Neutrino physics (theory) I

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TAE 2024
Benasque (10-11 Sep 2024)

Lecture 1

Introduction: neutrinos and their history

Neutrinos in the Standard Model

Neutrino oscillations

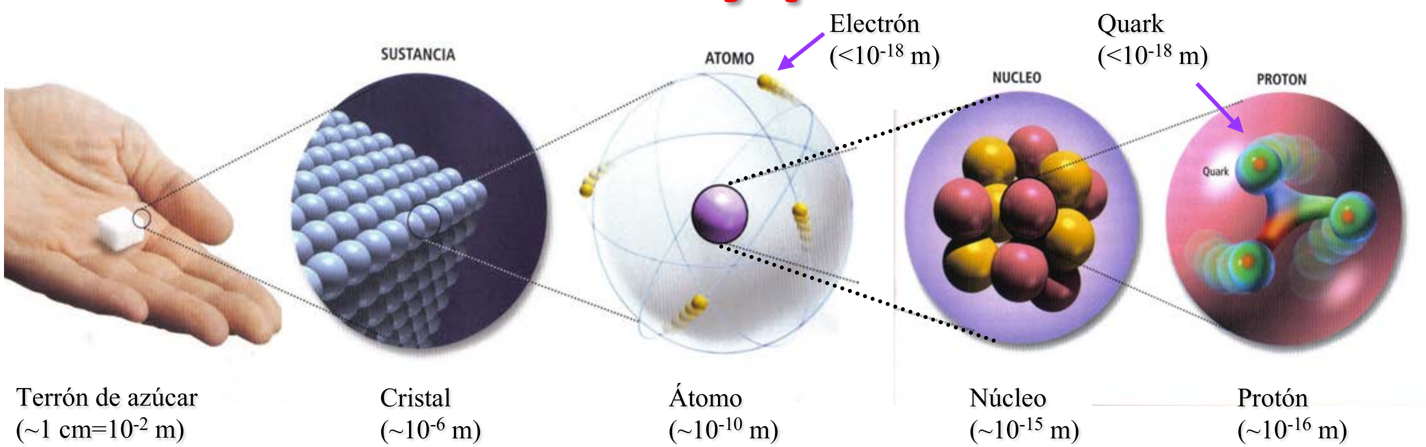
Lecture 2

Values of neutrino masses

Introduction to neutrino cosmology

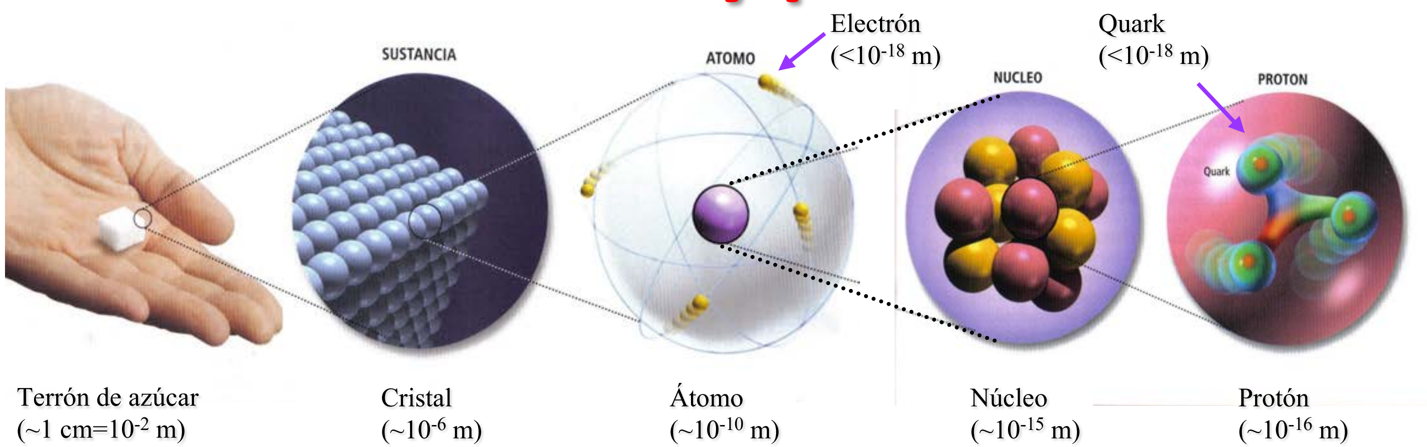
Cosmological bounds on some neutrino properties

Elementary particles



	Quarks		Leptons	
	Charge +2/3	Charge −1/3	Charge −1	Charge 0
1st Family	up u	down d	electron e	e-neutrino ν_e
2nd Family	charm c	strange s	muon μ	μ-neutrino ν_μ
3er Family	top t	bottom b	tau τ	τ-neutrino ν_τ
	Gravity			
	Weak interaction			
	Electromagnetism			
	Strong interaction			

Elementary particles



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3er Family	top t	bottom b	tau τ	τ-neutrino ν_τ

Weak interaction

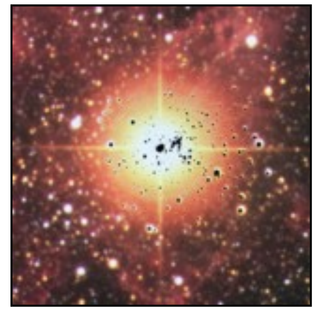
neutral spin ½ particles
+ 3 antineutrino states

Where do neutrinos come from?

✓ Nuclear reactors

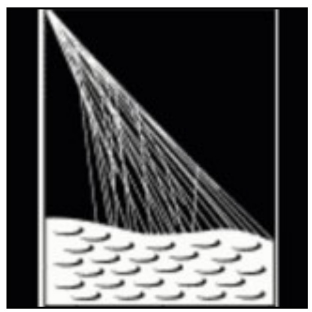
Sun ✓

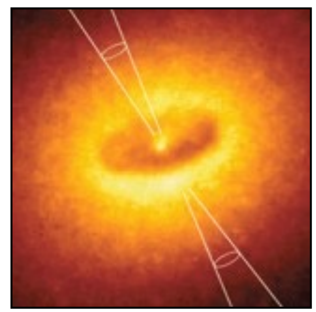
✓ Particle accelerators



Supernovae
SN 1987A ✓

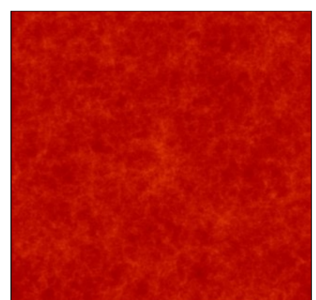
✓ Earth Atmosphere (Cosmic rays)



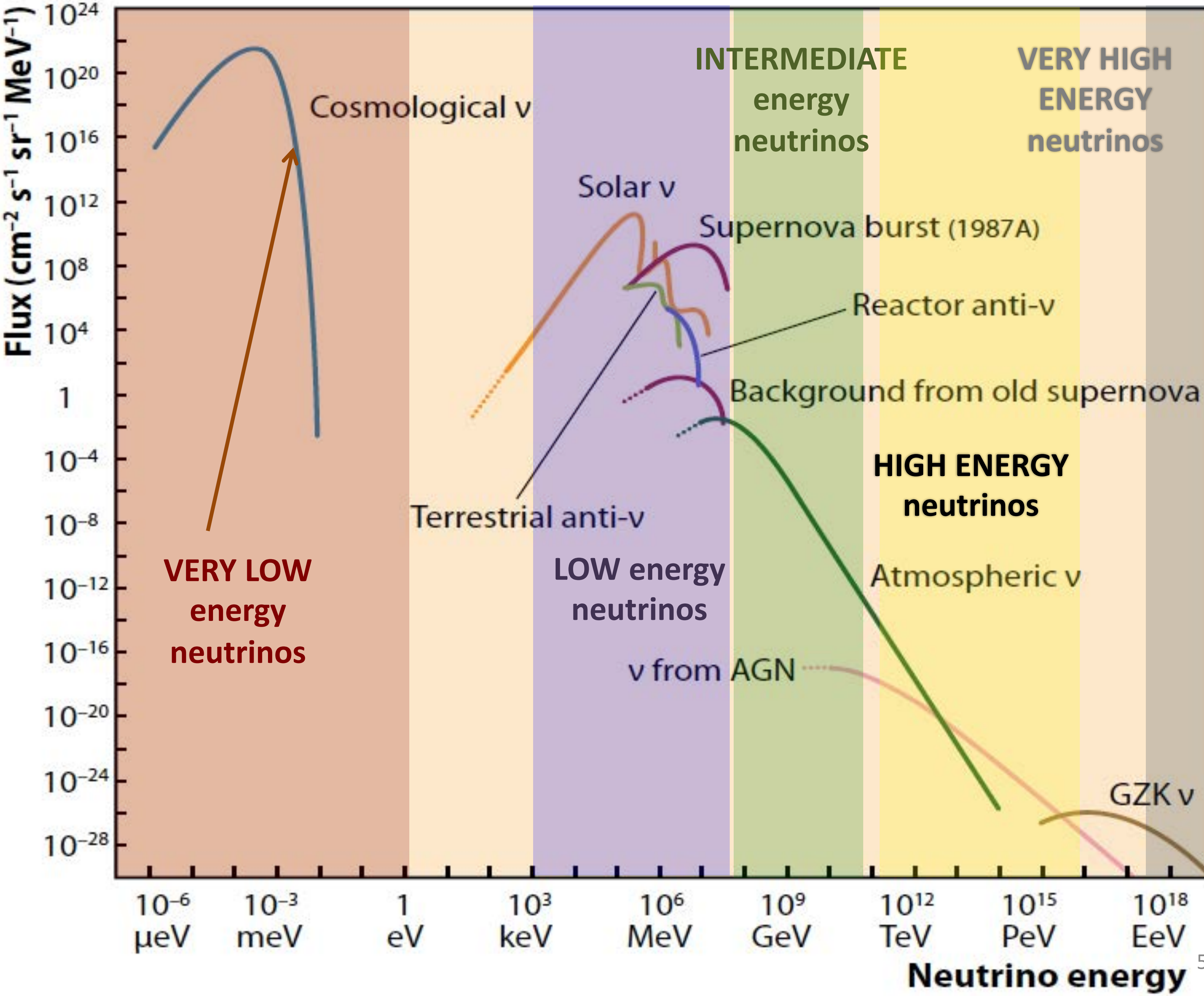


Accelerators in astrophysical sources ?✓

✓ Earth interior (Natural Radioactivity)

