Evidence for a second finite temperature QCD transition from center vortices
+ some speculations

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(3) University of Adelaide, Australia

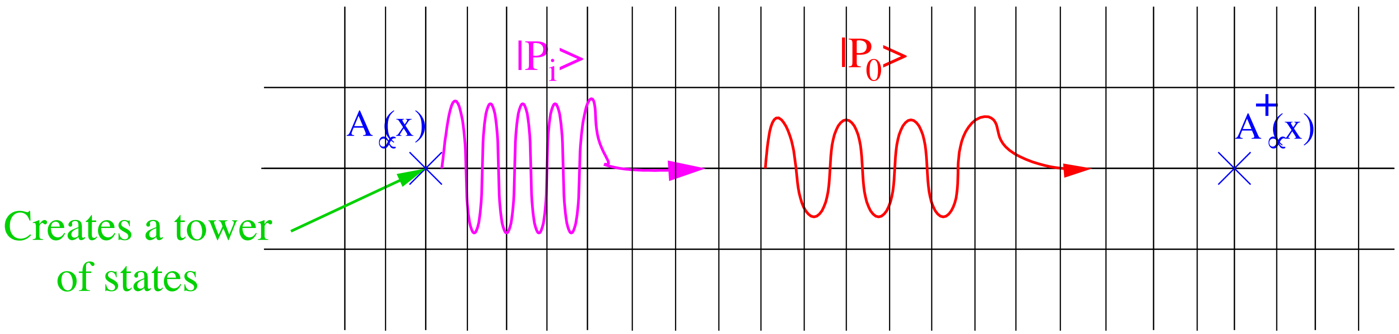
FASTSUM Collaboration

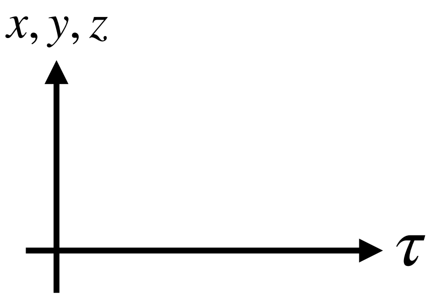
Overview

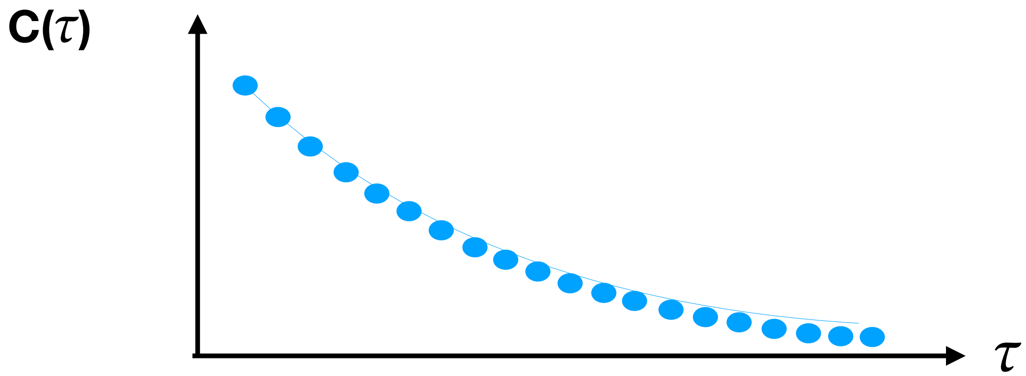
- FASTSUM approach
    - Anisotropic
- Maximal Centre Gauge
    - Vortices — Faber, Greensite, Olejník Phys.Lett.B 474 (2000) 177
- Measurements — *First study of Centre Vortices in thermal QCD*
    - Vortex & Branching Point Density
    - Cluster Extent
    - Correlations
- 2 Transition(s) in QCD ?
- *Some Speculations*

FASTSUM Approach:

Anisotropic Lattice







Spectral Quantities:

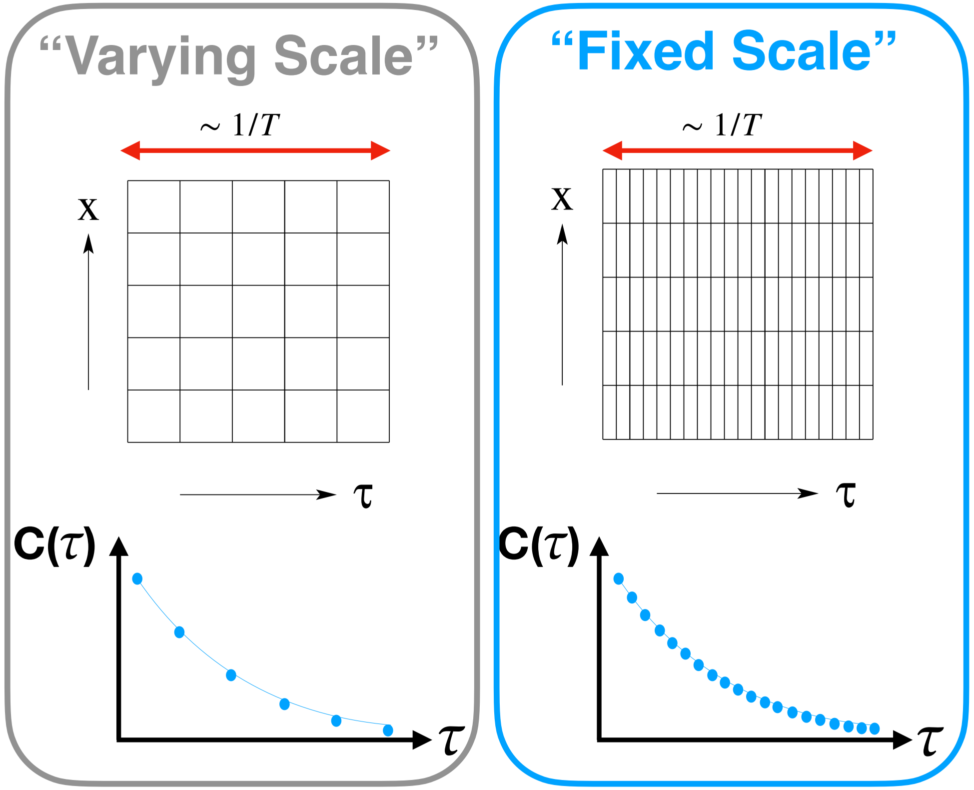
- Bottomonium
- Charmed mesons
- Heavy Baryons
- Light Hadrons

Interquark potential

Conductivity

FASTSUM Approach:

Anisotropic Lattice + *Fixed* Scale

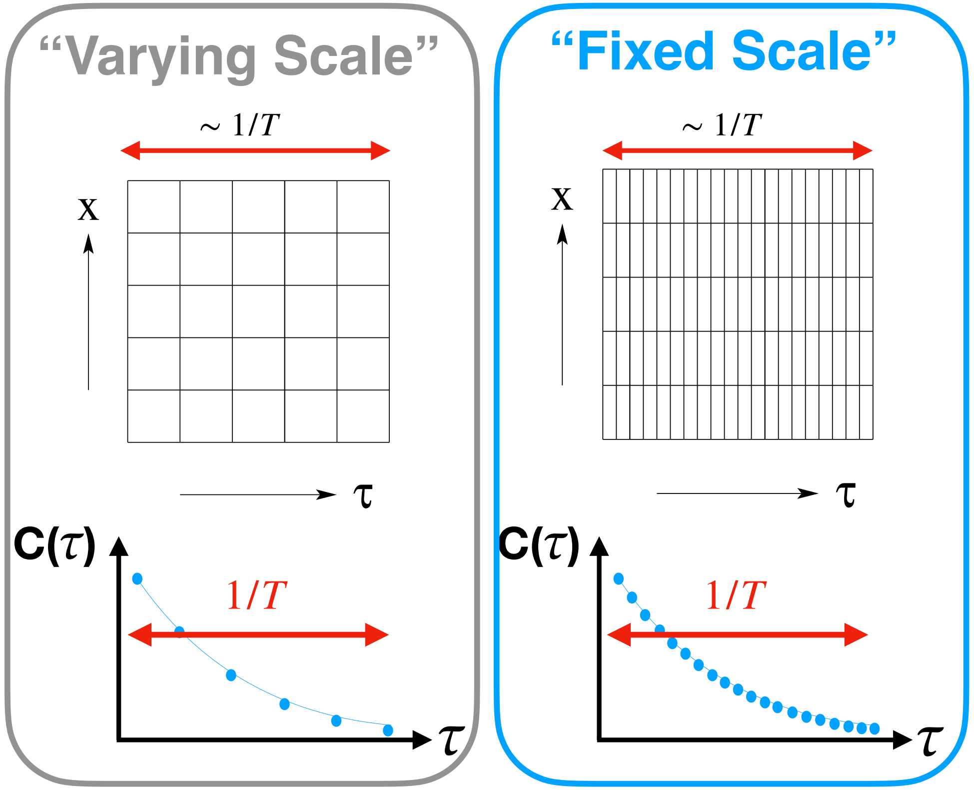


$$\sum_i \langle i | e^{-H L_\tau} | i \rangle = \sum_i \langle i | e^{-H/T} | i \rangle$$

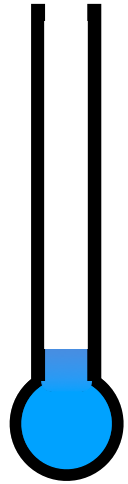
$$T = \frac{1}{L_\tau} = \frac{1}{a_\tau N_\tau}$$

FASTSUM Approach:

Anisotropic Lattice + *Fixed* Scale



$$a_\tau \to 0 \qquad N_\tau \to 0$$



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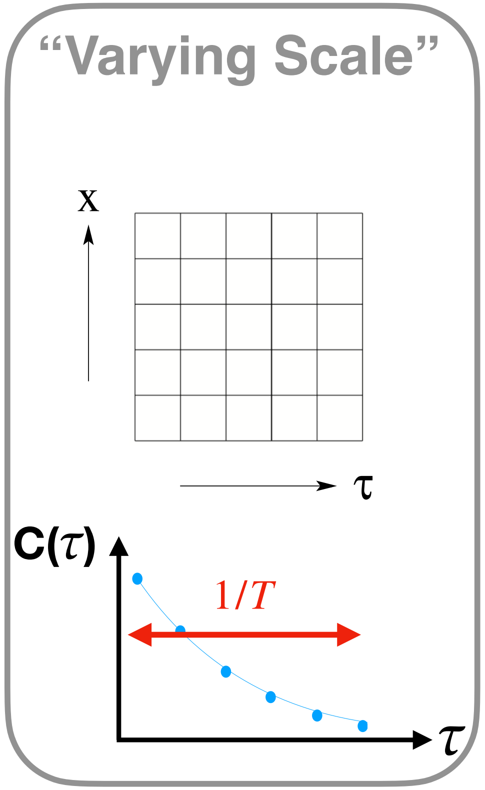
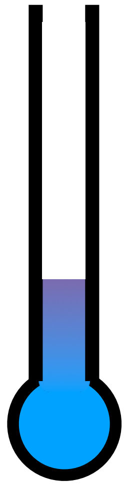


$a_\tau \to 0$ $\qquad$ $N_\tau \to 0$

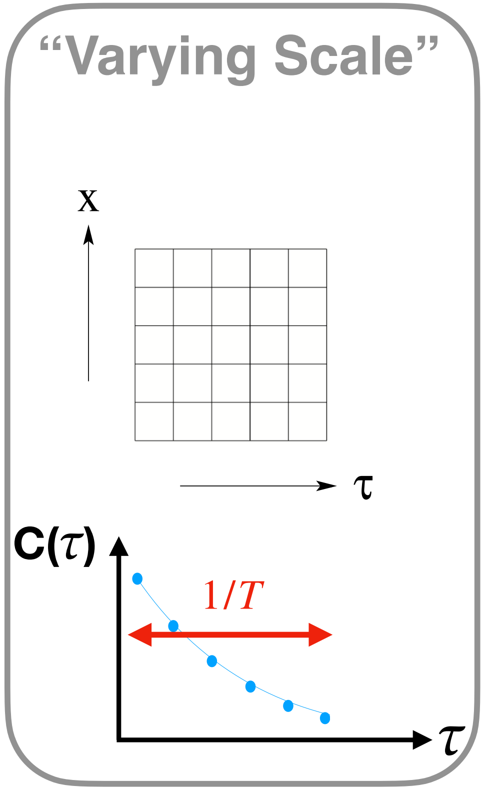


Going hotter…

$$T = \frac{1}{L_\tau} = \frac{1}{a_\tau N_\tau}$$

FASTSUM Approach:

Anisotropic Lattice + *Fixed* Scale



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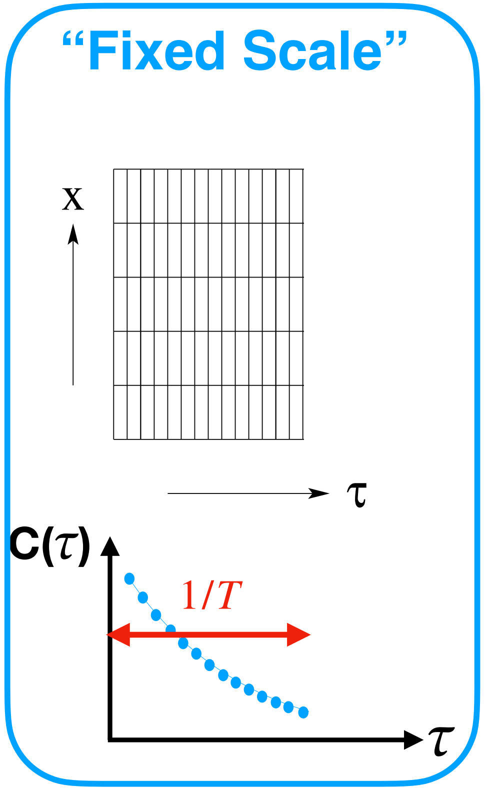
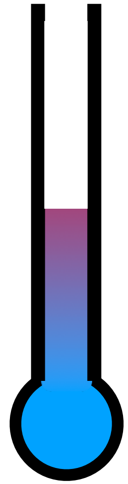


