Hadron physics

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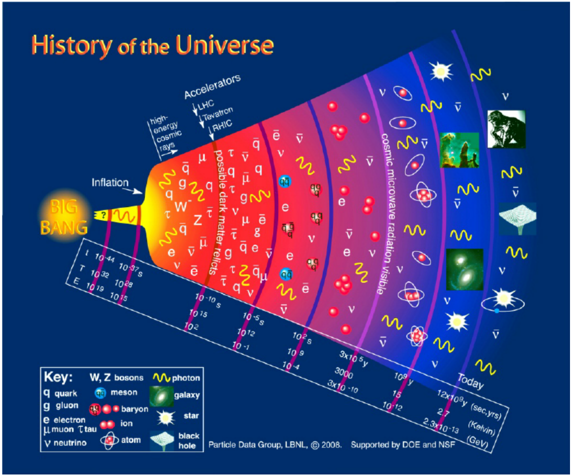
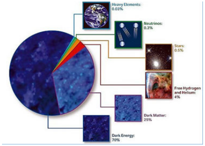
Content

Hadrons

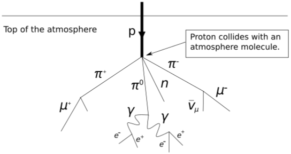




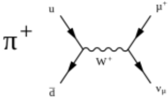
Ordinary matter (5%) formed after 100 s of the Big Bang

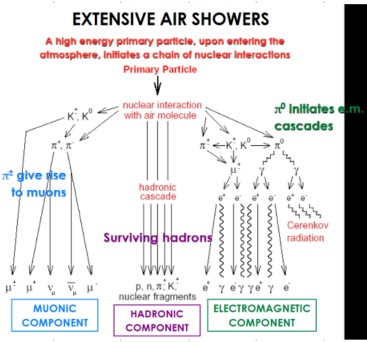
Hadrons

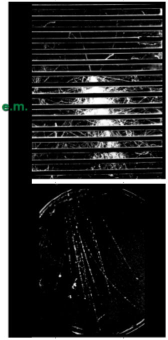
Rutherford, 1909, proton
Chadwick, 1932, neutron
Cosmic rays



$$\pi^+ \to \mu^+ + \nu_\mu$$
$$\pi^- \to \mu^- + \bar{\nu}_\mu$$
$$\pi^0 \to 2\gamma$$



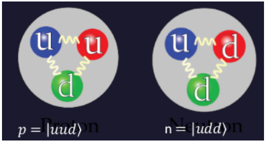


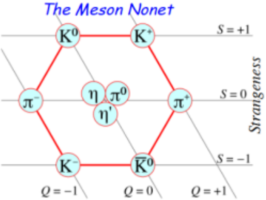


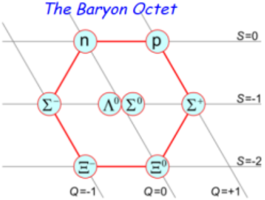
1947, Discovery of the π, K, César Lattes, Giuseppe Occhialini, Cecil Powell, George Rochester and Cliford Butler in Cloud Chambers

Quark model

The Eightfold way. *Three quarks for a master mark!*
Murray Gell-Mann and George Zweig, 1964







$m_N \simeq 938 - 940$ MeV

Isospin $\begin{pmatrix} p \\ n \end{pmatrix}$

$$Q = I_3 + \frac{1}{2}Y, \; Y = B + S$$

Gell-Mann-Nishijima

$$\pi^+ = -|u\bar{d}\rangle$$

$$\pi^- = |\bar{u}d\rangle$$

$$\pi^0 = \frac{1}{\sqrt{2}}|u\bar{u} - d\bar{d}\rangle$$

$m_\pi \simeq 135 - 140$ MeV

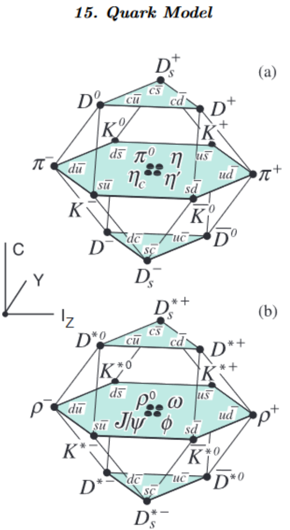
$$K^+ = |u\bar{s}\rangle$$

$$K^0 = |u\bar{d}\rangle$$

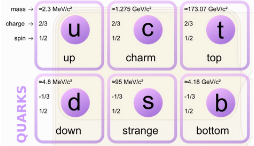
$m_K \simeq 495$ MeV

Three quarks generation

J/ψ discovery in 1974, SLAC & Brookhaven, Burton Richter & Samuel Ting



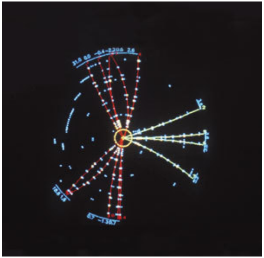
Charmed mesons, 1976 $D^+ = |c\bar{d}\rangle$



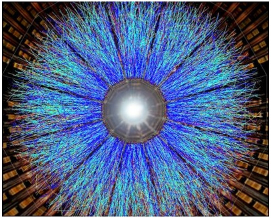
Color

QCD (Quantum Chromodynamics) arises from the quark model (Gell-Mann) to organize quarks (u,d,s) with spin (↑↓). However, Spin-Statistics Theorem:

$$\Delta^{++} \quad J^P = \frac{3}{2}^{++} \qquad u\uparrow u\uparrow u\uparrow$$



QGP - CERN, 1991, BNL-RHIC, 2005



Experimental evidences, Feynman, 1972

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

$$R = \frac{e^2 \sum_q Q_q^2}{e^2} = \sum_q Q_q^2 = 3 \times (\frac{4}{9} + \frac{1}{9} + \frac{4}{9} + \frac{1}{9} + \frac{1}{9} + ...)$$

$$= \frac{11}{3} \qquad \text{(including quarks up to b)}$$

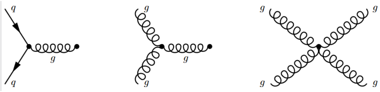
Other: Decay $\pi^0 \to 2\gamma$

Quantum Chromodynamics (QCD)

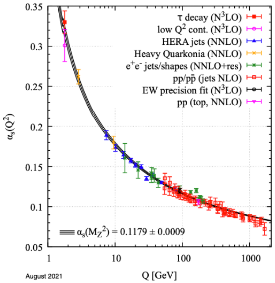
Strong interaction

$$\mathcal{L}_{\rm QCD} = \bar{\psi}_i \left(i\gamma^\mu (D_\mu)_{ij} - m\,\delta_{ij} \right) \psi_j - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

Non-abelian gauge theory. Symmetry group SU(3). *Color: Red, green, blue*



$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + b_0 \alpha_s(\mu^2) \log(Q^2/\mu^2) + \mathcal{O}(\alpha_s^2)}$$



- Assimptotic Freedom - David Gross, Frank Wilczek, David Politzer
- Confinement - evidence from LQCD

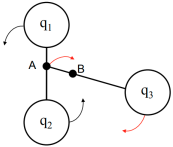
Baryon wave function

- Wavefunction for a baryon (fermion) is required to be antysimmetric under the exchange of two quarks:

$$\psi(total) = \phi(space) \cdot \phi(spin) \cdot \phi(flavour) \cdot \phi(colour) \quad (1)$$

- All observed particles are colour singlets:

$$\psi(colour) = \frac{1}{\sqrt{6}}(rgb - rbg + gbr - grb + brg - bgr) \quad (2)$$



$1, 2$ have l respect to A (CM_{12}) $(1, 2), 3$ have l' respect to B (CM_{123})

- This means that for the ground-state baryon $l = l' = 0$

$$\phi(spin) \cdot \phi(sabor)$$

must be symmetric.

Baryon wave function

Baryons in SU(2)

$$2 \otimes 2 = 3_S + 1_A \tag{3}$$

Flavour:

$$\left.\begin{array}{c} |1,1\rangle = uu \\ |1,0\rangle = \frac{1}{\sqrt{2}}(ud+du) \\ |1,-1\rangle = dd \end{array}\right\} I = 1 \quad (3_S)$$

$$|0,0\rangle = \frac{1}{\sqrt{2}}|ud-du\rangle \quad (1_A) \tag{4}$$

$$2 \otimes 2 \otimes 2 = (3_S + 1_A) \otimes 2 = (3_S \otimes 2) + (1_A \otimes 2) \tag{5}$$

$$I = \frac{1}{2} \qquad \left.\begin{array}{c} \frac{1}{\sqrt{2}}(ud-du)u \\ \frac{1}{\sqrt{2}}(ud-du)d \end{array}\right\} 1_A \otimes 2 \equiv 2_{M_A}; \qquad \left(\begin{array}{c} p \\ n \end{array}\right)_{M_A} \rightarrow \phi_{M_A}(\text{flavour})$$

Baryon wave function

$3_s \otimes 2 = 4_S + 2_{M_S}$ $(I = \frac{1}{2}, \frac{3}{2})$; $2 \otimes 2 \otimes 2 = 4_S + 2_{M_S} + 2_{M_A}$

$$I = \frac{3}{2} : \begin{pmatrix} uuu \\ \frac{1}{\sqrt{3}}(uud + udu + duu) \\ \frac{1}{\sqrt{3}}(udd + dud + ddu) \\ ddd \end{pmatrix} = \begin{pmatrix} \Delta^{++} \\ \Delta^{+} \\ \Delta^{0} \\ \Delta^{-} \end{pmatrix} \quad \text{(only flavour)}$$

(flavour, spin) Δ : $(4_S, 4_S)$:

$$\Delta^{++}(s_z = \frac{3}{2}) = (uuu)(\uparrow\uparrow\uparrow) = u\uparrow u\uparrow u\uparrow$$

$$\Delta^{+}(J_z = \frac{1}{2}) = \frac{1}{\sqrt{3}}(uud + udu + duu)\frac{1}{\sqrt{3}}(\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow) =$$

$$\frac{1}{3}[(u\uparrow u\uparrow d\downarrow + u\uparrow u\downarrow d\uparrow + u\downarrow u\uparrow d\uparrow) + \text{permutations}] \qquad (6)$$

$$I = \frac{1}{2} : \begin{pmatrix} \frac{1}{\sqrt{6}}(2uud - udu - duu) \\ \frac{1}{\sqrt{6}}(udd + dud - 2ddu) \end{pmatrix} ; \quad \begin{pmatrix} p \\ n \end{pmatrix}_{M_S} \rightarrow \phi_{M_S}(\text{flavour}) \qquad (7)$$

(Standard) Hadron wave function

(flavour, spin) N: $\frac{1}{\sqrt{2}}(2_{M_S}, 2_{M_A})$ [show]

$$p(s_z = \frac{1}{2}) = \frac{1}{3\sqrt{2}}\{2(u\uparrow u\uparrow d\downarrow + d\downarrow u\uparrow u\uparrow + u\uparrow d\downarrow u\uparrow)$$
$$- (u\uparrow u\downarrow d\uparrow + d\uparrow u\downarrow u\uparrow + u\downarrow d\uparrow u\uparrow)$$
$$- (u\downarrow u\uparrow d\uparrow + d\uparrow u\uparrow u\downarrow + u\uparrow d\uparrow u\downarrow)\}$$
$$= \frac{1}{3\sqrt{2}}\{(2u\uparrow u\uparrow d\downarrow - u\uparrow u\downarrow d\uparrow - u\downarrow u\uparrow d\uparrow) + \text{permutations}\} \quad (8)$$

Similarly, for mesons, one has,

$$\rho^0(s_z = 0) = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})\frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow) = \frac{1}{2}(u\uparrow\bar{u}\downarrow + u\downarrow\bar{u}\uparrow - d\uparrow\bar{d}\downarrow - d\downarrow\bar{d}\uparrow)$$
$$\pi^+(s_z = 0) = -u\bar{d}\frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow) = -\frac{1}{\sqrt{2}}(u\uparrow\bar{d}\downarrow - u\downarrow\bar{d}\uparrow) \quad (9)$$

Magnetic dipole moment

$$\vec{\mu} = \frac{Qe}{2mc} g\vec{s} \,, \tag{10}$$

for a point particle, with spin $s_z = 1/2$, $g = 2$. Then,

$$\mu_u = \frac{2}{3}\frac{e\hbar}{2m_u c} = \frac{e\hbar}{3m_u c}; \quad \mu_d = -\frac{1}{3}\frac{e\hbar}{2m_u c} = -\frac{e\hbar}{6m_u c} \tag{11}$$

In the quark model, the magnetic dipole moment is, $\vec{\mu}_{\text{hadron}} = \sum_{i=1}^{n} \vec{\mu}_i$. For a baryon with spin $s_z = 1/2$,

$$\mu_B = \langle B \uparrow | \sum_{i=1}^{3} \mu_i (\sigma_z)_i | B \uparrow \rangle \tag{12}$$

$$\mu_p = \frac{1}{18}[4(\mu_u + \mu_u - \mu_d) + (\mu_u - \mu_u + \mu_d) + (\mu_u - \mu_u + \mu_d)]3 = \frac{1}{3}(4\mu_u - \mu_d)$$

For the neutron, $\mu_n = \frac{1}{3}(4\mu_d - \mu_u)$. Since $\mu_d = -\frac{1}{2}\mu_u$,

$$\frac{\mu_n}{\mu_p} = -\frac{2}{3}; \quad (exp. -0.685) \tag{13}$$

Isospin

Isospin is a good approximate symmetry in the strong interactions.
$m_p = 938.28$ MeV; $m_n = 939.57$ MeV

$$\sigma(p+p \longrightarrow p+p) \sim \sigma(p+n \longrightarrow p+n); \quad E < 300 \text{ MeV} . \tag{14}$$

