Aspects of IR Phase in Thermal QCD

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People involved: [various stages/aspects]

Andrei Alexandru (George Washington), Peter Markoš (Comenius), Robert Mendris (Shawnee)

Beijing (Yi-Bo Yang et al), Keh-Fei Liu (Kentucky), Massimo D'Elia (Pisa), Claudio Bonanno (Madrid)

Literature: [with degrees of separation]

0-th	0-th	1-st	1-st	2-nd
1906.08047	2305.09459	1405.2968	hep-lat/0607031	1807.03995
1502.07732	2310.03621	1412.1777	hep-lat/0610121	1809.07249
2103.05607	2404.12298		hep-lat/0703010	2110.11266
2110.04833			0803.2744	2205.11520
				2207.13569
				2212.09806
basics	new evidence (thermal)	evidence (large N_f)	$< F^2 >$ & Dirac spectrum	math tools

Technical credits: Dimitris Petrellis

Setup: μ_B=0

Asymptotically Free SU(3) Gauge Theories w Fundamental Quarks

theories $\mathcal{T}$: $$S = -\frac{1}{2g^2}\,\mathrm{tr}\,F_{\mu\nu}F_{\mu\nu} + \sum_{f=1}^{N_f}\bar{\psi}_f(D+m_f)\psi_f$$

For AF theories we know how to take the continuum limit.

Consider these at arbitrary temperature T , $N_f < 16.5$, m_f .

Important to understand phases/types of dynamics: QCD lives here, conformal FTs etc

Standard symmetry approach <u>very limited</u>:

- ❑ multiple massless flavors: flavored chiral symmetry considerations
  flavor-singlet axial anomaly considerations
- ❑ massless flavors: scale invariance/anomaly considerations
- ❑ no/infinitely heavy flavors: scale invariance/anomaly and center considerations

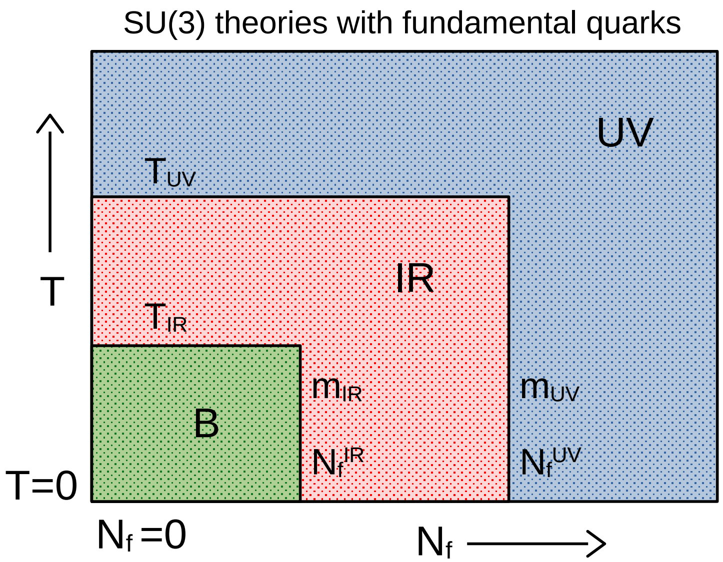
But physics of phases moved on since 60-s, 70-s and 80-s! Topological phases & more.

PERHAPS WE SHOULD DO THINGS/THINK DIFFERENTLY?

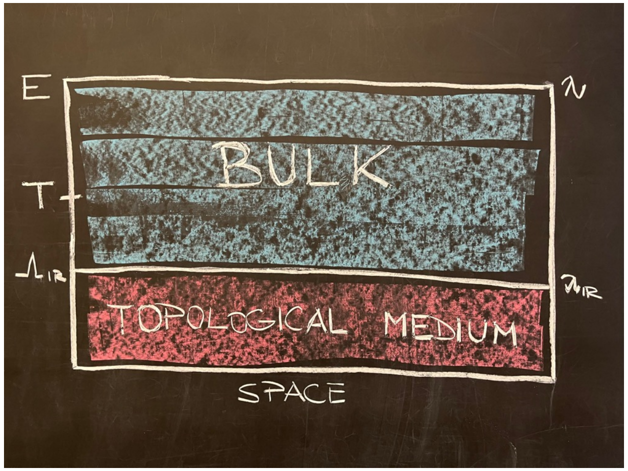
HOW TO EXPLORE ENTIRE $\mathcal{T}$?

Phases <u>without</u> the Symmetry (but also with)

AA & IH 1906.08047, 1502.07732



IR phase



PHASE TRANSITIONS IN $\mathcal{T}$:

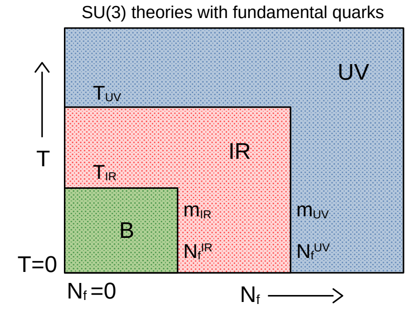
TRANSITIONS BETWEEN SINGLE-COMPONENT AND MULTI-COMPONENT SYSTEM

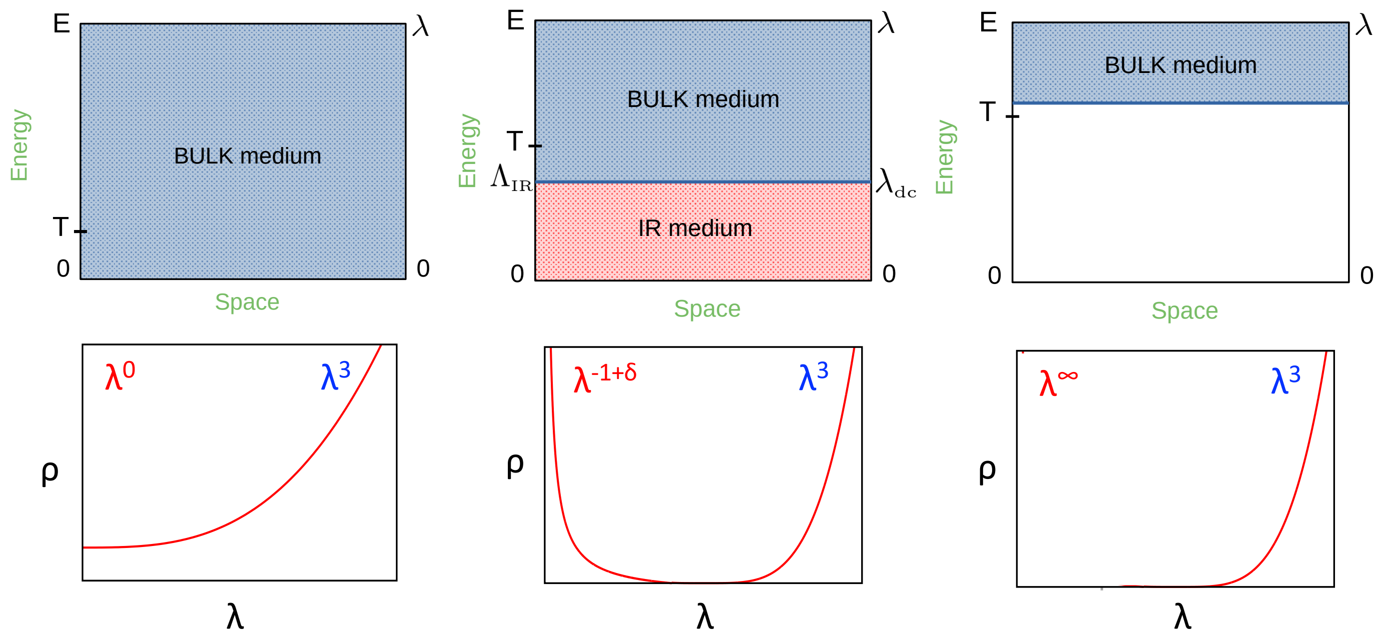
No symmetry involved per se. But hold on, it will appear in a new light…

- IR Phase has <u>IR medium</u> and independent <u>bulk</u>
- COMPONENTS SEPARATED IN DIRAC SPECTRUM
- Reminiscent of Tisza/Landau 2-component theory of liquid helium

  Physics elements appear to be dual to each other though.

Phases without the Symmetry (but also with)…





1906.08047

Dirac spectral density:

$$D\psi_\lambda = i\lambda\,\psi_\lambda \quad , \quad D = D[A] \qquad \rho(\lambda, V) \equiv \frac{\#\,\text{modes near}\,\lambda}{V\,\mathrm{d}\lambda}$$

1) DISTRIBUTION OF QUARK DOFs ACROSS ENERGY-LIKE SCALES
2) GAUGE-INVARIANT SCALE-DEPENDENT GLUE OPERATOR

Phases without the Symmetry (but also with)…

AA & IH 1906.08047

$$\text{phase} \;=\; \begin{cases} \text{B} & \text{if} \quad p = 0 \\ \text{IR} & \text{if} \quad p < 0 \\ \text{UV} & \text{if} \quad p > 0 \end{cases} \qquad \rho(\lambda) \propto \lambda^p \;, \quad \lambda \to 0$$

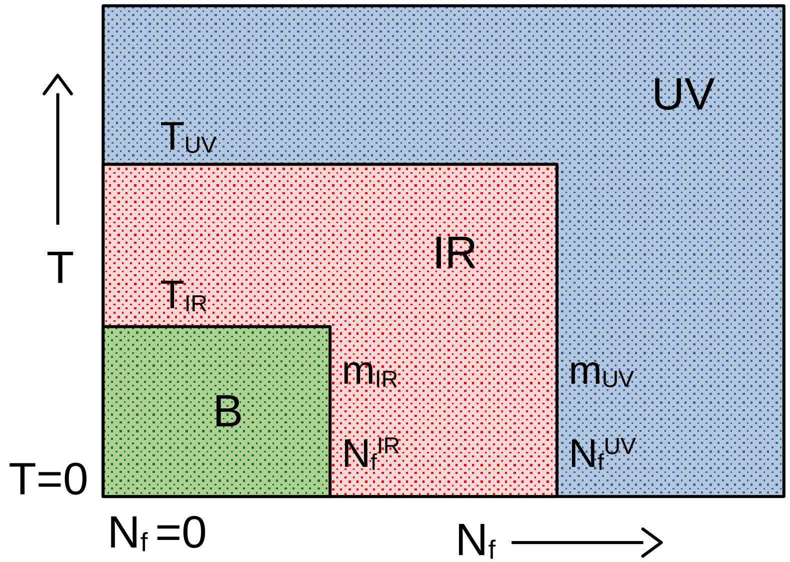
B = IR scale-broken

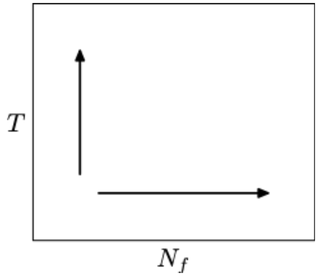
IR = IR scale-symmetric

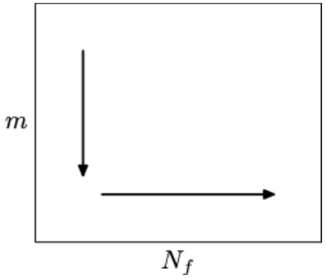
UV = IR trivial

$\rho(\lambda)$ = Dirac spectral density

SU(3) theories with fundamental quarks







Changes consistent with directions of arrows can induce transitions from B→IR or from IR→UV.