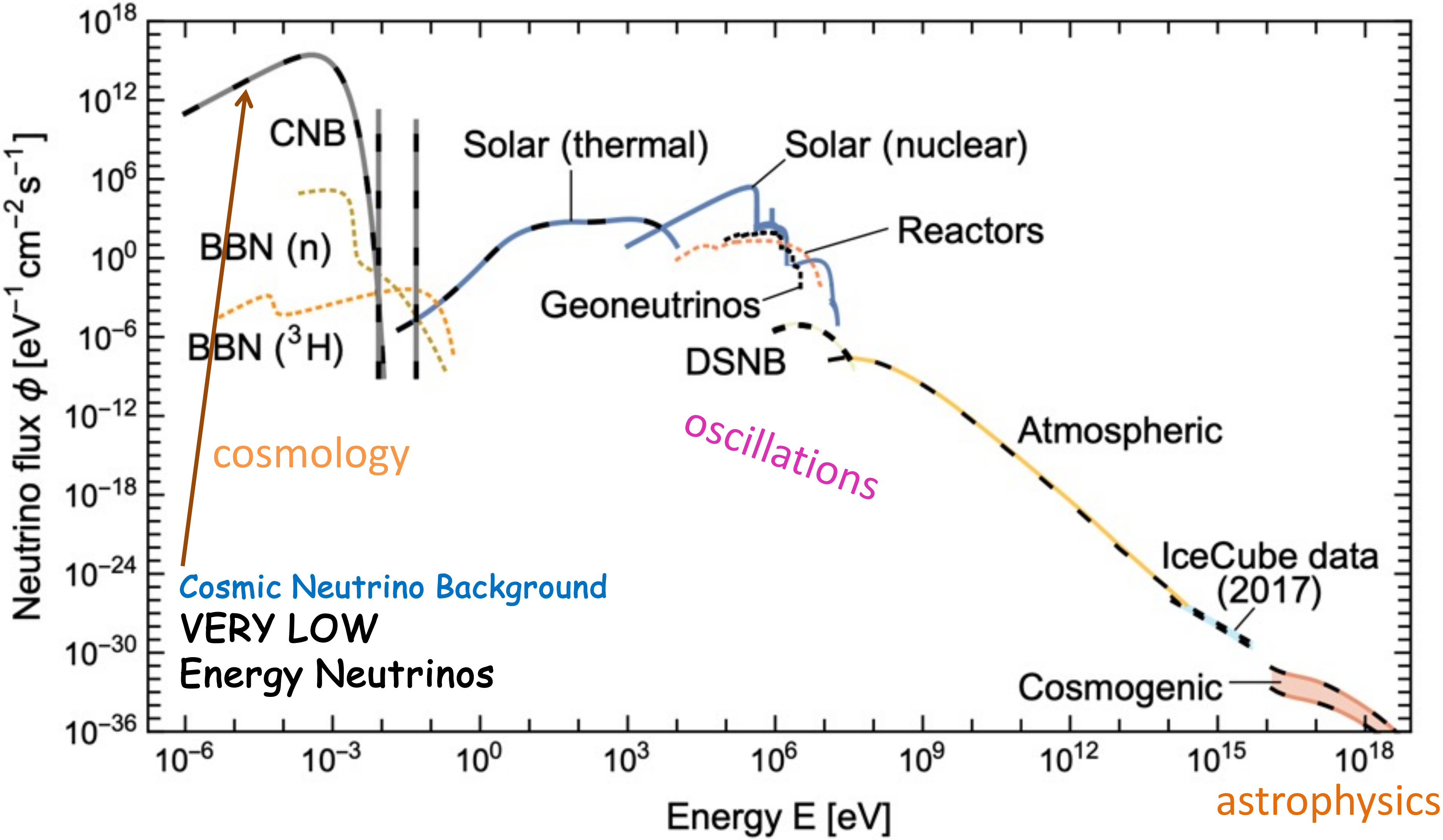


EARLY UNIVERSE
(today 336 ν/cm^3)
Indirect evidence



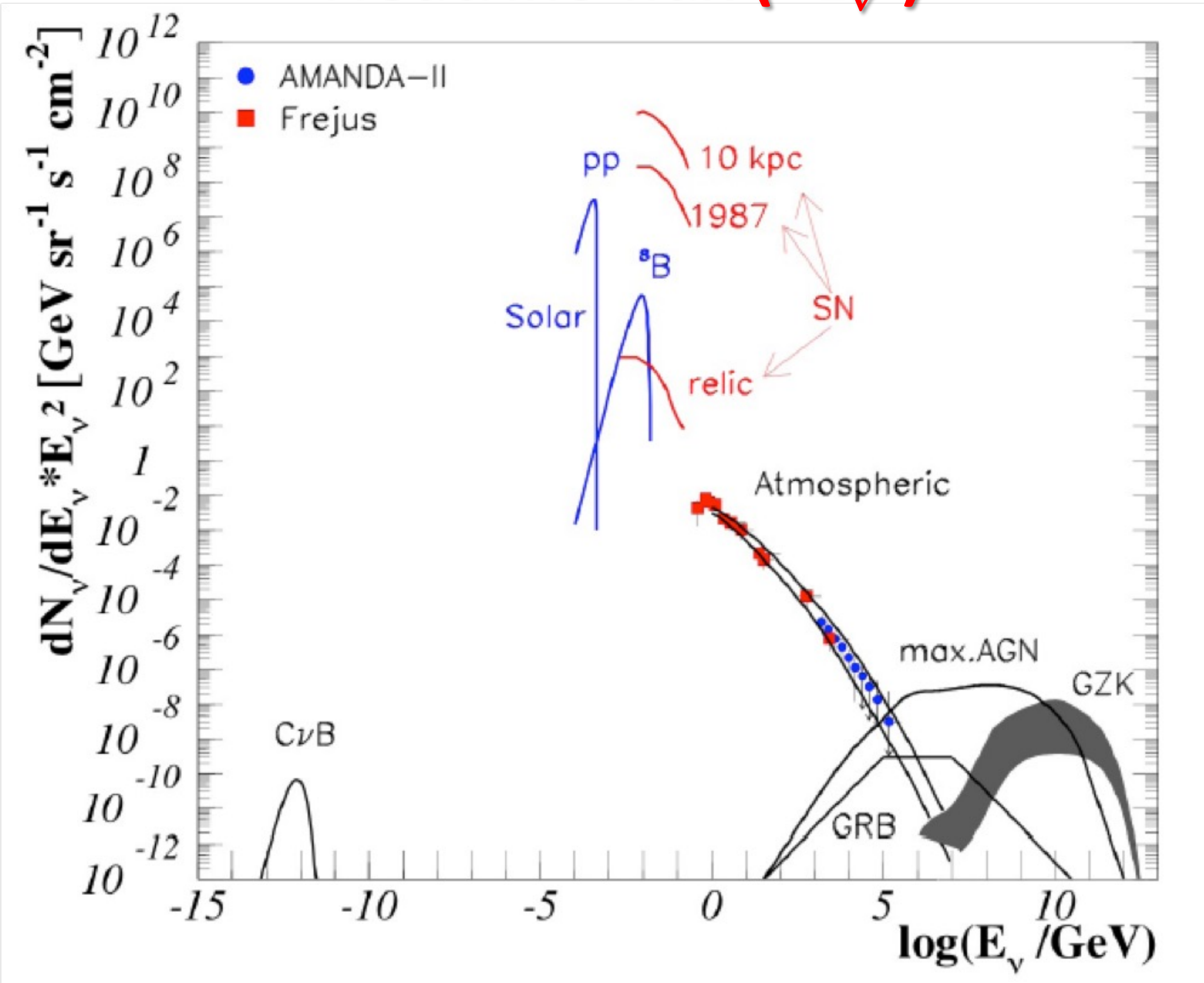
Flux (cm⁻² s⁻¹ sr⁻¹ MeV⁻¹)
Cosmological ν
INTERMEDIATE energy neutrinos
VERY HIGH ENERGY neutrinos
Solar ν
Supernova burst (1987A)
Reactor anti-ν
Background from old supernova
HIGH ENERGY neutrinos
Terrestrial anti-ν
VERY LOW energy neutrinos
LOW energy neutrinos
Atmospheric ν
ν from AGN
GZK ν
μeV
meV
eV
keV
MeV
GeV
TeV
PeV
EeV
Neutrino energy

Grand Unified Neutrino Spectrum at Earth



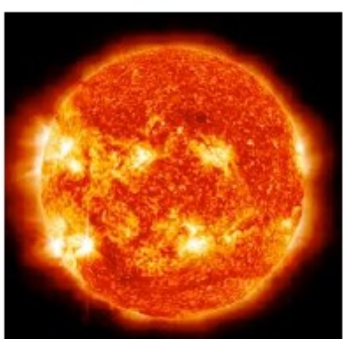
Vitagliano, Tamborra, and Raffelt, Rev Mod Phys **92** (2020) 45006

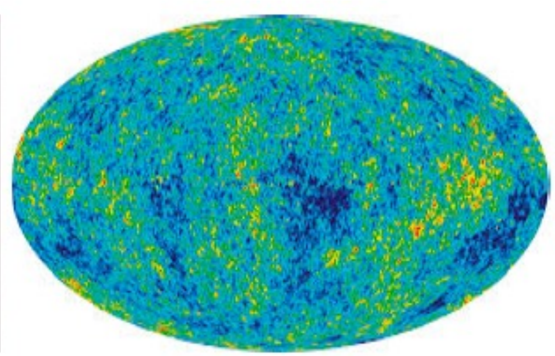
Neutrino flux ($\times E_\nu^2$)



Why neutrinos are so important?

- They can **probe astrophysical environments** that other techniques cannot: supernova explosions, the core of the Sun,...
- Their role is crucial for the **evolution of the Universe** (Big Bang Nucleosynthesis, structure formation…)
- They could help explaining the **matter-antimatter asymmetry** of the Universe (leptogenesis mechanism)
- They provide an evidence for **physics beyond the SM**!!!



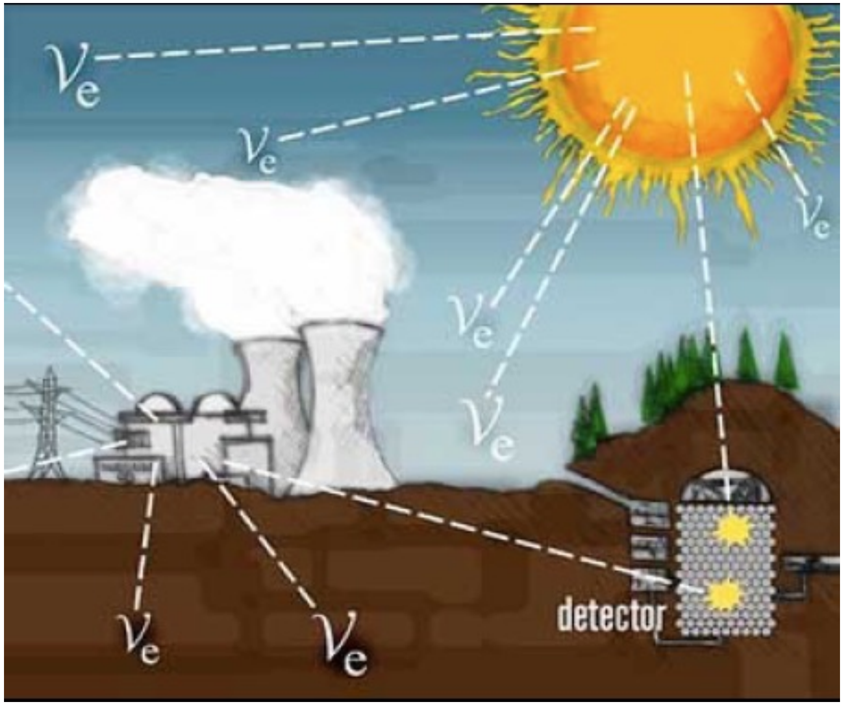



Detecting neutrinos

Neutrinos are so weakly interacting that **detecting** and **studying** them is an extremely difficult task

We need :

- Very intense neutrino sources
- A lot of patience (time)
- Large detectors
- Very low backgound signal (underground experiments)

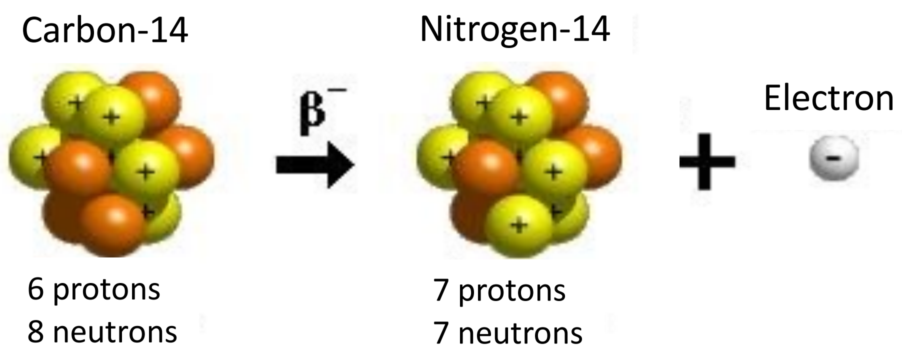


See Patricia's lectures on experiments

A brief history of neutrinos

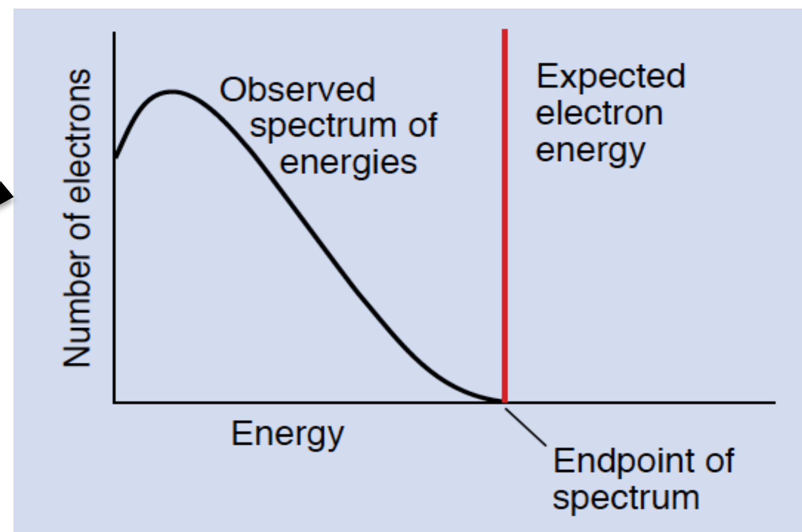
Neutrino proposal

Experiments by James Chadwick (1913) and others, measuring the **electrons** from radioactive β **decays** of atomic nuclei



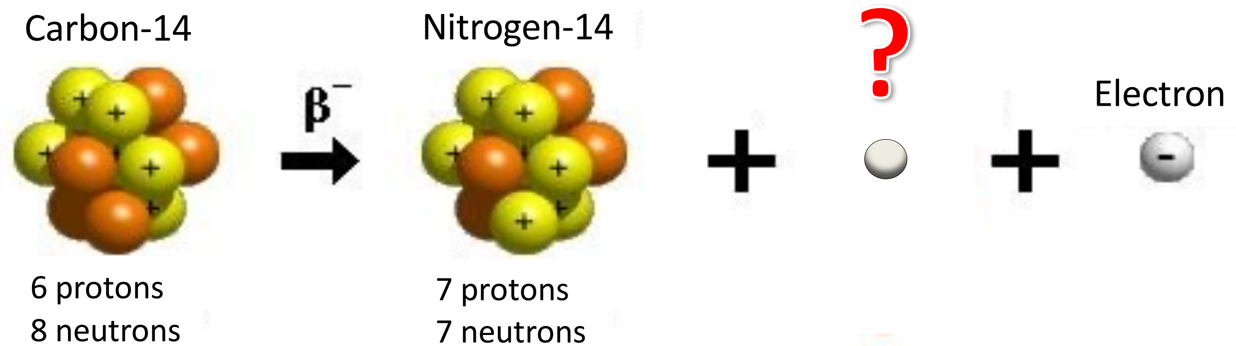
From energy and momentum conservation, the emitted electrons should have a **fixed energy**

Niels Bohr even suggested that energy may not be conserved in individual nuclear processes



Neutrino proposal

1930: **Wolfgang Pauli,** in a famous letter to colleagues, postulated as a **desperate way** to solve the observations, the **existence of a new particle** (with spin ½ from conservation of angular momentum)

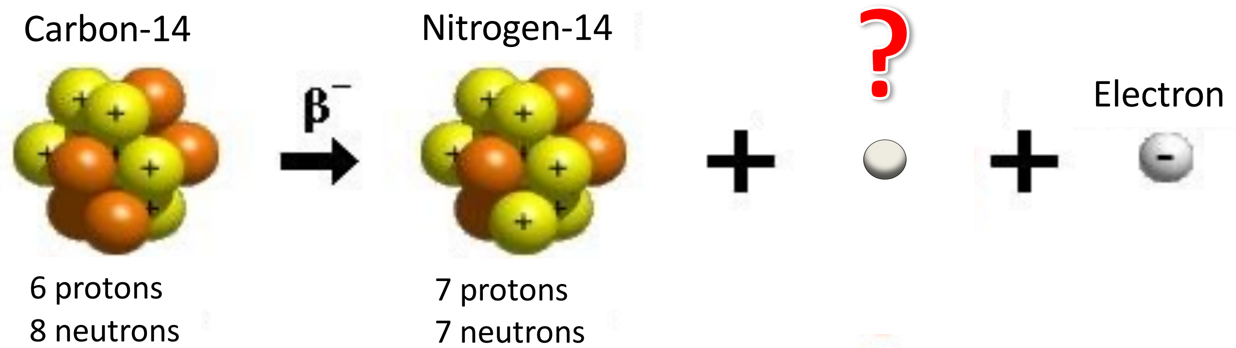


"Dear radioactive ladies and gentlemen,

I have come upon a desperate way out regarding . . . [some fairly obscure data], as well as to the continuous β-spectrum, in order to save . . . The energy law. To wit, the possibility that there could exist in the nucleus electrically neutral particles which I shall call neutrons, which have spin 1/2 and satisfy the exclusion principle and which are further distinct from light-quanta in that they do not move with light velocity. . . . The continuous β-spectrum would then become understandable from the assumption that in β-decay a neutron is emitted along with the electron, in such a way that the sum of the energies of the neutron and the electron is constant."

Neutrino proposal

1930: **Wolfgang Pauli,** in a famous letter to colleagues, postulated as a **desperate way** to solve the observations, the **existence of a new particle** (with spin ½ from conservation of angular momentum)



The **new particle** would interact very weakly with matter, which explains why it had not been observed, and would carry part of the decay energy, so that the energy spectrum of electrons is continuous.

