TAE 2024 – Benasque

GAMMA-RAY TUTORIAL: EXERCISES

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PROBLEM 1: Gamma-ray fluxes and attenuation.
The gamma-ray luminosity at 100 GeV of an AGN located at redshift $z = 0.1$ (a luminosity distance of about 463 Mpc) is 3 10^{45} erg/s.

a) How many gamma rays of this energy will be detected by the Fermi Large Area Telescope (Fermi LAT) on board the NASA Fermi satellite in fifteen years of mission?

b) Would it be possible for Fermi LAT to detect this same object in the same exposure time and at the same energy if it were located at the so-called "cosmic gamma-ray horizon" ($\tau = 1$)?

Hint: Assume isotropic emission at the source. Use Fig. 1 for the Fermi-LAT collection area and differential point-source sensitivity. Assume that the AGN is located at galactic coordinates $(l, b) = (0.24°, 31.6°)$. Use Fig. 2 for the optical depth accounting for EBL attenuation.

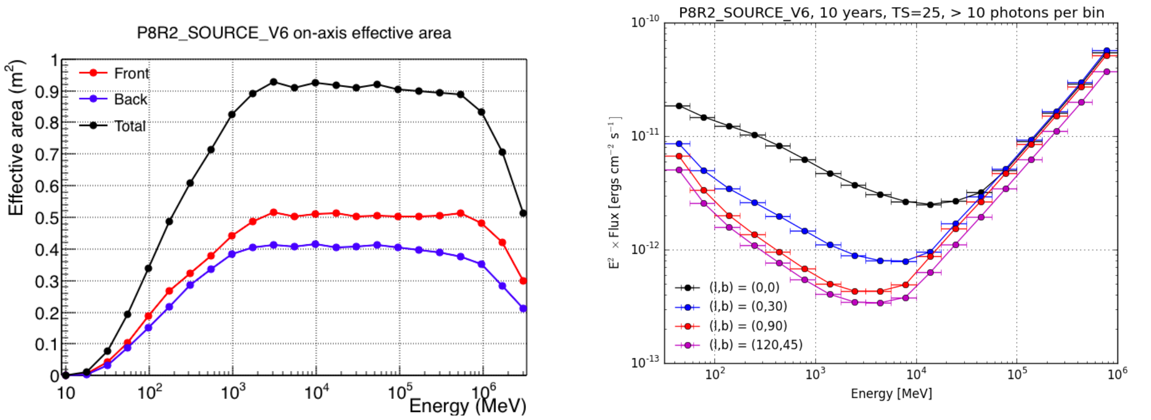

Figure 1: Left: Fermi-LAT collection area (take the black solid line as the reference one). Right: Fermi-LAT point-source differential sensitivity.

PROBLEM 2. Cherenkov light energy thresholds.
Extended air showers of particles in the atmosphere giving rise to Cherenkov radiation can be initiated not only by gamma rays (electromagnetic cascades) but also by charged nuclei of different kind (hadronic showers). Muons resulting from the decay of charged mesons are one of the possible final products of these hadronic showers. Thanks to a very distinct signal imprinted in the camera of Cherenkov telescopes, these muons can be used to calibrate our instruments.

i) What is the energy threshold for muons in order to produce Cherenkov light a) in the air, and b) in the water? Provide examples of both air Cherenkov and water Cherenkov telescopes currently in operation.

ii) Why there is basically not proton-induced backgrounds for air Cherenkov telescopes below ~ 40 GeV, while this is *potentially* an important source of background for water-based Cherenkov telescopes down to ~ 1 GeV?

Hints: The relation between Cherenkov angle θ, the relativistic velocity $\beta = v/c$ (with c being the speed of light), present in $E = \frac{m}{\sqrt{1-\beta^2}}$, and the refractive index, n, is given by $\cos\theta = \frac{1}{n\beta}$. $n_{air} = 1.000273$ and $n_{water} = 1.33$. The muon and proton masses are, respectively, $m_\mu \simeq 105$ MeV and $m_p \simeq 938$ MeV.

PROBLEM 3: IGRB for Fermi-LAT and CTA.