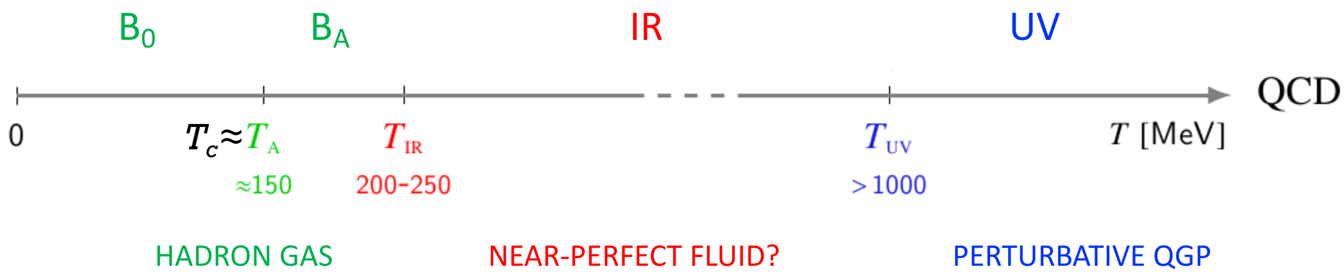
See also 1502.07732

- Most known detail comes from B→IR thermal case: IR PHASE OF THERMAL QCD
- Important also B→IR light-flavor case (T=0): IR PHASE = STRONGLY COUPLED PART OF CONFORMAL WINDOW

  1405.2968, 1412.1777, 1906.08047
- No hard evidence for traditional UV phase!

Phases without the Symmetry (but also with)… QCD thermal case

AA & IH 1906.08047

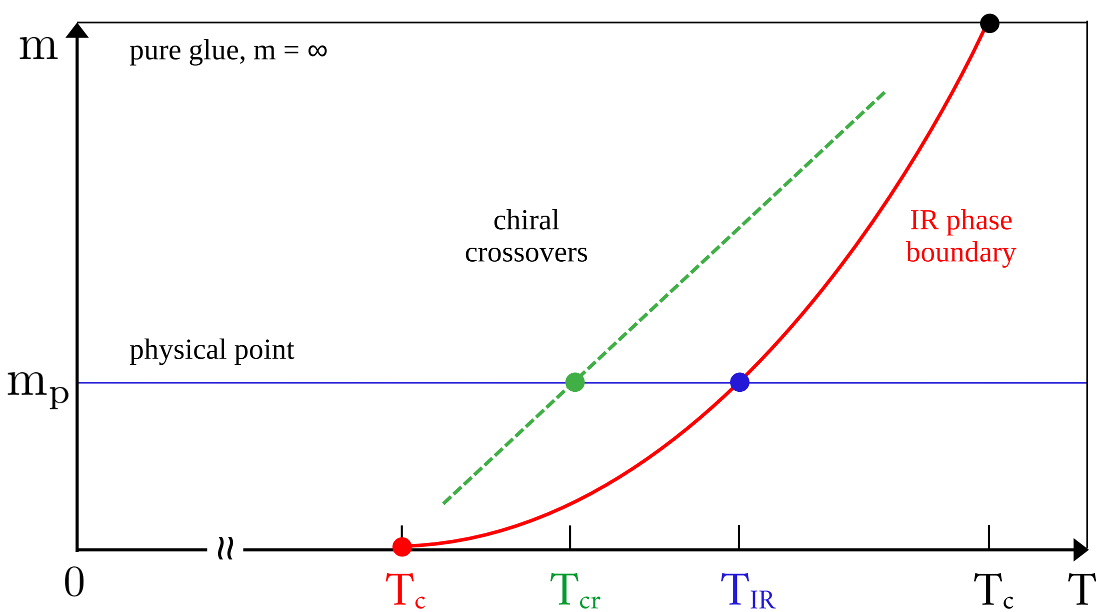


N_f = 2+1
physical point
staggered quarks
overlap Dirac probe

$$T_c \quad < \quad T_{IR} \quad < \quad T \quad < \quad T_{UV}$$

≈155 MeV 200-230 MeV perturb

2404.12298, 2305.09459



real-world QCD, such as

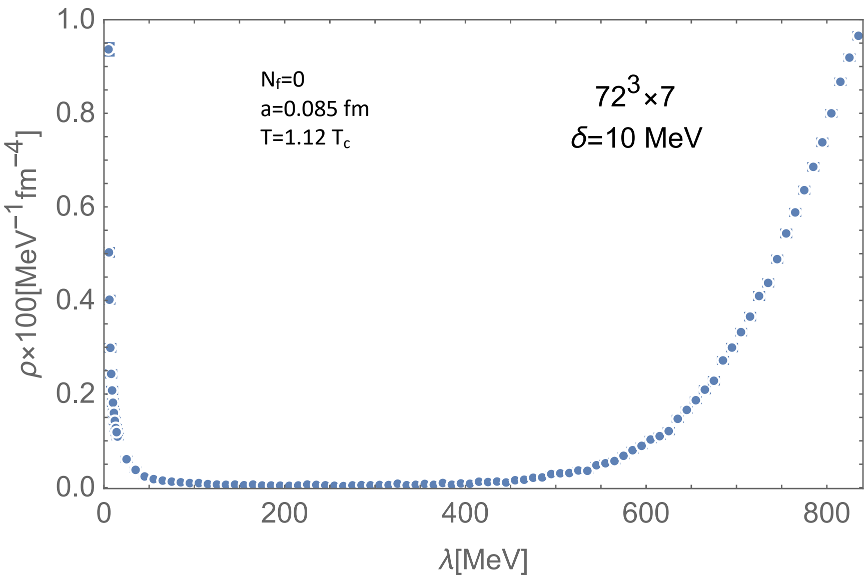
N_f = 2+1 , quark mass m

IR phase trends from

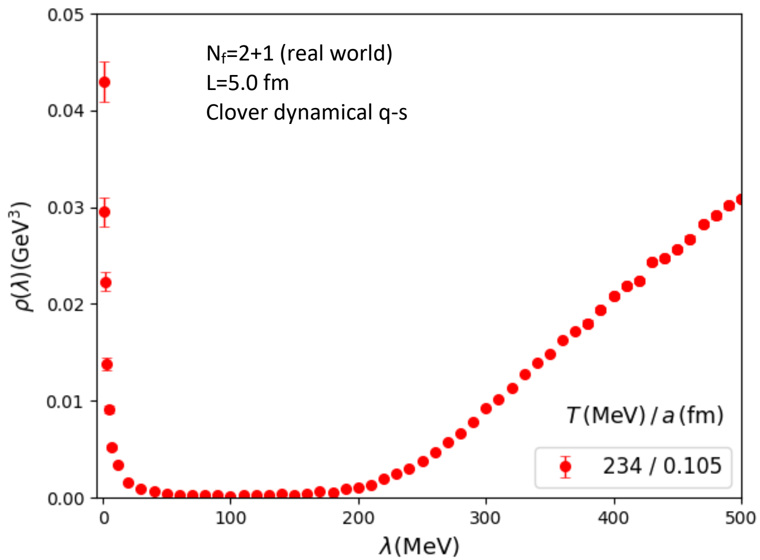
AA & IH 1502.07732

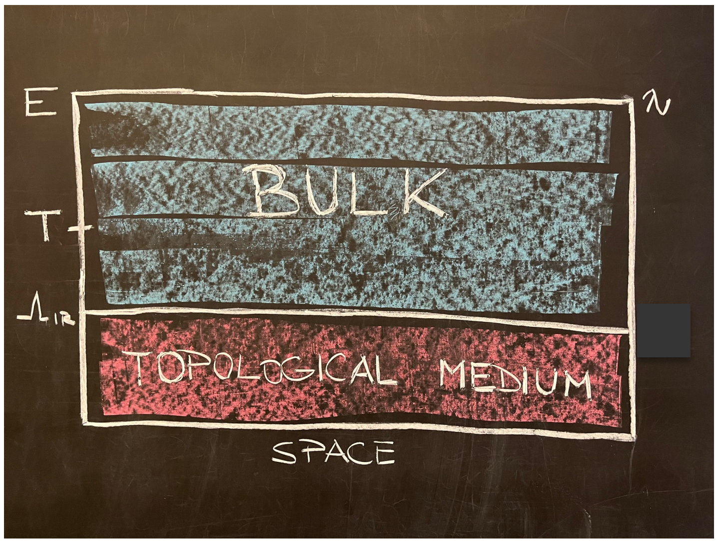
Phases without the Symmetry (but also with)… QCD thermal case

AA & IH unpublished



X. Meng et al 2305.09459





- **IR medium decouples from bulk**

quark-gluon thermal medium becomes 2-component

- **Glue of IR medium is scale invariant**

IR phase transition = restoration of IR scale invariance

Hypothesis:

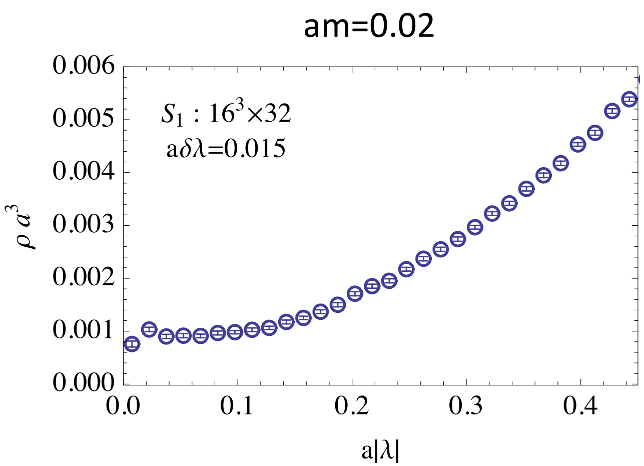
IR phase describes the near-perfect fluid [RHIC, ALICE] state of matter.

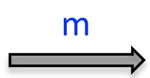
AA & IH 1906.08047

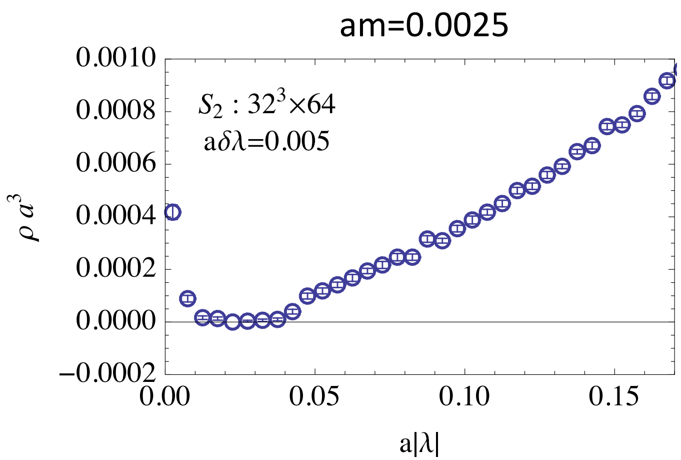
Experimental signatures on ALICE3?