$a + b \longrightarrow c + d$

$$|i\rangle = \sum_{I,m} |I_a I_b; Im\rangle\langle I_a I_b; Im|I_a I_b; m_a m_b\rangle \quad |f\rangle = \sum_{I',m'} |I_c I_d; I'm'\rangle\langle I_c I_d; Im|I_c I_d; m_c m_d\rangle$$

$$\langle f|t|i\rangle = \sum_I C(I_a I_b I; m_a m_b m) C(I_c I_d I; m_c m_d m) \mathcal{T}^{(I)} , \tag{15}$$

Example

$$\langle \pi^- p|\mathcal{T}|\pi^- p\rangle = \frac{1}{3}\mathcal{T}^{(3/2)} + \frac{2}{3}\mathcal{T}^{(1/2)} , \tag{16}$$

$$|\pi^- p\rangle = \sqrt{\frac{1}{3}}|\frac{3}{2} - \frac{1}{2}\rangle - \sqrt{\frac{2}{3}}|\frac{1}{2} - \frac{1}{2}\rangle . \tag{17}$$

Isospin

$$\langle \pi^+ p | \mathcal{T} | \pi^+ p \rangle = \mathcal{T}^{(3/2)} \ . \tag{18}$$

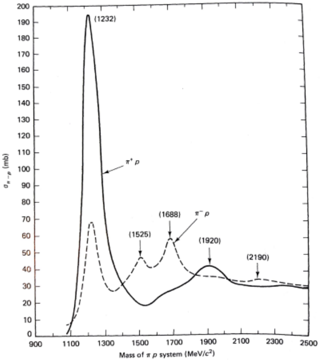
One obtains, [show]

$$\sigma(\pi^+ p \to \pi^+ p) : \sigma(\pi^- p \to \pi^- p) : \sigma(\pi^- p \to \pi^0 n) = 9 : 1 : 2 \tag{19}$$

For energies, $\sqrt{s} \sim 1232$ MeV, the cross section is dominated by the $\Delta(1232)$ resonance ($I = 3/2$), $\mathcal{T}^{(3/2)} >> \mathcal{T}^{(1/2)}$.

$$\frac{\sigma(\pi^+ p)}{\sigma(\pi^- p)} = 3$$

(Total cross sections, Gasiorowicz, S. (1966))



SU(3) flavour symmetry

Isospin breaking, $(m_d - m_u)/\Lambda_{QCD} \sim 1\%$. Color is an exact SU(3) symmetry.
Since $m_s > m_u, m_d$ we expect that SU(3) flavour symmetry is not exact.

$$\begin{pmatrix} u' \\ d' \\ s' \end{pmatrix} = \hat{U} \begin{pmatrix} u \\ d \\ s \end{pmatrix}; \quad U^\dagger U = 1; \quad \hat{U} = e^{i\alpha_k \lambda_k} \tag{20}$$

The three quarks states can be represented as,

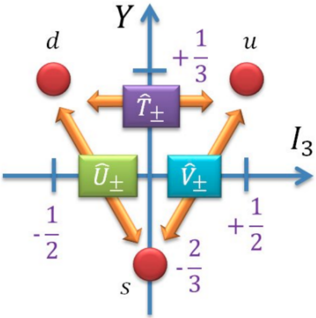
$$u = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}; \quad d = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}; \quad s = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{21}$$

$$u \longleftrightarrow d \quad \lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \longleftarrow \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$u \longleftrightarrow s \quad \lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \nwarrow \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

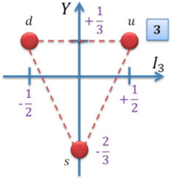
$$d \longleftrightarrow s \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \swarrow$$

SU(3) flavour symmetry



$$\hat{T}_\pm = \frac{1}{2}(\lambda_1 \pm i\lambda_2)$$
$$\hat{V}_\pm = \frac{1}{2}(\lambda_4 \pm i\lambda_5)$$
$$\hat{U}_\pm = \frac{1}{2}(\lambda_6 \pm i\lambda_7)$$

with $I_3 = \frac{1}{2}\lambda_3$ and $Y = \frac{1}{\sqrt{3}}\lambda_8$



Quarks:

$$\hat{I}_3 u = +\frac{1}{2}u \qquad \hat{I}_3 d = -\frac{1}{2}d \qquad \hat{I}_3 s = 0$$
$$\hat{Y} u = +\frac{1}{3}u \qquad \hat{Y} d = +\frac{1}{3}d \qquad \hat{Y} s = -\frac{2}{3}s$$

Since $(m_s - m_d)/\Lambda_{QCD} \lesssim \frac{1}{2}$, the symmetry is broken, but still very useful.

Quark model

Most quark models contain:

Confining interaction (harmonic oscillator or linear) + spin-spin interaction

Color-magnetic interaction (analog of the 'hyperfine' interaction in QED) for S-wave:

$$H_I = -\alpha_S M \sum_{i>j} (\vec{\sigma}\lambda_a)_i (\vec{\sigma}\lambda_a)_j \quad (22)$$

Example. Empirical formula for the masses of mesons with zero angular orbital momenta:

$$M = m_1 + m_2 + a\frac{\vec{\sigma}_1 \cdot \vec{\sigma}_2}{m_1 m_2} \quad (23)$$

$$\vec{\sigma}_1 \cdot \vec{\sigma}_2 = 4[\vec{s}^{\,2} - \vec{s}_1^{\,2} - \vec{s}_2^{\,2}]/2 = 2\left[s(s+1) - \frac{3}{2}\right] = \begin{cases} 1 & s=1 \\ -3 & s=0 \end{cases} \quad (24)$$

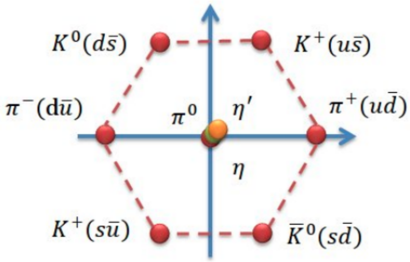
Quark model

$3 \otimes \bar{3} = 8 + 1$. The lightest mesons divide into:

- Pseudoscalar mesons with $s = 0$
- Vector mesons with $s = 1$

Taking $m_u = m_d = 310$ MeV, $m_s = 483$ MeV, $a = 160$ MeV, one gets,

	$\pi^\pm$	ρ	K	$K^\star$	η	ϕ
th.	140	780	484	890	559	1032
ex.	138	776	496	892	549	1020



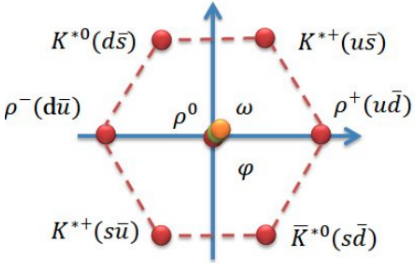
Empirically, we find

$$\pi^0 = \frac{1}{\sqrt{2}}\left(|u\bar{u}\rangle - |d\bar{d}\rangle\right)$$

$$\eta \approx \frac{1}{\sqrt{6}}\left(|u\bar{u}\rangle + |d\bar{d}\rangle - 2\,|s\bar{s}\rangle\right)$$

$$\eta' \approx \frac{1}{\sqrt{3}}\left(u\bar{u} + d\bar{d} + s\bar{s}\right) \quad \boxed{\text{singlet}}$$

Quark model



$$\rho^0 = \frac{1}{\sqrt{2}}\left(|u\bar{u}\rangle - |d\bar{d}\rangle\right)$$
$$\omega \approx \frac{1}{\sqrt{2}}\left(|u\bar{u}\rangle + |d\bar{d}\rangle\right)$$
$$\phi \approx s\bar{s}$$

Within the quark model ...

- The different masses within the octect comes mostly from the fact that $m_s > m_u$.
- The different masses of the π and ρ mesons can be mostly attributed to spin-spin interactions
- The η' meson predicted mass differs significantly from the anomalously large observed value of 958 MeV. This is the so-called $\eta - \eta'$ puzzle.

Remark: LQCD cannot generate the ρ meson with only $q\bar{q}$ operators.

Experiment

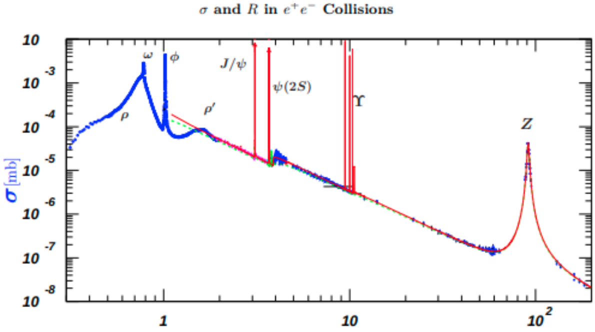


Figure 52.2: World data on the total cross section of $e^+e^- \to hadrons$ and the ratio $R(s) = \sigma(e^+e^- \to hadrons, s)/\sigma(e^+e^- \to \mu^+\mu^-, s)$. $\sigma(e^+e^- \to hadrons, s)$ is the experimental cross section corrected for initial state radiation and electron-positron vertex loops, $\sigma(e^+e^- \to \mu^+\mu^-, s) = 4\pi\alpha^2(s)/3s$. Data errors are total below 2 GeV and statistical above 2 GeV. The curves are an educative guide: the broken one (green) is a naive quark-parton model prediction, and the solid one (red) is 3-loop pQCD prediction (see "Quantum Chromodynamics" section of this *Review*, Eq. (9.7) or, for more details [99], Breit-Wigner parameterizations of J/ψ, $\psi(2S)$, and $\Upsilon(nS), n = 1,2,3,4$ are also shown. The full list of references to the original data and the details of the R ratio extraction from them can be found in [100]. Corresponding computer-readable data files are available at http://pdg.lbl.gov/current/xsect/. (Courtesy of the COMPAS (Protvino) and HEPDATA (Durham) Groups, August 2019. Corrections by P. Janot (CERN) and M. Schmitt (Northwestern U.))

Quark model

Possible quantum numbers of $q\bar{q}$ ($S = 0, 1$). Since the total spin, $\vec{J} = \vec{L} + \vec{S}$, Parity and C-Parity are preserved,

- Parity, $P = (-)^L(-) = (-)^{L+1}$
- C-Parity, $C = (-)^L(-)(-)^{S+1} = (-)^{L+S}$

For ordinary mesons, we have:

	$S = 0$	$S = 1$
$L = 0$	0^{-+}	1^{--}
$L = 1$	1^{+-}	$0^{++}, 1^{++}, 2^{++}$
$L = 2$	2^{-+}	$1^{--}, 2^{--}, 3^{--}$
$L = 3$	3^{+-}	$2^{++}, 3^{++}, 4^{++}$
.	.	.

Note, that we cannot have with $q\bar{q}$,

$$J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, 3^{-+}, \cdots \qquad (25)$$

The $\pi_1(1400)$ has been observed with $J^{PC} = 1^{-+}$ in the $\pi^- p \longrightarrow \eta\pi^- p$ reaction. "Hibrid", $q\bar{q}g$. GlueX experiment at Jefferson Lab.

Godfrey-Isgur Quark Model

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Mesons in a relativized quark model with chromodynamics

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(Received 12 December 1983; revised manuscript received 10 May 1985)

We show that mesons—from the π to the Υ—can be described in a unified quark model with chromodynamics. The key ingredient of the model is a universal one-gluon-exchange-plus-linear-confinement potential motivated by QCD, but it is crucial to the success of the description to take into account relativistic effects. The spectroscopic results of the model are supported by an extensive analysis of strong, electromagnetic, and weak meson couplings.

$$H\,|\Psi\rangle=(H_0+V)\,|\Psi\rangle=E\,|\Psi\rangle$$

$$H_0\to\sum_{i=1}^{2}\left[m_i+\frac{p^2}{2m_i}\right] \tag{2a}$$

and

$$V_{ij}(\mathbf{p},\mathbf{r})\to H_{ij}^{\rm conf}+H_{ij}^{\rm hyp}+H_{ij}^{\rm so}+H_A \tag{2b}$$

where

$$H_{ij}^{\rm conf}=-\left[\tfrac{3}{4}c+\tfrac{3}{4}br-\frac{\alpha_s(r)}{r}\right]\mathbf{F}_i\cdot\mathbf{F}_j \tag{3}$$

includes the spin-independent linear confinement and Coulomb-type interactions,

$$H_{ij}^{\rm hyp}=-\frac{\alpha_s(r)}{m_im_j}\left[\frac{8\pi}{3}\mathbf{S}_i\cdot\mathbf{S}_j\delta^3(\mathbf{r})+\frac{1}{r^3}\left[\frac{3\mathbf{S}_i\cdot\mathbf{r}\mathbf{S}_j\cdot\mathbf{r}}{r^2}-\mathbf{S}_i\cdot\mathbf{S}_j\right]\right]\mathbf{F}_i\cdot\mathbf{F}_j$$

is the color hyperfine interaction, and

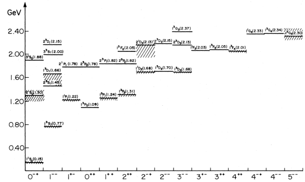
$$H_{ij}^{\rm so}=H_{ij}^{\rm so(cm)}+H_{ij}^{\rm so(tp)} \tag{5}$$

is the spin-orbit interaction with

$$H_{ij}^{\rm so(cm)}=-\frac{\alpha_s(r)}{r^3}\left[\frac{1}{m_i}+\frac{1}{m_j}\right]\left[\frac{\mathbf{S}_i}{m_i}+\frac{\mathbf{S}_j}{m_j}\right]\cdot\mathbf{L}\,(\mathbf{F}_i\cdot\mathbf{F}_j)\,, \tag{6}$$

