Universidad Zaragoza 1542

Cosmology

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IPARCOS

CENTRO DE CIENCIAS DE BENASQUE PEDRO PASCUAL

Introduction

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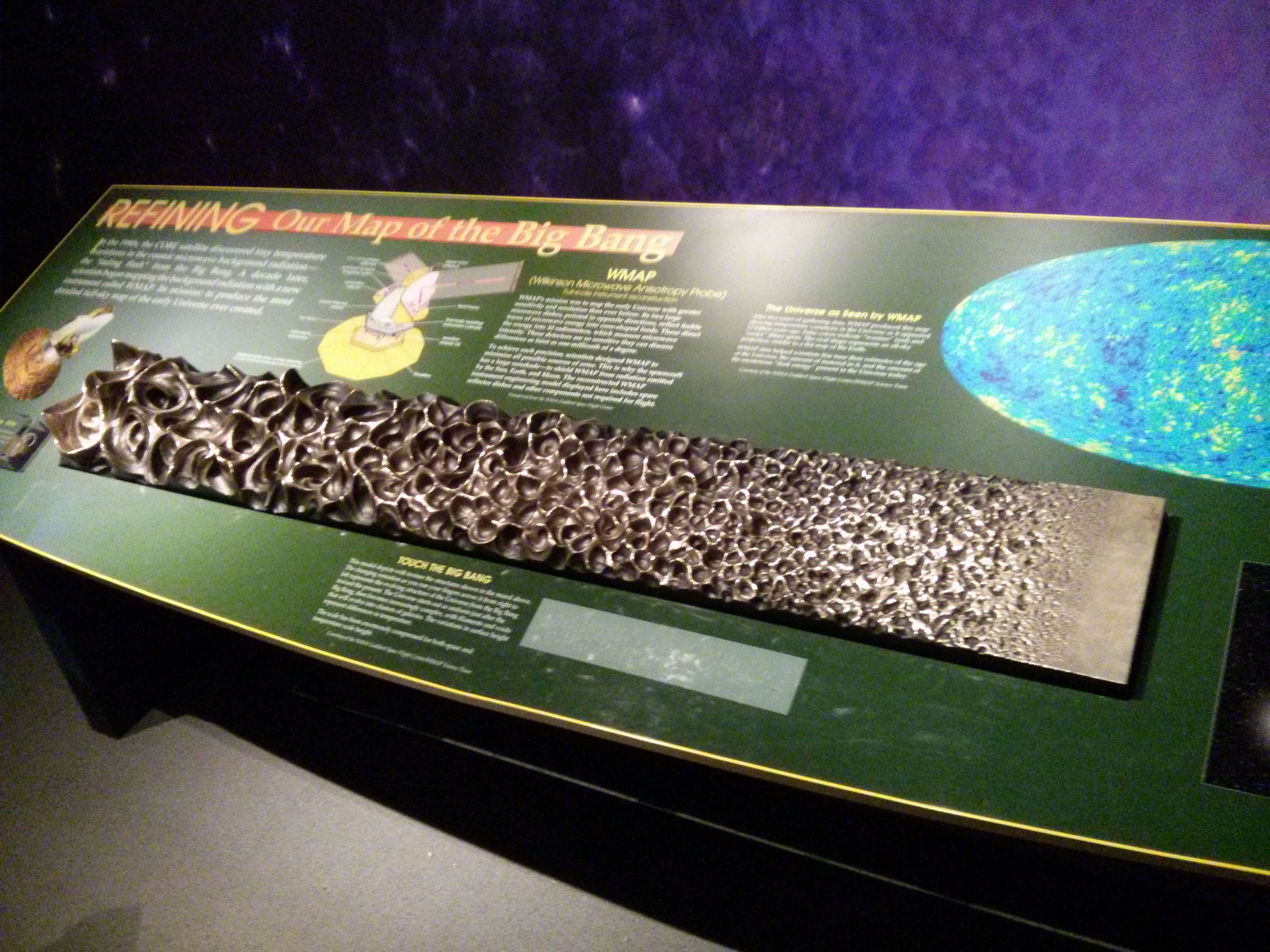
4 hours of theory + 1tutorial

Some bibliography:

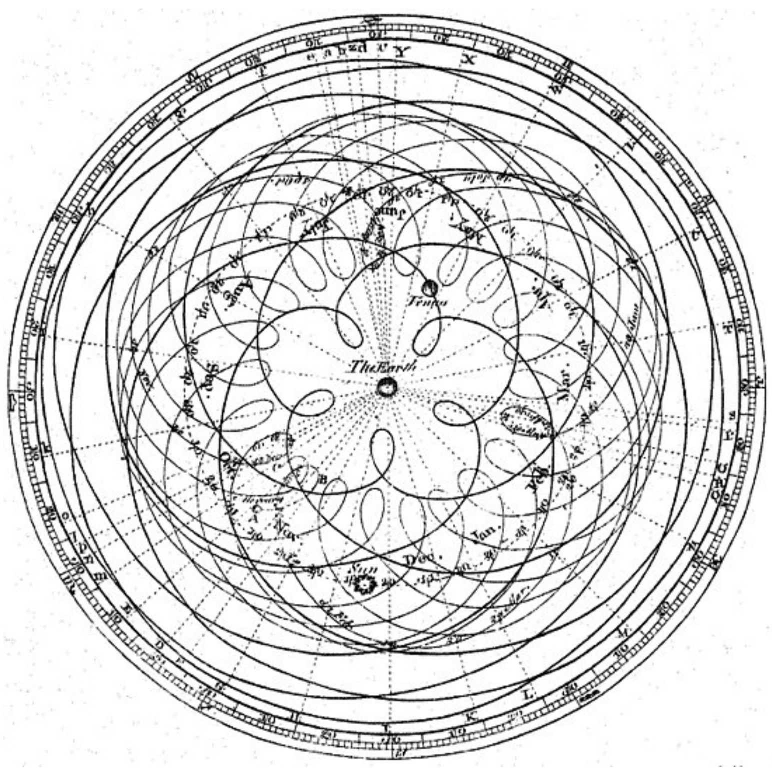
- *Modern Cosmology*, S. Dodelson
- *A course in Cosmology*, D. Huterer
- *Cosmology*, D. Baumann
- *Cosmological Physics,* J. Peacock
- *Physical foundations of Cosmology,* V. Mukhanov

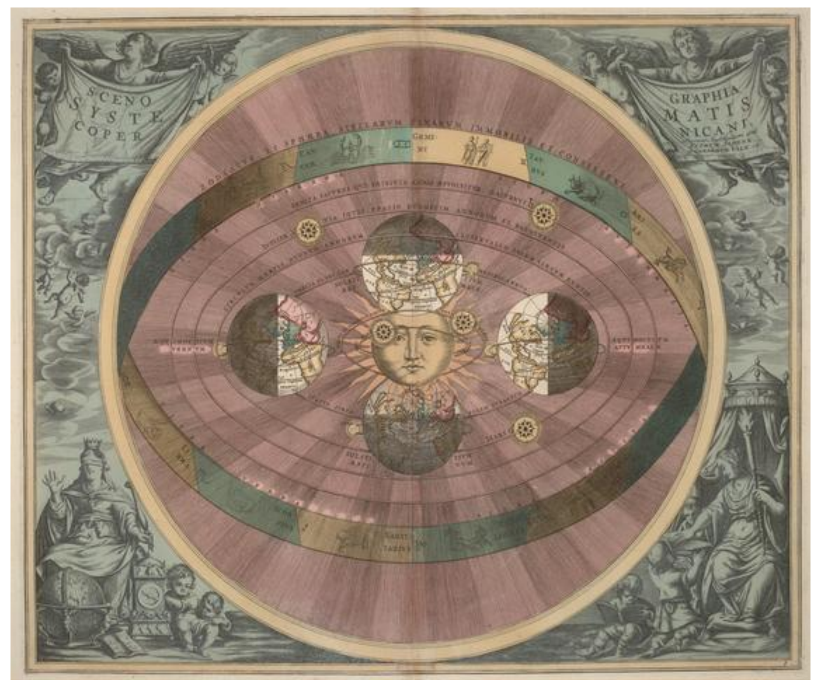

REFINING Our Map of the Big Bang
WMAP
(Wilkinson Microwave Anisotropy Probe)
The Universe as Seen by WMAP
TOUCH THE BIG BANG

Contents

Introduction

Cosmology: Study and explain the Universe as a whole (long history)

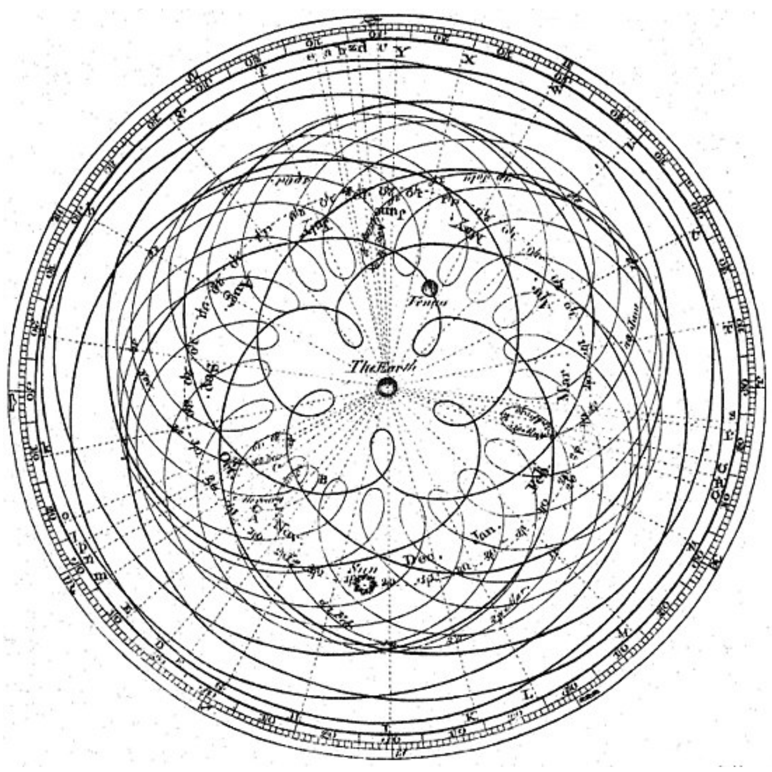


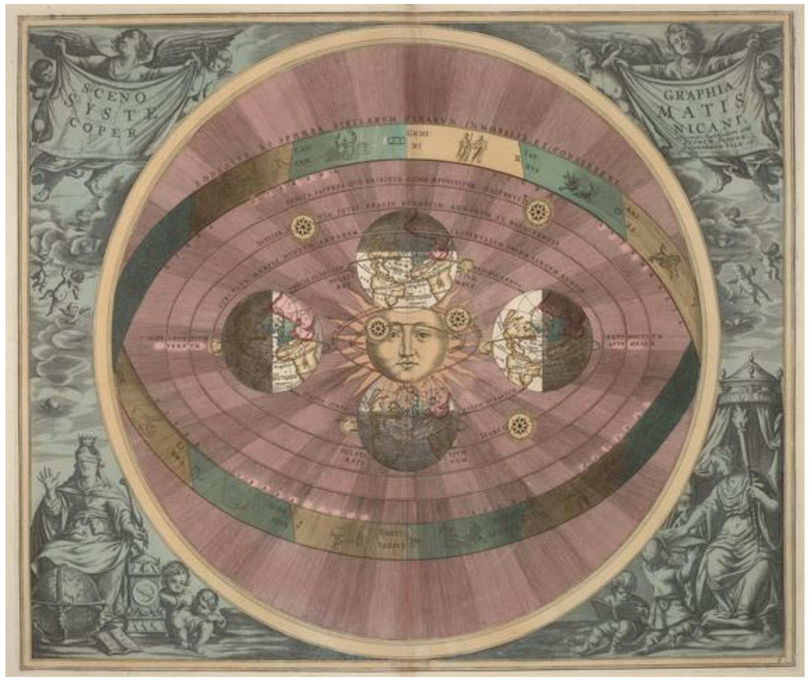


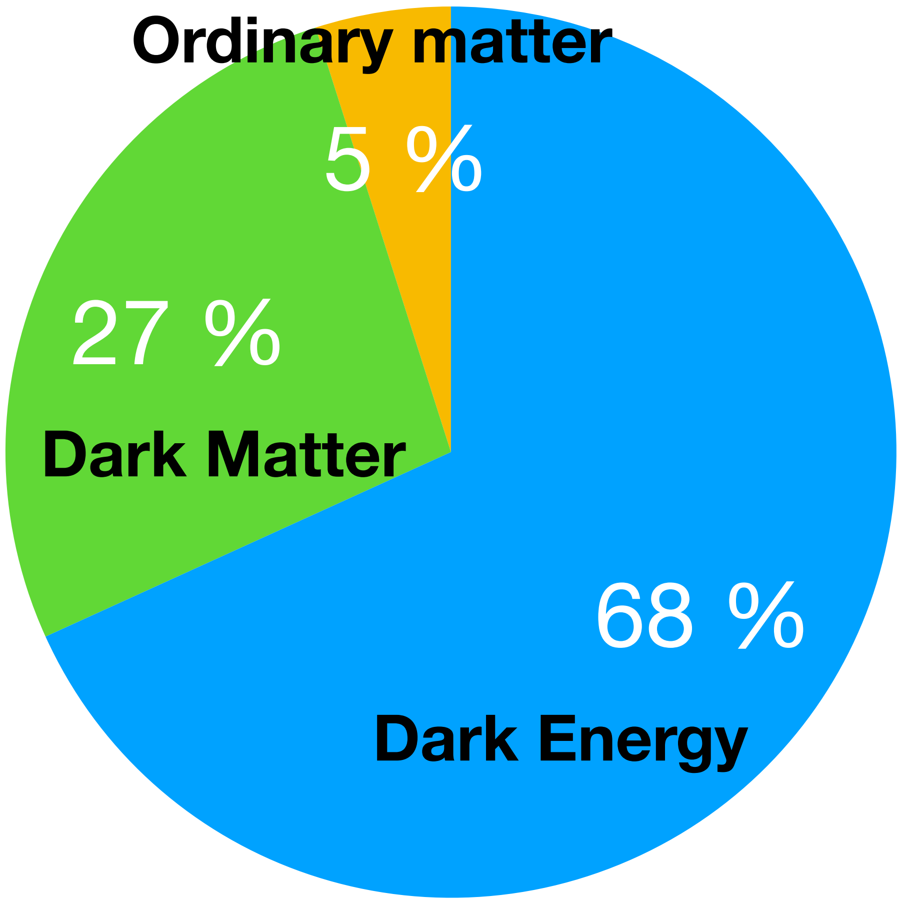
How did we get to the current standard model of cosmology?

Introduction

Cosmology: Study and explain the Universe as a whole (long history)

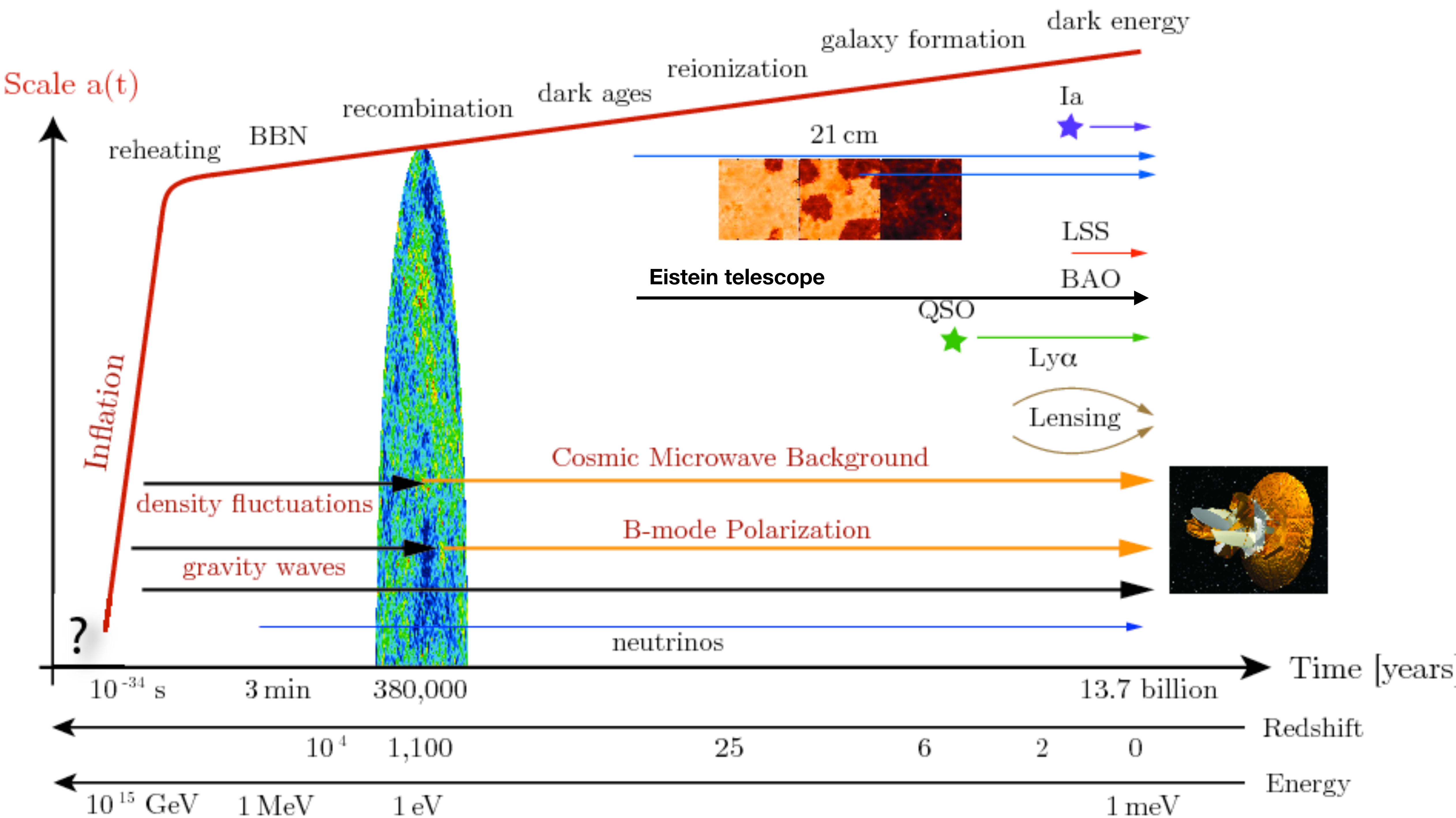






How did we get to the current standard model of cosmology?