Phases without the Symmetry (but also with)… many flavors

$N_f=12$, $T=0$ Configs: A. Hasenfratz et al, 1207.7162 staggered with nHYP AA & IH 1405.2968 1411.1777





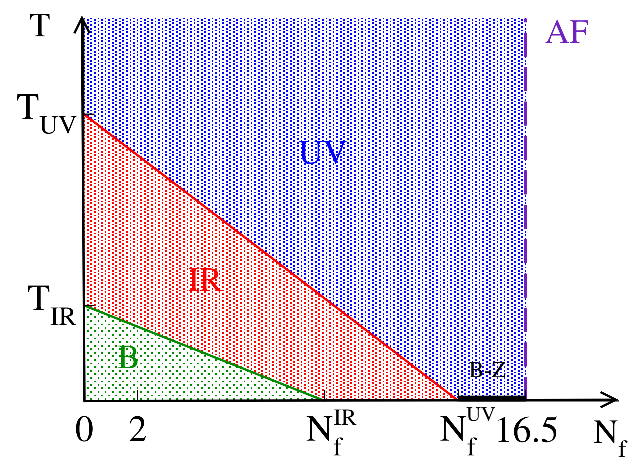


Lowering quark mass at sufficient number of flavors can generate IR phase

Conjecture: AA & IH 1906.08047

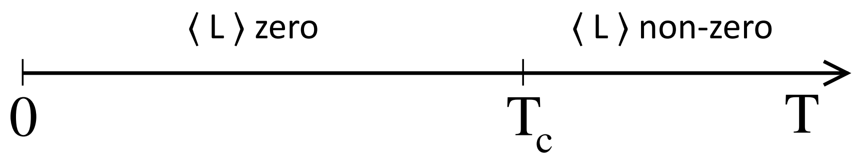
Conformal window has a strongly coupled part with p < 0.

$$N_f^c \equiv N_f^{\rm IR} < N_f < N_f^{\rm UV} \leq 16.5$$

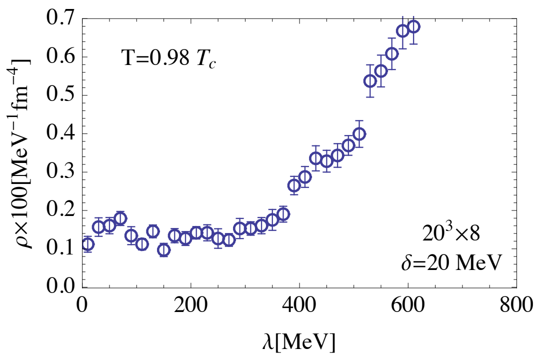


Scale Invariance? How???

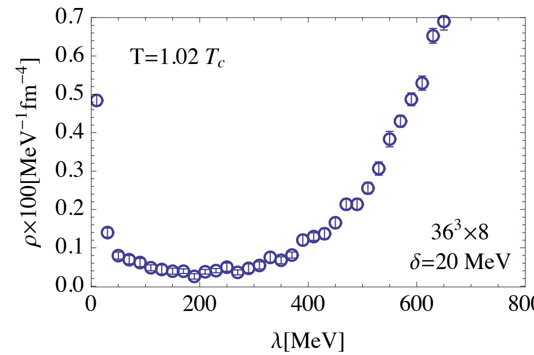
N_f=0 classically scale invariant



⟨ L ⟩ = Polyakov loop
1-st order transition



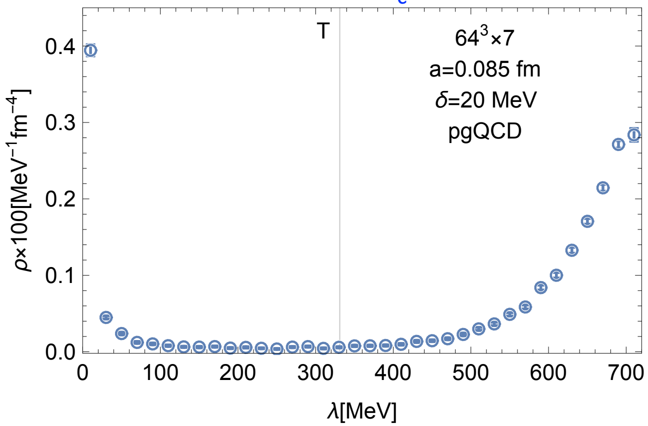
T



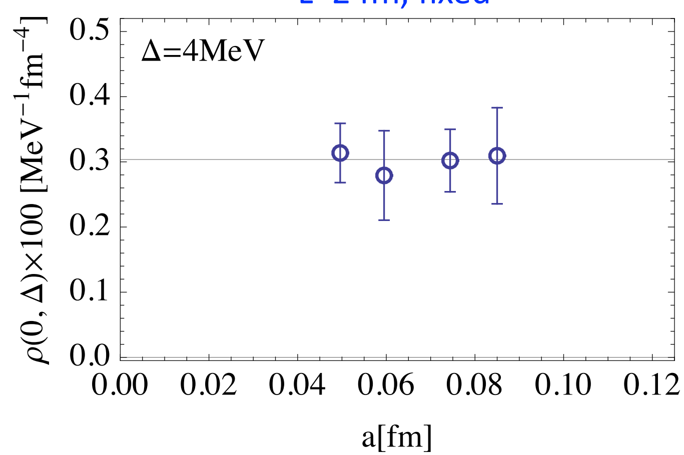
AA & IH
1502.07732

Well, perhaps some sort of an artifact? Edwards et al, hep-lat/9910041

T=1.12 T_c



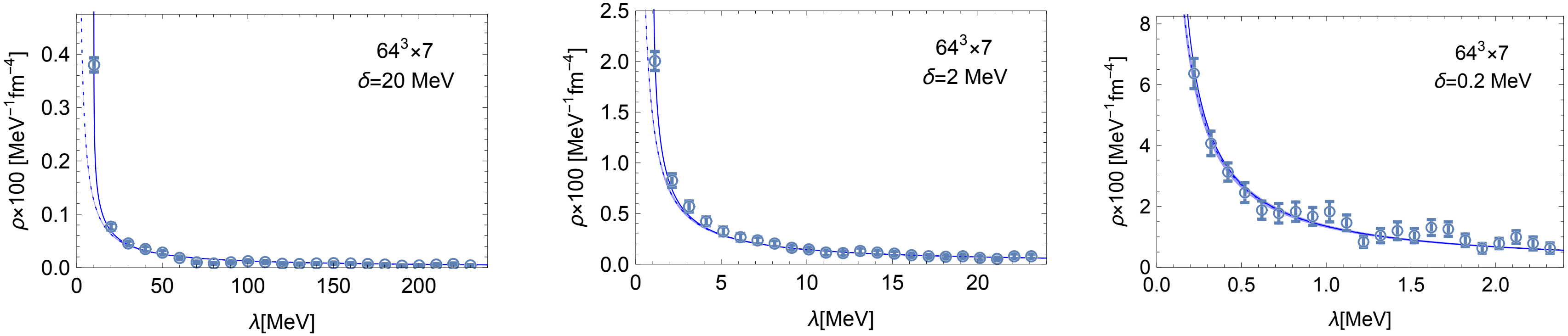
L=2 fm, fixed

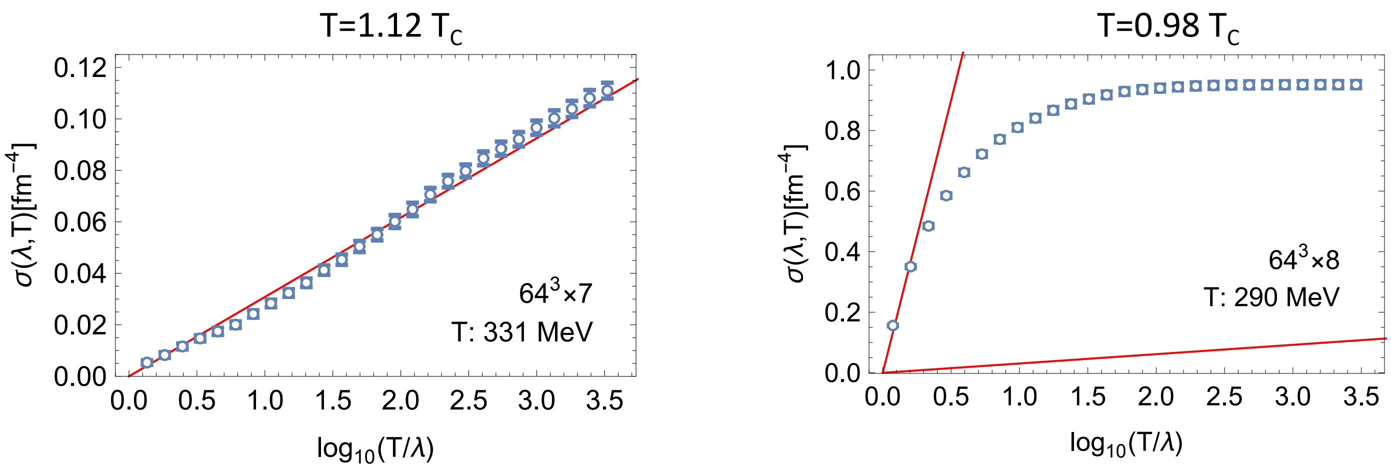


No artifact! Strength of IR peak scales! 1502.07732

Scale Invariance? How???...

Fits to $\rho(\lambda) \propto 1/\lambda$ [N_f=0, T=1.12 T_c] AA & IH 1906.08047





Data: (1) IR SCALE-INVARIANT DENSITY (λ < T) OVER 3 ORDERS OF MAGNITUDE IN SCALE

(2) NEGATIVE POWER-LAW ACCUMULATION OF DIRAC MODES IN IR: $\rho(\lambda) \propto \lambda^p$ $p \gtrsim -1$

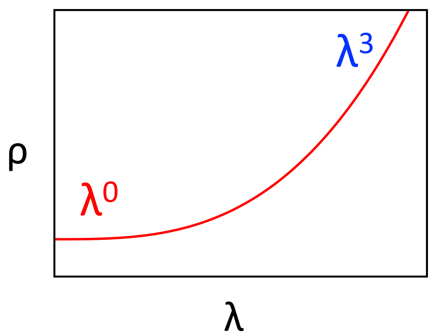
Proposal: THIS REFLECTS THE UNDERLYING IR SCALE-INVARIANT GLUE 1906.08047

POINT: SYMMETRY OF A COMPONENT, NOT OF AN ENTIRE SYSTEM "…without and with…"

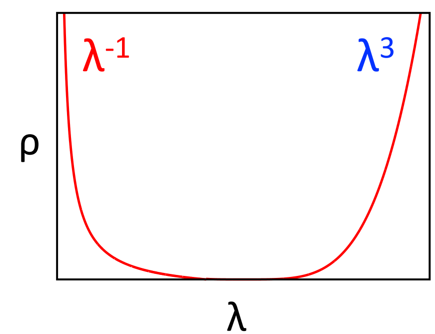
Scale Invariance? How???...

T=0 classically scale invariant theory → quantum fluctuations / scale anomaly → scales got generated world of hadrons etc

scale-broken quantum T=0 theory → thermal agitation / increasing T → scale-invariant but only for $\Lambda < \Lambda_{IR} < T$



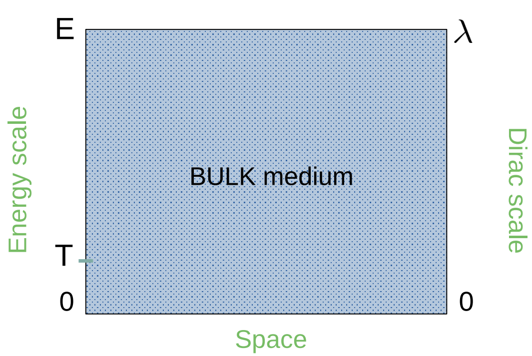
thermal agitation →



$\lambda^{-1} \rightarrow \lambda^{-1+\delta}$

$\delta = \delta(a) \rightarrow 0$?