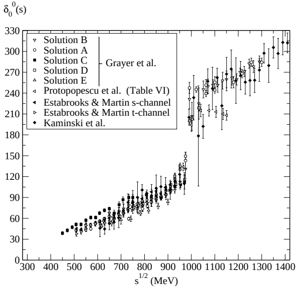
its color-magnetic piece and with

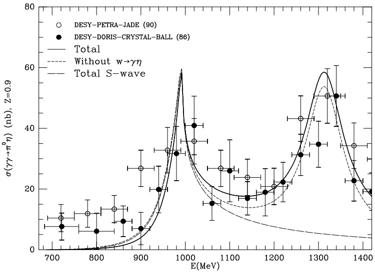
$$H_{ij}^{\rm so(tp)}=\frac{-1}{2r}\frac{\partial H_{ij}^{\rm conf}}{\partial r}\left[\frac{\mathbf{S}_i}{m_i^{\,2}}+\frac{\mathbf{S}_j}{m_j^{\,2}}\right]\cdot\mathbf{L} \tag{7}$$



Godfrey-Isgur Quark Model

- Difficulties to explain the masses of some excited states ...
- In particular the **scalar mesons** $J^{PC} = 0^{++}$, σ, κ, $f_0(980)$, $a_0(980)$, which decay in two pseudoscalar mesons, $\pi\pi/\pi K/K\bar{K}$

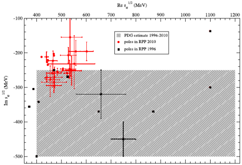


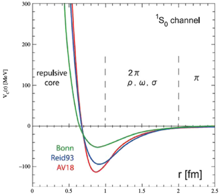


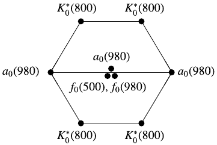
- Other light mesons difficult to explain: $f_0(1500)$, $f_0(1370)$ ($J^{PC} = 0^{++}$)
- Mesons observed with quantum numbers that **cannot be obtained with** $q\bar{q}$, $\pi_1(1400)$, $\pi_1(1600)$, $J^{PC} = 1^{-+} \cdots$
- Other examples in the baryon sector: $N(1440)$, $\Lambda(1405) \cdots$

The σ meson

What is the **nature** of the scalar resonances? Is there a light scalar nonet?



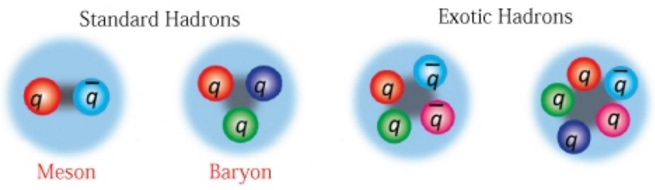




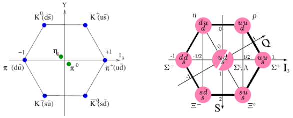
- 70's. Analyticity, unitarity and crossing symmetry constraints required a broad σ pole (Guillou, Morel, Navalet, Basdevant, Froggatt, Peterson, Roy).
- Glueball scenario. Not favored by LQCD, large N_c, chiral symmetry.
- Alternative scenario to $q\bar{q}$, Jaffe, 1976, MIT bag model, tetraquark?.

Rev. Part. Phys.'73. *It is clear that the behavior of the* δ_0^0 *is much too complicated to allow a description in terms of one or several Breit-Wigner resonances. We therefore list the positions of the poles of the T matrix.*

Hadrons

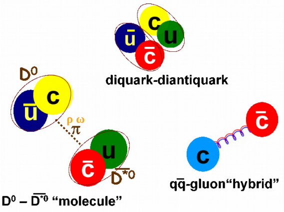


- 'Regular' hadrons: $q\bar{q}$, qqq



- Exotics: $q\bar{q}q\bar{q}$, $qqqq\bar{q}$, qqg,...

  Not $q\bar{q}$: $J^{PC} = 0^{+-}, 1^{-+}, 2^{+-}, 3^{-+}$,...

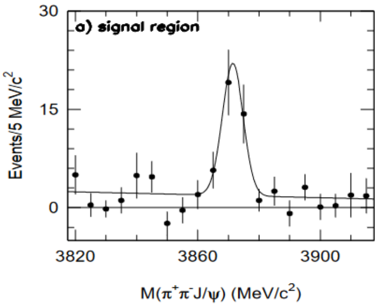


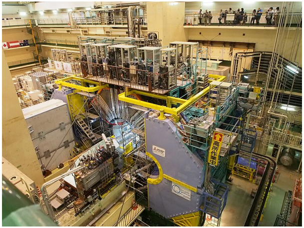
Exotics: The X(3872)

PRL03, BELLE (close to $D^0\bar{D}^{*0}$ th.)

ABSTRACT

We report the observation of a narrow charmoniumlike state produced in the exclusive decay process $B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}J/\psi$. This state, which decays into $\pi^{+}\pi^{-}J/\psi$, has a mass of $3872.0 \pm 0.6(\mathrm{stat}) \pm 0.5(\mathrm{syst})$ MeV, a value that is very near the $M_{D^0} + M_{D^{*0}}$ mass threshold. The results are based on an analysis of 152M B-$\bar{B}$ events collected at the $\Upsilon(4S)$ resonance in the Belle detector at the KEKB collider. The signal has a statistical significance that is in excess of 10σ.





The measured mass of the state is within errors of the $D^0\bar{D}^{*0}$ mass threshold (3871.3 ± 0.5 MeV [7]). This would be expected for a loosely bound $D\bar{D}^*$ multiquark "molecular state," such as proposed by De Rujula, Georgi and Glashow in 1977 [13].

CONCLUSION

We have observed a strong signal (8.6σ) for a state that decays to $\pi^+\pi^-J/\psi$ with

$$M = 3871.8 \pm 0.7\ (\mathrm{stat}) \pm 0.4\ (\mathrm{syst})\ \mathrm{MeV}$$
$$\Gamma < 3.5\ \mathrm{MeV}\ .$$

This mass value is about 60 MeV higher than potential model predictions for a 1D charmonium state and equal, within errors, to $M_{D^0} + M_{D^{*0}}$. This coincidence with the $D^0\bar{D}^{*0}$ mass threshold suggests that this may be a DD^* multiquark state.

Exotics: The $D_{s0}(2317)$ and $D_{s1}(2460)$

BABAR, CLEO’03

PHYSICAL REVIEW LETTERS

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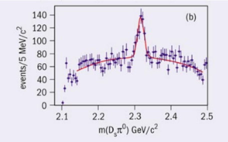
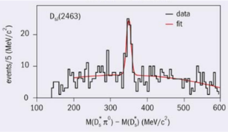
Observation of a Narrow Meson State Decaying to $D_s^+\pi^0$ at a Mass of $2.32\ \mathrm{GeV}/c^2$

B. Aubert *et al.* (BABAR Collaboration)

Phys. Rev. Lett. **90**, 242001 – Published 17 June 2003

- Babar observed
  $D_{s0}{}^{*+}(2317)\to D_s{}^+\,\pi^0$
  Phys. Rev. Lett. 90(2003)242001
- Cleo observed
  $D_{s1}{}^{+}(2460)\to D_s{}^{+*}\pi^0$
  Phys. Rev. D 68(2003)032002
- D_s in final state
  > most probable assignment
  $[c\ \bar{s}]$ L=1 states
- ~100 MeV too low compared to early quark models
  Godfrey, Isgur, PRD 32(1985)189

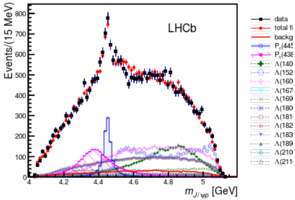


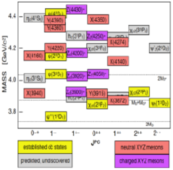


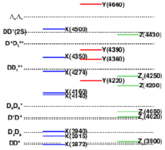
Similar masses to the DK, DK^* thresholds

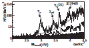
Exotics: $P_c(4450)$, pentaquarks, and other XYZ

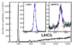
LHCb, PRL15



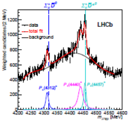




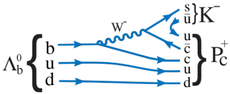




State	M_0 [MeV]	Γ [MeV]
$P_c(4459)^0$	$4458.8 \pm 2.9^{+4.7}_{-1.1}$	$17.3 \pm 6.5^{+8.0}_{-5.7}$



Clear evidence of exotic states!



- Hidden-charm charged tetraquarks $Z_c^+ \sim c\bar{d}u\bar{c}$ $(D^{(*)}\bar{D}^{(*)})$.
  Hidden-strange candidate? $a_0(980)$? ... more?
- Hidden-charm (strange) pentaquarks $P^+_{c(s)} \sim c\bar{c}uud(s)$, $(\bar{D}^{(*)}\Sigma_c^{(*)}(\Xi_c^{(*)}))$.
  Hidden-strange candidate? $N^*(1535)$, (strange) $\Lambda(1405)$, ...more?

(I,S)	z_R (MeV)		g_a	
$(1/2,0)$		$\bar{D}^*\Sigma_c$	$\bar{D}^*\Lambda_c^+$	
	4418	2.75	0	
$(0,-1)$		$\bar{D}_s^*\Lambda_c^+$	$\bar{D}^*\Xi_c$	$\bar{D}^*\Xi_c'$
	4370	1.23	3.14	0
	4550	0	0	2.53

TABLE IV: Pole position and coupling constants for the bound states from $VB \to VB$.

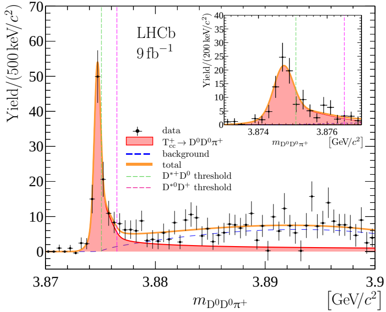
PRL 105, 232001 (2010) PHYSICAL REVIEW LETTERS week ending 3 DECEMBER 2010

Prediction of Narrow N^* and Λ^* Resonances with Hidden Charm above 4 GeV

Jia-Jun Wu,[1,2] R. Molina,[2,3] E. Oset,[2,3] and B. S. Zou[1,3]
[1]Institute of High Energy Physics, CAS, Beijing 100049, China
[2]Departamento de Física Teórica and IFIC, Centro Mixto Universidad de Valencia-CSIC, Institutos de Investigación de Paterna, Apartado 22085, 46071 Valencia, Spain
[3]Theoretical Physics Center for Science Facilities, CAS, Beijing 100049, China
(Received 5 July 2010; published 29 November 2010)

The interaction between various charmed mesons and charmed baryons is studied within the framework of the coupled-channel unitary approach with the local hidden gauge formalism. Several meson-baryon dynamically generated narrow N^* and Λ^* resonances with hidden charm are predicted with mass above 4 GeV and width smaller than 100 MeV. The predicted new resonances definitely cannot be accommodated by quark models with three constituent quarks and can be looked for in the forthcoming PANDA/FAIR experiments.

Exotics: T_{cc}^{+} signal in $D^0 D^0 \pi^+$

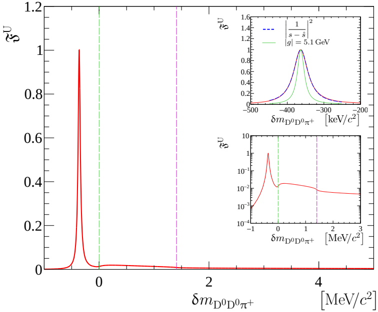


Nature, 18 (2022). BEFORE resolution:

$$m_{\text{exp}} = 3875.09 \text{ MeV} + \delta m_{\text{exp}}$$

$$\delta m_{\text{exp}} = -273 \pm 61 \pm 5^{+11}_{-14} \text{ keV};$$

$$\Gamma = 410 \pm 165 \pm 43^{+18}_{-38}\ keV$$



Nature, 13 (2022). AFTER resolution:

$$\delta m_{\text{exp}} = -360 \pm 40^{+4}_{-0} \text{ keV};$$

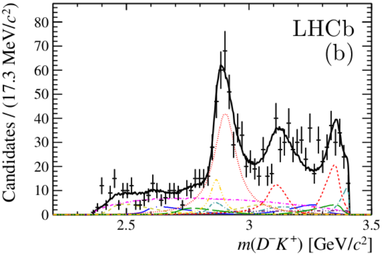
$$\Gamma = 48 \pm 2^{+0}_{-14} keV$$

Remarkably close to the $D^{*+}D^0/D^{*0}D^+$ thresholds!

Flavor exotic tetraquark $T_{cs}(2900)$

LHCb (2020) Two states $J^P = 0^+, 1^-$ decaying to $\bar{D}K$. First clear example of an heavy-flavor exotic tetraquark, $\sim \bar{c}\bar{s}ud$.

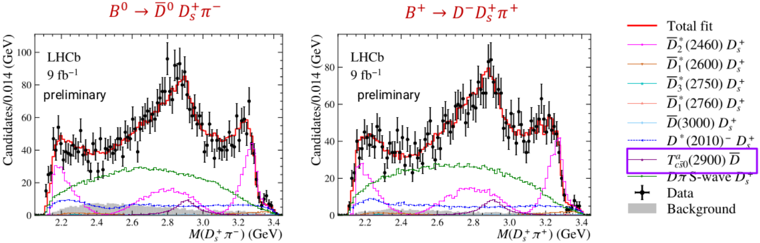
$$X_0(2866) : M = 2866 \pm 7 \quad \text{and} \quad \Gamma = 57.2 \pm 12.9\,\text{MeV},$$
$$X_1(2900) : M = 2904 \pm 5 \quad \text{and} \quad \Gamma = 110.3 \pm 11.5\,\text{MeV}.$$



PRL20, PRD20

New exotic tetraquark seen in $D_s^+\pi^+$

LHCb (2022) One state decaying $T_{c\bar{s}}(2900)$ decaying to $D_s^+\pi^-$ and $D_s^+\pi^+$ has been observed $\sim c\bar{s}u\bar{d}$.