The so-called isotropic gamma-ray background (IGRB) is the sum of fluxes from all *unresolved* sources beyond our Galaxy.

ii) The IGRB differential flux spectrum has been measured by the Fermi LAT and, in the $100\ MeV < E_\gamma < 100\ GeV$ energy range, it is well described by $dN/dE = 1.19 \cdot 10^{-6}\ (E/100\ MeV)^{-2.32}\ MeV^{-1}\ cm^{-2}\ s^{-1}$ [1]. Calculate the total IGRB flux expected above 1 GeV, $F_{IGRB}(E > 1 GeV)$.

iii) It has been calculated [2] that unresolved blazars contribute only 20% to the IGRB flux in this same $100\ MeV < E_\gamma < 100\ GeV$ energy range. Should we expect to detect the blazar contribution to the IGRB with the future CTA-North *at* 1 TeV? Why?

Hint: Use Fig. 2 for the optical depth and CTA differential sensitivity. For simplification, assume that the bulk of this unresolved emission is produced by blazars located at $z = 0.1$ exhibiting a differential spectrum well described by $dN/dE \propto E^{-2.4}$ at low energies where the EBL is not relevant [2].

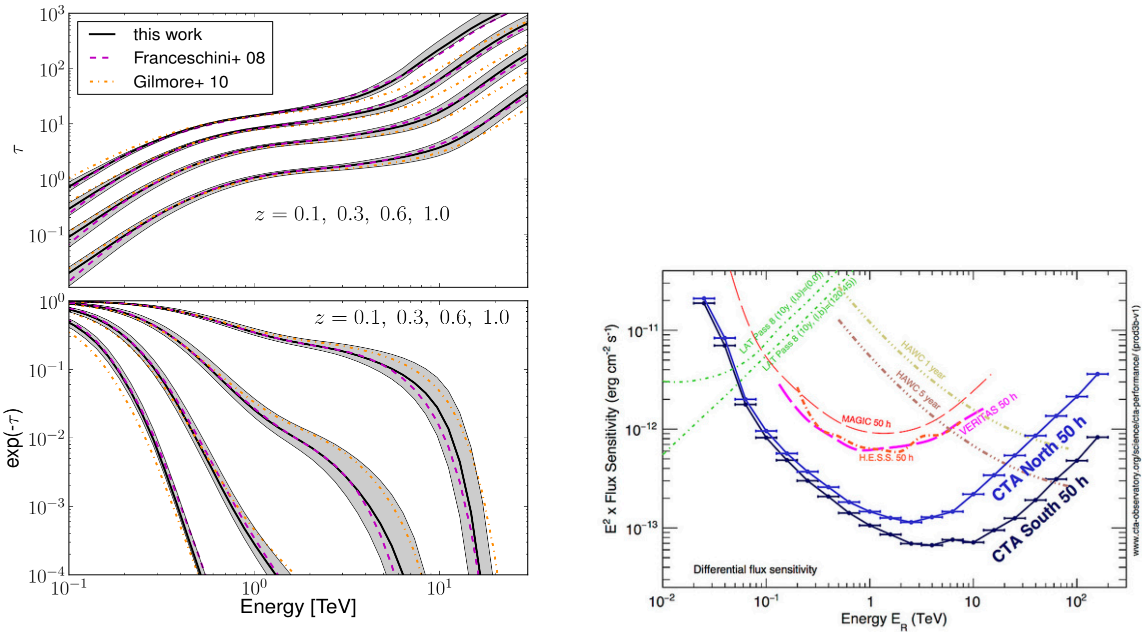

Figure 2: Left: Optical depths as given by the model of Ref. [3] for different redshifts (from top to bottom, curves increase in redshift). Right: Predicted CTA differential sensitivity.

PROBLEM 4. J-factors for Fermi LAT and IACTs.