Fisrt proposal of a **new part**icle that is **NOT** part of ordinary matter

> *"I have done something very bad today by proposing a particle that cannot be detected. It is something that no theorist should ever do."*

Pauli (1930)

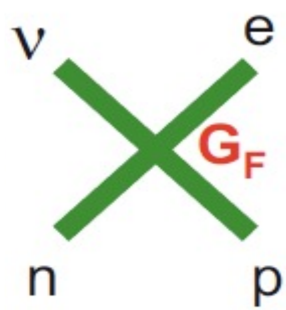
Enrico Fermi and the neutrino

Enrico Fermi (1934) proposes the first theory of **nuclear β decay** : the theory of **weak interactions**

the **NEUTRINO** is born

(Italian: “il piccolo neutro”, el little neutral particle)

$$n \longrightarrow p + e^{-} + \bar{\nu}_e$$



Is it posible to detect neutrinos?

Bethe and Peierls (1934) calculate the cross section **for the processes**

$$\nu + n \rightarrow e^- + p$$

$$\bar{\nu} + p \rightarrow e^+ + n$$

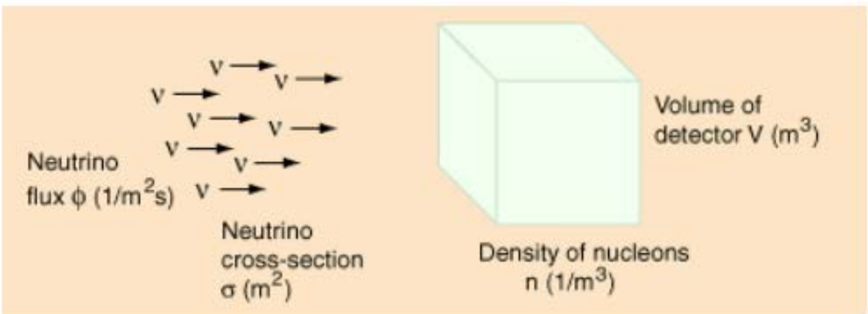
$$\sigma \sim 10^{-44}\,\mathrm{cm}^2$$

Interactions of ν / sec = Flux (ν $cm^{-2}s^{-1}$) × cross section (cm^2)
× particles in target

mean free path of neutrinos → E_ν = 2 MeV

λ_ν (**water**) ≈ 15 light-years (all nucleons)
λ_ν (**lead**) ≈ 1.5 light-years (all nucleons)

Event number in a neutrino experiment:



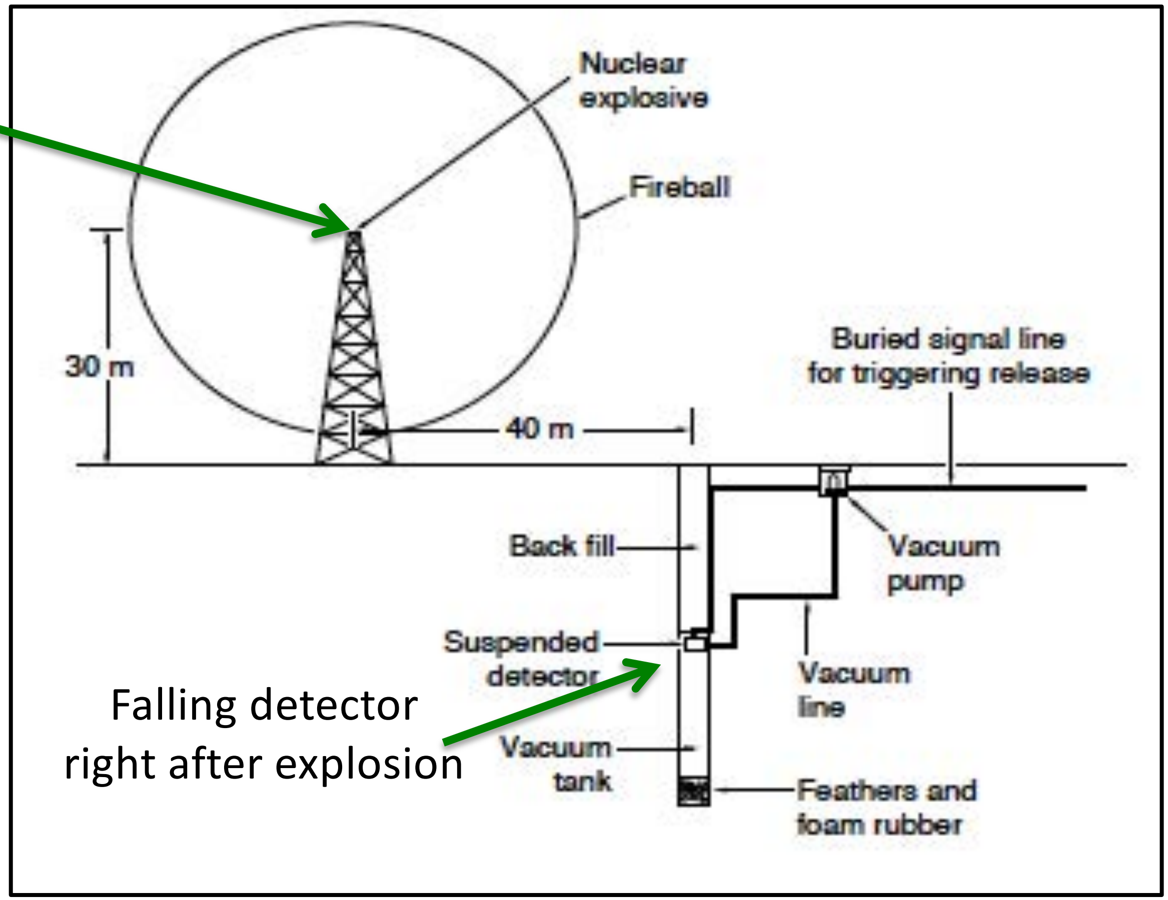
$$N = \varphi\, \sigma\, N_{targ}\, \Delta t$$

with a 1000 kg detector and a flux of 10^{10} ν/s: few ν events/day

solar / **reactor** neutrino flux ~ 7×10^{10} ν cm^{-2} s^{-1} / 10^{20} ν s^{-1}

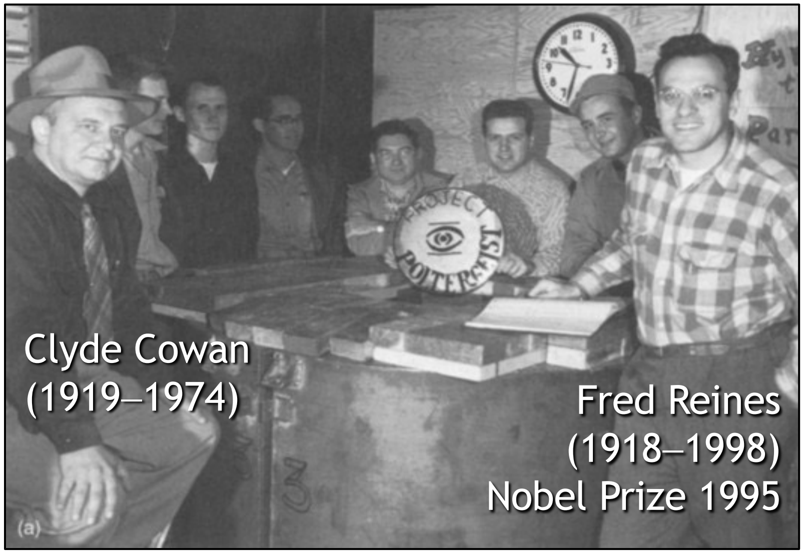
First attempts to detect neutrinos

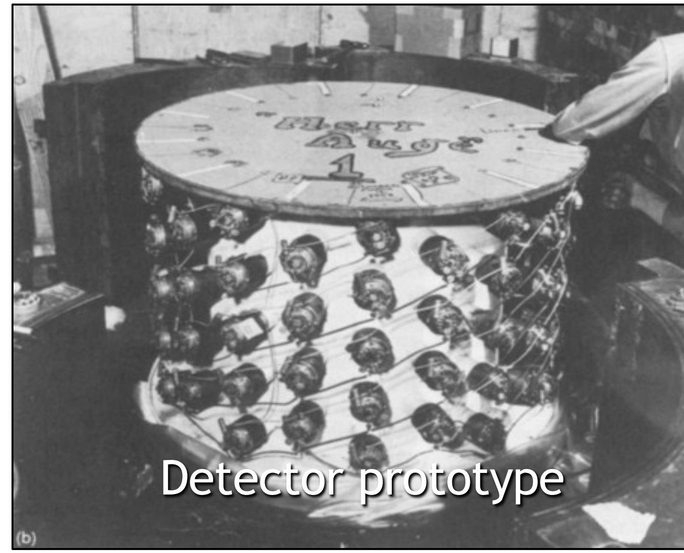
After a first idea (1952) to detect neutrinos in a **nuclear explosion**

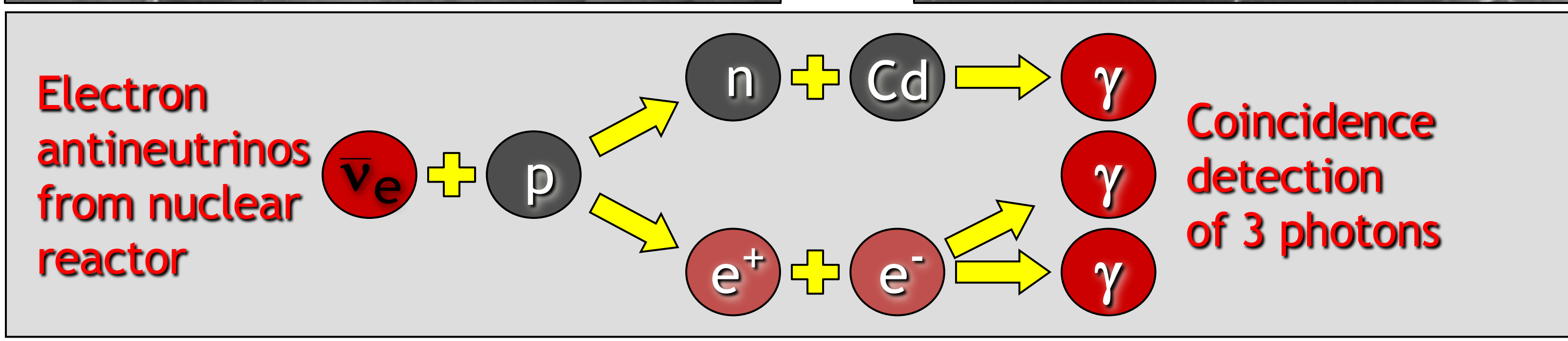


First attempts to detect neutrinos

1st experiment to detect neutrinos from a **nuclear reactor** (1953)

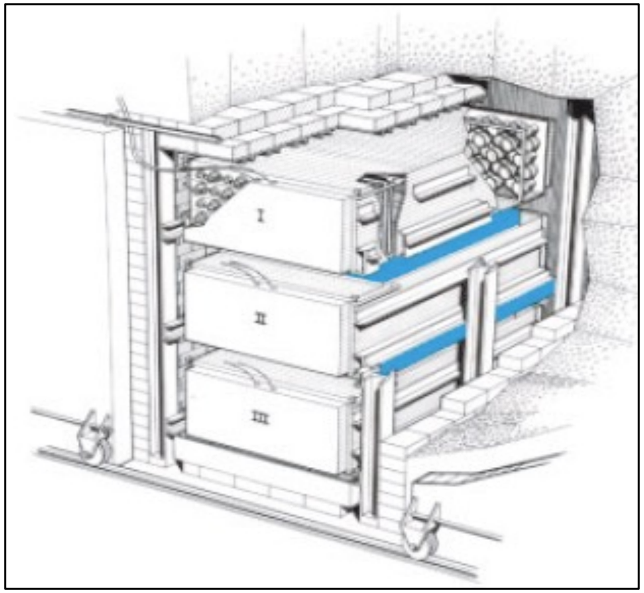


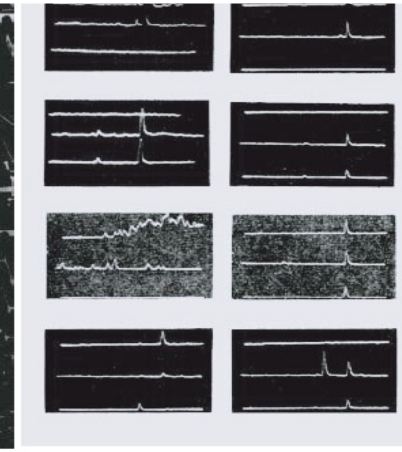




First detection of neutrinos

Evidence for neutrino detection from the **Savannah River nuclear reactor** (1956)





Fred Reines
(1918–1998)
Nobel Prize in Physics 1995

Telegram to Pauli:

"We are happy to inform you that we have definitely detected neutrinos from fission fragments by observing inverse beta decay of protons. Observed cross section agrees well with expected six times ten to minus forty-four square centimeters"

More than ONE neutrino flavour?

1959: Pontecorvo suggested the existence of a **different neutrino state**, associated to muon decay, and proposed an experiment to check it.