Timeline



D. Baumann, 2009, arXiv:0907.5424

Modern Cosmology

The current standard model of cosmology is based on 3 premises:

1) Gravitation is described by **general relativity**

2) The background metric of the Universe is described by the **cosmological principle**

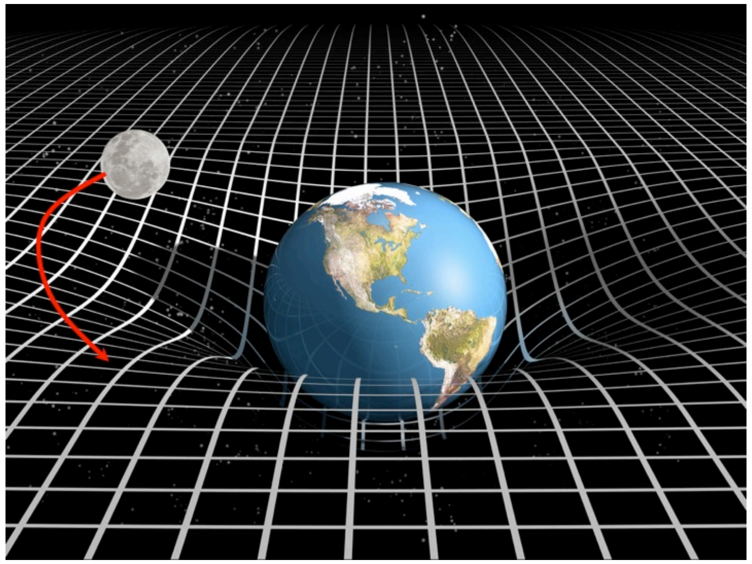
3) Before the hot big bang phase, the Universe undertook an accelerated expansion phase called **Cosmological Inflation**.

1) General relativity

Gravitation defined by Einstein's general relativity. Therefore, Einstein equations:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

We need to define the metric and the energy content of the Universe in order to proceed



2) Cosmological principle: FLRW metric

If we assume that at large-scales (> 100 Mpc) the Universe is:

- **Isotropic:** properties are the same in every direction
- **Homogeneous:** isotropic at every point.

Then we derive the Friedman-Lemaître-Robertson-Walker metric

$$ds^2 = c^2 dt^2 - a^2(t) \left[d\chi^2 + S_k^2(\chi)\, d^2\Omega \right].$$

$$S_k(\chi) = \begin{cases} \sin\chi & \text{si } k > 0 \ (k = 1) \\ \chi & \text{si } k = 0 \\ \sinh\chi & \text{si } k < 0 \ (k = -1) \end{cases}$$

a(t): scale factor; χ: comoving distance;

Friedmann equations

From FLRW metric we derive the Friedmann equations:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3}$$

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}$$

G: Newton constant

ρ: energy density

P: pressure

Λ: cosmological constant

H: Hubble parameter

We need to describe each fluid contributing to the energy-momentum tensor. Each fluid has a particular equation of state: $p = w\rho$

$$T_{\mu\nu} = (\rho + p/c^2)u_\mu u_\nu + p g_{\mu\nu}$$

Energy-momentum tensor of a perfect fluid

Expansion of the Universe

We define the critical density for the k=0 case and without cosmological constant, then we can describe three different curvature cases:

$$\rho_{crit} = \frac{3H^2}{8\pi G} \qquad \Omega_X = \frac{\rho_X}{\rho_{crit}}$$

$$H^2 = H_0^2 \left[\Omega_m \, a^{-3} + \Omega_r \, a^{-4} + \Omega_k \, a^{-2} + \Omega_{DE} \, a^{-3(1+w)} \right]$$

Ω<1: k>0. Closed Universe

Ω=1: k=0. Flat Universe

Ω>1: k=0. Open Universe

The evolution of each energy component depends on its equation of state.

$$p = w\rho$$

$$\dot{\rho} + 3(\rho + \frac{P}{c^2})\frac{\dot{a}}{a} = 0$$

$$\rightarrow \rho \propto a^{-3(1+w)}$$

Matter: w= 0

Radiation: w= 1/3

Λ: w= -1

Dark Energy w< -1/3

Quintaessence: $w=w_0+w_a(1+z)$

Universe expansion

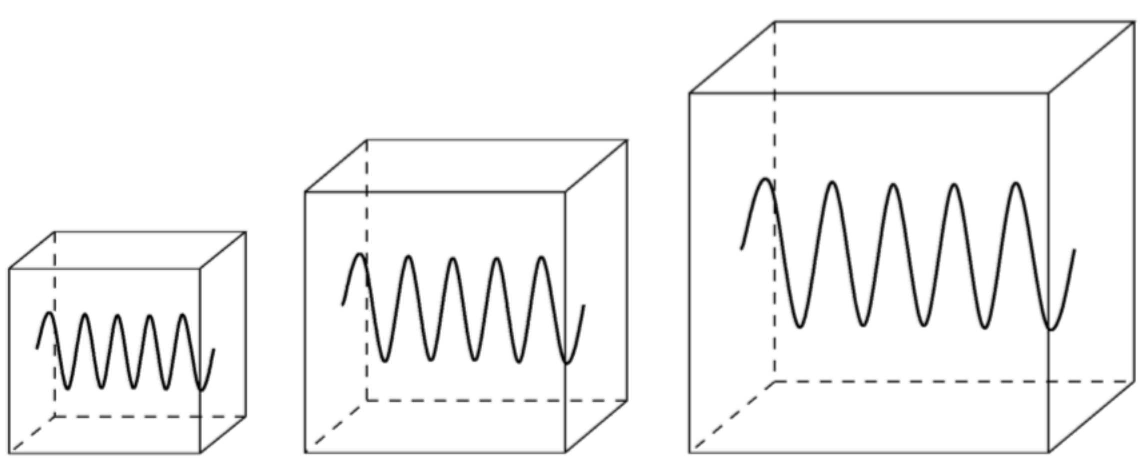
Physical distance related with comoving distance: $\boldsymbol{r}(t) = a(t)\,\boldsymbol{x}$.

By setting today, r=χ, then we normalise the value of a: $a(t_0) = 1$.

Velocity of a comoving particle: $\boldsymbol{v}(\boldsymbol{r},t) = \frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{r}(t) = \frac{\mathrm{d}a}{\mathrm{d}t}\,\boldsymbol{x} \equiv \dot{a}\,\boldsymbol{x} = \frac{\dot{a}}{a}\,\boldsymbol{r} \equiv H(t)\,\boldsymbol{r}$,

Due to the expansion, 2 comoving observers will measured the emitted light by the other with a different frequency than the emitted one, given by redshift z.

$$\frac{\lambda_e}{a(t_e)} = \frac{\lambda_o}{a(t_0)} \qquad a(t_e) = 1/(1+z)$$



Cosmological distances

We can estimate the distances in the Universe using this formalism so we can directly study the energy content

$$H^2 = H_0^2 \left[\Omega_m \, a^{-3} + \Omega_r \, a^{-4} + \Omega_k \, a^{-2} + \Omega_{DE} \, a^{-3(1+w)} \right]$$

$$\chi(z) = \frac{c}{H_0} \int_{a=(1+z)^{-1}}^{a=1} \frac{da}{\sqrt{\Omega_m \, a + \Omega_r + \Omega_k \, a^2 + \Omega_\Lambda \, a^4}} = \frac{c}{H_0} \int_0^z \frac{dz}{E(z)}$$

$$d = cz/H_0$$ Local distance

$$D = \chi(z)$$ Comoving distance

$$D_\theta = \chi(z)/(1+z)$$ Angular diameter distance $D_\theta \equiv \frac{d_p}{\theta_A}$

$$D_L = \chi(z)\,(1+z) = D_\theta / a^2$$ Luminosity distance $D_L \equiv \sqrt{\frac{L}{4\pi F_0}}$

$$d_H = \int_{t_i}^{t} \frac{dt}{a(t)}$$ Particle horizon

In optical photometry, we use magnitudes: $m_x = -2.5 \log_{10} \left(\frac{F_x}{F_{x,0}} \right)$

$$\mu \equiv m - M = 5 \log D_L(z) + 25$$ Distance modulus

m: Apparent magnitude

M: absolute magnitude

Universe history eras

The dynamics of the Universe and the growth of structure depend on which energy component is dominating at each time of history in the Universe

$$\rho \propto a^{-3(1+w)}$$

Matter: w= 0

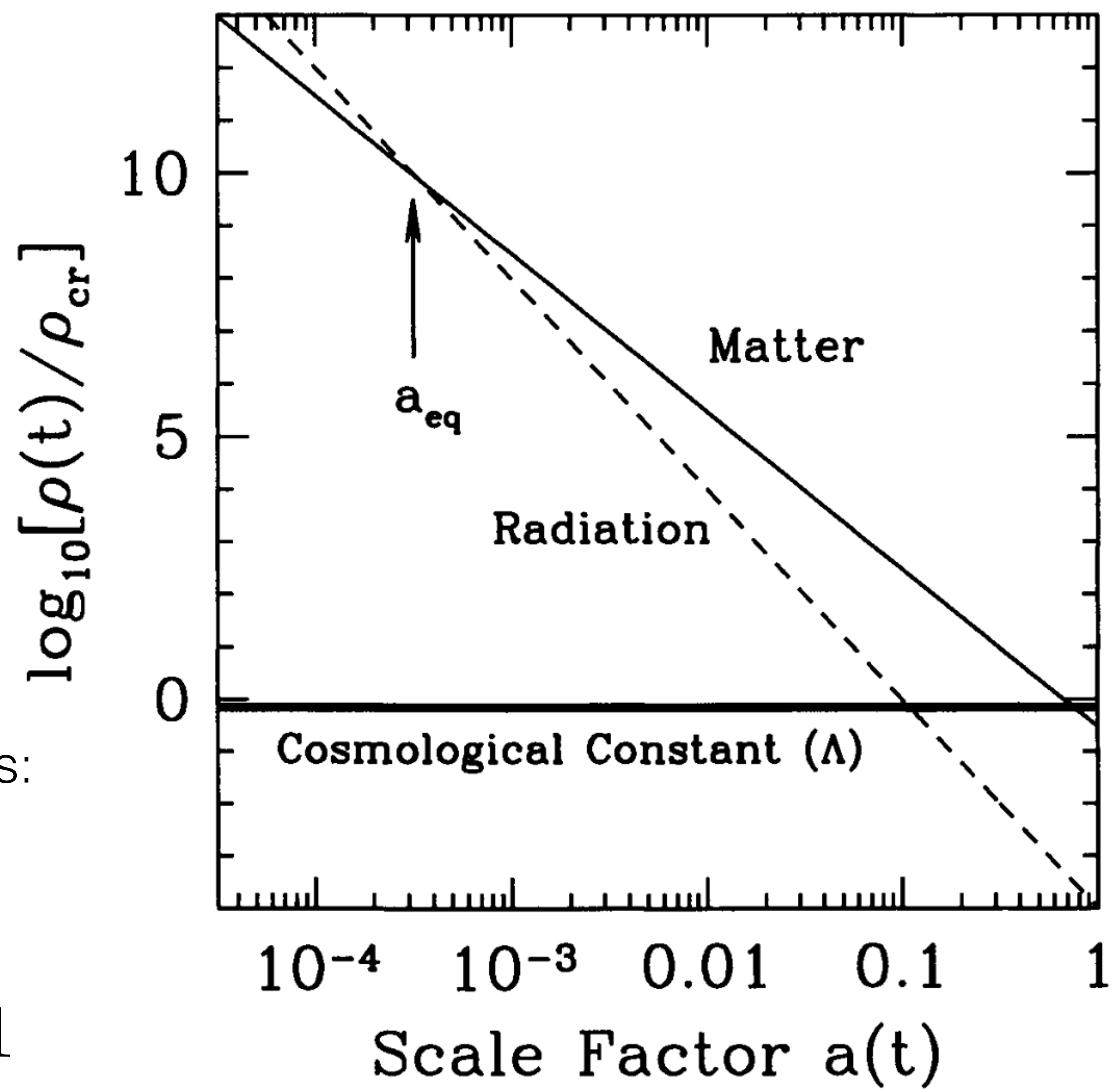
Radiation: w= 1/3

Λ: w= -1

If only one component dominates:

$$a \propto t^{\frac{2}{3(1+w)}}$$

$$a \propto e^{Ht} \quad \text{if} \quad w = -1$$



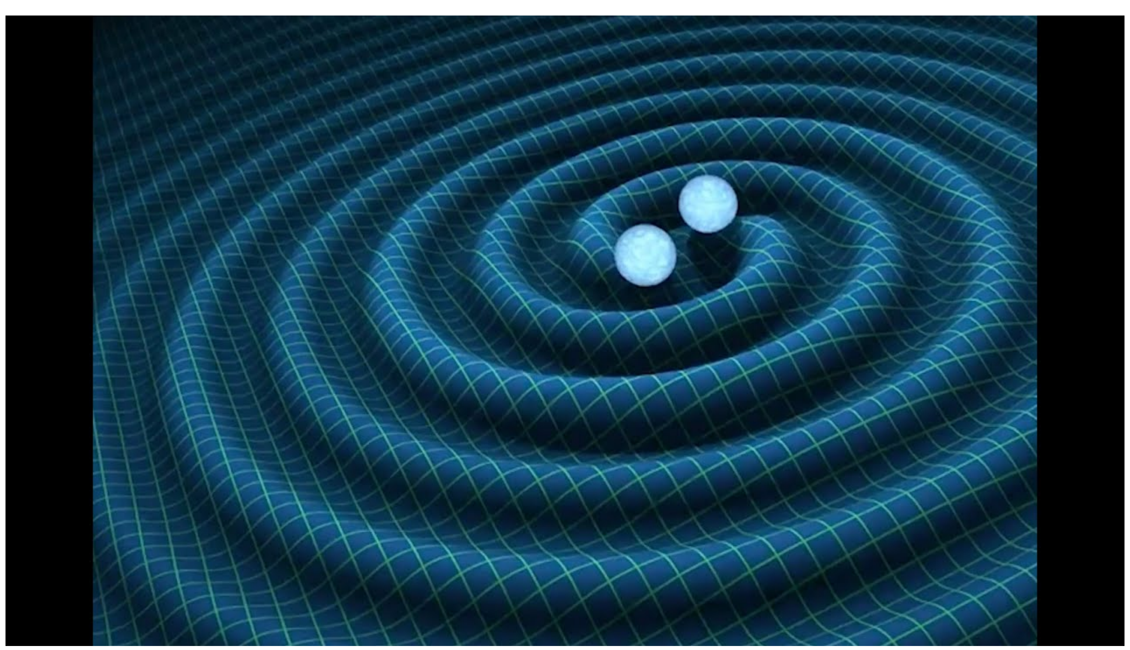
Observational tests of cosmology

We need to study **all cosmology** at **all times** and **all scales**.

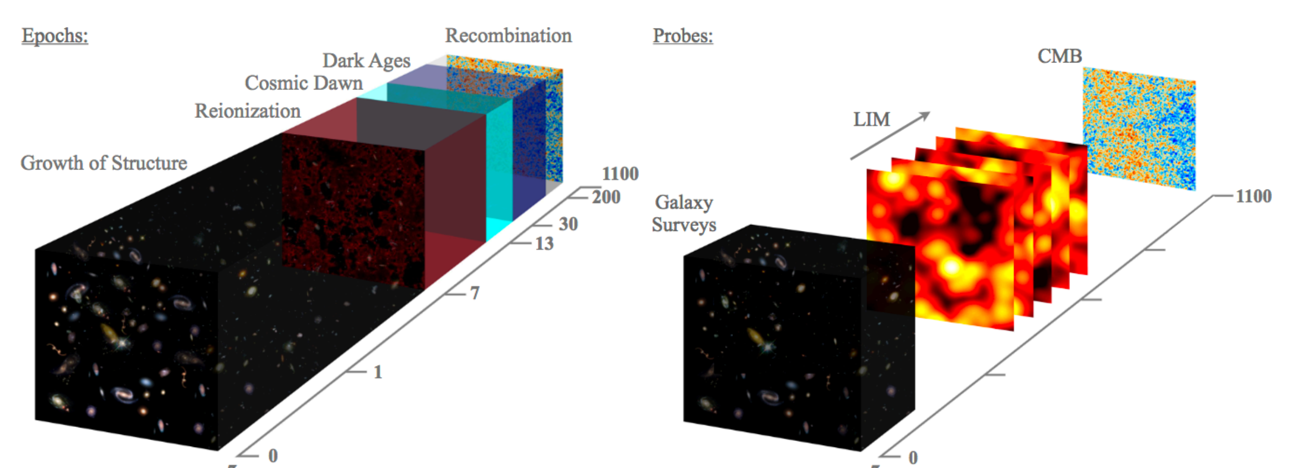
Study transient objects in the sky and the distribution of matter using different wavelengths.

Principal observational methods

Time-domain cosmology



Large-scale structure of the Universe

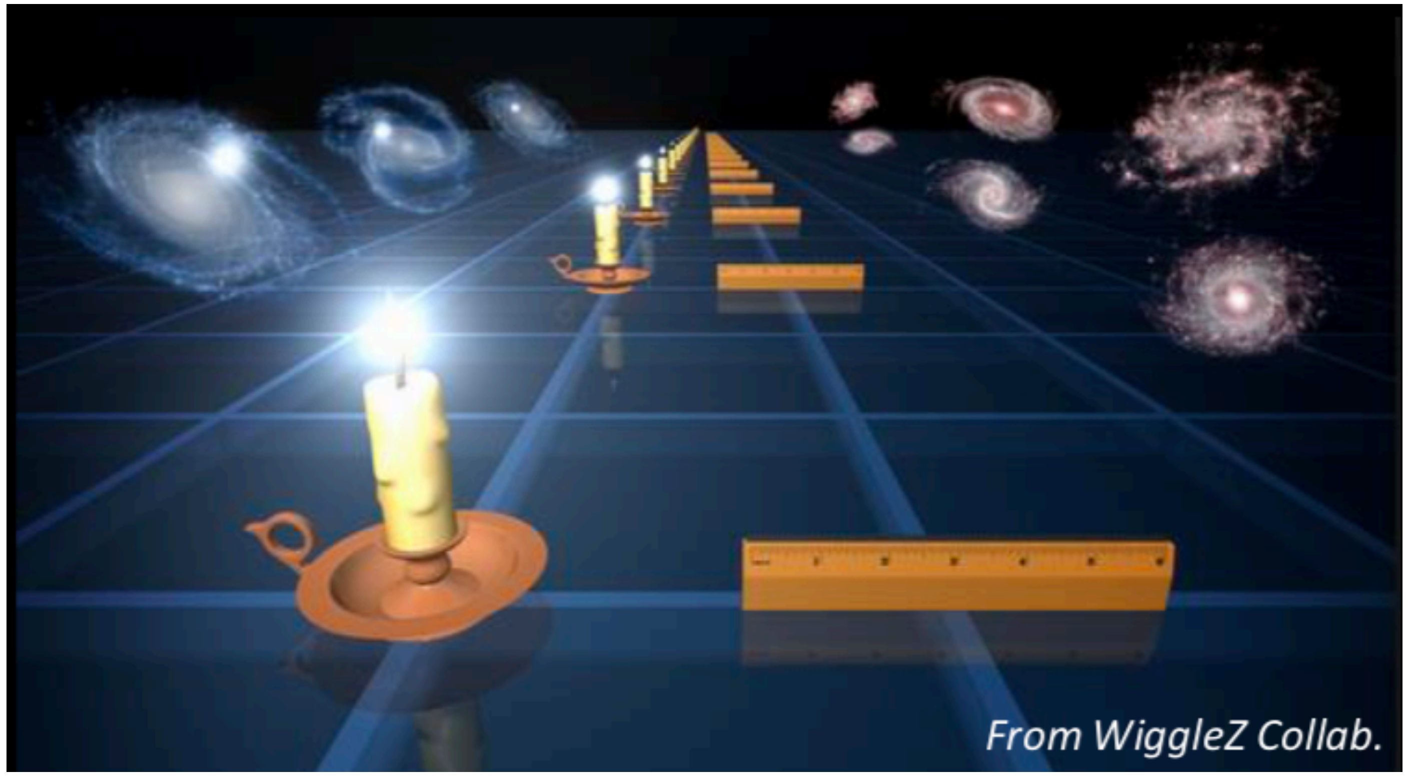


Cosmography

Using the luminosity or the angular diameter distance we can test the cosmological background model.

