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**Exercises on Axions**  
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**Exercise 1.** Let us suppose that the Dark Matter's main component is an ultralight boson with mass  $m_b c^2 = 1\mu eV$ .

- (a) What is the de Broglie wavelength  $\lambda_{dB}$  of such a boson?
- (b) Estimate how many bosons one would find in a volume of  $\lambda_{dB}^3$ .

Consider that the virialised velocity of Dark Matter in the Milky Way is of the order  $10^{-3}c$ , and the local Dark Matter density  $\rho_{dm}$  is  $0.45 GeV/cm^3$ .

**Exercise 2.** The term in the Lagrangian describing the axion-photon interaction is

$$\mathcal{L}_{A\gamma} = \frac{g_{A\gamma}}{4} A F^{\mu\nu} \tilde{F}_{\mu\nu} \quad (1)$$

- (a) Derive the form of 2, as a function of  $\mathbf{E}$  and  $\mathbf{B}$

$$\mathcal{L}_{A\gamma} = g_{A\gamma} A \mathbf{E} \cdot \mathbf{B} \quad (2)$$

- (b) Does this term violate CP-symmetry? Explain.

**Exercise 3.** We are thinking of conducting a simple LSW experiment, using two magnets each of length  $L$  and strength  $B$ . The laser we plan to use will have an angular frequency of  $\omega$ . Up to what ALP mass do we maintain maximum sensitivity?

**Exercise 4.** The signal power  $P_{sig}$  of microwave cavity experiments looking for relic axions is determined by a combination of theoretical parameters of the axion physics (among which the axion mass  $m_a$ , the coupling constant  $g_{a\gamma\gamma}$ , the local Dark Matter density  $\rho_{DM}$ ) and experimental parameters of each experiment (cavity resonance frequency  $\omega_c$ , cavity volume  $V$  and mode  $C_{mnl}$ , external magnetic field strength  $B$ , quality factor  $Q$ ...).

The ADMX experiment, sensitive to DFSZ axions, presents a power of the order of  $P_{sig} \sim 10^{-23}W$ . If a Dark Matter axion with  $m_a = 3.3\mu eV$  with a DFSZ coupling exists, what is the average number of photons emitted from the cavity every second? Assume that if the cavity is tuned to the correct frequency.

**Exercise 5.** An axion helioscope uses a uniform magnetic field with a strength of  $B = 9T$  along a length  $L = 10m$ .

- (a) What axion masses is the helioscope sensitive to when the magnet bores are kept in vacuum?
- (b) In a second phase of the experiment, the vacuum is filled with gas at a very low pressure. What range of masses is the experiment sensitive to?
- (c) What are the values if the magnet is filled with He at a pressure of  $p = 5.49 mbar$ ?

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## Bibliography

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