$N_\tau \to 0$

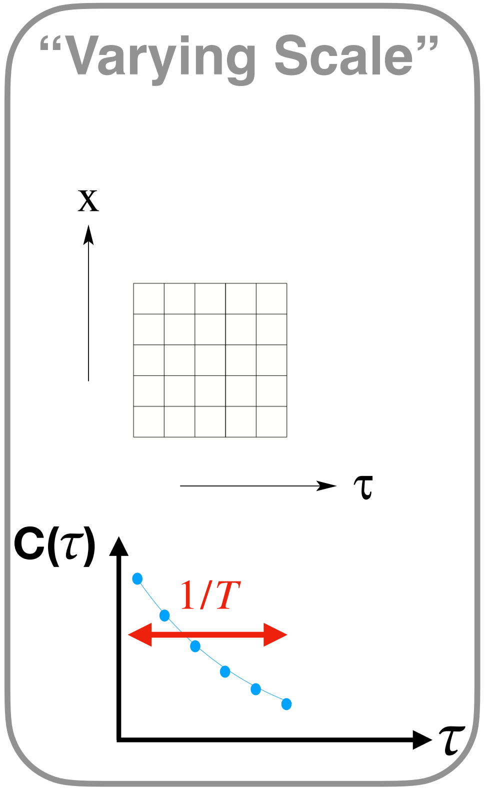


Going hotter…

$$T = \frac{1}{L_\tau} = \frac{1}{a_\tau N_\tau}$$

FASTSUM Approach:

Anisotropic Lattice + *Fixed* Scale



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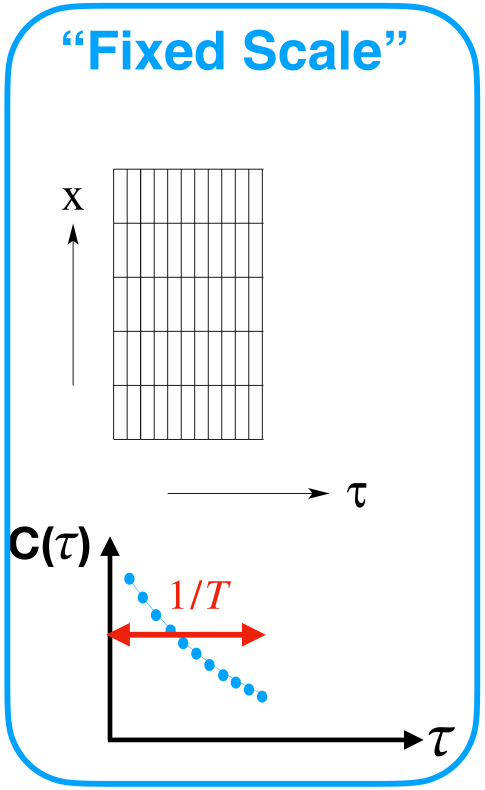
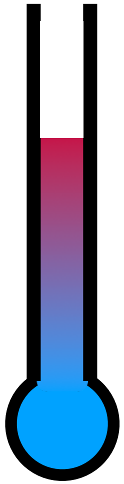


$$N_\tau \to 0$$



Going hotter…

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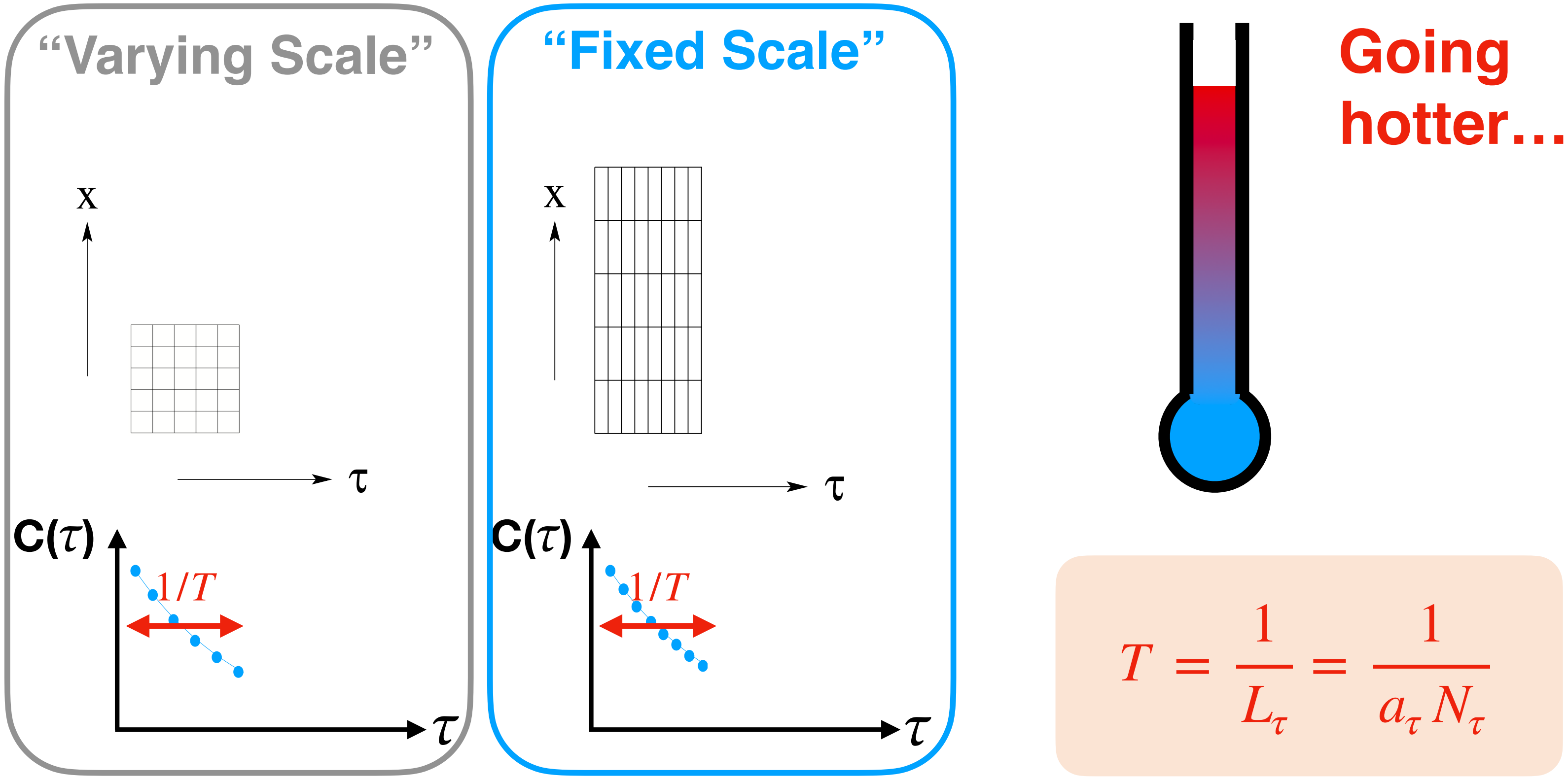
FASTSUM Approach:

Anisotropic Lattice + *Fixed* Scale



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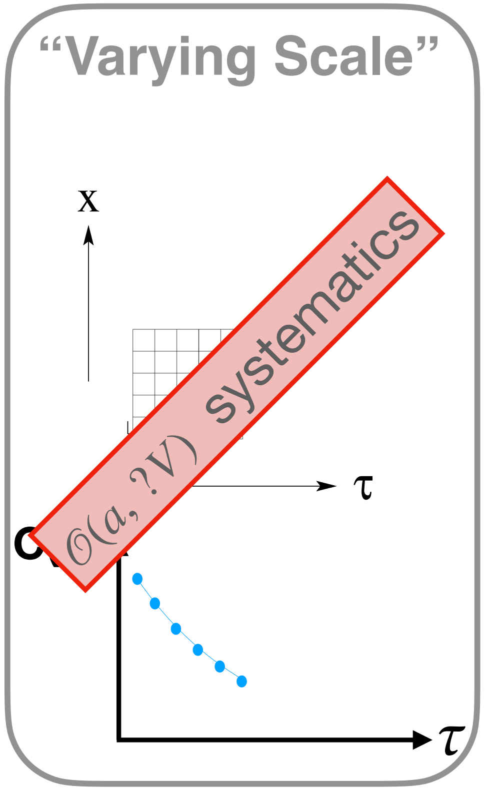
$N_\tau \to 0$

$$T = \frac{1}{L_\tau} = \frac{1}{a_\tau N_\tau}$$

FASTSUM Approach:

Anisotropic Lattice + *Fixed* Scale



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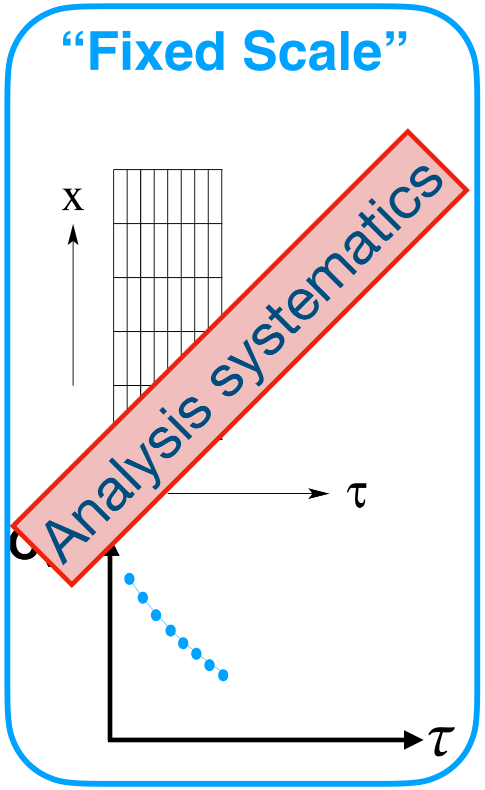
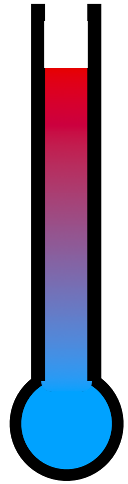


$N_\tau \to 0$

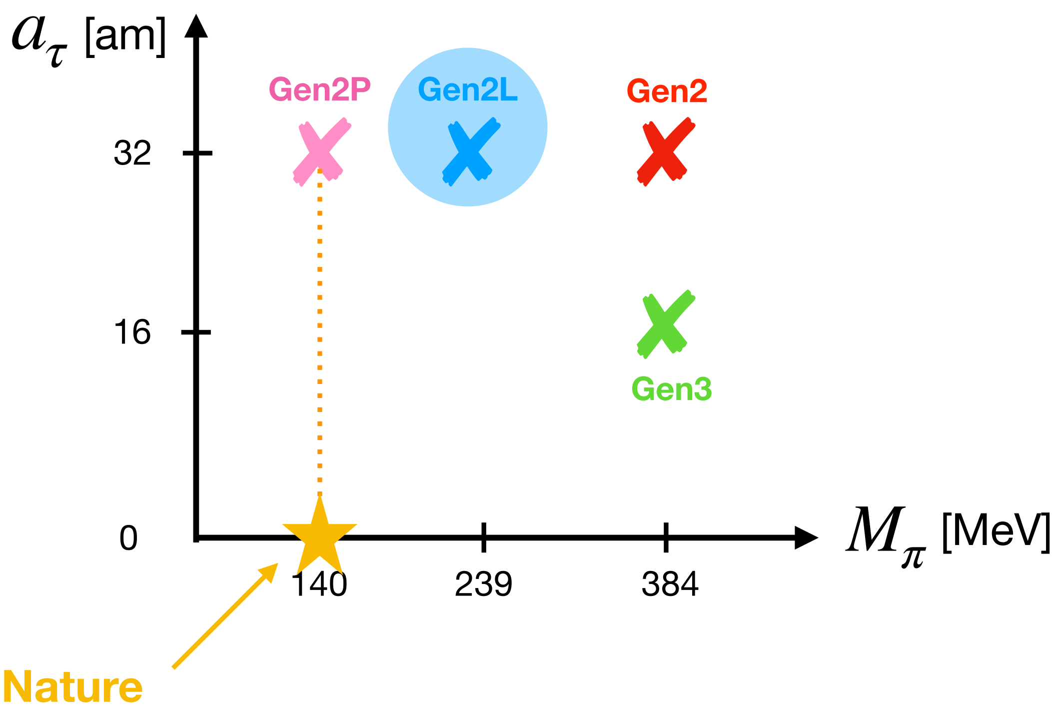


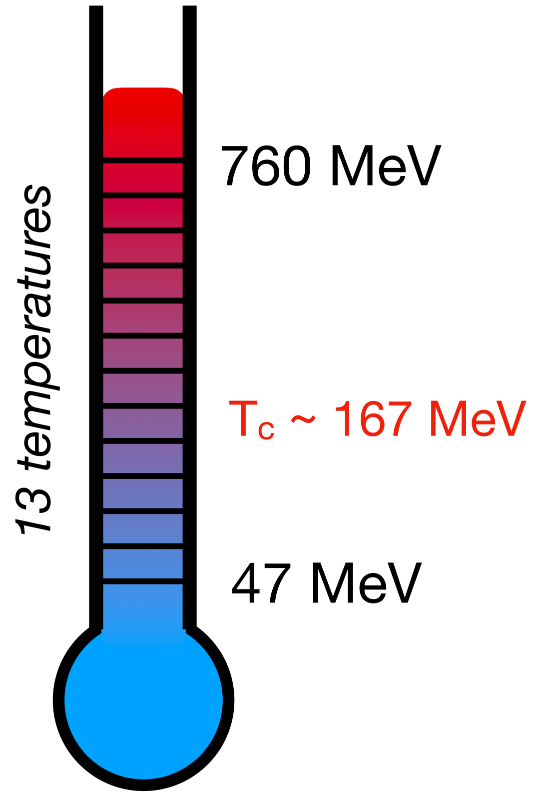
Going hotter…

$$T = \frac{1}{L_\tau} = \frac{1}{a_\tau N_\tau}$$

FASTSUM Approach:

Lattice Parameters





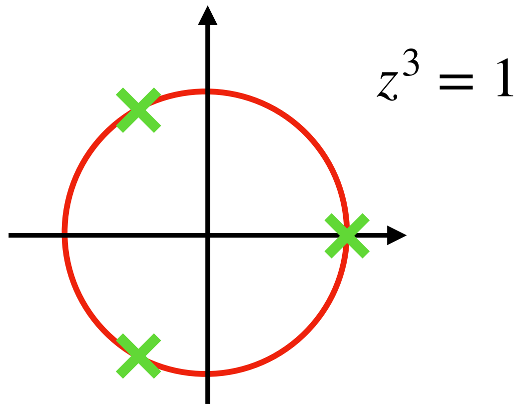
Generation 2L
(2+1) flavour
a_s ~ 0.112 fm

Gauge Action:
Anisotropic,
Symanzik-improved

Fermion Action:
Wilson-clover,
tree-level tadpole,
stout-smeared links

FASTSUM Phys.Rev.D105(2022)3, 034504

Maximal Centre Gauge



Choose gauge transform Ω st

$$U \longrightarrow U' = \Omega\, U\, \Omega' \approx z\, V \qquad \text{where } z \in Z(3) \text{ i.e. } z^3 = 1$$

$$\text{i.e.} \quad \approx e^{i\,2\pi/3\,n}\, V \qquad \text{where } n = \{-1,\, 0,\, +1\}$$

