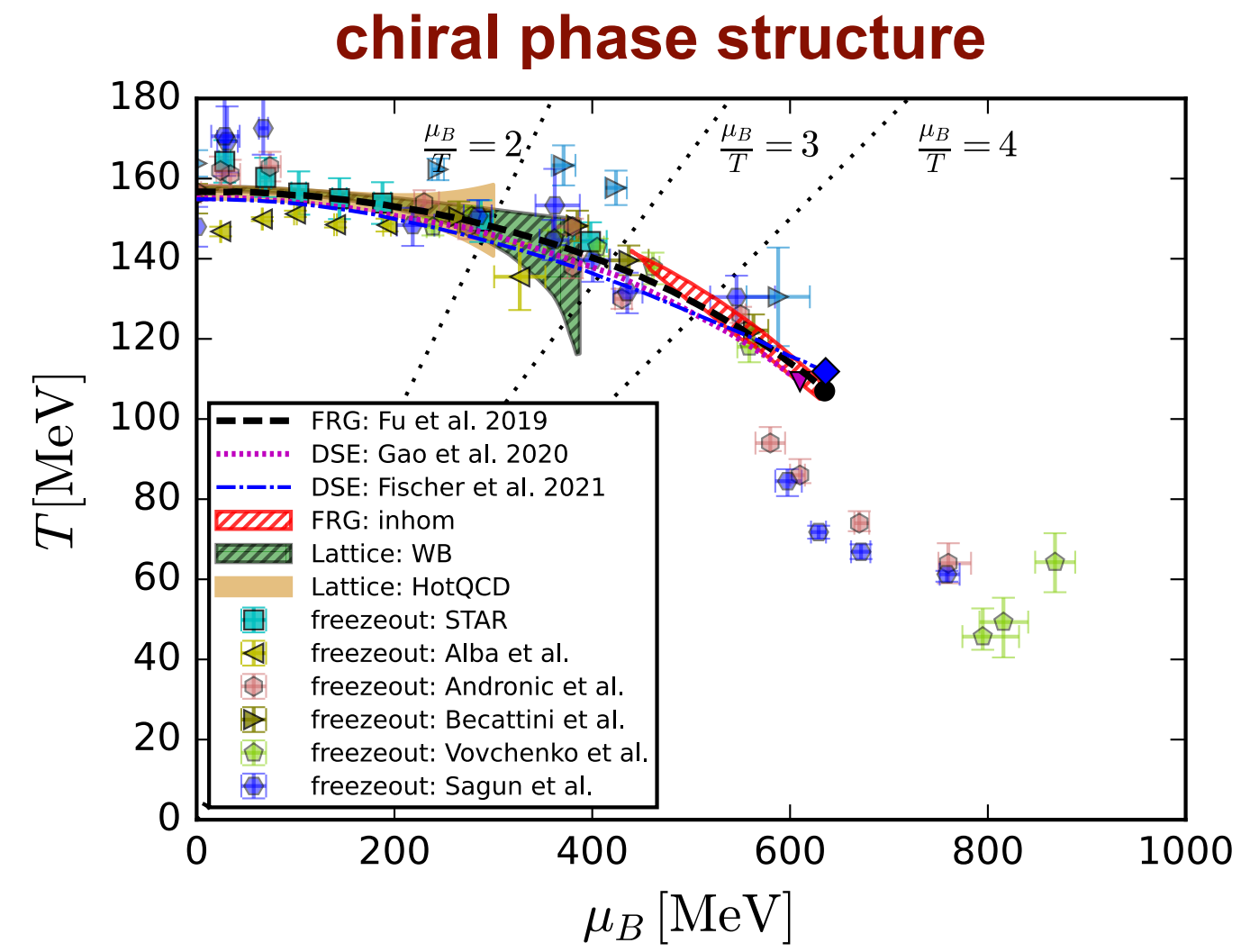


Ripples of the CEP

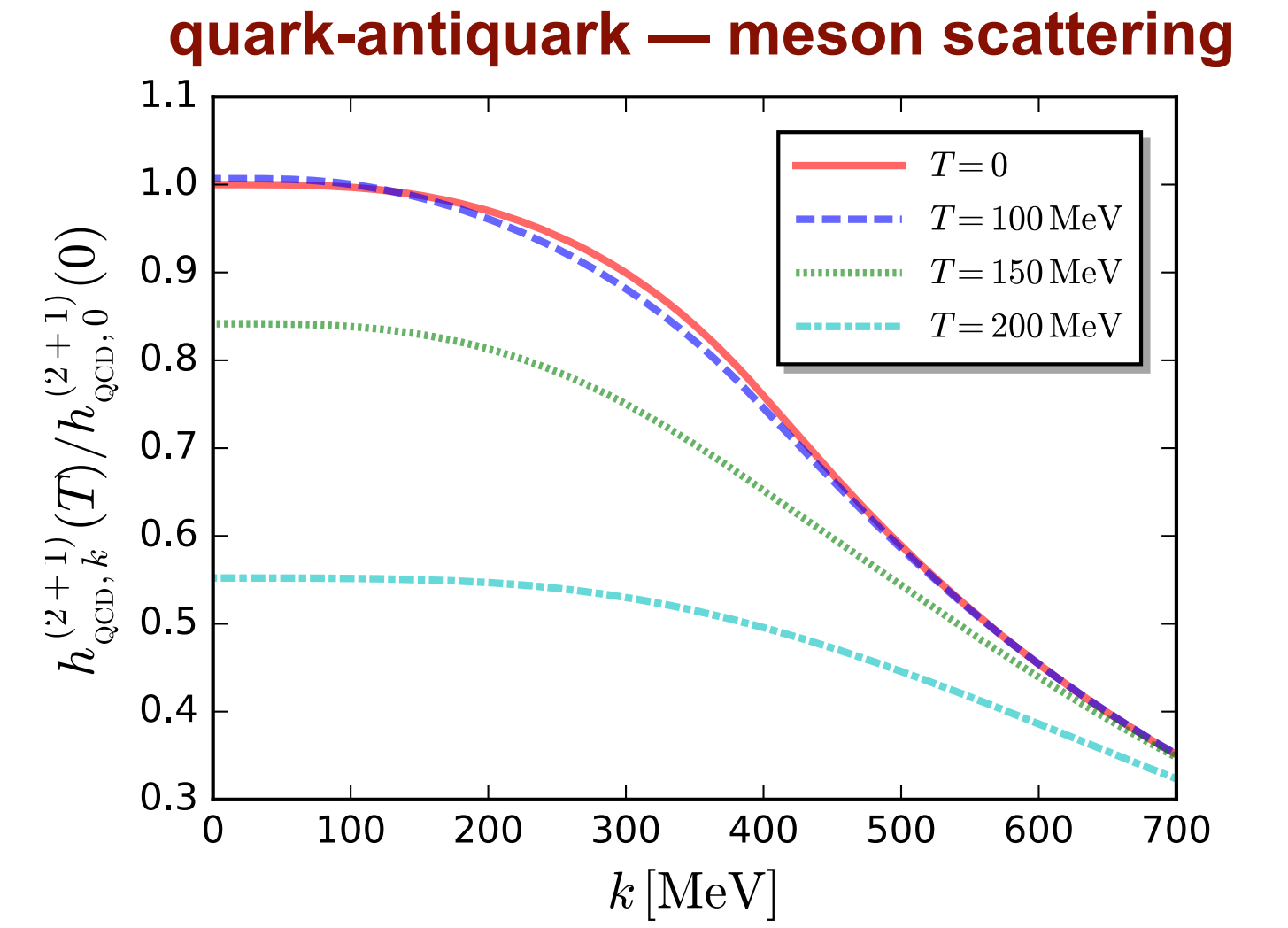
Fu, Luo, JMP, Rennecke, Wen, Yin, PRD 111, L031502