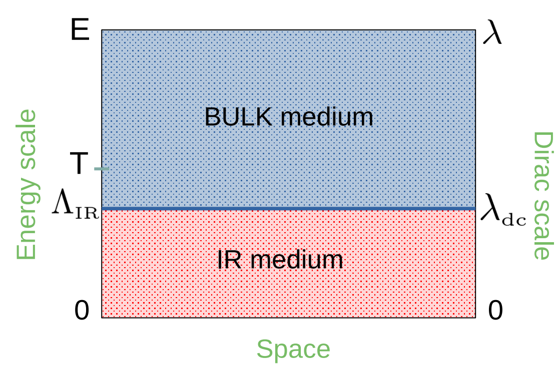
$a \rightarrow 0$



thermal agitation →

IR-BULK SEPARATION

AA & IH 1906.08047



NON-INVARIANT

INVARIANT

AT $T=T_{IR}$ THERMAL QCD GENERATES IR MEDIUM: AUTONOMOUS SUBSYSTEM REVERTING SCALE ANOMALY

Important Aspects of IR Phase [focus on thermal QCD]

I. IR PHASE OF THERMAL QCD

1906.08047, 2404.12298, 2305.09459

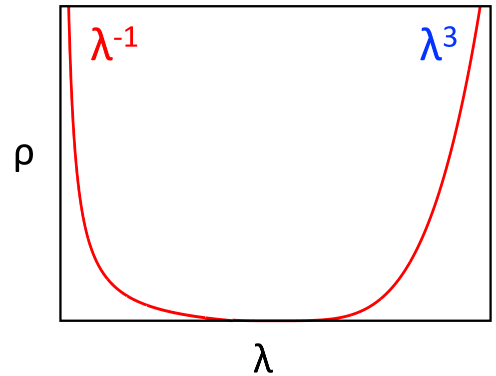
strictly above the χ-crossover T_c

$T_c < T_{IR} < T < T_{UV}$

$\approx$155 MeV 200-230 MeV perturb

II. WHY IR?

- Power-law accumulation of DOFs in IR
  AA & IH 1906.08047
- Thermal QCD in IR phase:
  ⟶ highly unusual scales $\Lambda < 1$ MeV
  in fact, IR-bottomless
  ⟶ partial deconfinement 1502.07732



$\lambda^{-1} \rightarrow \lambda^{-1+\delta}$

$\delta=\delta(a) \rightarrow 0$

$a \rightarrow 0$?

III. WHY PHASE?

At T_{IR} :

1906.08047
2103.05607
2110.04833

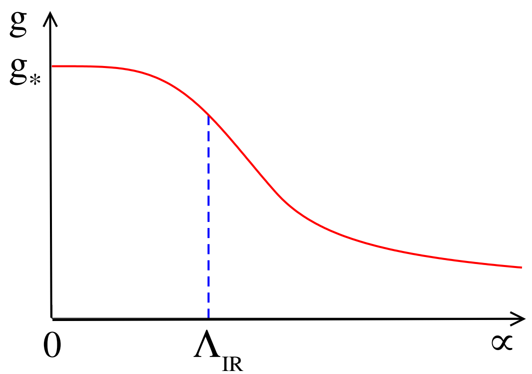
(i) IR DOFs BECOME AN AUTONOMOUS SUBSYSTEM
[IR-BULK decoupling, system splits into components]

(ii) SCALE-INVARIANT GLUE IN IR COMPONENT

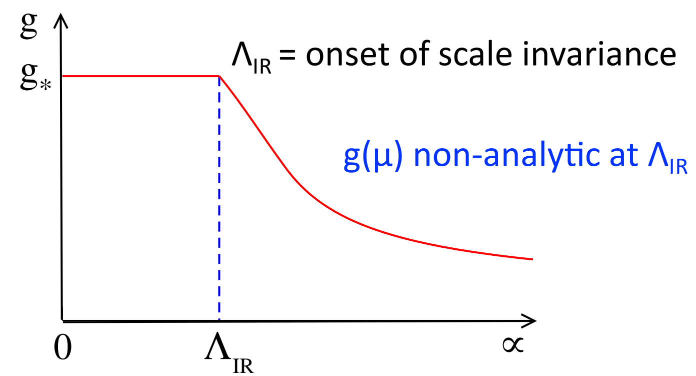
(iii) NON-ANALYTICITIES in DIRAC SPECTRA APPEAR
[generate T non-analyticity at T_{IR}]

(iv) INFINITE GLUE SCREENING LENGTHS APPEAR

Important Aspects of IR Phase: comments on non-analyticity



asymptotic vs exact



Q: Do non-analyticities in Dirac spectra exist and, if so, how do they arise?

Their existence would also facilitate IR-BULK decoupling & T-non-analyticity at T_{IR}.

But no evidence of non-analyticity in $\rho(\lambda)$ except for $\lambda=0$ and $\lambda=\infty$!

Hint: Anderson-like mobility edges $\lambda_A>0$

Garcia-Garcia & Osborn hep-lat/0611019, Kovacs & Pitler 1006.1205, Giordano, Kovacs & Pitler 1312.1179

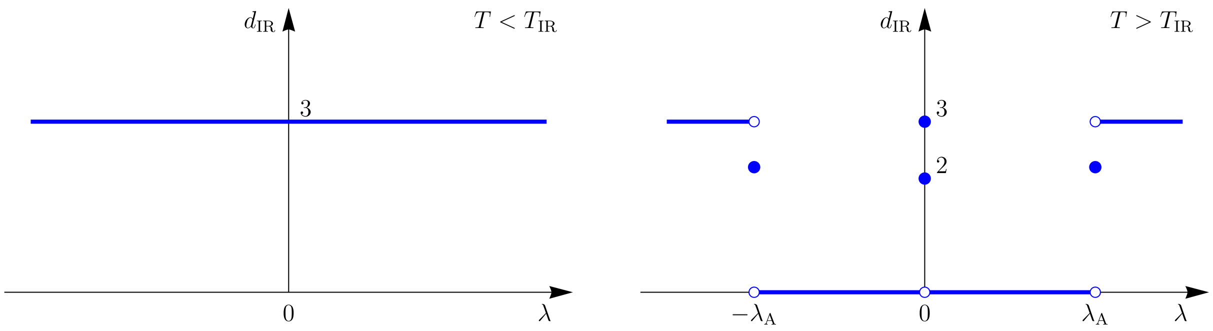
Focus on spatial IR dimension d_{IR} of modes!

WHAT IS IR DIMENSION OF MODES? Concept didn't exist.

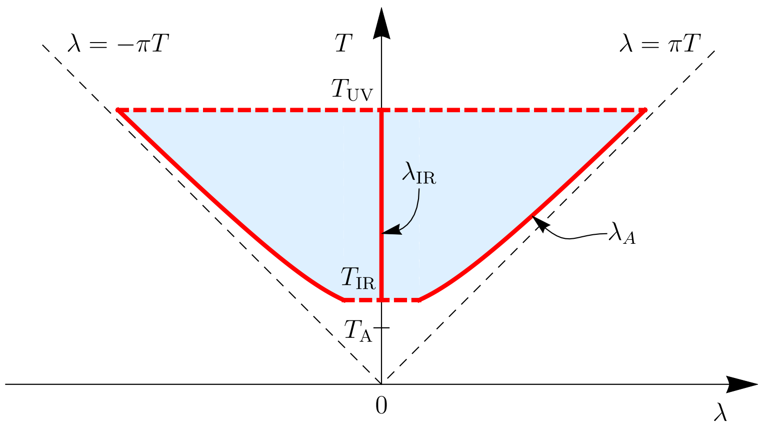
IH & RM 1807.03995 — effective-number theory

IH, PM and RM 2205.11520 — effective-dimension theory

Important Aspects of IR Phase: comments on non-analyticity…



Pattern of non-analyticity
2103.05607, 2310.03621



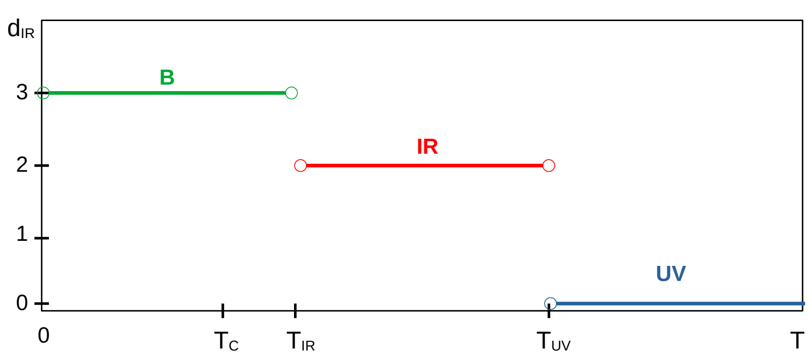
Dirac spectral phase diagram of QCD (pgQCD)

AA & IH 2110.04833 metal-to-critical scenario accommodates all listed properties

new mobility edge λ_{IR}=0 gives long-range physics

old mobility edge λ_A facilitates decoupling

Model in Kovács 2311.04208 supports decoupling.



spatial effective dimension of IR glue field

follows from above and what I will discuss below

Scale Invariance? How???...

Restoration of scale invariance in IR component the most intriguing aspect here!

1) How to better formalize the notion of scale-based component?

2) How to better formalize the notion of scale invariance in IR component?

The route goes through scale anomaly since violations are quantified that way.

$$T_{\mu\mu} = \frac{\beta(g)}{2g}\,\langle F^2\rangle \,+\, \big(1+\gamma_m(g)\big)\, m\langle\bar{\psi}\psi\rangle$$

Gell-Mann [Minkowski & Fritsch]

C. Collins, A. Duncan, and S. D. Joglekar (1977)

Can one assign scalar condensates involved to a component? YES!

Scale Invariance? How???...

$$\mathrm{tr}_{cs}\hat{D}_{x,x}(U) - \mathrm{tr}_{cs}\hat{D}_{x,x}(\mathbb{I}) \;=\; c_{\mathrm{S}}\, a^4\, \mathrm{tr}_c\, F_{\mu\nu}F_{\mu\nu}(x,A) \;+\; \mathcal{O}(a^6)$$

IH hep-lat/0610121, hep-lat/0607031

$\hat{D} \;\equiv\; aD$ Lattice Dirac operator: hypercubic symmetries + classical limit

$$D\psi_\lambda = \lambda\psi_\lambda \quad , \quad \lambda = \lambda_{\mathrm{R}} + i\lambda_{\mathrm{I}} \;\in\; \mathbb{C}$$

A classical continuum glue field

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu] \qquad A_\mu \in su(3)$$

U transcription of A onto hypercubic lattice, $\mathbb{I}$ free-field configuration

$$c_{\mathrm{S}} \neq 0 \quad \Longrightarrow \quad \text{defines} \quad F^2(x) \equiv \mathrm{tr}_c\, F_{\mu\nu}F_{\mu\nu}(x)$$

$$F^2(x,U) = \frac{1}{c_{\mathrm{S}}\, a^3}\, \mathrm{tr}_{cs}\Big[D_{x,x}(U) - D_{x,x}(\mathbb{I})\Big] \quad \longrightarrow \quad \langle\, F^2 \,\rangle = \frac{a}{c_{\mathrm{S}}}\,\frac{T}{L^3}\,\Big\langle\, \mathrm{Tr}\Big[D(U) - D(\mathbb{I})\Big]\,\Big\rangle$$

Scale Invariance? How???...

$$\langle F^2 \rangle = \frac{a}{c_s} \int_{\mathbb{R}^2[\mathbb{C}]} d\mathcal{S}\, \lambda\; \rho_s^{ef}(\lambda) \qquad -m\langle\bar{\psi}\psi\rangle = m \int_{\mathbb{R}^2[\mathbb{C}]} d\mathcal{S}\, \frac{1}{\lambda + m}\; \rho_s^{ef}(\lambda)$$

$$\rho_s^{ef} \equiv \rho_s - \rho_{s0} \qquad d\mathcal{S} = d\lambda_R d\lambda_I \qquad \rho_s(\lambda) = \langle n(\lambda, d\mathcal{S})\rangle / (V_4\, d\mathcal{S})$$

[surface spectral density]

Glue degrees of freedom expressed via Dirac spectral density [unified language]

1) glue part (glue condensate) is UV dominated

2) quark part is IR dominated

Scale Invariance? How???... Overlap

AA+IH+KFL arXiv:0803.2744

Overlap Dirac operators:
Neuberger, 1998

$$a\,\frac{D(\Delta)}{\Delta} = 1 + \frac{\hat{D}_W - \Delta}{\sqrt{(\hat{D}_W - \Delta)^\dagger(\hat{D}_W - \Delta)}} \qquad \Delta \in (0,2)$$

- D_W is massless Wilson-Dirac operator
- γ_5 – Hermiticity (complex pairs, antiparticles)
- Ginsparg-Wilson relation: chiral symmetry and $\Delta(\lambda+\lambda^*) = a\,\lambda^*\lambda$

$$\rho_s(\sigma,\varphi) = \frac{\rho(\sigma)}{\sigma}\,\delta\Big(\varphi - \cos^{-1}\frac{a\sigma}{2\Delta}\Big) \qquad \sigma^2 = \lambda^*\lambda \qquad 0 \le \sigma \le \frac{2\Delta}{a}$$

$$\langle\, F^2\,\rangle_{a,L} \;=\; \frac{a^2}{c_{\mathrm{S}}\Delta}\int_0^{(\frac{2\Delta}{a})^-} d\sigma\,\sigma^2\,\rho_{\mathrm{eff}}(\sigma,a,L) \;+\; T\,\frac{2\Delta}{c_{\mathrm{S}}}\,\frac{\langle n_0\rangle_{a,L}}{L^3}$$

$$\rho_{\mathrm{eff}}(\sigma) \;\equiv\; \rho(\sigma) - \rho_0(\sigma) \qquad\qquad n_0 \;=\; \text{number of zeromodes}$$

Fully regularized formula for gluon condensate from overlap!