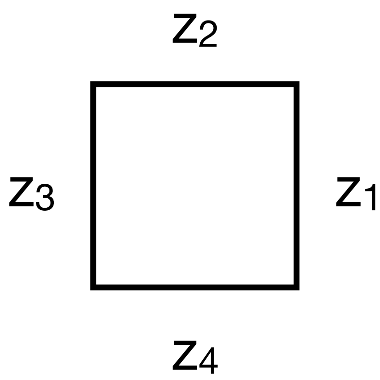
$$V \approx \text{Identity}$$

Non-pert

"Perturbative"

Can factorise $U' = \Omega\, U\, \Omega = e^{i\,2\pi/3\,n}\, V_{\text{pert}} = z\, V_{\text{pert}}$

Product around MCG Plaq = $U^{MCG}_{\text{plaq}} = \prod_{i=1}^{4} z_i \in Z(3)$



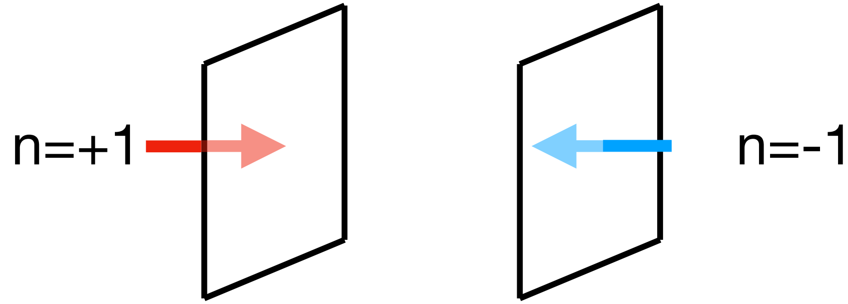
$\Longrightarrow U^{MCG}_{\text{plaq}}$ either

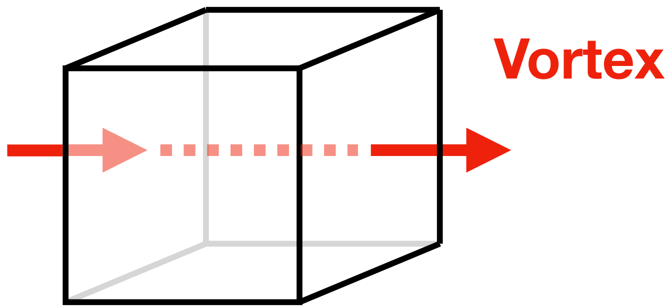
- $e^{\pm\, i\, 2\pi/3}$ "pierced"
- 1 not pierced

Maximal Centre Gauge

Vortices, Flux & Branching Points

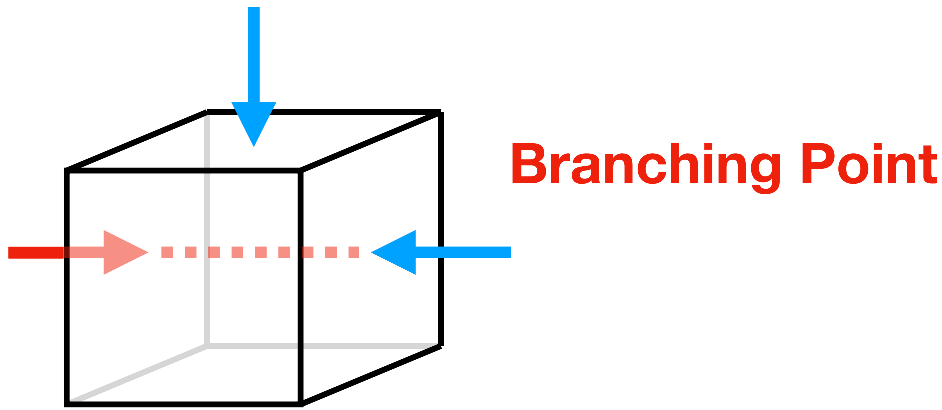
U^{MCG}_{plaq} either $\begin{cases} e^{\pm i\,2\pi/3} & \text{“pierced”} \\ 1 & \text{not pierced} \end{cases}$





$N_{tot} = +1 - 1 = 0$

i.e. $e^{2\pi i/3} \times e^{-2\pi i/3} = 1$



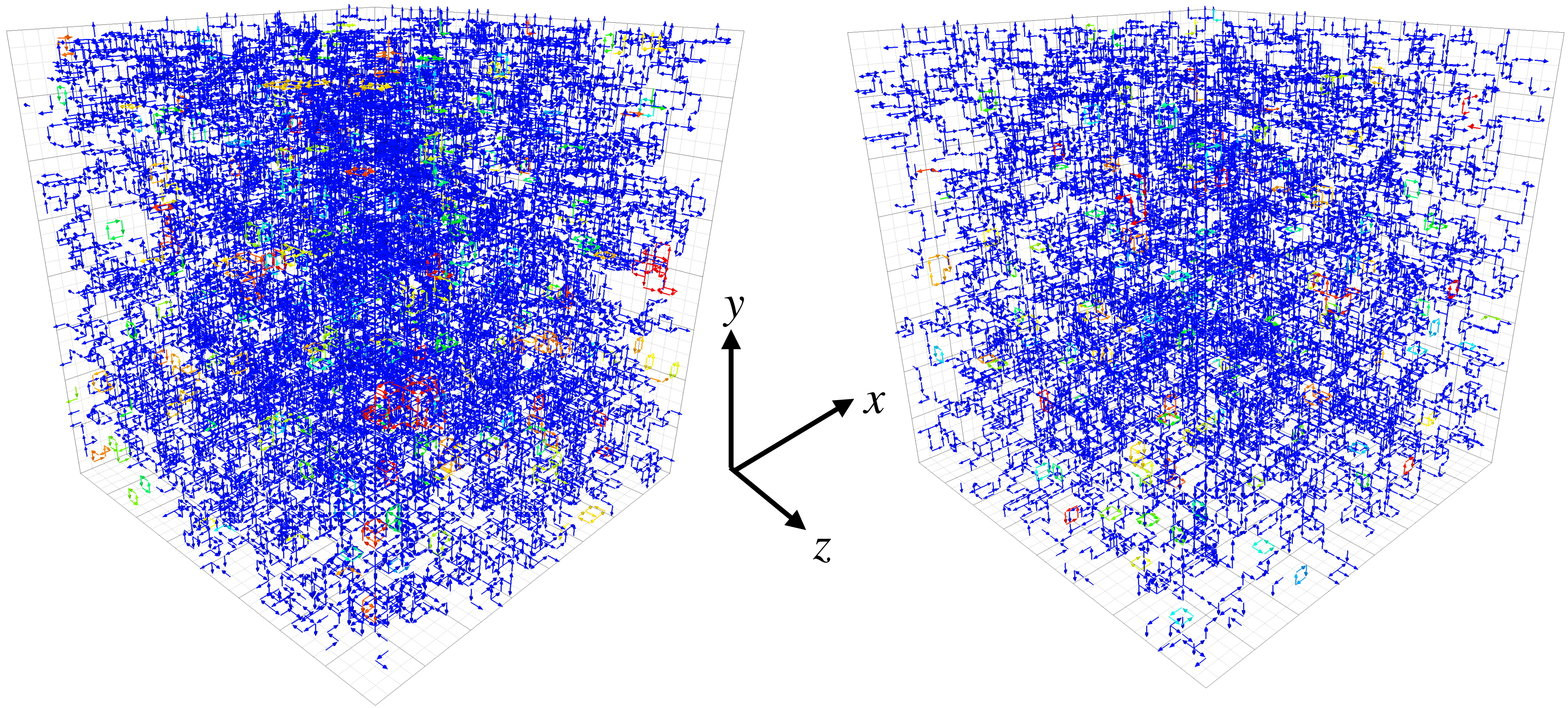
$N_{tot} = +1 + 1 + 1 \text{ mod } 3 = 0$

i.e. $e^{2\pi i/3} \times e^{2\pi i/3} \times e^{2\pi i/3} = 1$

Conservation of Flux modulo 3

0. Visualisation

Space-Space-Space

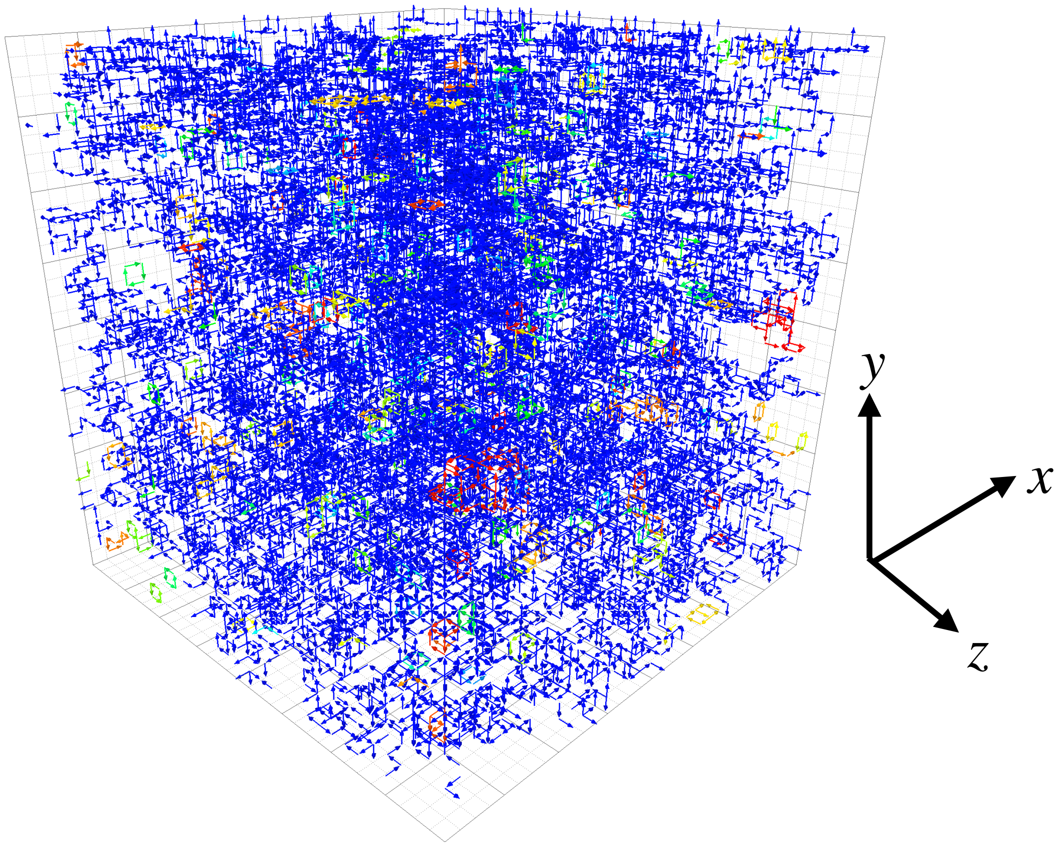


Nt=64 T=95MeV

Nt=8 T=760MeV

0. Visualisation

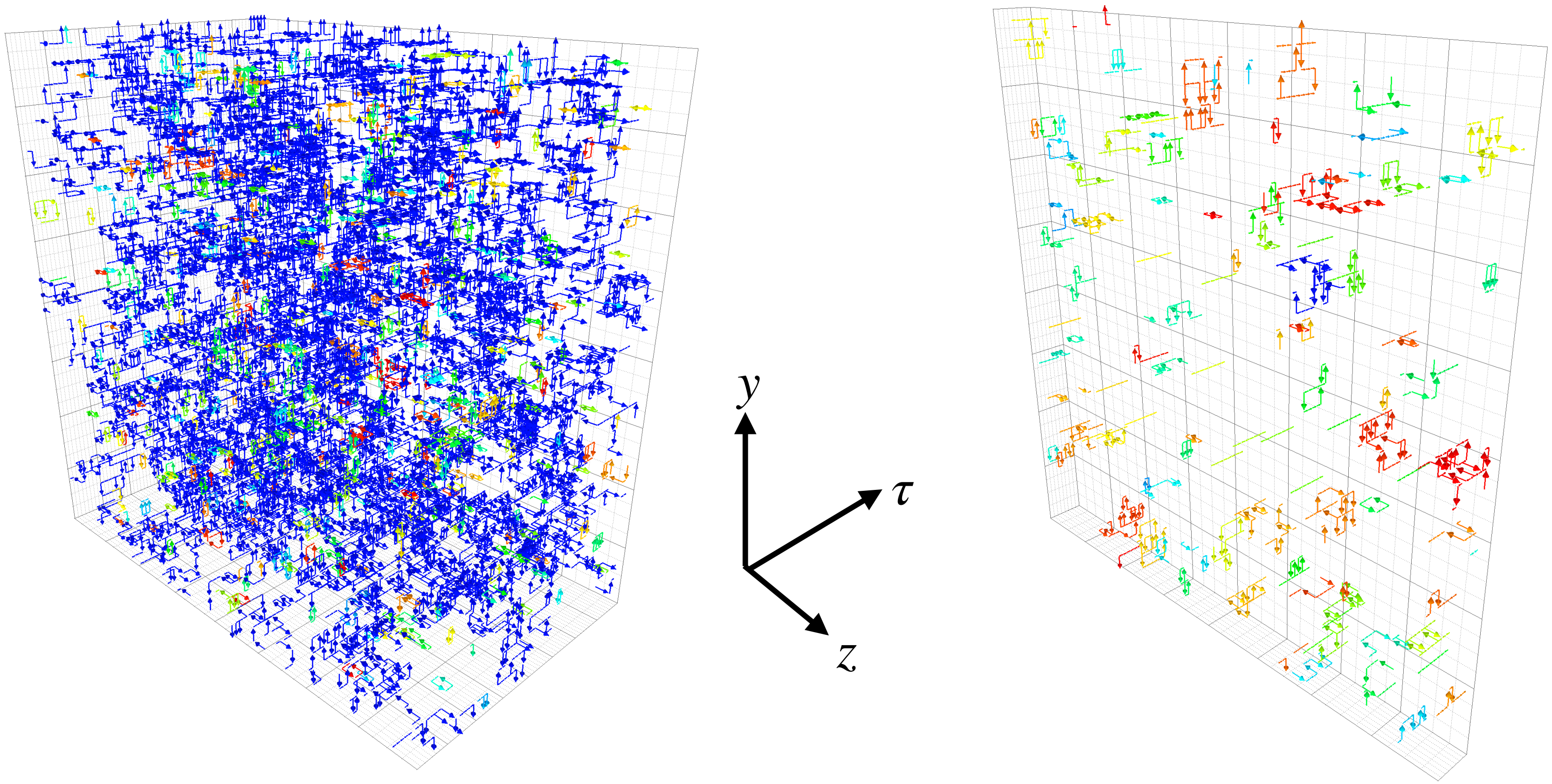
Space-Space-Space



Nt=64 T=95MeV

0. Visualisation

Space-Space-Time



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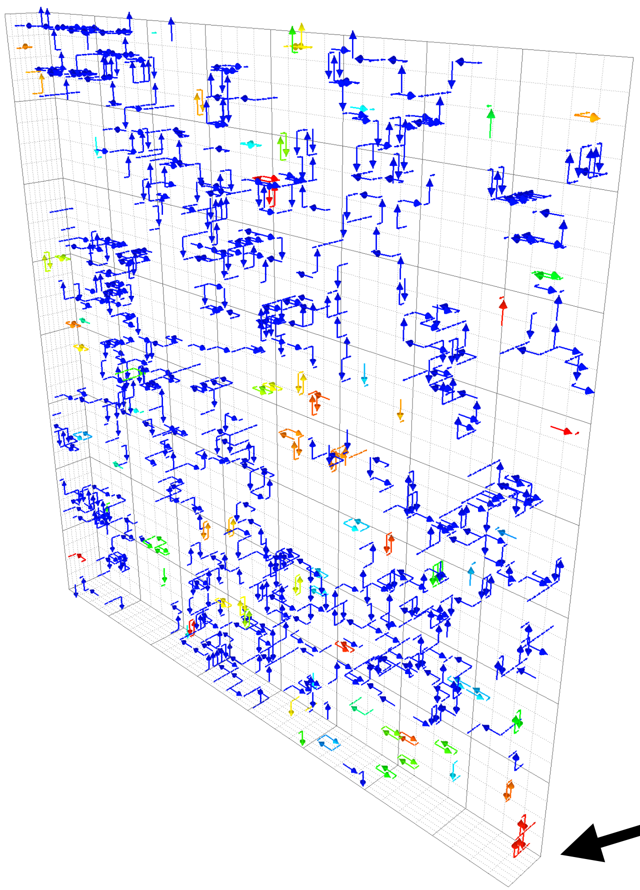
Space-Space-Time