QCD-assisted low energy effective theory

Direct QCD input

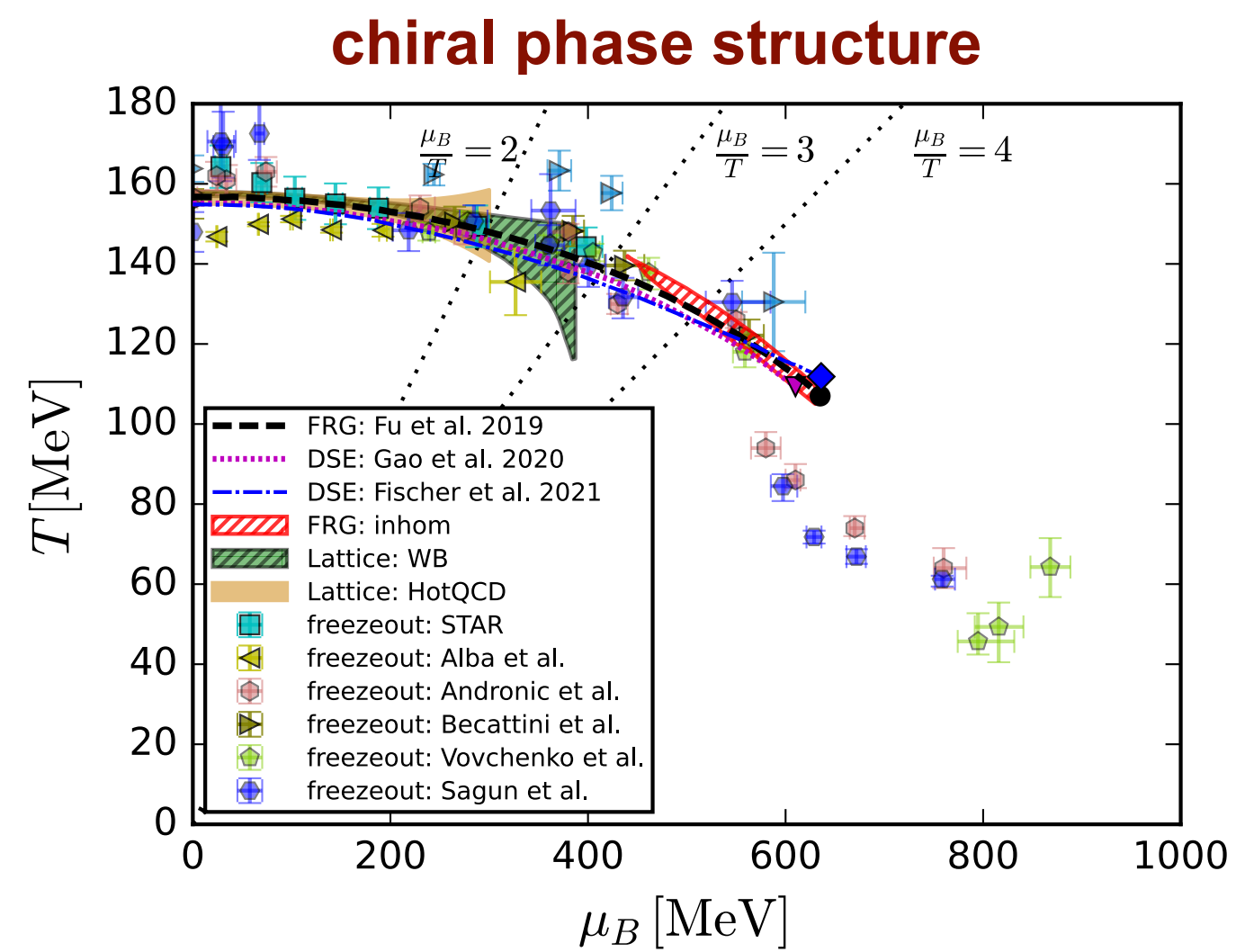


+

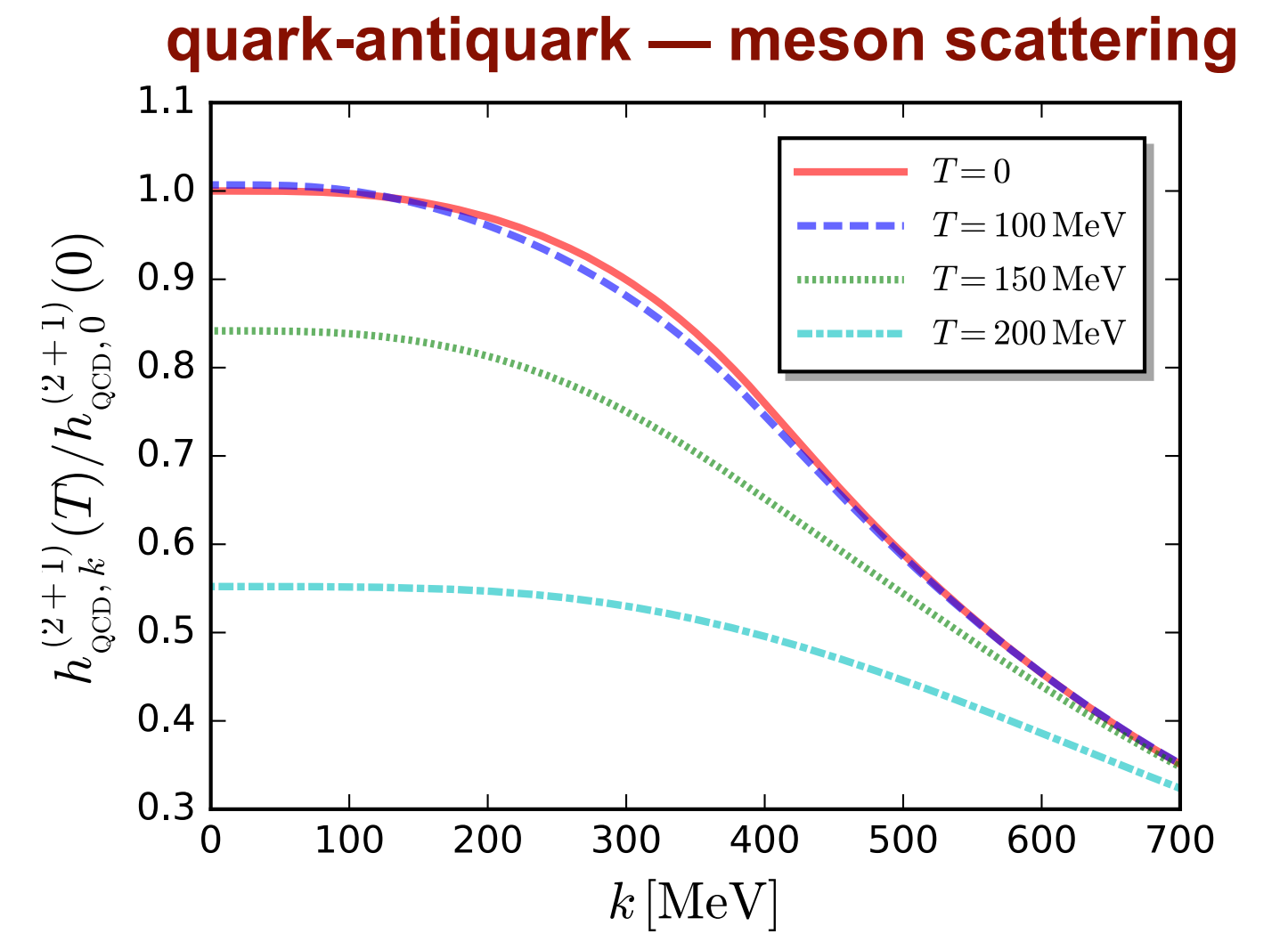


QCD-assisted low energy effective theory

Direct QCD input



+

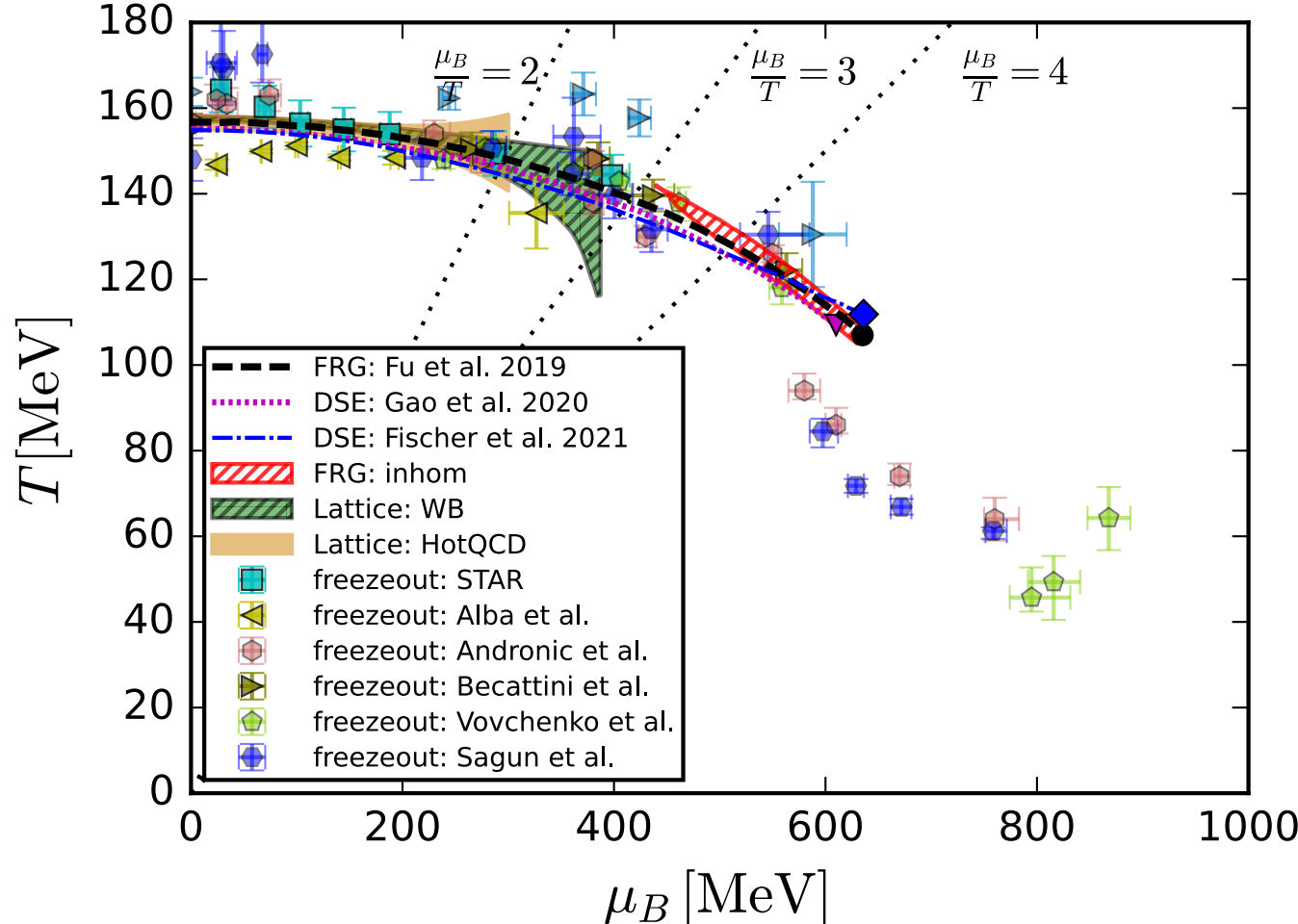


low energy quantum, thermal & density fluctuations via fRG (QCD-assisted PQM model)

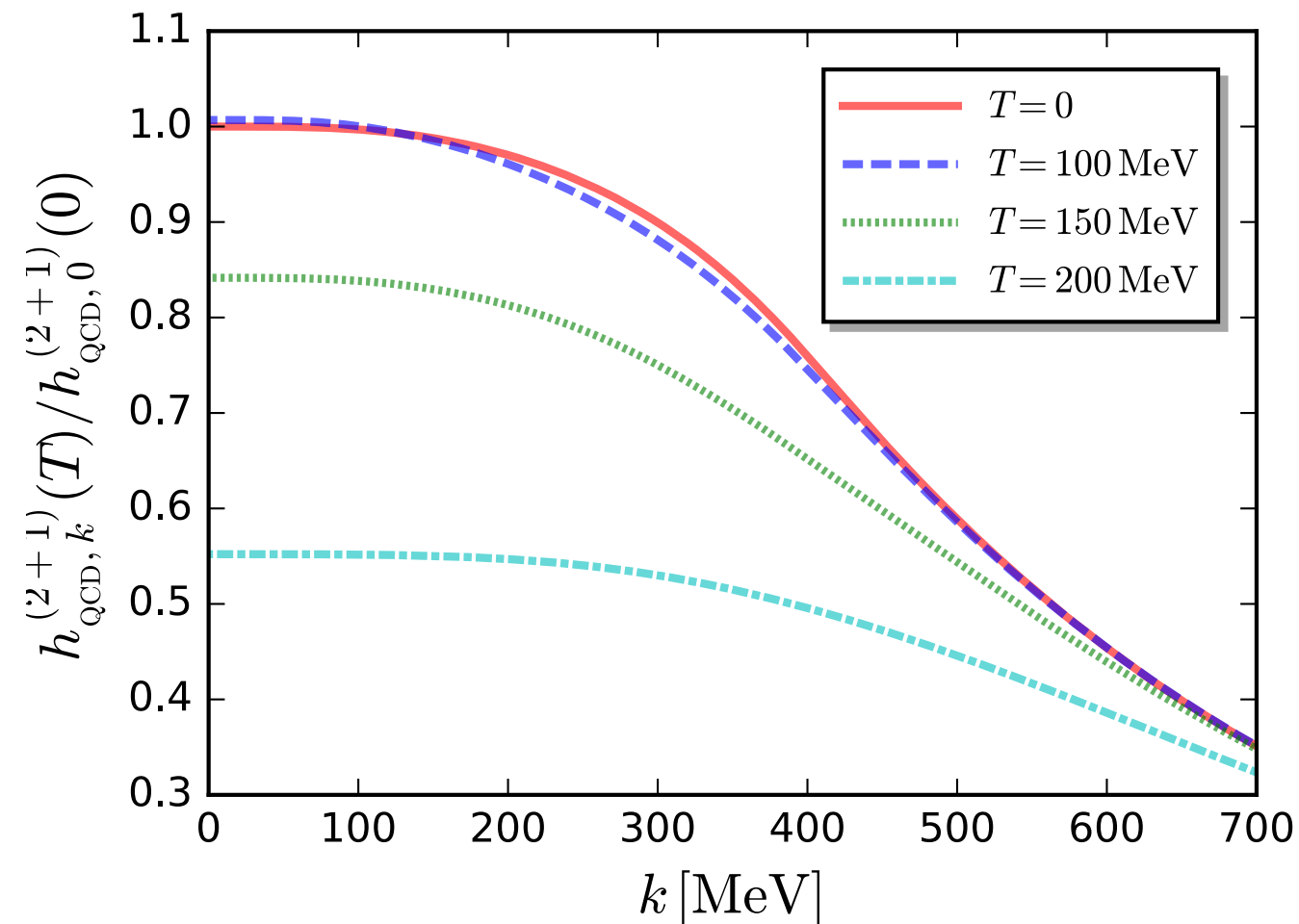
QCD-assisted low energy effective theory

Direct QCD input

chiral phase structure



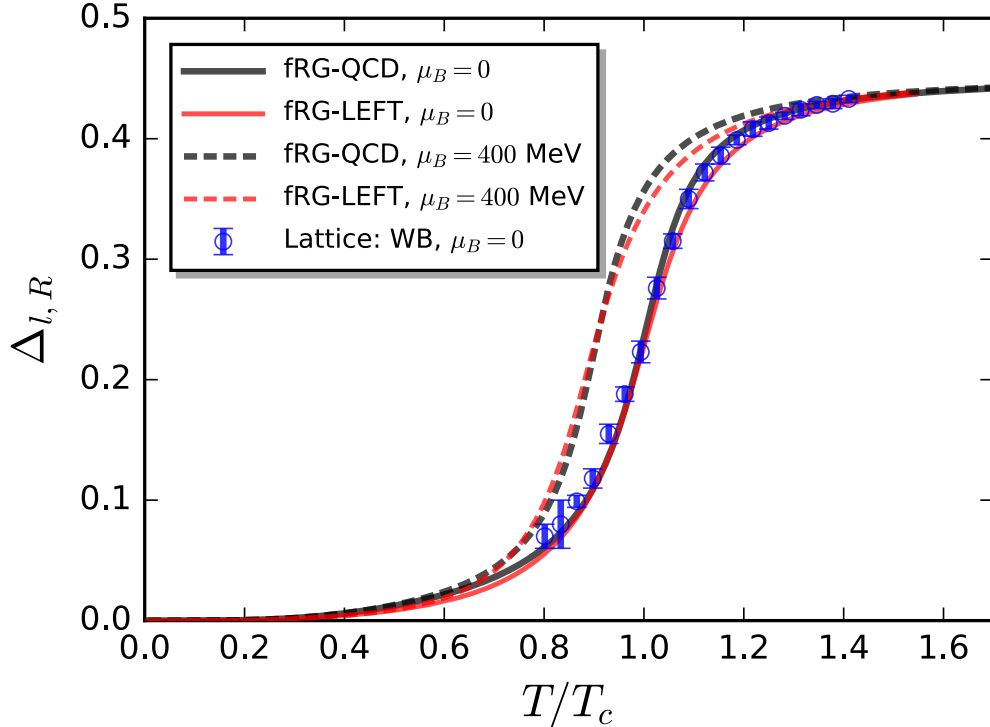
quark-antiquark — meson scattering



+

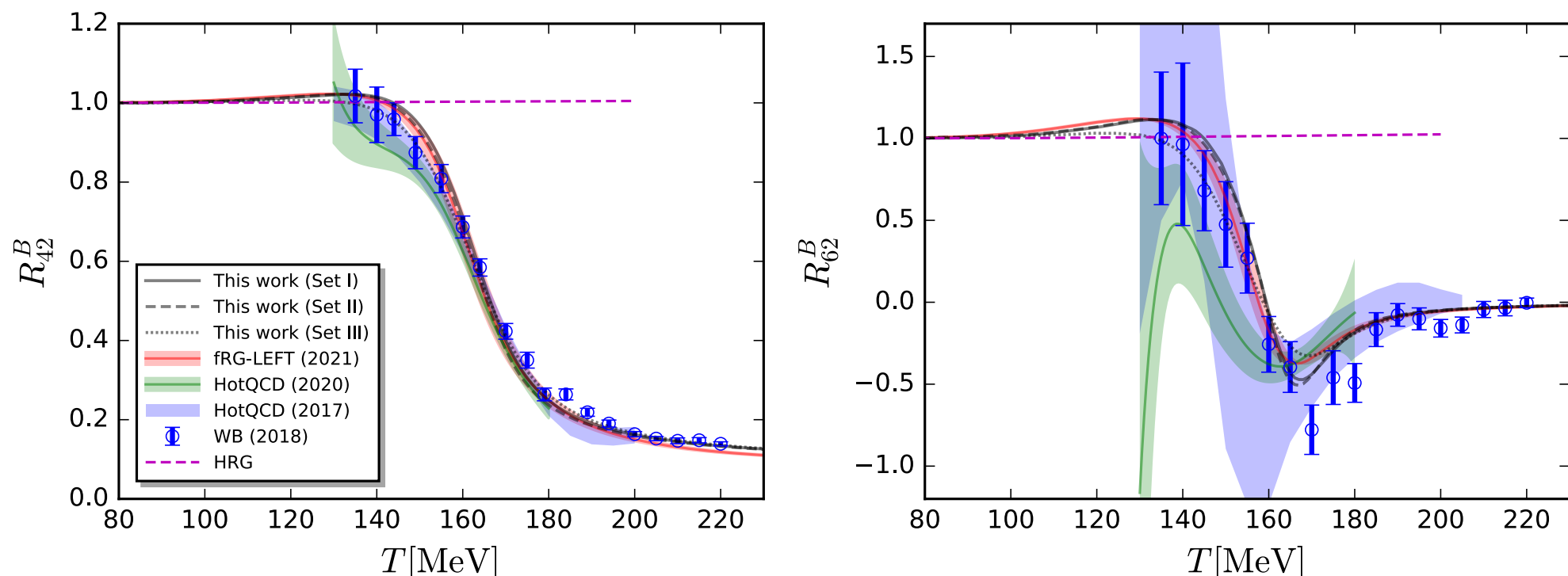
low energy quantum, thermal & density fluctuations via fRG (QCD-assisted PQM model)

renormalised chiral condensate



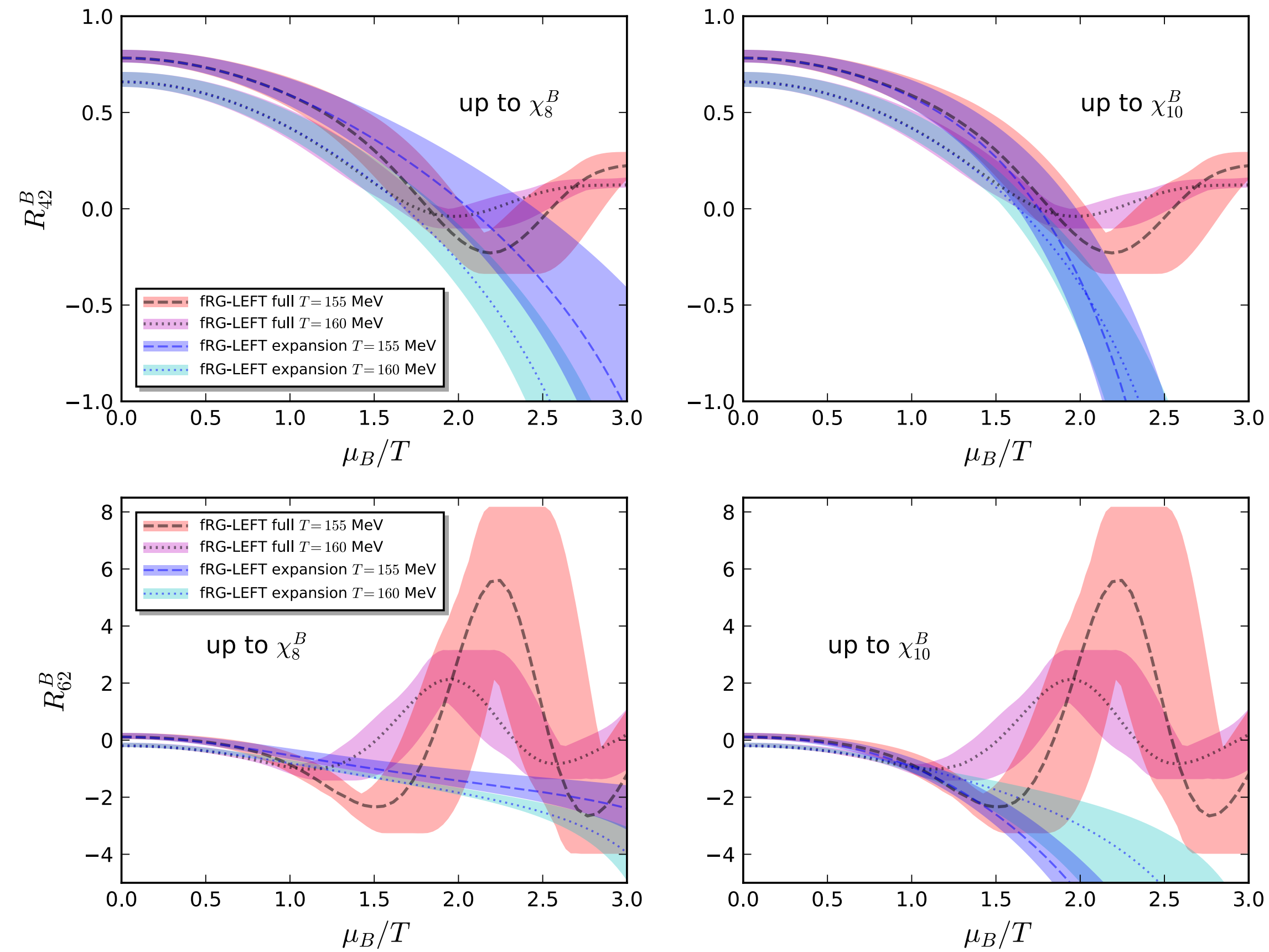
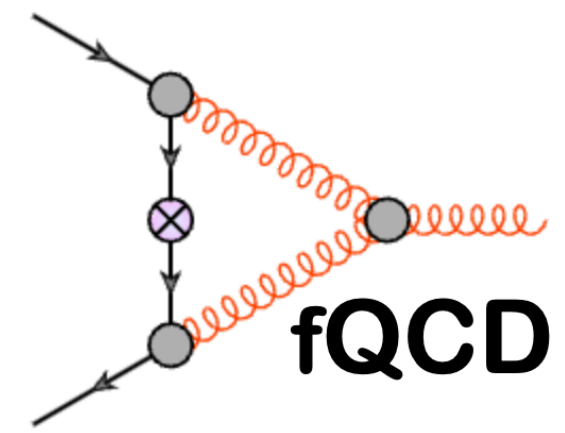
Benchmarks with lattice and fQCD
at
vanishing density and fQCD at finite density