Scale Invariance? How???... Overlap...Gluon Condensate

Take L to infinity for simplicity: $$\langle F^2 \rangle_a = \frac{a^2}{c_S \Delta} \int_0^{(\frac{2\Delta}{a})^-} d\sigma\, \sigma^2\, \rho_{\text{eff}}(\sigma, a)$$

Naïve: $$\rho^{\text{eff}}(\sigma) = \frac{c}{\sigma} + \bar{\rho}^{\text{eff}}(\sigma) \longrightarrow \langle F^2 \rangle = \frac{2\Delta}{c_S}\, c$$

Fancy: analytically continue $\rho^{\text{eff}}(\sigma,a)$ into the complex plane -> $\rho^{\text{eff}}(z,a)$

Let σ_k be the centra of annuli where $\rho^{\text{eff}}(z,a)$ is analytic and C_k the associated circle in complex plane. Then from its Laurent expansion:

$$\langle F^2 \rangle = \frac{2\Delta}{c_S} \frac{1}{2\pi i} \lim_{a \to 0} \sum_k \oint_{C_k} dz \rho^{\text{eff}}(z, a)$$

analog of Banks-Casher for $\langle F^2 \rangle$

Any computational use for all this? Actually, likely yes. To be continued...

Scale Invariance? How???... Scale Anomaly

$$T_{\mu\mu} = \frac{\beta(g)}{2g}\,\langle F^2\rangle \;+\; \big(1+\gamma_m(g)\big)\, m\langle\bar\psi\psi\rangle$$

$$\langle F^2\rangle \;=\; \langle F^2\rangle_{\rm IR} \,+\, \langle F^2\rangle_{\rm B} \qquad\qquad \langle\, F^2 \,\rangle_{\rm IR} \;=\; \frac{a^2}{c_{\rm S}\Delta}\int_0^{\sigma_{\rm IR}} d\sigma\,\sigma^2\,\rho(\sigma)$$

Crucial subtlety:

$$\big\langle\, F^2_{\rm IR}\,\big\rangle \;\longrightarrow\; 0 \quad \text{for}\;\; a\to 0$$

NEED BOTH!

$$\big\langle\, F^2_{\rm IR} F^2_{\rm B}\,\big\rangle_c \;\longrightarrow\; 0 \quad \text{for}\;\; L\to \infty$$

NO CIGAR WITHOUT DECOUPLING!

GLUE CONTRIBUTION OF IR MEDIUM TO SCALE ANOMALY VANISHES!

FORMAL STATEMENT OF IR SCALE INVARIANCE FOR IR PHASE!

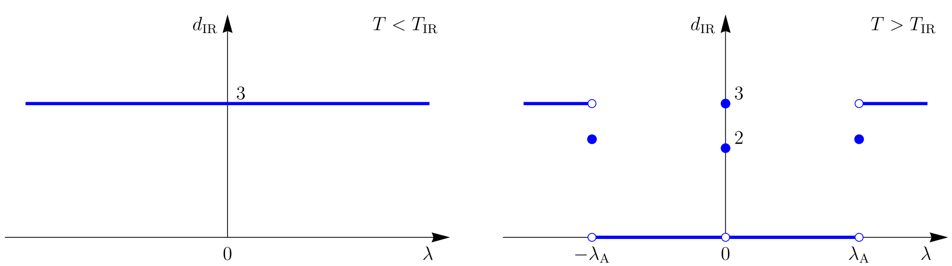
Scale Invariance? How???... IR-Bulk separatipon

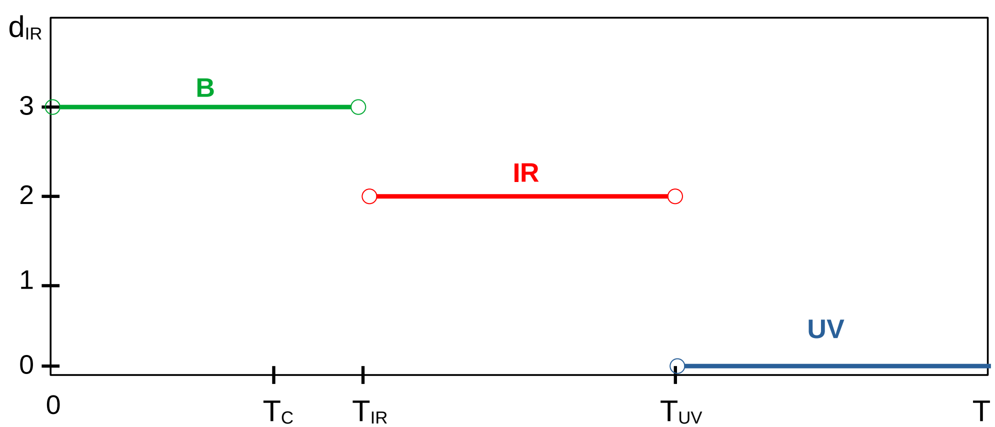
Apart from above, everything depends on expansion of local density:

$$F^2(x,U,\sigma_1,\sigma_2) = \frac{1}{c_S\, a^3} \sum_{\sigma_1<\sigma_i<\sigma_2} \left[\psi^\dagger_{\sigma_i}\psi_{\sigma_i}(x)\,\sigma_i \;-\; \frac{a^4 T}{L^3}\sigma_{i0}\right]$$

<u>All of IR Phase stuff can be recast in terms of F^2</u>

For example, dimension of $F^2(x,U,0,\sigma_{\rm IR}) \;\equiv\; F^2_{\rm IR}(x,U)$

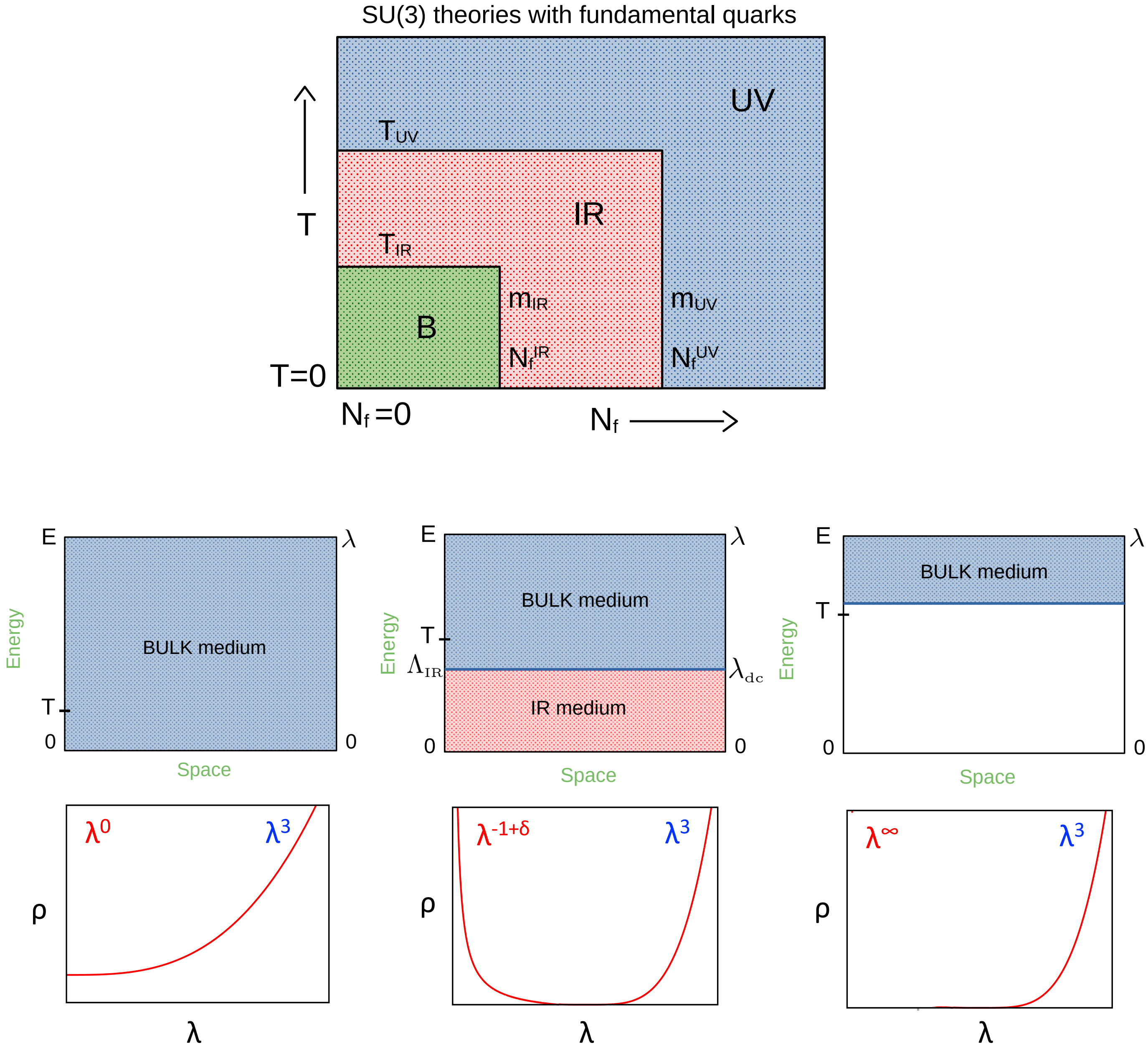




Translates into non-analytic T-dependence

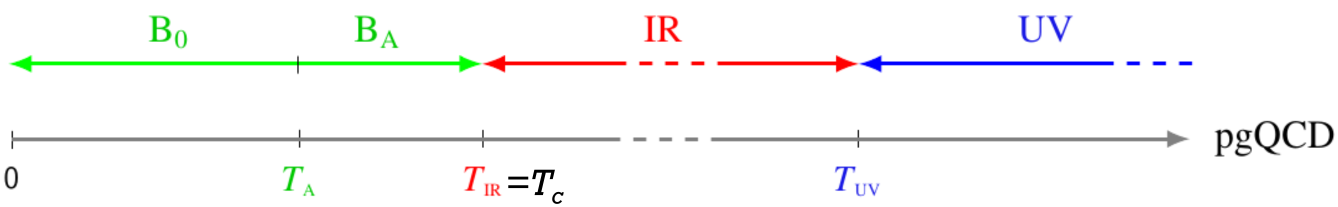
etc etc etc

IR PHASE ALL & WELL



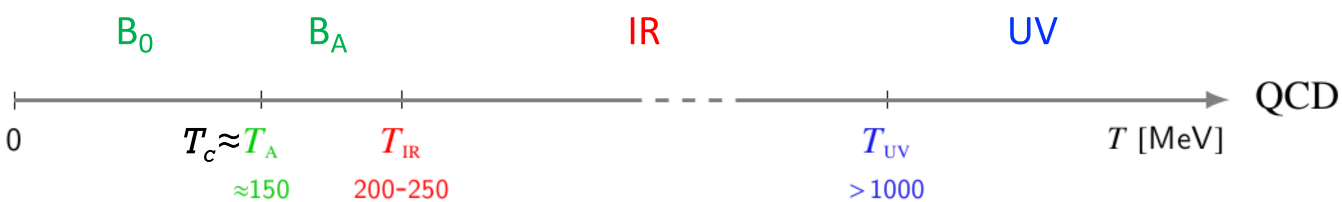
Phase Diagrams of Thermal QCD via IR Scale Invariance of Glue AA & IH 1906.08047

■ Thermal phase diagram for $N_f=0$: Polyakov line transition coincides w IR-phase transition



IR BROKEN p = 0 IR SYMMETRIC p ≈ -1 IR TRIVIAL p ≫ 0

■ Thermal phase diagram of N_f=2+1 ``real-world": chiral crossover below IR-phase transition



STRONG COUPLING p = 0
HADRON GAS

STRONG COUPLING p ≈ -1
NEAR-PERFECT FLUID?