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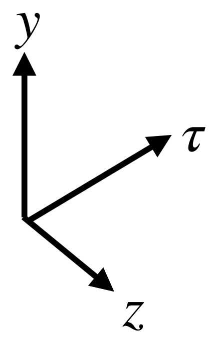
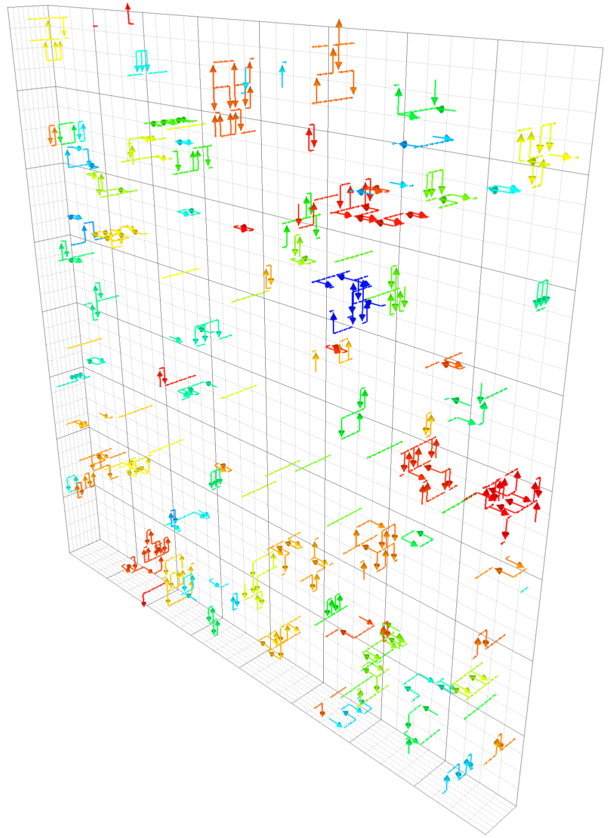


CROPPED

to 8 timeslices

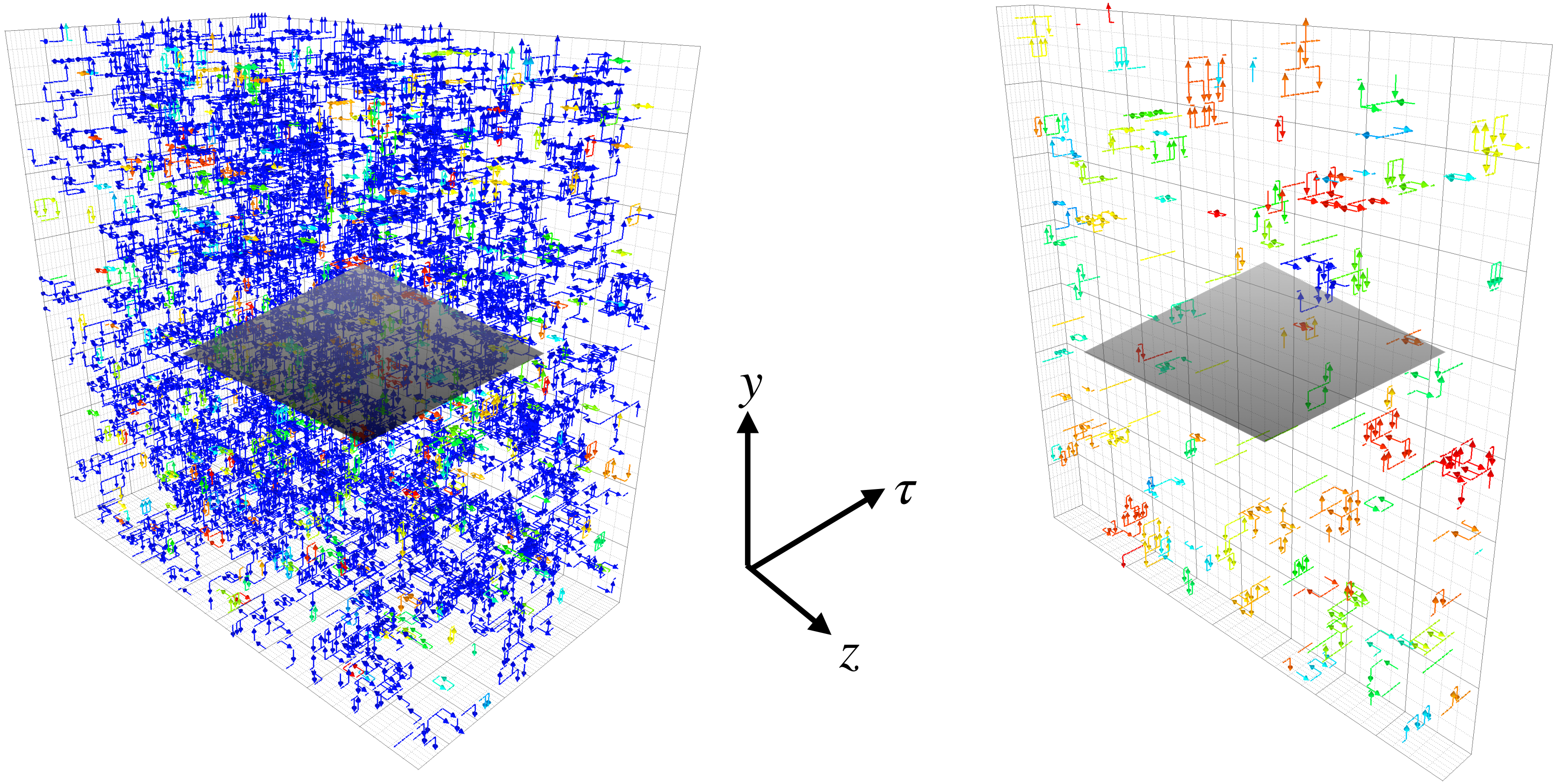


Nt=8 T=760MeV

Connection with Percolation and Area Law

Engelhardt, Langfeld, Reinhardt, Tennert Phys.Rev.D 61 (2000) 054504

Mickley, Kamleh, Leinweber 2405.10670



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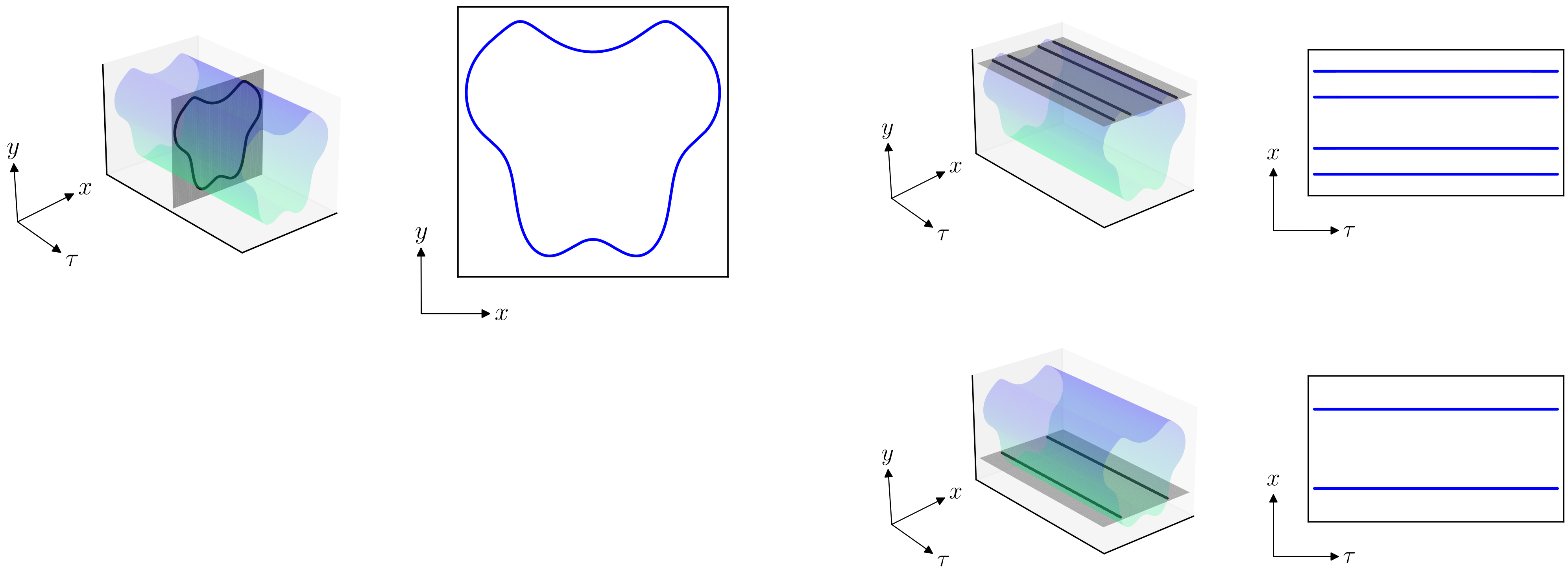
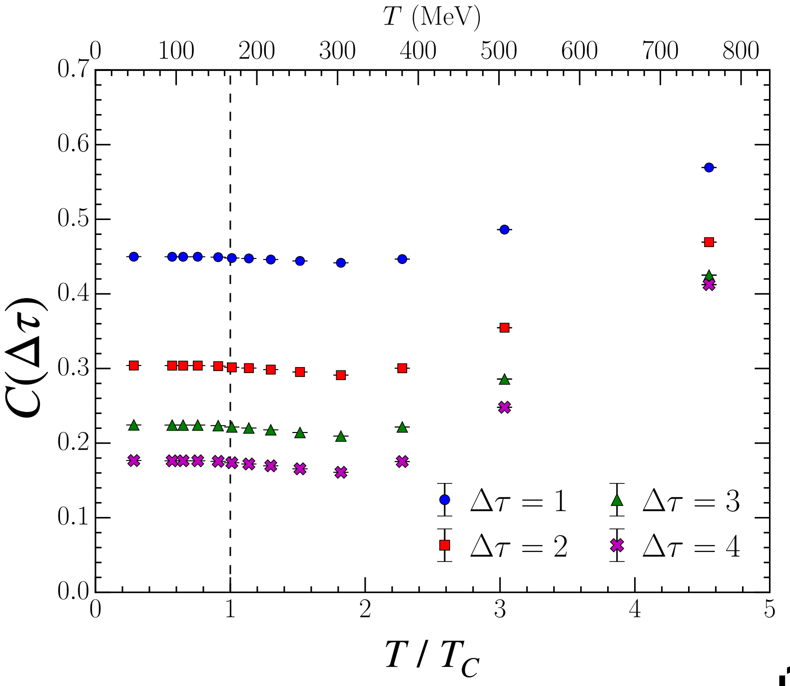
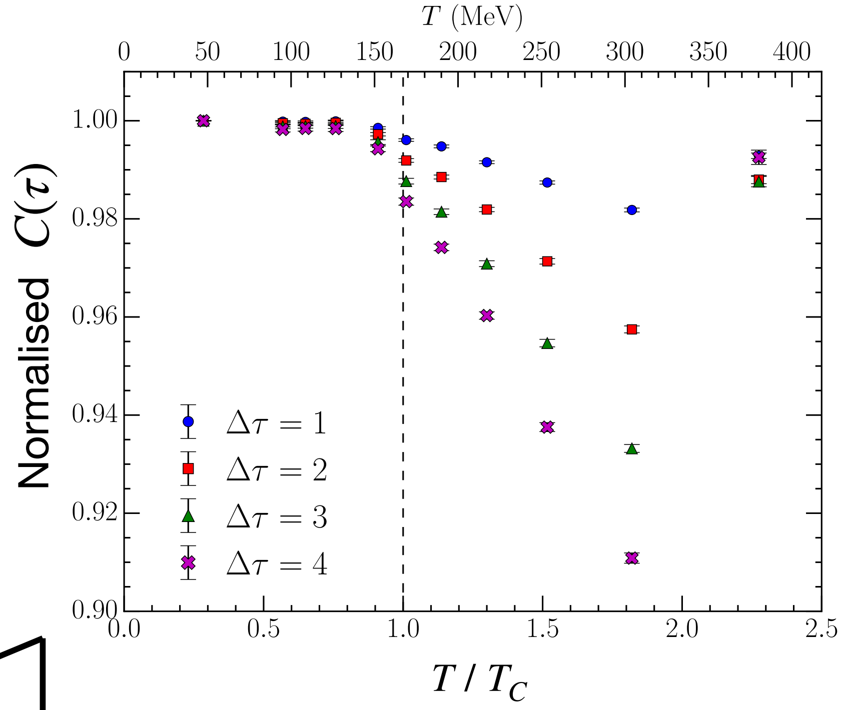
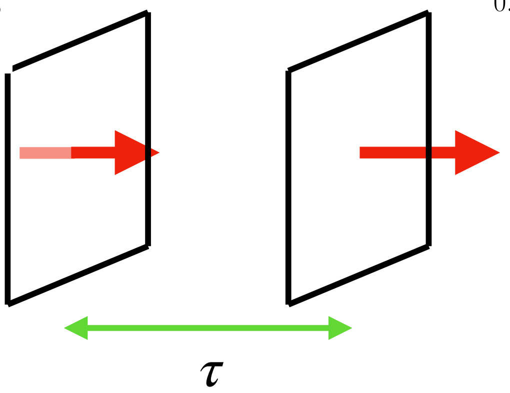