baryon number fluctuations



Fluctuations of conserved charges

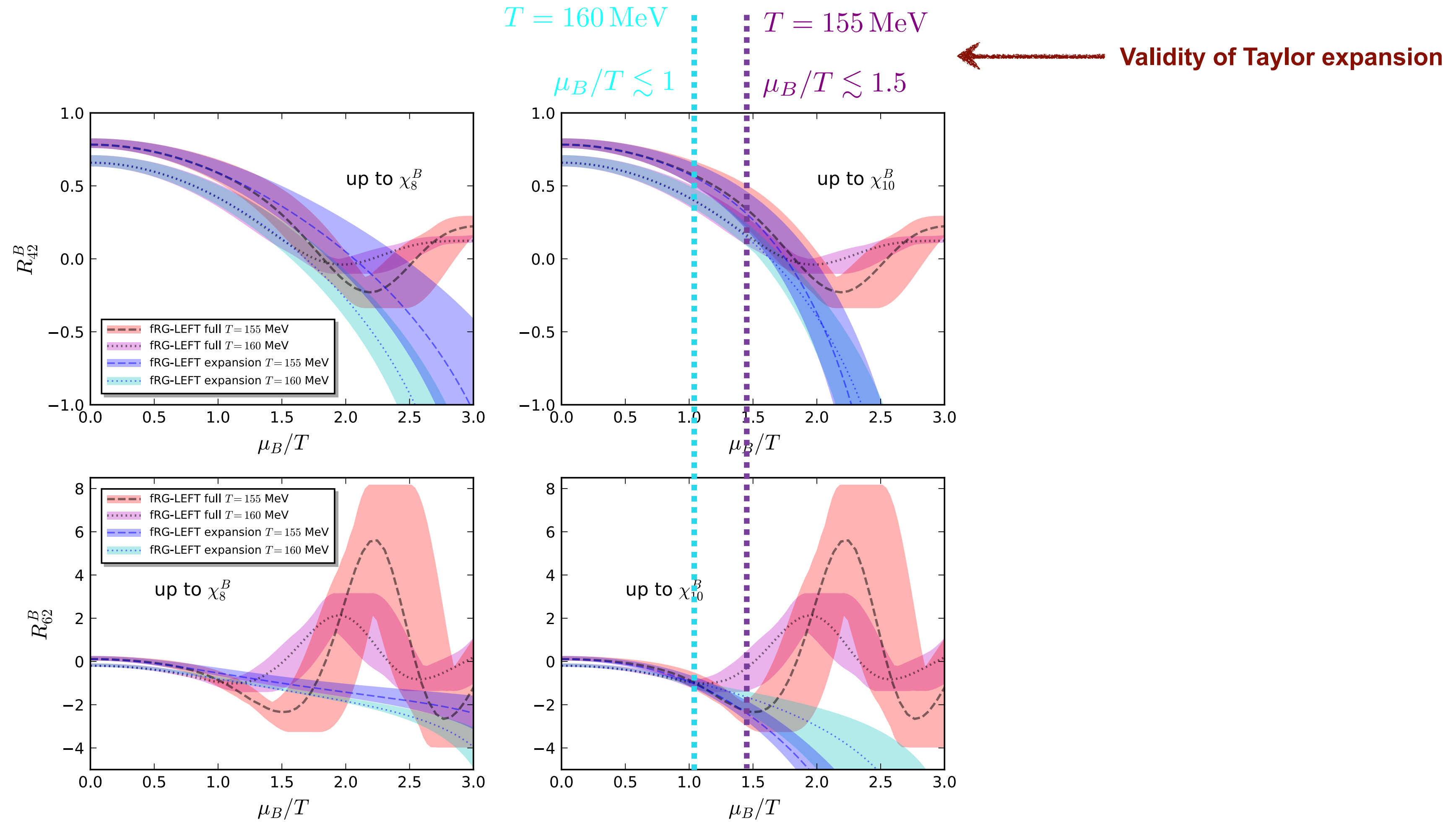
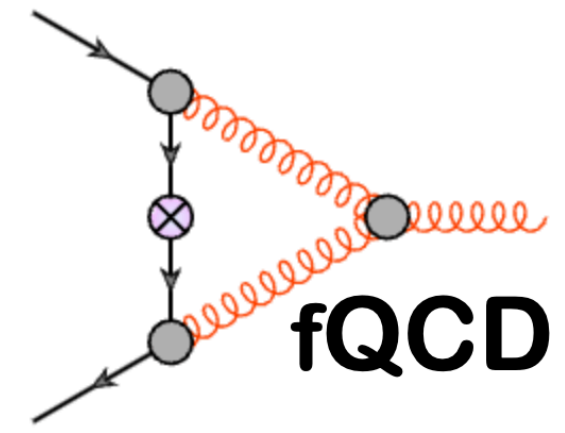
Fluctuations of conserved charges



Great opportunity for a combined analysis of high density QCD (Exp. data + lattice QCD + functional QCD)

Fluctuations of conserved charges

Fluctuations of conserved charges

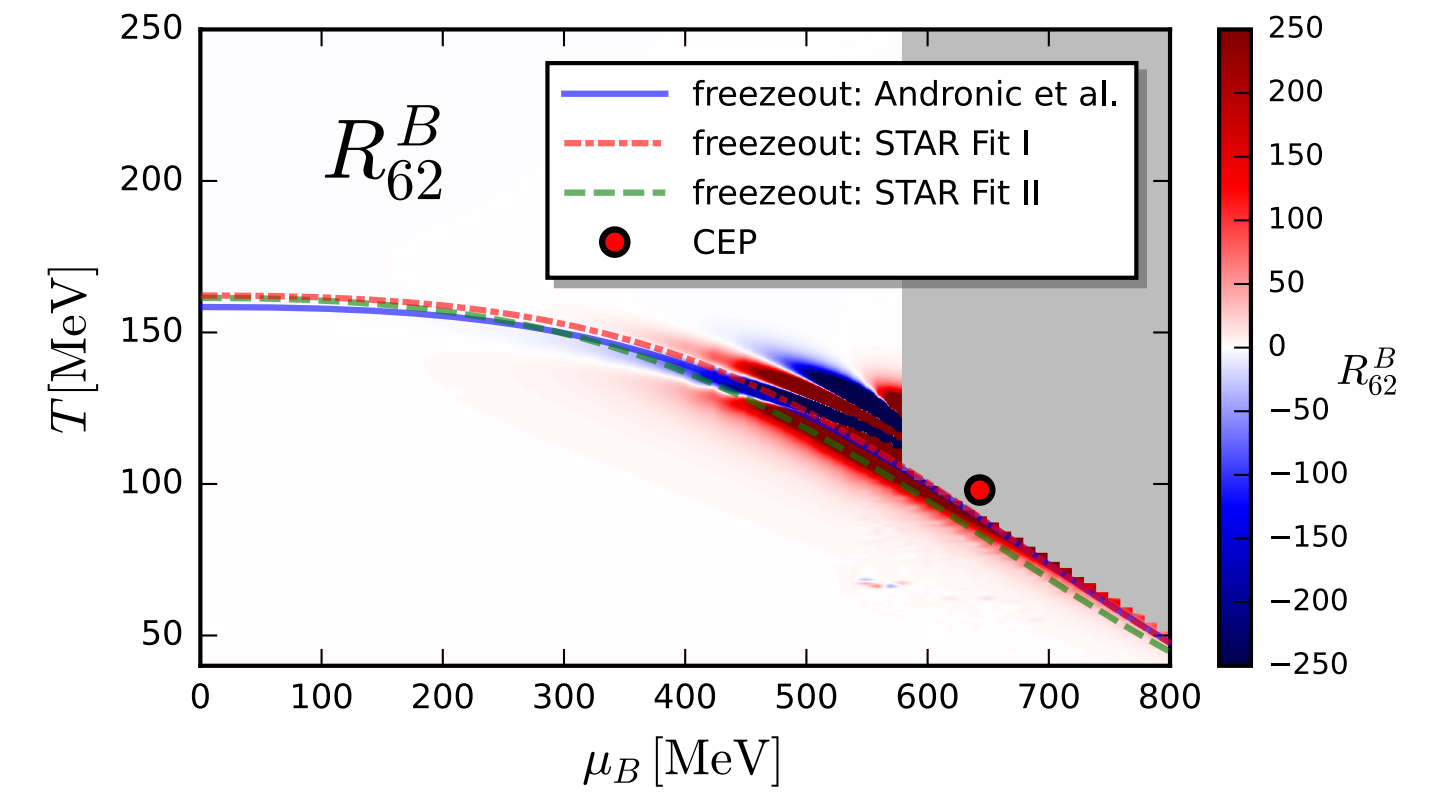
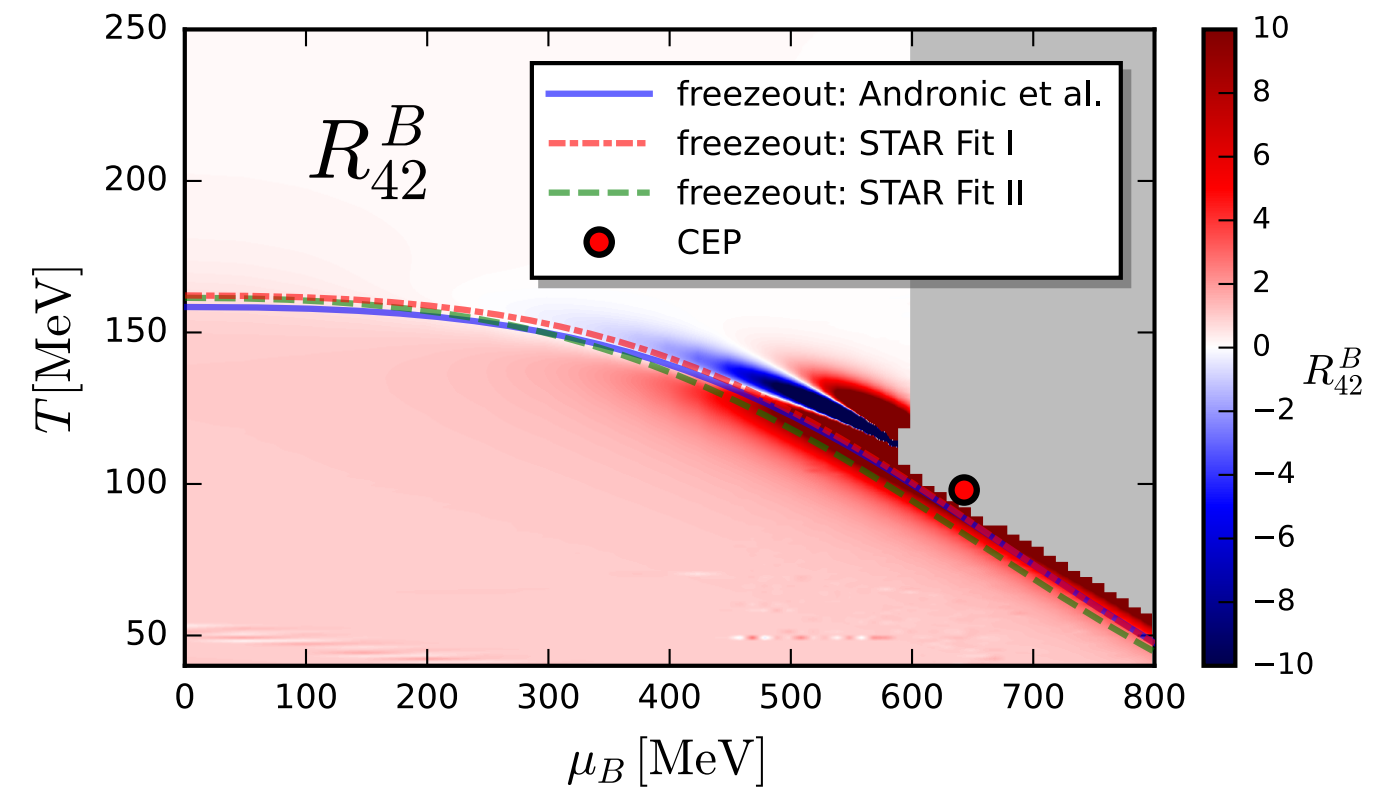


Great opportunity for a combined analysis of high density QCD (Exp. data + lattice QCD + functional QCD)