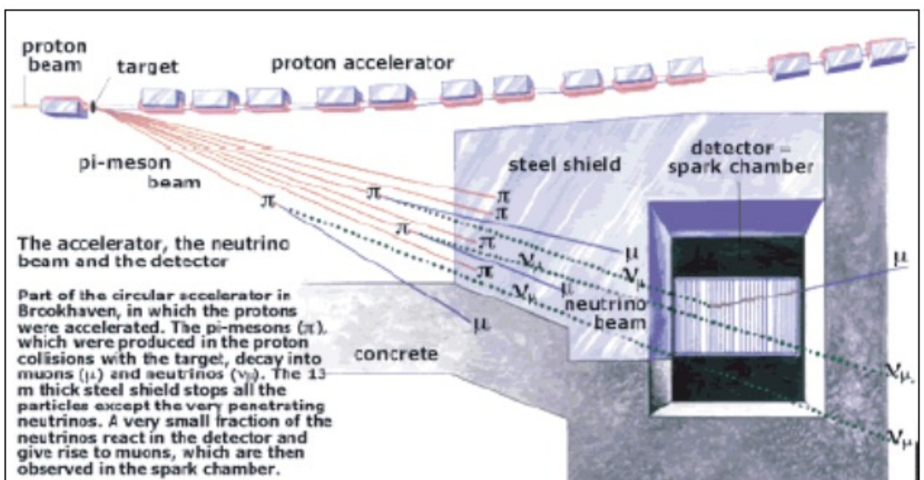
$$\nu_{\rm acc} + n \to p + (e^- \text{ or } \mu^- ?)$$

1963: Discovery of **muon neutrinos** (ν_μ) by Lederman, Schwartz & Steinberger

$$\pi^+ \to \mu^+ + \nu_\mu$$

$$\nu_\mu + n \to p + \mu^-$$

not e^-



Leon M. Lederman · Melvin Schwartz · Jack Steinberger

Nobel Prize in Physics 1988

More than TWO neutrino flavours? ν_e ν_μ ν_τ

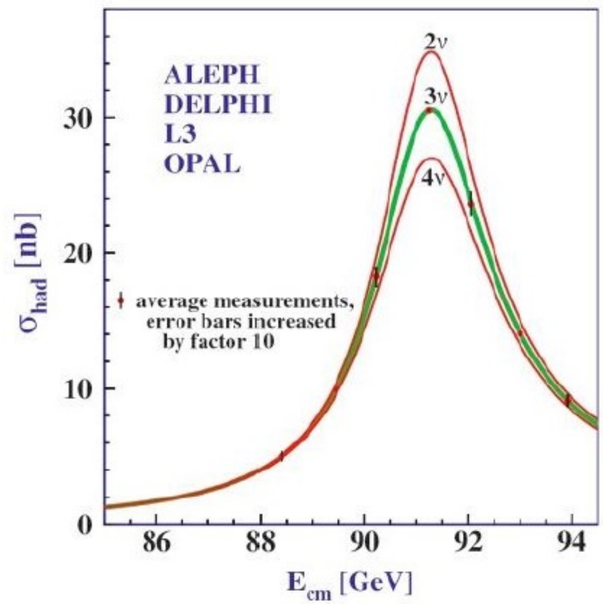
1978: Discovery of the τ lepton at SLAC → imbalance of energy in τ decay suggests the existence of a third neutrino

1989: measurements at LEP accelerator of the invisible decay width of Z boson

$$\Gamma_{\rm inv} \equiv \Gamma_Z - \Gamma_{\rm had} - 3\Gamma_{\rm lep}$$

$$N_\nu = \Gamma_{\rm inv}/\Gamma_{\rm SM}(Z \to \nu_i \overline{\nu}_i)$$

$\to N_\nu = 2.984 \pm 0.008$



2000: Discovery of ν_τ by the DONUT Collaboration:

800 GeV p ⇒ Ds meson ($\equiv c\bar{s}$) → ν_τ τ ⇒ τ detected

Parity violation in weak interactions

1956: T.D. Lee and C.N. Yang proposed **parity violation** in weak interactions.

1957: using a radioactive source of ^{60}Co, Chien-Shiung Wu et al. determined that **weak interaction violates parity conservation** **maximally**

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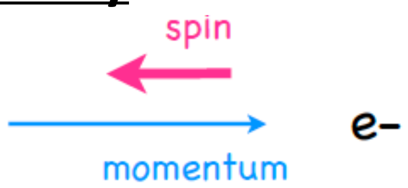
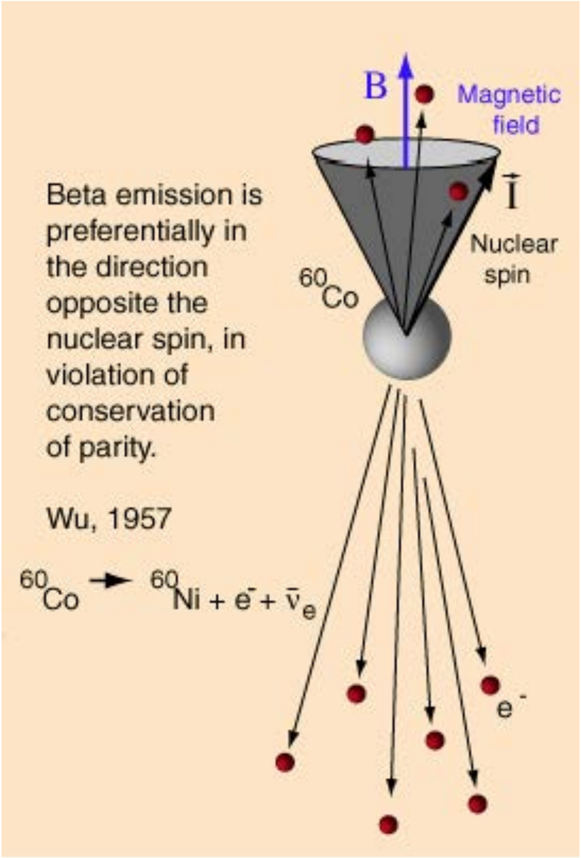


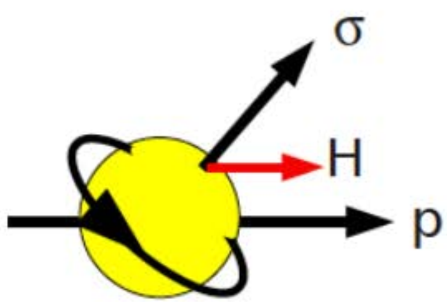
1957: Nobel Prize in Physics (Lee & Yang)
1978: Wolf Prize in Physics (C.S. Wu)

1958: Goldhaber et al. showed that **neutrinos can only be emitted with their spin anti-parallel to their momentum** direction.



Helicity and Chirality

Helicity is the projection of spin along the momentum direction



$$\hat{H} = \frac{\vec{\sigma} \cdot \vec{p}}{|\vec{p}|}$$

→ Lorentz-invariant only for massless particles

Chirality is an asymmetry property: a chiral object is not identical to its mirror image, cannot be superimposed on it.

$$P_{\mathrm{L,R}} = \frac{1 \mp \gamma_5}{2}, \; \psi_{\mathrm{L,R}} = P_{\mathrm{L,R}} \psi$$

→ Handedness = Chirality

→ Lorentz-invariant although not directly measurable

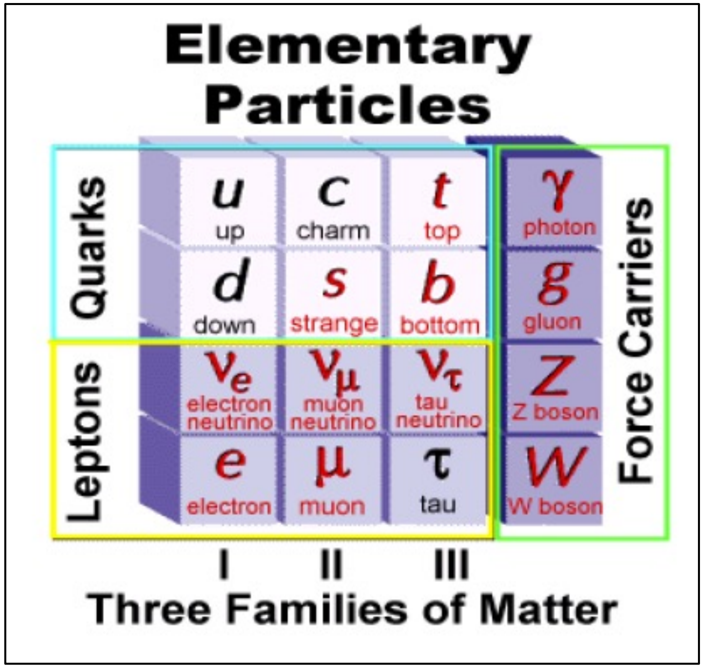
Massless particles: Helicity = Chirality

Massive particles: Chiral states contain contributions from both helicity states

Ultra-relativistic particles: LH (RH) chiral projection dominated by a - (+) helicity state

Neutrinos in the Standard Model

Neutrinos in the Standard Model of particle physics



Each type (**flavour**) of neutrino is associated to a charged lepton

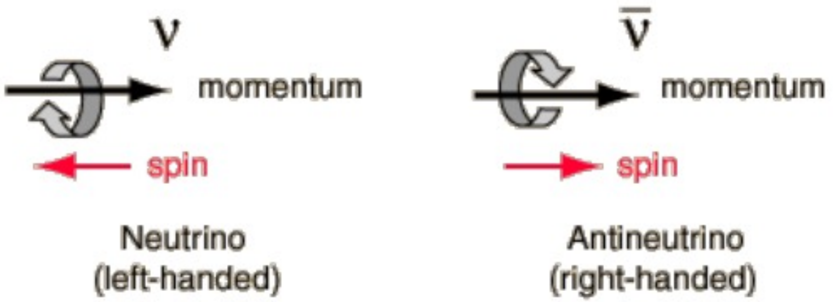
They belong to **SU(2) lepton doublets**

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

only sensitive to the **weak force**

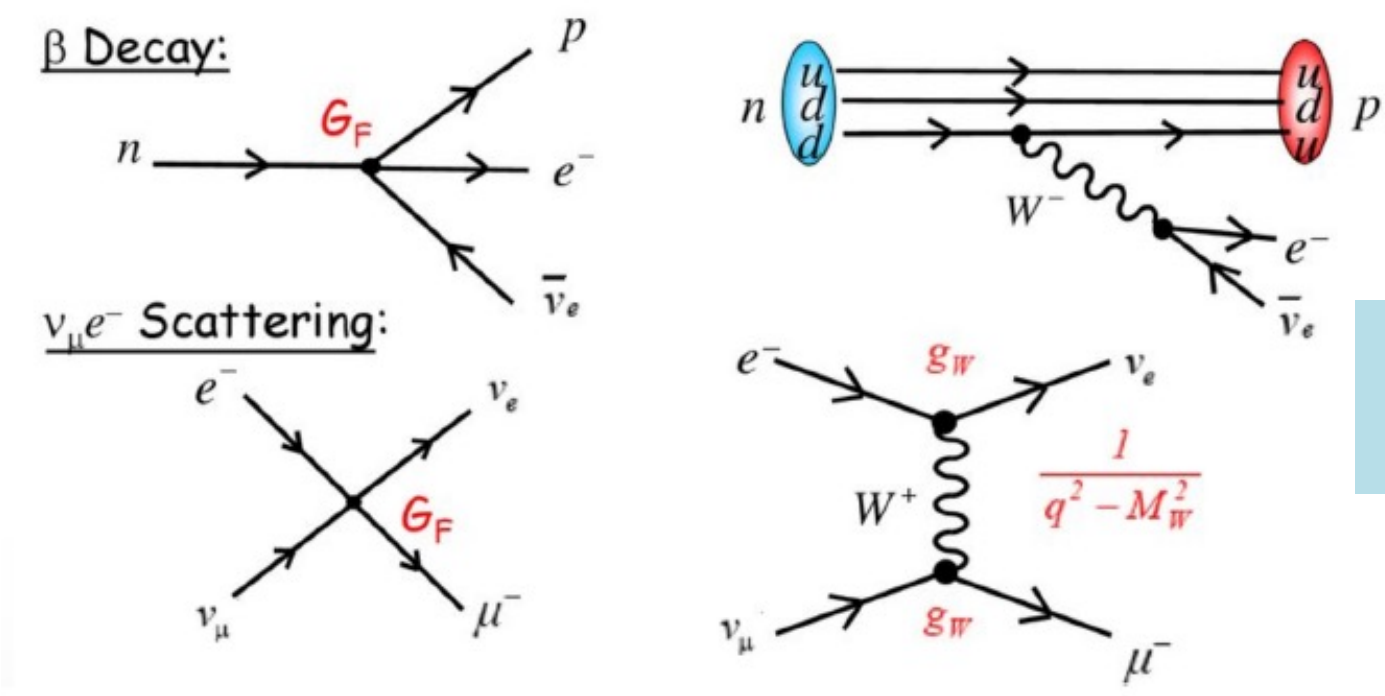
- Only two types of neutrinos have been observed in nature: **left-handed neutrinos** AND **right-handed antineutrinos**

In the SM, there exist (e_R, μ_R, τ_R) but **no SU(2) neutrino singlets**



Weak interactions of neutrinos

- Neutrinos were first detected through their weak interactions with **charged leptons**, or Charged Current (CC) interactions.
- CC weak interactions **first described by Fermi** as point-like 4-fermion vertex.
- SM: CC interactions are **mediated by the vector boson W** (W-, W+)

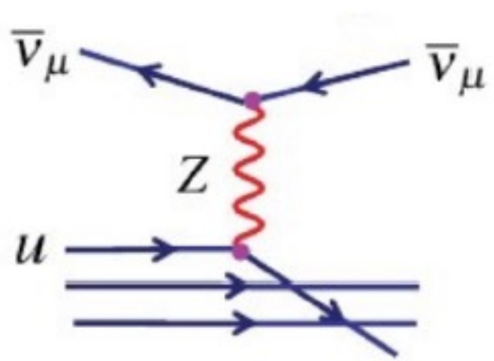


W bosons couple to leptons in the same doublet

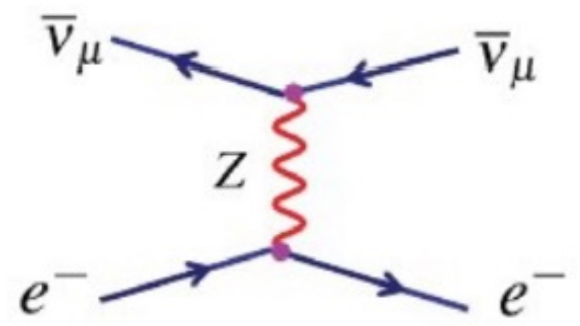
Weak interactions of neutrinos

- In the SM: predicted **Neutral Current (NC) interactions** mediated by a **neutral vector boson**, the Z^0
- NC **first observed in 1973** with the Gargamelle bubble chamber

$$\overline{\nu}_\mu + N \to \overline{\nu}_\mu + \text{hadrons}$$



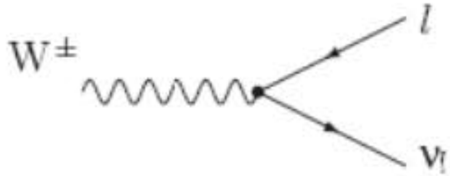
$$\overline{\nu}_\mu + e^- \to \overline{\nu}_\mu + e^-$$



→ First evidence for Z boson

Neutrino interactions in the SM

Charged Current (CC):



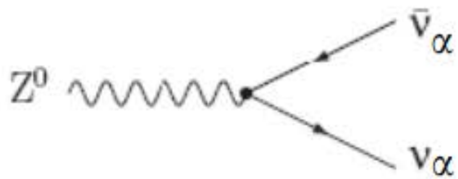
$$W^- \to l_\alpha^- + \overline{\nu}_\alpha$$

$$W^+ \to l_\alpha^+ + \nu_\alpha$$

$$(\alpha = e, \mu, \tau)$$

$$\mathcal{L}_{\rm CC} = -\frac{g}{\sqrt{2}} \sum_\alpha \overline{\nu}_{\alpha L} \gamma^\mu l_{\alpha L} W_\mu + \text{h.c.}$$

Neutral Current (NC):



$$Z^0 \to \nu_\alpha \overline{\nu}_\alpha$$

$$\mathcal{L}_{\rm NC} = -\frac{g}{2\cos\theta_W} \sum_\alpha \overline{\nu}_{\alpha L} \gamma^\mu \nu_{\alpha L} Z_\mu^0$$

in the SM, only LH neutrinos and RH antineutrinos participate in weak interactions