y
x
τ

1. Temporal Vortex Correlators

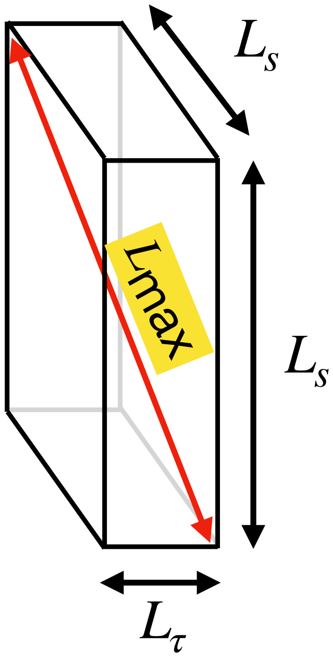






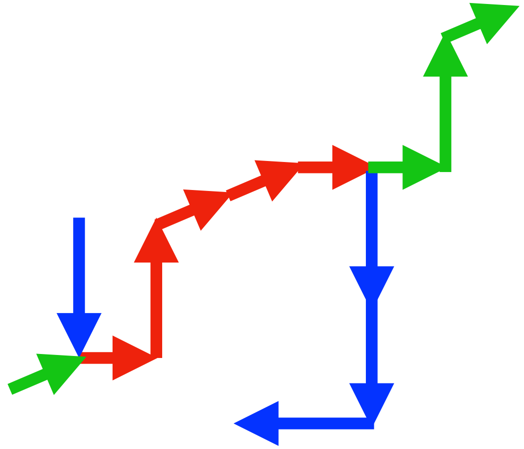
2. Cluster Extent

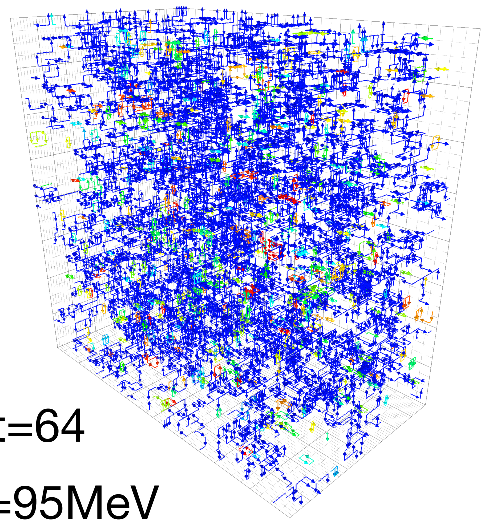
Space-Space-Time



$$\text{Normalised Cluster Extent} = \frac{\text{Cluster Extent}}{\frac{1}{2} L_{\text{max}}}$$

Periodic B.C.'s





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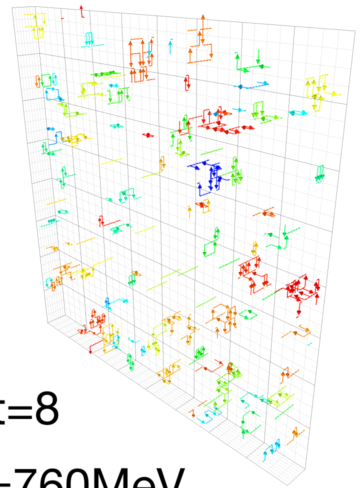

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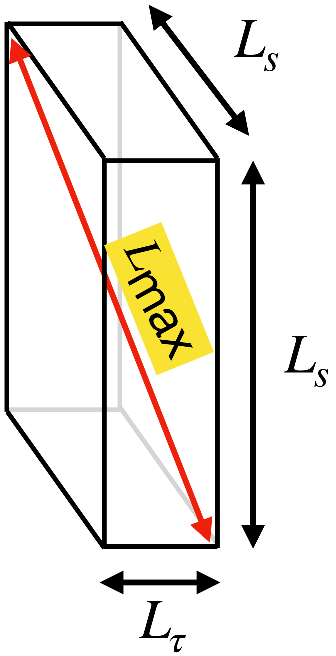
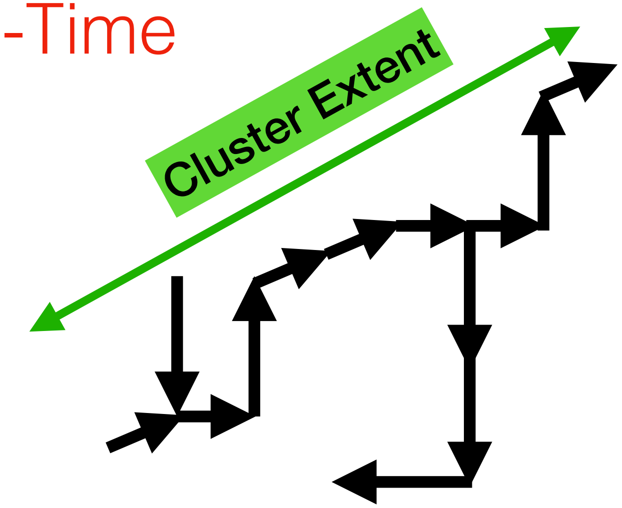
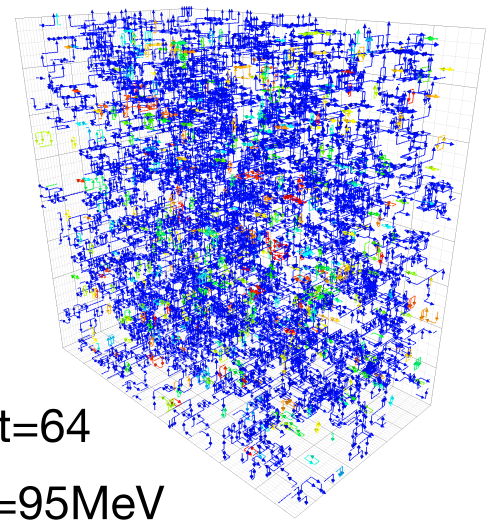
Space-Space-Time





$$\text{Normalised Cluster Extent} = \frac{\text{Cluster Extent}}{\frac{1}{2} L\text{max}}$$

Periodic B.C.'s



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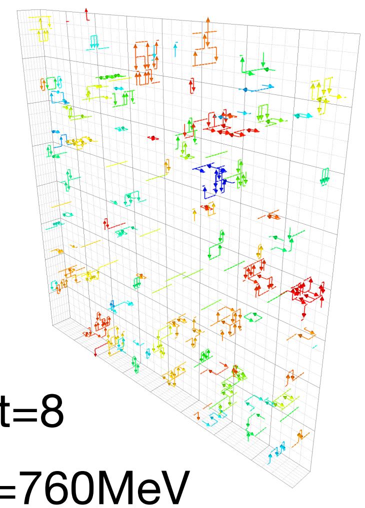

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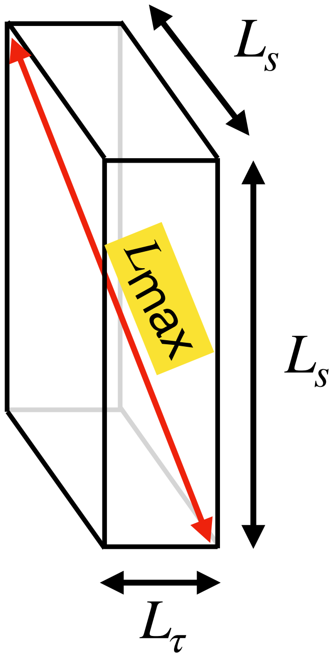
2. Cluster Extent

Space-Space-Time



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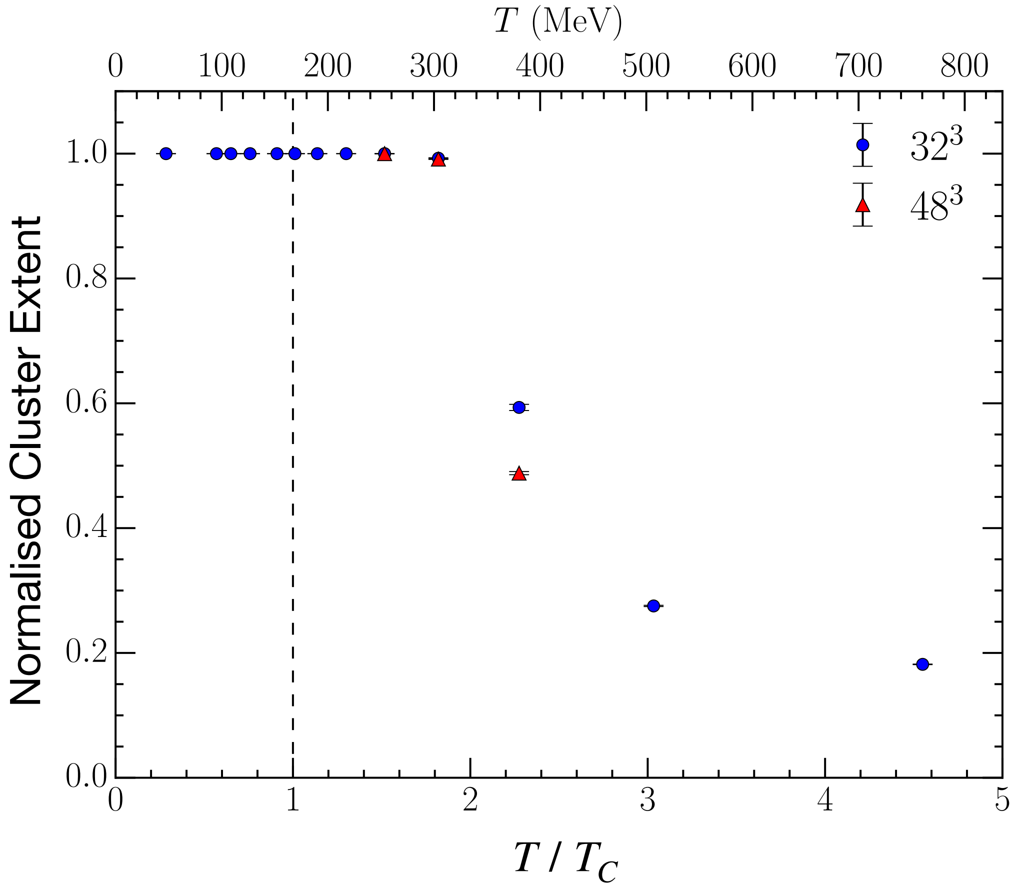
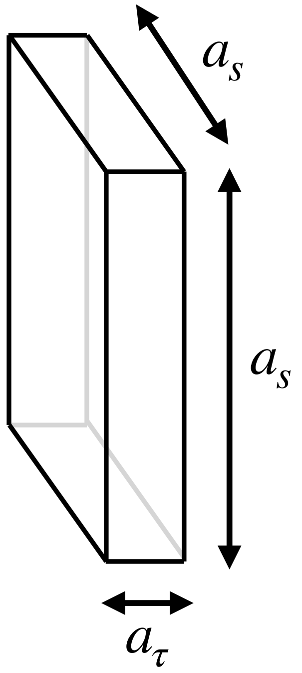


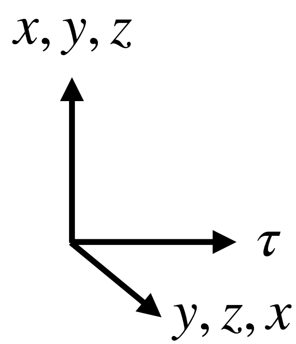
Maximal Centre Gauge

Anisotropic Lattices

Reminder: $a_\tau \ll a_s$

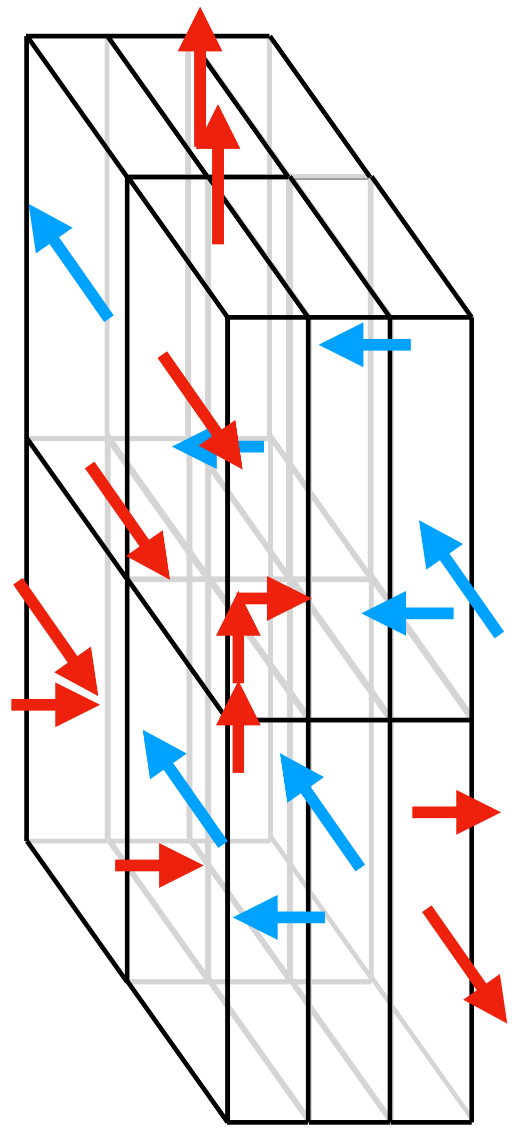
Fundamental 3-Vol:





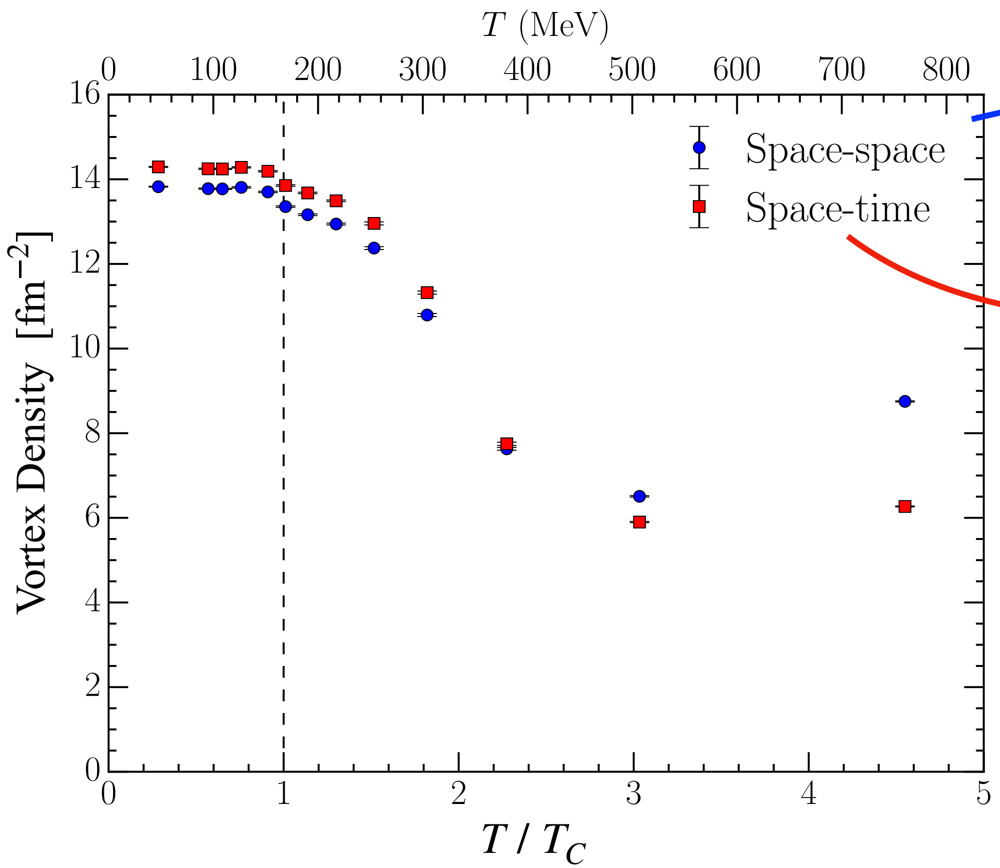
Check:

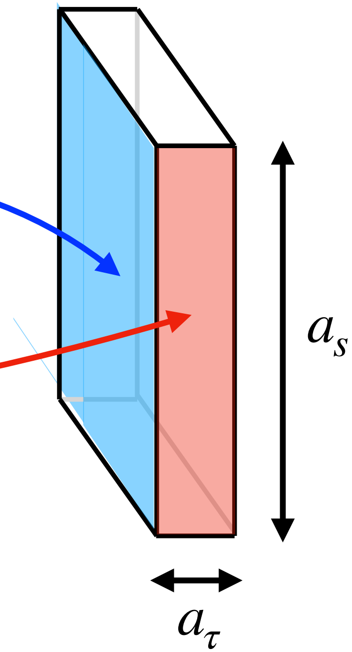
No. of Vortices / Area is isotropic



3. Vortex Density

Number per Area





Method:

1. MCG with isotopic functional
2. MCG with anisotropic

T_C from Chiral Condensate

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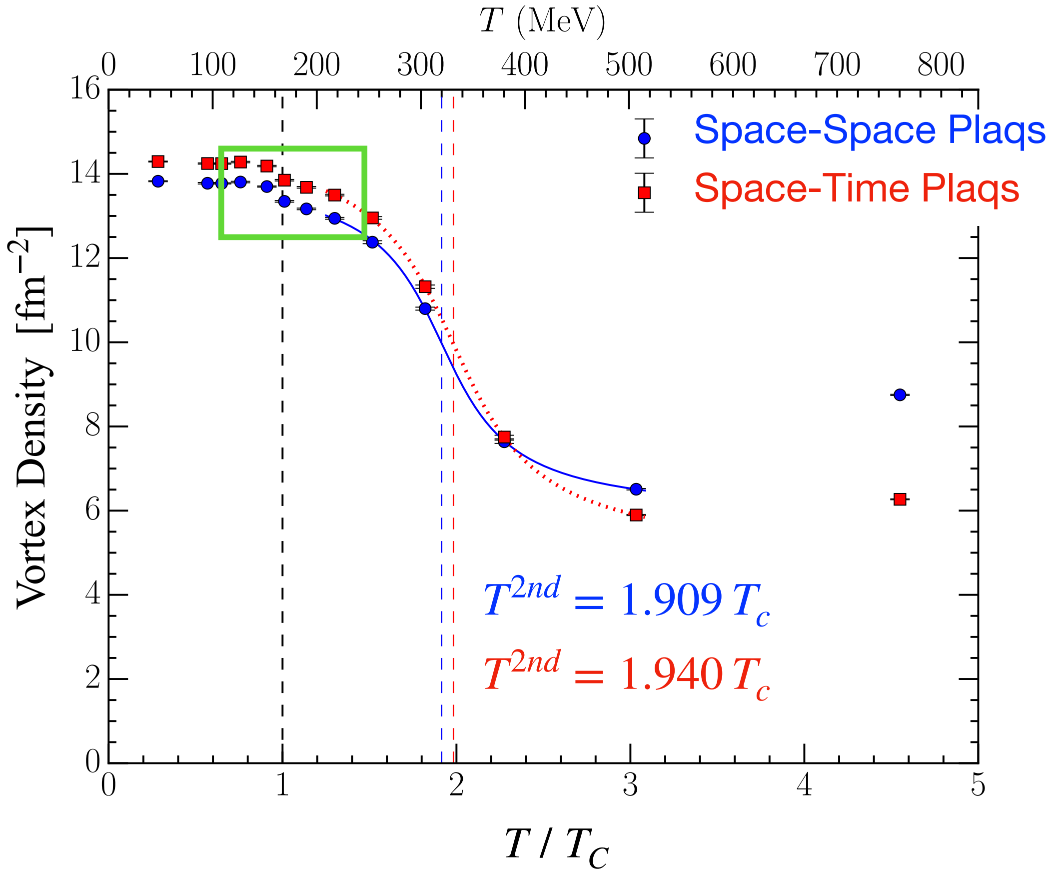
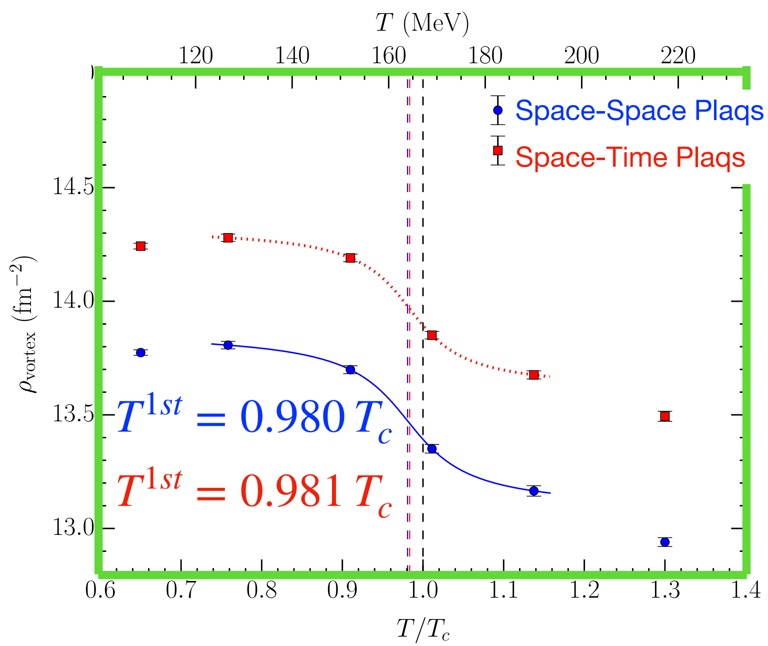
3. Vortex Density