- The analysis favors $J^P = 0^+$
- Mass, $m = 2908 \pm 11 \pm 20$ MeV
- Width, $\Gamma = 136 \pm 23 \pm 11$ MeV

PRD23

D^*K^* th.: 2903 MeV

$D_s^*\rho$ th.: 2890 MeV

Symmetries

Lorent invariance requires S (action), L, Lagrangian, to transform as scalars.

$$L = \int d^3x \mathcal{L}(\phi(x), \partial_\mu \phi(x), t)$$
$$S = \int dt\, L = d^4x \mathcal{L}(\phi(x), \partial_\mu \phi(x), t) \tag{26}$$

Then, under an infinitesimal variation,

$$\phi \rightarrow \phi + \delta\phi$$
$$\partial_\mu \phi \rightarrow \partial_\mu \phi + \delta(\partial_\mu \phi) \tag{27}$$

$$\text{Hamilton Principle} \quad \delta S = 0 \longrightarrow \frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) = 0 \tag{28}$$

Example. Free fermion.

$$\mathcal{L} = \bar{\psi}(i\gamma_\mu \partial^\mu - m)\psi \longrightarrow (i\gamma^\mu \partial_\mu - m)\psi = 0 \quad \text{Dirac equation} \tag{29}$$

Symmetries

Given L symmetric, under an infinitesimal field transformation,

$$\phi \longrightarrow \phi + \delta\phi; \quad \mathcal{L}(\phi + \delta\phi) = \mathcal{L}(\phi)$$

$$0 = \frac{\partial \mathcal{L}}{\partial \phi}\delta\phi + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}\delta(\partial_\mu \phi)$$

$$\Longrightarrow \delta^\mu J_\mu = 0, \quad J_\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)}\delta\phi_i \tag{30}$$

(Noether current) A conserved current leads to charge conservation,

$$Q = \int d^3x J_0(x); \frac{d}{dt}Q = 0 \tag{31}$$

Massless fermions. Consider the vector and axial symmetries,

$$\mathcal{L} = i\bar{\psi}_j \partial\!\!\!/ \psi_j, \quad j = u, d \tag{32}$$

$$\Lambda_V : \psi \longrightarrow e^{-i\frac{\vec{\tau}}{2}\cdot\vec{\alpha}\psi} \simeq (1 - i\frac{\vec{\tau}}{2}\cdot\vec{\alpha})\psi; \quad \Lambda_A : \psi \longrightarrow e^{-i\gamma_5\frac{\vec{\tau}}{2}\cdot\vec{\alpha}\psi} \simeq (1 - i\gamma_5\frac{\vec{\tau}}{2}\cdot\vec{\alpha})\psi$$

Chiral symmetry

The Lagrangian is invariant under these symmetries $SU(2)_V \times SU(2)_A$, and the vector and axial-vector currents are preserved.

$$V^a_\mu = \bar{\psi}\gamma_\mu \frac{\tau^a}{2}\psi; \quad A^a_\mu = \bar{\psi}\gamma_\mu\gamma_5\frac{\tau^a}{2}\psi \tag{33}$$

However, when we introduce a mass term,

$$\delta\mathcal{L} = -m(\bar{\psi}\psi)\ , \tag{34}$$

the lagrangian is invariant under Λ_V, but not under Λ_A, since,

$$\Lambda_A : m(\bar{\psi}\psi) \rightarrow \bar{\psi}\psi - 2i\vec{\alpha}\cdot\left(\bar{\psi}\frac{\vec{\tau}}{2}\gamma_5\psi\right) \tag{35}$$

Still, $m_u, m_d << \Lambda_{QCD} \simeq 200$ MeV. PCAC. Partial conservation axial current.

$$\langle 0|A^a_\mu(x)|\pi^b(q)\rangle = if_\pi q_\mu \delta^{ab} e^{-iq\cdot x}; \quad A^\pi_\mu = f_\pi \partial_\mu \phi(x) \tag{36}$$

Linear sigma model

Gell-Mann and Levy, 1960. Chiral limit. Nucleons and pions.

$$\mathcal{L} = i\bar{\psi}\partial\!\!\!/\psi - g_\pi(\bar{\psi}\gamma_5\vec{\tau}\psi\vec{\pi} + \bar{\psi}\psi\sigma) - \frac{\lambda}{4}((\pi^2+\sigma^2) - f_\pi)^2 + \frac{1}{2}\partial_\mu\pi\partial^\mu\pi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma$$

Transformation of the fields under Λ_A (infinitesimal),

$$\begin{aligned}\psi &\longrightarrow \left(1 - i\gamma_5\frac{\vec{\tau}}{2}\cdot\vec{\alpha}\right)\psi \\ \pi_i &\longrightarrow \pi_i + \alpha_i\sigma \\ \sigma &\longrightarrow \sigma - \alpha_i\pi_i\end{aligned} \tag{37}$$

can be shown by using combinations of quark fields,

$$\vec{\pi} : i\bar{q}\vec{\tau}\gamma_5 q; \quad \sigma : \bar{q}q \tag{38}$$

and the combination below remains invariant,

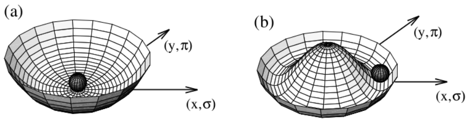
$$\pi^2 + \sigma^2 \longrightarrow \pi^2 + \sigma^2 \ . \tag{39}$$

Linear sigma model

The potential,

$$V = V(\pi^2 + \sigma^2) = \frac{\lambda}{4}((\pi^2 + \sigma^2) - f_\pi)^2 \tag{40}$$

has a minimum when $\sigma = f_\pi$ for $\pi = 0$ and has a form of a mexican hat.



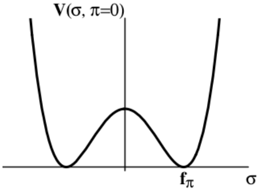


Figure: (a) No esp. breaking; (b) Esp. breaking (Goldstone theorem and pseudoscalar mesons).

Ground state,

$$\sigma \simeq \sigma_0 + \delta\sigma; \pi \simeq \delta\pi$$

$$V(\sigma, \pi) \simeq \lambda f_\pi^2 (\delta\sigma)^2 + \mathcal{O}(\delta^3)$$

$$< \sigma >= \sigma_0 = f_\pi$$

$$< \pi >= 0$$

$$M_N = g_\pi \sigma_0 = g_\pi f_\pi$$

$$m_\sigma^2 = \lambda f_\pi^2 \neq 0;\ m_\pi = 0$$

Explicit symmetry breaking

$$\partial\mathcal{L} = -m\bar{q}q \quad \text{(QCD)}; \quad \partial\mathcal{L} = \epsilon\sigma \quad \text{(LSM)} \tag{41}$$

Potential with explicit symmetry breaking,

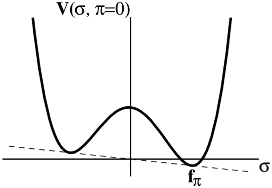
$$V = V(\pi^2+\sigma^2) = \frac{\lambda}{4}((\pi^2+\sigma^2)-v_0)^2 - \epsilon\sigma \tag{42}$$

If we require that the new minimum is still f_π to preserve the Goldberger-Treiman relation ($g_{\pi NN} = g_a \frac{M_N}{f_\pi}$),

$$v_0 = f_\pi - \frac{\epsilon}{2\lambda f_\pi^2}$$

$$m_\sigma^2 = \left.\frac{\partial^2 V}{\partial\sigma^2}\right|_{\sigma_0} = 2\lambda f_\pi + \frac{\epsilon}{f_\pi}$$

$$m_\pi^2 = \left.\frac{\partial^2 V}{\partial\pi^2}\right|_{\sigma_0} = \frac{\epsilon}{f_\pi} \neq 0 \rightarrow \epsilon = f_\pi m_\pi^2$$



Explicit symmetry breaking

$$M_N = g_\pi \sigma_0 = g_\pi \left(v_0 + \frac{\epsilon}{2\lambda f_\pi^2} \right), \quad \Sigma_{\pi N} = \delta M_N = g_\pi \frac{\epsilon}{2\lambda f_\pi^2} \simeq g_\pi f_\pi \frac{m_\pi^2}{m_\sigma^2}$$

Experimental value from pionic atom data, $\Sigma_{\pi N} = 57 \pm 7$ MeV. Friedman, Gal, PLB19. We expect that,

$$\langle 0|\epsilon\sigma|0\rangle = \langle 0| - m\bar{q}q|0\rangle \quad (43)$$

By using, $\epsilon = m_\pi^2 f_\pi$, and $\langle 0|\sigma|0\rangle = f_\pi$,

$$m_\pi^2 f_\pi^2 = -\frac{m_u + m_d}{2}\langle 0|\bar{u}u + \bar{d}d|0\rangle \quad (GOR) \quad (44)$$

Exact expression of the pion-nucleon sigma term,

$$\Sigma_{\pi N} = \frac{m_u + m_d}{2}\langle N|\bar{u}u + \bar{d}d|N\rangle \quad (45)$$

Alfaro, Fubini, Furlan, Rosseti, *Currents in hadron physics* 1973; Gasser, Leutwyler, PRD82; H. Höhler, *Pion-nucleon scattering*, 1983

Heavy quark spin and flavor symmetries

- Heavy quark symmetries arises when $m_Q >> \Lambda_{QCD}$. $Q\bar{q}$ system, $\Delta p \simeq \Lambda_{QCD} \longrightarrow \Delta v = \frac{\Delta p}{m_Q} \rightarrow 0$ in the limit $m_Q \rightarrow \infty$. The heavy quark behaves as a static source.
- The heavy quark interacts with gluons through the chromoelectric charge. Spin dependent interactions, $\mu^c \propto 1/m_Q$. Heavy Quark Spin Symmetry (HQSS) and Heavy Quark Flavor Symmetry (HQFS) Isgur, Nathan and Wise, Mark B.PLB89, PLB90, PRL91, Manohar, Aneesh V. and Wise, Mark B., "Heavy Quark Physics"
- SU(2) spin symmetry, heavy hadrons organize into doublets with approximately similar mass and it is possible to work in this basis

$$|S_H, L, J\rangle \quad \text{HQSS basis} \tag{46}$$

- Doublets

$$(D, D^*) \quad (\eta_c, J/\psi) \quad (D_s, D_s^*) \cdots \tag{47}$$

$Q\bar{q}$-like systems

$D_{(s)}$ mesons, $j_l = L \pm \frac{1}{2}$

- $L = 0 : J^P = 0^-, 1^-$ (D, D^*), (D_s, D_s^*)
- $L = 1$: Two Doublets:
  - $j_l^P = \frac{1}{2}^+$. $J^P = (0,1)^+$. $D^*(2300), D_1(2420)$?
  - $j_l^P = \frac{3}{2}^+$. $J^P = (1,2)^+$. $D_1(2430), D_2^*(2460)$?
  - $j_l^P = \frac{1}{2}^+$. $J^P = (0,1)^+$. $D_{s0}(2317), D_{s1}(2460)$?
  - $j_l^P = \frac{3}{2}^+$. $J^P = (1,2)^+$. $D_{s1}(2536), D_{s2}^*(2573)$?
- However, there are some puzzles:
  - Exp. masses and widths of the $D_{s0}(2317)$ and $D_{s1}(2460)$ not compatible with $q\bar{q}$ expectations. Role of the DK, D^*K channel.
  - Masses of the D_s counterparts are expected to be 100 higher since $m_s/m_d \simeq 20$. Godfrey, Isgur, Kokoski, PRD79,85 However,

$$B(D_{s0}(2317))_{DK} \simeq B(D_{s1}(2460))_{D^*K} \simeq 40 \text{ MeV} \tag{48}$$

  - Measurement of the $D_0^*(2300)$ mass varies from $2300 - 2400$ MeV. Possible two pole structure related to $D\pi$, $D_s\bar{K}$ channels. Not confirmed yet by LQCD.