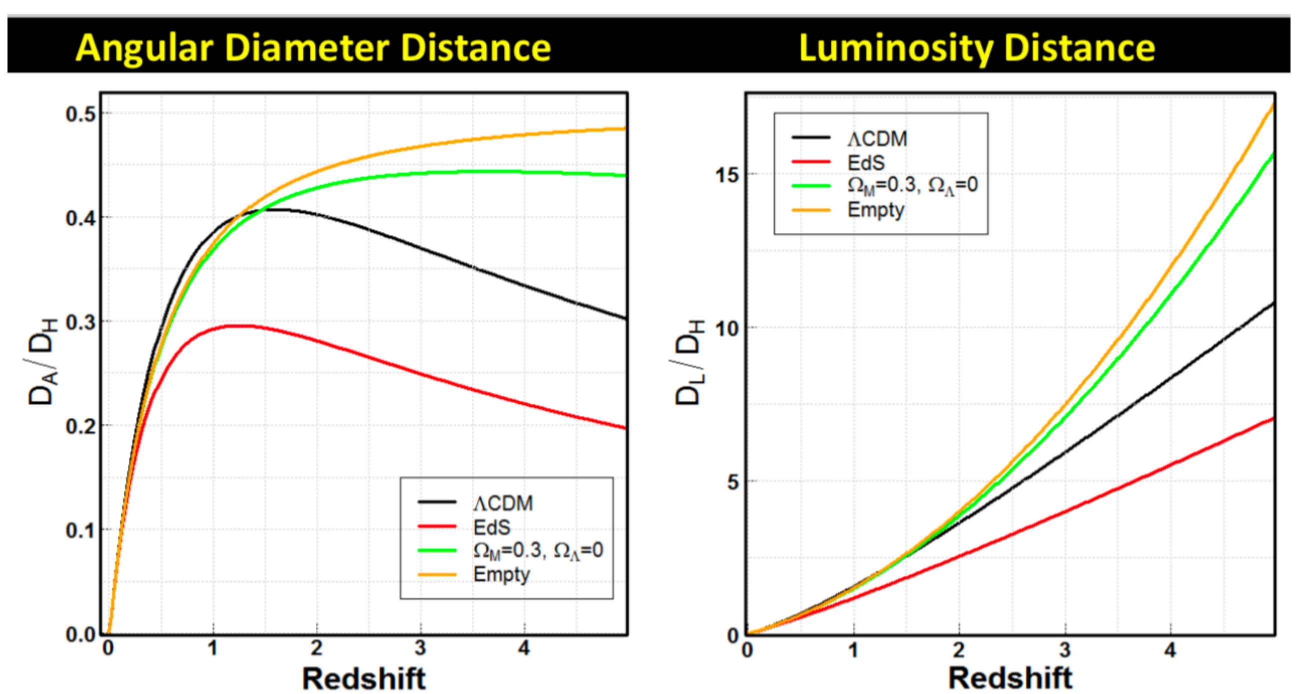
Standard candles

$$F = \frac{L}{4\pi D_L^2}$$



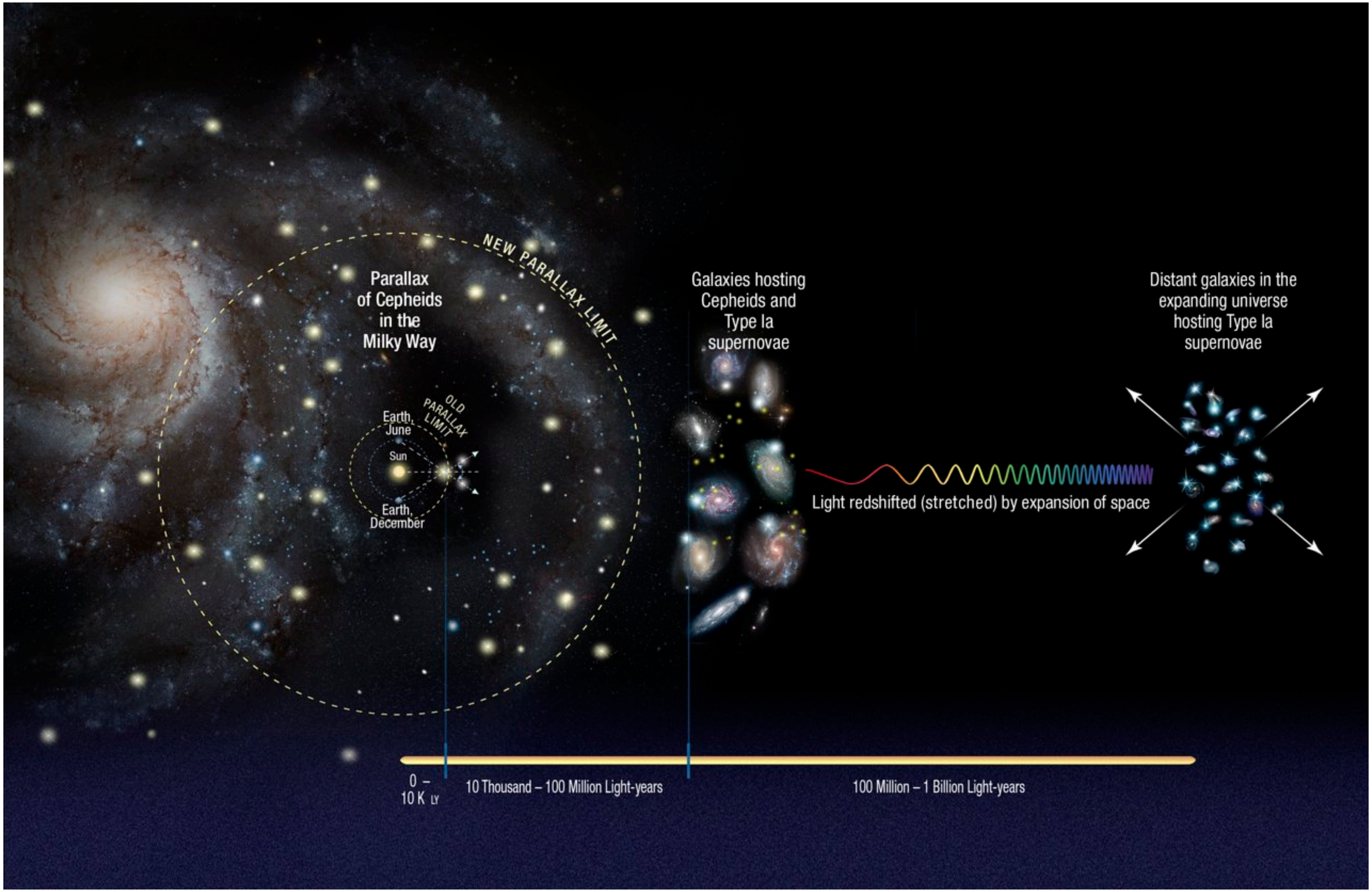
Standard rulers

$$D_A = \frac{R}{\theta}$$



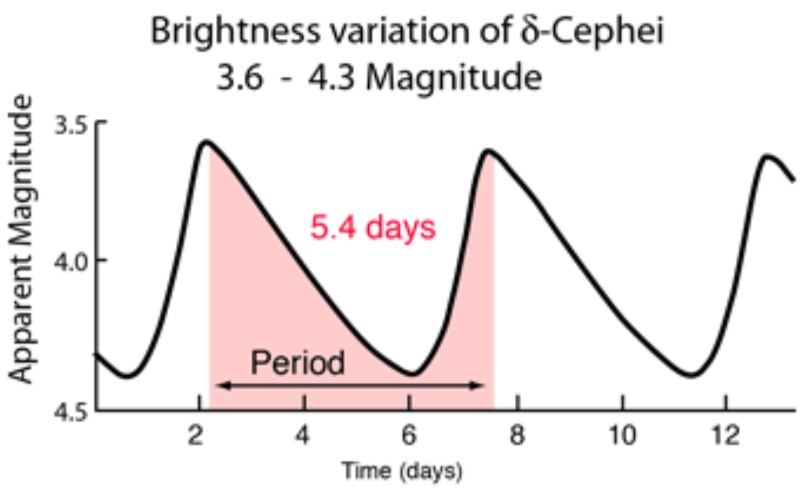
Cosmic distance ladder

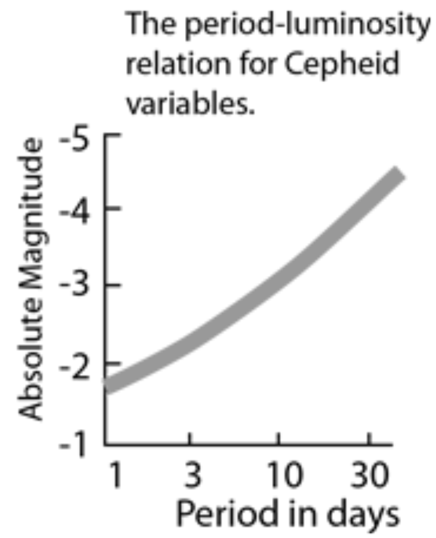
Most distance measurements are carried using the distance ladder technique.

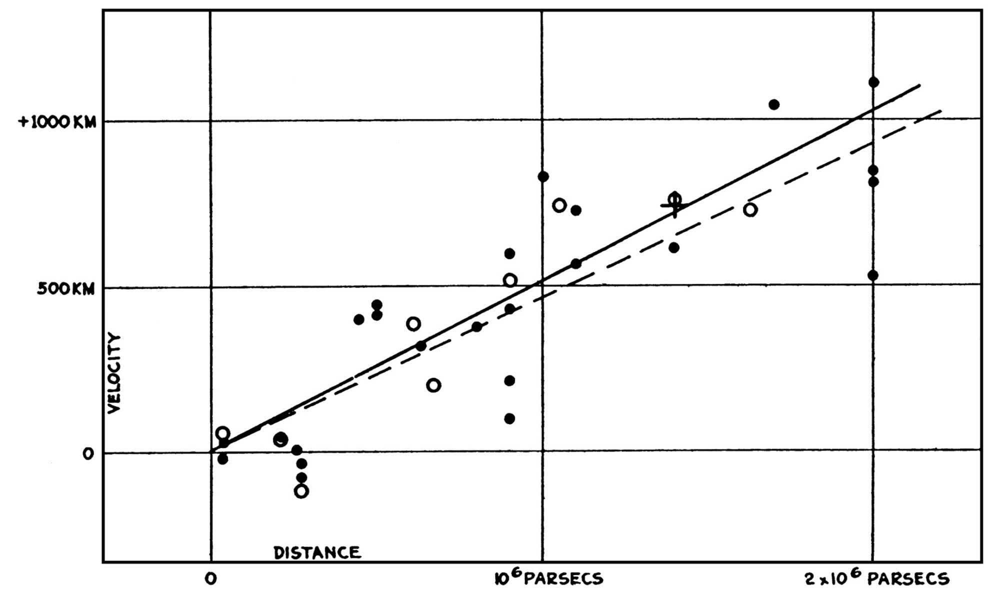


Hubble-Lemaître law

Standard candles: If we know luminosity -> We measure distance



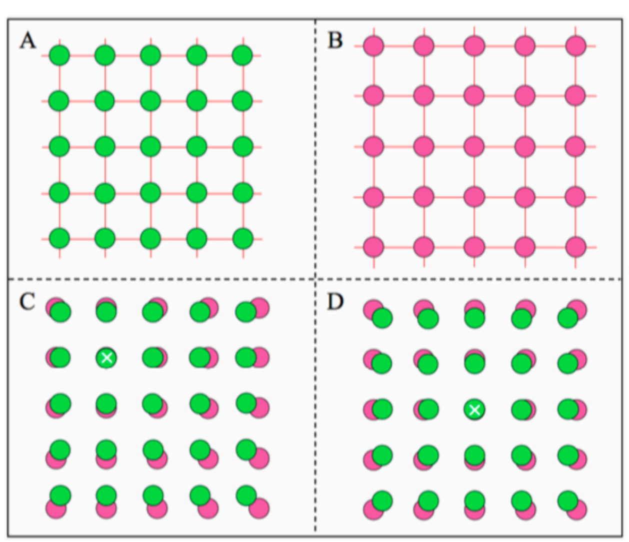


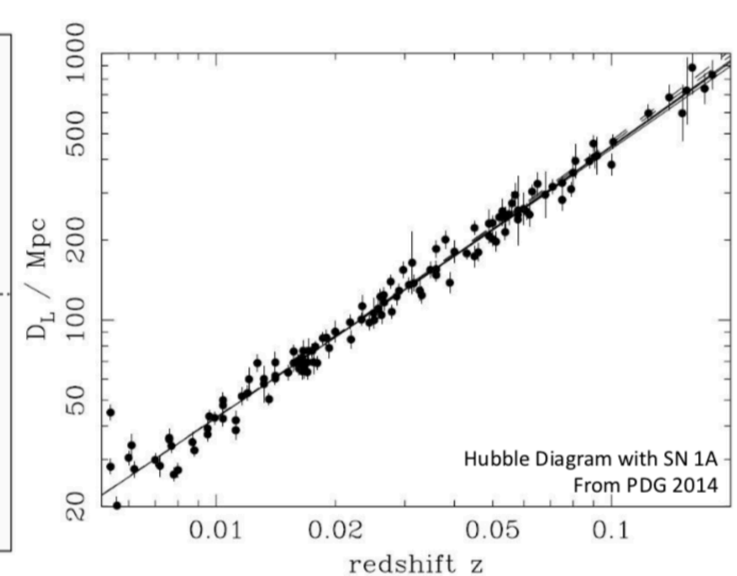


We observe galaxies “receding” at a velocity proportional to the distance to them

$$cz \sim v = H\,d = \frac{\dot{a}}{a}\,d$$

Universe expands,
confirming FLRW metric





Expansion of the Universe (beginning)

During the 1920s, thanks to Vesta Slipher redshift measurements, Henrietta Leavitt cepheids period-luminosity relation, Georges Lemaître found Friedmann's solution and compared with data, funding the expansion but then with more data Edwin Hubble measured the distance-redshift relation.

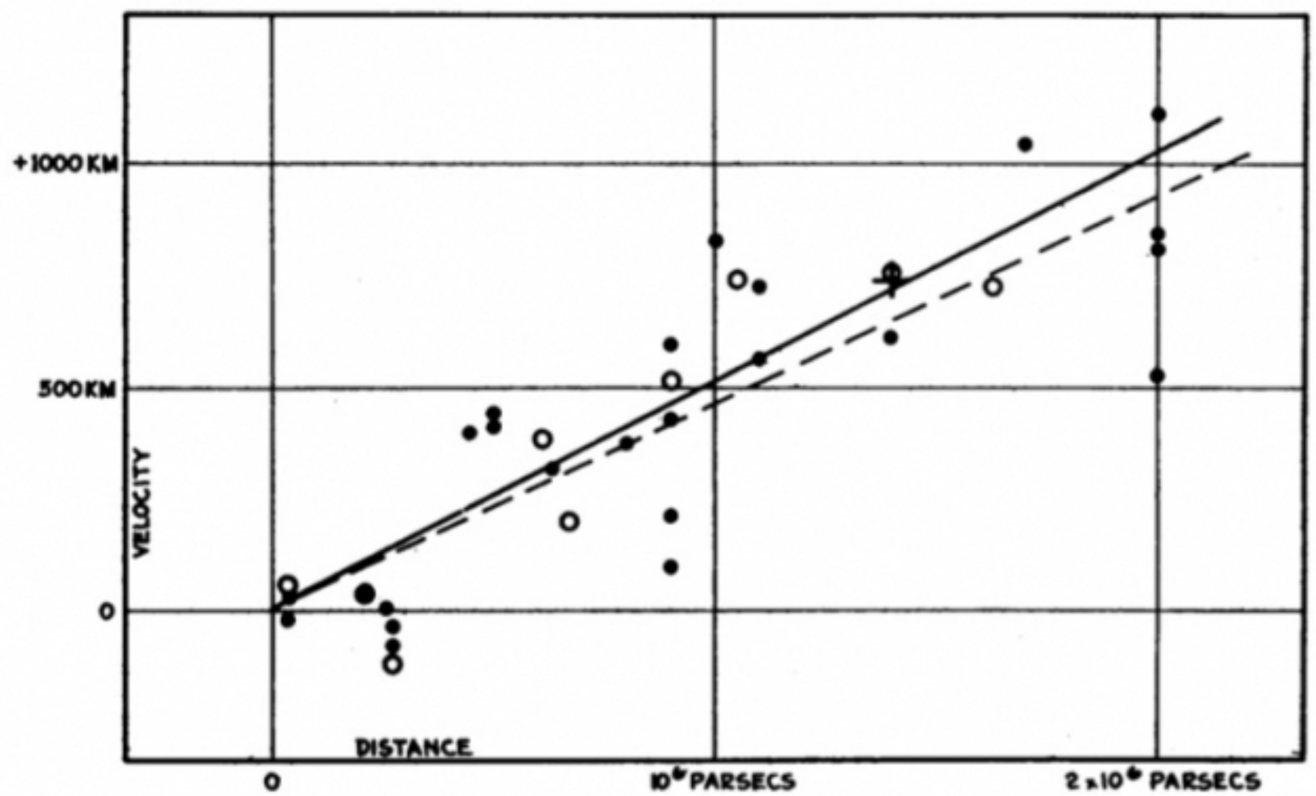
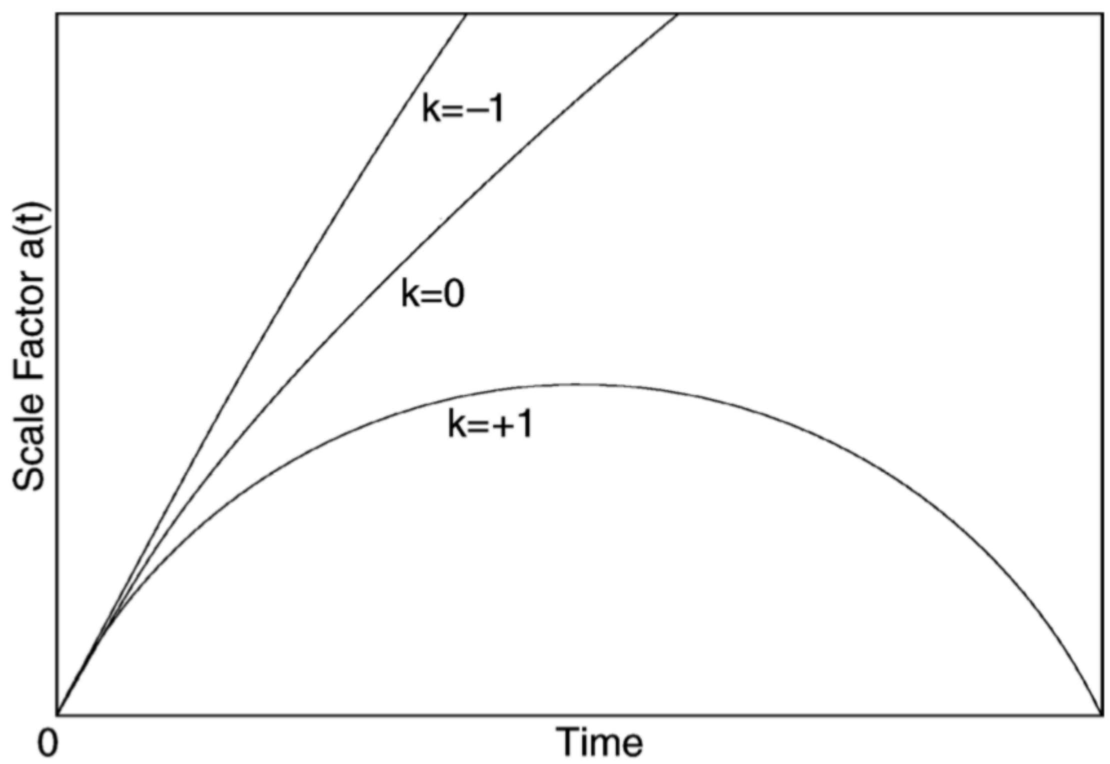


FIGURE 1
Velocity-Distance Relation among Extra-Galactic Nebulae.