Ripples of the critical point

baryon number fluctuations in the phase structure

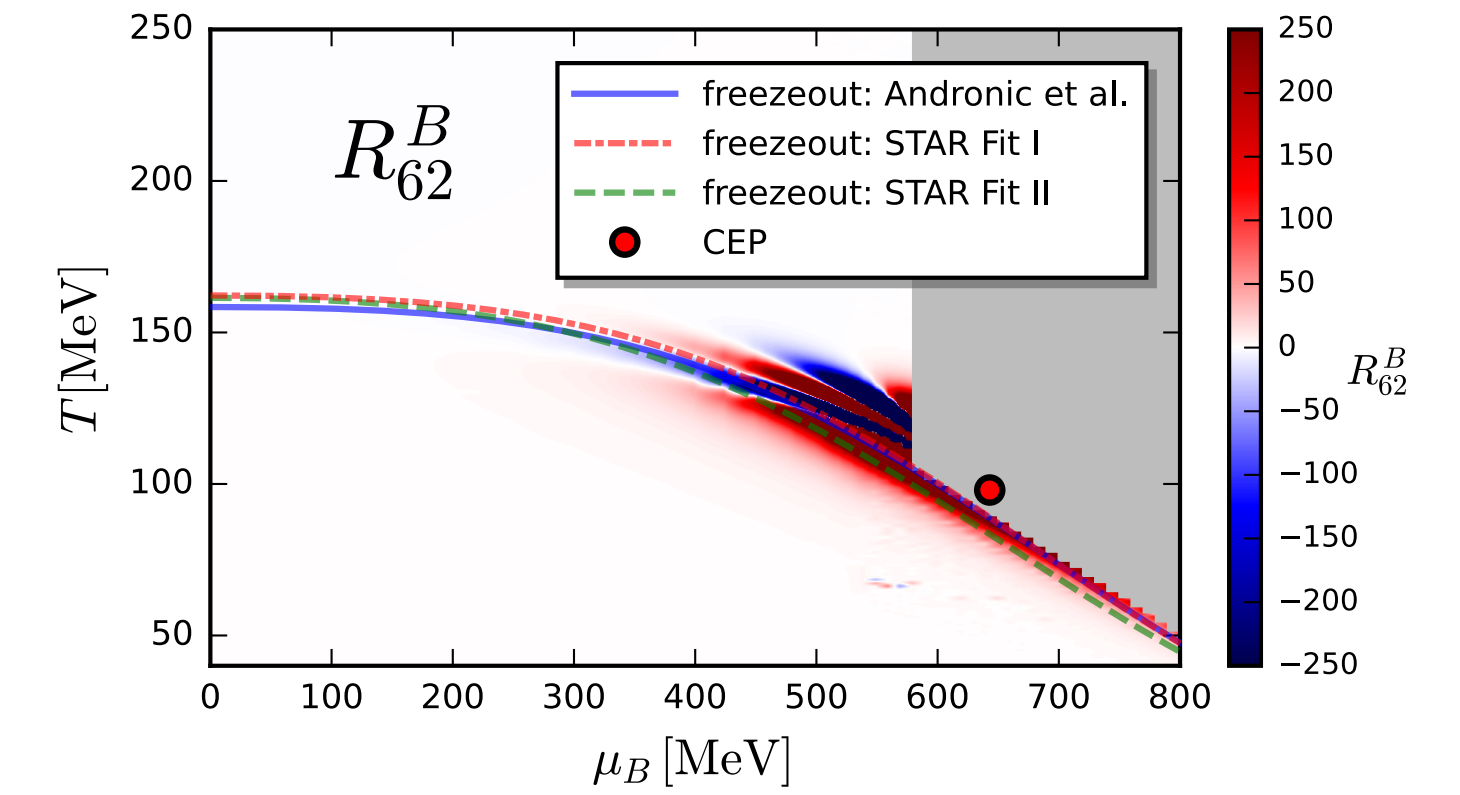
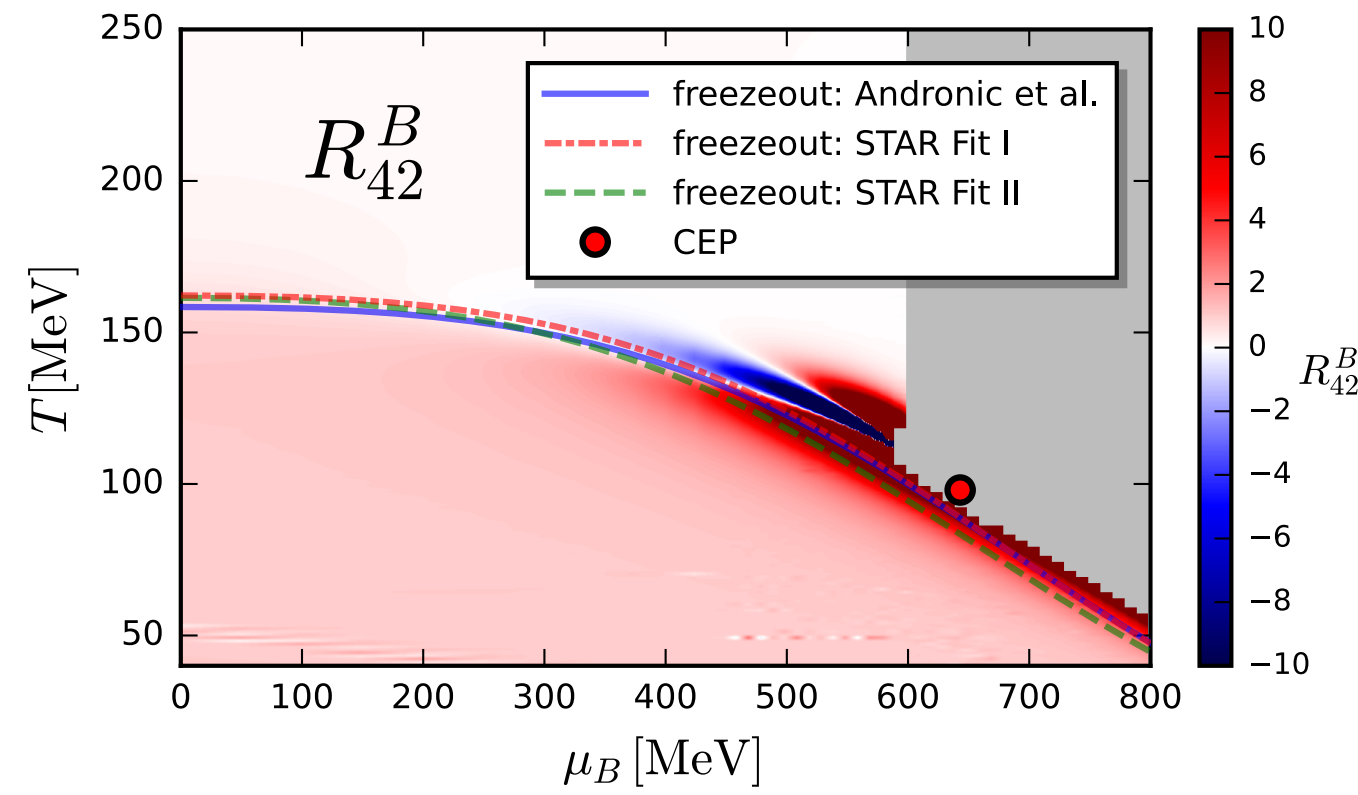
$$(T_{\text{CEP}}, \mu_{B_{\text{CEP}}}) = (98, 643) \text{ MeV}$$



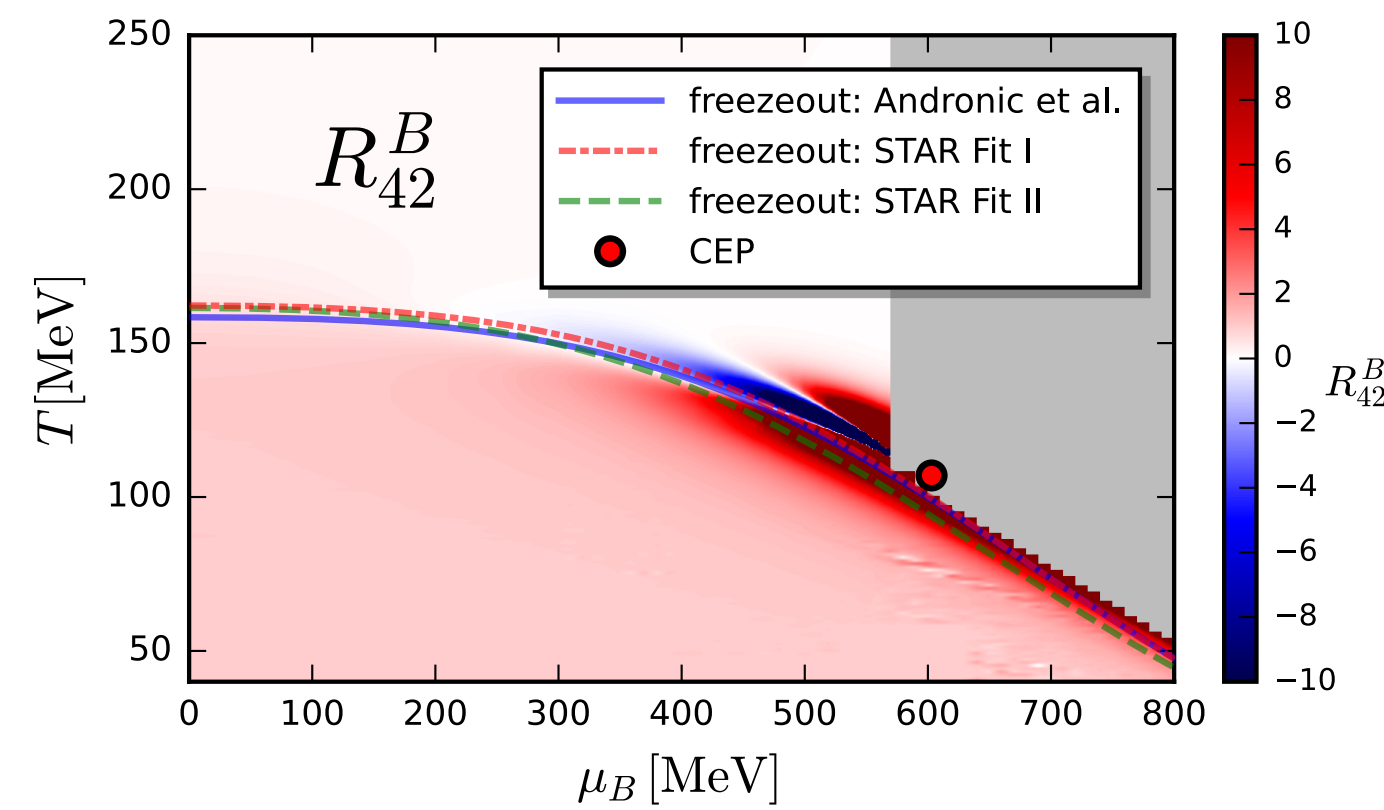
Ripples of the critical point

baryon number fluctuations in the phase structure

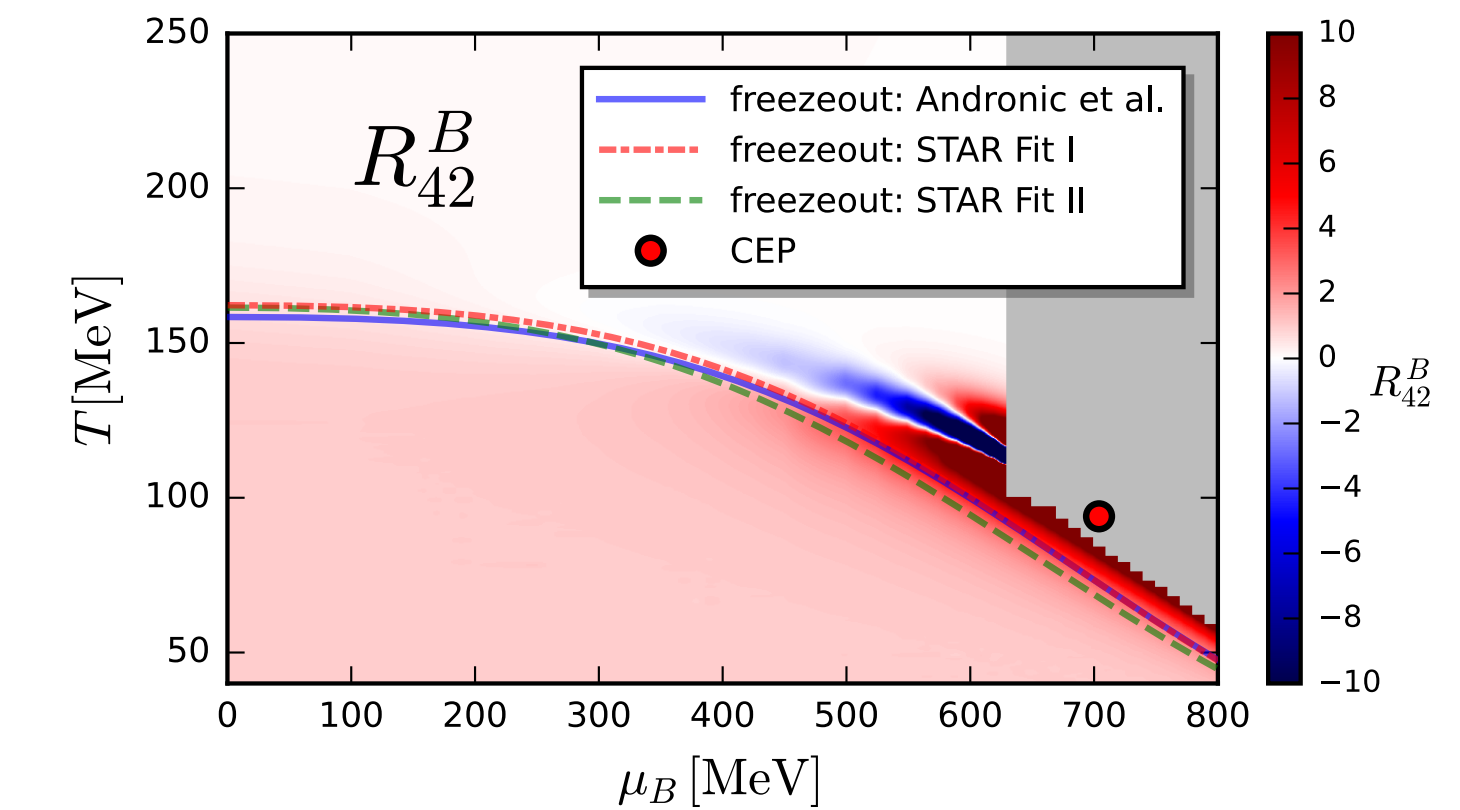
$$(T_{\text{CEP}}, \mu_{B_{\text{CEP}}}) = (98, 643) \text{ MeV}$$



Variations of the CEP in the allowed estimate regime



$$(T_{\text{CEP}}, \mu_{B_{\text{CEP}}}) = (108, 604) \text{ MeV}$$



$$(T_{\text{CEP}}, \mu_{B_{\text{CEP}}}) = (94, 704) \text{ MeV}$$

Ripples of the critical point

Canonical corrections via subensemble acceptance

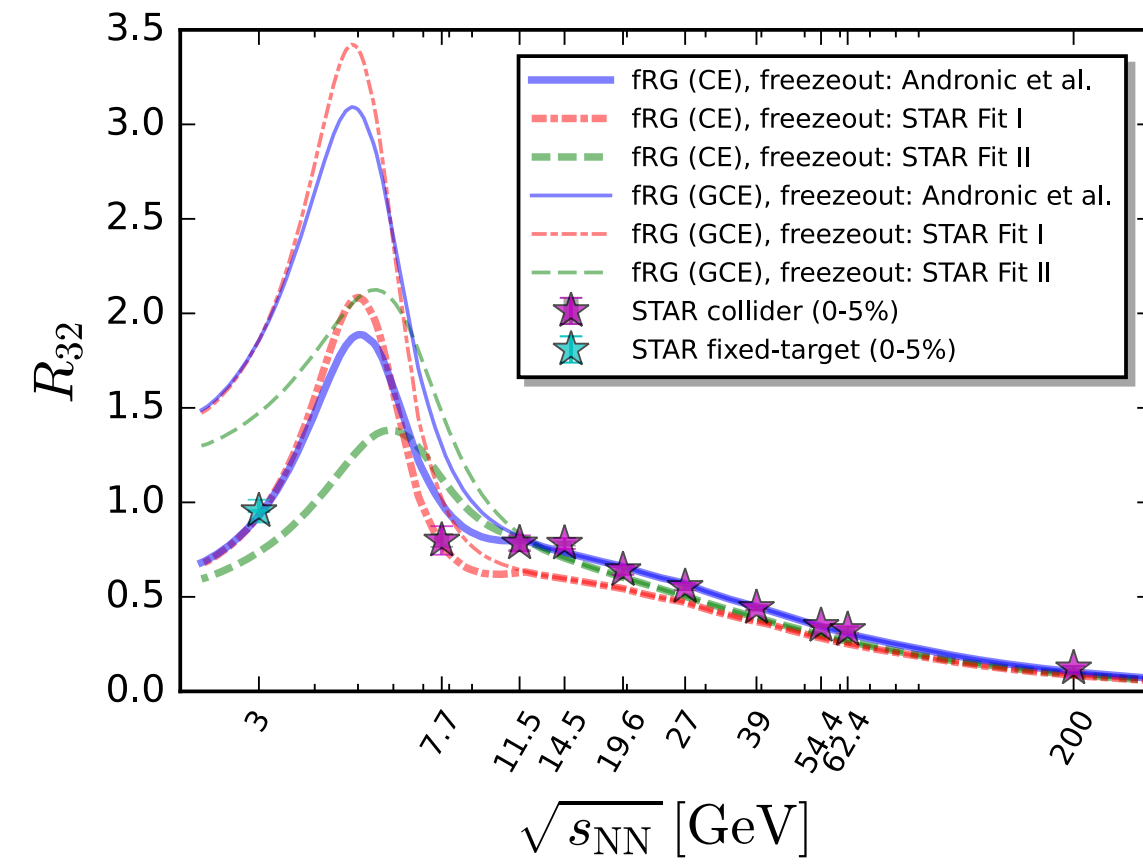
Vovchenko, Savchuk, Poberezhnyuk, Gorenstein, Koch, PLB 811, 135868 (2020)

fixing the subensemble volume

subensemble volume system volume

$$V_1 = \alpha V$$

$$\bar{R}_{32}^B = (1 - 2\alpha)R_{32}^B$$



Ripples of the critical point

Canonical corrections via subensemble acceptance

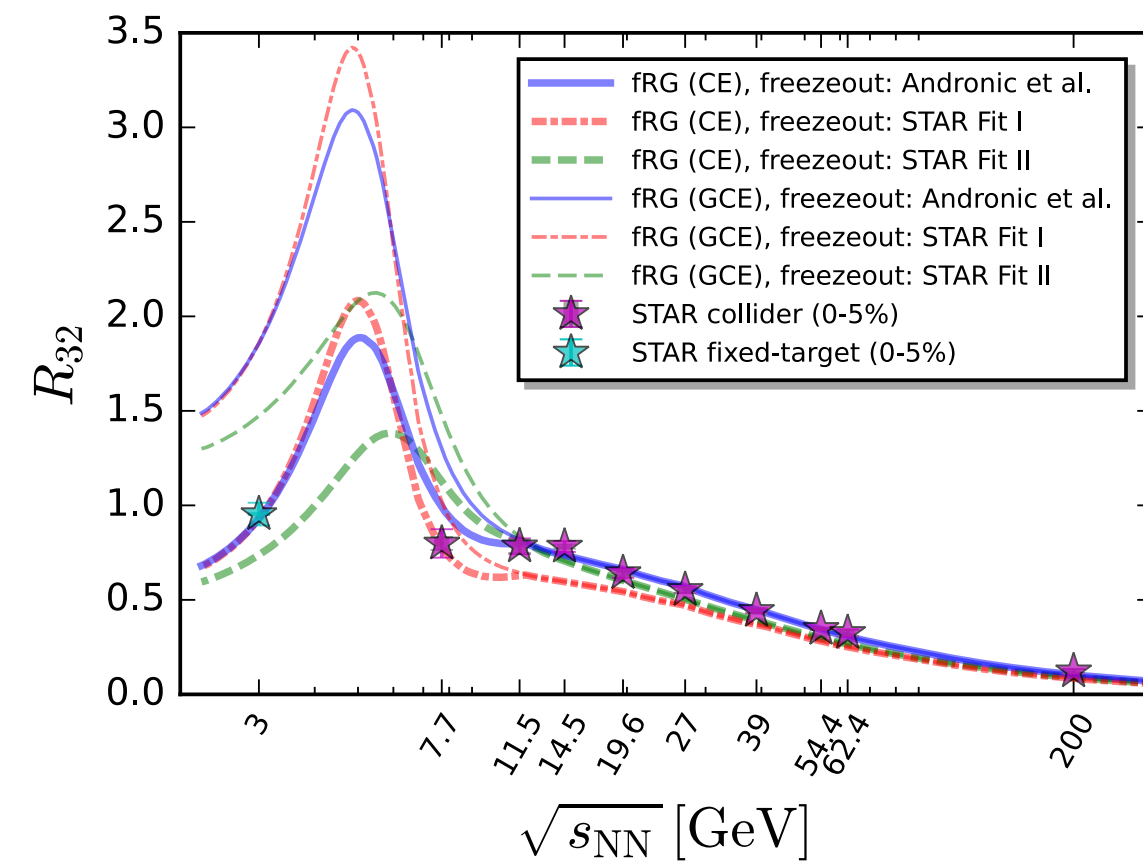
Vovchenko, Savchuk, Poberezhnyuk, Gorenstein, Koch, PLB 811, 135868 (2020)

fixing the subensemble volume

subensemble volume system volume

$$V_1 = \alpha V$$

$$\bar{R}_{32}^B = (1 - 2\alpha)R_{32}^B$$



qualitative adjustment

$$\alpha(\bar{s}) = a \left(1 - \sqrt{\bar{s}}\right) \theta(1 - \bar{s})$$

$$a = 0.33 \quad \sqrt{\bar{s}} = \frac{\sqrt{s_{\text{NN}}}}{11.9 \text{ GeV}}$$

Ripples of the critical point

Canonical corrections via subensemble acceptance

Vovchenko, Savchuk, Poberezhnyuk, Gorenstein, Koch, PLB 811, 135868 (2020)

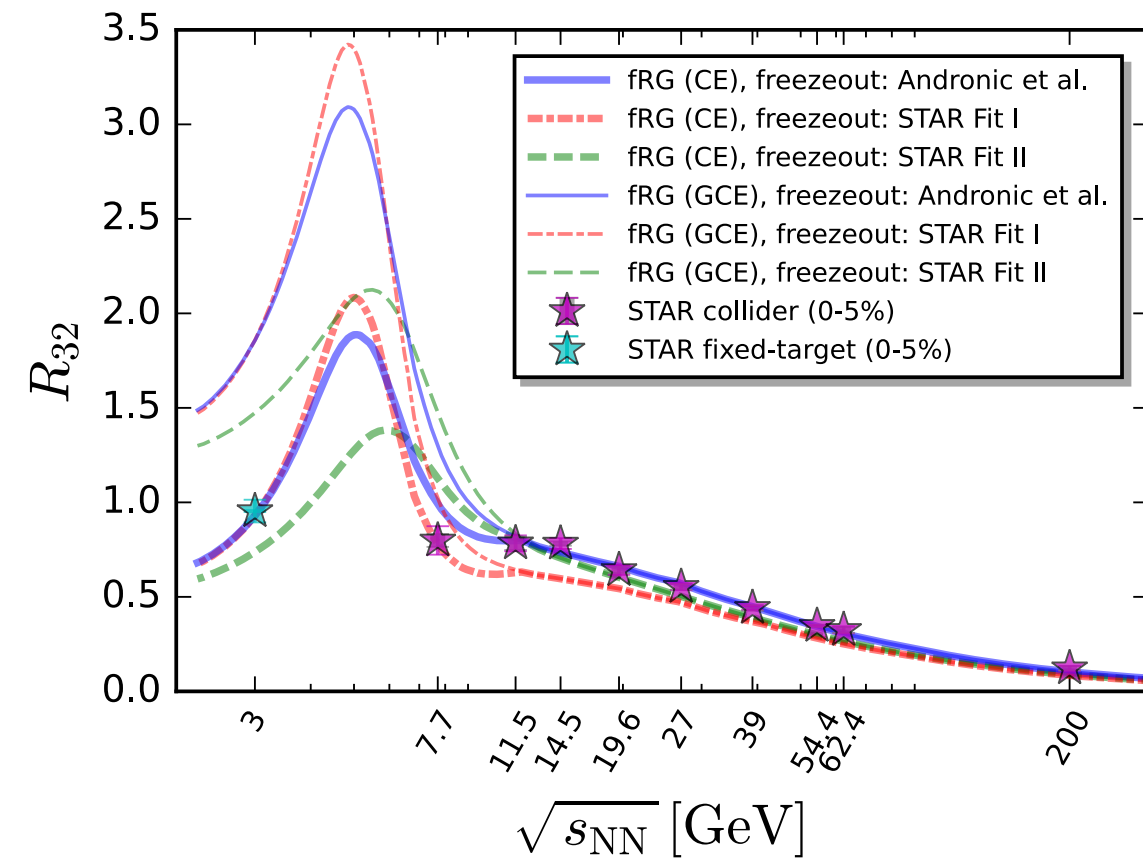
fixing the subensemble volume

baryon & proton number fluctuations

subensemble volume system volume

$$V_1 = \alpha V$$

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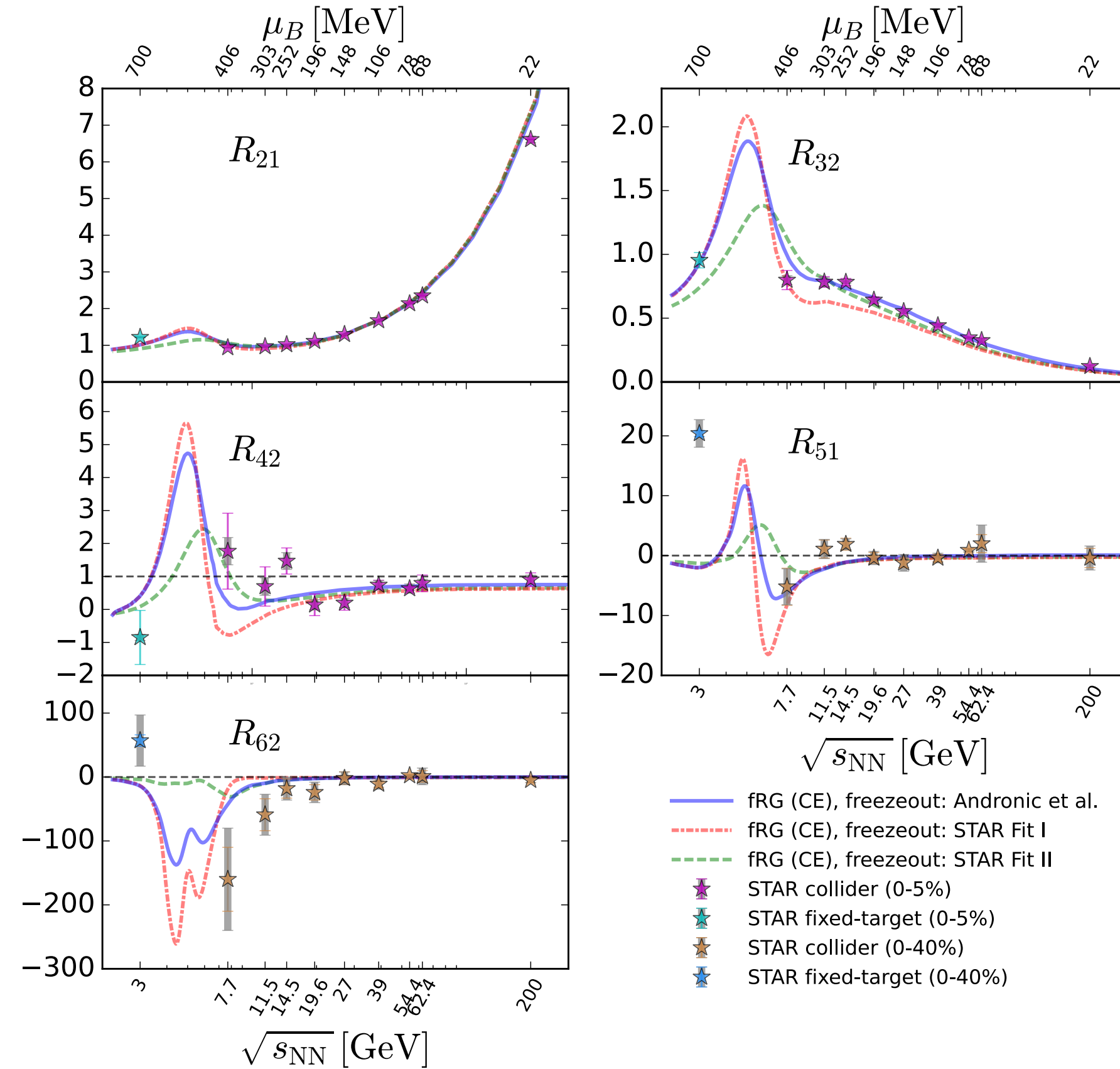


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Ripples of the critical point

Canonical corrections via subensemble acceptance

Vovchenko, Savchuk, Poberezhnyuk, Gorenstein, Koch, PLB 811, 135868 (2020)