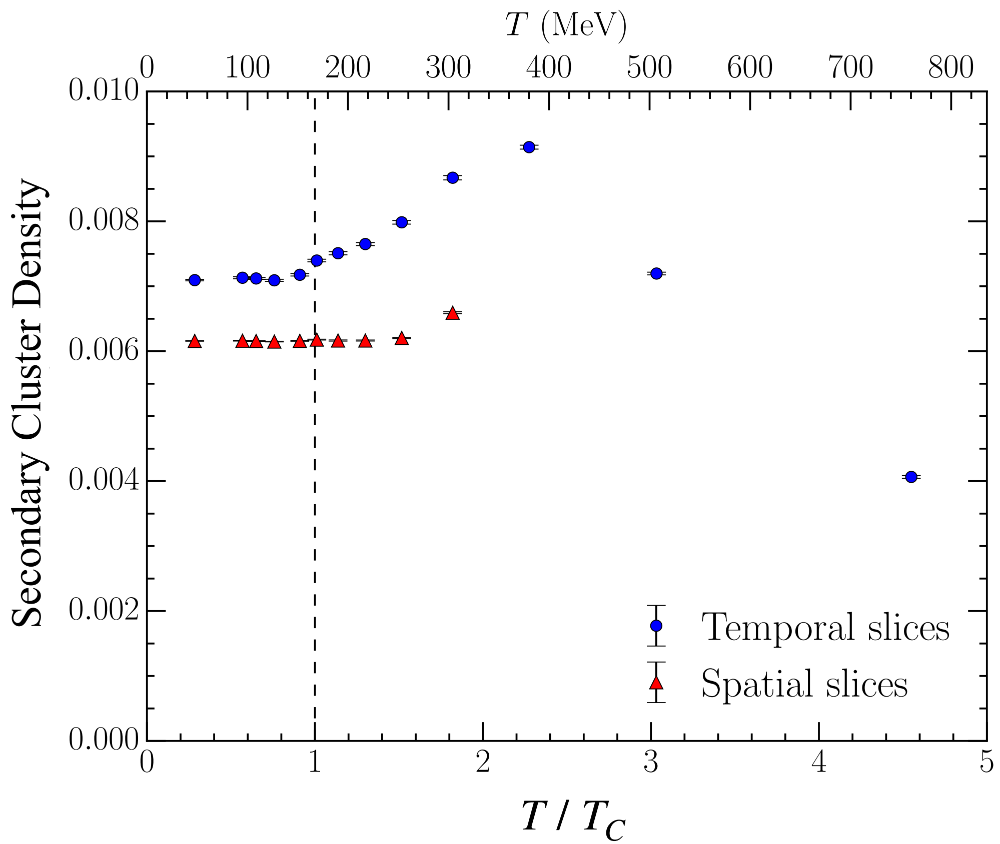
Fits around transitions

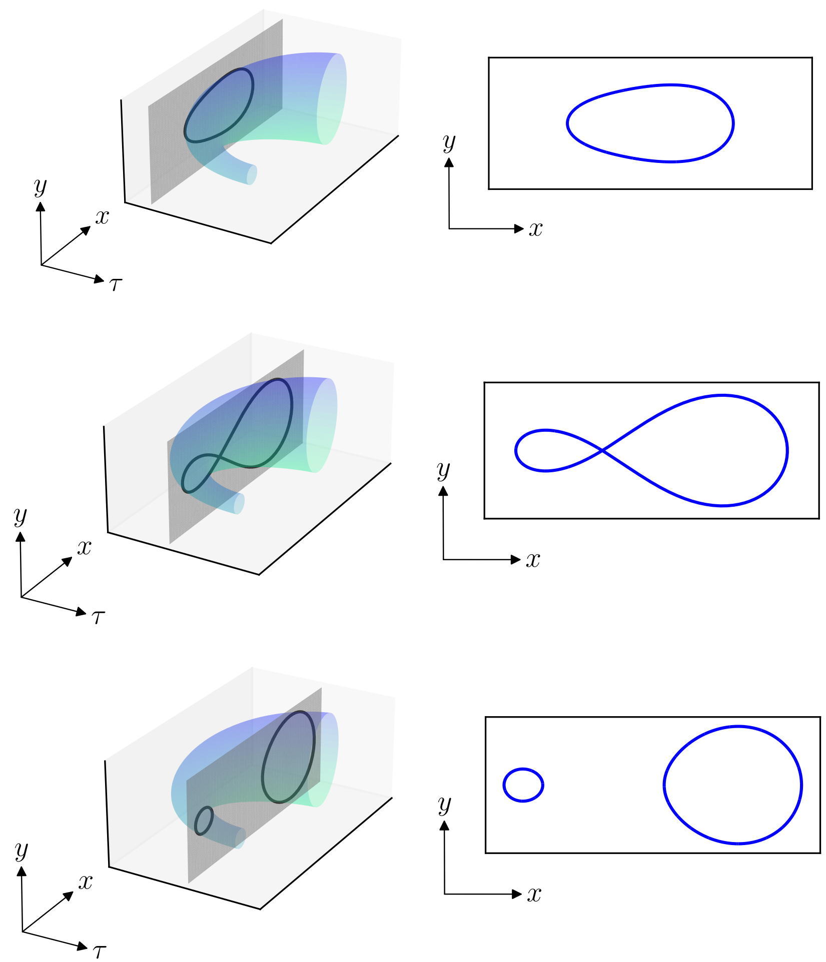


Fit around T_C



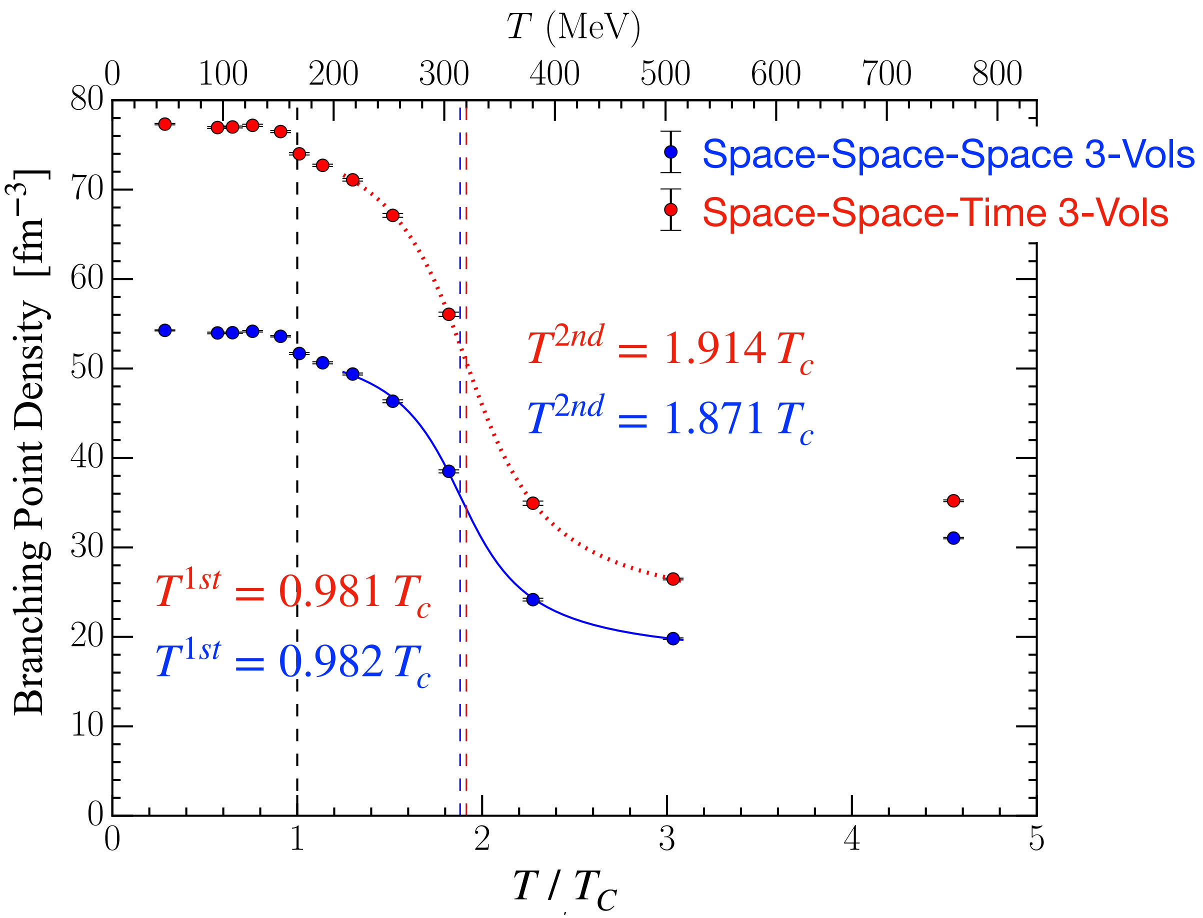
4. Secondary Clusters

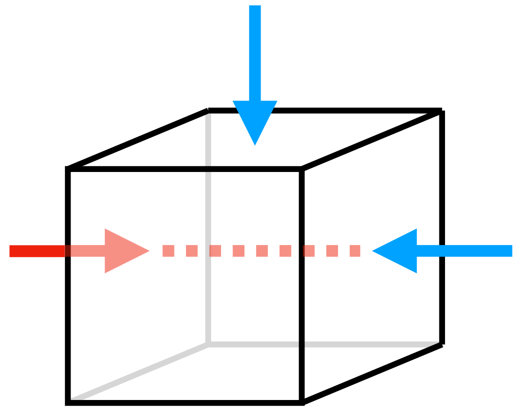




5. Branching Point Density

Number per 3-Volume





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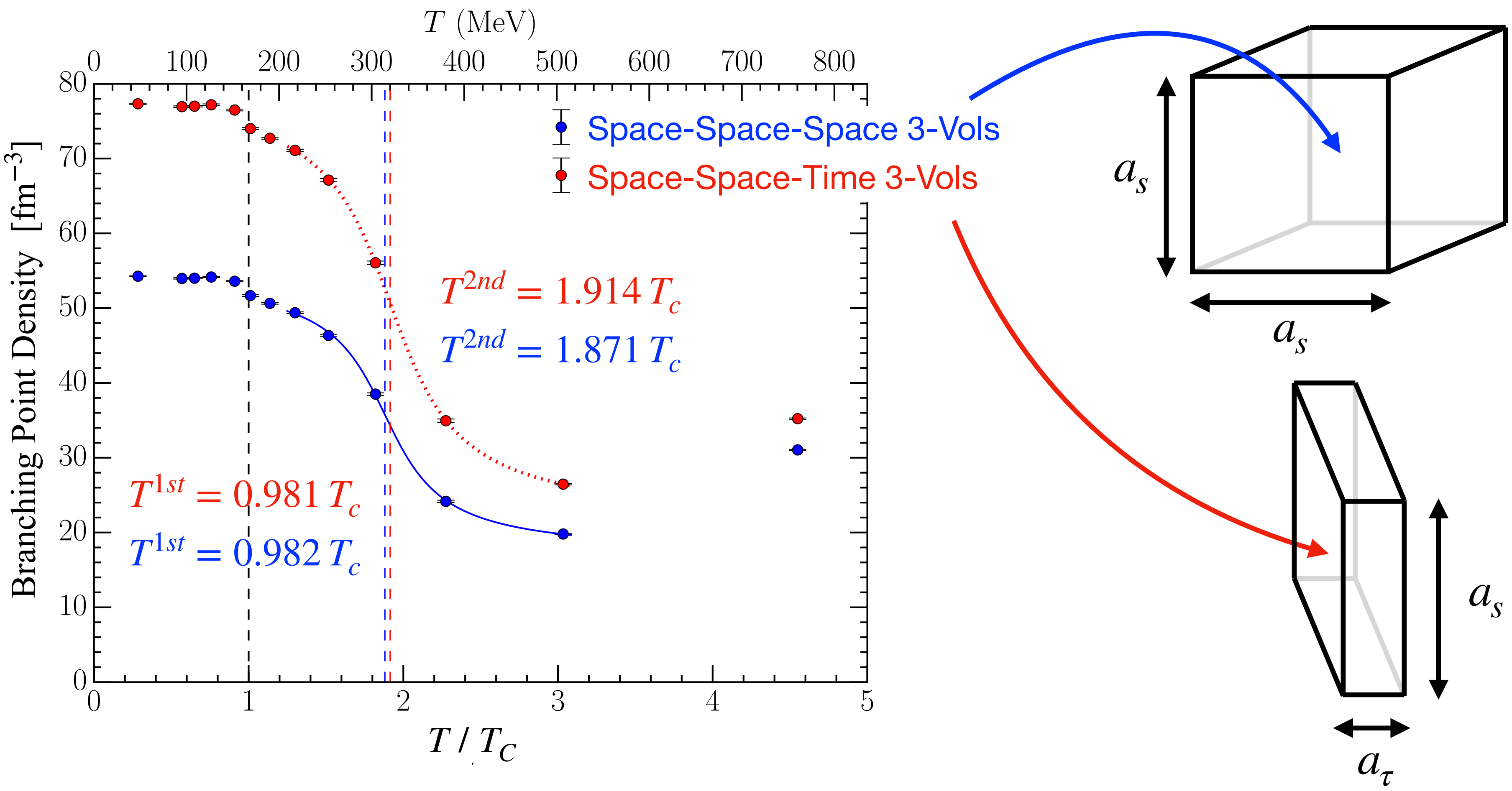
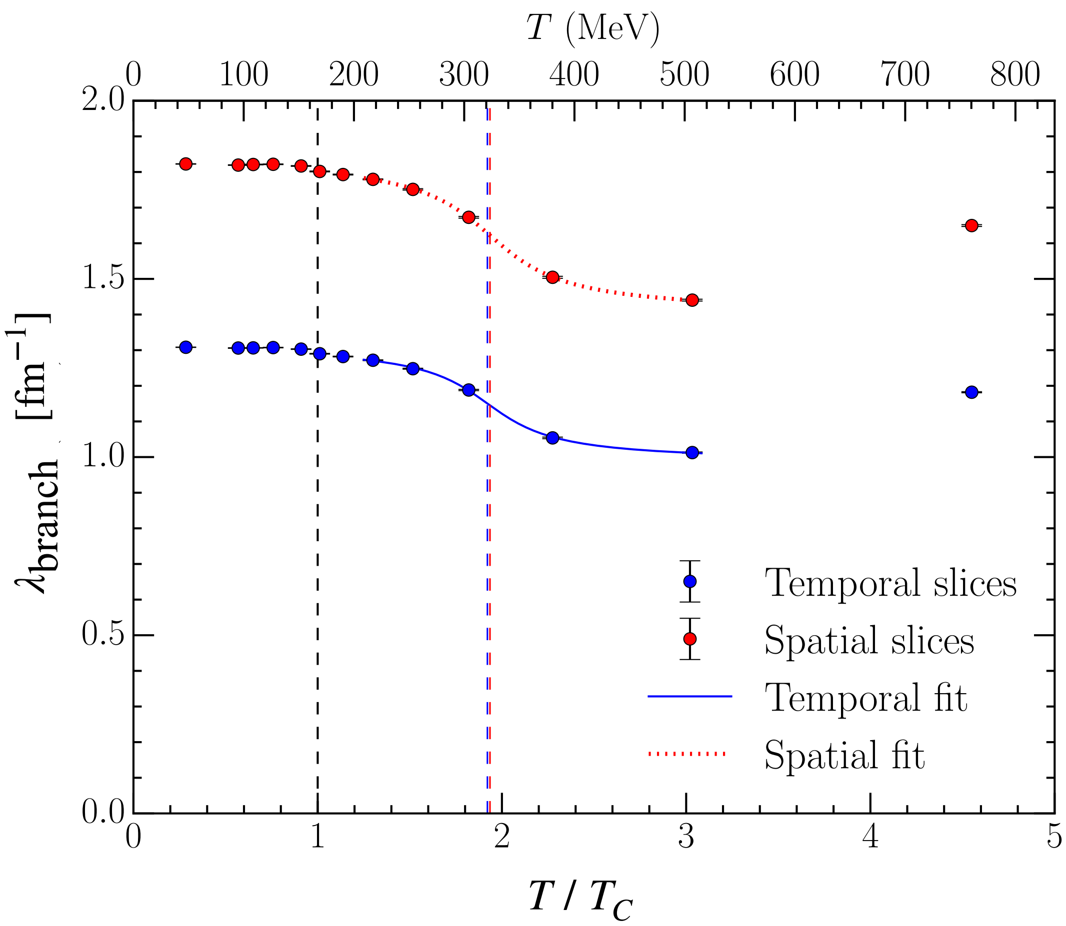
Number per 3-Volume



6. Linear Branching Point Density

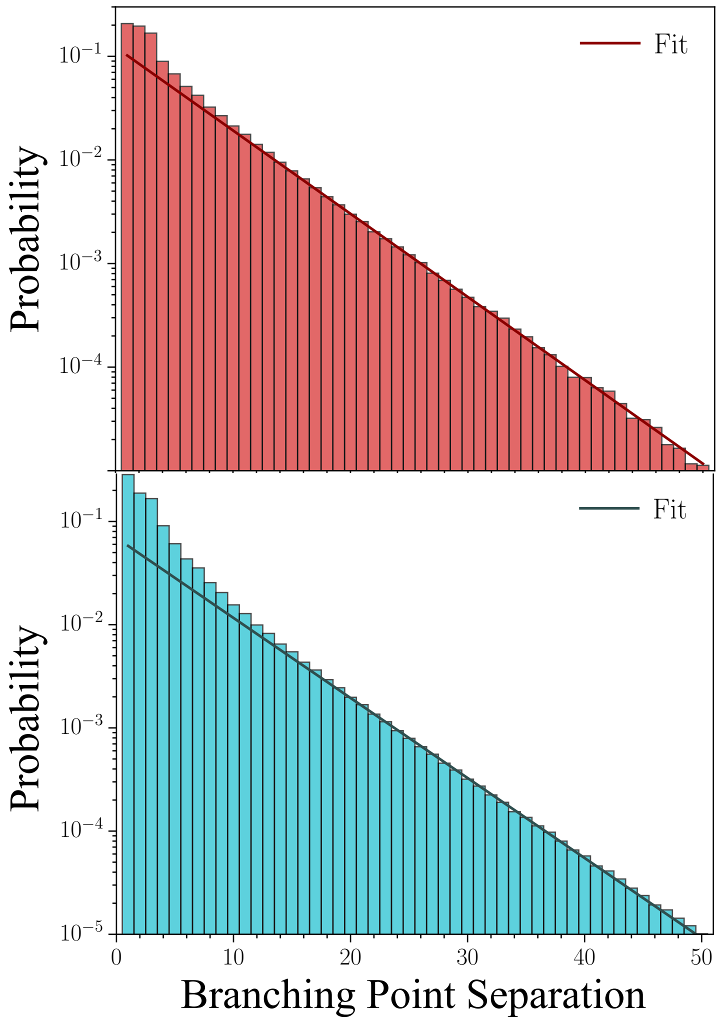
Number of branches per unit vortex length



$$\lambda_{\text{branch}} = \frac{\text{Number of branching points}}{\text{Total length of the vortex path (in fm)}}$$

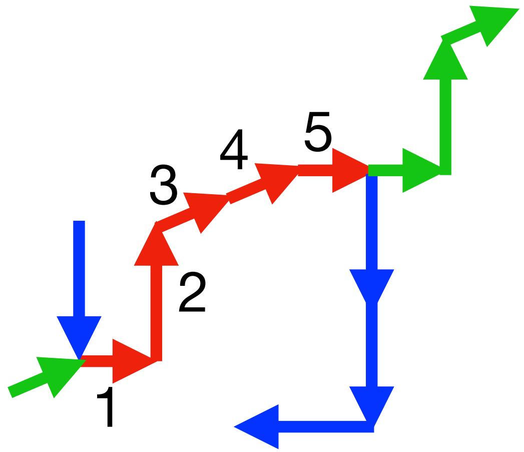
7. Chain Length Probability

Space-Space-Time



Nt=8
T=760MeV

Nt=64
T=95MeV



Estimates of “Transition” Temperatures

Quantity	Fit	T_1/T_c	T_1 (MeV)	T_2/T_c	T_2 (MeV)
ρ_{vortex}	Space-space	0.981(8)(18)	163.9(1.3)	1.913(6)(34)	319.7(1.0)
	Space-time	0.983(8)(18)	164.1(1.4)	1.983(6)(36)	331.4(1.0)
ρ_{branch}	Temporal slices	0.982(8)(18)	164.1(1.3)	1.881(5)(34)	314.3(0.9)
	Spatial slices	0.983(8)(18)	164.2(1.3)	1.915(6)(34)	320.0(0.9)
λ_{branch}	Temporal slices	0.984(9)(18)	164.5(1.6)	1.921(7)(35)	321.0(1.2)
	Spatial slices	0.983(9)(18)	164.3(1.6)	1.935(8)(35)	323.4(1.4)