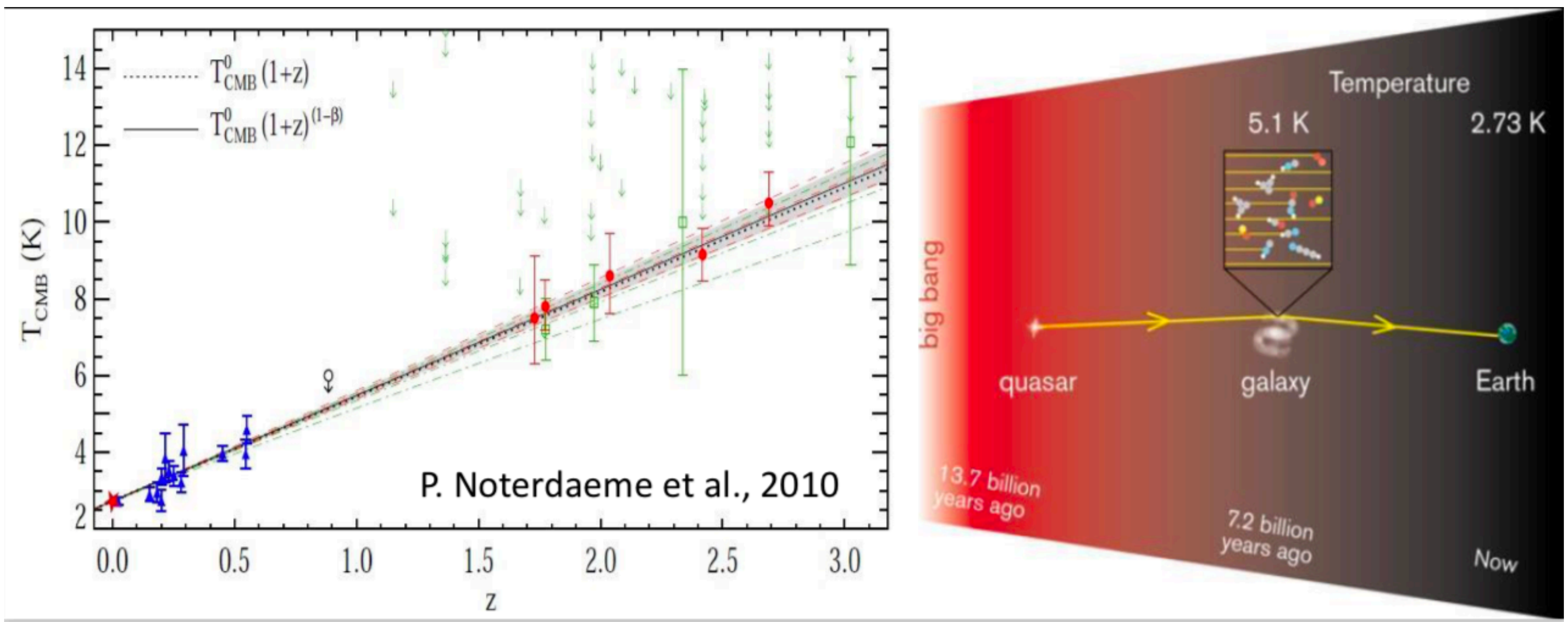
One of Lemaître conclusions was to formulate the primeval atom of the Universe in which going back on time, the Universe must have been in a dense state.

Thermal history

According to the Big Bang model, there should be a remnant radiation cooled down to a few K today.

Observing this radiation -> significant support of the model

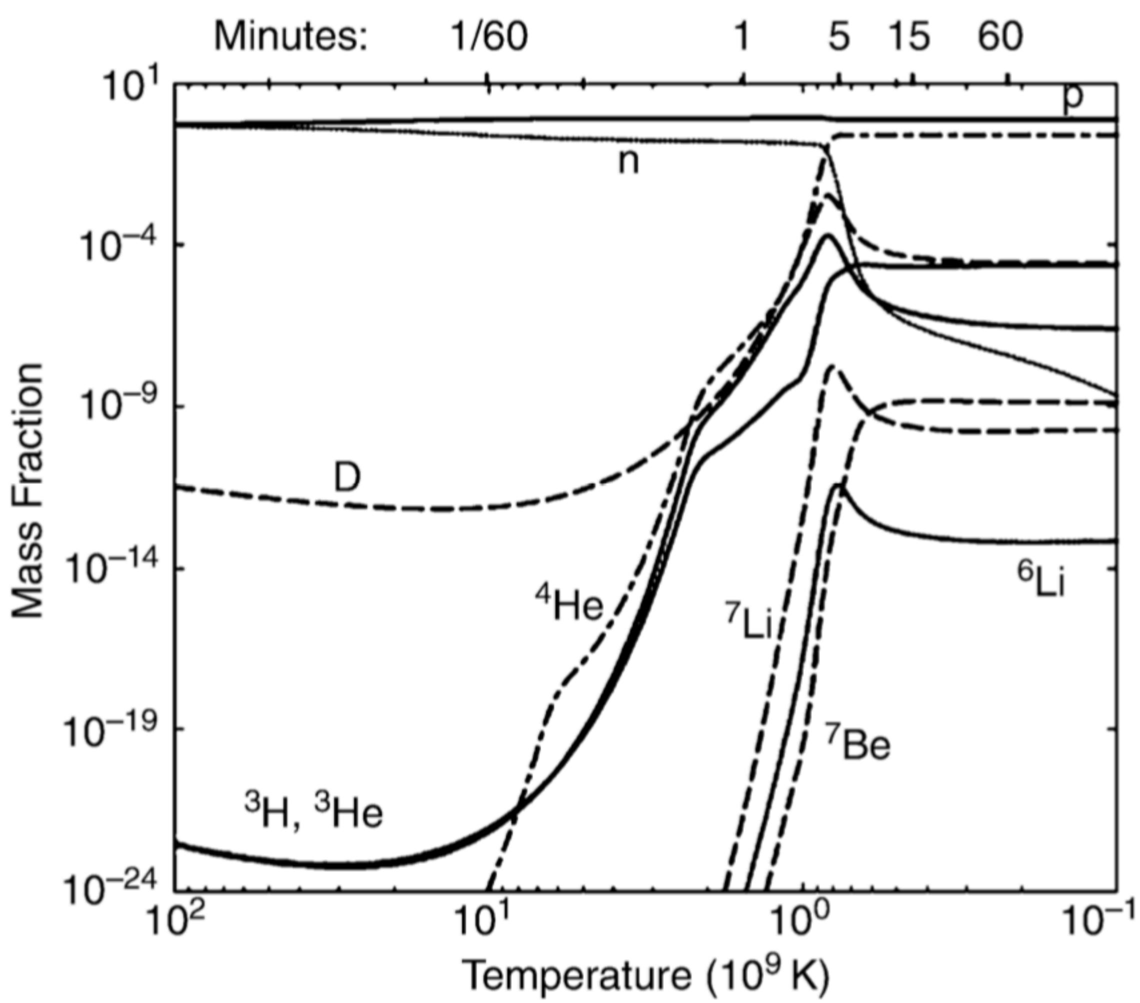
$$T = T_0(1+z)$$

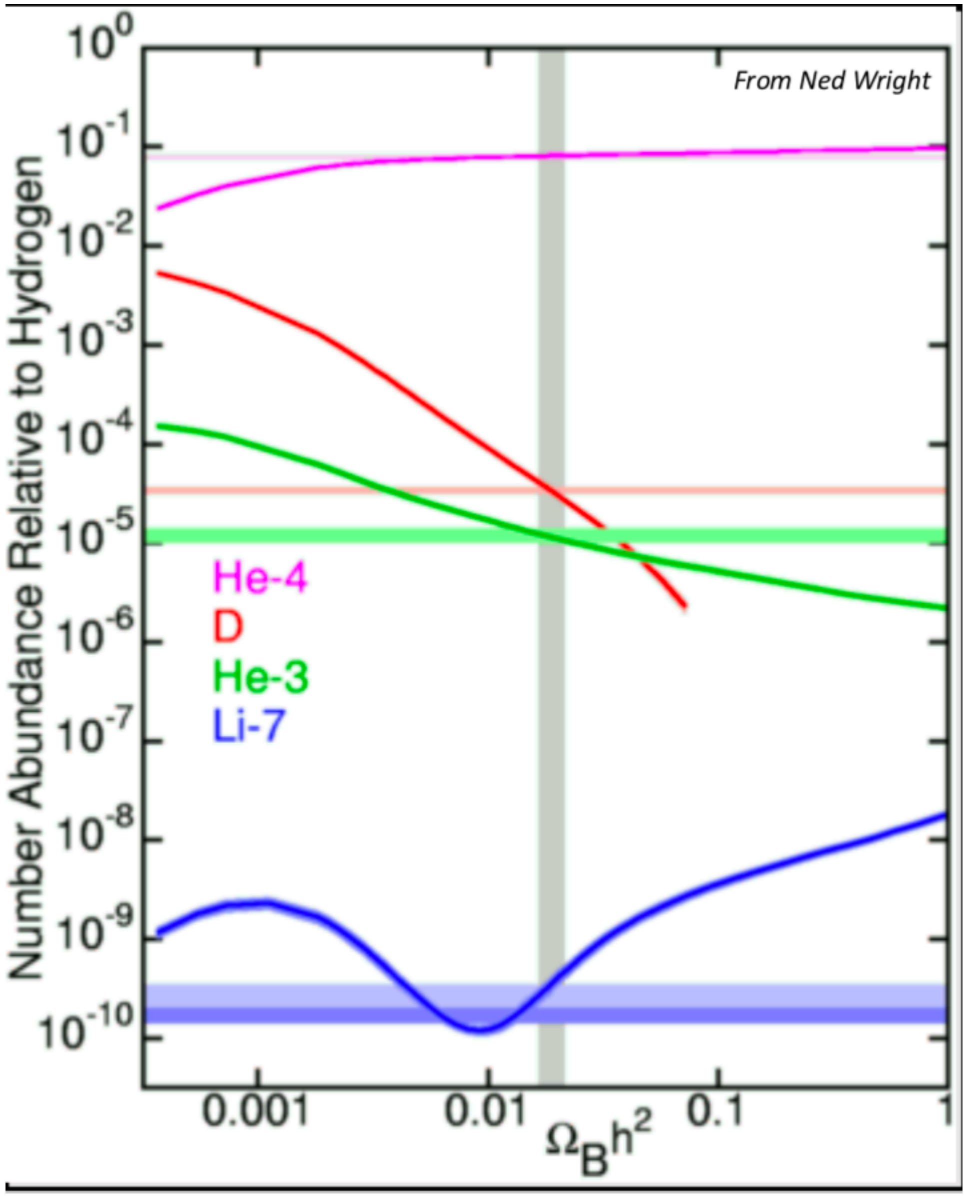


The evolution of the CMB temperature has been measured with redshift, agreeing with the expected result.

Thermal History (Primordial nucleosynthesis)

We can predict the abundances of light elements, depending on the amount of baryons in the Universe. The abundances given by the observed baryon density at present agree with astrophysical abundances of this elements.





Thermal history (recombination & decoupling)

- At high redshifts, photons and electrons are tightly coupled through Thompson scattering:

$$\gamma + e^- \rightleftarrows \gamma + e^-$$

The scattering rate is: $\Gamma = c n_e \sigma_T = \frac{c n_{B_0} \sigma_T}{a^3} = \frac{5 \cdot 10^{-21} s^{-1}}{a^3}$ (where we assumed $n_e = n_p = n_B$) and $n_{B_0} = \rho_{c_0} \Omega_b / m_p$.

As long as $\Gamma > H$ the particles will remain tightly coupled. -> Hint for decoupling.

- Before decoupling, recombination happens when atoms are formed through:

$$H + \gamma \rightleftarrows p + e^-$$

- Maxwell-Boltzmann equation determine number densities: $n_i = g_i \left(\frac{m_i T}{2\pi} \right) e^{-(m_i - \mu_i)/T}$

Taking the ratio to cancel the chemical potentials and assuming $m_p \sim m_H >> m_e$

we reach the Saha equation: $$\frac{n_H}{n_p n_e} = \left(\frac{m_e}{T} 2\pi \right)^{-3/2} e^{\frac{Q}{T}}$$

where $Q = m_p + m_e - m_H \sim 13.6\ eV$

Thermal history (recombination & decoupling)

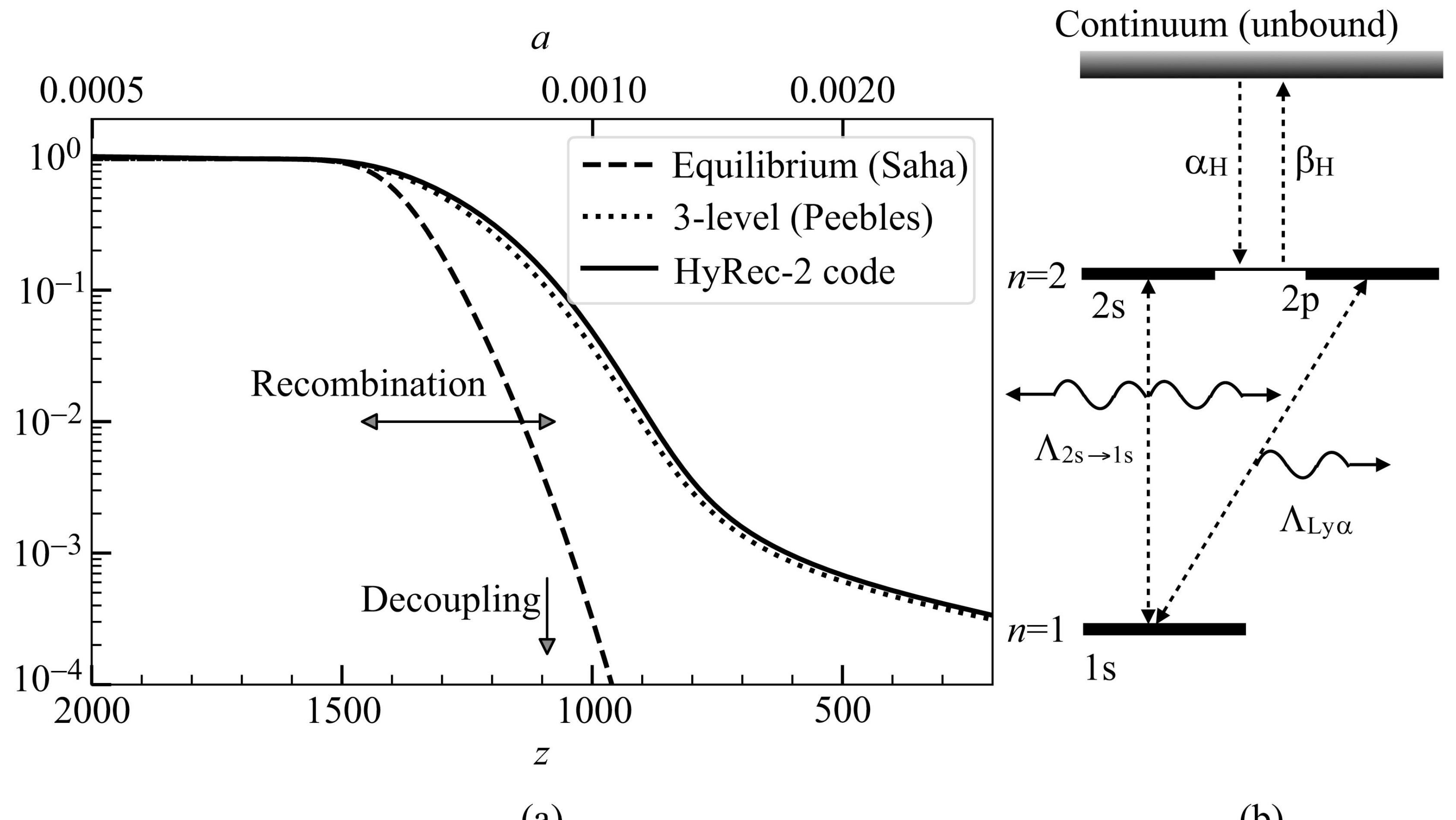
- It is really important to define the ionisation fraction:

$$\frac{n_p}{n_p + n_H} = \frac{n_p}{n_B} = \frac{n_p}{n_B} \equiv x_e(t)$$

Rearranging the equation and introducing the baryon to photon ratio η_γ we can have something solvable (using $T = T_0/a$)

$$\frac{1 - x_e}{x_e^2} = n_\gamma \eta_\gamma \left(\frac{m_e T}{2\pi} \right)^{-3/2} e^{Q/T}$$

Recombination: $z_{rec} \sim 1275$



Thermal history (recombination & decoupling)

- For decoupling, we can just estimate when the interaction rate becomes smaller than the Hubble rate so we can set

$$\Gamma(z_{dec}) = H(z_{dec})$$

- We need to include the effect of recombination in $\Gamma = c n_e \sigma_T = \frac{c n_{B_0} \sigma_T}{a^3} = \frac{5 \cdot 10^{-21} s^{-1}}{a^3}$ by including the ionisation fraction: $\Gamma = 5 \cdot 10^{-21} x_e(z)(1+z)^3$

- By the recomibination, the Universe is already in the matter domination phase so $H(z) = H_0 \sqrt{\Omega_m (1+z)^3}$ and if we use the proper solution for $x_e(z)$ we get

$$z_{dec} = 1090$$

To summarize:

Phase	z	T	T
Matter-radiation equality	3250	50000 yrs	9000 K
Recombination	1275	290000 yrs	3500 K
Decoupling	1090	380000 yrs	3000 K

Thermal history (reionization)

- After decoupling, a 5% of CMB photons get lost (which will impact in the amplitude of the power spectrum) because of **reionization**.

- First stars and quasars produce the ionisation of electrons in the intergalactic media around z=11. The freed electrons will produce again some Thompson scattering with the CMB photons.

