Witten effect II: the Aharonov-Bohm effect strikes back

arXiv:2410.23355 with Shi Chen, Aleksey Cherman, Gongjun Choi and arXiv:25XX.XXXXX

Maria Neuzil

Benasque Science Center

February 11, 2025

Introduction

Goal of this talk:

Provide an unconventional perspective on the famous θ term of $U(1)$ Maxwell gauge theory

This perspective ultimately leads to vast generalizations of the *idea* of θ terms, but here we focus on understanding the case of Maxwell theory

Something new to say about a 100-year-old theory! :D

Maxwell θ term: a reminder

4d Maxwell theory:

- $U(1)$ gauge theory with gauge field a (the photon)
- Admits a "θ term." In Euclidean space, it is:

$$S(\theta) = \frac{1}{2g^2} \int \underbrace{\mathrm{d}a \wedge \star \mathrm{d}a}_{\sim F^{\mu\nu} F_{\mu\nu}} - \frac{\mathrm{i}\theta}{8\pi^2} \int \underbrace{\mathrm{d}a \wedge \mathrm{d}a}_{\sim \epsilon^{\rho\sigma\mu\nu} F_{\rho\sigma} F_{\mu\nu}}$$

Facts:

- θ is 2π-periodic: theory with $\theta + 2\pi$ equivalent to theory with θ
- θ term is total derivative: does *not* affect equations of motion for a
  - So is it completely invisible? No!
  - θ has a famous consequence: the **"Witten Effect"**

Outline

Part I: The Witten Effect, Old and New

1. Review: the standard Witten effect
2. Re-interpretation of Witten effect: symmetry surface attachment
3. Charge Witten effect vs. topological Witten effect

Part II: New origins for Aharonov-Bohm interference

4. Topological Witten effect $\implies$ Aharonov-Bohm effect

Part III: Conclusion and teaser

Review: the standard Witten effect

Witten effect: θ term leads to "fractionalization" of electric charge
(Witten 1979)

To understand what this means, it's helpful to think about the global symmetries of Maxwell theory:

- $U(1)_e$ electric 1-form symmetry
- $U(1)_m$ magnetic 1-form symmetry

"1-form symmetry" means that the objects charged under these symmetries are 1-dimensional, i.e. loops

- Wilson loop: worldline of probe particle with electric charge $e \in \mathbb{Z}$
- **'t Hooft loop**: worldline of probe particle with magnetic charge $m \in \mathbb{Z}$
- In general, a loop could have both e, m charge

Symmetry charges vs. physical charges

$U(1)$ symmetry charges like e are **always** integers — so what does it mean that "electric charge gets fractionalized?"

Must distinguish between:

- **symmetry charge**: labels like $e, m \in \mathbb{Z}$ that correspond to representations of the group
- **physical charge**: quantity that really dictates the physics, e.g. the charge you get from Gauss' law:

$$M \equiv \int \frac{\mathrm{d}a}{2\pi} = m\,, \quad E \equiv \int -i\frac{\star \mathrm{d}a}{g^2} \underset{\text{calculation}}{\overset{\text{short}}{=}} \underbrace{e + \frac{\theta}{2\pi}m}_{\text{fractional } \textit{physical} \text{ electric charge!}}$$

$\theta = 0 \implies$ no distinction between symmetry vs. physical charge

Role of anomaly in the standard Witten effect

Physical electric charge: $E = e + \frac{\theta}{2\pi} m$

Notice: the *electric* charge is being modified by something that depends on the *magnetic* charge
$\implies$ the electric and magnetic symmetries are talking to each other

In fact, there is a **mixed 't Hooft anomaly** between $U(1)_e$ and $U(1)_m$ that explains this!

By direct calculation, you can show that the θ-induced electric charge fractionalization originates from the anomaly

Recap so far: the standard Witten effect

θ term of Maxwell theory leads to physical consequences

- "Physical" electric charge gets fractionalized, $E = e + \frac{\theta}{2\pi} m$
- "Physical" electric charge shows up in physical places like Gauss' law

What ingredients did we need for that story?