- Interactions conserve the **total lepton number** L:

$$L(l^-) = L(\nu) = -L(l^+) = -L(\bar{\nu}) = 1$$

- **Family lepton numbers** $L(\nu_e)$ $L(\nu_\mu)$ $L(\nu_\tau)$ are also conserved

Fermion masses in the SM lagrangian

- In the SM, fermion masses appear in the lagrangian with terms like:

$$m\overline{\psi}\psi \qquad \rightarrow \text{Dirac mass term}$$

decomposing into its chiral states: $\psi \equiv \psi_L + \psi_R$

$$\Rightarrow \quad m\overline{\psi}\psi = m\overline{\psi}_L\psi_R + m\overline{\psi}_R\psi_L$$

→ forbidden: not invariant under SU(2): it couples ψ_L with ψ_R ($I_W=1/2$)

→ solved by Higgs mechanism: after SSB, Dirac mass terms appear from Yukawa couplings:

$$\mathcal{L}_{\text{Yukawa}} = Y\overline{\psi}_L\phi\psi_R + \text{h.c.} \qquad \langle\phi^0\rangle = v$$

→ OK for most of particles but SM neutrino has only a L-chiral state (no ψ_R)

a **Dirac mass term** for neutrinos can **not** be built in the Standard Model

Majorana neutrino mass

✦ We build a R-chiral field from a L-chiral field by charge conjugation:

$$\psi_R \equiv \psi_L^C = \hat{C}\overline{\psi}^T \qquad \hat{C} = i\gamma^2\gamma^0$$

→ the total neutrino field is: $\psi = \psi_L + \psi_R = \psi_L + \psi_L^C$

→ taking the charge conjugate: $\psi^C = (\psi_L + \psi_R)^C = \psi_L^C + \psi_L = \psi$

$$\psi = \nu = \nu_L + \nu_L^C$$

neutrino = antineutrino

Majorana mass term: $-\mathcal{L}_M = \frac{1}{2}m\left(\overline{\nu_L^C}\nu_L + \overline{\nu_L}\nu_L^C\right)$

Not invariant under U(1) transformations

However: this mass term not invariant under weak isospin (I_W=1)

→ solved with a Higgs triplet BUT it is not included in the SM.

→ solved with a dim-5 operator (Weinberg operator) BUT non-renormalizable

Neutrino mass in the SM

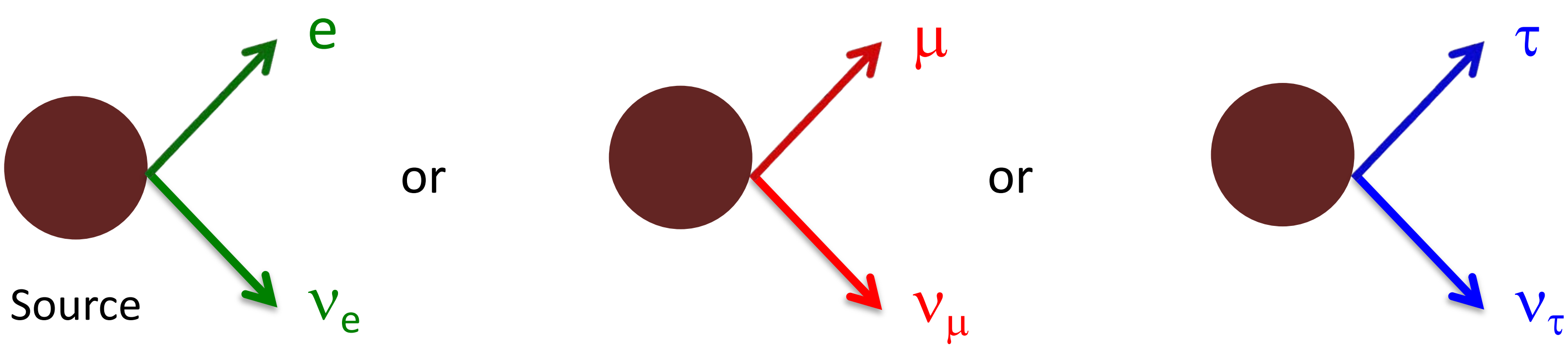
- Since the SM does not contain right-handed neutrinos: a Dirac mass term as for the rest of fermions is not allowed.
- The SM only contains one Higgs doublet: no Higgs triplet to build a Majorana mass term
- The SM is renomalizable and, therefore, dim-5 terms as the Weinberg operator are not allowed.

Neutrinos are strictly MASSLESS in the Standard Model!

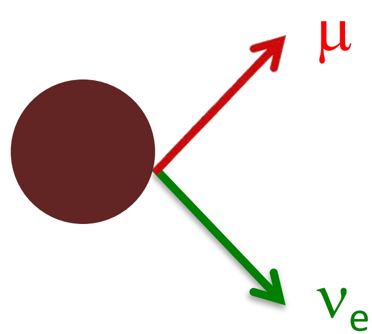
Weak interactions conserve flavour

Neutrinos are always produced together with an **associated charged lepton** (**e**, μ, τ)

FLAVOUR IS PRESERVED

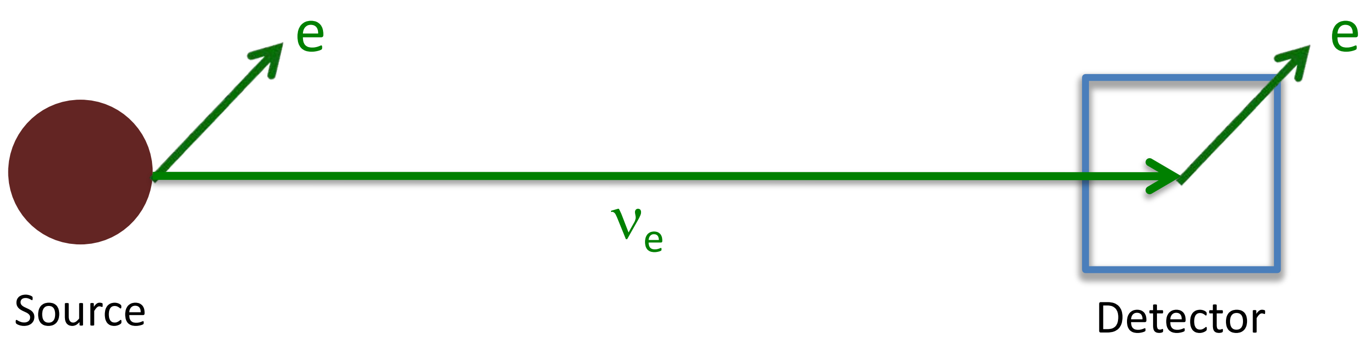


NEVER

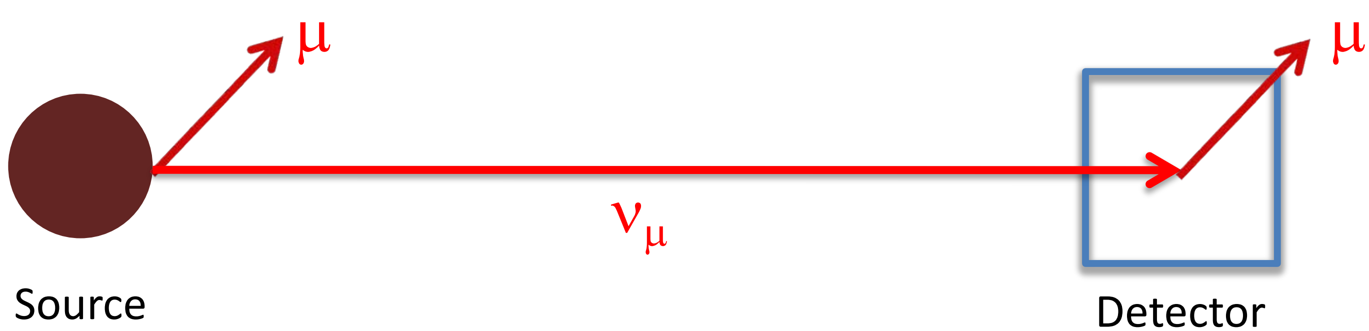


Production and detection of neutrinos

Neutrino detection at experiments is related to the corresponding **charged lepton**

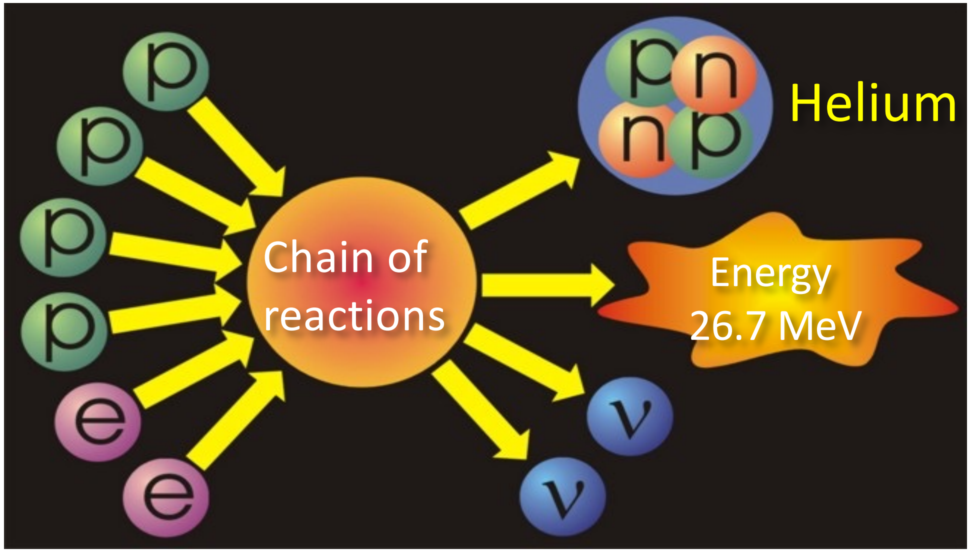


FLAVOURS DO NOT MIX



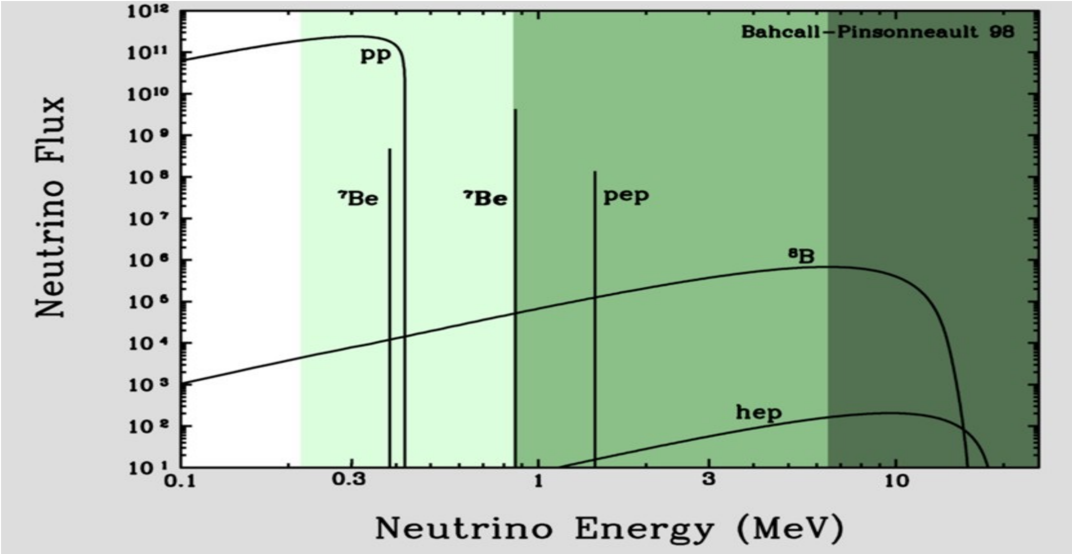
Neutrino oscillations

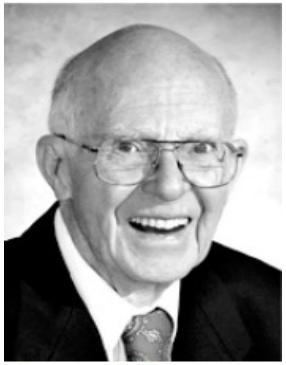
Solar neutrinos



Solar luminosity :
98 % light (photons)
2 % neutrinos

Solar neutrino
flux @ Earth :
6.6×10^{10} neutrinos $cm^{-2}s^{-1}$





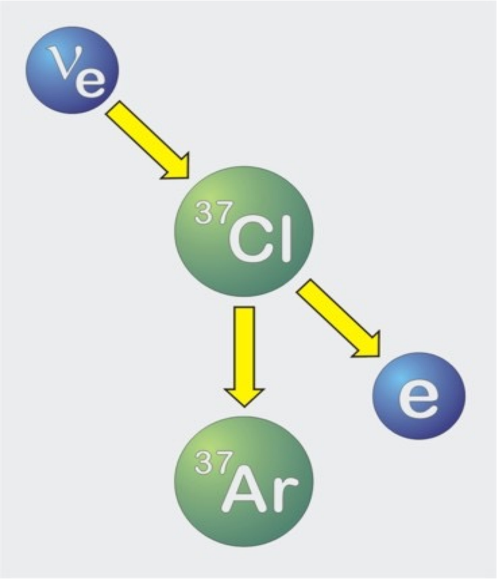
Raymond Davis Jr.

Seeing the Sun with neutrinos

1968: First observation of **solar neutrinos** in an **underground experiment** (Homestake gold mine, South Dakota)

615 Tn of C_2Cl_4

$$\nu_e + {}^{37}Cl \to {}^{37}Ar + e^-$$



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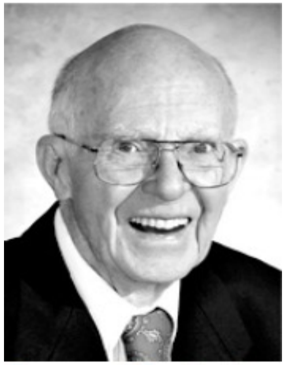

Raymond Davis Jr.