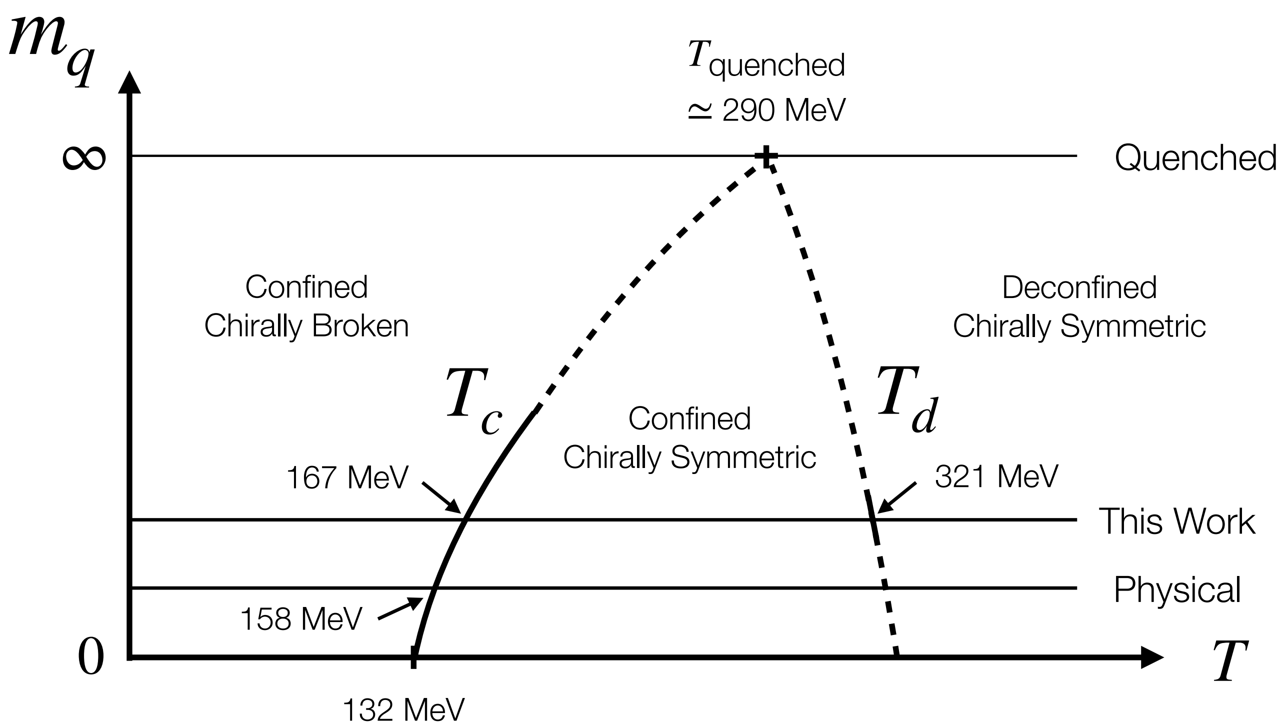
10 measurements of 7 properties

$T_d =$ 321(6) MeV = 1.92(5) T_c

(i.e. $N_\tau^d \approx 19$)

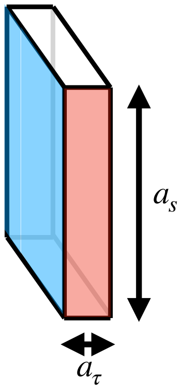
T_C from Chiral Condensate FASTSUM Phys.Rev.D 105 (2022) 3, 034504

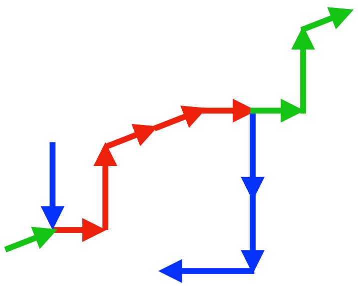
Proposed QCD Phase Diagram

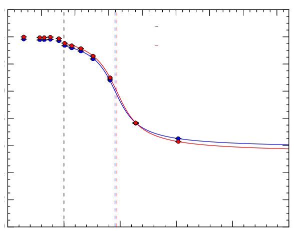


Overview

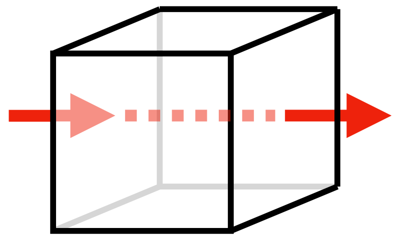
Part 1

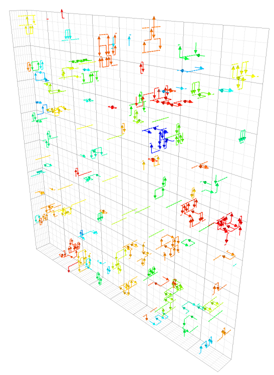






- FASTSUM approach
  - Anisotropic
- Maximal Centre Gauge
  - Vortices
- Measurements
  - Temporal Correlations
  - Cluster Extent
  - Vortex Density
  - Secondary Clusters
  - Branching Point Density
  - Linear Branching Point
  - Chain Length Probability
- Transition(s) in QCD ?
  - Recent Proposals of new QCD phase





Speculative Entropic Arguments

Consider Lattice QCD as a Stat Mech model.

1d Ising Model: $Z = \sum e^{-\beta S}$ where $S = -J \sum_{<ij>} s_i s_j$

Peierls argument: consider kink:

$$\uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow$$

Kink costs energy = $2J$ But there are N kink positions

$$Z = Z_{0\text{ kinks}} + Z_{1\text{ kinks}} + \ldots = Z_{0\text{ kinks}} + e^{\ln N - 2J\beta}$$

Dominates as $N \to \infty$

Ground state full of kinks — Topological in Nature

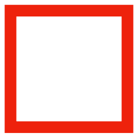
Speculative Entropic Arguments

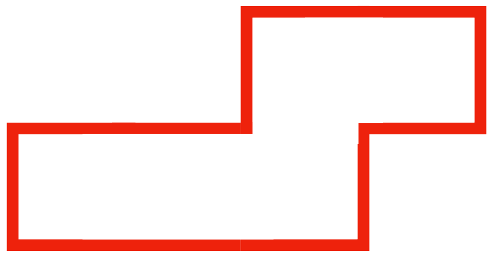
Centre Vortices in QCD

J. Greensite, Progress in Particle and Nuclear Physics 51 (2003) I-83

Kinks in 1D Ising model are equivalent to Centre Vortex clusters

i.e. dominant states are (closed) loops of vortices

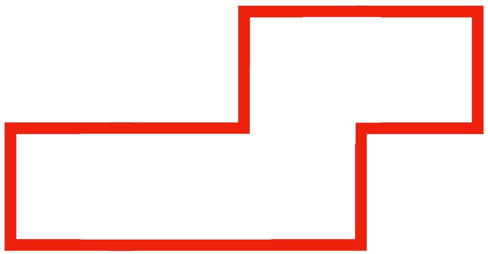
- Simplest:  i.e. plaq (in dual lattice)
- Energy $\propto$ Length
- Large loops have more d.o.f.
  - *i.e. much higher multiplicity*



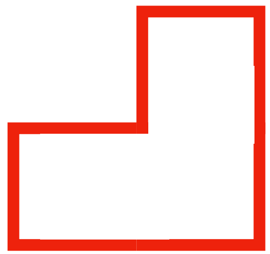
Connected versus Disconnected

Centre Vortices in QCD

Consider 2 disconnected clusters:



Length L_1



Length L_2

Can represent these in terms of connection matrix:

$$\mathscr{C}_{ij} = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ \cdot & \cdot & \cdot & \cdot & & \\ & & & & & \end{pmatrix}$$

= 1 for “connected” elements i, j

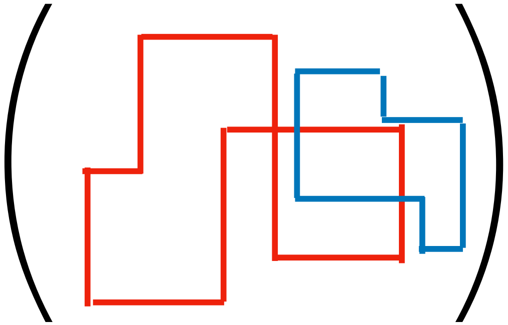
i.e. neighbours

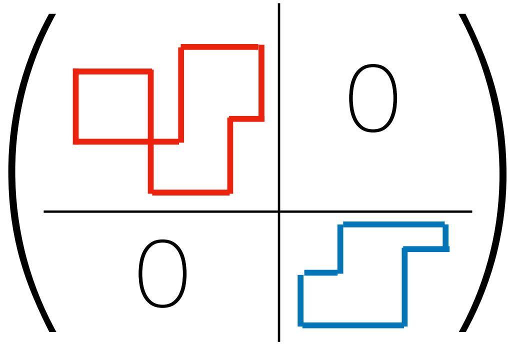
= 0 for “disconnected” elements i, j

Connected versus Disconnected

Centre Vortices in QCD

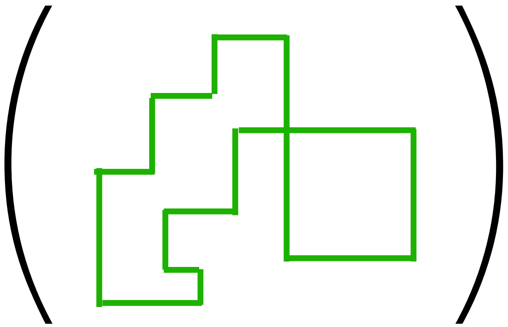
Vortex cluster is a cluster in $\mathscr{C}_{ij}$ connecting (non-zero) elements

$\mathscr{C}_{ij} =$ 

Re-order: $\mathscr{C}'_{ij} =$ 

(ignoring bulk zeros)

Now consider larger cluster of length $L = L_1 + L_2$

$\mathscr{C}'_{ij} =$ 

This has much higher multiplicity

$\longrightarrow$ Large, connected clusters dominate

i.e. Percolation

Speculative Entropic Arguments

Continuum Limit in QCD

Above was Stat Mech, L in lattice units

Write $L^{phys} = L\,a$ = fixed and multiplicity $N(L) = e^{f(L)}$

Schematically: $$Z = \sum_L e^{f(L)-\beta L} = \sum_L e^{f(L^{phys}/a) - \beta L^{phys}/a}$$

$$\approx \sum_L e^{f(L^{phys}/a) + (\ln a)\, L^{phys}/a}$$

If $f(L) \nearrow$ sufficiently fast with L $\longrightarrow$ $\exists$ percolation $\longrightarrow$ Confinement

Speculative Entropic Arguments

Finite Temperature

4d volume = $V_3\, N_\tau \;\; = \;\; N_s^3 \times N_\tau$

(Fixed a) thermodynamic limit is $N_s \nearrow$ with N_τ constant

Also require periodic boundary conditions

$\longrightarrow$ *Simplest* vortex loops tend to run parallel with temporal direction

Consider adding more structure:

Since $N_s^3 \gg N_\tau$

$\longrightarrow$ *Far higher multiplicity to add this structure in spatial direction*

$\longrightarrow$ Percolation remains in spatial direction after it switches off in temporal

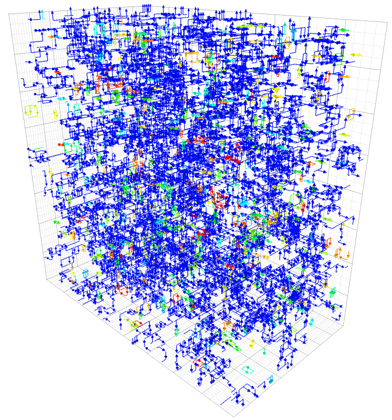
Overview

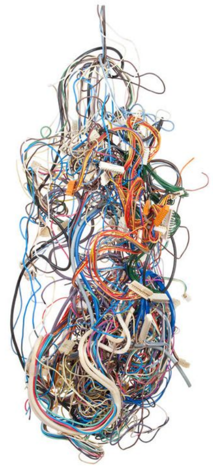
Speculative Entropic Argments

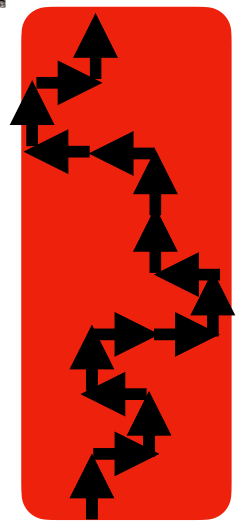
- Peierls Entropic Argument
    - Following Greensite
- Percolation in large dimension favoured
    - Large number of small clusters disfavoured
- Continuum Limit
    - Percolation always dominates?
- Finite Temperature
    - Percolation remains in spatial direction

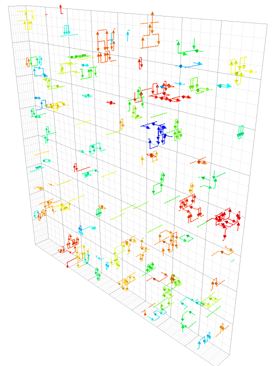
⟶ We are expanding around wrong state

- Topological Nature of vortex clusters
- Multiplicity of vortex clusters



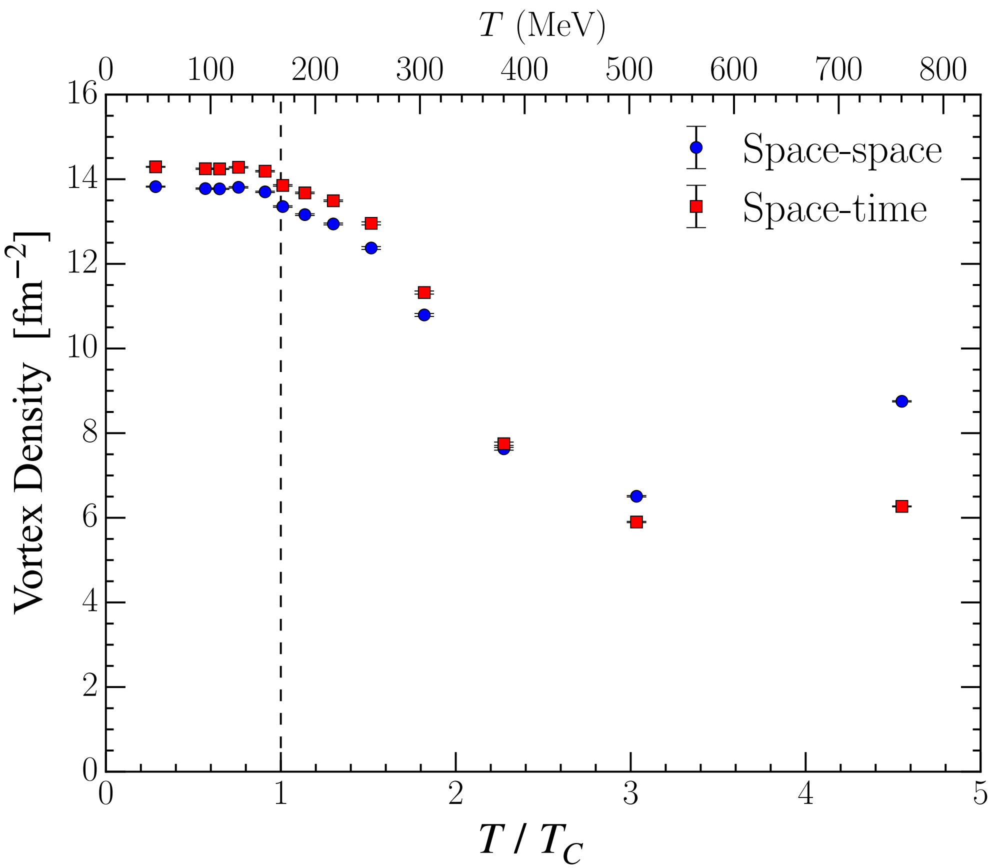






3. Vortex Density

Number per Area



Error bars are tiny

No broad distribution

in centre vortex density!

⟶ centre vortices dominate