- The direct impact of reionisation in the ΛCDM model is through the optical depth of the reionization called τ which is given by:

$$\tau = \int_{t_{reion}}^{t_0} \Gamma(t) dt$$

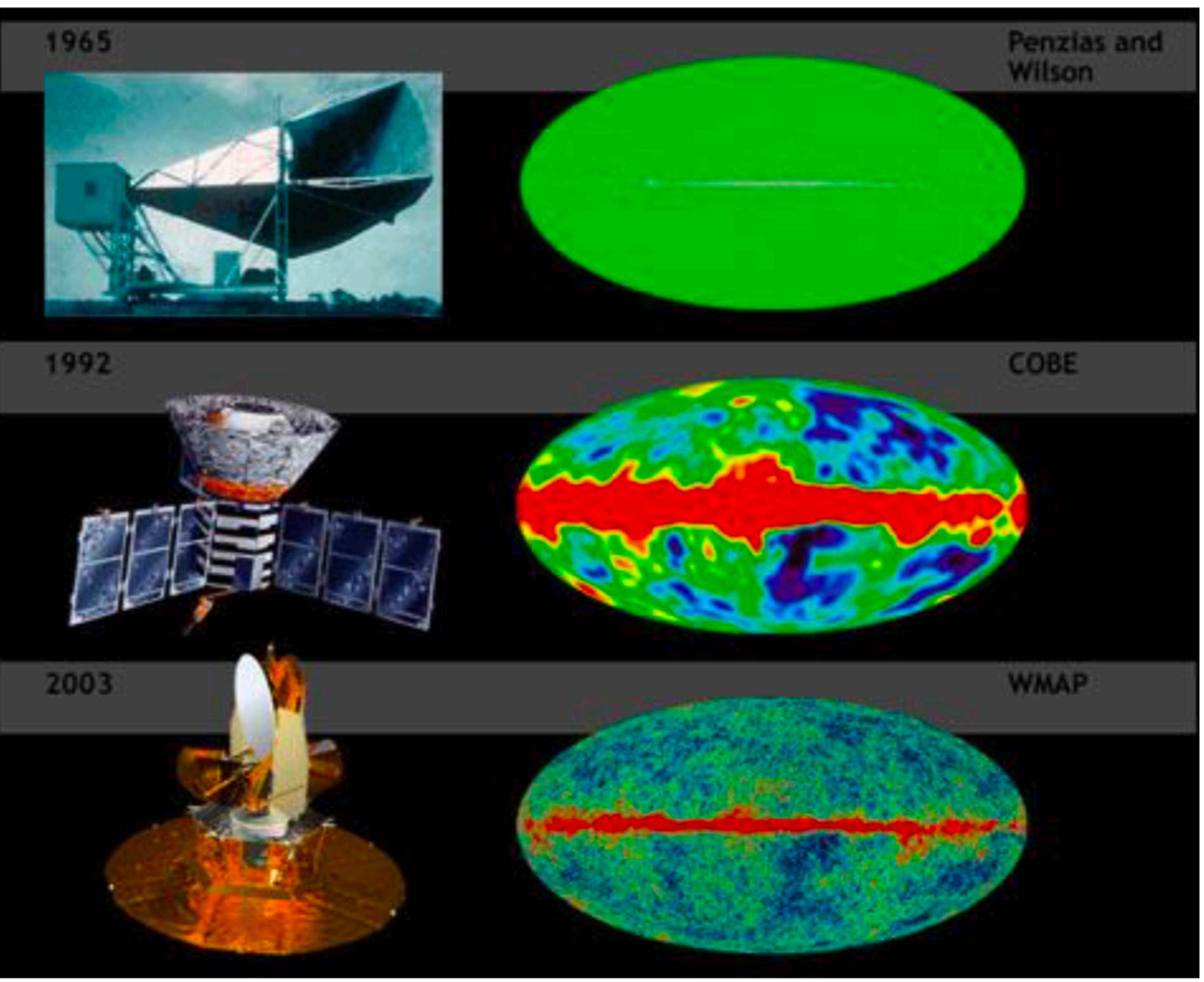
The impact in the amplitude of the power spectrum is given by $A_s e^{-2\tau}$. It has a similar type of nuisance as the galaxy bias when studying the clustering of galaxies.

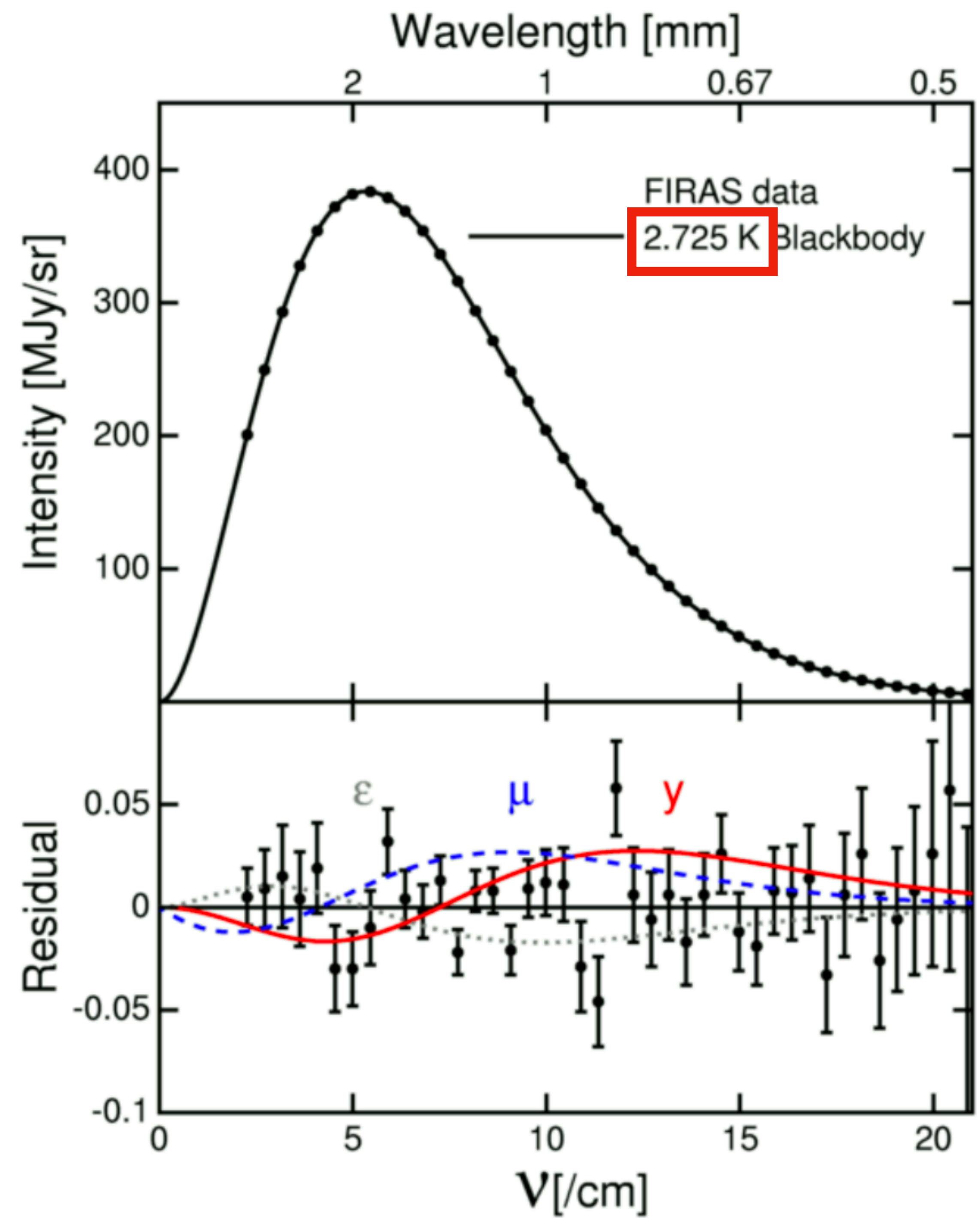
Universe timeline

Event	Time	Redshift	Temp (eV)	Temp (K)
Inflation	10^{-35} s ??	?	?	?
Baryogenesis	?	?	~ 100 GeV ??	?
QCD phase transition	10 µs	1.5×10^{12}	150 MeV	10^{12}
Dark-matter freezeout	?	?	~ 100 MeV ??	?
Neutrino decoupling	0.7 s	6×10^{9}	1 MeV	10^{10}
Electron–positron annihilation	2 s	3×10^{9}	0.5 MeV	6×10^{9}
Big Bang nucleosynthesis	2 min	5×10^{8}	0.1 MeV	10^{9}
Matter–radiation equality	50,000 yr	3500	0.8 eV	9000
Recombination	290,000 yr	1275	0.3 eV	3500
Photon decoupling	380,000 yr	1090	0.25 eV	3000
Dark energy dominates	10 Gyr	0.3	3.1×10^{-4} eV	3.6
Present day	13.8 Gyr	0	2.35×10^{-4} eV	2.725

Cosmic Microwave Background radiation

Discovery in 1965 by Penzias and Wilson. Decoupled from matter at 3000K (decoupling). Blackbody distribution with a temperature close to 3K with huge precision.





CMB observations: COBE

FIRAS

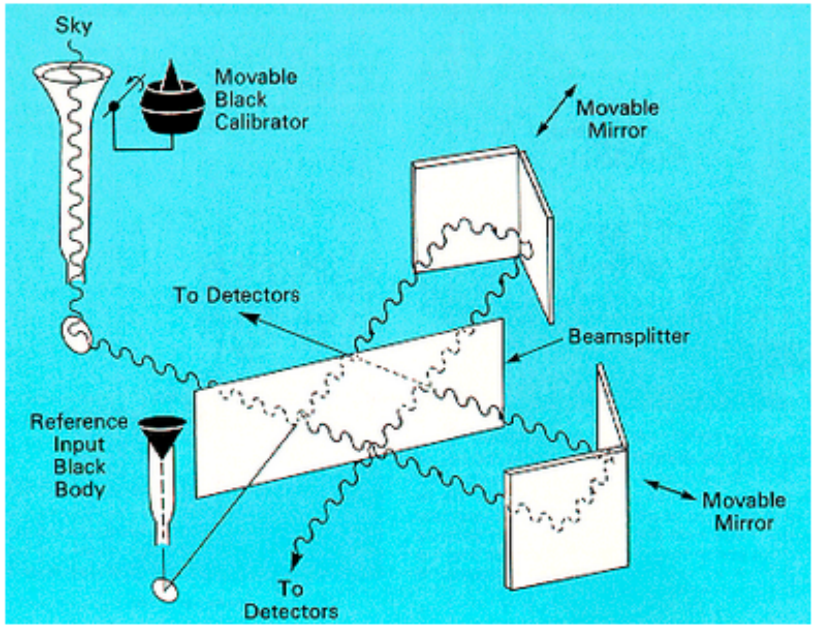
Measurement of Blackbody spectrum of CMB -> Interferometry made with a reference blackbody

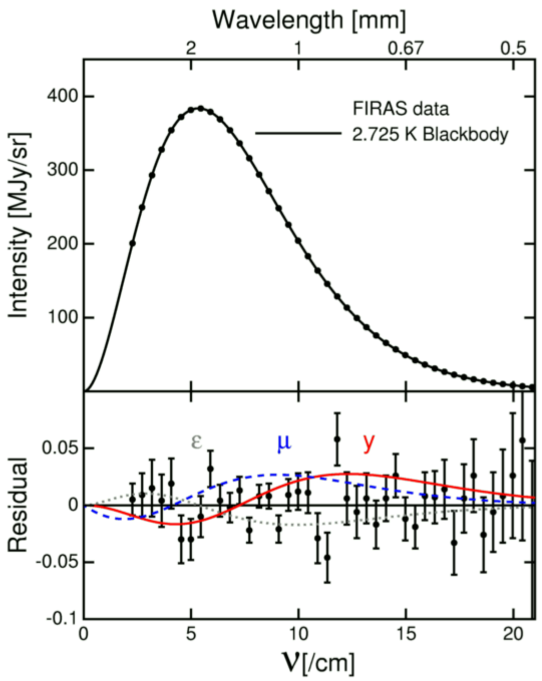
FoV of 7 degrees and cooling system at 1.5 K

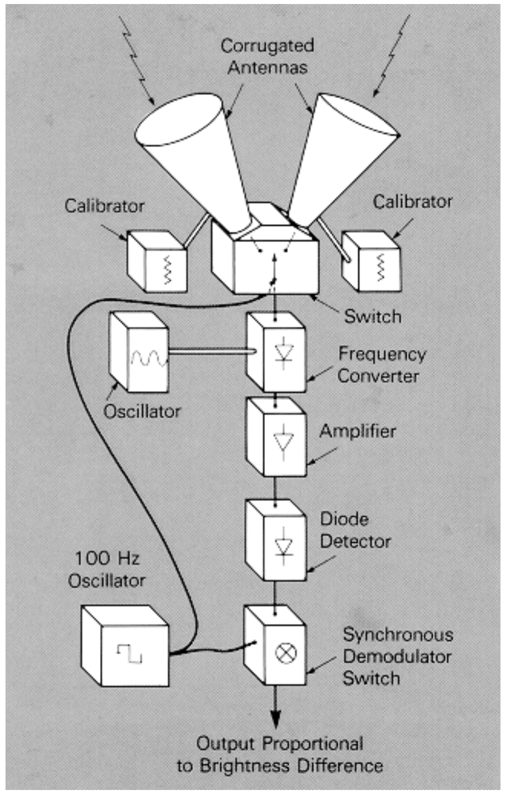
DMR

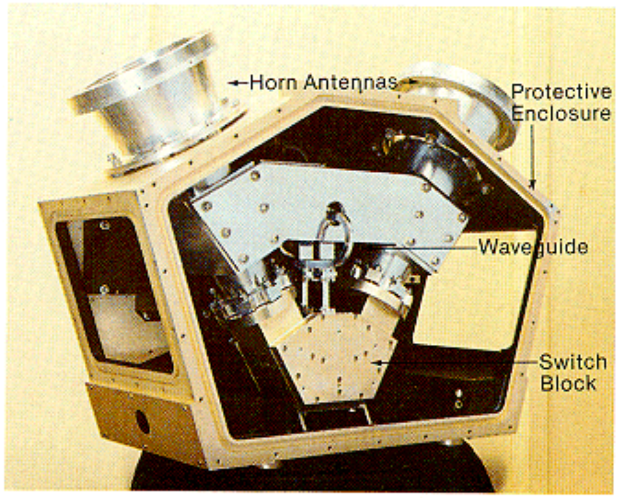
Differential measurement of power with a 60 degree angle in the sky.

Detection of anisotropies





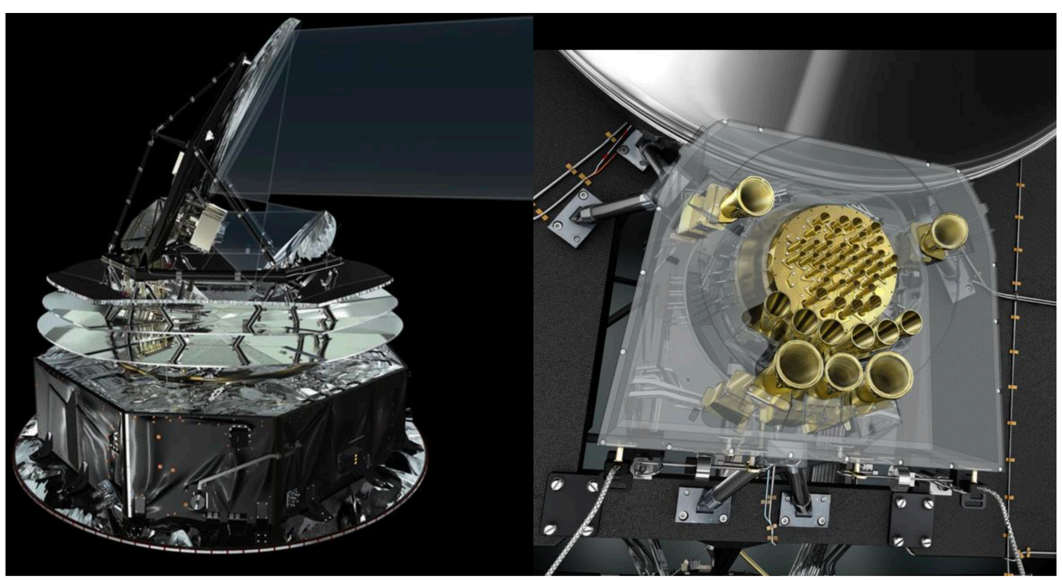


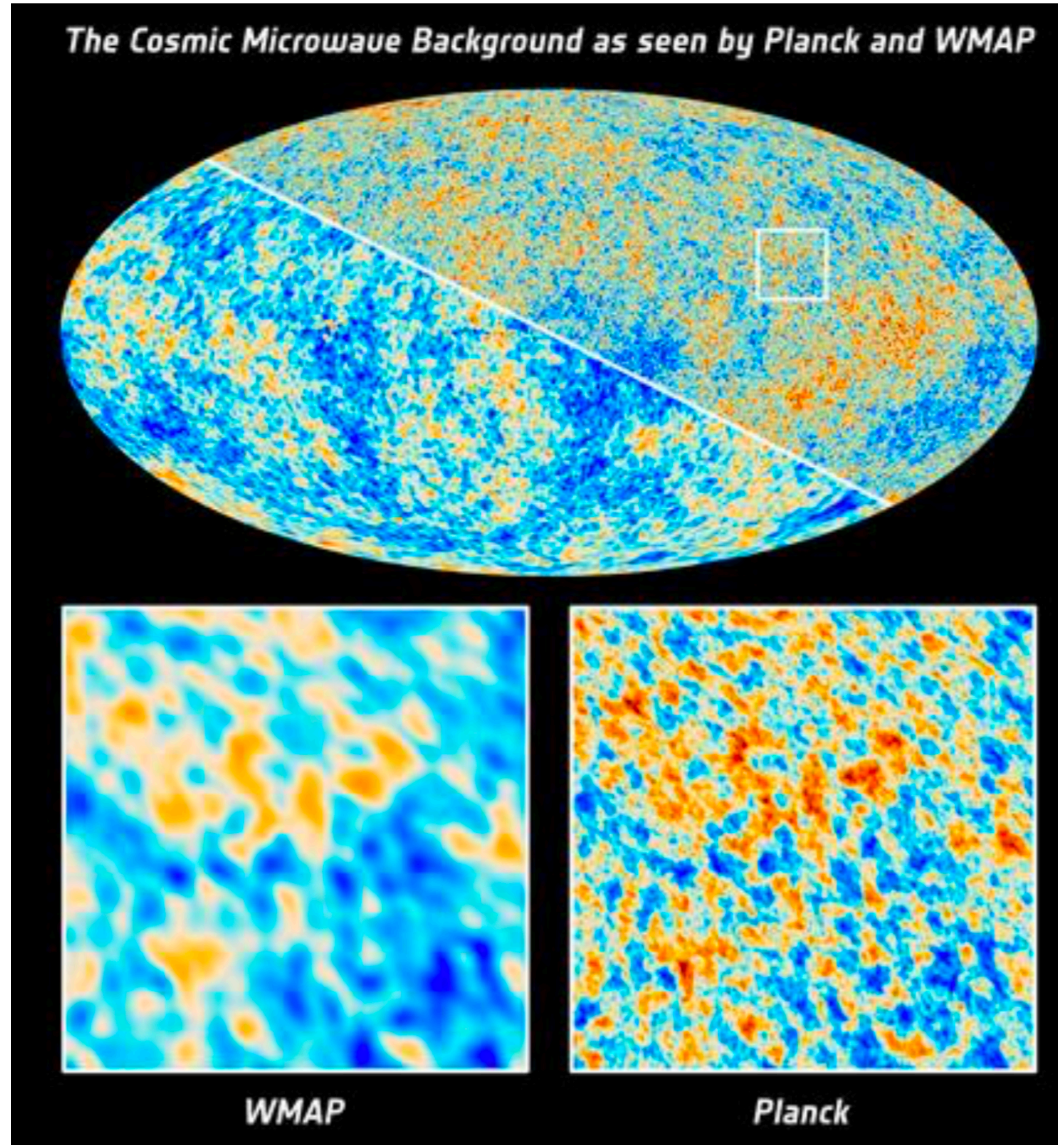


The 9.6 mm DMR receiver partially assembled. Corrugated cones are antennas.

