Cosmology tutorial - TAE 2024

1 Growth of matter: CDM and baryons

a) Show the evolution of the gravitational potential for a matter-dominated Universe, a radiation-dominated Universe and a dark-energy dominated Universe. b) Show that the coupled ODE of baryons and cold dark matter are:

$$\ddot{\delta_b} + \frac{4}{3t}\dot{\delta}_b = 4\pi G(\bar{\rho}_b\delta_b + \bar{\rho}_c\delta_c) \quad (1)$$

$$\ddot{\delta_c} + \frac{4}{3t}\dot{\delta}_c = 4\pi G(\bar{\rho}_b\delta_b + \bar{\rho}_c\delta_c) \quad (2)$$

c) Show that $\delta_b \to \delta_c$ in the late Universe and that we can consider a single presureless fluid with $\delta_m t$

2 Growth of matter structure

a) What is the evolution of dark matter perturbations for scales above the Jeans scale but below the Hubble radius b) Show that the equation above can be written as:

$$\frac{d}{da}\left(a^3 H \frac{d\delta_m}{da}\right) = 4\pi G\rho_{m_0}\frac{\delta_m}{Ha^2} \quad (3)$$

c) For a Universe with a mix of matter, curvature and a cosmological constant show that the solutions are:

$$\delta_m \propto \begin{cases} H \\ H\int \frac{da}{(aH)^3} \end{cases} \quad (4)$$

and show what happens for the different fluid domination eras. d) For a Universe dominated by radiation and presureless matter write the Hubble parameter $H(y)$ in terms of $y = a/a_{eq}$. e) Verify that the solutions of:

$$\frac{d^2\delta_m}{dy^2} + \frac{2+3y}{2y(y+1)}\frac{d\delta_m}{dy} = \frac{3}{2}\frac{\delta_m}{y(y+1)} \quad (5)$$

are:

$$\delta_m \propto \begin{cases} 1+\frac{3}{2}y \\ \left(1+\frac{3}{2}y\right)\ln\left(\frac{\sqrt{1+y}-1}{\sqrt{1+y}+1}\right)x - 3\sqrt{1+y} \end{cases} \quad (6)$$

and show the growing and decaying modes are early and late times.

3 Power spectrum

a) Show that the correlation function is $\xi(r) = \int \frac{dk}{k} \frac{k^3}{2\pi^2} P(k) \frac{\sin kr}{kr}$. Hint: $\langle \delta_k \delta_k'^* \rangle = (2\pi)^3 \delta_D(\mathbf{k} - \mathbf{k}') P(k)$
b) Find the fourier transform of a top-hat filter.
c) Show that the Zeldovich-Harrison power spectrum $P(k) \propto k^{n_s}$ is scale invariant when $n_s = 1$

4 Cosmological tools

Go to TAE-2024-cosmo and use the notebooks to run cobaya