Nobel Prize 2002

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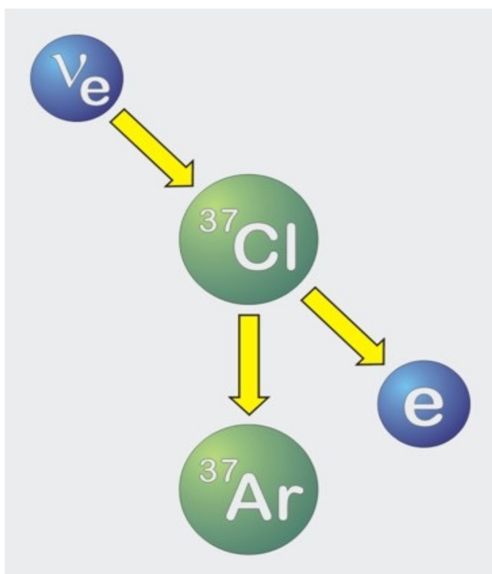
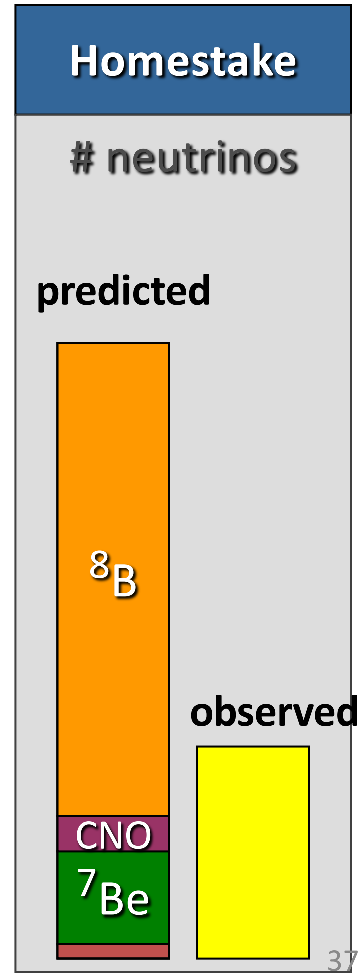


the Solar neutrino problem



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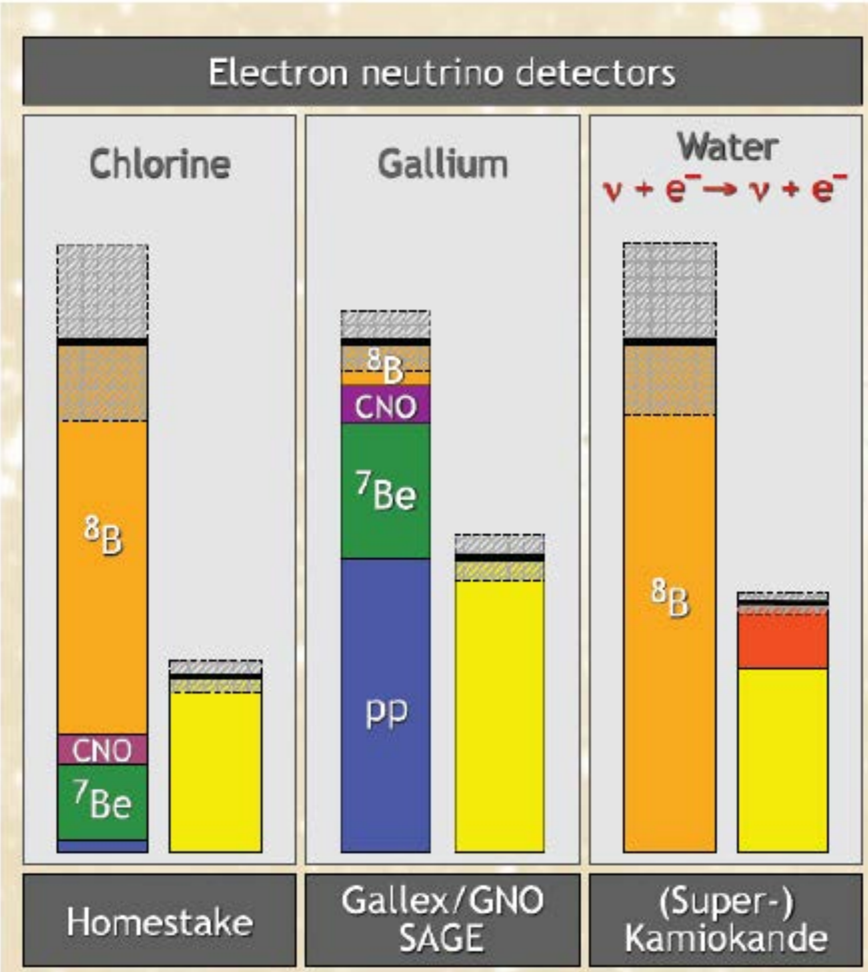


~30% ~50% ~40%

1980s-1990s: Confimed by the following solar neutrino experiments

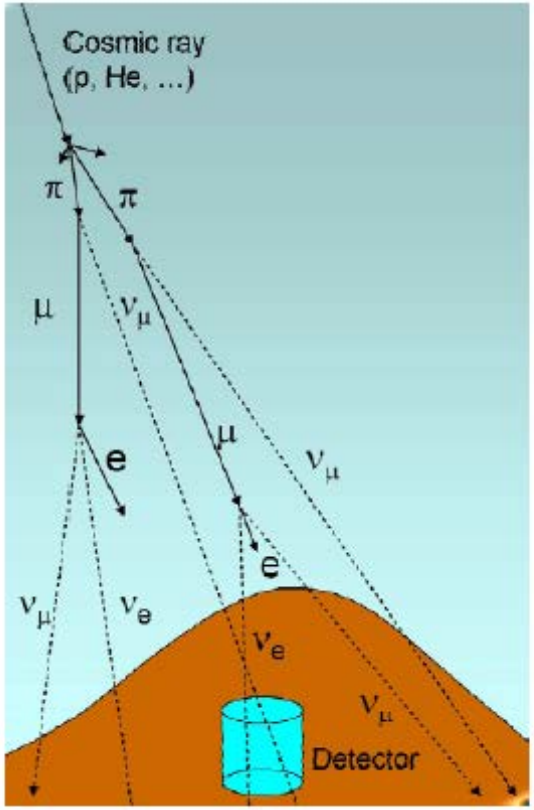
Explanation?

→ theory (SM, SSM) was wrong

→ experiments were wrong (all of them?)

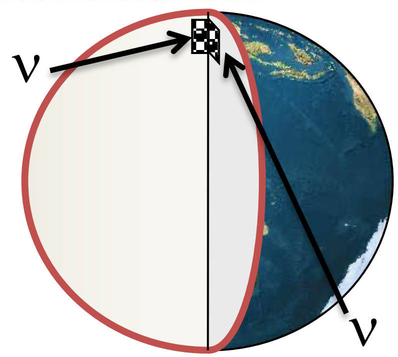
→ something was happening to neutrinos

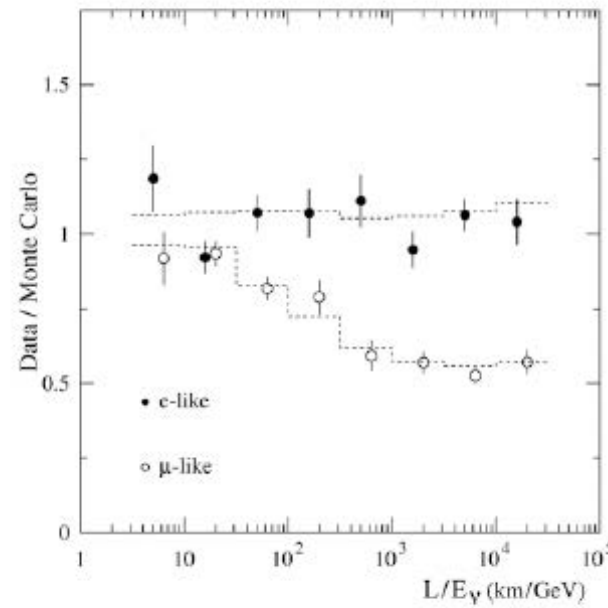
The atmospheric neutrino anomaly



1985: First indications of a deficit in the observed number of atmospheric ν_μ at the IMB experiment.

1994: Kamiokande finds the ν_μ deficit depends on the distance travelled by the neutrino and its energy.



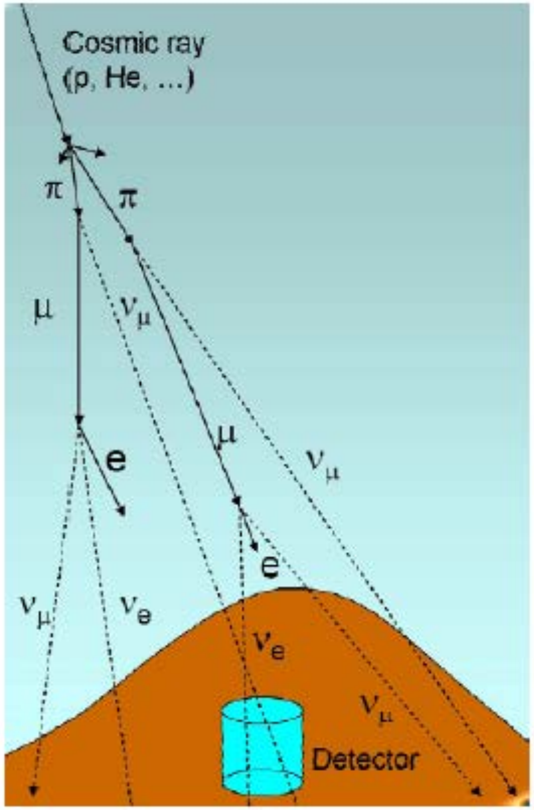


1998: Discovery of atmospheric neutrino oscillations in Super-Kamiokande.

oscillation channel $\nu_\mu \to \nu_\tau$

➡ first evidence for non-zero **neutrino masses.**

The atmospheric neutrino anomaly



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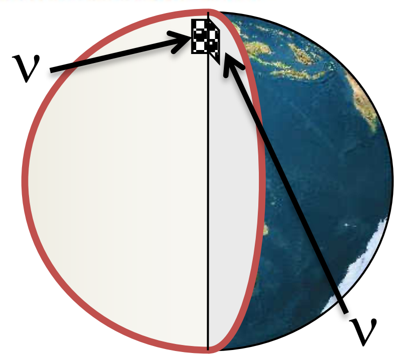

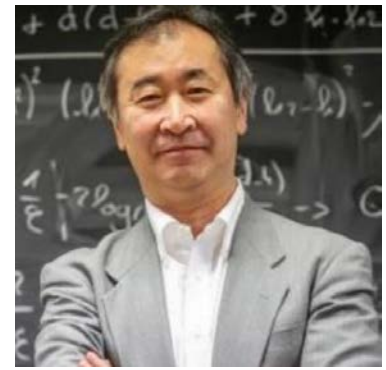

Takaaki Kajita

Nobel Prize in Physics 2015

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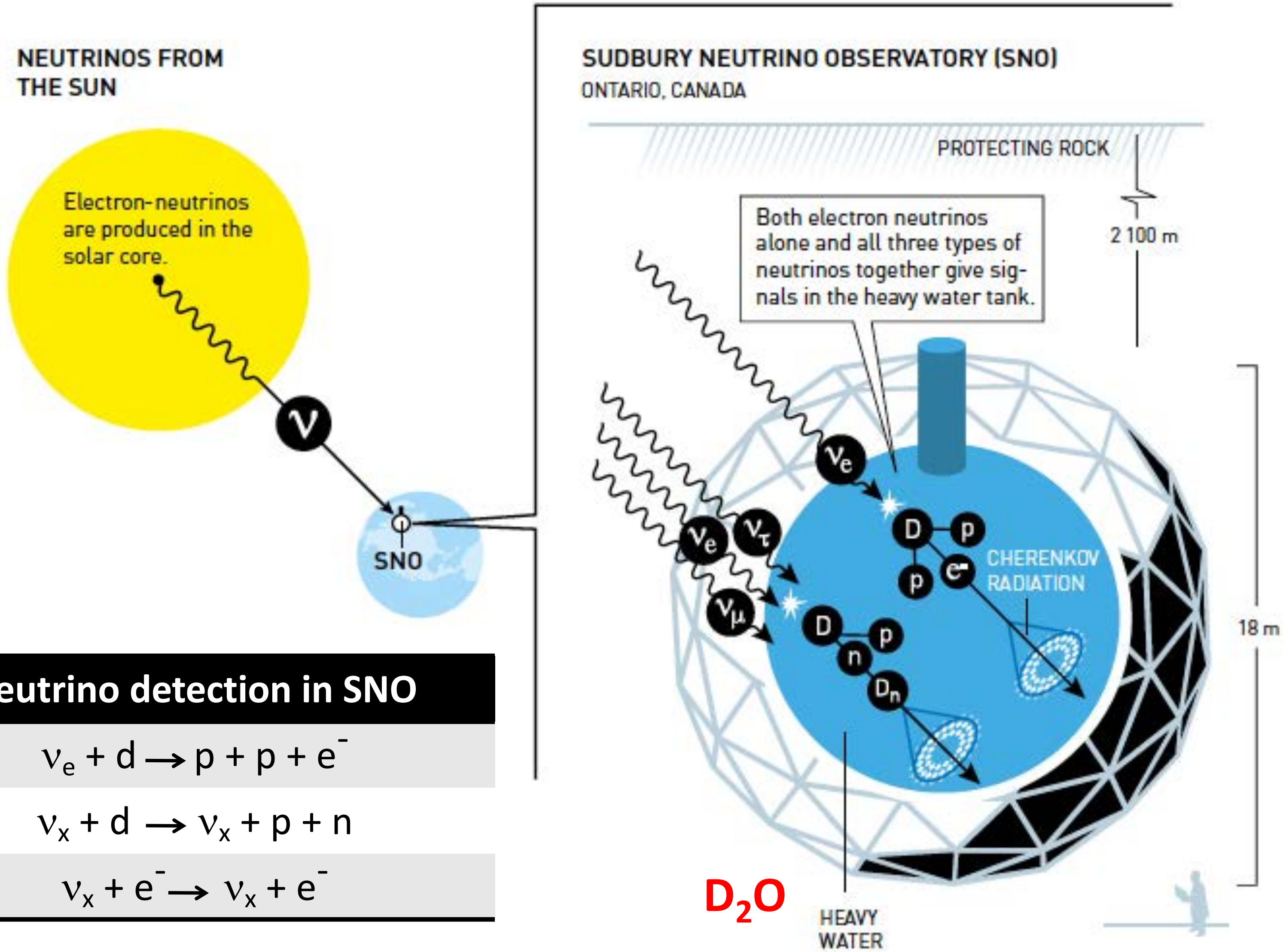
oscillation channel $\nu_\mu \rightarrow \nu_\tau$