- $U(1)_e$ electric and $U(1)_m$ magnetic symmetry: makes the symmetry charges e, m exist in the first place
- Mixed 't Hooft anomaly between $U(1)_e$ and $U(1)_m$: makes the charge fractionalization happen, since the two symmetries talk to each other

Looking ahead

"θ term + two symmetries with mixed 't Hooft anomaly lead to the Witten effect of charge fractionalization"

What if we don't have all those ingredients?

Spoiler alert: we can generalize the Witten effect!

To make this generalization, we first will observe that θ actually has *another* consequence:

θ **attaches topological surfaces to 't Hooft loop operators**

Symmetry surface attachment

Claim: θ attaches topological surface operators to 't Hooft loops

- These topological surface operators will be the generators of the $U(1)_m$ 1-form symmetry

Can see this by considering correlation functions of magnetically-charged 't Hooft loops T_m on $\mathbb{R}^4$. Schematically:

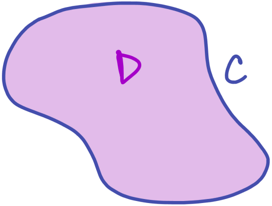
$$\begin{aligned}\left\langle T_m \cdots \right\rangle_\theta &= \frac{1}{\mathcal{Z}(\theta)} \int e^{-S(\theta)}\, T_m \cdots \\ &= \frac{1}{\mathcal{Z}(0)} \int e^{-S(0)}\, T_m\, U(\theta) \cdots = \left\langle T_m\, U(\theta) \cdots \right\rangle_0\end{aligned}$$

Effect of θ on 't Hooft loops

Result:

$$\left\langle T_m(C)\cdots\right\rangle_\theta \overset{}{=} \text{short calculation} = \left\langle T_m(C)\exp\left(\frac{i\theta m}{2\pi}\int_D \mathrm{da}\right)\cdots\right\rangle_0$$

this surface operator is the $U(1)_m$ 1-form sym generator



Interpretation: can trade θ term for a surface attached to 't Hooft operators

Bonus observation

This symmetry surface attachment has a Hamiltonian analog: θ shuffles which $U(1)_m$-twisted sector a particular fixed-charge $\mathcal{H}$ acts on

There's more to be said, but we don't need it for the punchline of this talk

Ask me after or consult the upcoming paper if you are interested...

Old perspective vs. new perspective on θ

Summary: Maxwell θ has two effects

- Fractionalizes physical electric charge
- Attaches symmetry surfaces to 't Hooft loops
    - Equivalently, "shuffles $U(1)_m$-twisted sectors"

Consider the ingredients we needed to discuss each story:

- For charge fractionalization: $U(1)_m$ and $U(1)_e$ symmetries with mixed 't Hooft anomaly
- For surface attachment: just $U(1)_m$!

"Charge Witten effect" vs. "topological Witten effect"

Motivates fine-graining the definition of the Witten effect:

"Charge Witten effect:" θ leads to charge fractionalization

- Ingredients: 2 symmetries $+$ mixed anomaly

"Topological Witten effect:" θ attaches surfaces to 't Hooft loops

- Ingredient: 1 symmetry

$$\text{charge Witten effect} \quad \Longleftrightarrow \quad \begin{cases} \text{topological Witten effect} \\ \text{extra symmetry with mixed anomaly} \end{cases}$$

For generalizations, topological Witten effect requires fewer assumptions.
But definition is quite abstract... are there concrete physical consequences?

Outline

Part I: The Witten Effect, Old and New

1 Review: the standard Witten effect

2 Re-interpretation of Witten effect: symmetry surface attachment

3 Charge Witten effect vs. topological Witten effect

Part II: New origins for Aharonov-Bohm interference

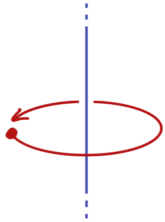
4 Topological Witten effect $\Longrightarrow$ Aharonov-Bohm effect

Part III: Conclusion and teaser

Topological Witten effect $\implies$ Aharonov-Bohm effect

Punchline: physical consequence of topological Witten effect is a generalized **Aharonov-Bohm effect**

Recall textbook Aharonov-Bohm effect: electrically-charged particle travels around solenoid and gets a phase shift



Topology of the **operator** (solenoid/vortex) and the path of the **excitation** (particle worldline) is important: they **link**

Physical consequences of topological Witten effect?