WEAK COUPLING p ≫ 0
PERTURBATIVE QGP

■ Notes: (a) no hard evidence for existence of UV phase [Lives as likely an oversimplified but historically first picture of high-T QCD. Strict UVp: 50-50 chance!]

(b) other transitions/crossovers in the bulk possible

Useful talks

Original talk: https://indico.cern.ch/event/764552/contributions/3420459/attachments/1865996/3068382/WuHan_jun_2019_infra.pdf

Useful talk: https://drive.google.com/file/d/1vZ0AY0WsZAfF9iV7-Br-E_2NiwaZzRGp/view

See also recent talks:

https://indico.cern.ch/event/1293041/contributions/5946693/attachments/2914234/5113815/Horvath_confXVI_Aug_2024_w_refs.pdf

https://indico.ectstar.eu/event/213/contributions/5060/attachments/3345/4711/Trento_Sep_2024.pdf

https://bodri.elte.hu/budqcd2024/slides/Ivan_Horvath_budqcd2024.pdf

Studies of Dirac Spectra in Other Contexts

$U_A(1)$ problem and other

Dick et al 1502.06190

Kaczmarek et al 2102.06136

Aoki et al 2011.0149

Ding et al 2010.14836

Kehr et al 2304.13617

Glozman et al 2204.05083

Bonanno & Giordano 2312.02857

Kaczmarek et al 2301.11610

Kovacs & Vig 1706.03562

Rohrhofer et al 1902.03191

Cardinali et al 2107.02745

Giordano 2404.03546

Pandey et al 2407.09253

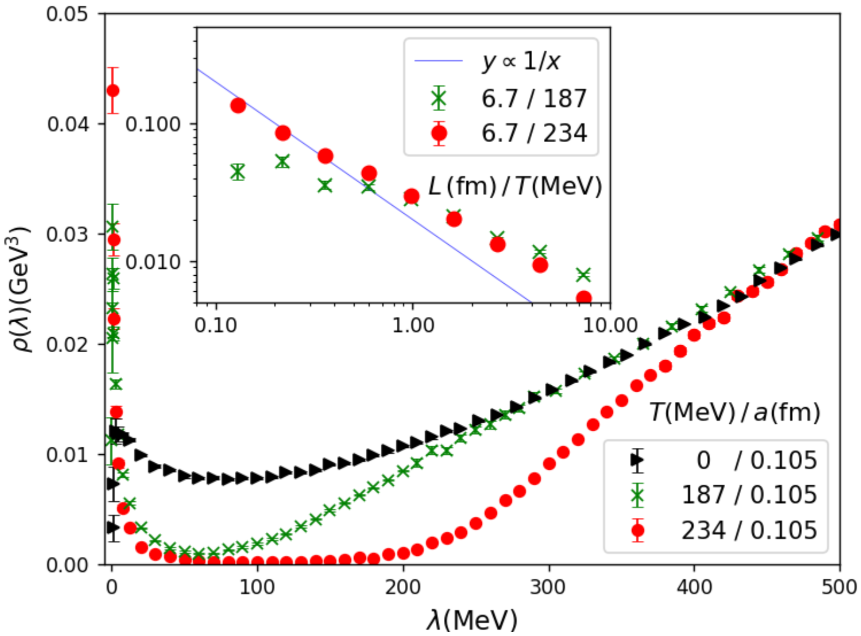
Kovács 2311.04208

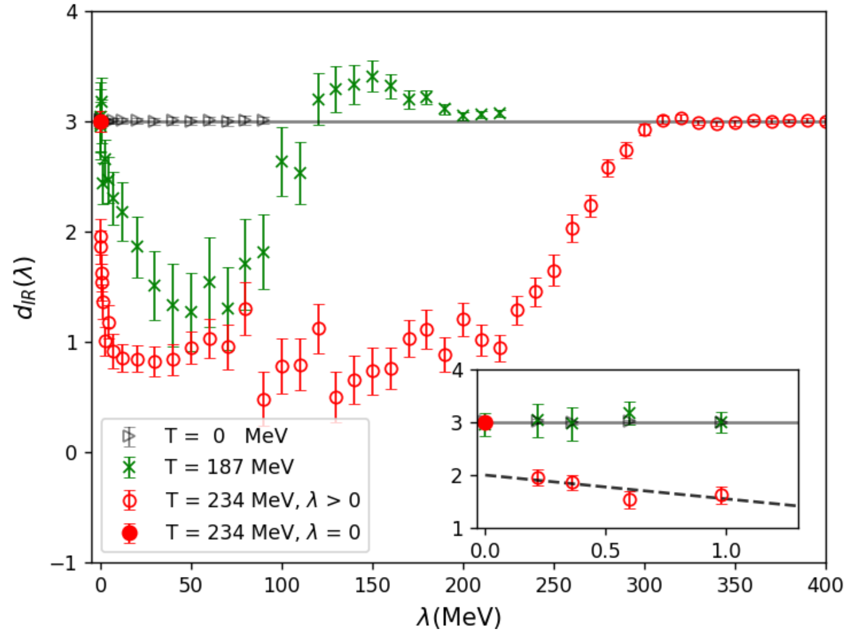
and other…

BACKUPS/DETAILS

A. IR Phase: Dirac operator T_c vs T_{IR}

N_f=2+1 physical point, *a*=0.105 fm   X. Meng et al 2305.09459





$T = 0\,\mathrm{MeV} \quad \rightarrow \quad p = 0 \;\; (\log)$

$T = 187\,\mathrm{MeV} \quad \rightarrow \quad p = 0 \;\; (\log)$?

$T = 234\,\mathrm{MeV} \quad \rightarrow \quad p \gtrsim -1$

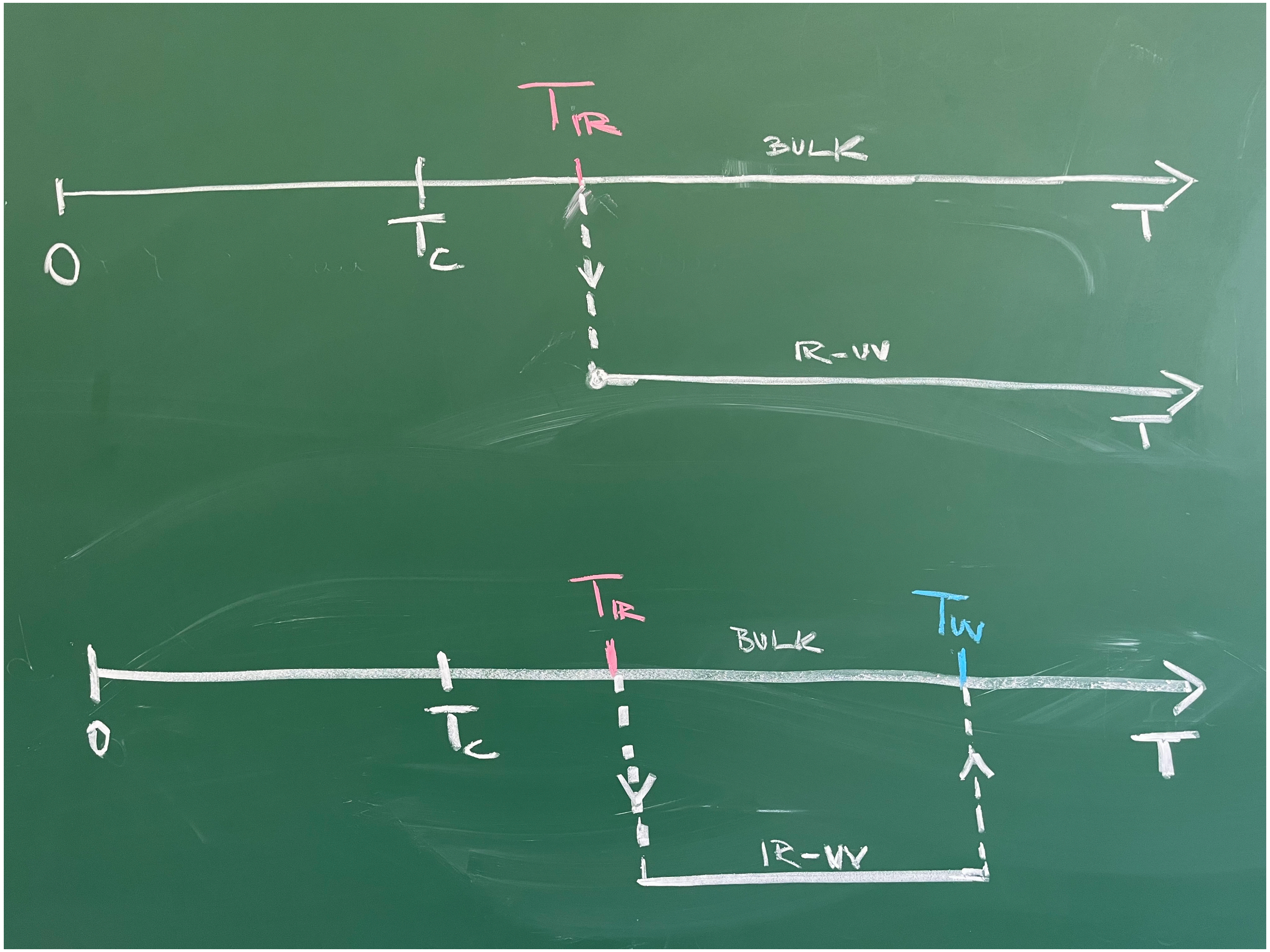
$T = 0\,\mathrm{MeV} \quad \rightarrow \quad d_{\mathrm{IR}}(0^+) \simeq 3$

$T = 187\,\mathrm{MeV} \quad \rightarrow \quad d_{\mathrm{IR}}(0^+) \simeq 3$

$T = 234\,\mathrm{MeV} \quad \rightarrow \quad d_{\mathrm{IR}}(0^+) \simeq 2$

SUPPORTS THE ORIGINAL PROPOSITION THAT T_{IR} IS STRICTLY ABOVE THE CHIRAL CROSSOVER!