➡ first evidence for non-zero **neutrino masses.**

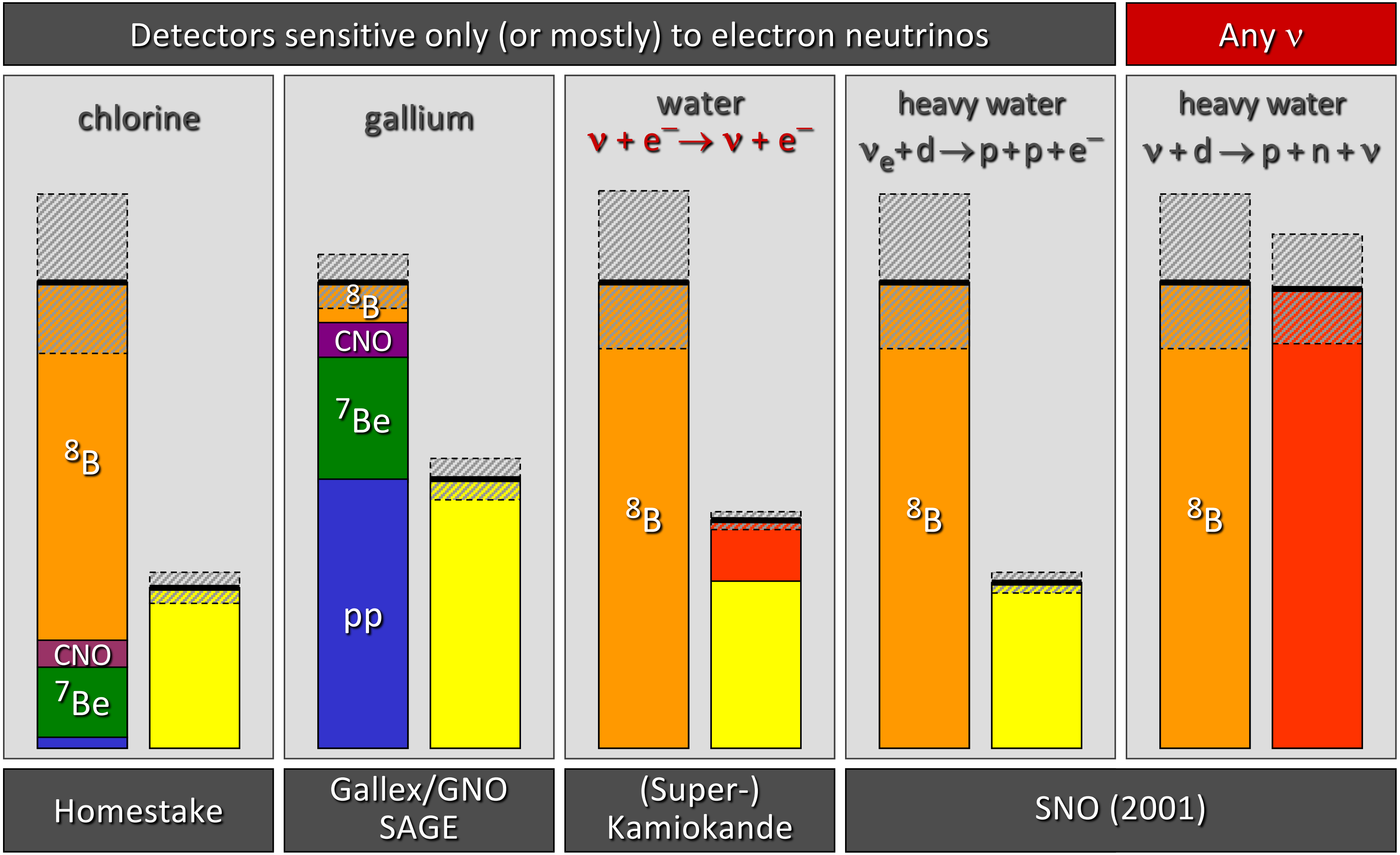
Sudbury neutrino observatory (SNO)

Arthur B. McDonald
Nobel Prize in Physics 2015



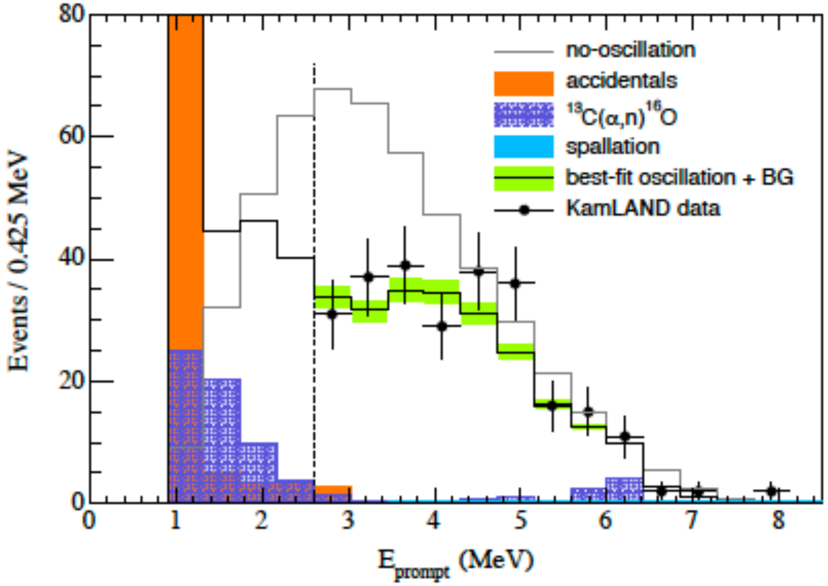
neutrino detection in SNO
$\nu_e + d \rightarrow p + p + e^-$
$\nu_x + d \rightarrow \nu_x + p + n$
$\nu_x + e^- \rightarrow \nu_x + e^-$

SNO: confirmation of solar neutrino conversion



Other important results on neutrino oscillations

2002: The reactor experiment KamLAND observed neutrino oscillations consistent with the solar anomaly.



KamLAND Coll, PRL 90 (2003) 021802

2002: Results of the accelerator experiment K2K consistent with ν_μ oscillations as in the atmospheric anomaly (MINOS, T2K, NOvA).

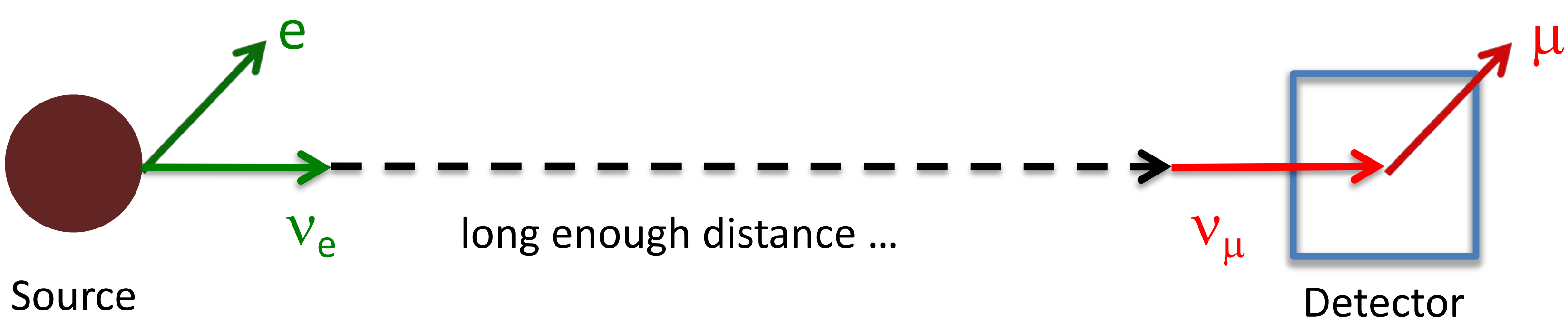
2011: $\nu_\mu \rightarrow \nu_e$ oscillations observed in long-baseline accelerator experiments.

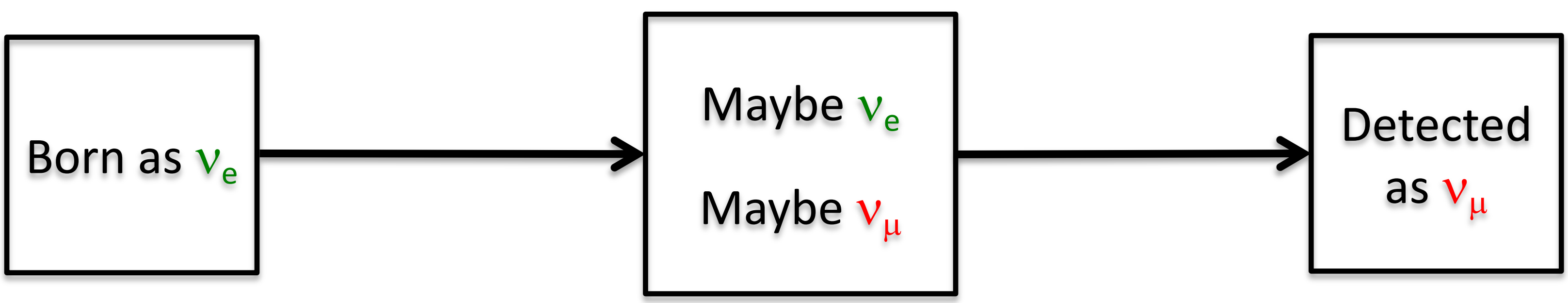
2011: Double Chooz confirmed reactor antineutrino oscillations in a baseline of ~1 km (Daya Bay, RENO).

3-neutrino oscillations have been observed in **solar**, **atmospheric**, **reactor** and **accelerator** neutrino experiments

Flavour neutrino conversions?

Experimental data can only be explained **if neutrinos change type** (**flavour**) during their propagation

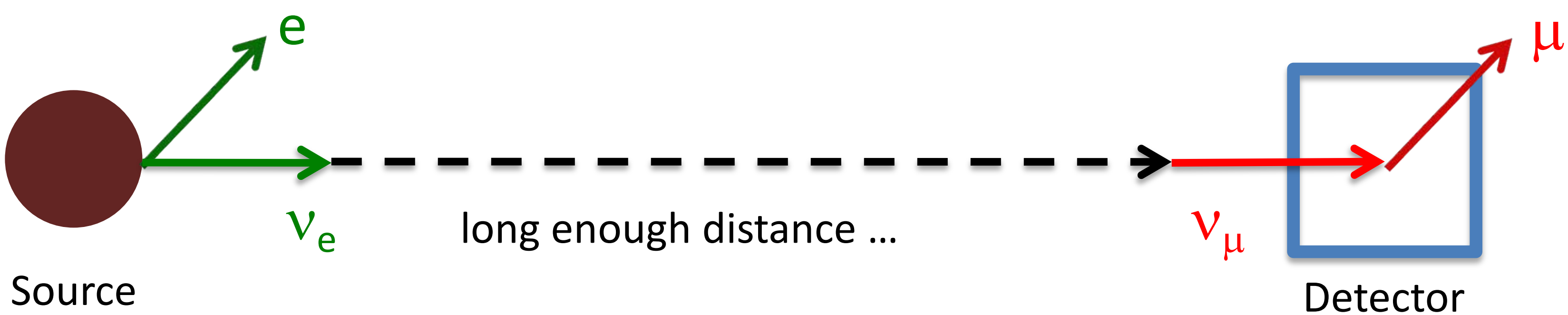


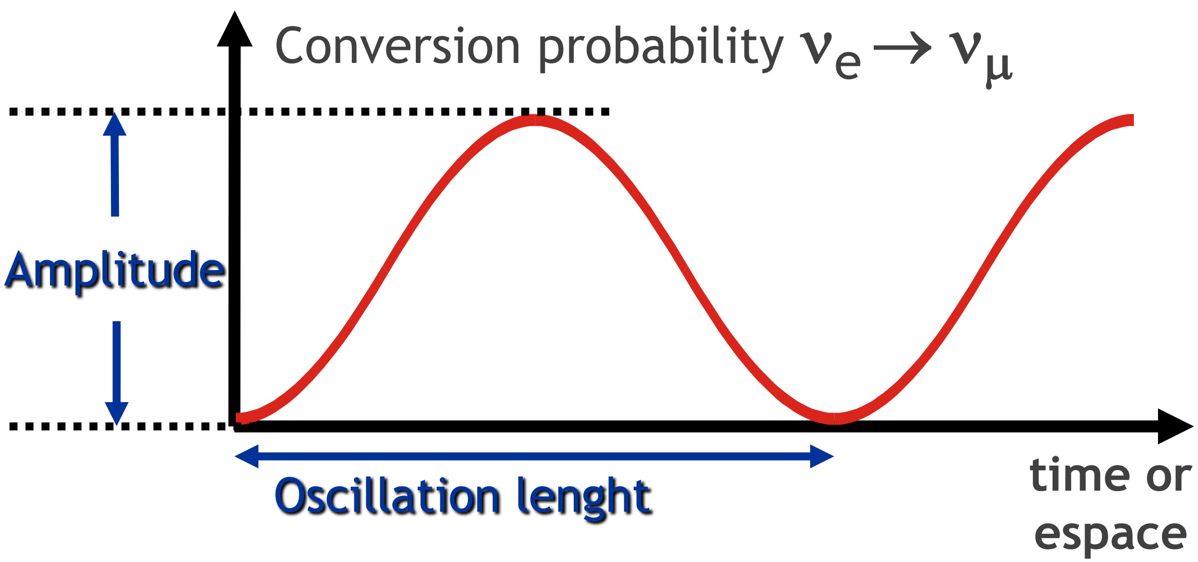


Flavour neutrinos are produced and detected, but mass eigenstates propagate (ν_1, ν_2)

Flavour neutrino conversions?

Experimental data can only be explained **if neutrinos change type** (**flavour**) during their propagation





Bruno Pontecorvo (1913–1993)

Neutrino mixing

- Mixing described by the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix:

$$\nu_{\alpha L} = \sum_k U_{\alpha k} \nu_{kL}$$

mixing matrix

- NxN unitary matrix: NxN real parameters

  → N(N-1)/2 mixing angles + N(N+1)/2 phases (not all observables!)