Claim:

The topological Witten effect of θ induces an Aharanov-Bohm effect for strings

How do we show this?

In the case of pure Maxwell theory, it is difficult to see...

- $U(1)_m$ is spontaneously broken on $\mathbb{R}^4$, so there is no way to "isolate" magnetic strings. Doesn't make sense to ask how θ affects them...

Physical consequences of topological Witten effect?

Idea: consider the **Higgs phase** of charge-1 **Abelian Higgs model**

- Lots of motivation to study this model: superconductors, BSM physics (e.g. dark photons)...

$$\mathcal{Z} = \int \mathcal{D}a\mathcal{D}\varphi\mathcal{D}\varphi^\dagger \; \exp\left\{ -S_{\text{Maxwell}}(a;\theta) - \int |D\varphi|^2 - \lambda \int \star \left(\varphi\varphi^\dagger - v^2\right)^2 \right\}$$

In the Higgs phase:

- $U(1)_e$ electric symmetry is destroyed
  - Can't fractionalize electric charge because it is ill defined
  - **Standard charge-fractionalization Witten effect is gone**
- $U(1)_m$ magnetic symmetry is intact; no SSB
  - No SSB of $U(1)_m \implies$ magnetic confinement $\implies$ there are **magnetic string excitations with finite tension** that we can study

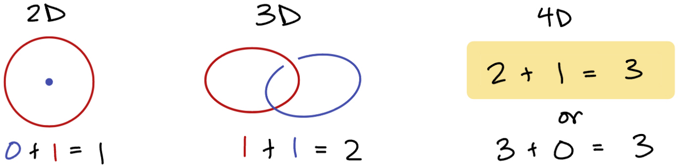
Topological Witten effect in charge-1 Abelian Higgs model

Claim: **a magnetic string excitation traveling around a 't Hooft loop operator pick up a phase shift** $e^{i\theta}$

Recall that for an Aharonov-Bohm phase, topology is important: need objects that **link**

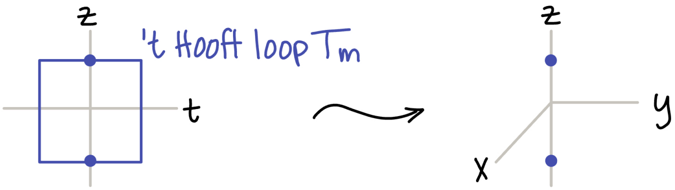
Sanity check: does it make sense to "link" a magnetic string excitation (2d worldvolume) and a 't Hooft loop (1d operator)?

Yes: linking is well-defined if dimensions of object add up to $d-1$:

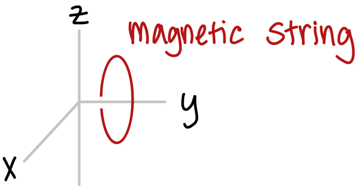


Visualizing linking in 4d

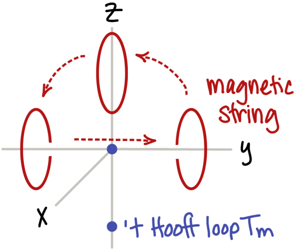
Put the 't Hooft loop T_m in the tz plane, visualize a timeslice:



Timeslice of the magnetic string worlsheet is just a loop:

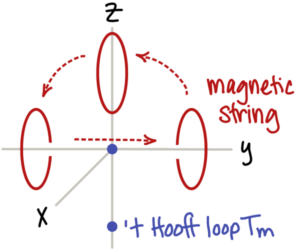


Topological Witten effect in charge-1 Abelian Higgs model



3d timeslice of theory with $\theta \neq 0$

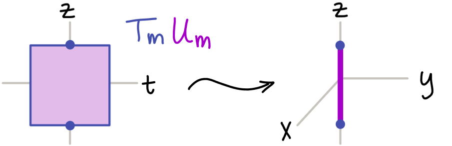
Topological Witten effect in charge-1 Abelian Higgs model