CMB observations: Planck

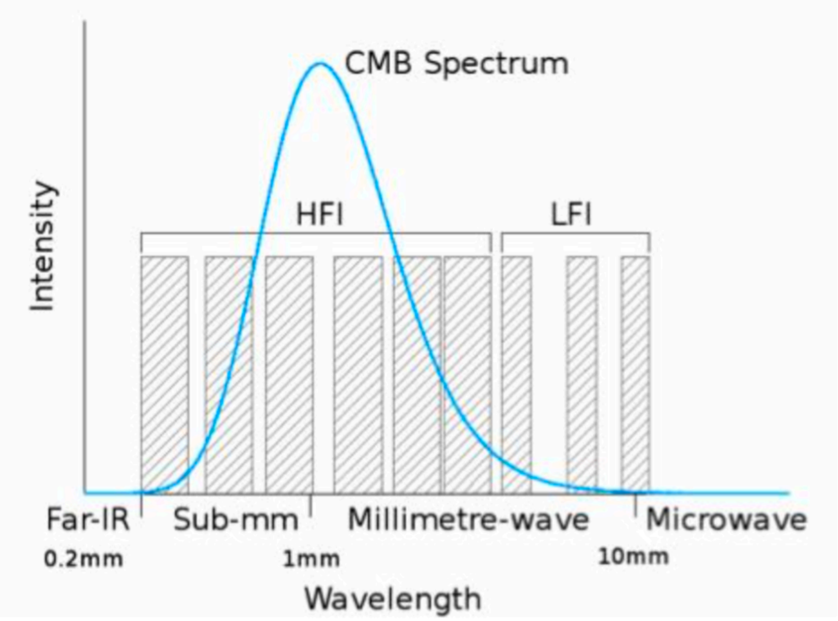
1.5 meters diameter with 2 instruments, high frequency (> 100 GHz) and low frequency (<100 GHz)

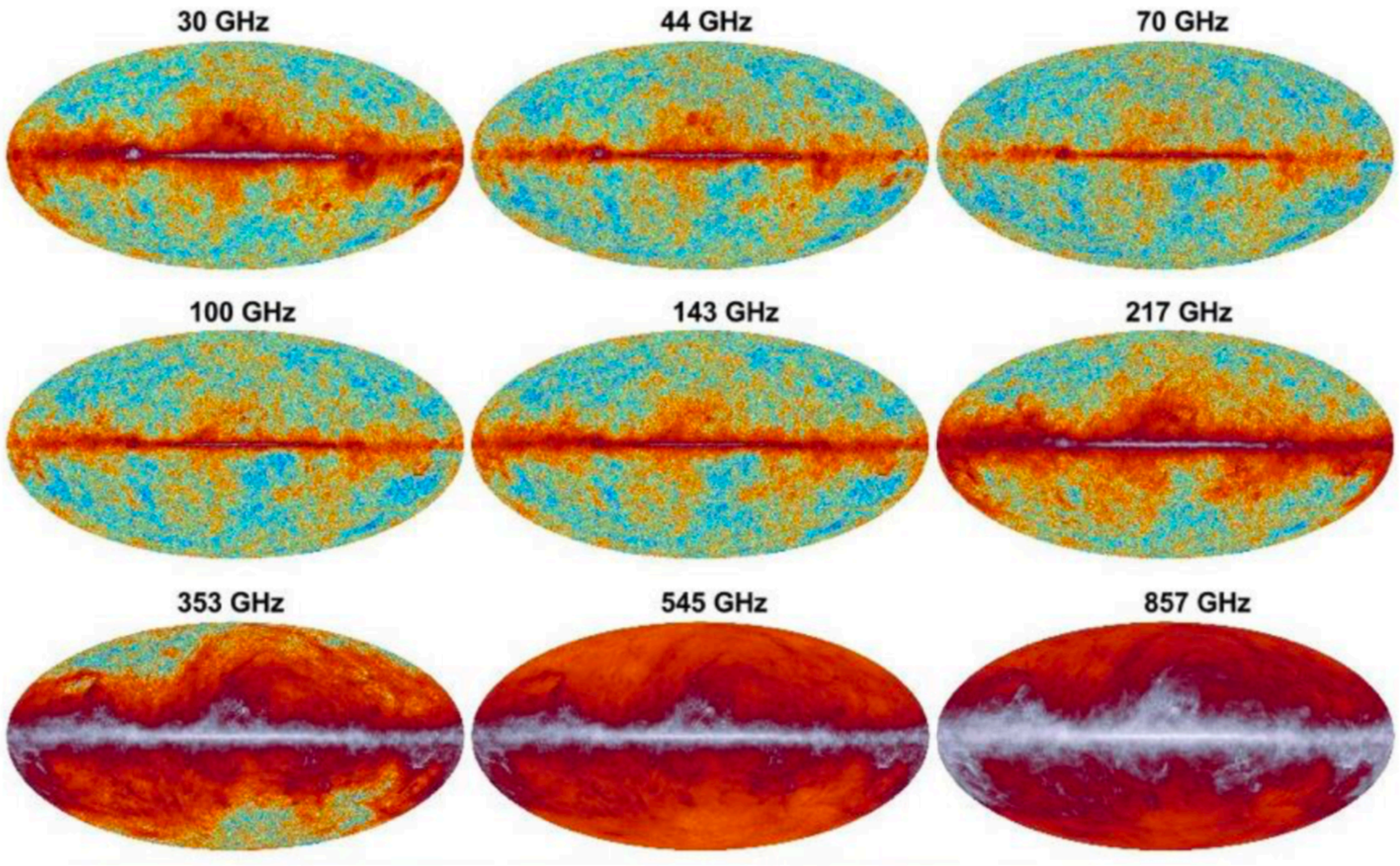


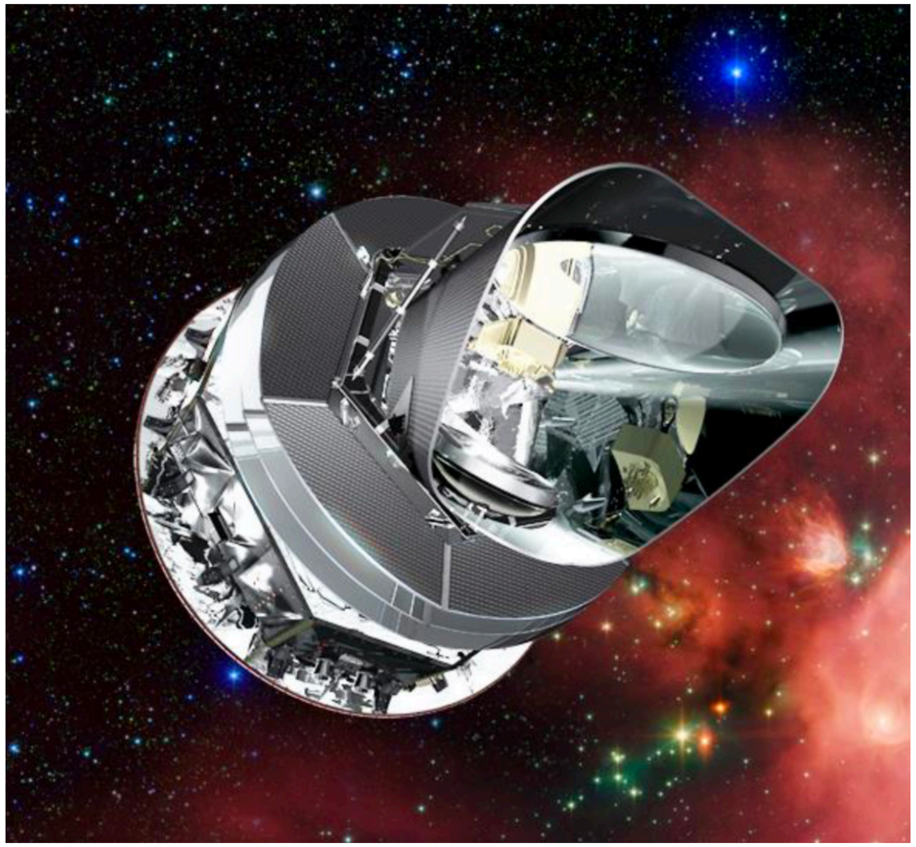


CMB observations: Planck

Observation made in multiple frequencies to reduce the impact of systematic emissions

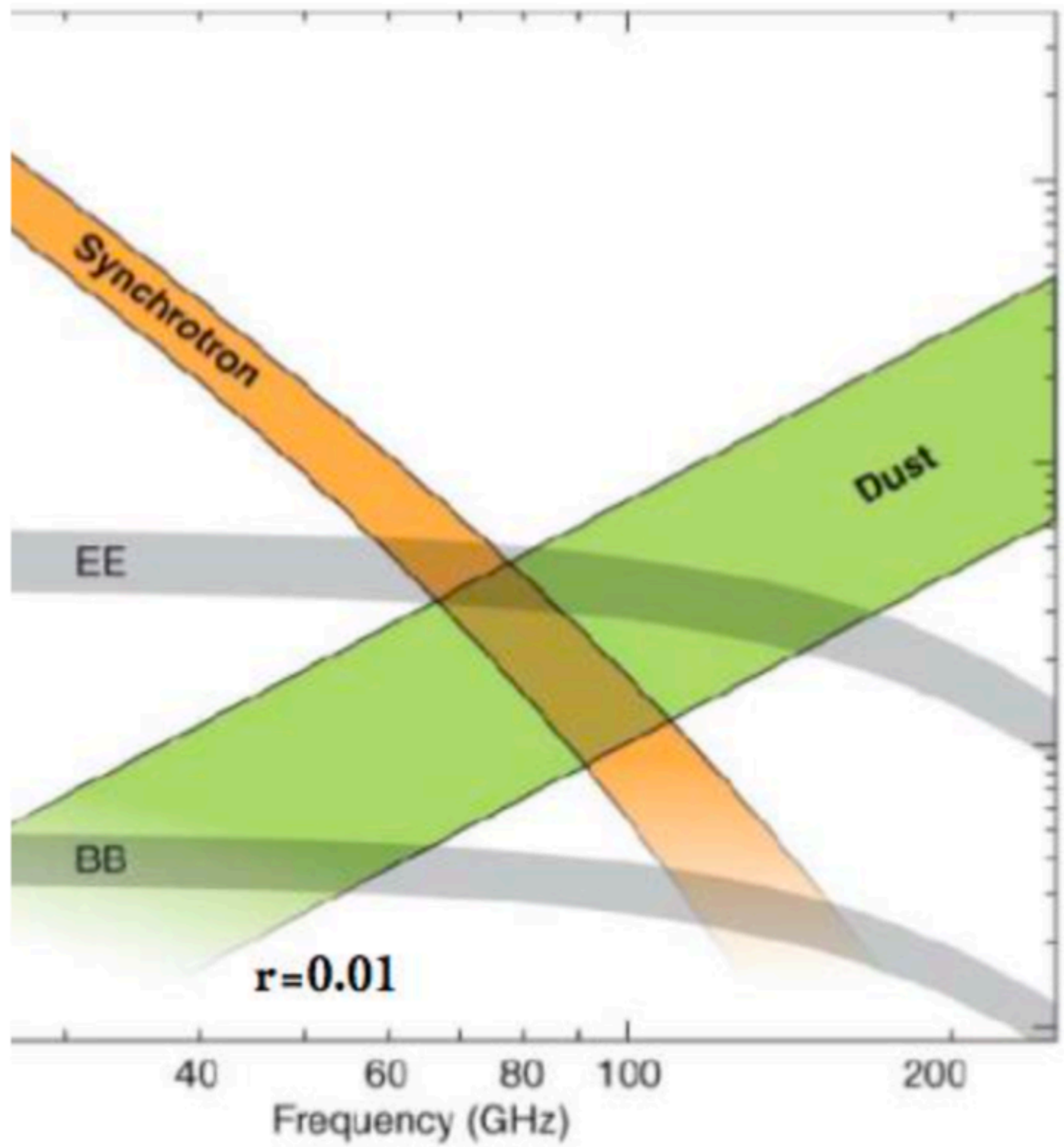






CMB observations: Systematics

Systematic signals have different behaviours with frequency (and they are also smooth.)

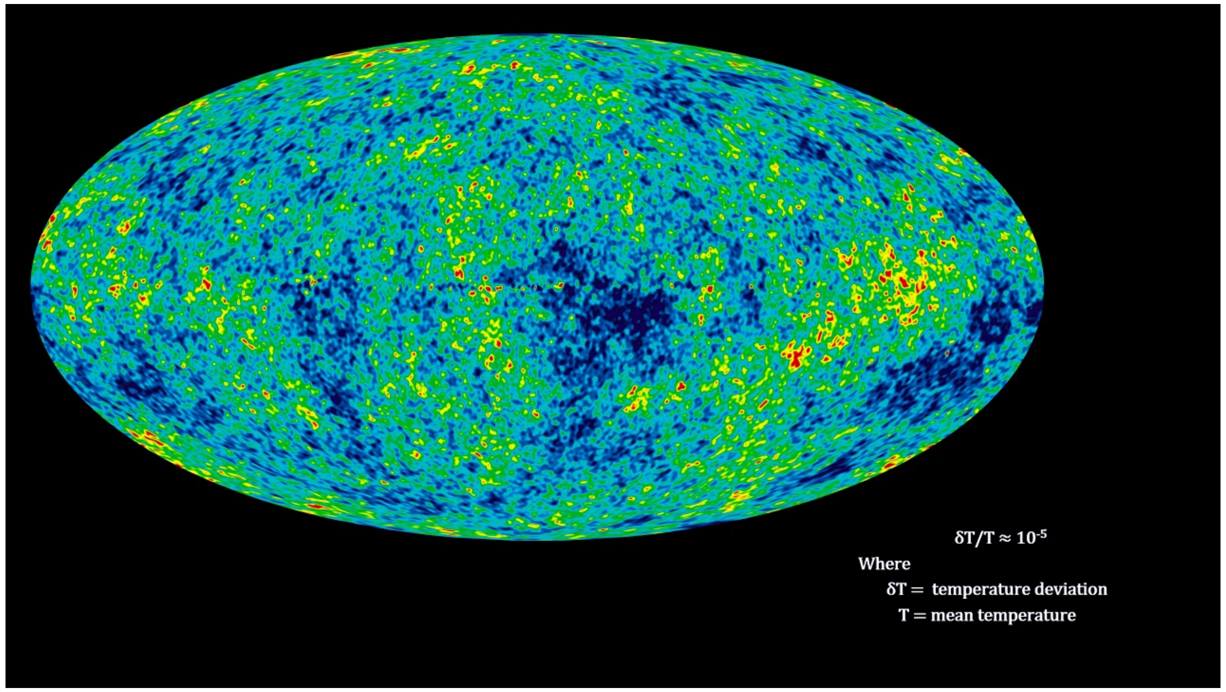


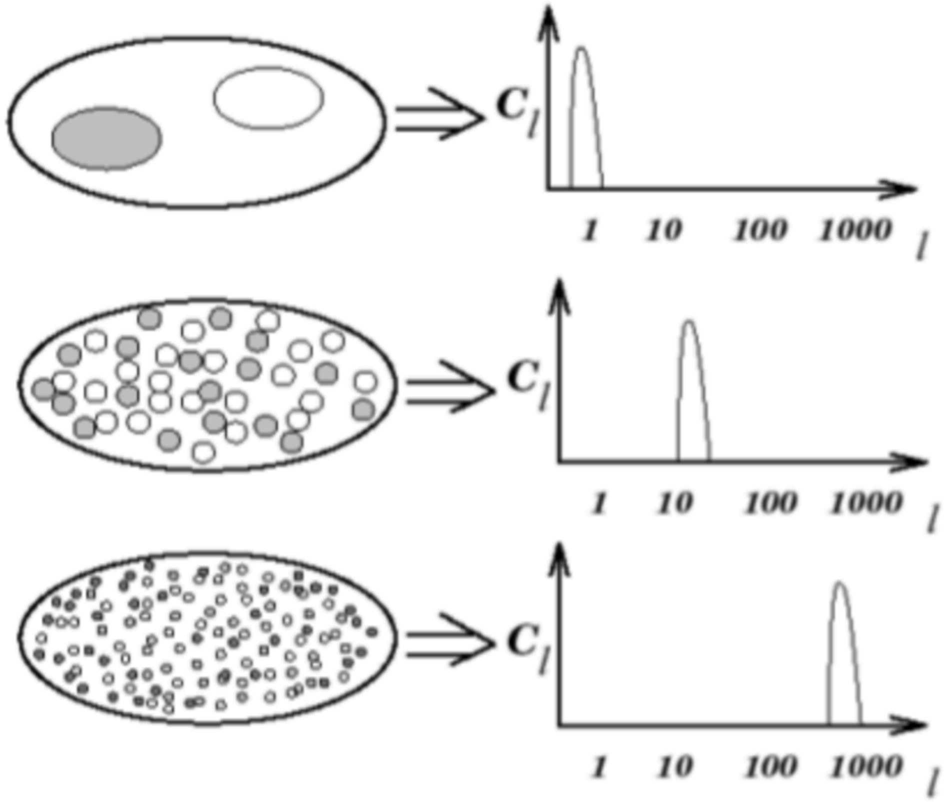
Cosmic Microwave Background radiation anisotropies

Once the observations are cleaned, the measured angular power spectrum of CMB anisotropies contains cosmological information. Basis to establish ΛCDM model

$$\frac{\Delta T}{T}(\theta,\phi) = \sum_{l,m} a_{lm} Y_{lm}(\theta,\phi) \qquad C_l \equiv \left\langle |a_{lm}|^2 \right\rangle$$