fixing the subensemble volume

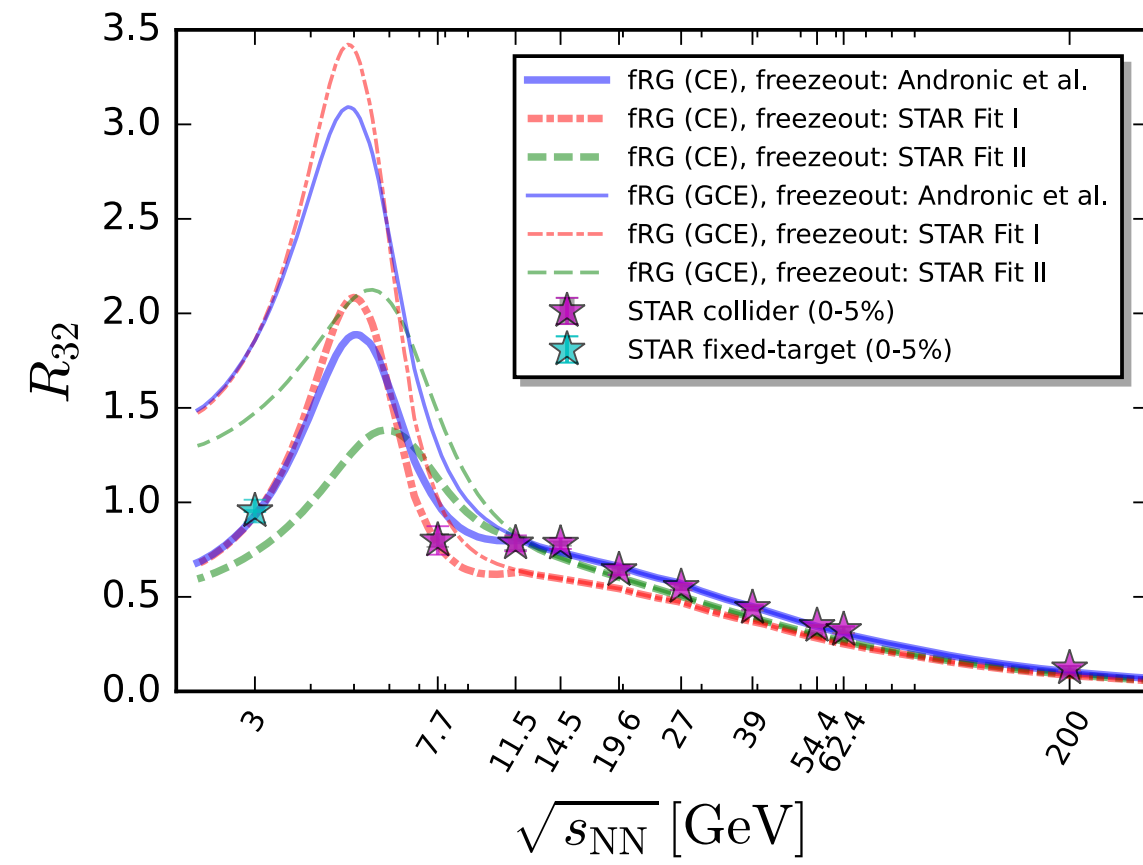
baryon & proton number fluctuations

Sneak preview

subensemble volume system volume

$$V_1 = \alpha V$$

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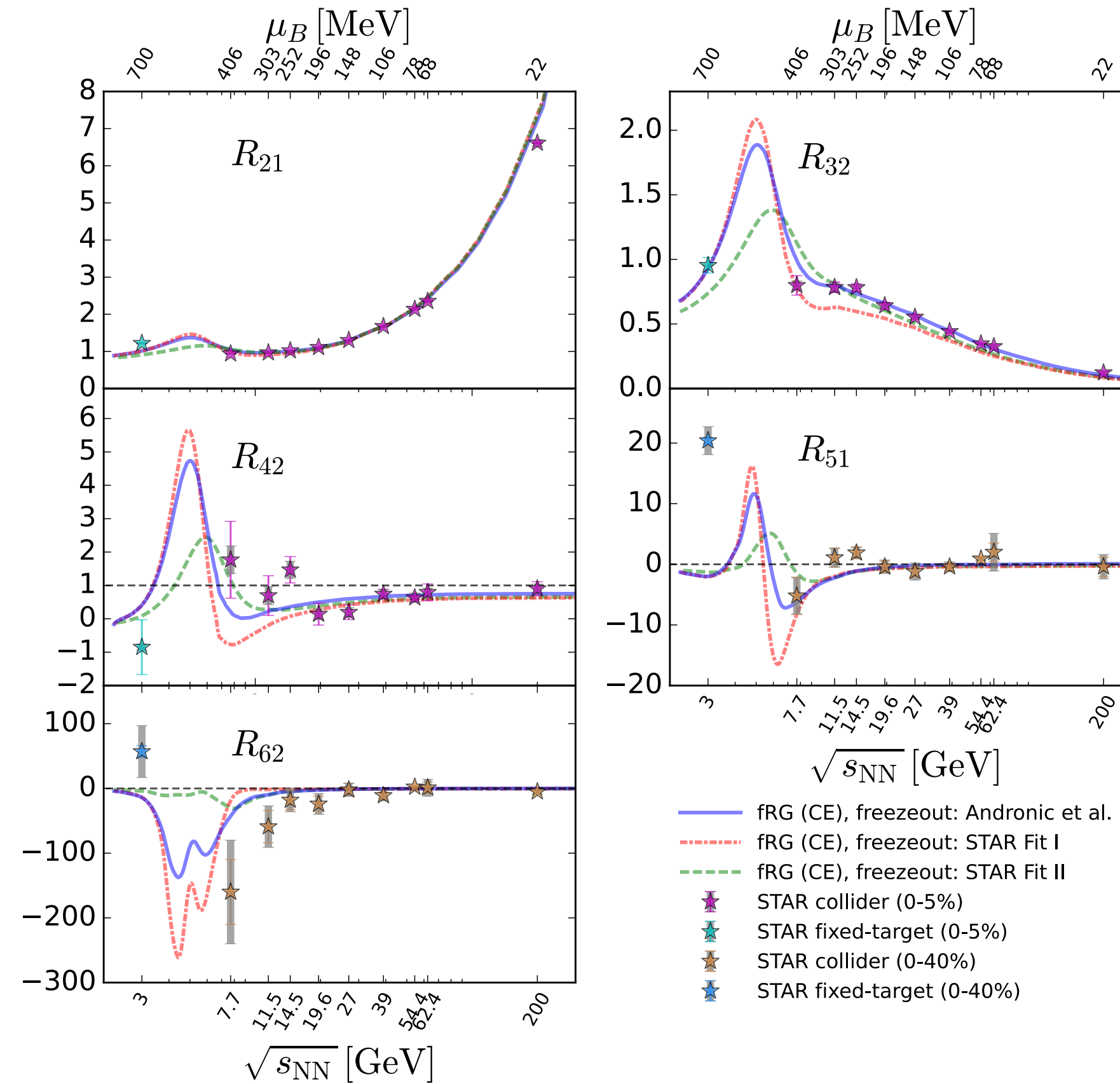


qualitative adjustment

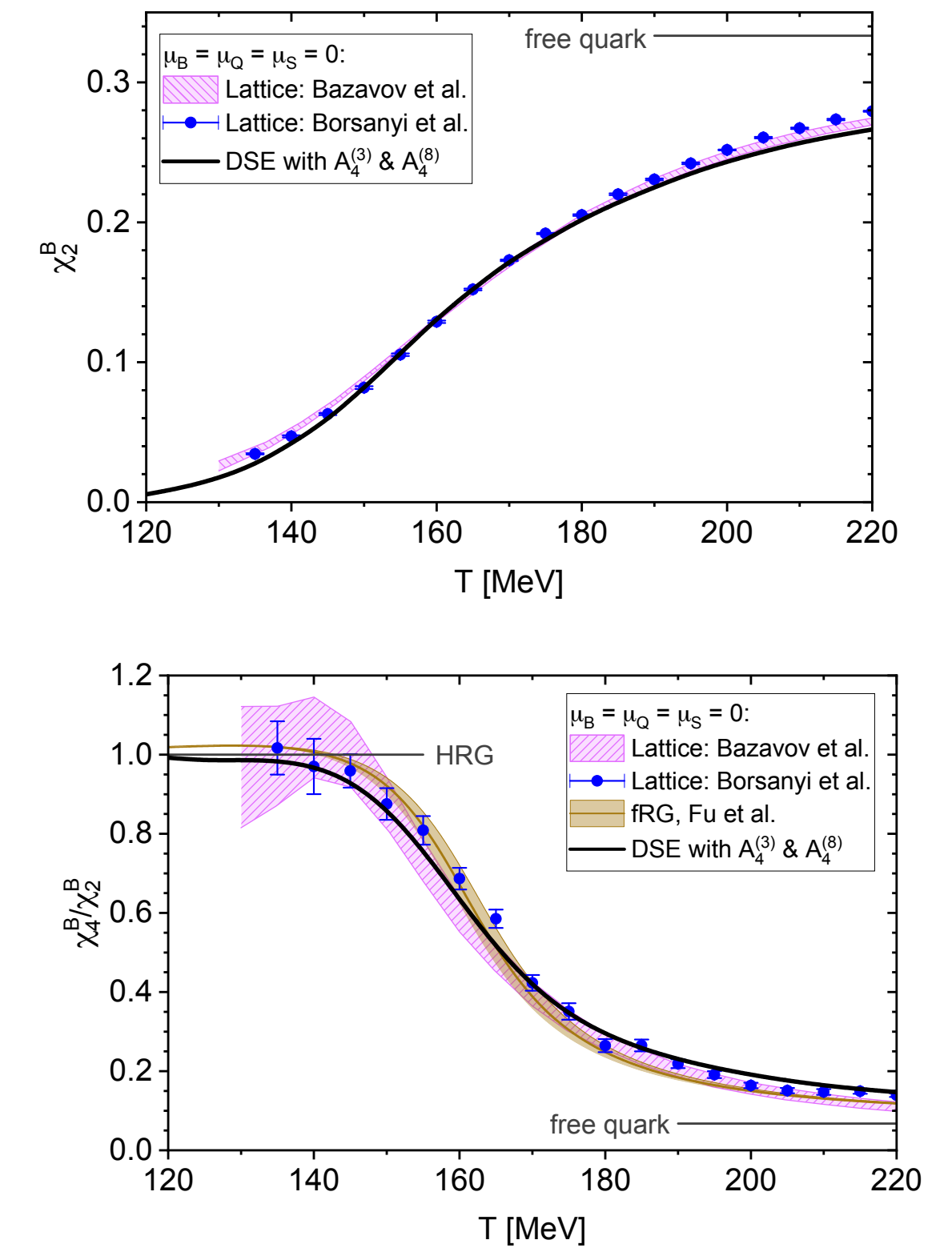
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$$a = 0.33$$

