BSM in Early Universe Cosmology

1st UNDARK School Benasque
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Refs: 1) Kolb and Turner
2) Gorbunov and Rubakov
3) Dodelson and Schmidt

Contents

Lectures I and II
* Thermal History (slides)
* Cosmological Dynamics
* Early Universe Thermodynamics
  - Distributions
  - Time/Temperature relation.
  - Conservation of entropy
  - An application to neutrino decoupling.

Lectures III and IV
* ~~The~~ Boltzmann equation. Interaction Rates ~~and~~
  - Thermal state of the plasma
  - Production of axions (hot)
  - Evolution of massive stable relics: WIMPS
  - Production and evolution of ~~B~~ CP asymmetries the case of Baryons.

Exercise: ~~Solve~~ Obtain the helium abundance in the Universe BSM and from first principles

Cosmological Dynamics

FRWL metric: $ds^2 = dt^2 - a^2 d\vec{x}^2$

$G_{00} = 8\pi G\, T_{00} \Rightarrow \left(\frac{\dot{a}}{a}\right)^2 = \boxed{H^2 = \frac{8\pi G}{3}\rho}$ ← 1st Friedman equation

$Tr\left[G_{ii} = 8\pi G\, T_{ii}\right] \Rightarrow \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$

$\nabla_\mu T^{\mu\nu} = 0 \quad : \nu = 0 \Rightarrow \boxed{\frac{d\rho}{dt} = -3H(\rho + p)}$ ← Continuity equation

Recall that $T_{\mu\nu} = -\eta_{\mu\nu}\, p + (\rho + p)\, u_\mu u_\nu$ $\quad u^\mu = (1, \vec{0})$ (rest frame)

evolution of key fluids: $p = \omega \times \rho$: ω : equation of state

Matter: $\omega = 0$ $\rho = \rho_0 \cdot \frac{a^{-3}}{a_0^{-3}}$

Radiation $\omega = \frac{1}{3} \Rightarrow \rho \propto a^{-4}$ //

time - Hubble relation: $H = \frac{\dot{a}}{a} = \frac{da/dt}{a} \Rightarrow dt = \frac{1}{H}\frac{da}{a}$

Matter Domination: $H = H_0 \cdot a^{-3/2} \Rightarrow t = \frac{2}{3}\frac{1}{H}$

Radiation Domination: $H = H_0 \cdot a^{-2} \Rightarrow t = \frac{1}{2H}$

Energy density today: $\rho_c = \frac{3H_0^2}{8\pi G} = 8.1 \cdot 10^{-47}\, GeV^4 \cdot h^2$

$= 1.9 \cdot 10^{-29}\, g/cm^3 \cdot h^2$ //

* Equilibrium Thermodynamics

Motivation $\frac{\delta T}{T} \sim 10^{-5}$ FIRAS!

$$n \equiv g \cdot \int \frac{d^3\vec{p}}{(2\pi)^3} f(\vec{p}, \vec{x}) \qquad \text{number density}$$

$$\rho \equiv g \int \frac{d^3\vec{p}}{(2\pi)^3} \cdot E \cdot f(\vec{p}, \vec{x}) \qquad \text{energy density}$$

$$p \equiv g \int \frac{d^3\vec{p}}{(2\pi)^3} \cdot \frac{|\vec{p}|^2}{3E} f(\vec{p}, \vec{x}) \qquad \text{pressure}$$

The early Universe should have been very homogeneous and isotropic. From the CMB we know that $\frac{\Delta T}{T} \sim 10^{-5}$ and the earlier we go the more true it is.

This implies: $f(\vec{x}, \vec{p}) = f(|\vec{p}|)$: Namely, it can only depend upon the magnitude of the momentum.

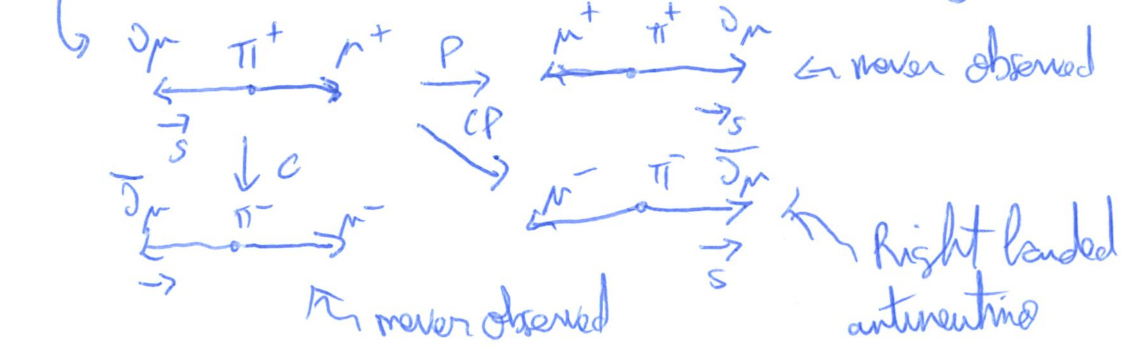
$g \equiv$ internal number of degrees of freedom:

What are their values:

$g_{e^-} = 2 \qquad g_{e^+} = 2 \qquad g_\gamma = 2 \qquad g_{\nu_L} = 1? \qquad g_{\bar{\nu}_R} = 1.$

Left-handed neutrino

but $g_{\nu_R} = 0$ and $g_{\bar{\nu}_L} = 0$.