Anisotropy — Weight — Spherical harmonics



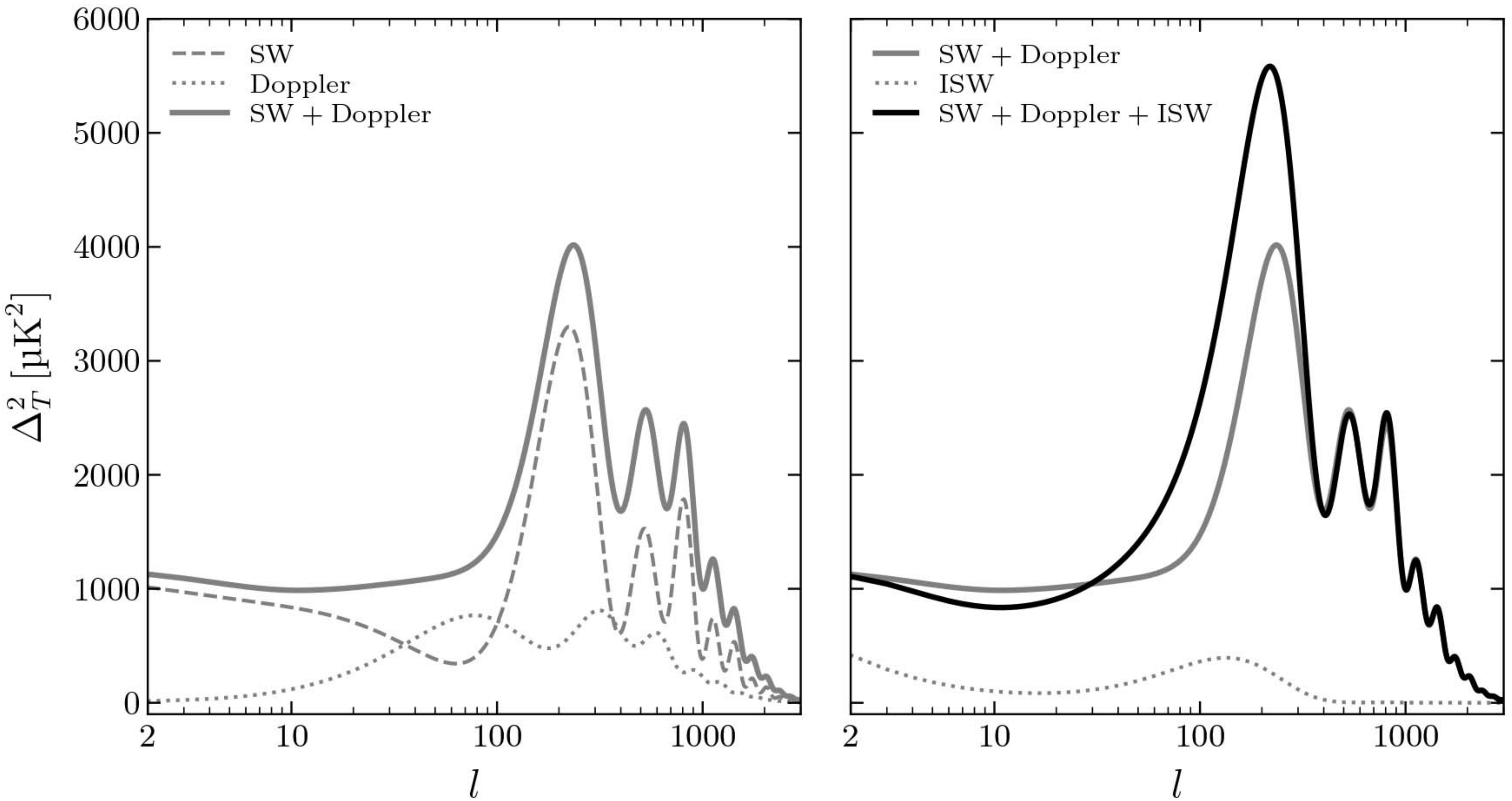


CMB anisotropies power spectrum

There are 3 main contributions to the temperature anisotropies that lead to the power spectrum:

$$\frac{\delta T}{T}(\hat{\mathbf{n}}) = \left(\frac{1}{4}\delta_\gamma + \psi\right)_* - (\hat{\mathbf{n}}\cdot\mathbf{v_b})_* + \int_{\eta_*}^{\eta_\mathbf{o}} \mathbf{d}\eta(\phi' + \psi')$$

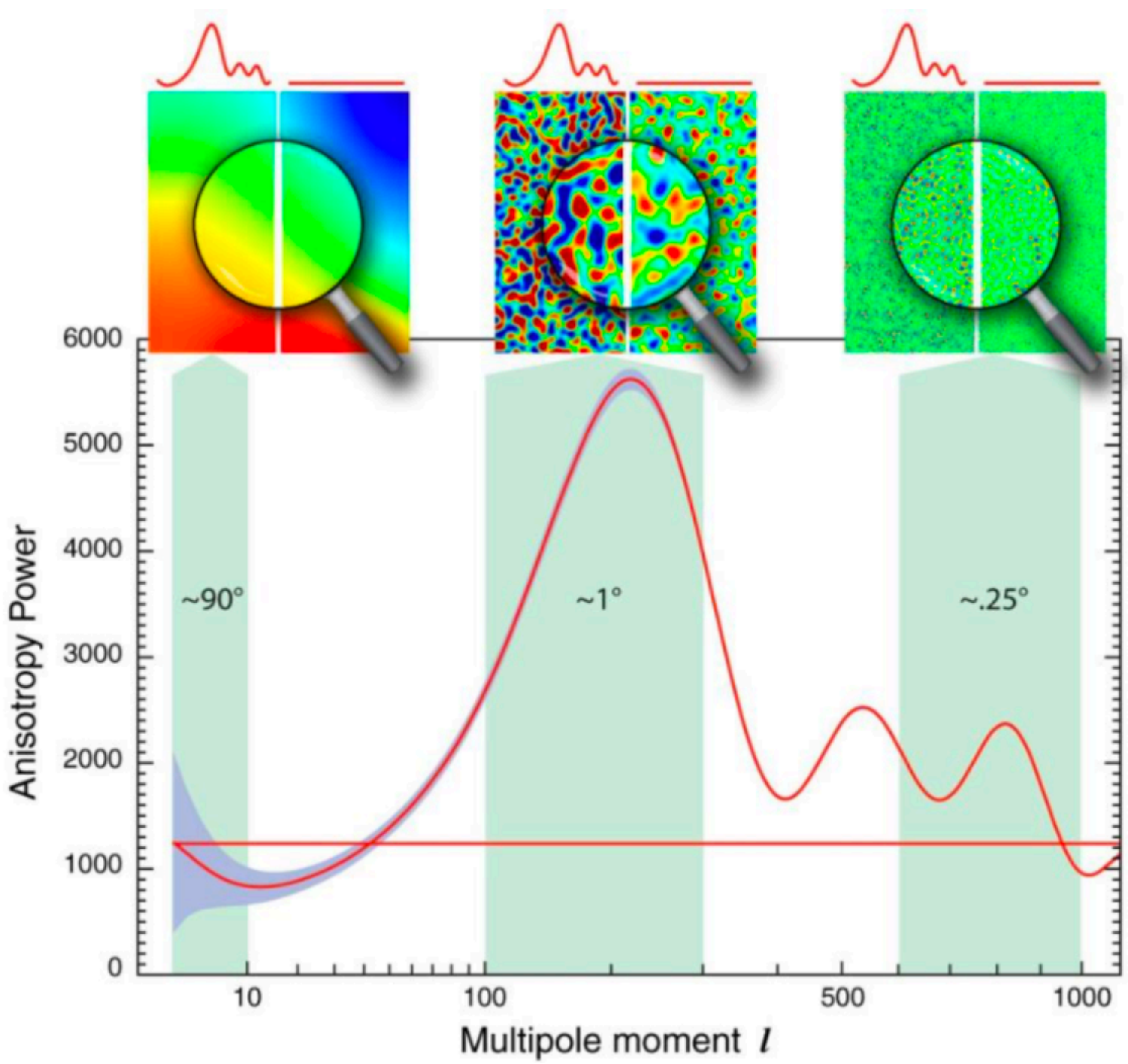
SW **Doppler** **ISW (0 if potential is constant)**

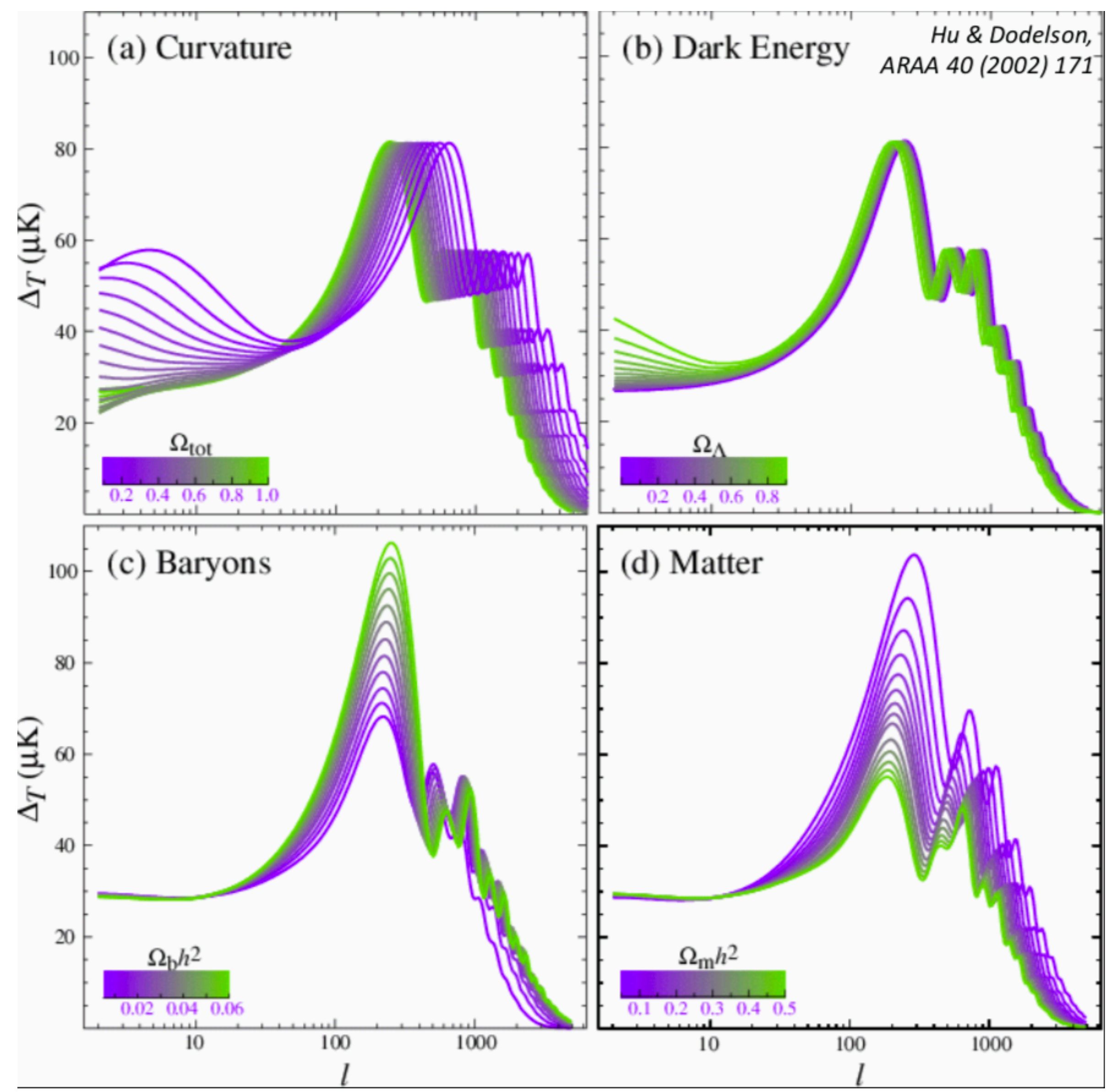


CMB anisotropies power spectrum

Cosmological model success to predict the CMB power spectrum

Role of dark energy at time on early Universe small, important for secondary effects on the photons (integrated Sachs-Wolfe effect)

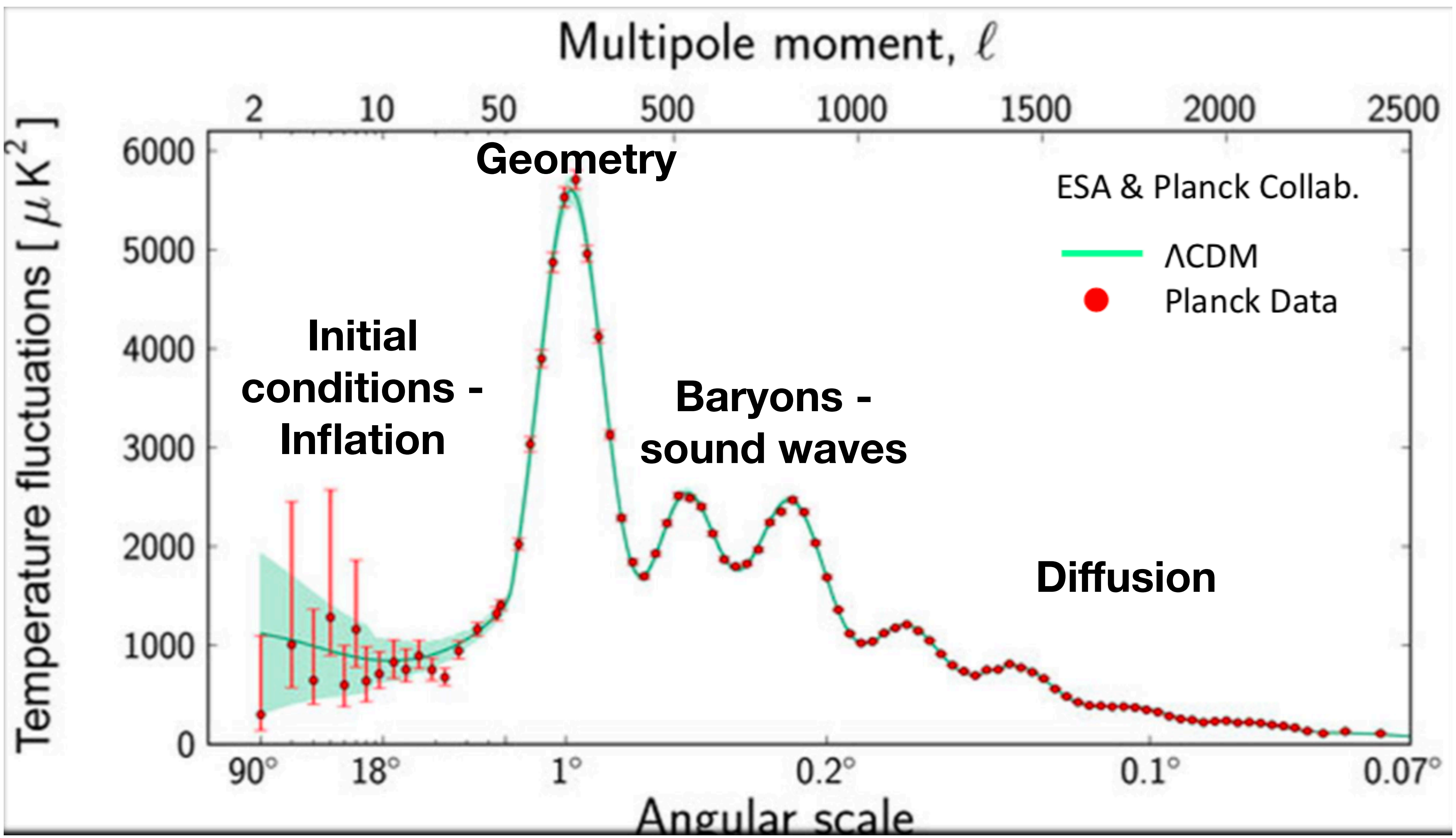




CMB anisotropies as cosmological probe

Position of first peak clear indicator of flatness of the Universe (k=0)

Relation between peaks gives us relation between total matter density and bayrons density.

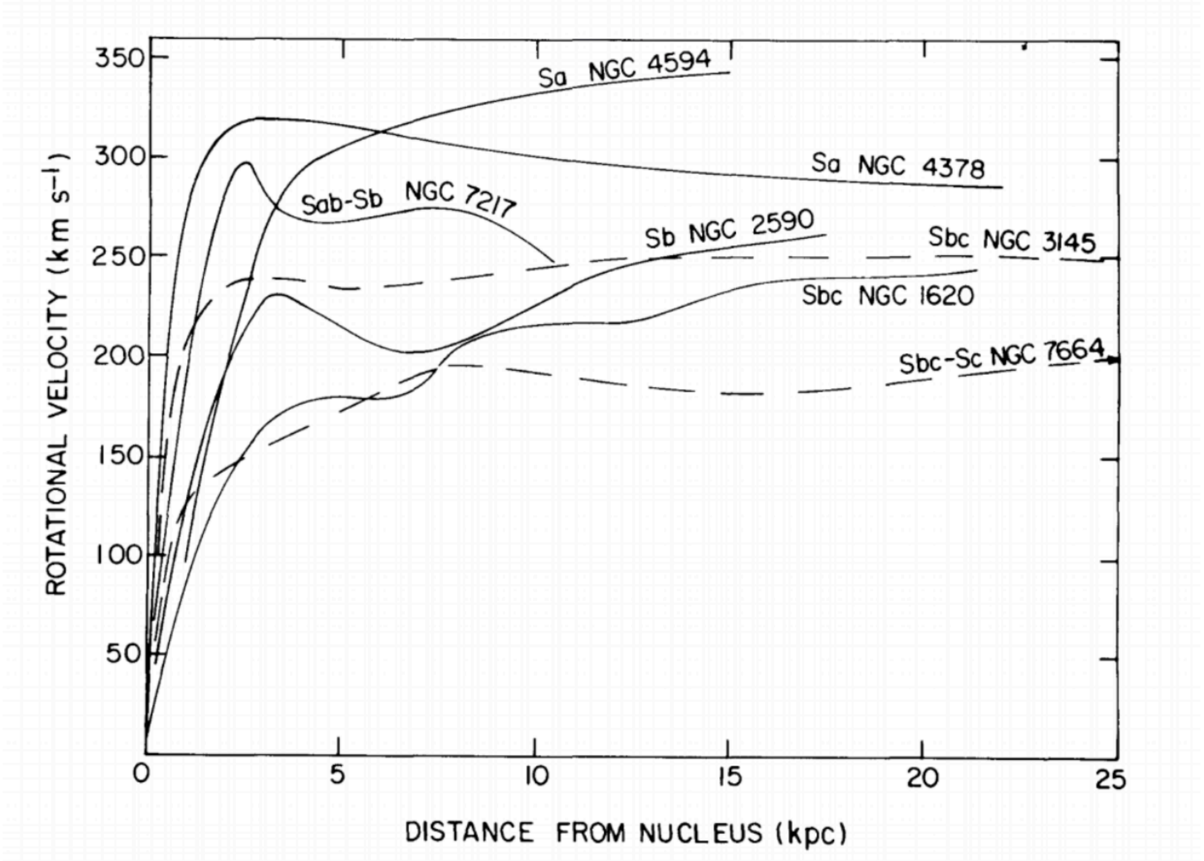


Dark matter (more detail in D. Blas lectures 13/09)

In parallel to all these, another mysterious component of the Universe has been proposed, dark matter.