$$\sqrt{\bar{s}} = \frac{\sqrt{s_{NN}}}{11.9 \text{ GeV}}$$

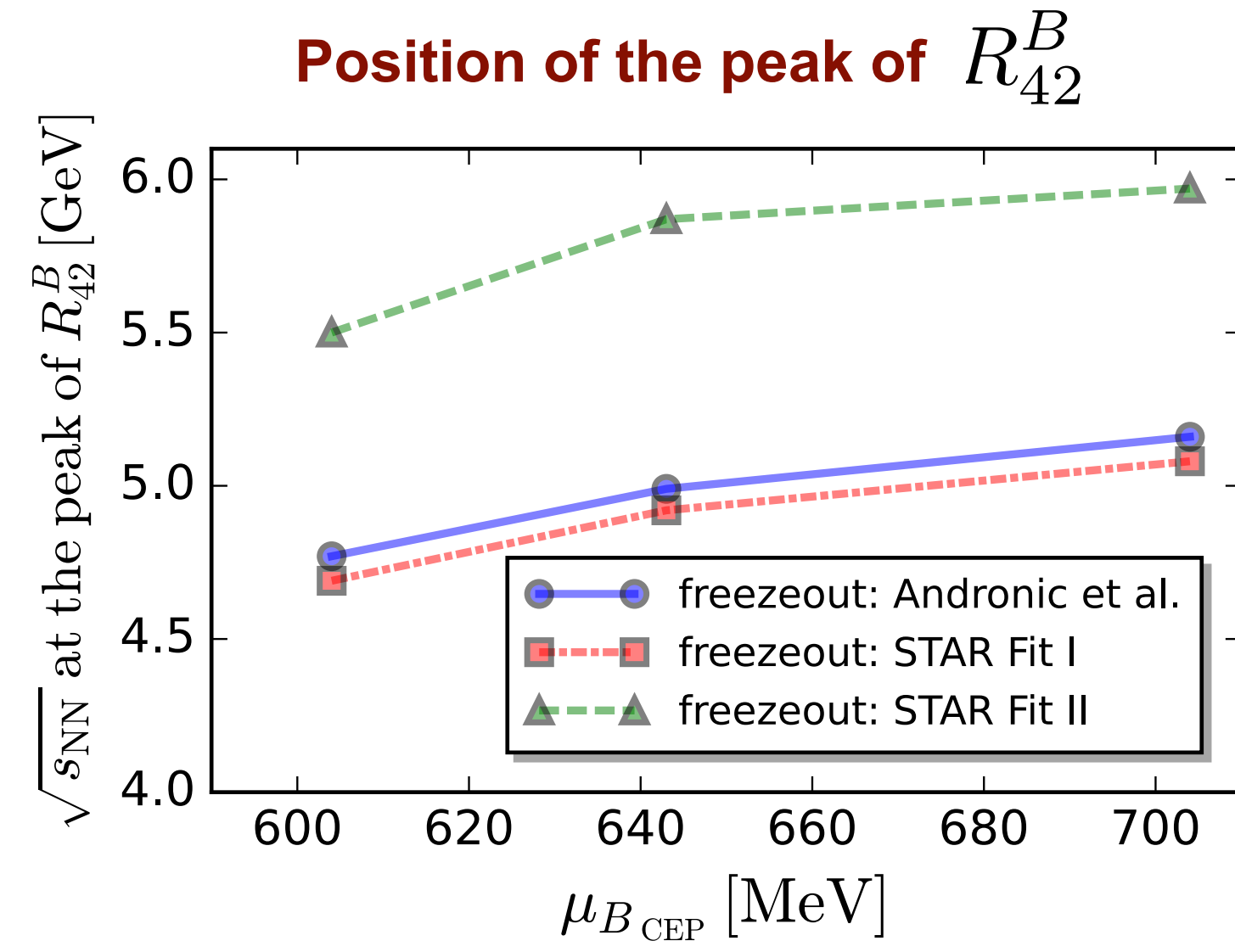


baryon fluctuations with functional QCD

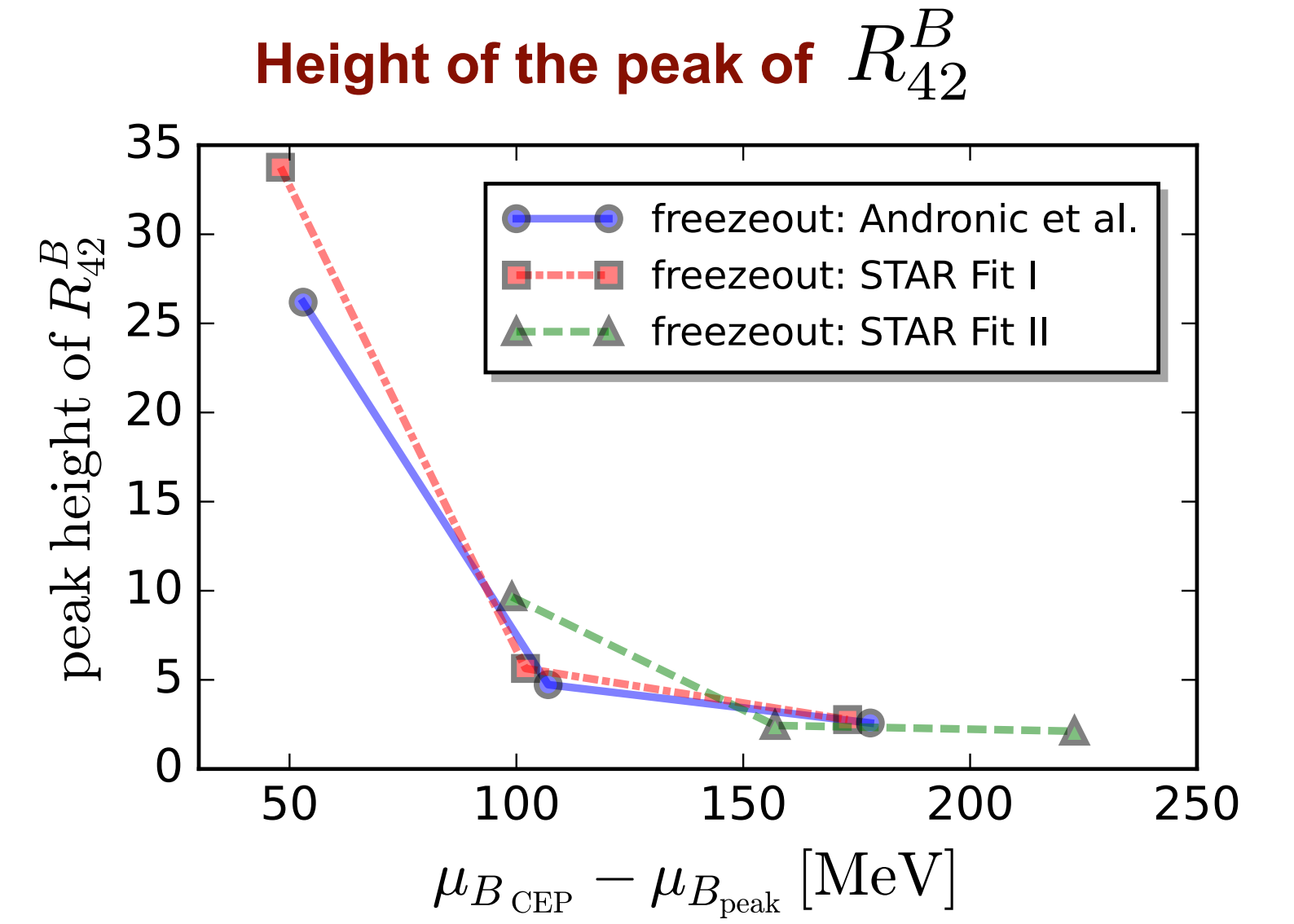


Gao, Liu, Lu, JMP, in preparation

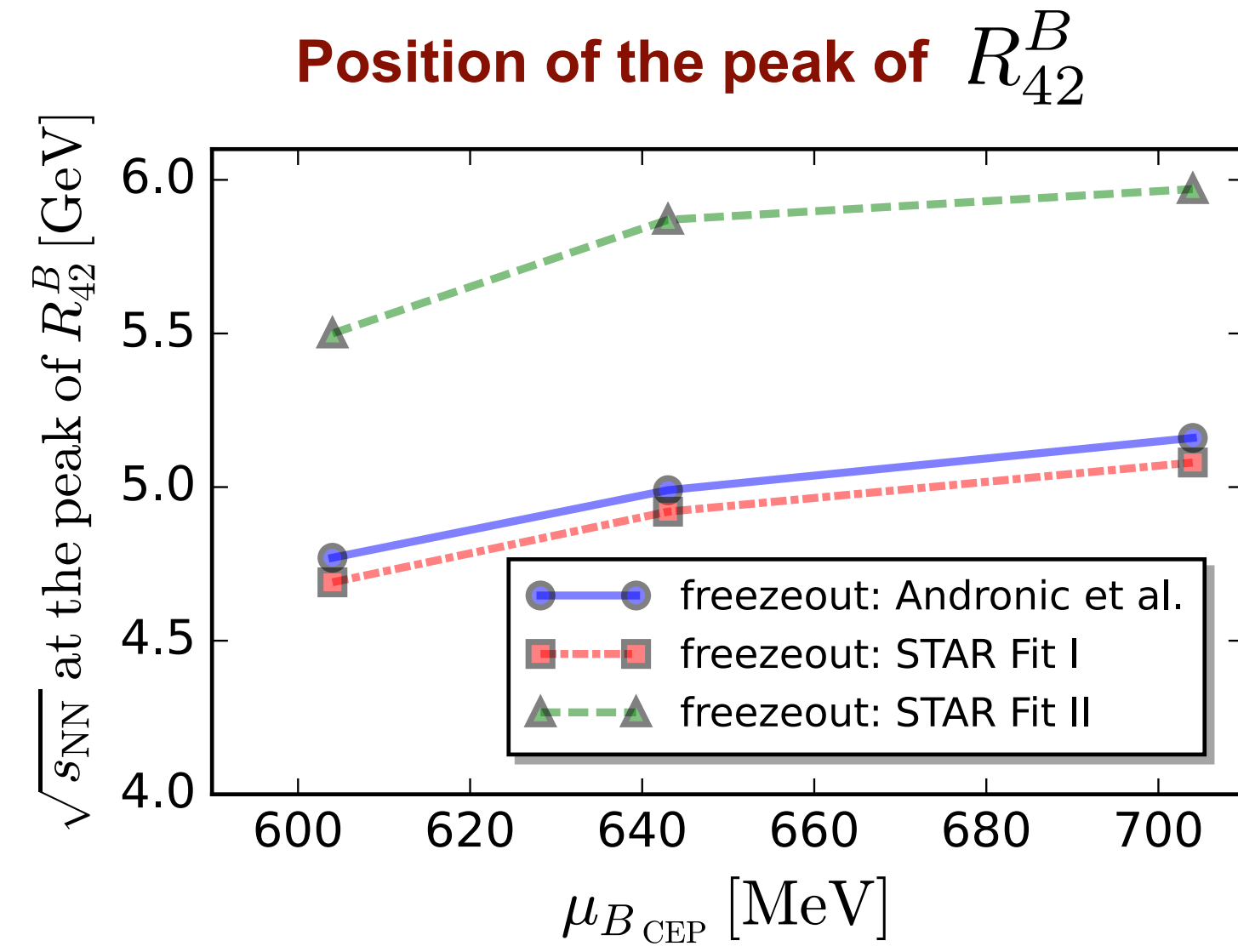
Ripples of the critical point



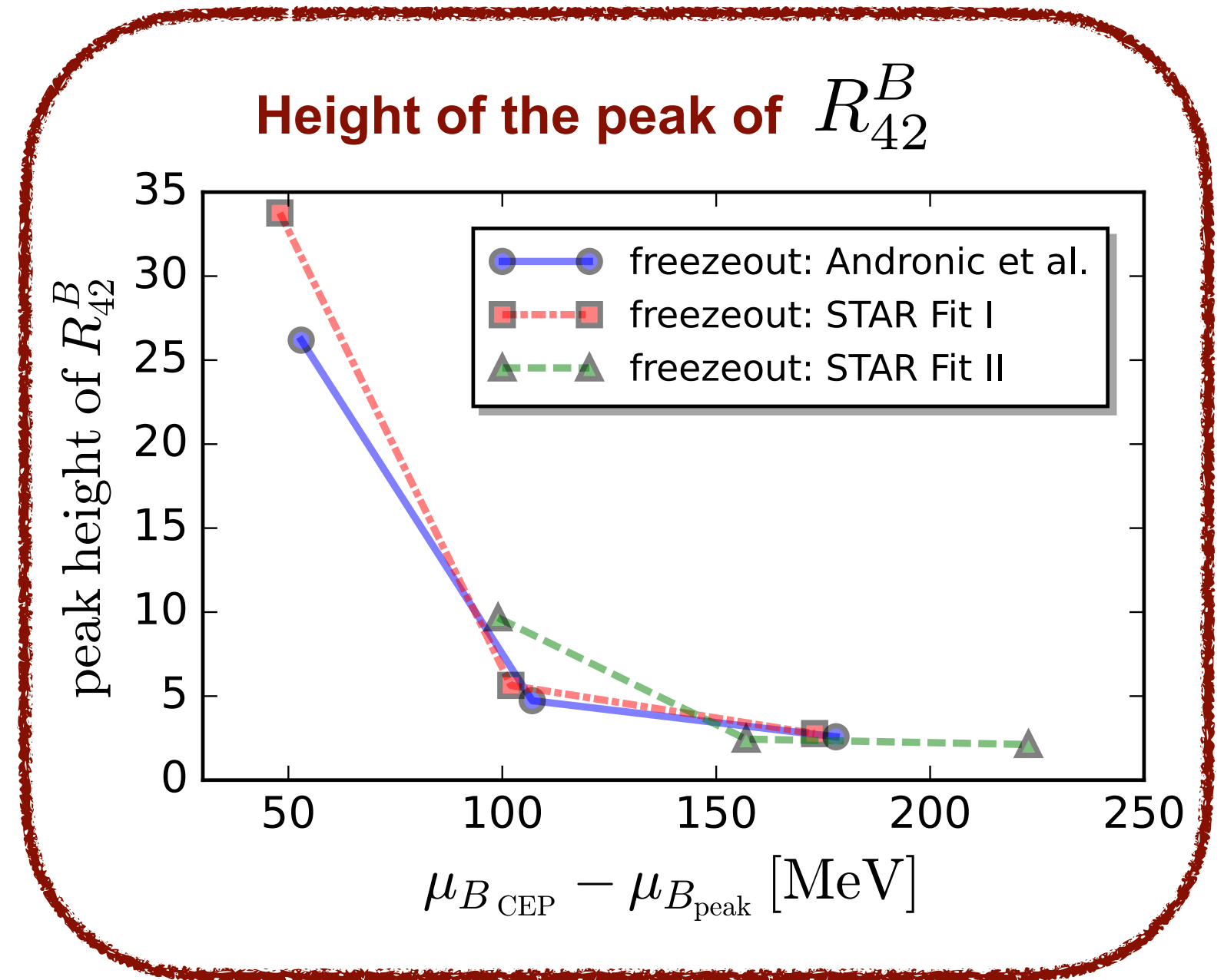
Reconstructing the CEP



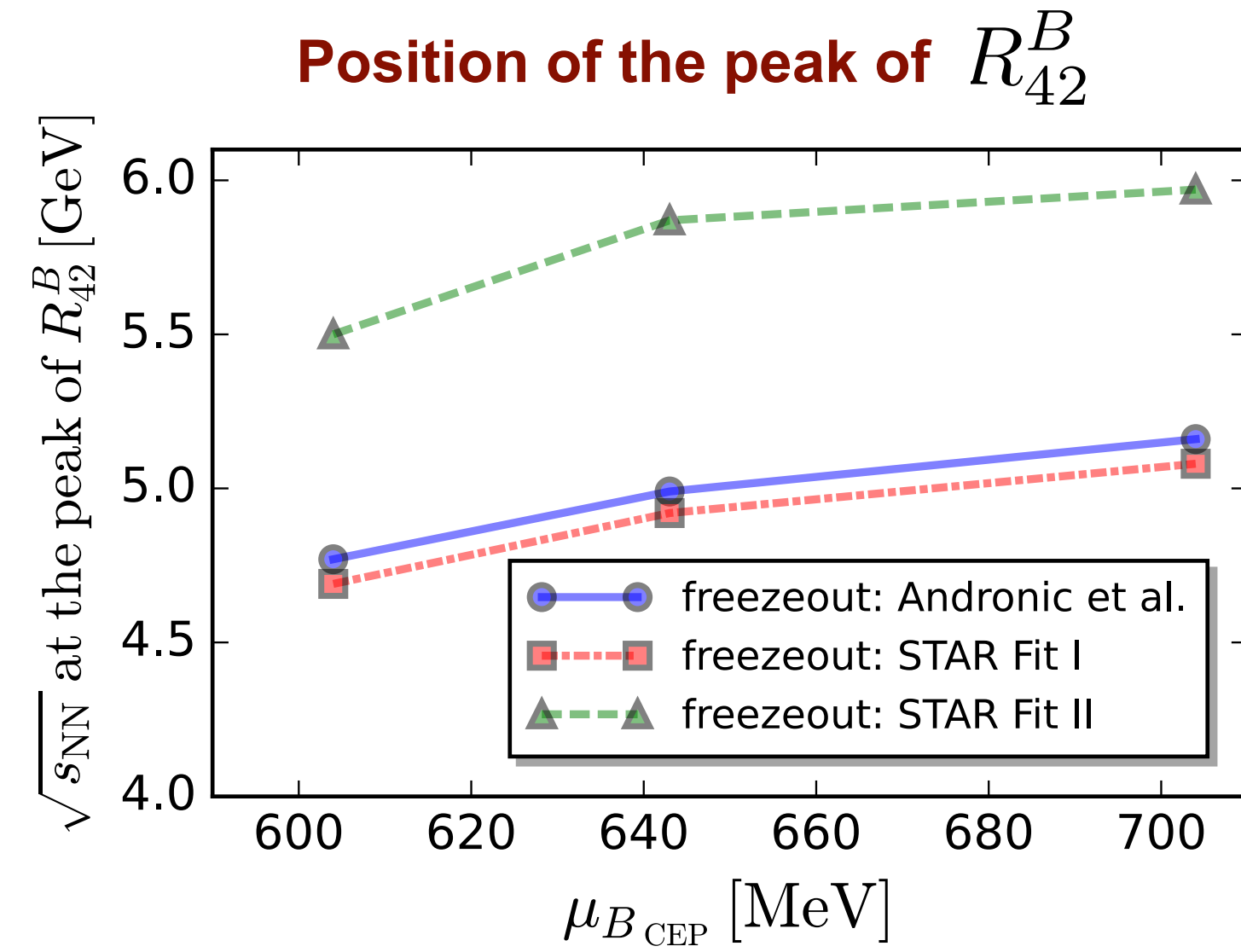
Ripples of the critical point



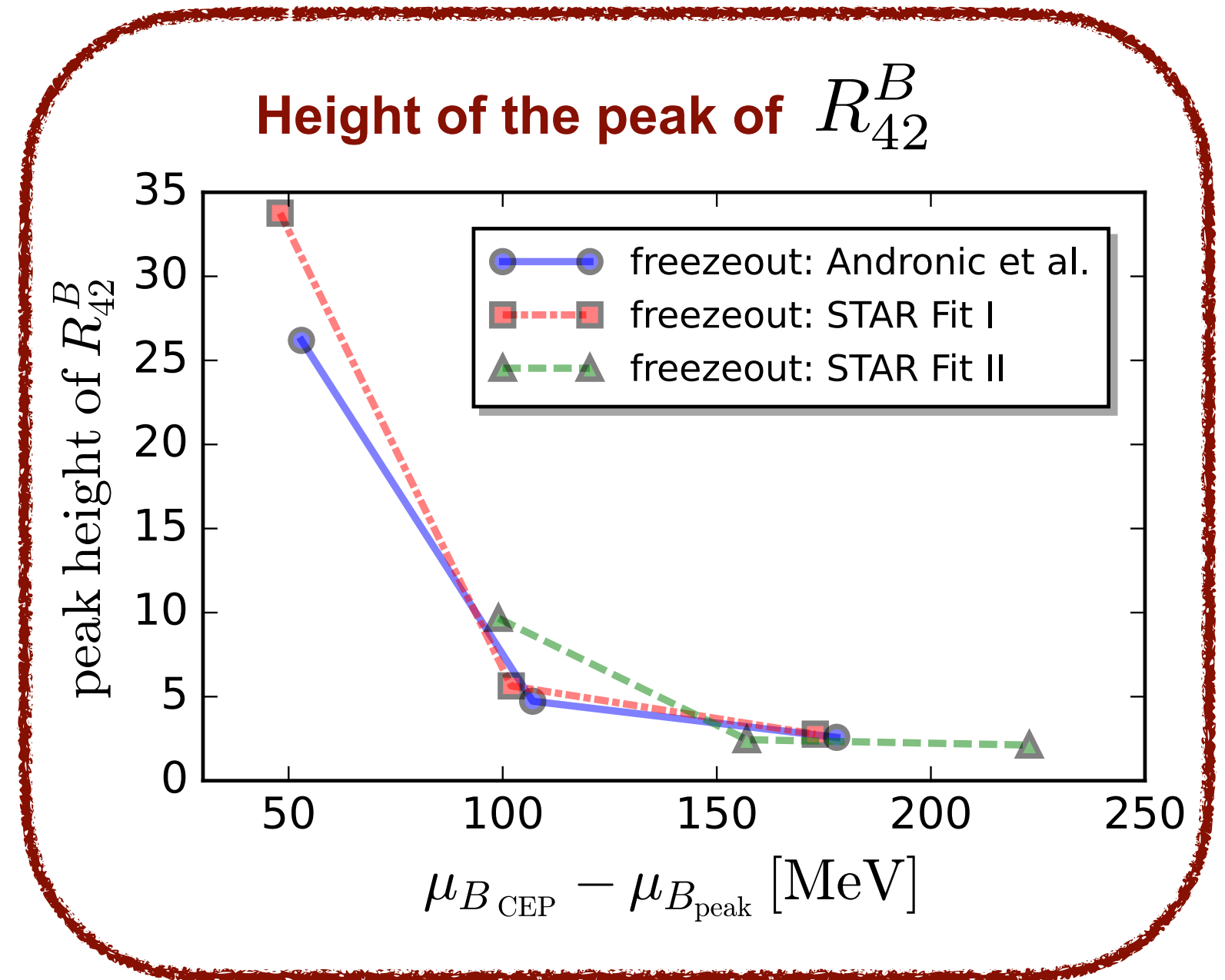
Reconstructing the CEP



Ripples of the critical point



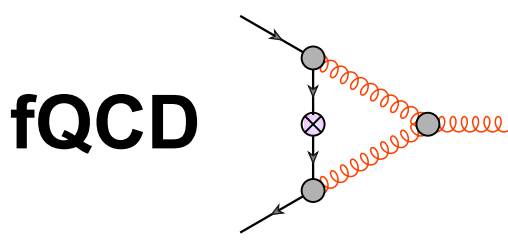
Reconstructing the CEP



Unfolding the high density regime with new phases & physics

Great opportunity for a combined high precision analysis of high density QCD (Exp. data + lattice QCD + functional QCD)