Thermal Distribution Functions

$$f = \frac{1}{1 + e^{\frac{E}{T} - \frac{\mu}{T}}} \qquad \text{Fermi-Dirac}$$

$$f = \frac{1}{-1 + e^{\frac{E-\mu}{T}}} \qquad \text{Bose-Einstein} \qquad (\mu/T \leq 0)$$

for massless particles

$$f = e^{-\frac{E}{T} + \frac{\mu}{T}} \qquad \text{Maxwell-Boltzmann}$$

Interpretation from 1st law of thermodynamics:

$$dE = T\,dS - P\,dV + \sum_i \mu_i\, dN_i$$

T: Energy needed to increase the entropy

μ_i: Energy needed to increase the number density.

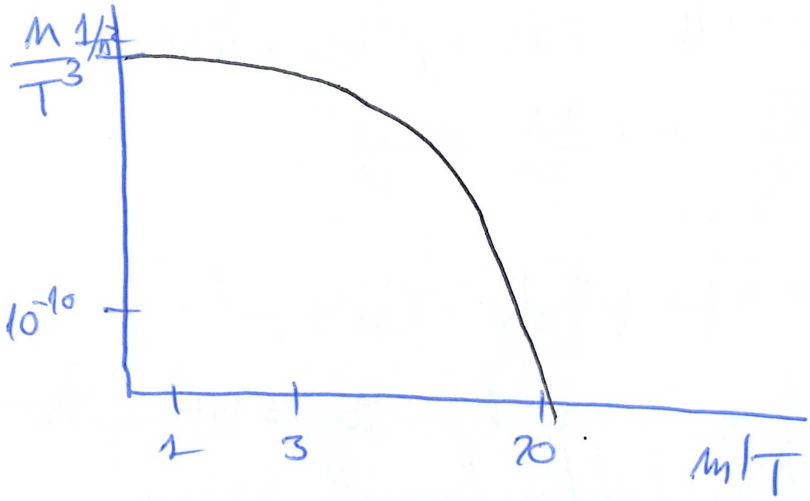
Relativistic limit: $\mu \ll T$, $T \gg m$

	FD	BE	MB
$n =$	$g\frac{3}{4}\frac{\zeta_3}{\pi^2}T^3$	$g\frac{\zeta_3}{\pi^2}\cdot T^3$	$g\frac{1}{\pi^2}T^3$
$\rho =$	$g\frac{7}{8}\frac{\pi^2}{30}T^4$	$g\frac{\pi^2}{30}\cdot T^4$	$g\frac{3}{\pi^2}T^4$
	$\langle E \rangle = 3.15\,T$	$\langle E \rangle = 2.7\,T$	$\langle E \rangle = 3T$

$P = \frac{1}{3}\rho$

Non-Relativistic Limit $T \ll m$

$$n = g\left(\frac{mT}{2\pi}\right)^{3/2} e^{-\left(\frac{m}{T}-\frac{\mu}{T}\right)} \qquad \rho \simeq m\cdot n \qquad p \simeq n\cdot T$$



Exercise: Find T such that $\frac{n}{T^3} \simeq 10^{-10}$ for $m = 1\,\mathrm{GeV}$

Consequence:

$T < 50\,\mathrm{MeV}$ only baryons in the Universe

Consequence: The early Universe is dominated by ultrarelativistic particles:

Key equation: $$H^2 = \frac{8\pi G}{3}\rho = \frac{8\pi}{M_{pl}^2}\cdot\frac{\pi^2}{30} g_* T^4$$

$$\boxed{H \simeq 1.66\cdot\sqrt{g_*}\cdot\frac{T^2}{M_{pl}}}$$

g_* ~~of E~~

EW PT $\rightarrow$ $g_* = 106.75$

10 MeV $\longrightarrow$ $g_* = 10.75$

Today$|_{SM}$ $\rightarrow$ $g_* = 3.36$

$$\boxed{g_* \equiv \sum_i g_{B_i} + \frac{7}{8}\sum_i g_{F_i}}$$

$$\boxed{H = 0.7\,s^{-1}\cdot\sqrt{\frac{g_*}{10.75}}\cdot\left(\frac{T}{\mathrm{MeV}}\right)^2}$$