For a given dark matter halo, the *total* J-factor, i.e. the one obtained for the *whole* object, can be computed as follows:

$$J_T = \frac{1}{4\pi D^2}\int_V \rho^2_{DM}(r)\, dV = \frac{1}{4\pi D^2}\frac{M\, c_{200}(M_{200})^3}{[f(c_{200}(M_{200}))]^2}\frac{200\,\rho_{crit}}{9}\left(1-\frac{1}{(1+c_{200}(M_{200}))^3}\right)\ , \quad (1)$$

with D the distance from the Earth to the center of the halo, r is the galactocentric distance inside it, $f(x) = \ln(1+x) - x/(1+x)$ and $\rho_{crit} = 275.027\ h^2\ M_\odot/kpc^3$ is the critical density of the Universe. The above expression assumes NFW density profiles [4] for the dark matter distribution inside the halo. The so-called halo concentration-mass relation is given by [5]:

$$c_{200}(M_{200}, z=0) = \sum_{i=0}^{5} c_i \times \left[\ln\left(\frac{M_{200}}{h^{-1}M_\odot}\right)\right]^i , \quad (2)$$

where $c_i = [37.5153, -1.5093, 1.636\cdot 10^{-2}, 3.66\cdot 10^{-4}, -2.89237\cdot 10^{-5}, 5.32\cdot 10^{-7}]$. Adopt $h = 0.7$.

i) Compute J_T (in units of $\mathrm{GeV}^2\ \mathrm{cm}^{-5}$ and $\mathrm{M}_\odot\ \mathrm{kpc}^{-5}$) for the Draco dwarf galaxy, the Andromeda galaxy and the Virgo galaxy cluster, assuming that the dark matter density profile is well described by NFW in all cases. Rank the objects according to their annihilation fluxes. Take the following values as the masses and distances of these objects:

Target	Distance (kpc)	M_{200} ($\mathrm{M}_\odot$)
Draco	82	$2\cdot 10^8$
Andromeda	778	$1.5\cdot 10^{12}$
Virgo	$15.4 \cdot 10^3$	$5.4\cdot 10^{14}$

ii) In terms of level of annihilation flux of these objects at Earth, how would the ranking be modified by including subhalos in the above computation? Adopt the following parametrization for the so-called substructure boost, B, that modifies the overall flux as $F_{total} = F_{no-subhalos} \times (1+B)$ [6]:

$$\log_{10} B(M) = \sum_{i=0}^{5} b_i \left[\log_{10}\left(\frac{M_{200}}{M_\odot}\right)\right]^i . \quad (3)$$

where b_i =[-0.186, 0.144, $-8.8\cdot 10^{-3}$, $1.13\cdot 10^{-3}$, $-3.7\cdot 10^{-5}$ $-2\cdot 10^{-7}$].

iii) Calculate the minimum value of the J-factor that would be ideally needed in order to have a detection. Do it for both Fermi LAT and a typical Cherenkov telescope. Use the following numbers as nominal values for Fermi LAT and IACTs:

Parameter	Fermi LAT	IACTs
Energy range	0.1-300 GeV	0.1-10 TeV
Effective area, A_{eff}	~ 1 m^2	$\sim 10^5$ m^2
Observing time, T_{obs}	$\sim 10^8$ s	$\sim 10^6$ s

Hint: The annihilation flux is $\frac{d\phi_\gamma}{dE} = J_T \times \phi_{PP}$, where $\phi_{PP} = \frac{<\sigma v>}{2m_\chi^2}\frac{dN_\gamma}{dE_\gamma}$ is the particle physics factor. Assume $\int dE\ \frac{dN_\gamma}{dE_\gamma} \sim \frac{m_\chi}{GeV}$, where m_χ is the dark matter particle mass. Adopt a WIMP mass of 300 GeV that annihilates entirely to $b\bar{b}$ with the thermal relic cross section value, i.e. $3\times 10^{-26} cm^2\ s^{-1}$. A minimum of 10 photons is needed for detection.

iv) At least a priori, which object would be more suitable for its observation with Cherenkov telescopes like MAGIC, and why?

Hint: The actual angular extension of the annihilation signal is well approximated by the angular size subtended by the object scale radius. The analysis of extended sources is not trivial for IACTs, as their FoV is $\sim 4°$.