Kolomeitsev and Lutz, PLB04, Albaladejo, Fernandez-Soler, Nieves, Guo

Scattering (Lippmann-Schwinger equation)

$$H = H_0 + V, \tag{49}$$

$$\begin{aligned}(H_0 + V)|\psi\rangle = E|\psi\rangle &\Longrightarrow (E - H_0)|\psi\rangle = V|\psi\rangle \\ H_0|\phi\rangle = E|\phi\rangle &\Longrightarrow (E - H_0)|\phi\rangle = 0\end{aligned} \tag{50}$$

Subtracting both equations,

$$(E - H_0)(|\psi\rangle - |\phi\rangle) = V|\psi\rangle \Longrightarrow \boxed{|\psi^{(+)}\rangle = |\phi\rangle + \frac{1}{E - H_0 + i\epsilon} V|\psi^{(+)}\rangle} \tag{51}$$

We define the scattering amplitude, $V|\psi^{(+)}\rangle = T|\phi\rangle$.

$$\begin{aligned}T|\phi\rangle \equiv V|\psi^{(+)}\rangle &= V|\phi\rangle + V\frac{1}{E - H_0} V|\psi^{(+)}\rangle \\ &= V|\phi\rangle + V\frac{1}{E - H_0} T|\phi\rangle \, .\end{aligned} \tag{52}$$

$$\boxed{T = V + V\frac{1}{E - H_0} T} \tag{53}$$

Scattering equation

Scattering of two mesons. Relativistic, in the momentum base, (quantum field theory)

$$T(p,p') = V(p,p') + i \int \frac{d^4q}{(2\pi)^4} V(p,q') I(q) T(q,p') \quad (54)$$

$$I(q) = \frac{1}{q^2 - m_1^2 + i\epsilon} \frac{1}{(P-q)^2 - m_2 + i\epsilon} \quad (55)$$

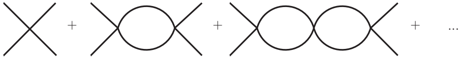
$P^\mu = p_1^\mu + p_2^\mu$, C. M. frame, $|\vec{p}_1| = |\vec{p}_2| = p$. $P_0 = \sqrt{s}$. When V is soft can be taken outside of the integral, in the so-call on-shell aproximation. *Bethe-Salpeter*

$$\boxed{T = V + VGT}; \quad G \equiv i \int \frac{d^4q}{(2\pi)^4} I(q) \quad (56)$$

This equation can be also written as,

$$\boxed{T^{-1} = V^{-1} - G} \quad (57)$$

Scattering equation and unitarity



Unitarity in coupled channels S-matrix ψ_+ and ψ_- are the *in* and *out* scattering states. ϕ_i and ϕ_f are the initial and final states asymptotic states coming from $-\infty$ and going to ∞.

$$|\psi_+\rangle = \Omega_+|\phi_i\rangle \quad |\psi_-\rangle = \Omega_-|\phi_f\rangle \quad S \equiv \Omega_-^\dagger \Omega_+ \tag{58}$$

$$\mathcal{P}_{i\to f} = |\langle\psi_-|\psi_+\rangle|^2 = |\langle\phi_f|\Omega_-^\dagger\Omega_+|\phi_i\rangle|^2 = |\langle\phi_f|S|\phi_i\rangle|^2 \tag{59}$$

$$S = I + i(2\pi)^4\delta^4(P_i - P_f)T \tag{60}$$

Probability conservation

$$SS^\dagger = S^\dagger S = I \Longrightarrow (2\pi)^4\delta^4(P_f - P_i)\{i(t_{fi} - t_{if}^\star) + \sum_n (2\pi)^4\delta(P_n - P_i)t_{nf}^* t_{ni}\}$$

- **Unitarity in coupled channels:**

$$\boxed{\mathrm{Im}t_{if} = t_{in}\sigma_{nn}t^*_{nf}} \qquad (61)$$

with $\sigma_{nn}(s) = -\frac{p_n}{8\pi\sqrt{s}}\theta(s-(m_{1n}+m_{2n})^2)$ and p_n is the on-shell c.m. momentum of the intermediate meson (our normalization).

K-matrix formalism: $T^{-1} = K^{-1} - i\sigma$, where $K^{-1} = \mathrm{Re}T^{-1}$. And,

$$\sigma = T^{-1}\mathrm{Im}TT^{*-1} = \frac{1}{2i}T^{-1}(T-T^*)T^{*-1} = \frac{1}{2i}(T^{-1*}-T^{-1}) = -\mathrm{Im}T^{-1}.$$

Therefore, $T^{-1} = \mathrm{Re}T^{-1} + i\mathrm{Im}T^{-1} = \mathrm{Re}T^{-1} - i\sigma$, or

$$\boxed{T = [\mathrm{Re}T^{-1} - i\sigma]^{-1}} \qquad (62)$$

$$T^{-1} = V^{-1} - G \Longrightarrow \mathrm{Im}T^{-1} = -\mathrm{Im}G = \frac{p}{8\pi\sqrt{s}} \quad \text{(BS one channel)} \qquad (63)$$

S-matrix parametrization

In the one channel case, since $\mathrm{Im}T^{-1} = \frac{p}{8\pi\sqrt{s}}$, the relation of the S-matrix with the scattering amplitude in our convention is,

$$S = 1 - i\frac{p}{4\pi\sqrt{s}}T \quad \text{(one channel)} \tag{64}$$

For one channel, $S = e^{2i\delta}$, with δ the phase shift. Low energy expansion (Effective range approximation, ERE)

$$p\cot\delta(p) = -\frac{1}{a} + \frac{1}{2}rp^2 + O(p^4)\,, \tag{65}$$

a is the scattering length, and r the effective range. Two coupled channel case, the S matrix is paramtrized as,

$$S = \begin{pmatrix} \eta e^{2i\delta_1} & i(1-\eta^2)^{1/2}e^{i(\delta_1+\delta_2)} \\ i(1-\eta^2)^{1/2}e^{i(\delta_1+\delta_2)} & \eta e^{2i\delta_2} \end{pmatrix}$$

and for the T matrix one has,

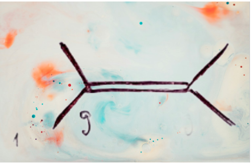
$$(T)_{11} = -\frac{4\pi E}{ip_1}[(S)_{11} - 1]\,, \quad (T)_{22} = -\frac{4\pi E}{ip_2}[(S)_{22} - 1], \quad (T)_{12} = -\frac{4\pi E}{i\sqrt{p_1 p_2}}(S)_{12}\,.$$

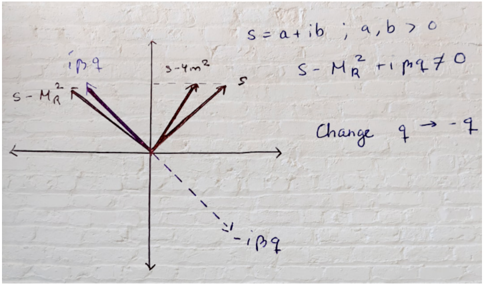
Poles in the second Riemann Sheet

Why do we need to go to the II Riemann Sheet?

$$t = \frac{g^2}{s - M_R^2 + i\sqrt{s}\,\Gamma(s)}$$

Resonance decaying (s-wave) into two mesons 1 and 2. Take $\sqrt{s}\,\Gamma(s) = \beta\, q$.

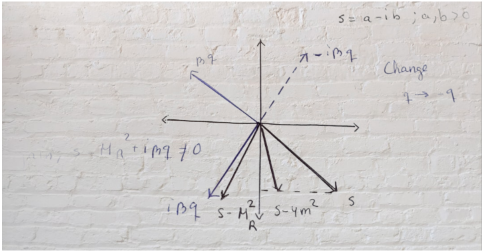




$$q = \frac{1}{2}\sqrt{s - 4m^2}$$

For particles with equal mass. Since $s = a + i\,b$, $a, b > 0$, in principle there is no pole.

Poles in the second Riemann Sheet



If $s = a - i\,b$ one also does not find a solution, unless we change q by $-q$.
Bethe-Salpeter equation:

$$T = \frac{1}{V^{-1} - G}$$

Loop function. The imaginary part can be evaluated with Cutosky rules

$$\mathrm{Im}\, G = -\frac{q}{8\pi\sqrt{s}}$$

Resonances are found in the II Riemann sheet

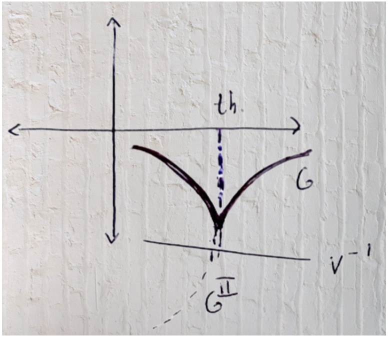
Poles in the second Riemann Sheet

Virtual state. If we have a bound state very close to threshold and below with V^{-1}, and we sum $\delta > 0$, $V \rightarrow V + \delta$, $T = \frac{1}{(V+\delta)^{-1} - G}$ will have no pole, but, ...

Since $G^{II} = G + i\frac{2}{8\pi\sqrt{s}}q$, and below the threshold q is purely imaginary:

$$G^{II} = G - \gamma(4m^2 - s)^{1/2}$$

We will find a pole again in the II sheet but below the threshold.



Poles in the second Riemann sheet

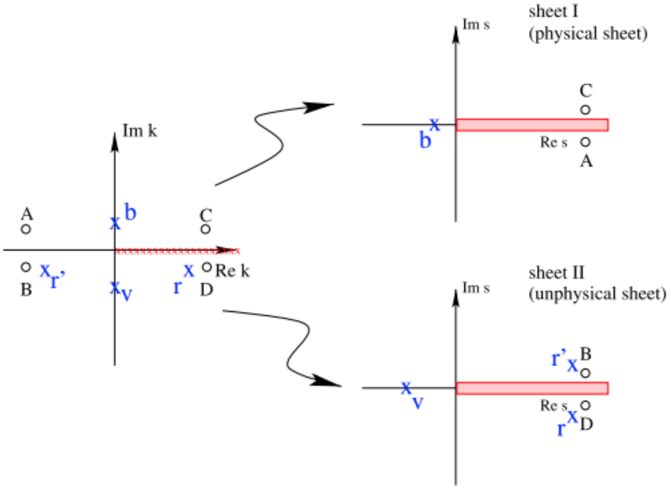


Figure: Momenta and energy complex plane. Hanhart, Pelaez, Rios, PLB14.

Poles in the second Riemann Sheet

The **Schwartz reflection theorem** states that if a function $f(z)$ is analytic in a region of the complex plane, including a portion of the real axis in which f is real, then,

$$[f(z^\star)]^\star = f(z)$$

The loop function G satisfies these conditions, therefore, for $\mathrm{Re}(\sqrt{s}) > m_l + M_l$, we have,

$$G(\sqrt{s} - i\,\epsilon) = [G(\sqrt{s} + i\epsilon)]^\star = G(\sqrt{s + i\epsilon}) - i2\,\mathrm{Im}G(\sqrt{s} + i\epsilon) \qquad (66)$$

Since the beginning of R2 is equal to the end of R1, we have

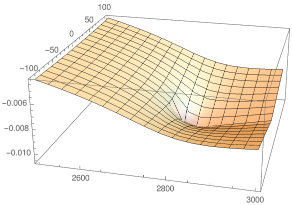
$$G^{II}(\sqrt{s} + i\,\epsilon) = G^{I}(\sqrt{s} - i\epsilon) = G^{I}(\sqrt{s + i\epsilon}) - i2\,\mathrm{Im}G^{I}(\sqrt{s} + i\epsilon) \qquad (67)$$

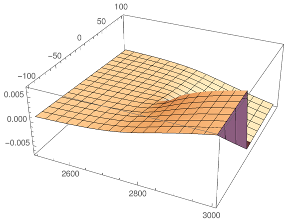
Since the analytical continuation is unique,

$$G^{II}(\sqrt{s}) = G^{I}(\sqrt{s}) + i\frac{q}{4\pi\sqrt{s}}, \quad \mathrm{Im}\,q > 0 \qquad (68)$$

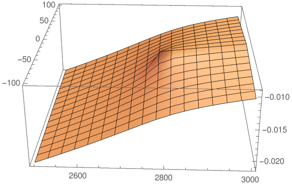
Poles in the second Riemann Sheet

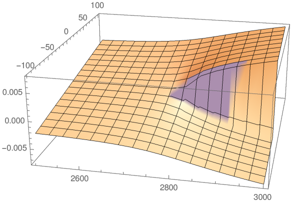
Real and imaginary part of G^{I} ($D^{*}\rho$ system)





Real and imaginary part of G^{II}





Scattering amplitude in the finite volume

Infinite volume: $T = \frac{1}{V^{-1}-G}$. (1 ch. 2 mesons), two-meson-loop function G:

$$G = G^{co}(E) = \int_{q<q_{max}} \frac{d^3q}{(2\pi)^3} \frac{\omega_1+\omega_2}{2\omega_1\omega_2} \frac{2M_i}{E^2-(\omega_1+\omega_2)^2+i\epsilon} \tag{69}$$

where $\omega_i = \sqrt{m_i^2 + |\vec{q}_i\,|^2}$ is the energy and $\vec{q}$ stands for the momentum of the meson in the channel i. In the finite volume, the momenta is quantized,

$$\vec{q}_i = \frac{2\pi}{L}\vec{n}_i; \qquad T \longrightarrow \tilde{T}; \qquad G(E) \longrightarrow \tilde{G}(E)\,, \tag{70}$$

$$\textbf{Finite volume}: \quad \tilde{T} = \frac{1}{V^{-1}-\tilde{G}} \tag{71}$$

[1] M. Doring, U. G. Meißner, E. Oset and A. Rusetsky, Eur. Phys. J. A47, 139 (2011). M. Doring, J. Haidenbauer, U. G. Meißner, and A. Rusetsky, Eur. Phys. J. A47, 163 (2011)

Scattering amplitude in the finite volume

$$\tilde{G}(E) = \frac{1}{L^3} \sum_{\vec{q}_i} I(E, \vec{q}_i) \ , \tag{72}$$

$$I(E, \vec{q}_i) = \frac{\omega_1(\vec{q}_i) + \omega_2(\vec{q}_i)}{2\omega_1(\vec{q}_i)\omega_2(\vec{q}_i)} \frac{1}{(E)^2 - (\omega_1(\vec{q}_i) + \omega_2(\vec{q}_i))^2} \tag{73}$$

and $\vec{q} = \frac{2\pi}{L}\vec{n}$, $\omega_{1,2}(\vec{q}) = \sqrt{m_{1,2}^2 + \vec{q}^2}$.

Roca and Oset, PRD12
$a_1(1260)$ $(\pi\rho)$

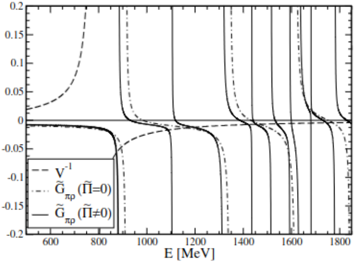


FIG. 2. Loop function in the box $\tilde{G}_{\pi\rho}$ (solid line) and V^{-1} (dashed line) for $L = 2m_\pi^{-1}$ and $p_{\max} = 1$ GeV. The dashed dotted line corresponds to the case with stable ρ, $\tilde{\Pi}(s_\rho)$=0.