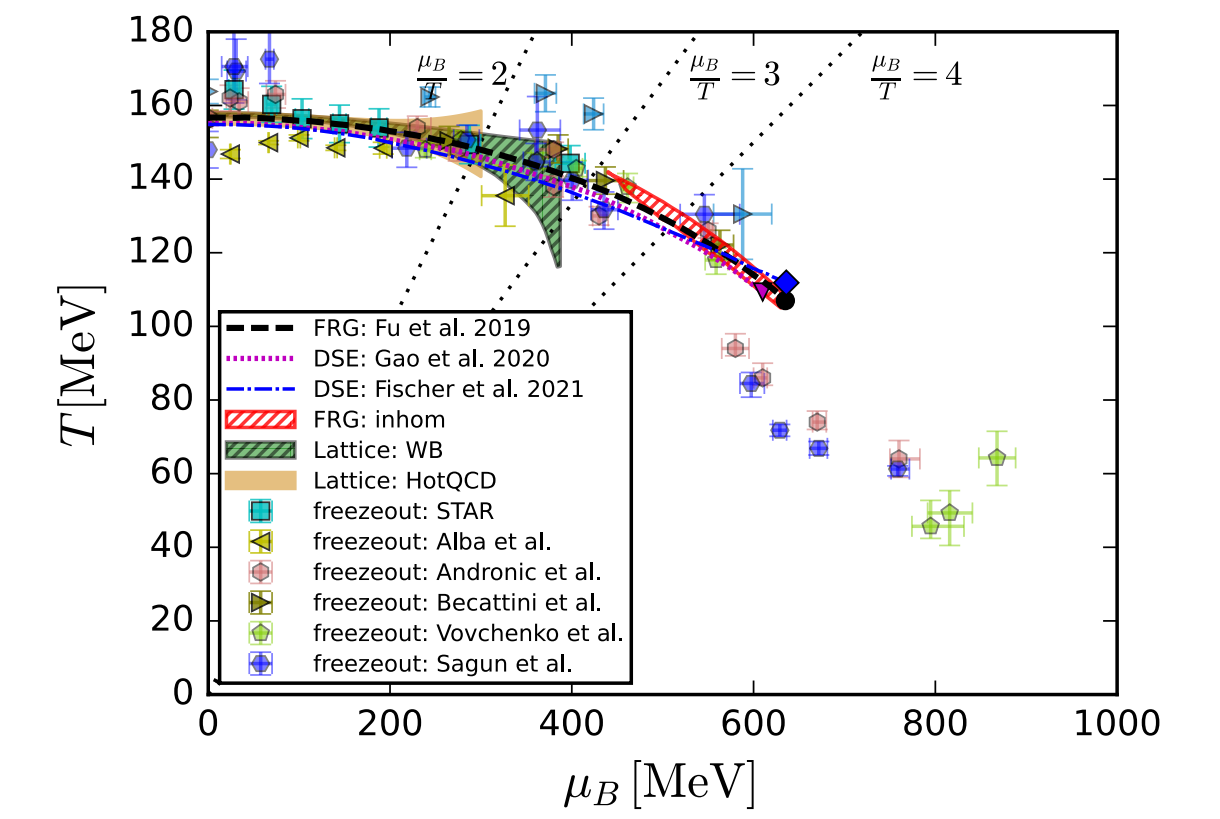
Summary



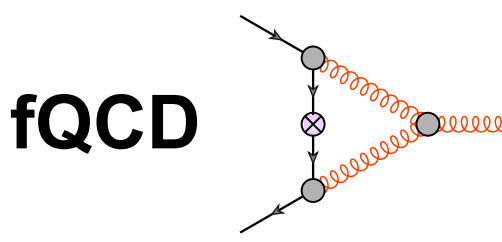
- Functional QCD provides direct 1st principle results for the phase structure at finite density

- Predictions: $\frac{\mu_B}{T} \lesssim 4$

- Estimates: $4 \lesssim \frac{\mu_B}{T} \lesssim 8$



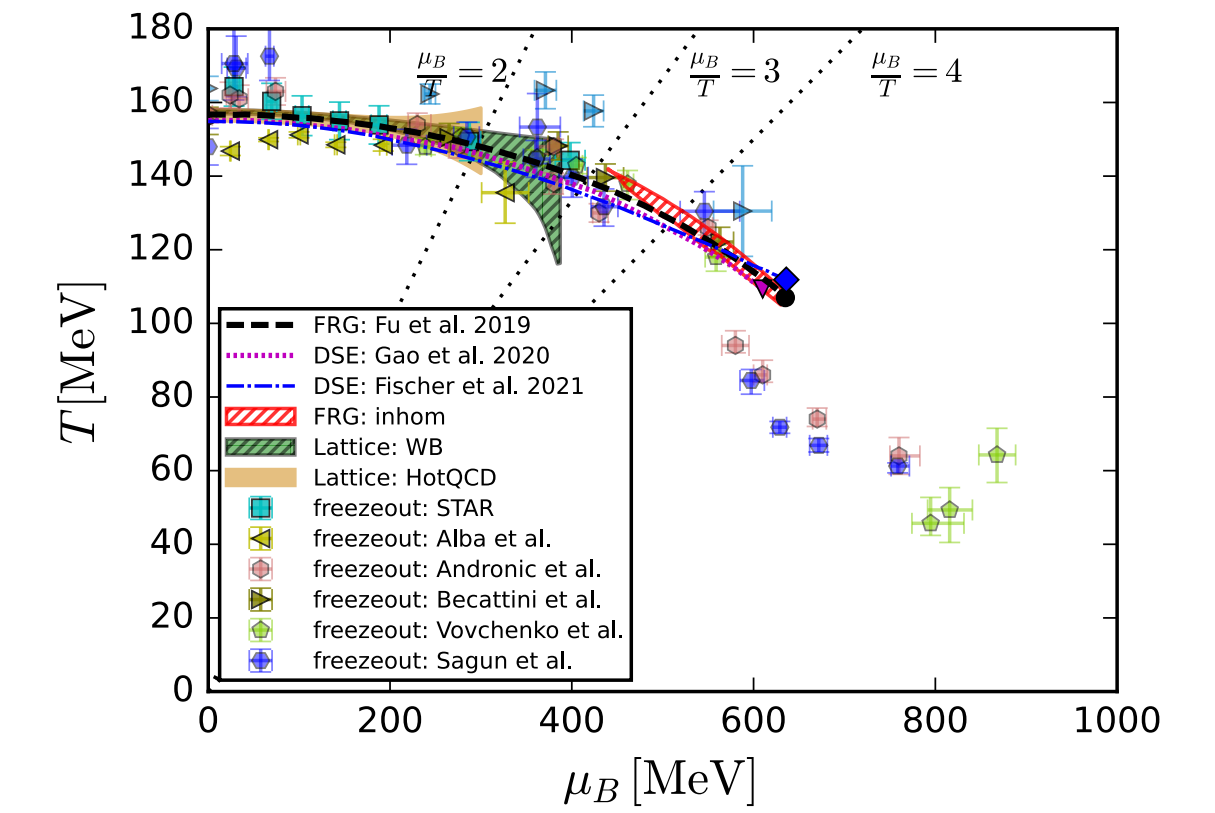
Summary



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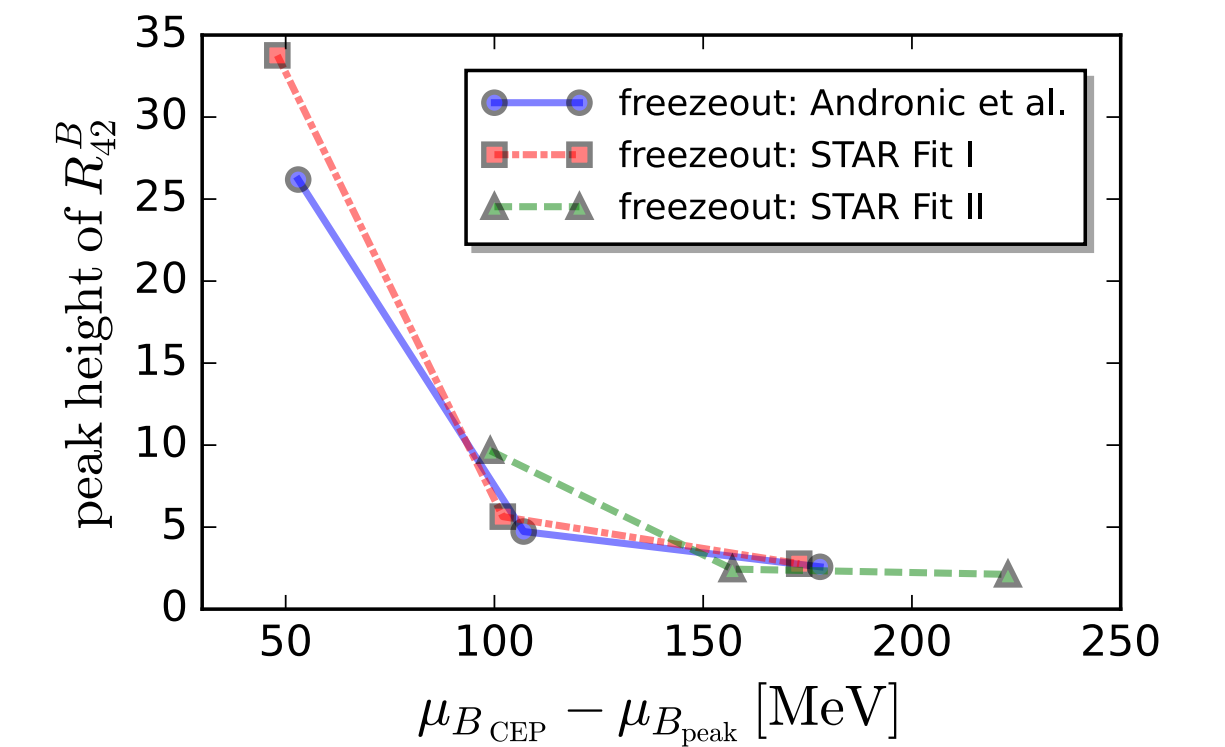


- Functional QCD results support the use of low energy effective theories for phenomenological applications

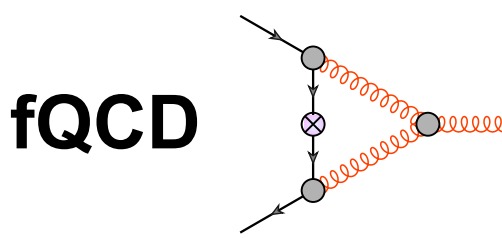
- Explanation for the convergence of CEP locations of extrapolation approaches

- Likely scenario: tiny critical regime

- Access to observables such as fluctuations of conserved charges



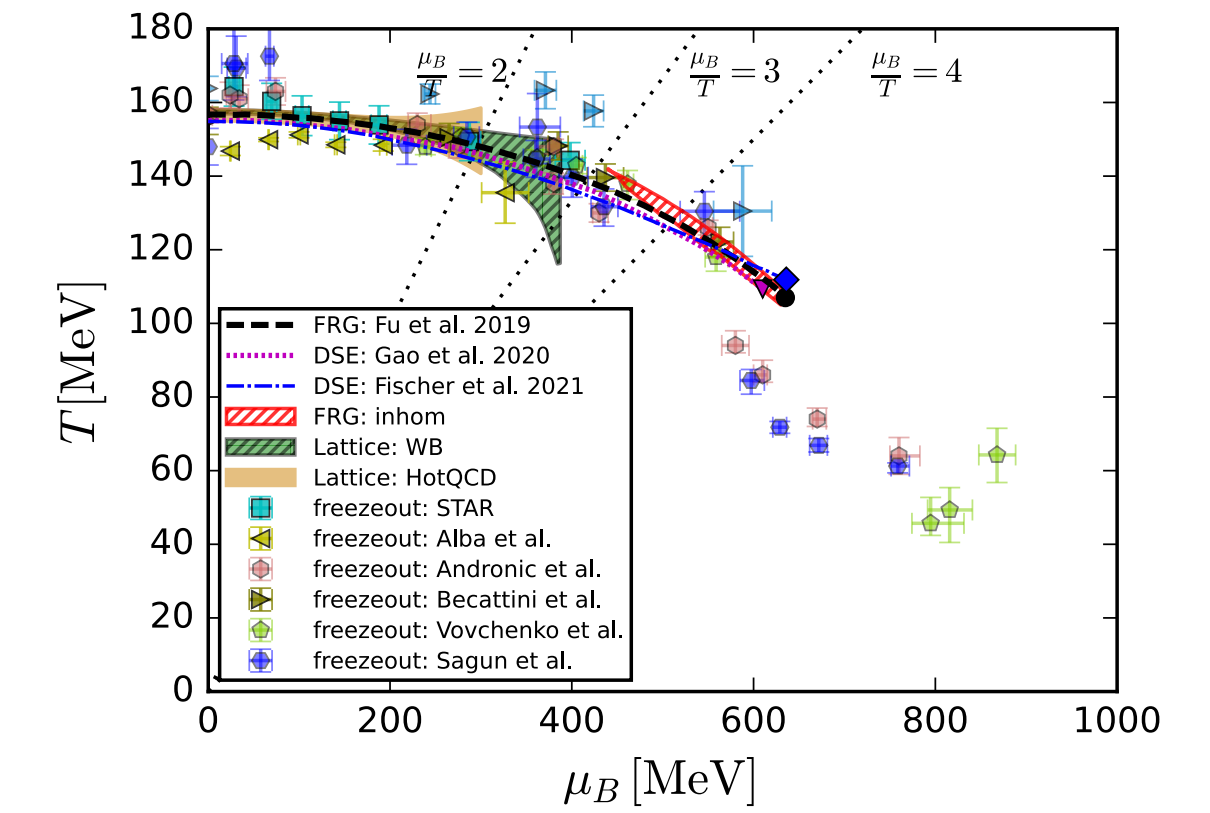
Summary



- Functional QCD provides direct 1st principle results for the phase structure at finite density

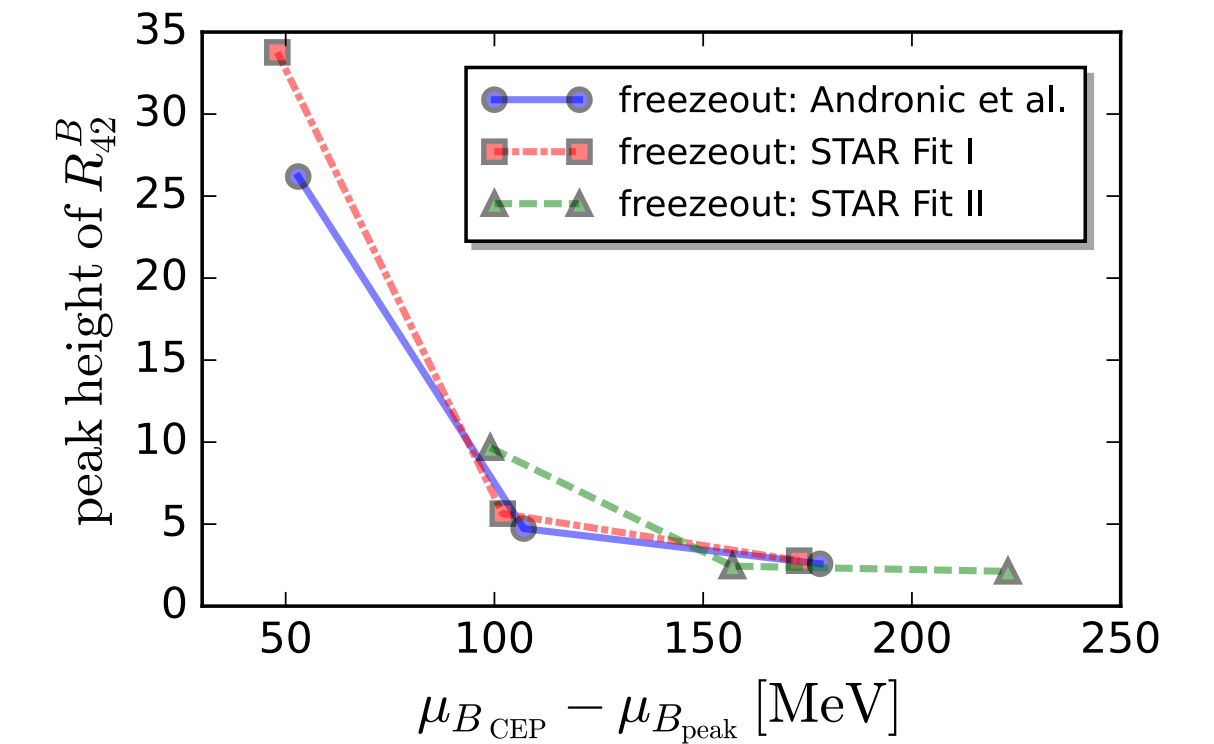
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- Functional QCD results support the use of low energy effective theories for phenomenological applications

- Explanation for the convergence of CEP locations of extrapolation approaches
- Likely scenario: tiny critical regime
- Access to observables such as fluctuations of conserved charges



- Systematic error estimates with the LEGO[®] principle

CEP Estimate \rightarrow CEP Prediction

Diquarks/baryons: ✓ Density channel/mode: (✓) Moat/inhomogeneous regime: ((✓))

Stay tuned