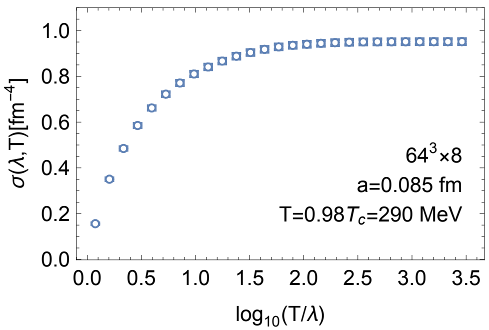
AA & IH 1906.08047

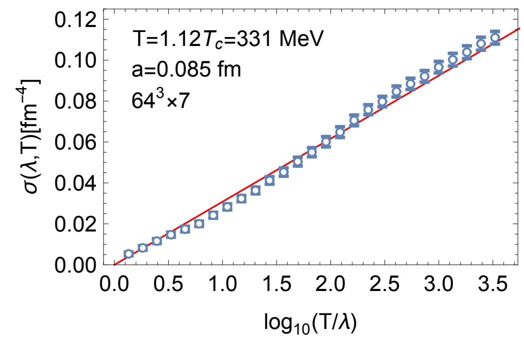


T_IR
BULK
0
T_c
T
IR-UV
T
T_IR
BULK
T_UV
0
T_c
T
IR-UV

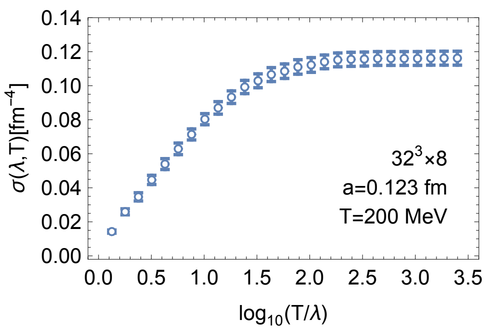
A. IR Phase: real-world QCD

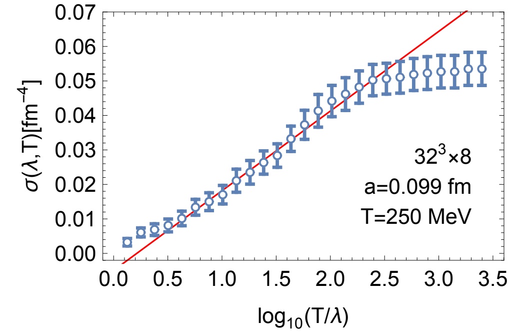
$N_f=0$





$N_f=2+1$





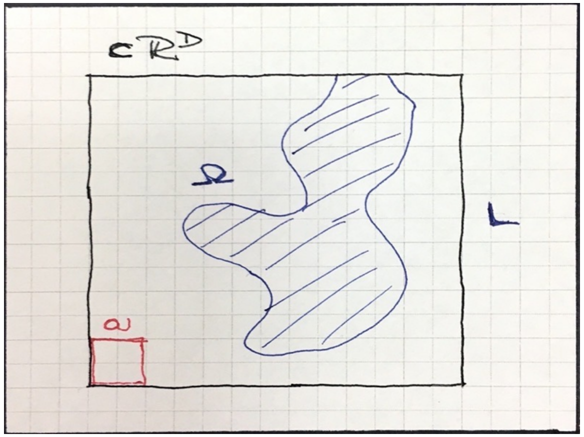
Real-world QCD is $N_f=2+1$ at physical quark masses of stouted staggered quarks (Wuppertal-Budapest) here.

Conjecture: REAL-WORLD QCD HAS IR PHASE WITH p≈-1

$$200\,\mathrm{MeV} \;<\; T_{\mathrm{IR}} \;<\; 250\,\mathrm{MeV}$$

AA & IH 1906.08047

A. IR Phase: non-analyticity, effective dimensions



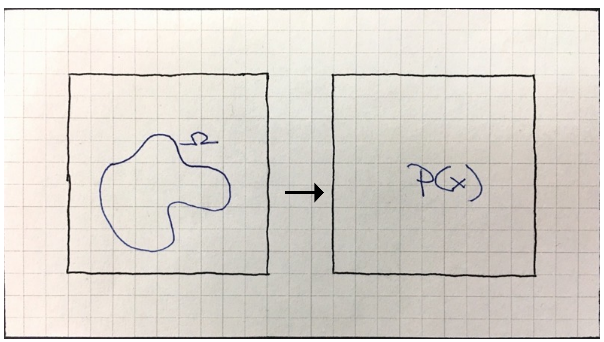
characterize fine [UV] and global [IR] features of fixed sets

all points/elements of regularized space: $N \propto (L/a)^D$

points/elements covering Ω: N_+

$$\text{UV:} \quad N_+(a,L) \propto a^{-d_{\mathrm{UV}}(L)} \;, \quad a \to 0$$

$$\text{IR:} \quad N_+(a,L) \propto L^{d_{\mathrm{IR}}(a)} \;, \quad L \to \infty$$



$$P(x) \quad \Longrightarrow \quad \Omega_{\mathrm{eff}}$$

But how to proceed when instead of fixed Ω we have P(x)?

1) Count how many points $\mathcal{N} = \mathcal{N}[P] = \mathcal{N}(p_1, p_2, \ldots, p_N)$ are effectively selected by *P*.
2) Select Ω_{eff} as $\mathcal{N}$ most probable points on the lattice
3) Proceed as Minkowski/box-counting with N_+

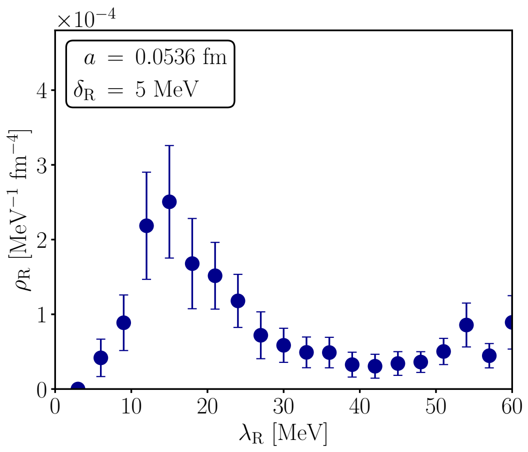
Consistent realization of this program leads to unique effective dimensions IH, PM and RM 2205.11520

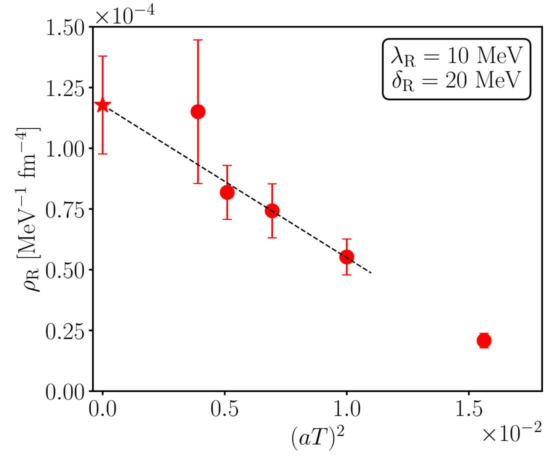
$$\mathcal{N}_\star[P] = \sum_{i=1}^{N} \mathfrak{n}_\star(N p_i) \quad , \quad \mathfrak{n}_\star(c) = \min\{c, 1\}$$ IH & RM 1807.03995

$$\text{Box:} \quad N \longrightarrow N_+ \qquad \text{Effective:} \quad N \longrightarrow \mathcal{N}_\star[P]$$

A. IR Phase: Dirac operator describing dynamical quarks

AA, Bonanno, D`Elia, IH 2404.12298

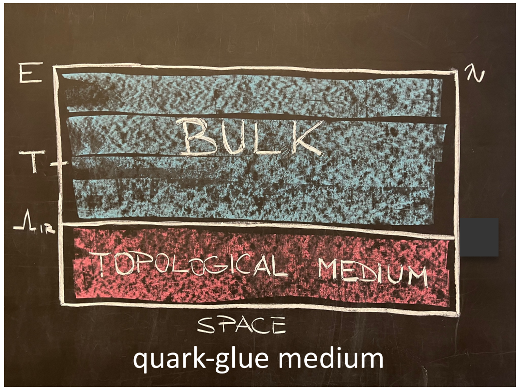




Real-world QCD is N_f=2+1 at physical quark masses of stouted staggered quarks here.

Lattice Dirac operator = stouted staggered [not overlap]

- IR structure exists in Dirac operator describing dynamical quarks
- not a lattice artifact
- IR medium is a quark-glue medium
- green light to study IR phase using overlap probe: correct & efficient



Scale Invariance and IR Phase: Anomaly & Stuff…

$$\text{anomalous part } T_{\mu\mu} \;=\; \frac{\beta(g)}{2g}\langle F^2\rangle \;+\; \gamma_m(g)\, m\, \langle\bar\psi\psi\rangle$$

$$\langle F^2\rangle \;=\; \langle F^2\rangle_{\rm IR} \;+\; \langle F^2\rangle_{\rm B}$$

$$\langle\, F^2\,\rangle_{\rm IR} \;=\; \frac{a^2}{c_{\rm S}\Delta}\int_0^{\sigma_{\rm IR}} d\sigma\, \sigma^2\, \rho(\sigma)$$

Crucial subtlety:

$$\left\langle\, F^2_{\rm IR}\,\right\rangle \;\longrightarrow\; 0 \quad \text{for } \; a \to 0$$

NEED BOTH!

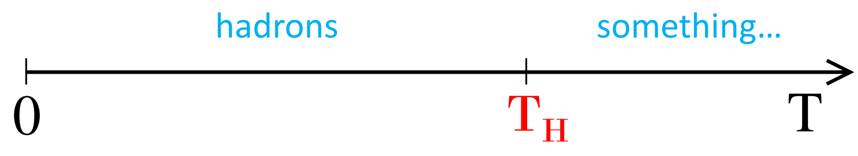
$$\left\langle\, F^2_{\rm IR} F^2_{\rm B}\,\right\rangle_c \;\longrightarrow\; 0 \quad \text{for } \; L \to \infty$$

NO CIGAR WITHOUT DECOUPLING!

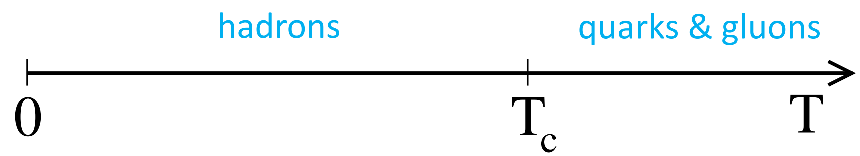
GLUE CONTRIBUTION OF IR COMPONENT TO SCALE ANOMALY VANISHES!

Formal statement if IR scale invariance in IR phase!

Stories of Temperature & QCD

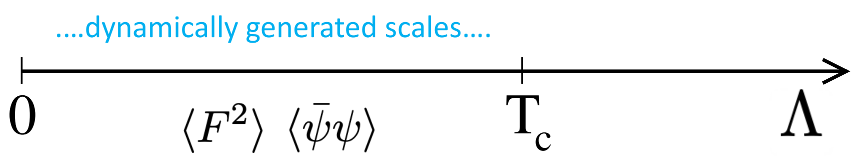


Hagedorn, 1965-71



QCD, 1971-73

Effects of Temperature:



[energy scale]

Scales Story: thermal agitation erodes condensates and melts them upon T reaching T_c

DOFs Story: thermal agitation reduces IR dof-s and depletes them when T reaches T_c

[DOFs = quark & glue]

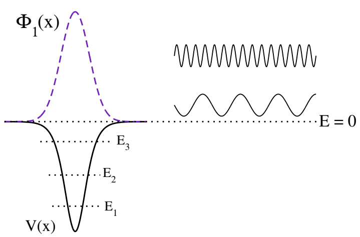
Anderson Localization & Transitions

courtesy of P.Markos

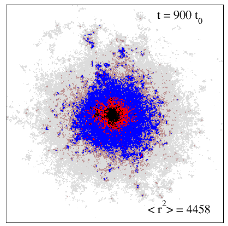
Quantum mechanics: eigenstates of quantum particle could be

bounded extended . . . and localized

[P. W. Anderson 1958]



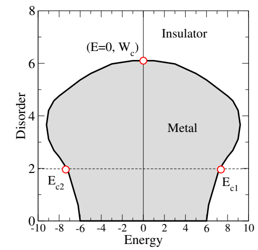
$E < 0$ $E > 0$

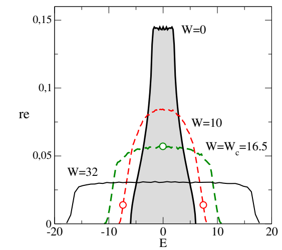


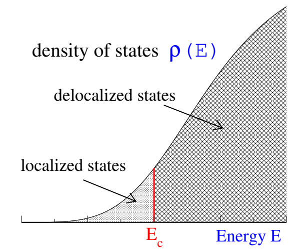
Localization is a consequence of

- disorder
- wave character of particle
- $\Phi(\vec{r}) \propto \exp\,[-r/\lambda]$

3D Anderson model: phase diagram, density of state, mobility edge





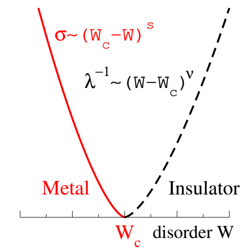


Critical exponents:

conductivity: $\sigma(E) \sim (E_c - E)^s$

localization length: $\lambda(E) \sim (E - E_c)^{-\nu}$

$s = (d-2)\nu$ . . . critical exponents.