$t_{EW} \simeq 10^{-10}\,s$

$t_{QCD} \simeq 10^{-4}\,s$

$t_{MeV} \simeq 1\,s$

Key quantity: entropy density

$$\boxed{s = \frac{\rho + p - \mu n}{T}}$$

This comes from the 1st law: $dE = TdS - pdV + \mu dN$
applied to densities $\rho = \frac{E}{V}$ $\quad n = \frac{N}{V}$ $\quad s = \frac{S}{V}$
then:

$$dV\left[-\rho - p + sT + \mu n\right] = V\left[d\rho - Tds + \mu dn\right]$$

apply it for $dV \neq 0$ {the eq is valid for any system}
and then reapply it to find the formula for s.

For the case of $\mu = 0$ and in thermal equilibrium
one can find that $s \cdot a^3 = \text{constant}$.

the key is to use $\frac{dp}{dT} = \frac{\rho + p}{T}$ (only in TE)

$$\boxed{s = \frac{4\pi^2}{90} T^3 g_{*s}}$$

This is very useful because it means we should normalize
it for conserved quantities.

Examples: today: $\eta_b \equiv \frac{n_B}{n_\gamma} \simeq 6.1 \times 10^{-10}$.
In the early Universe we will then have:

$$\boxed{\eta_b\big|_1 = \eta_b\big|_2 \times \frac{g_{*s1}}{g_{*s2}}} \quad \text{at EW} \quad \boxed{\frac{\eta_B}{\eta_{B\,\text{today}}} \simeq 27}$$

similarly for the dark matter abundance.

$\cancel{g_{*s} T_\gamma^3 = \text{cons}}$ at $T \ll m_e \Rightarrow \boxed{T_\gamma \cdot a = \text{cte}}$

Neutrino Decoupling and temperature of the CNB

Neutrinos and e^+e^- stop interacting at $T \simeq 1\,MeV$ this is above m_e and means that e^+e^- only heated up the γ fluid. Not the neutrino one.

Using entropy conservation we can easily solve for T_γ/T_ν. Entropy is separately conserved in both sectors:

$$s_{\nu 1} a_1^3 = s_{\nu 2} a_2^3$$

$$s_{\gamma 1} a_1^3 = s_{\gamma 2} a_2^3$$

Taking their ratio we get and taking $T_1 = T_\nu = T_\gamma$

$$\frac{g_\nu \frac{7}{8}}{g_\gamma + g_e \frac{7}{8}} = \left(\frac{T_\nu}{T_\gamma}\right)^3 \frac{g_\nu \frac{7}{8}}{g_\gamma} \Rightarrow \left(\frac{T_\gamma}{T_\nu}\right)^3 = \frac{g_\gamma + g_e \frac{7}{8}}{g_\gamma} \Rightarrow$$

$$\frac{T_\gamma}{T_\nu} = \left(\frac{2 + 7/2}{2}\right)^{1/3} = \left(\frac{11}{4}\right)^{1/3} \simeq 1.402$$

The CNB is colder than the CMB!

Today

$$g_* = 2 + 6 \cdot \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} = 3.36$$

$$g_*^s = 2 + 6 \cdot \frac{7}{8}\left(\frac{4}{11}\right) = 3.91$$

But critically we can now define:

$$N_{eff} = \frac{8}{7}\left(\frac{11}{4}\right)^{4/3} \left[\frac{\rho_{rad} - \rho_\gamma}{\rho_\gamma}\right]$$

in the SM $\simeq 3.04$

We can now express its contribution to Neff from any massless relic:

$$\Delta N_{eff} = \frac{8}{7}\left(\frac{11}{4}\right)^{4/3} \cdot \frac{\rho_{BSM}}{\rho_\gamma}$$

~~T_{BSM}~~ $a^3 s_{BSM} = \text{constant}$. ~~Hence: $T_{BSM}/2 =$~~

Taking a temperature $T_{eq} = T_{BSM} = T_\gamma$ one then finds:

$$\left(\frac{T_{BSM}}{T_\gamma}\right)^3 = \frac{g_{*s}^{SM}(T_\gamma)}{g_{*s}^{SM}(T_{eq})} \quad \text{valid for } T_\gamma < T_{eq}$$

and therefore:

$$\Delta N_{eff} = \frac{8}{7} f^{BF/FD} \times \left(\frac{11}{4}\right)^{4/3} \cdot \frac{g_{BSM}}{2} \left[\frac{g_{*s}^{SM}(T_\gamma)}{g_{*s}^{SM}(T_{eq})}\right]^{4/3}$$

for a sterile neutrino thermal at $T \simeq 10$ MeV:

$$f^{BF/FD} = \frac{7}{8} \qquad g_{BSM} = 2$$

$$\frac{g_{*s}^{SM}(T_0)}{g_{*s}^{SM}(T_{eq})} = \cancel{2 + 6 \cdot \frac{7}{8}\left(\frac{4}{11}\right)^{4/3}} = \frac{3.9}{10.75}$$