PROBLEM 5. The Galactic center excess interpreted as dark matter annihilation. The gamma-ray flux corresponding to the Galactic center excess (GCE) as observed by the Fermi LAT between 1-3 GeV, integrated within the innermost one degree of the Galaxy, is $\phi_{GCE} \sim 10^{-10}$ erg cm^{-2} s^{-1}. Its spectrum is compatible with a WIMP mass $m_\chi = 49$ GeV annihilating to b quarks with roughly half of the thermal relic cross section value [7].

i) Calculate the corresponding J-factor necessary to account for the observed GCE flux.
Hint: The annihilation flux is $\frac{d\phi_\gamma}{dE} = J \times \phi_{PP}$, where $\phi_{PP} = \frac{<\sigma v>}{2m_\chi^2} \frac{dN_\gamma}{dE_\gamma}$ is the particle physics factor. Assume as a reasonably good approximation that $\int dE \, \frac{dN_\gamma}{dE_\gamma} \sim \frac{m_\chi}{GeV}$.

ii) Is this value of the J-factor compatible with the one expected from our Galaxy integrated for that same inner region?
Hint: The thermal relic cross section is $3 \times 10^{-26} cm^2 \ s^{-1}$. Assume an NFW profile with a normalization at the Solar galactocentric radius $\rho(r = r_\odot = 8 \ kpc) = 0.4 \ GeV/cm^3 = 1.0477 \times 10^7 M_\odot/kpc^3$ and a scale radius $r_s = 20 \ kpc$. The NFW density profile is the most widely used in the literature [4]:

$$\rho(r) = \frac{\rho_0}{\left(\frac{r}{r_s}\right)\left[1 + \left(\frac{r}{r_s}\right)\right]^2}, \tag{4}$$

where ρ_0 and r_s represent a characteristic density and a scale radius, respectively. As a good approximation, one can use that the J-factor for the GCE region is $J_{GCE} = \frac{1}{4\pi D^2} \int_V \rho_{DM}^2(r) \, dV$, where the volume integral is performed up to the physical radius that corresponds to the GCE region.

iii) One of the most promising dwarf candidates recently discovered by the Dark Energy Survey (DES) is Reticulum II. Assuming a dark matter origin for the GCE (with the properties given above), should we expect to observe a dark matter-induced gamma-ray signal from Reticulum II with the Fermi LAT?
Hint: Assume that this object is located at 32 kpc and its current total mass is $M_{200} \sim 10^7 \ M_\odot$. Its concentration, significantly boosted due to tidal forces, is $c_{200} \sim 40$ [6]. The Fermi LAT sensitivity at 1 GeV is $F_{min} \sim 10^{-6} \ MeV \ cm^{-2} \ s^{-1}$. As an approximation (the object is a dwarf so NFW is formally not applicable) one can use the following expression for the *total* J-factor, i.e. the one obtained for the *whole* object:

$$J_T = \frac{1}{4\pi D^2} \int_V \rho_{DM}^2(r) \, dV = \frac{1}{4\pi D^2} \frac{M_{200} \, c_{200}(M_{200})^3}{[f(c_{200}(M_{200}))]^2} \frac{200 \, \rho_{crit}}{9} \left(1 - \frac{1}{(1 + c_{200}(M_{200}))^3}\right), \tag{5}$$

with D the distance from the Earth to the center of the halo, r is the galactocentric distance inside it, M_{200} is in solar masses, $f(x) = \ln(1+x) - x/(1+x)$, $c_{200}(M_{200})$ refers to the concentration-mass model (use, e.g., the one in equation 2), and ρ_{crit} is the critical density of the Universe, i.e. $\rho_{crit}(z = 0) = 1.05 \times 10^{-5} \ h^2 \ GeV/cm^3 = 275.027 \ h^2 \ M_\odot/kpc^3$.

References

[1] M. Ackermann et al., ApJ, **799** (2015) 86 [astro-ph/1410.3696].

[2] M. Ajello et al., ApJL, **800** (2015) 27 [astro-ph/1501.05301].

[3] A. Domínguez et al., MNRAS, **410** (2011) 2556 [astro-ph/1007.1459].

[4] J. F. Navarro, C. S. Frenk and S. D. M. White,

[5] Sánchez-Conde M. A., Prada F., 2014, MNRAS, 442, 2271

[6] Moliné Á., Sánchez-Conde M. A., Palomares-Ruiz S., Prada F., 2017, MNRAS, 466, 4974

[7] Calore F., Cholis I., McCabe C., Weniger C., 2015, PhRvD, 91, 063003