Scattering amplitude in the finite volume

For the discrete energies satisfying,

$$T(E) = \left(V^{-1}(E) - G(E)\right)^{-1} = \left(\tilde{G}(E) - G(E)\right)^{-1} . \tag{74}$$

$$\textbf{Phase shift} \quad T(E) = \frac{-8\pi E}{p\cot\delta(p) - i\,p} , \tag{75}$$

Thus,

$$p\cot\delta(p) = -8\pi E\left\{\tilde{G}(E) - \left(G(E) + \frac{ip}{8\pi E}\right)\right\} [\text{ above threshold }] .$$

Above threshold, $G(E) + ip/(8\pi E) = \mathrm{Re}\, G(E)$. Effective range expansion,

$$p\cot\delta(p) = \frac{1}{a} + \frac{1}{2}rp^2 + O(p^4) , \tag{76}$$

Below threshold, $p = i\,\gamma$

$$\frac{1}{a} - \frac{1}{2}r\gamma^2 + \cdots = -8\pi E\left\{\tilde{G}(E) - \left(G(E) - \frac{\gamma}{8\pi E}\right)\right\} \tag{77}$$

$$[\text{ below threshold }] . \tag{78}$$

Relation to the Lüscher approach

We start from,

$$\frac{1}{2\,\omega_1\,\omega_2}\,\frac{\omega_1+\omega_2}{E^2-(\omega_1+\omega_2)^2+i\epsilon} = \frac{1}{2E}\,\frac{1}{p^2-\vec{q}^{\,2}+i\epsilon} - \frac{1}{2\omega_1\,\omega_2}\,\frac{1}{\omega_1+\omega_2+E}$$
$$-\frac{1}{4\,\omega_1\,\omega_2}\,\frac{1}{\omega_1-\omega_2-E} - \frac{1}{4\,\omega_1\,\omega_2}\,\frac{1}{\omega_2-\omega_1-E}$$

Lüscher90, the contribution from the last three terms is exponentially suppressed in the difference $\tilde{G}(E)-G(E)$ and can be neglected.

$$\tilde{G}(E)-G(E) = \left\{\frac{1}{L^3}\sum_{\vec{q}}^{|\vec{q}|<q_{\rm max}} - \int^{|\vec{q}|<q_{\rm max}} \frac{d^3\vec{q}}{(2\pi)^3}\right\}$$
$$\times\,\frac{1}{2E}\,\frac{1}{p^2-\vec{q}^{\,2}+i\epsilon}+\cdots = \frac{1}{2E}\,\frac{1}{L^3}\sum_{\vec{q}}^{|\vec{q}|<q_{\rm max}}\frac{1}{p^2-\vec{q}^{\,2}}$$
$$+\frac{1}{4\pi^2 E}\left(q_{\rm max}+\frac{p}{2}\log\frac{q_{\rm max}-p}{q_{\rm max}+p}\right)+\frac{ip}{8\pi\,E}+\cdots\,, \qquad (79)$$

Relation to the Lüscher approach

Doring, Meissner, Oset, Rusetsky, EPJA11

$$\lim_{q_{\max}\to\infty}\left\{\frac{1}{L^3}\sum_{\vec{q}}^{|\vec{q}|<q_{\max}}\frac{1}{p^2-\vec{q}^2}-\frac{q_{\max}}{2\pi^2}\right\}=-\frac{1}{2\pi^{3/2}L}\,\mathcal{Z}_{00}(1,\hat{p}^2)\,,\qquad \hat{p}=\frac{pL}{2\pi}$$

where $\mathcal{Z}_{00}$ stands for the Lüscher zeta-function. One obtains, the Lüscher equation

$$p\cot\delta(p)=\frac{2\pi}{L}\,\pi^{-3/2}\,\mathcal{Z}_{00}(1,\hat{p}^2)$$

and is cutoff-independent up to exponentially small correction.

Boost, Asymmetric Boxes and Partial Wave Decomposition

Doering, Meißner, Oset, Rusetsky (2012)

$\vec{q}_1, \vec{q}_2 = \vec{P} - \vec{q}_1$, $s \equiv W^2 = (P^0)^2 - \vec{P}^2$, and $\vec{q}^*$ the momenta in the CM frame

$$\int \frac{d^3\vec{q}^{\,*}}{(2\pi)^3} I(|\vec{q}^{\,*}|) \rightarrow \tilde{G}(P) = \frac{1}{\eta L^3} \frac{\sqrt{s}}{P^0} \sum_{\vec{n}} I(|\vec{q}^{\,*}(\vec{q})|). \quad (80)$$

$\vec{q}^{\,*}_{1,2} = \vec{q}_{1,2} + \left[\left(\frac{\sqrt{s}}{P^0} - 1 \right) \frac{\vec{q}_{1,2} \cdot \vec{P}}{|\vec{P}|^2} - \frac{q^{*0}_{1,2}}{P^0} \right] \vec{P}$; with $\vec{q} = \frac{2\pi}{L}(n_x, n_y, \frac{n_z}{\eta})$, $\vec{P} = \frac{2\pi}{L}(N_x, N_y, \frac{N_z}{\eta})$.

$$\tilde{T}_{lm,l'm'}(p,p') = V_l(p,p')\delta_{ll'}\delta_{mm'} + \sum_{l''m''} V_l(p, q^{\mathrm{on},*})\tilde{G}_{lm,l''m''}(q^{\mathrm{on},*})\tilde{T}_{l''m'',lm}(q^{\mathrm{on},*}, p') \quad (81)$$

$$\boxed{\det(\delta_{ll'}\delta_{mm'} - V_l(q^{\mathrm{on},*}, q^{\mathrm{on},*})\tilde{G}_{lm,l'm'}(q^{\mathrm{on},*})) = 0} \quad (82)$$

Irreducible representations for asymmetric boxes and boost $\vec{P} = \frac{2\pi}{\eta L}(0,0,1)$,

$$\mathrm{I} = \mathrm{L} = 0 \longrightarrow \mathrm{A}^+ : -1 + \mathrm{V}_0 \mathrm{G}_{00,00} = 0$$

$$\mathrm{I} = \mathrm{L} = 1 \longrightarrow \mathrm{A}_2^- : -1 + \mathrm{V}_1 \mathrm{G}_{10,10} = 0; \mathrm{E}^- : -1 + \mathrm{V}_1 \mathrm{G}_{11,11} = 0$$

Meson-Meson scattering in ChPT

- ChPT expansion of the amplitude for meson-meson scattering

$$t(s) = t_2(s) + t_4(s) + ...t_{2k} = \mathcal{O}(p^{2k}) \quad (83)$$

- Lowest-order Chiral Lagrangian

$$\mathcal{L}_2 = \frac{f^2}{4}\langle \partial_\mu U^\dagger \partial^\mu U + M(U + U^\dagger)\rangle \quad (84)$$

$$\begin{aligned}\mathcal{L}_4 = \quad & L_1\langle \partial_\mu U^\dagger \partial^\mu U\rangle^2 + L_2\langle \partial_\mu U^\dagger \partial_\nu U\rangle\langle \partial^\mu U^\dagger \partial^\nu U\rangle \\ & + L_3\langle \partial_\mu U^\dagger \partial^\mu U \partial_\nu U^\dagger \partial^\nu U\rangle + L_4\langle \partial U^\dagger \partial^\mu U\rangle\langle U^\dagger M + M^\dagger U\rangle \\ & + L_5\langle \partial_\mu U^\dagger \partial^\mu U(U^\dagger M + M^\dagger U)\rangle + L_6\langle U^\dagger M + M^\dagger U\rangle^2 \\ & + L_7\langle U^\dagger M - M^\dagger U\rangle^2 + L_8\langle M^\dagger U M^\dagger U + U^\dagger M U^\dagger M\rangle \end{aligned} \quad (85)$$

where $U(\phi) = \exp(i\sqrt{2}\Phi/f)$, and

$$\Phi(x) = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}_\mu \tag{86}$$

$$M = \begin{pmatrix} m_\pi^2 & 0 & 0 \\ 0 & m_\pi^2 & 0 \\ 0 & 0 & 2m_K^2 - m_\pi^2 \end{pmatrix} \tag{87}$$

[1] J. Gasser and H. Leutwyler, Annals Phys. **158**, 142 (1984)

[2] J. Gasser and H. Leutwyler, Nucl. Phys. B **250**, 465 (1985)

[3] J. A. Oller, E. Oset and J. R. Pelaez, Phys. Rev. D **59**, 074001 (1999)

Pseudoscalar meson masses at LO

$$M_{0\pi}^2 = 2\, m_{ud} B_0 \,,$$
$$M_{0K}^2 = (m_{ud} + m_s) B_0 \,,$$
$$M_{0\eta}^2 = \frac{2}{3}(m_{ud} + 2\, m_s) B_0 \,. \quad (88)$$

Quark condensate value in the chiral limit,

$$\Sigma_0 = -\langle 0|\bar{q}q|0\rangle_0 = B_0 f_0^2, \quad (89)$$

with $q \in \{u, d, s\}$. Chiral trajectories:

$$m_{0K}^2 = -\frac{1}{2} m_{0\pi}^2 + C\, B_0 \,, \quad (90)$$

for $\mathrm{Tr}\mathcal{M} = C$ and

$$m_{0K}^2 = +\frac{1}{2} m_{0\pi}^2 + k\, B_0 \,. \quad (91)$$

Meson masses at NLO

The physical masses can be expressed as a function the leading order masses (M_0), LEC's (L^r) and pseudoscalar decay constants (f).

$$M_\pi^2 = M_{0\,\pi}^2 \left[1 + \mu_\pi - \frac{\mu_\eta}{3} + \frac{16 M_{0\,K}^2}{f_0^2}\left(2L_6^r - L_4^r\right) + \frac{8 M_{0\,\pi}^2}{f_0^2}\left(2L_6^r + 2L_8^r - L_4^r - L_5^r\right)\right],$$

$$M_K^2 = M_{0\,K}^2 \left[1 + \frac{2\mu_\eta}{3} + \frac{8 M_{0\,\pi}^2}{f_0^2}\left(2L_6^r - L_4^r\right) + \frac{8 M_{0\,K}^2}{f_0^2}\left(4L_6^r + 2L_8^r - 2L_4^r - L_5^r\right)\right],$$

$$M_\eta^2 = M_{0\,\eta}^2 \left[1 + 2\mu_K - \frac{4}{3}\mu_\eta + \frac{8 M_{0\,\eta}^2}{f_0^2}(2L_8^r - L_5^r) + \frac{8}{f_0^2}(2M_{0\,K}^2 + M_{0\,\pi}^2)(2L_6^r - L_4^r)\right]$$
$$+ M_{0\,\pi}^2 \left[-\mu_\pi + \frac{2}{3}\mu_K + \frac{1}{3}\mu_\eta\right] + \frac{128}{9 f_0^2}(M_{0\,K}^2 - M_{0\,\pi}^2)^2(3L_7 + L_8^r),$$

$$\mu_P = \frac{M_{0\,P}^2}{32\pi^2 f_0^2}\log\frac{M_{0\,P}^2}{\mu^2}, \qquad P = \pi, K, \eta\,,$$

where f_0 is the pion decay constant in the chiral limit.

Unitarity amplitude in coupled channels

Expansion of T^{-1} in powers of p^2:

$$T \simeq T_2 + T_4 + \dots$$
$$T^{-1} \simeq T_2^{-1}[1 + T_4 T_2^{-1} + \dots]^{-1} \simeq T_2^{-1}[1 - T_4 T_2^{-1} \dots]$$
$$T = T_2 T_2^{-1}[\mathrm{Re}T^{-1} - i\sigma]^{-1} T_2^{-1} T_2 = T_2[T_2 \mathrm{Re}T^{-1} T_2 - iT_2\sigma T_2]^{-1} T_2$$

Inserting the expansion of T^{-1} in the first member of [],

$T_2 = \mathrm{Re}T_2$, $\mathrm{Im}T = \mathrm{Im}T_4 = T_2\sigma T_2$.

$T_2 \mathrm{Re}T^{-1} T_2 = T_2 \mathrm{Re}(T_2^{-1}(1 - T_4 T_2^{-1}))T_2 = T_2 - \mathrm{Re}T_4$.