~~2+6~~

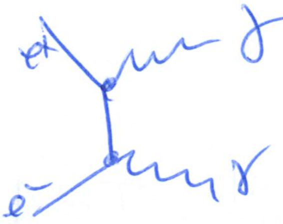
$\Rightarrow \Delta N_{eff} = 1$

Interactions and their Rates

Consider the $1+2 \to 3+4$ process. Then the rate of interaction for 1 is : $\Gamma_1 = n_2 \langle \sigma v \rangle_{1+2\to 3+4}$

Lets apply this to two cases:

$e^+e^- \to \gamma\gamma$:



$\alpha \equiv \frac{e^2}{4\pi} = \frac{1}{137}$

$\sigma \simeq \frac{\alpha^2}{s}$ (by dimensional analysis and for $\sqrt{s} \gg m_e^2$)

at $T \gg m_e$ then $v_e \simeq 1$ and one gets

$\langle \sigma v \rangle \simeq \frac{\alpha^2}{T^2}$ because $s = 8E_{cm}^2$

$n \sim T^3$ and one gets $\Gamma \simeq \alpha^2 \cdot T$

One needs to compare this with the Hubble expansion rate in order to understand if the process ~~is~~ is efficient or not.

$N_{int} \simeq \frac{\Gamma}{H} \simeq \frac{t_H}{\tau}$ if $\frac{\Gamma}{H} \gg 1 \to$ Thermal equilibrium

if $\frac{\Gamma}{H} \ll 1 \to$ out of equilibrium

In a Radiation dominated Universe we get

$$\frac{\Gamma}{H} \simeq \alpha^2 \cdot \frac{M_{pl}}{T}$$

this means those processes are active at least at $T < \alpha^2 \cdot M_{pl} \simeq$ ~~[illegible] GeV~~ $\simeq 10^{14}$ GeV

In addition at $T = 1$ MeV $\frac{\Gamma}{H} \simeq 10^{20}$!

$\Rightarrow$ EM interactions are highly efficient indeed in the early Universe. Main reason we considered equilibrium thermodynamics

<u>The neutrino case</u>

$e^+e^- \to \nu\bar{\nu}$: $\sigma \simeq G_F^2 \cdot s$ 

$E \ll M_W$

then $\langle \sigma v \rangle \simeq G_F^2 \cdot T^2$ $\quad \Gamma \simeq G_F^2 \cdot T^5$

comparing it with Hubble we get

$T_{dec} \simeq 2\text{-}3$ MeV $\leftarrow$ Neutrino Decoupling

Key things to remember from Lectures I and II

The early Universe is governed by relativistic particles.

$$H \simeq 1.66 \cdot \sqrt{g_*} \cdot \frac{T^2}{M_{pl}} \qquad M_{pl} = 1.22 \cdot 10^{19}\ \text{GeV}$$

$g_* \equiv$ degrees of freedom in radiation.

Entropy density is conserved in thermal equilibrium or in the absence of interactions.

$$s = \frac{2\pi^2}{45} T^3 \cdot g_{*s} \rightarrow$$ degrees of freedom contributing to radiation in the form of entropy

$g_{*s}|_0 \simeq 3.91$ $\qquad$ at $T \gg 100$ GeV $\quad g_* = g_{*s} = 106.75$

$g_*|_0 \simeq 3.36$

Relativistic particles have $\langle E \rangle \simeq 3T$

and $\rho \propto T^4 \qquad n \sim T^3$.

Non-relativistic ones are reduced exponentially:

$$n \simeq g \left(\frac{mT}{2\pi}\right)^{3/2} e^{\mu/T} \cdot e^{-\frac{m}{T}} \leftarrow \text{key!}$$

$\frac{\Gamma}{H} \gg 1$ Equilibrium

$\frac{\Gamma}{H} \ll 1$ Out of equilibrium.

EM interactions are very efficient: $\frac{\Gamma}{H} \simeq 10^{20}$ at MeV

also: even with tiny couplings

a thermal population of particles may have been produced! Very stringent BSM constraint!

Lecture III

The Liouville equation:

$$\frac{df}{dt} = C[f] \quad \text{Collision term}$$

f: distribution function

$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial p}\frac{dp}{dt} = C[f]$$

$$\frac{\partial f}{\partial t} - Hp\,\frac{\partial f}{\partial p} = C[f]$$

$\frac{dp}{dt} = -Hp$ from: the Geodesic eq:

$$\frac{d^2x^\mu}{ds^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds} = 0$$

Upon integration over $g_i \frac{d^3\vec{p}}{(2\pi)^3}$ one gets:

$$\frac{dn_i}{dt} + 3Hn_i = \int g_i C[f]\,\frac{d^3\vec{p}}{(2\pi)^3} \equiv \frac{\delta n}{\delta t}$$

and upon integrating over $g_i\, E\, \frac{d^3\vec{p}}{(2\pi)^3}$ one gets:

$$\frac{d\rho_i}{dt} + 3H(\rho_i + p_i) = \int C[f]\, E g_i \frac{d^3\vec{p}}{(2\pi)^3} \equiv \delta\rho/\delta t.$$

Want to explore the simplest cases for the processes:

decays and inverse decays.

annihilations (and inverse annihilations).

concentrating on $1+2 \leftrightarrow 3+4$ processes we have:

$$C[f_1] \equiv -\frac{1}{2E_1}\,\frac{1}{g_1}\underset{\substack{\text{sum over}\\ \text{all spin states}}}{\sum} \int d\Pi_2\, d\Pi_3\, d\Pi_4\, (2\pi)^4 \delta^4(p_1+p_2-p_3-p_4)$$

$$S\times\Big[f_1 f_2 [1\pm f_3][1\pm f_4]\,|M_{1+2\to 3+4}|^2 - f_3 f_4 [1\pm f_2][1\pm f_3]\,|M_{3+4\to 1+2}|^2\Big]$$

$d\Pi_i \equiv \dfrac{d^3\vec{p}_i}{2E_i(2\pi)^3}$ Lorentz invariant phase space density

$S \equiv$ Symmetry factor $\times\frac{1}{2!}$ for each pair of initial or final state particles and

$\times 2$ for each pair of initial state ones.

end result is: $S = \frac{1}{2}$ for final state particles and that's it which are identical:

This agrees with Gondolo and Gelmini but also with Dolgov hep-ph/0202122 and hep-ph/9703315

$+$ is for bosons $a^\dagger|n\rangle = \sqrt{1+n}\,|n+1\rangle$ ← Kinetic Theory Wagoner '78

$-$ is for fermions $a^\dagger|n\rangle = \sqrt{1-n}\,|n+1\rangle$

M is the matrix element of a given process. Typically calculated in perturbation theory as: $iM = \langle out|\, i\mathcal{L}\,|in\rangle$

$\mathcal{L}$ = Lagrangian.

Things to note:

1): In the presence of ~~stationary solu~~ efficient interactions stationary solutions to the Liouville equation $\frac{df}{dt}=0$ represent thermal equilibrium configurations

If T invariance is respected, this is most easily seen as it requires:

$$f_1 f_2 [1 \pm f_3][1 \pm f_4] - f_3 f_4 [1 \pm f_2][1 \pm f_3] = 0$$

which will only be true if $T_1 = T_2 = T_3 = T$

← Kinetic / Thermal equilibrium.

and if $\mu_1 + \mu_2 = \mu_3 + \mu_4$. ← Chemical equilibrium

2) In most scenarios quantum statistical factors will not be important. They represent <20% corrections to the rates.

E.g. $n + \nu_e \leftrightarrow p + e^-$ 5%

$\nu \bar{\nu} \leftrightarrow e^+ e^-$ 10% etc.

Then things simplify greatly and one has:

$$\frac{dn_1}{dt} + 3Hn_1 = -\sum_{\text{spins}} d\Pi_1 \, d\Pi_2 \, d\Pi_3 \, d\Pi_4 (2\pi)^4 \delta^4(p_1+p_2-p_3-p_4)$$

$$\times |M|^2 S \cdot \left[f_1 f_2 (1 \pm f_3)(1 \pm f_4) - f_3 f_4 (1 \pm f_1)(1 \pm f_2) \right]$$

Ignoring statistical factors $\rightarrow$ $\hookrightarrow = |M|^2 S \left[f_1 f_2 - f_3 f_4 \right]$

writing it as MB functions $\rightarrow$ $\hookrightarrow = |M|^2 S \left[e^{\frac{\mu_1}{T}} e^{\frac{\mu_2}{T}} e^{-\frac{(E_1+E_2)}{T}} - e^{\frac{\mu_3}{T}} e^{\frac{\mu_4}{T}} e^{-\frac{(E_1+E_2)}{T}} \right]$

Because $E_1+E_2 = E_3+E_4$

We can then relate this to the cross section

Remember: $d\sigma \equiv \frac{1}{2E_A 2E_B} \frac{1}{|v_A - v_B|} \prod_f d\Pi_f \cdot |M(A+B) \rightarrow f|^2 (4\pi)^2 \delta^4(p_A + p_B - \sum p_f)$

Furthermore noting that $e^{\frac{\mu_i}{T}} \equiv \frac{n_i}{n_i^{(0)}}$ where $n_i^{(0)}$ is the number density without a chemical potential

Putting everything together one then finds:

$$\frac{dn_1}{dt} + 3Hn_1 = n_1^{(0)} n_2^{(0)} \langle \sigma v \rangle \left(\frac{n_3 n_4}{n_3^{(0)} n_4^{(0)}} - \frac{n_1 n_2}{n_1^{(0)} n_2^{(0)}} \right)$$

Saha type equation

show formula on the slides

Further comments:

3) We clearly see now that $\Gamma = n_2 \langle \sigma v \rangle$ and that it compares directly with H

4) If the net change of particles is small compared with the particle production and destruction rates then we are in thermal equilibrium and these things should vanish.

Detailed balance: it allows to relate backward and forward rates.

Example: QCD axion production in the early Universe:

$$\mathcal{L} = \frac{g_s^2}{32\pi^2} \frac{a}{f_a} \cdot G_{\mu\nu} \tilde{G}^{\mu\nu}$$

solves the strong CP problem and leads to

$$V(a) \simeq f_\pi^2 m_\pi^2 \left(1 - \cos\left(\frac{a}{f_a}\right)\right)$$

$$m_a \cdot f_a = m_\pi f_\pi \qquad m_\pi = 130\,\text{MeV}$$
$$f_\pi \simeq 100\,\text{MeV}$$

Main production mechanism:

$\pi\pi \to \pi a$ relevant for $T \lesssim 300\,\text{MeV}$

Rate: $\Gamma \simeq 10^{-3} \frac{1}{(f_a/f_\pi)^2} \cdot T^5$ // di Luzio et al 2211.05073.

Comparing $\frac{\Gamma}{H} \simeq 1$ one gets:

$T_{eq} \simeq 300$ MeV for $\boxed{f_a \gtrsim 10^8 \text{ GeV}}$

in other words; $\boxed{m_a \lesssim 0.2 \text{ eV}}$

Finally, one can rewrite the equation in a simpler form:

$$\frac{dn_a}{dt} + 3Hn_a = -\langle\Gamma\rangle(n_a - n_a^{eq}) - \langle\sigma v\rangle(n_a^2 - n_a^{2\,eq})$$

$Y_a \equiv \frac{n_a}{s}$ $\Rightarrow$ if s is conserved one finds

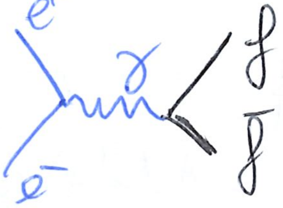
$$\frac{dY_a}{dt} = -\langle\Gamma\rangle(Y_a - Y_a^{eq}) \, s\langle\sigma v\rangle(Y_a^2 - Y_a^{2\,eq})$$

it is very convenient to rewrite this as a function of Temperature. This leads to:

$$\frac{dY_a}{dT} = \frac{\langle\Gamma\rangle}{HT}(Y_a - Y_a^{eq}) + \frac{s\langle\sigma v\rangle}{HT}(Y^2 - Y_a^{2\,eq})$$

Another example:

Millicharged particles see e.g. hep-ph/0001179



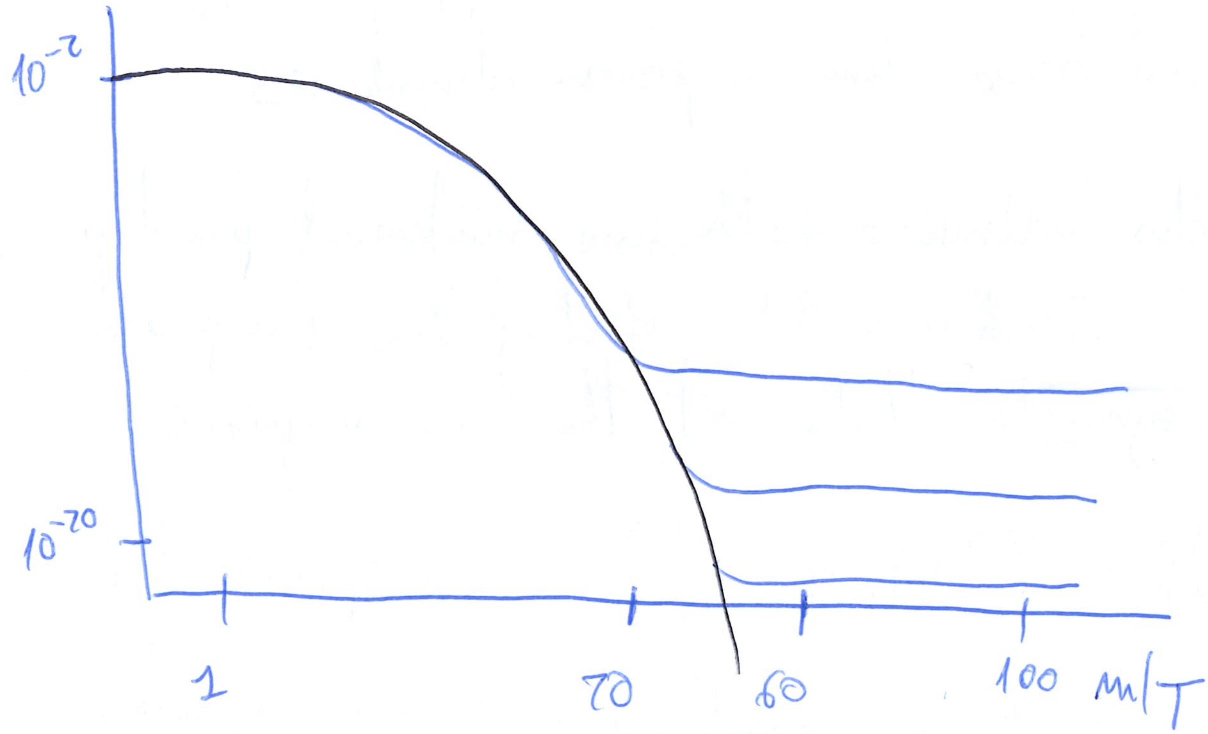
$e^+e^- \to f\bar f$ $\quad \Gamma \simeq \alpha\alpha' T$

hence $\alpha' < 10^{-20}$ or $\epsilon < 10^{-10}$!

The WIMP case:

$$\frac{dn}{dt} + 3Hn = -\langle\sigma v\rangle\left(n_\chi^2 - n_\chi^{eq\,2}\right)$$

$$\frac{dY}{dT} = \frac{s\langle\sigma v\rangle}{H\cdot T}\left(Y_\chi^2 - Y_\chi^{eq\,2}\right)$$



Instantaneous freeze-out approximation:

$$\Gamma \simeq H \simeq n_\chi\langle\sigma v\rangle \Rightarrow Y_\chi \simeq \frac{H}{s\langle\sigma v\rangle}\Big|_{\text{at freeze out}}$$

$$\boxed{\Omega_{DM}h^2 = 0.12\ \frac{x_f}{30}\cdot\frac{\sqrt{g_*}}{g_s/\sqrt{g_*}}\cdot\frac{2.2\cdot10^{-26}\,\mathrm{cm}^3/s}{\langle\sigma v\rangle}}$$

Recall that $s_0 = 2900\ \mathrm{cm}^{-3}$ $\quad\rho_{crit} = 1.9\cdot10^{-29}\ \mathrm{g/cm}^3$
$= 1\times10^4\ \mathrm{eV/cm}^3$

$$\boxed{Y_B = 8.7\cdot10^{-11}} \qquad \boxed{Y_{DM} = 8.7\cdot10^{-11}\times5.3\times\frac{m_p}{m_{DM}}}$$

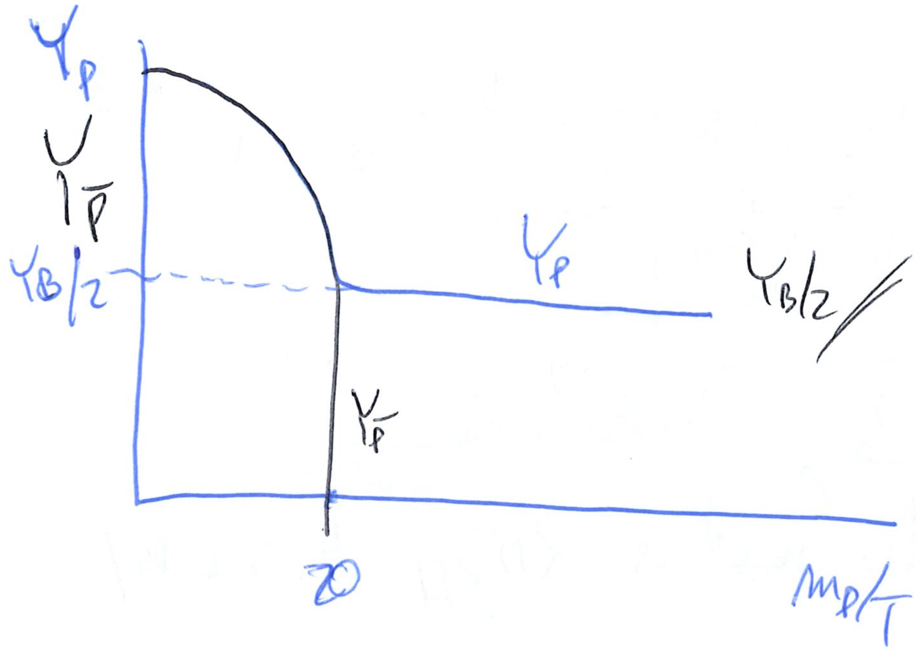
$2.2\times10^{-26}\,\mathrm{cm}^3/s \simeq 2.57\cdot10^{-9}\,\mathrm{GeV}^{-2} \simeq \frac{g^2}{M_W^2}$ WIMP miracle!

on the other hand, particles interacting strongly would have: $\sigma \simeq \cancel{\frac{1}{R}}\pi R^2 \simeq \frac{1}{m_\pi^2} \simeq 0.01\,\mathrm{GeV}^{-2}$ and hence tiny a priori abundances.

Consider a Universe with same number of p and $\bar p$ then $\Omega_p h^2 \simeq 10^{-9}$! But if there is a particle asymmetry that's not the case anymore.

$$\frac{dY_{\bar p}}{dT} = \frac{s\langle\sigma v\rangle}{HT}\left(Y_p Y_{\bar p} - Y_p^{eq\,2}\right) \quad \text{since } Y_p \text{ is constant}$$

$Y_{\bar p}$ decays exponentially fast over temperature after p dominate.



Considerations: stable particles which do not interact strongly enough will have $\Omega_{DM} h^2 > 0.12$.

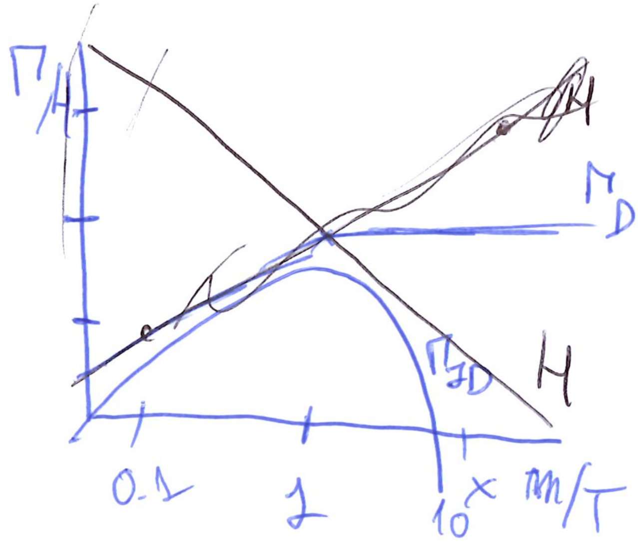
Remember it is very easy to produce particles in the early Universe!

Lets consider the case of a decaying particle X

$$\frac{dY_X}{dT} = -\frac{\langle\Gamma\rangle}{HT}\left(Y_X - Y_X^{eq}\right)$$

decay ... inverse decay

what matters the most is $\langle\Gamma\rangle/H$ at $T \simeq m_X/3$



$$\Gamma_D \simeq \frac{\Gamma_X}{\frac{1}{m} + \frac{m}{Tm}}$$

$$\Gamma_{ID} \simeq \Gamma_D \cdot e^{-\frac{m_X}{T}}$$

Lets consider this particle can decay into Baryons and anty baryons violating B number and at different rates:

Then:

$$\frac{dY_B}{dt} = \langle\Gamma_X\rangle \{\varepsilon \times (Y_X - Y_X^{eq}) - \Gamma_{B\text{-breaking}} Y_B$$

Hence Γ_X / H needs to be small

$Y_X - Y_X^{eq}$ needs to be $\neq 0$! (out of equilibrium)

and of c $\cancel{B}$ violation

Shakarov conditions:

* Violating C and CP
* Violating B number
* Out of equilibrium.

Baryogenesis via decays is by far the simplest.

Imagine $\Gamma(X \to B) - \Gamma(X \to \bar{B}) = \Gamma_X \varepsilon$

Then if the particle decays out of equilibrium one can do a simple estimate if X dominates the Universe: $Y_B = Y_X \cdot \varepsilon$ at $T = T_{RH}$

corresponding to $\rho_X \simeq \rho_{rad}$

$m_X \cdot Y_X s = \rho_{rad}$

$$Y_X \simeq \frac{\rho_{rad}}{m_X s_{rad}} \simeq \frac{3\, T_{RH}}{4\, m_X}$$

$$Y_B \simeq \varepsilon \cdot \frac{3}{4} \cdot \frac{T_{RH}}{m_X}$$

Case: leptogenesis $m_X \simeq 10^9\, GeV$ $T_{RH} \simeq 10^9\, GeV$

$$\varepsilon \leq \frac{3}{8\pi} \frac{M_1 (m_3 - m_1)}{v_H^2}$$

hence $M_1 > 10^8\, GeV$!

Davidson Ibarra bound.

But remember that any B violating interaction in thermal equilibrium will erase any asymmetry.

Finish with the overall description of the usual BSM bounds.