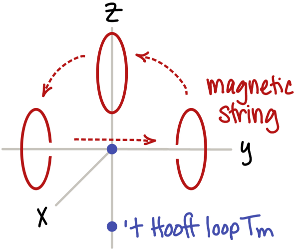
3d timeslice of theory with $\theta \neq 0$

Method: trade θ term for surface operator

$$\Big\langle T_m(C)\ldots\Big\rangle_\theta = \Big\langle T_m(C)\underbrace{e^{\left(\frac{i\theta m}{2\pi}\int_D \mathrm{d}a\right)}}_{\text{1-form sym generator ``}U_m\text{''}}\ldots\Big\rangle_0$$

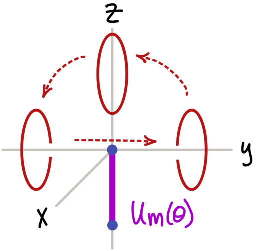


Topological Witten effect in charge-1 Abelian Higgs model



3d timeslice of theory with $\theta \neq 0$

=



3d timeslice of theory with $\theta = 0$

Symmetry surface U_m acts on magnetic string when they intersect and gives a phase

Topological Witten effect in charge-1 Abelian Higgs model

Punchline:

A magnetic string excitation traveling around a 't Hooft loop operator picks up an Aharanov-Bohm phase

Related: note that 't Hooft operators themselves create magnetic string excitations

- This phase can be interpreted as an interference effect between magnetic strings with intersecting worldsheets

$\implies$ this phase affects scattering processes of magnetic strings

Conclusion

- We reviewed the way the Maxwell θ term leads to electric charge fractionalization, AKA the "charge" Witten effect
- We considered a new perspective for the effect of θ: it attaches surfaces to 't Hooft loops
- We defined this new perspective to be **"topological Witten effect"**
  - Requires fewer ingredients — good for generalizations!
- Topological Witten effect has physical consequences: **Aharonov-Bohm phases** for strings

Teaser

- Secretly this whole story of the topological Witten effect and the new way to look at θ terms is much more general than Maxwell theory or the Abelian Higgs model
- We have a formal way to construct θ terms in all kinds of theories
  - Uses technology from TQFTs
  - Find **many** new θ terms — in 4d $U(1)$ gauge theory, 4d $SU(N)$ Yang-Mills...
- Generalized Aharonov-Bohm effects show up in other examples

Thank you!

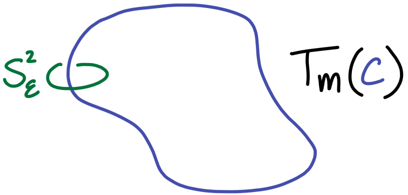
Backup slides

5 Symmetry surface attachment

Symmetry surface attachment

How to handle $T_m(C)$ in correlation functions:

View $T_m(C)$ as a defect that prescribes $\int_{S^2_\epsilon} \mathrm{d}a = 2\pi m$ for little spheres S^2_ϵ that link with C



Delete a tiny neighborhood of C from spacetime and by hand enforce the boundary condition on $\int \mathrm{d}a$

Symmetry surface attachment

On e.g. S^4 for simplicity, inserting a $T_m(C)$ "defect" gives:

$$\exp\left\{\frac{i\theta}{8\pi^2}\int_{\substack{S^4 \text{ minus} \\ \text{neighborhood} \\ \text{of } C}} \mathrm{d}a \wedge \mathrm{d}a\right\} = \cdots \underset{\text{think about topology}}{\text{use Stoke's theorem,}} \cdots$$

$$= \exp\left\{\frac{i\theta}{4\pi^2}\left(\int_{S^2_\epsilon} \mathrm{d}a\right)\left(\int_D \mathrm{d}a\right)\right\}$$

$\Downarrow$ enforce boundary condition

$$= \exp\left(\frac{i\theta m}{2\pi}\int_D \mathrm{d}a\right)$$