λ is a localization length here $\lambda \longrightarrow \ell$

(1) $\ell \propto \xi$

density-density correlation length within the mode

(2) $\langle\, \psi^2_{loc}(x)\psi^2_{ext}(y)\,\rangle_c \longrightarrow 0 \quad \text{for } L \to \infty$

Anderson-like transition in thermal QCD:
Disorder W $\longrightarrow$ Temperature T
Mobility edge $\lambda_A \neq 0$ invoked for understanding chiral phase transition: aka metal-to-insulator picture

Garcia-Garcia & Osborn hep-lat/0611019

Kovacs & Pitler 1006.1205

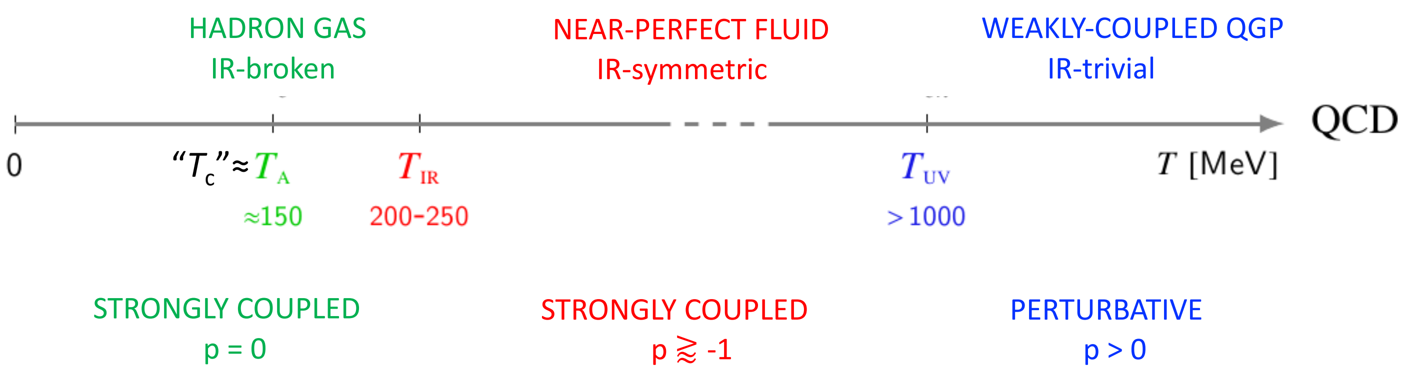
Giordano, Kovacs & Pitler 1312.1179

Shortly after AA & IH 1906.0804 we realized that λ_A is a very friendly feature to our claim and accepted it ☺.

WHAT WE HAVE HERE IS THE LACK OF COMMUNICATION…

PHASE STRUCTURE OF THERMAL QCD IN TERMS OF GLUE IR SCALE INVARIANCE

[AA & IH 1906.08047]



$$\text{phase} = \begin{cases} \text{B} & \text{if} \quad p = 0 \\ \text{IR} & \text{if} \quad p < 0 \\ \text{UV} & \text{if} \quad p > 0 \end{cases} \qquad \text{with} \qquad \rho(\lambda) \propto \lambda^p \quad \text{for} \quad \lambda \to 0$$

Original talk: https://indico.cern.ch/event/764552/contributions/3420459/attachments/1865996/3068382/WuHan_jun_2019_infra.pdf

Useful talk: https://drive.google.com/file/d/1vZ0AY0WsZAfF9iV7-Br-E_2NiwaZzRGp/view

Betting on Glue

[1906.08047 , 1502.07732]

Standard approaches to phases:

quark view: $\langle \bar{\psi}\psi \rangle$ large (hadron gas) — T_c — $\langle \bar{\psi}\psi \rangle$ small (quark-gluon plasma)

0 ... T_c ... T

quark scalar density

glue view: $\langle L \rangle$ small — T_c — $\langle L \rangle$ large

0 ... T_c ... T

Polyakov line

Quarks won the popularity contest [$T_c \approx 155$ MeV, crossover , Aoki et al, 2007]

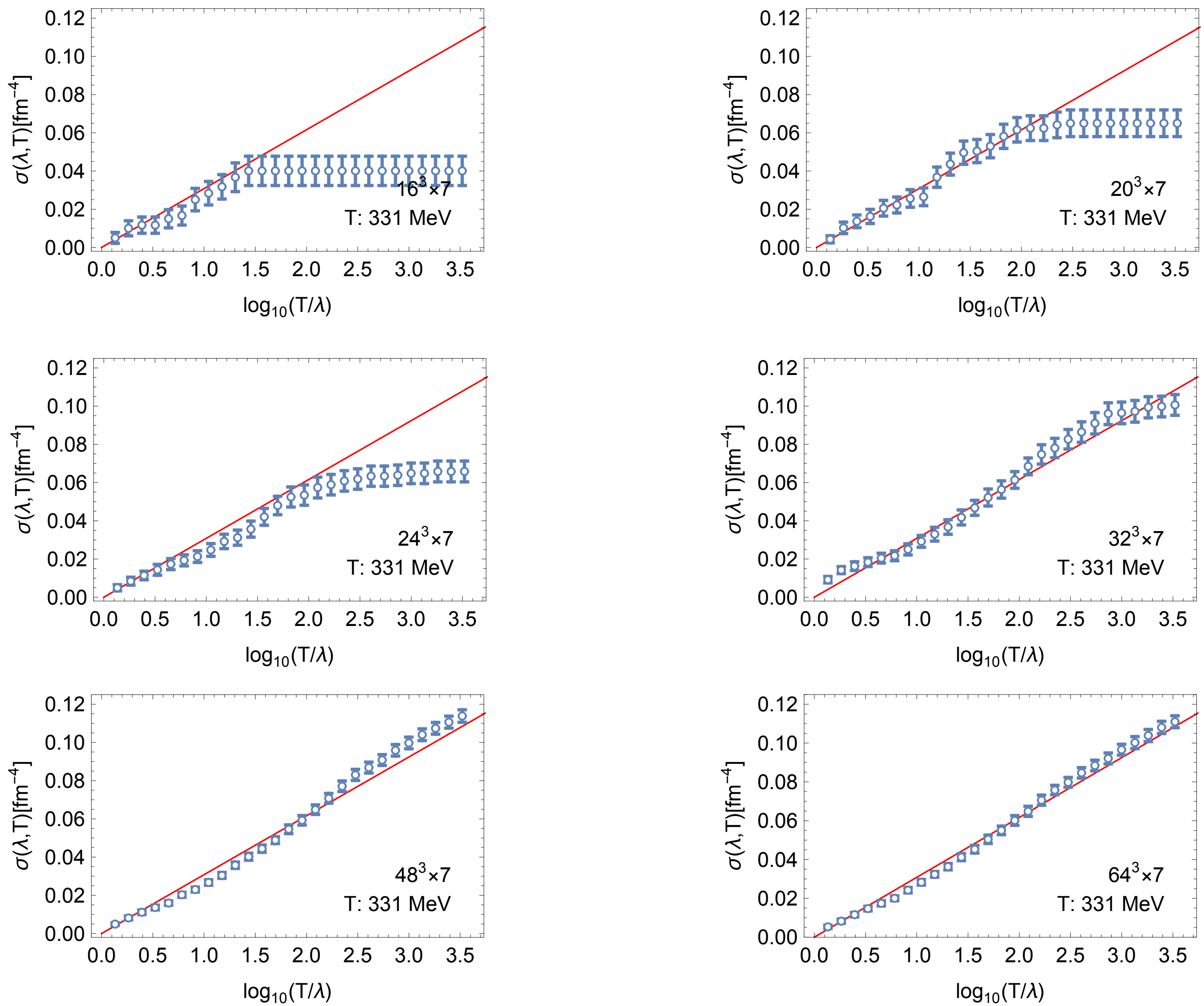
NEED NEW IDEA!

Point 1: $\langle \bar{\psi}\psi \rangle$ is cleaner because it reflects deeper IR of glue

Point 2: both $\langle \bar{\psi}\psi \rangle$ and $\langle L \rangle$ are limited in terms of reflecting glue

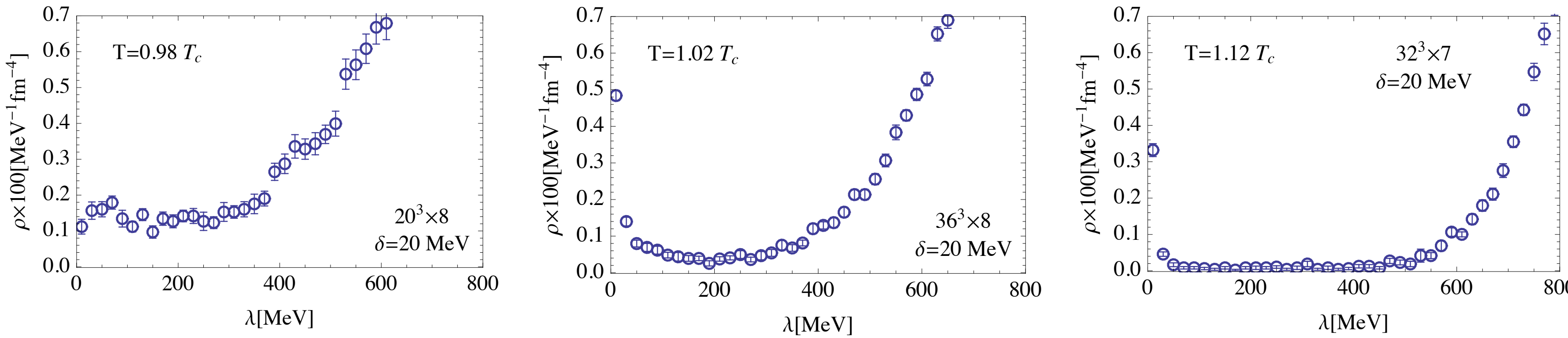
Point 3: need glue probe that is sensitive to any scale by construction
object with explicit scale dependence

Check this further & better… $\sigma(\lambda_1, \lambda_2) \equiv \int_{\lambda_1}^{\lambda_2} d\lambda \, \rho(\lambda)$ $\rho(\lambda) = c/\lambda \longrightarrow \sigma(\lambda, T) = c \ln(T/\lambda)$



- Peak in IR overlap spectrum upon crossing T_c (pure glue) [Edwards, Heller, Narayanan, Kiskis, 1999]

- Our version of it [AA & IH, 1502.07732]



▪ knee-jerk reaction was: quenched chiral condensate may diverge in high-temperature pure glue

▪ knee-jerk reaction should be: what on earth is glue doing to produce this? [1502.07732]

▪ didn't know but went on with it, e.g., around chiral crossover we got this: