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Beyond the Standard Model of Physics
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BSM Tutorial

Problem 1. Using

(1) $$\chi^\alpha = \varepsilon^{\alpha\beta}\chi_\beta, \quad \chi_\alpha = \varepsilon_{\alpha\beta}\chi^\beta$$

and the definitions $\chi\psi \equiv \chi^\alpha\psi_\alpha$, $\bar{\chi}\bar{\psi} \equiv \bar{\chi}_{\dot{\alpha}}\bar{\psi}^{\dot{\alpha}}$, demonstrate that

1) $\chi\psi = \psi\chi$, $\bar{\chi}\bar{\psi} = \bar{\psi}\bar{\chi}$
2) $(\chi\psi)^\dagger = \bar{\psi}\bar{\chi} = \bar{\chi}\bar{\psi}$.

Problem 2. Using

(2) $$A_{\alpha\dot{\beta}} = A_\mu(\sigma^\mu)_{\alpha\dot{\beta}}, \qquad A^\mu = \frac{1}{2}A_{\alpha\dot{\beta}}(\bar{\sigma}^\mu)^{\dot{\beta}\alpha},$$

(3) $$(\sigma^\mu)_{\gamma\dot{\delta}}(\bar{\sigma}_\mu)^{\dot{\beta}\alpha} = 2\delta^\alpha_\gamma\delta^{\dot{\beta}}_{\dot{\delta}}, \qquad \bar{\sigma}^{\mu\dot{\alpha}\alpha} = \varepsilon^{\alpha\beta}\varepsilon^{\dot{\alpha}\dot{\beta}}\sigma^\mu_{\beta\dot{\beta}}, \qquad \sigma^\mu_{\alpha\dot{\alpha}} = \varepsilon_{\alpha\beta}\varepsilon_{\dot{\alpha}\dot{\beta}}\bar{\sigma}^{\dot{\beta}\beta},$$

(4) $$\Psi_D = \begin{pmatrix} \chi_\alpha \\ \bar{\psi}^{\dot{\alpha}} \end{pmatrix}, \quad \gamma^\mu = \begin{pmatrix} 0 & (\sigma^\mu)_{\alpha\dot{\alpha}} \\ (\bar{\sigma}^\mu)^{\dot{\alpha}\alpha} & \end{pmatrix}$$

1) Check that $A_\mu B^\mu = \frac{1}{2}A_{\alpha\dot{\beta}}B^{\alpha\dot{\beta}}$
2) Write the Dirac Lagrangian in Van der Waerden notation.

Problem 3. Defining

(5) $$G(x^\mu, \theta, \bar{\theta}) = \exp\left[i\left(\theta^\alpha Q_\alpha + \bar{\theta}^{\dot{\beta}}\bar{Q}_{\dot{\beta}} - x_\mu P^\mu\right)\right]$$

and using the Hausdorff formula

(6) $$e^A e^B = \exp\left(A + B + \frac{1}{2}\left[A, B\right] + \ldots\right)$$

prove that

(7) $$G(x^\mu, \theta, \bar{\theta})G(a^\mu, \epsilon, \bar{\epsilon}) = G(x^\mu + a^\mu + i\epsilon\sigma^\mu\bar{\theta} - i\theta\sigma^\mu\bar{\epsilon}, \theta + \epsilon, \bar{\theta} + \bar{\epsilon})$$

Problem 4. Show the equivalence of

(8) $$(x_L)_{\alpha\dot{\alpha}} = x_{\alpha\dot{\alpha}} - 2i\theta_\alpha\bar{\theta}_{\dot{\alpha}}$$

and

(9) $$x^\mu_L = x^\mu - i\theta^\alpha(\sigma^\mu)_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}.$$

Problem 5. Using the definition of

(10) $$\bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} + i\theta^\alpha\partial_{\alpha\dot{\alpha}}$$

Show also that

(11) $$\bar{D}_{\dot{\alpha}}x^\mu_L = 0.$$

Problem 6. Consider the minimal composite Higgs model (non-custodial) in the unitary gauge. Assuming that

$$U(h) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(h/f) & i\sin(h/f) \\ 0 & i\sin(h/f) & \cos(h/f) \end{pmatrix} \tag{12}$$

and taking

$$\begin{aligned} D_\mu &= \partial_\mu - igW^i_\mu T^i - ig'\left[\frac{1}{\sqrt{3}}T^8\right]B_\mu, \quad i=1,2,3, \\ &= \partial_\mu - igW^\pm_\mu T^\pm + \dots \end{aligned} \tag{13}$$

with $T^a = \lambda^a/2$, $a = 1,\dots,8$ and

$$T^\pm = \frac{T^1 \pm iT^2}{\sqrt{2}}, \quad W^\pm = \frac{W^1_\mu \mp iW^2_\mu}{\sqrt{2}} \tag{14}$$

compute the would be mass term of $W^\pm$. You can use Mathematica for that.

Problem 7. Consider the following EFT for a light scalar φ

$$\mathcal{L}_{\text{EFT}} = \mathcal{L}_{\text{dim=4}} - \frac{\alpha}{6!}\varphi^6 - \frac{\beta_6}{4!}\varphi^3\partial^2\varphi - \frac{\gamma}{2}(\partial^2\varphi)^2. \tag{15}$$

where

$$\mathcal{L}_{\text{dim=4}} = \frac{1}{2}(\partial_\mu\varphi)^2 - m_l^2\varphi^2 - \frac{\lambda}{4!}\varphi^4. \tag{16}$$

Using equations of motions show that

$$\varphi^3\partial^2\varphi = -m_l^2\varphi^4 - \frac{1}{3!}\lambda\varphi^6, \tag{17}$$

$$(\partial^2\varphi)^2 = m_l^4\varphi^2 + \frac{m_l^2\lambda}{3}\varphi^4 + \frac{\lambda^2}{36}\varphi^6. \tag{18}$$

Problem 8. Imagine that we integrate out a theory with an exotic heavy vector-like lepton

$$E \sim (\mathbf{1},\mathbf{1})_{-1}. \tag{19}$$

Looking to article refered in the notes, to which operators contribute such particle at tree-level?

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