Thus,

$$T = T_2[T_2 - \mathrm{Re}T_4 - i\mathrm{Im}T_4]^{-1} T_2 \longrightarrow T = T_2[T_2 - T_4]^{-1} T_2 \qquad (92)$$

$\mathrm{Re}T_4 \simeq T_4^p + T_2 \mathrm{Re}G T_2$, $\mathrm{Im}G = \sigma$, $G \equiv$ Two meson func. loop,

$$\boxed{T = T_2[T_2 - T_4^p - T_2 G T_2]^{-1} T_2} \qquad (93)$$

UChPT (NLO) Oller, Oset, Pelaez (1999)

$V_{\rm IAM}$ is the kernel of the scattering equation:

$$T = T + VGT \longrightarrow T = [I - V_{\rm IAM}G]^{-1} V_{\rm IAM}, \quad \text{with} \tag{94}$$

$$\boxed{V_{\rm IAM} = [1 - V_4 (V_2)^{-1}]^{-1} V_2} \tag{95}$$

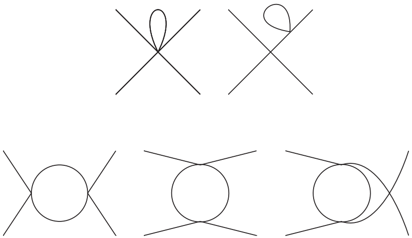
$$V_2 = \frac{m_\pi^2 - s}{f_\pi^2}$$

$$\begin{aligned} V_4 = -\frac{4}{f^4}&((2\,L_1 + L_3)(s - 2\,m_\pi^2)^2 \\ &+L_2((t - 2\,m_\pi^2)^2 + (u - 2\,m_\pi^2)^2) \\ &+2(2\,L_4 + L_5)m_\pi^2 (s - 2\,m_\pi^2) \\ &+4(2\,L_6 + L_8)m_\pi^4) \end{aligned}$$

$I = L = 1$:

$$\longrightarrow \boxed{V_{\pi\pi}^{11} = -2p^2/(3(f_\pi^2 - 8\hat{l}_1\, m_\pi^2 + 4\hat{l}_2\, E^2))} \tag{96}$$

and, $\hat{l}_1 = 2\,L_4 + L_5, \quad \hat{l}_2 = 2\,L_1 - L_2 + L_3.$



Partial wave decomposition

$$T_I = \sum_J (2J+1) T_{IJ} P_J(\cos\theta)$$

$$T_{IJ} = \frac{1}{2}\int_{-1}^{1} P_J(\cos\,\theta) T_I(\theta)\ \mathrm{d}\cos\theta$$

$$V^{00}_{\pi\pi}(s) = \frac{3(m_\pi^2 - 2s)^2}{6f_\pi^2(m_\pi^2 - 2s) + 8(L_a m_\pi^4 + s(L_b m_\pi^2 + L_c s))}$$

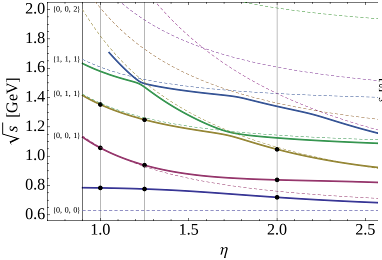
$$\begin{aligned} L_a &= -36\hat{l}_1 + 44\hat{l}_2 + 20(5L_2 + 6L_6 + 3L_8)\,, \\ L_b &= 12\hat{l}_1 - 40\hat{l}_2 - 80L_2\,, \\ L_c &= 11\hat{l}_2 + 25L_2\,, \end{aligned} \qquad (97)$$

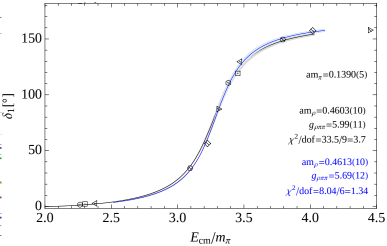
Application for the analysis of LQCD data

$$(T)_{11} = -\frac{8\pi E}{2ip_1}[(S)_{11} - 1]\,,$$
$$(T)_{22} = -\frac{8\pi E}{2ip_2}[(S)_{22} - 1]\,,$$
$$(T)_{12} = -\frac{8\pi E}{2i\sqrt{p_1 p_2}}(S)_{12}\,.$$

2 coupled channels $\pi\pi - K\bar{K}$, $\rho(770)$

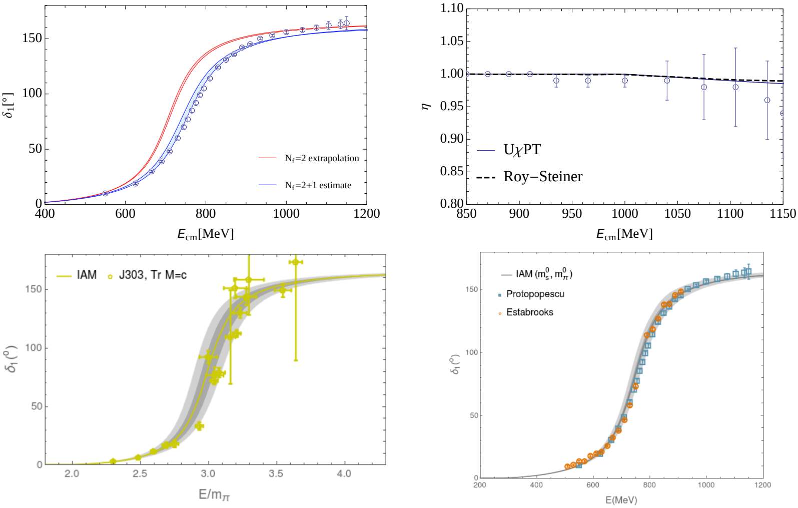
$$S = \begin{pmatrix} \eta e^{2i\delta_1} & i(1-\eta^2)^{1/2} e^{i(\delta_1+\delta_2)} \\ i(1-\eta^2)^{1/2} e^{i(\delta_1+\delta_2)} & \eta e^{2i\delta_2} \end{pmatrix}$$





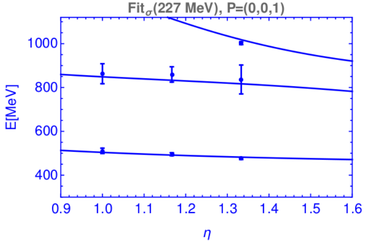
The $\rho(770)$ meson

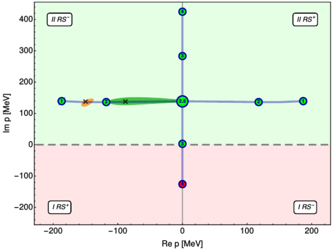
Guo, Alessandro, Molina, Doring, PRD16. Molina, Ruiz de Elvira, JHEP20

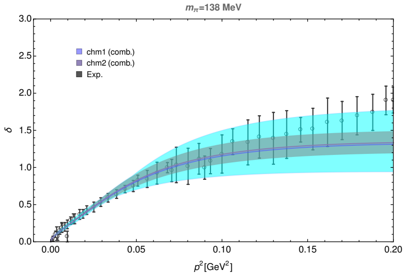


The σ meson

Guo, Alexandru, Molina, Mai, Doring, PRD18







Hadron physics II: The $T_{cs}(2900)$ and $T_{c\bar{s}}(2900)$

R. Molina

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Introduction

Flavour exotic states

Molina,Branz,Oset, PRD82(2010)

C, S	Channels	$I[J^P]$	$\sqrt{s}$	$\Gamma_A(\Lambda = 1400)$	$\Gamma_B(\Lambda = 1200)$	State	$\sqrt{s}_{exp}$	Γ_{exp}
$1, -1$	$D^*\bar{K}^*$	$0[0^+]$	2848	23	59	$X_0(2866)$ or $T_{cs}(2900)$	2866	57
		$0[1^+]$	2839	3	3			
		$0[2^+]$	2733	11	36			
$1, 1$	D^*K^*, $D_s^*\omega$	$0[0^+]$	2683	20	71			
	$D_s^*\phi$	$0[1^+]$	2707	4×10^{-3}	4×10^{-3}			
		$0[2^+]$	2572	7	23	$D_{s2}(2573)$	2572	20
$1, 1$	D^*K^*, $D_s^*\rho$	$1[0^+]$	Cusp structure around $D_s^*\rho$, D^*K^*			new $T_{c\bar{s}}(2900)$	2908	136
$1, 1$		$1[1^+]$	Cusp structure around $D_s^*\rho$, D^*K^*					
$1, 1$		$1[2^+]$	2786	8	11			
$2, 0$	D^*D^*	$0[1^+]$	3969	0	0			
$2, 1$	$D^*D_s^*$	$1/2[1^+]$	4101	0	0			

Table 1: Summary of the nine states obtained. The width is given for the model A, Γ_A, and B, Γ_B. All the quantities here are in MeV. Repulsion in $C = 0, S = 1, I = 1/2$; $C = 1, S = -1, I = 1$; $C = 1, S = 2, I = 1/2$; $C = 2, S = 0, I = 1$ and $C = 2, S = 2, I = 0$ is found.

Form factors in the $D^*D\pi$ vertex; Model A: $F_1(q^2) = \frac{\Lambda_b^2 - m_\pi^2}{\Lambda_b^2 - q^2}$, Titov, Kampfer EPJA7, PRC65 with $\Lambda_b = 1.4, 1.5$ GeV and $g = M_\rho/2f_\pi$. Model B: $F_2(q^2) = e^{q^2/\Lambda^2}$ Navarra, Nielsen, Bracco PRD65 (2002), $\Lambda = 1, 1.2$ GeV and $g_D = g^{exp}_{D^*D\pi} = 8.95$ (experimental value). Subtraction constant $\alpha = -1.6$.

The vector-vector interaction

The hidden gauge formalism

Starting from a nonlinear sigma model based on $G/H = SU(2)_L \otimes SU(2)_R/SU(2)_V$:

Bando,Kugo,Yamawaki

$$L = (f_\pi^2/4)\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) \; , \qquad U(x) = \exp[2i\pi(x)/f_\pi] \qquad (1)$$

and introduce new variables ξ_L, ξ_R and the field V_μ:

$$U(x) \equiv \xi_L^\dagger(x)\xi_R(x) \; , \; V_\mu = (1/2i)(\partial_\mu \xi_L \cdot \xi_L^\dagger + \partial_\mu \xi_R \cdot \xi_R^\dagger) \qquad (2)$$

Any linear combination $L = L_A + aL_V$ of the invariants:

$$L_V = -\frac{f_\pi^2}{4} Tr(D_\mu \xi_L \cdot \xi_L^\dagger + D_\mu \xi_R \cdot \xi_R^\dagger)^2 \quad L_A = -\frac{f_\pi^2}{4} Tr(D_\mu \xi_L \cdot \xi_L^\dagger - D_\mu \xi_R \cdot \xi_R^\dagger)^2$$

is equivalent to the original one, Eq. (1). A kinetic term is added, $-(1/4g^2)(V_{\mu\nu})^2$, and choosing $a = 2$ it is obtained

- 1)$m_\rho^2 = 2g_{\rho\pi\pi}^2 f_\pi^2$ (KSFR relation)
- 2)ρ dominance of the electromagnetic form factor of pions $(gV_\mu(\pi \times \partial^\mu \pi))$

And, fixing the gauge $\xi_L^\dagger = \xi_R \equiv \xi$ the Lagrangian becomes in the Weinberg's Lagrangian (nonlinear realization of the chiral symmetry)

The hidden gauge formalism

Bando, Kugo, Yamawaki, PRL54,1215

Lagrangian

$$\mathcal{L} = \mathcal{L}^{(2)} + \mathcal{L}_{III} \qquad (3)$$

$$\begin{aligned}
\mathcal{L}^{(2)} &= \frac{1}{4} f^2 \langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + \chi^\dagger U \rangle \qquad (4)\\
\mathcal{L}_{III} &= -\frac{1}{4}\langle V_{\mu\nu} V^{\mu\nu}\rangle + \frac{1}{2} M_V^2 \langle [V_\mu - \frac{i}{g}\Gamma_\mu]^2 \rangle \\
D_\mu U &= \partial_\mu U - ieQA_\mu U + ieUQA_\mu, \qquad U = e^{i\sqrt{2}P/f}
\end{aligned}$$

Upon expansion of $[V_\mu - \frac{i}{g}\Gamma_\mu]^2$, $\mathcal{L}'s$

$$\mathcal{L}_{V\gamma} = -M_V^2 \frac{e}{g} A_\mu \langle V^\mu Q\rangle, \mathcal{L}_{VPP} = -ig\langle V^\mu [P, \partial_\mu P]\rangle, \mathcal{L}_{\gamma PP} = ieA_\mu \langle Q[P, \partial_\mu P]\rangle, \ldots$$

$$\frac{F_V}{M_V} = \frac{1}{\sqrt{2}g}, \qquad \frac{G_V}{M_V} = \frac{1}{2\sqrt{2}g}, \qquad F_V = \sqrt{2}f, \qquad G_V = \frac{f}{\sqrt{2}}, \qquad g = \frac{M_V}{2f}$$

Local Hidden Gauge Approach
Vector-vector scattering

$$\mathcal{L}_{III} = -\frac{1}{4}\langle V_{\mu\nu} V^{\mu\nu}\rangle$$

$$\mathcal{L}_{III}^{(3V)} = ig\langle(\partial_\mu V_\nu - \partial_\nu V_\mu) V^\mu V^\nu\rangle$$

$$\mathcal{L}_{III}^{(c)} = \frac{g^2}{2}\langle V_\mu V_\nu V^\mu V^\nu - V_\nu V_\mu V^\mu V^\nu\rangle$$

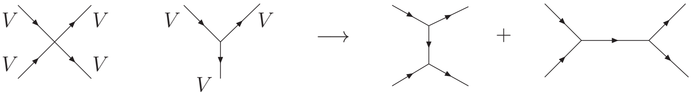
$\boldsymbol{V_{\mu\nu}} =$

$$\partial_\mu V_\nu - \partial_\nu V_\mu - ig[V_\mu, V_\nu]$$

$$g = \frac{M_V}{2f}$$

$\boldsymbol{V_\mu} =$

$$\begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} & \bar{D}^{*0} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} & D^{*-} \\ K^{*-} & \bar{K}^{*0} & \phi & D_s^{*-} \\ D^{*0} & D^{*+} & D_s^{*+} & J/\psi \end{pmatrix}_\mu$$



a)   b)   c)   d)

Local Hidden Gauge Approach

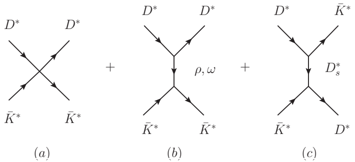


Figure 1: The $D^* \bar{K}^* \to D^* \bar{K}^*$ interaction at the tree level; (a) contact term; (b) exchange of light vectors; (c) exchange of a heavy vector.

Approximation

$$\epsilon_1^\mu = (0,1,0,0)$$
$$\epsilon_2^\mu = (0,0,1,0)$$
$$\epsilon_3^\mu = (|\vec{k}|,0,0,k^0)/m$$

$$k^\mu = (k^0, 0, 0, |\vec{k}|)$$

$$\vec{k}/m \simeq 0, \; k_j^\mu \epsilon_\mu^{(l)} \simeq 0$$

$$\epsilon_1^\mu = (0,1,0,0)$$
$$\epsilon_2^\mu = (0,0,1,0)$$
$$\epsilon_3^\mu = (0,0,0,1)$$

$$\mathcal{L}_{III}^{(3V)} = ig\langle(\partial_\mu V_\nu - \partial_\nu V_\mu)V^\mu V^\nu\rangle = ig\langle[V_\mu, \partial_\nu V_\mu]V^\nu\rangle$$

Spin projectors

$$\mathcal{P}^{(0)} = \frac{1}{3}\epsilon_\mu\epsilon^\mu\epsilon_\nu\epsilon^\nu; \quad \mathcal{P}^{(1)} = \frac{1}{2}(\epsilon_\mu\epsilon_\nu\epsilon^\mu\epsilon^\nu - \epsilon_\mu\epsilon_\nu\epsilon^\nu\epsilon^\mu)$$
$$\mathcal{P}^{(2)} = \{\frac{1}{2}(\epsilon_\mu\epsilon_\nu\epsilon^\mu\epsilon^\nu + \epsilon_\mu\epsilon_\nu\epsilon^\nu\epsilon^\mu) - \frac{1}{3}\epsilon_\mu\epsilon^\mu\epsilon_\nu\epsilon^\nu\} \; .$$

The $X_0(2866)$ or $T_{cs}(2900)$

Local Hidden Gauge Approach

Potential V: contact + vector-meson exchange (ρ, ω)

J	Amplitude	Contact	V-exchange	$\sim$ Total
0	$D^*\bar{K}^* \to D^*\bar{K}^*$	$4g^2$	$-\frac{g^2(p_1+p_4).(p_2+p_3)}{m^2_{D^*_s}} + \frac{1}{2}g^2(\frac{1}{m^2_\omega} - \frac{3}{m^2_\rho})(p_1+p_3).(p_2+p_4)$	$-9.9g^2$
1	$D^*\bar{K}^* \to D^*\bar{K}^*$	0	$\frac{g^2(p_1+p_4).(p_2+p_3)}{m^2_{D^*_s}} + \frac{1}{2}g^2(\frac{1}{m^2_\omega} - \frac{3}{m^2_\rho})(p_1+p_3).(p_2+p_4)$	$-10.2g^2$
2	$D^*\bar{K}^* \to D^*\bar{K}^*$	$-2g^2$	$-\frac{g^2(p_1+p_4).(p_2+p_3)}{m^2_{D^*_s}} + \frac{1}{2}g^2(\frac{1}{m^2_\omega} - \frac{3}{m^2_\rho})(p_1+p_3).(p_2+p_4)$	$-15.9g^2$

Table 2: Tree level amplitudes for $D^*\bar{K}^*$ in $I=0$. Last column: $(V_{\mathrm{th.}})$.

J	Amplitude	Contact	V-exchange	$\sim$ Total
0	$D^*\bar{K}^* \to D^*\bar{K}^*$	$-4g^2$	$\frac{g^2(p_1+p_4).(p_2+p_3)}{m^2_{D^*_s}} + \frac{1}{2}g^2(\frac{1}{m^2_\omega} + \frac{1}{m^2_\rho})(p_1+p_3).(p_2+p_4)$	$9.7g^2$
1	$D^*\bar{K}^* \to D^*\bar{K}^*$	0	$-\frac{g^2(p_1+p_4).(p_2+p_3)}{m^2_{D^*_s}} + \frac{1}{2}g^2(\frac{1}{m^2_\omega} + \frac{1}{m^2_\rho})(p_1+p_3).(p_2+p_4)$	$9.9g^2$
2	$D^*\bar{K}^* \to D^*\bar{K}^*$	$2g^2$	$\frac{g^2(p_1+p_4).(p_2+p_3)}{m^2_{D^*_s}} + \frac{1}{2}g^2(\frac{1}{m^2_\omega} + \frac{1}{m^2_\rho})(p_1+p_3).(p_2+p_4)$	$15.7g^2$

Table 3: Tree level amplitudes for $D^*\bar{K}^*$ in $I=1$. Last column: $(V_{\mathrm{th.}})$.

The interaction is attractive for $I=0$ and repulsive for $I=1$.

Local Hidden Gauge Approach

J	Amplitude	Contact	V-exchange	$\sim$ Total
0	$D^*K^* \to D^*K^*$	0	$\frac{g^2}{2}(\frac{1}{m_\rho^2} - \frac{1}{m_\omega^2})(p_1 + p_3).(p_2 + p_4)$	$0.11g^2$
0	$D^*K^* \to D_s^*\rho$	$4g^2$	$-\frac{g^2(p_1+p_4)(p_2+p_3)}{m_{D^*}^2} - \frac{g^2(p_1+p_3).(p_2+p_4)}{m_{K^*}^2}$	$-6.8g^2$
0	$D_s^*\rho \to D_s^*\rho$	0	0	0
1	$D^*K^* \to D^*K^*$	0	$\frac{g^2}{2}(\frac{1}{m_\rho^2} - \frac{1}{m_\omega^2})(p_1 + p_3).(p_2 + p_4)$	$0.11g^2$
1	$D^*K^* \to D_s^*\rho$	0	$\frac{g^2(p_1+p_4)(p_2+p_3)}{m_{D^*}^2} - \frac{g^2(p_1+p_3).(p_2+p_4)}{m_{K^*}^2}$	$-6.6g^2$
1	$D_s^*\rho \to D_s^*\rho$	0	0	0
2	$D^*K^* \to D^*K^*$	0	$\frac{g^2}{2}(\frac{1}{m_\rho^2} - \frac{1}{m_\omega^2})(p_1 + p_3).(p_2 + p_4)$	$0.11g^2$
2	$D^*K^* \to D_s^*\rho$	$-2g^2$	$-\frac{g^2(p_1+p_4)(p_2+p_3)}{m_{D^*}^2} - \frac{g^2(p_1+p_3).(p_2+p_4)}{m_{K^*}^2}$	$-12.8g^2$
2	$D_s^*\rho \to D_s^*\rho$	0	0	0

Table 4: Tree level amplitudes for $D^*K^*, D_s^*\rho$ in $I = 1$. Last column: $(V_{\mathrm{th.}})$ for $C = 1$, $S = 1$ and $I = 1$.

The interaction is attractive for both $I = 0$ and $I = 1$, favoring a $J^+ = 2^+$ state. (see PRD82 (2010) Molina, Branz, Oset, for $I = 0$)

New flavor exotic tetraquark ($C = 1, S = -1$)

Two-meson loop function

$$G_i(s) = \frac{1}{16\pi^2}\left(\alpha + \mathrm{Log}\frac{M_1^2}{\mu^2} + \frac{M_2^2 - M_1^2 + s}{2s}\mathrm{Log}\frac{M_2^2}{M_1^2} + \frac{p}{\sqrt{s}}\left(\mathrm{Log}\frac{s - M_2^2 + M_1^2 + 2p\sqrt{s}}{-s + M_2^2 - M_1^2 + 2p\sqrt{s}} + \mathrm{Log}\frac{s + M_2^2 - M_1^2 + 2p\sqrt{s}}{-s - M_2^2 + M_1^2 + 2p\sqrt{s}}\right)\right),$$

Bethe-Salpeter

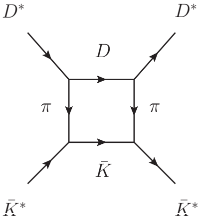
$$\boxed{T = [\hat{1} - VG]^{-1}V}$$

The states with $J^P = \{0,2\}^+$ decay into $D\bar{K}$

$$\mathcal{L}_{VPP} = -ig\langle[P, \partial_\mu P]V^\mu\rangle$$

$$F(q) = e^{((p_1^0 - q^0)^2 - \vec{q}^{\,2})/\Lambda^2}$$ Navarra, PRD65(2002)

with $q_0 = (s + m_D^2 - m_K^2)/2\sqrt{s}$.



Decay of the $T_{cs}(2900)$ to $D^*\bar{K}$

$I(J^P)$	M[MeV]	Γ[MeV]	Coupled channels	state
$0(2^+)$	2775	38	$D^*\bar{K}^*$	?
$0(1^+)$	2861	20	$D^*\bar{K}^*$	?
$0(0^+)$	2866	57	$D^*\bar{K}^*$	$T_{cs}(2900)$

Table 5: New results including the width of the D^*K channel. Molina, Oset, PLB20

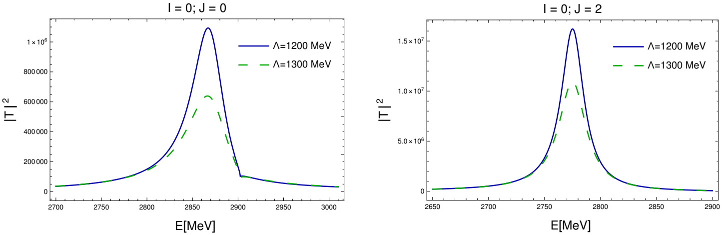


Figure 2: $|T|^2$ for $C = 1, S = -1, I = 0, J = 0$ and $J = 2$.

Decay of the $T_{cs}(2900)$ to $D^*\bar{K}$

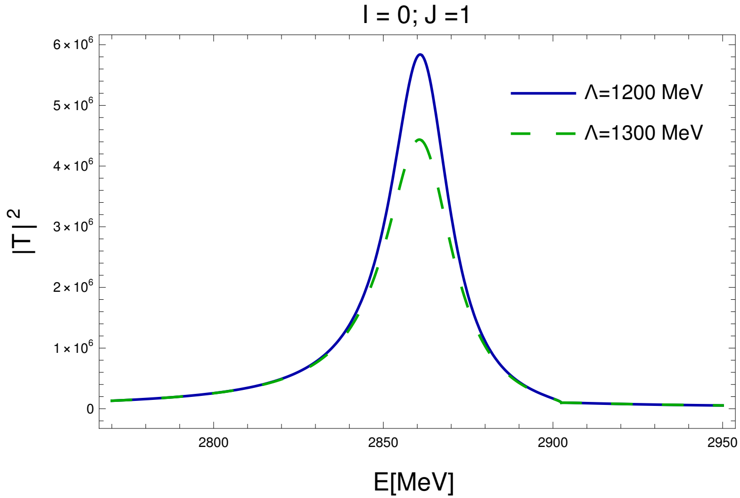


Figure 3: $|T|^2$ for $C = 1, S = -1, I = 0, J = 0$ and $J = 1$.

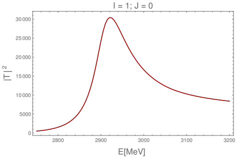
The $T_{c\bar{s}}(2900)$

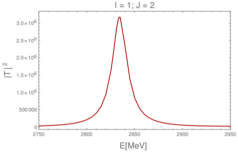
$C = 1, S = 1, I = 1$: The $T_{c\bar{s}}(2900)$

New results, $\alpha = -1.474$ to obtain the $T_{cs}(2900)$ state in $D^*\bar{K}^*$.

Convolution due to the vector meson mass distribution ρ, K^*

$$\tilde{G}(s) \quad = \quad \frac{1}{N}\int_{(M_1-4\Gamma_1)^2}^{(M_1+4\Gamma_1)^2} d\tilde{m}_1^2 (-\frac{1}{\pi})\mathcal{I}m\frac{1}{\tilde{m}_1^2 - M_1^2 + i\Gamma(\tilde{m})\tilde{m}_1} G(s, \tilde{m}_1^2, M_2^2) \ ,$$



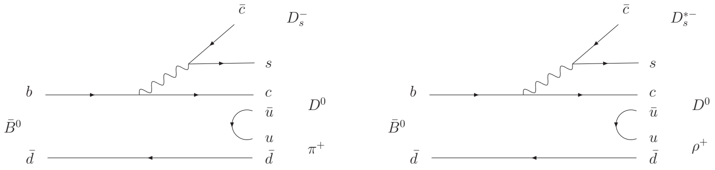


$I(J^P)$	M[MeV]	Γ[MeV]	Coupled channels	state
$1(0^+)$	2920	130	$D^*K^*, D_s\rho$	$T_{c\bar{s}}(2900)$
$1(1^+)$	2922	145		?
$1(2^+)$	2835	20		?

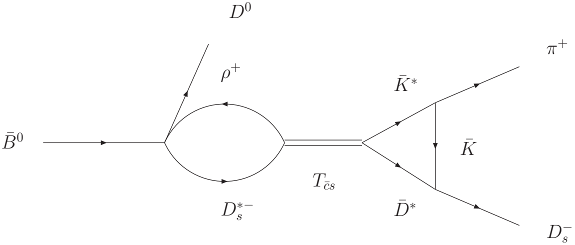
Table 6: PRD107(2023), Exp. $(m, \Gamma) = (2908 \pm 11 \pm 20, 136 \pm 23 \pm 11)$ MeV

Production of the $T_{\bar{c}s}(2900)$

$\bar{B}^0 \to D_s^- D^0 \pi^+$ in B decays



The $T_{\bar{c}s}(2900)$ can be produced by means of external emission



Production of the $T_{\bar{c}s}(2900)$ in B decays

$$T(E) = aG(E)_{D_s^*\rho}\, t_{D_s^*\rho\to\bar{D}^*\bar{K}^*}(E)\, t_L(E) + b \qquad (5)$$

$E = M_{inv}(\pi^+ D_s^-)$; a, b parameters; t_L amplitude for the triangle loop.

$$\boxed{\frac{d\Gamma}{dM_{Inv}} = \frac{1}{(2\pi)^3}\frac{1}{4M_B^2}\, p_D\, \tilde{p}_\pi |T|^2}$$