- Leptonic weak charged current:

$U = U_l^\dagger U_\nu$

$$j_\rho^{CC\dagger} = 2 \sum_\alpha \overline{\alpha_L} \gamma_\rho \nu_{\alpha L} = 2 \sum_\alpha \sum_k \overline{\alpha_L} \gamma_\rho U_{\alpha k} \nu_{kL}$$

- Lagrangian invariant under global phase transformations of Dirac fields:

$$\alpha \to e^{i\theta_\alpha} \alpha \,, \quad \nu_k \to e^{i\phi_k} \nu_k$$

$$j_\rho^{CC\dagger} \to 2 \sum_{\alpha,k} \overline{\alpha_L} e^{-i(\theta_\alpha - \phi_1)} \gamma_\rho U_{\alpha k} e^{i(\phi_k - \phi_1)} \nu_{kL}$$

N **N-1**

+ common rephasing of fields leaves current unchanged

2N-1 arbitrary phases can be eliminated from U: **(N-1)(N-2)/2 physical phases**

Neutrino mixing

✦ For **Majorana neutrinos**, the lagrangian is NOT invariant under global phase transformations of the Majorana fields:

$$\nu_k \to e^{i\phi_k}\nu_k \quad \Rightarrow \quad \nu_{kL}^T \mathcal{C}^\dagger \nu_{kL} \to e^{2i\phi_k}\nu_{kL}^T \mathcal{C}^\dagger \nu_{kL}$$

→ only N phases can be eliminated by rephasing charged lepton fields (neutrino fields can not be rephased!!):

$$j_\rho^{CC\dagger} \to 2\sum_{\alpha,k} \overline{\alpha_L} \underbrace{e^{-i\theta_\alpha}}_{N} \gamma_\rho U_{\alpha k}\nu_{kL}$$

N(N+1)/2 - N = N(N-1)/2 physical phases for Majorana neutrinos

→ N(N-1)/2 **physical phases:** (N-1)(N-2)/2 Dirac phases ← only these have an **effect on oscillations**

(N-1) Majorana phases

Neutrino mixing

- 2-neutrino mixing depends on 1 angle only (+1 Majorana phase)

$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

- 3-neutrino mixing is described by 3 angles and 1 Dirac (+2 Majorana) CP violating phases.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

atmospheric + LBL　　reactor + LBL　　solar + KamLAND

Neutrino oscillations

- Flavour states are admixtures of mass eigenstates: $\nu_{\alpha L} = \sum_k U_{\alpha k} \nu_{kL}$
- Neutrino evolution equation: $-i \frac{d}{dt} |\nu\rangle = H |\nu\rangle$

in the neutrino mass eigenstates basis ν_j :

$$H = \begin{pmatrix} E_i & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_3 \end{pmatrix}$$

→ neutrino mass eigenstates evolve as planes waves *:

$$|\nu_j(t)\rangle = e^{-iE_j t} |\nu_j\rangle$$

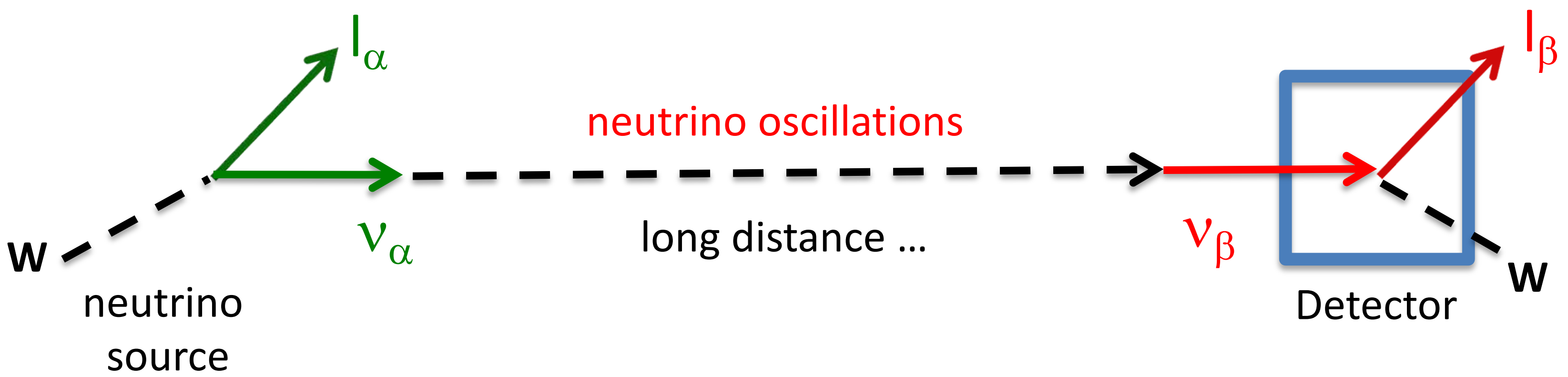
- For ultrarelativistic neutrinos: $E_j \simeq E + \frac{m_j^2}{2E}$

and t = L:

$$\rightarrow |\nu_j(t)\rangle = e^{-iEL} e^{-i\frac{m_j^2 L}{2E}} |\nu_j\rangle \rightarrow e^{-i\frac{m_j^2 L}{2E}} |\nu_j\rangle$$

* For a wave-packet treatment see: Giunti & Kim, Fundamentals of Neutrino Physics and Astrophysics. Oxford University Press, 2007

Neutrino oscillations picture



Production	Propagation	Detection
$\|\nu_\alpha\rangle = \sum_j U^*_{\alpha j}\|\nu_j\rangle$	$\nu_j : e^{-i\frac{m_j^2 L}{2E}}$	$\langle\nu_\beta\| = \sum_j \langle\nu_j\|U_{\beta j}$
coherent superposition of massive states	different propagation phases change ν_j composition	projection over flavour eigenstates

General properties of neutrino oscillations

- Conservation of probability: $$\sum_\beta P(\nu_\alpha \to \nu_\beta) = 1$$

- For **antineutrinos**: U →U*

- Neutrino oscillations violate flavour **lepton number conservation** but conserve total lepton number.

- Phases in the mixing matrix induce **CP violation**:

$$P(\nu_\alpha \to \nu_\beta) \neq P(\overline{\nu_\alpha} \to \overline{\nu_\beta})$$

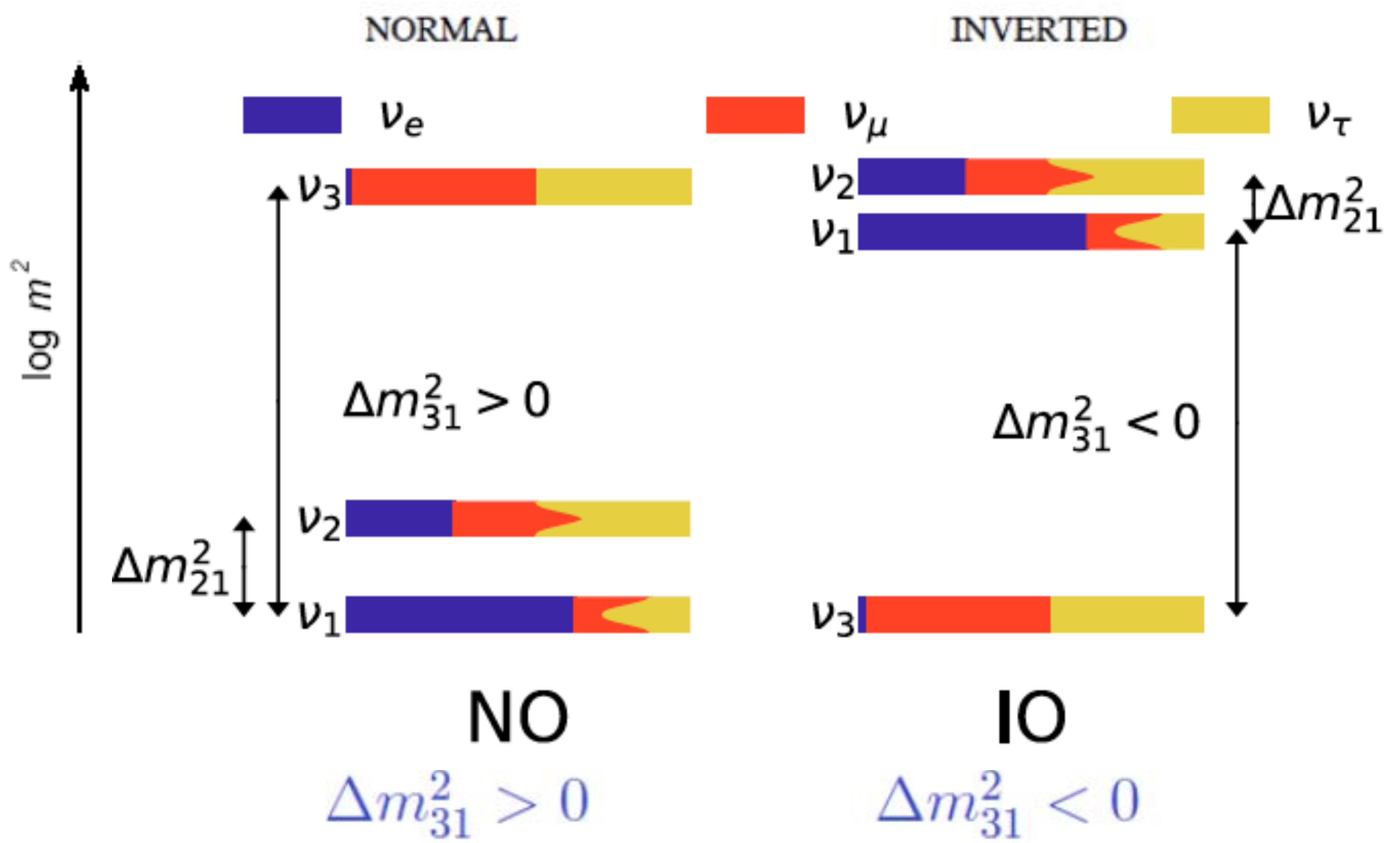
- Neutrino oscillations do not depend on the absolute neutrino mass scale and Majorana phases.

- Neutrino oscillations are sensitive only to **mass squared differences**:

$$\Delta m^2_{kj} = m^2_k - m^2_j$$

Two possible neutrino mass orderings

- Δm^2_{21} : solar + KamLAND (positive)
- Δm^2_{31} : atmospheric + LBL accelerator + SBL reactor (sign?)



2-neutrino oscillations

- Two-neutrino mixing matrix: $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

- Two-neutrino oscillation probability ($\alpha \neq \beta$):

$$P(\nu_\alpha \to \nu_\beta) = \left| U_{\alpha 1} U^*_{\beta 1} + U_{\alpha 2} U^*_{\beta 2} e^{-i \frac{\Delta m^2_{21} L}{2E}} \right|^2 = \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2_{21} L}{4E} \right)$$

- The **oscillation phase**:

$$\phi = \frac{\Delta m^2_{21} L}{4E} = 1.27 \frac{\Delta m^2_{21} [\mathrm{eV}^2] L[\mathrm{km}]}{E[GeV]}$$

→ short distances, $\phi << 1$: oscillations do not develop, $P_{\alpha\beta} = 0$

→ long distance, $\phi \sim 1$: oscillations are observable

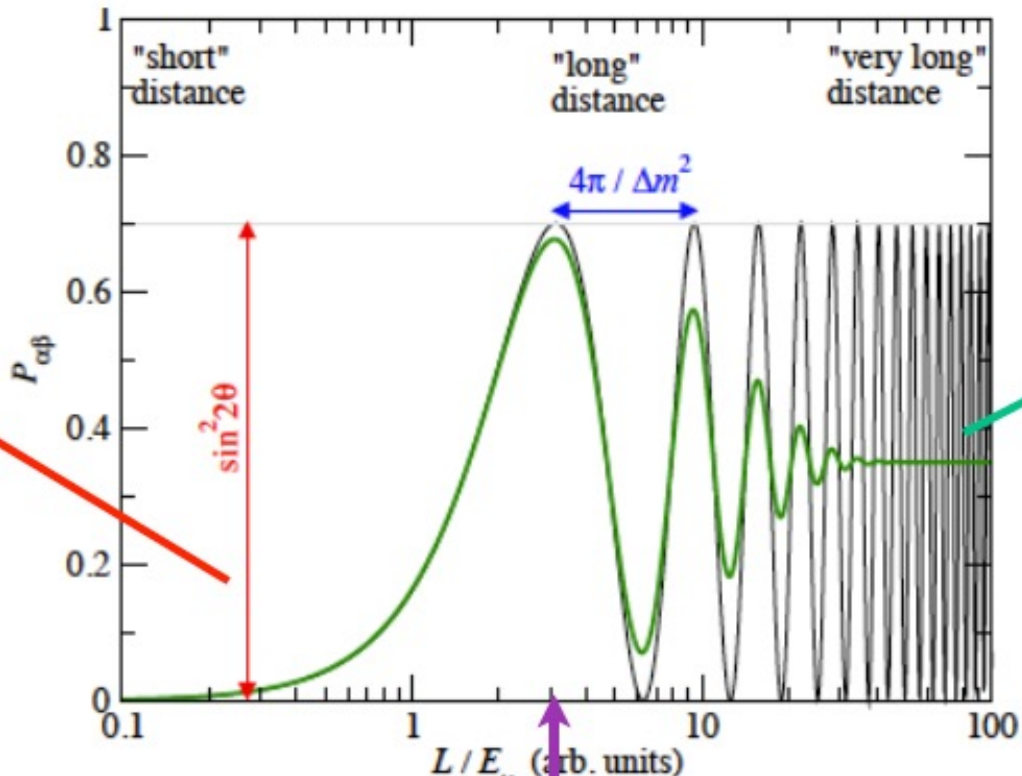
→ very long distances, $\phi >> 1$: oscillations are averaged out:

$$P_{\alpha\beta} \simeq \frac{1}{2} \sin^2 2\theta$$

2-neutrino oscillation probability

$$P_{\alpha\beta} = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

oscillation amplitude



averaged oscillations

oscillation length:

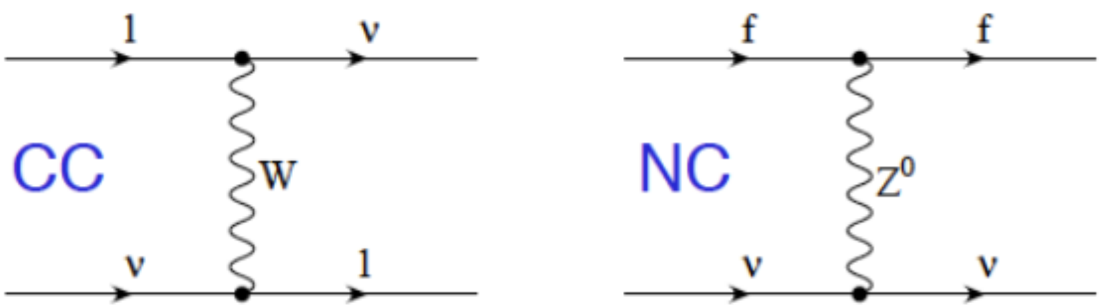
$$L_{\mathrm{osc}} = \frac{4\pi E}{\Delta m^2}$$

first oscillation maximum:

Matter effects on neutrino oscillations

✦When neutrinos pass trough matter, the interactions with the particles in the medium induce an **effective potential** for neutrinos.

[→ the coherent forward scattering amplitude leads to an index of refraction for neutrinos. **L. Wolfenstein, 1978**]



→ modifies the **mixing between flavor states and mass eigenstates** as well as the eigenvalues of the Hamiltonian, leading to a different oscillation probability with respect to vacuum oscillations.

End of 1st lecture