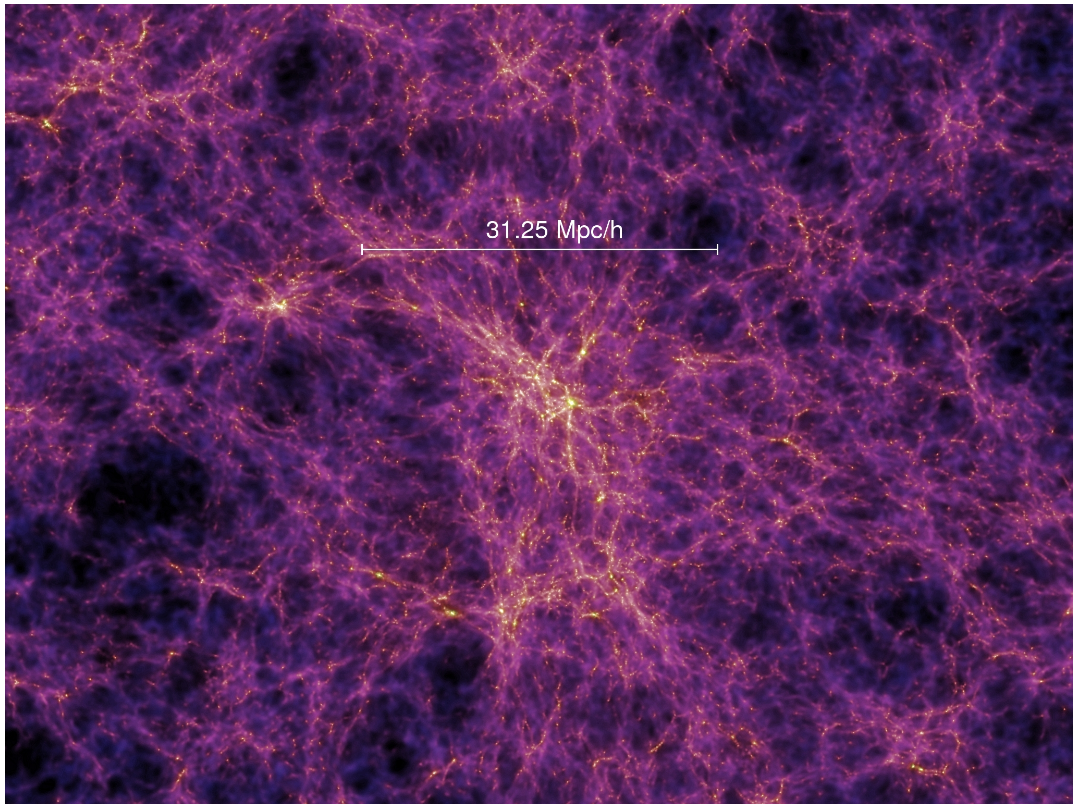
- In the 30s, F. Zwicky showed that the galaxies in the Coma cluster are moving much faster than they should according to the gravity from the visible matter in the cluster. He named the matter component holding the cluster “dunkle materie” (dark matter)

- Then in the 60s and 70s, Vera Rubin measured the rotational curves of spiral galaxies, finding a flat profile at large radii when according to the distribution of matter in the center of the galaxy the profile should fall.



Dark matter

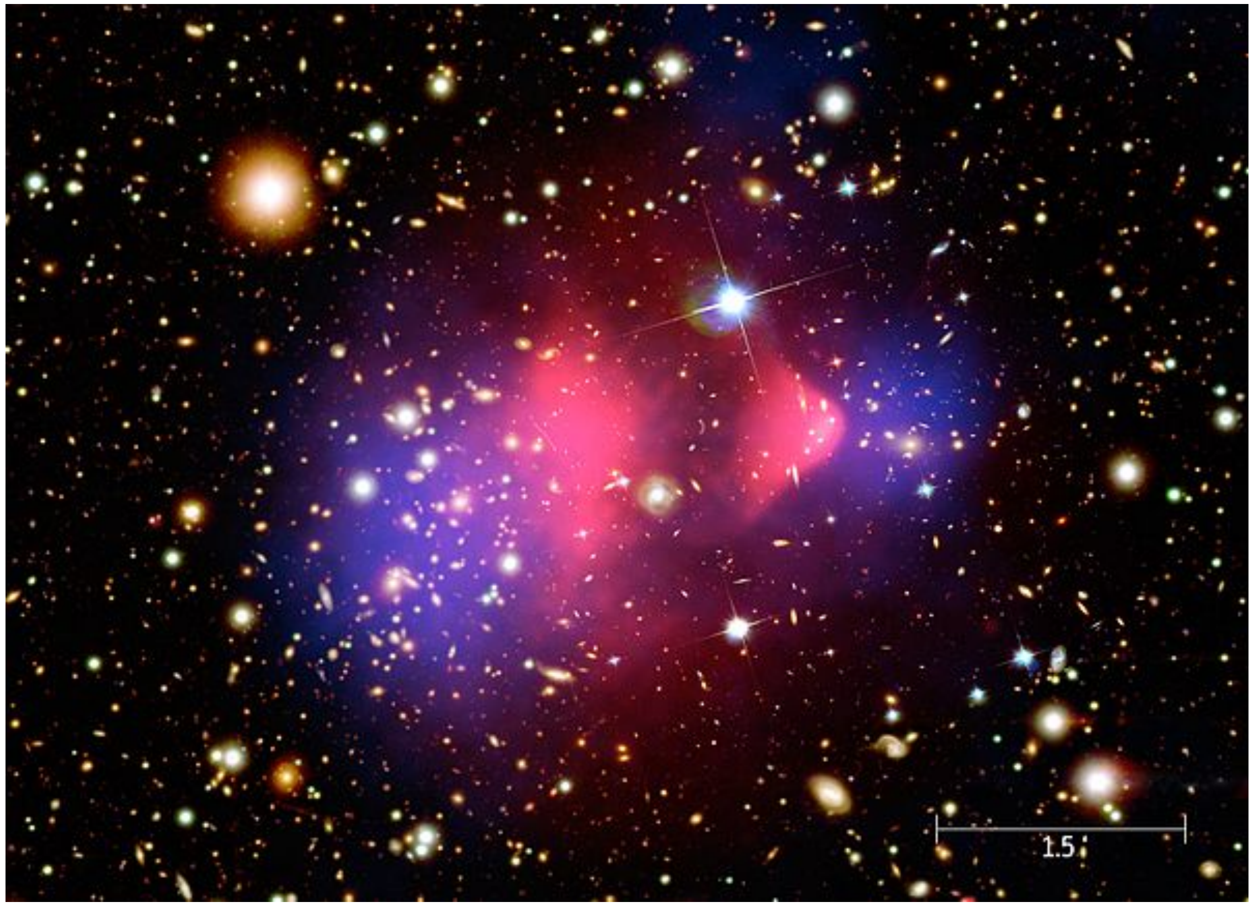
Once we started observing the cosmic web, also the presence of DM is needed to glue it and get the structures to growth up to what we see today.



This same evidence is why the theoretical fit to the CMB anisotropies need the existence of DM as the largest component of matter to

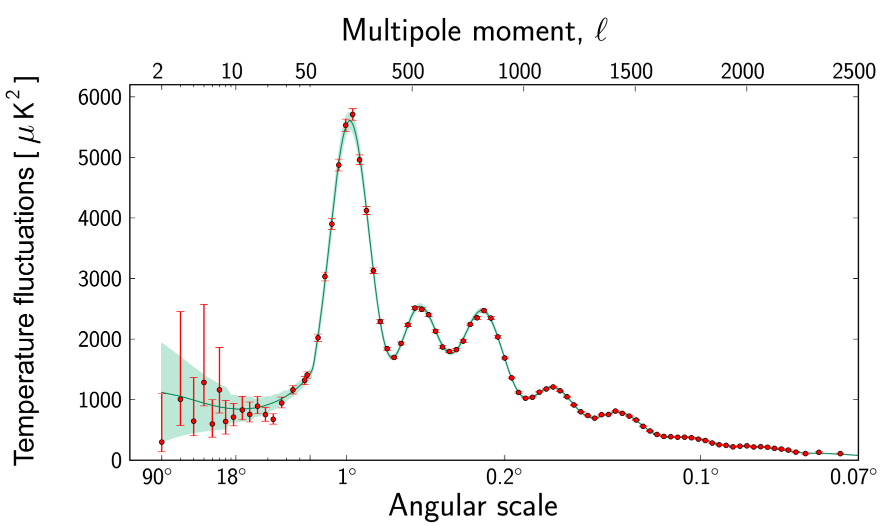
Dark matter

And also we can see the dark matter component is colisiones (cold) by comparing the distribution of a weak lensing image of the Bullet cluster (mapping all matter) with the hot gas distribution after the collision of the 2 clusters

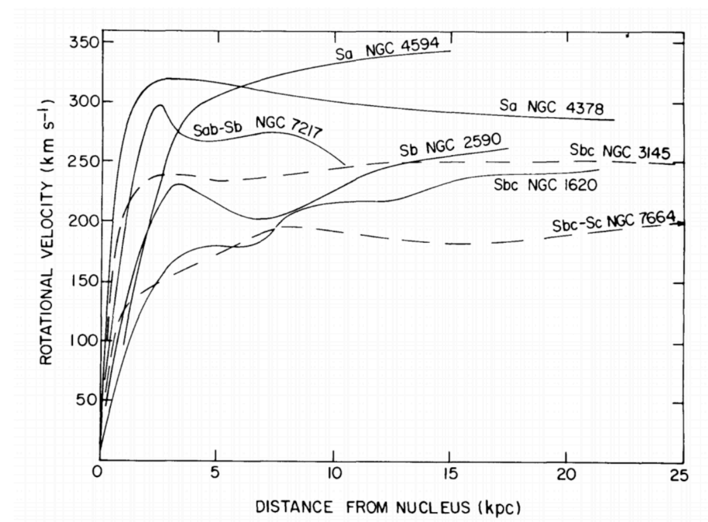


Dark matter

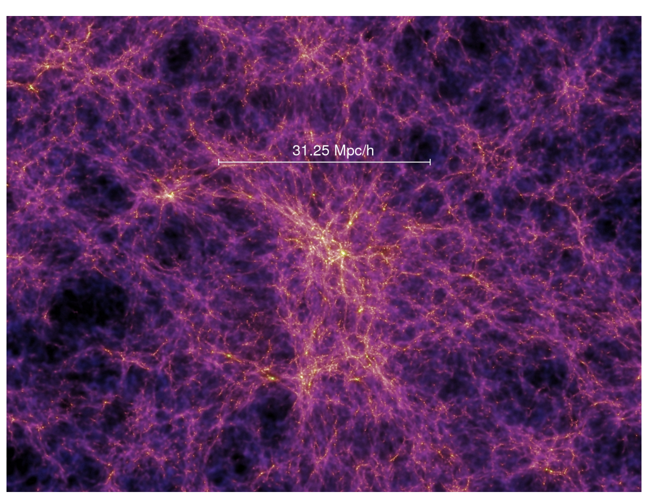
In summary, there are several astrophysical indirect evidences of DM but we still need to find the nature of this component.



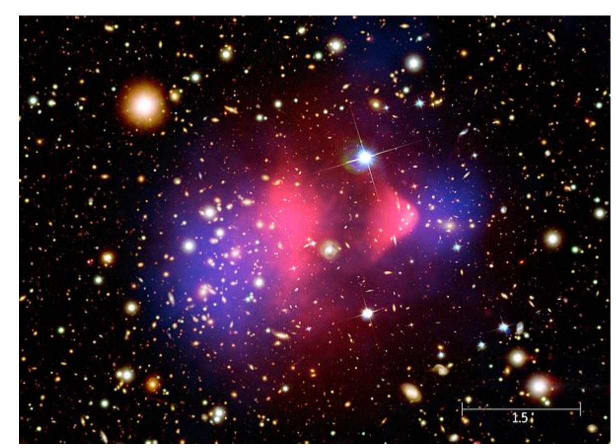
Planck Collaboration



V. Rubin et al. 1978



Cold Dark Matterer in Large-scale structure

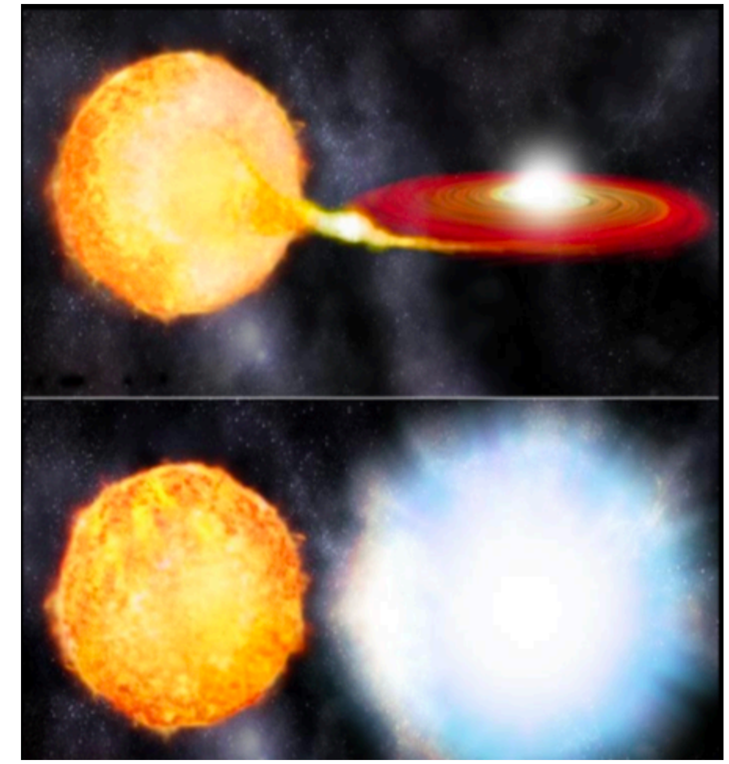


Bullet Cluster

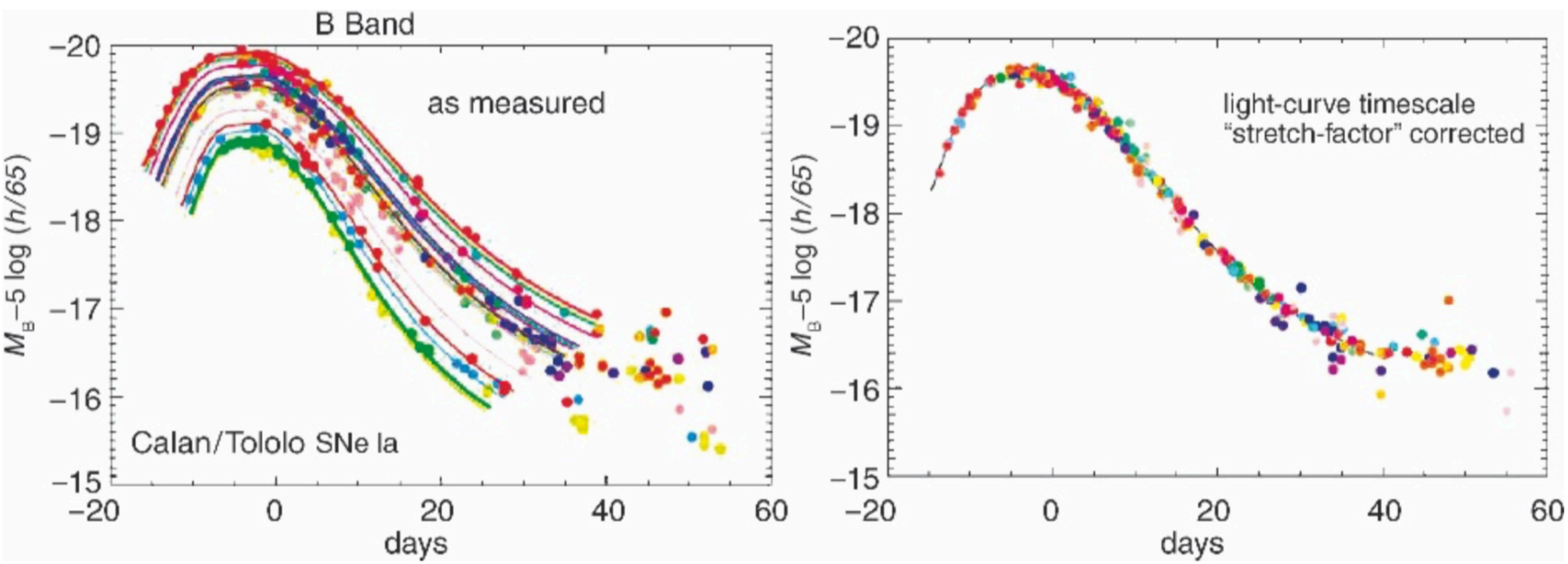
Standard candles at high redshift

The most successful standard candle in the last decades have been the type Ia SN (produced by a red giant-white dwarf binary)

We need first to classify the SN by using the SN spectrum (which can also give us the redshift) and we also need the photometric like curves



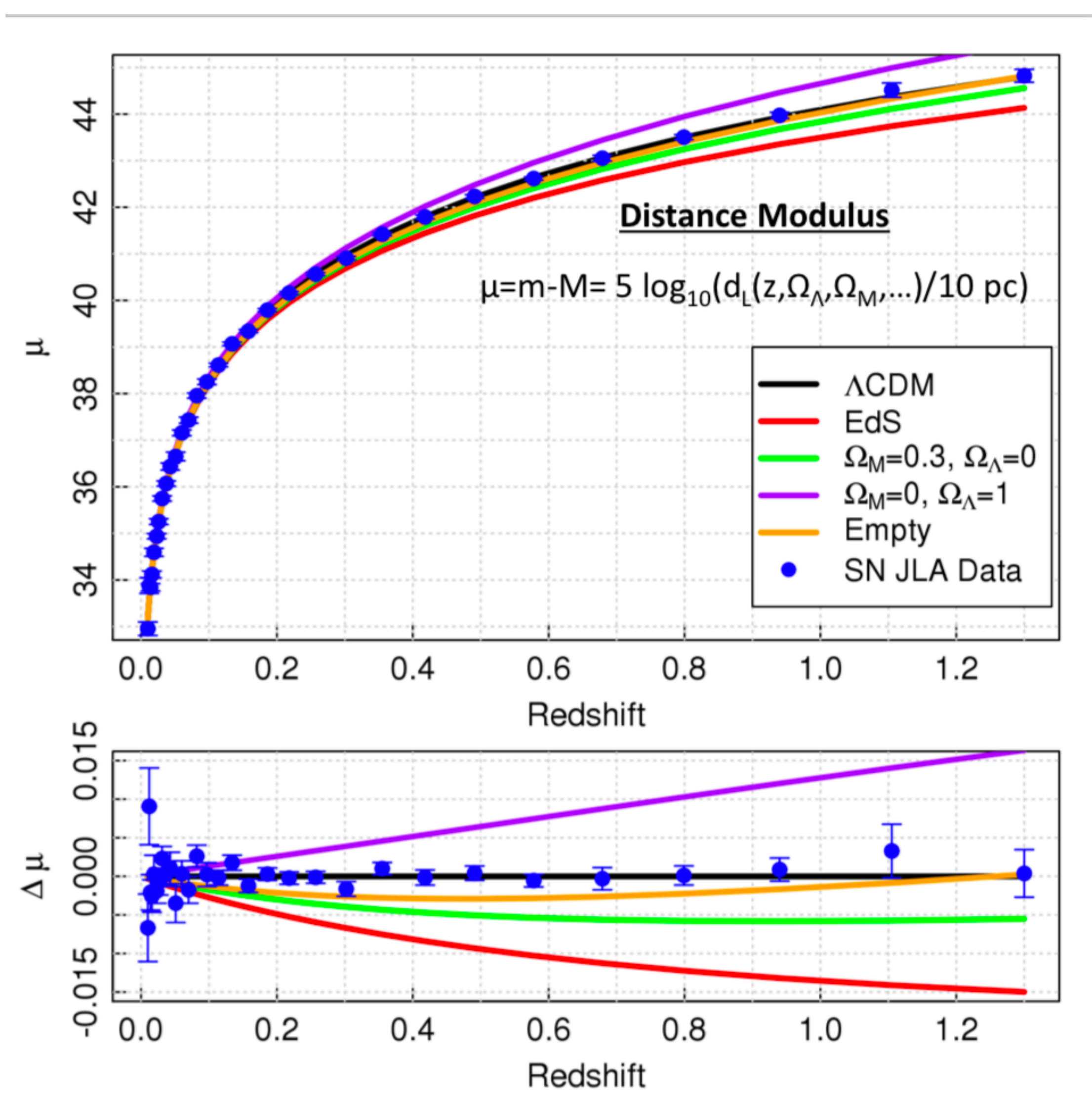
$$\mu = m_B^\star - (M_B - \alpha \times X_1 + \beta \times C)$$

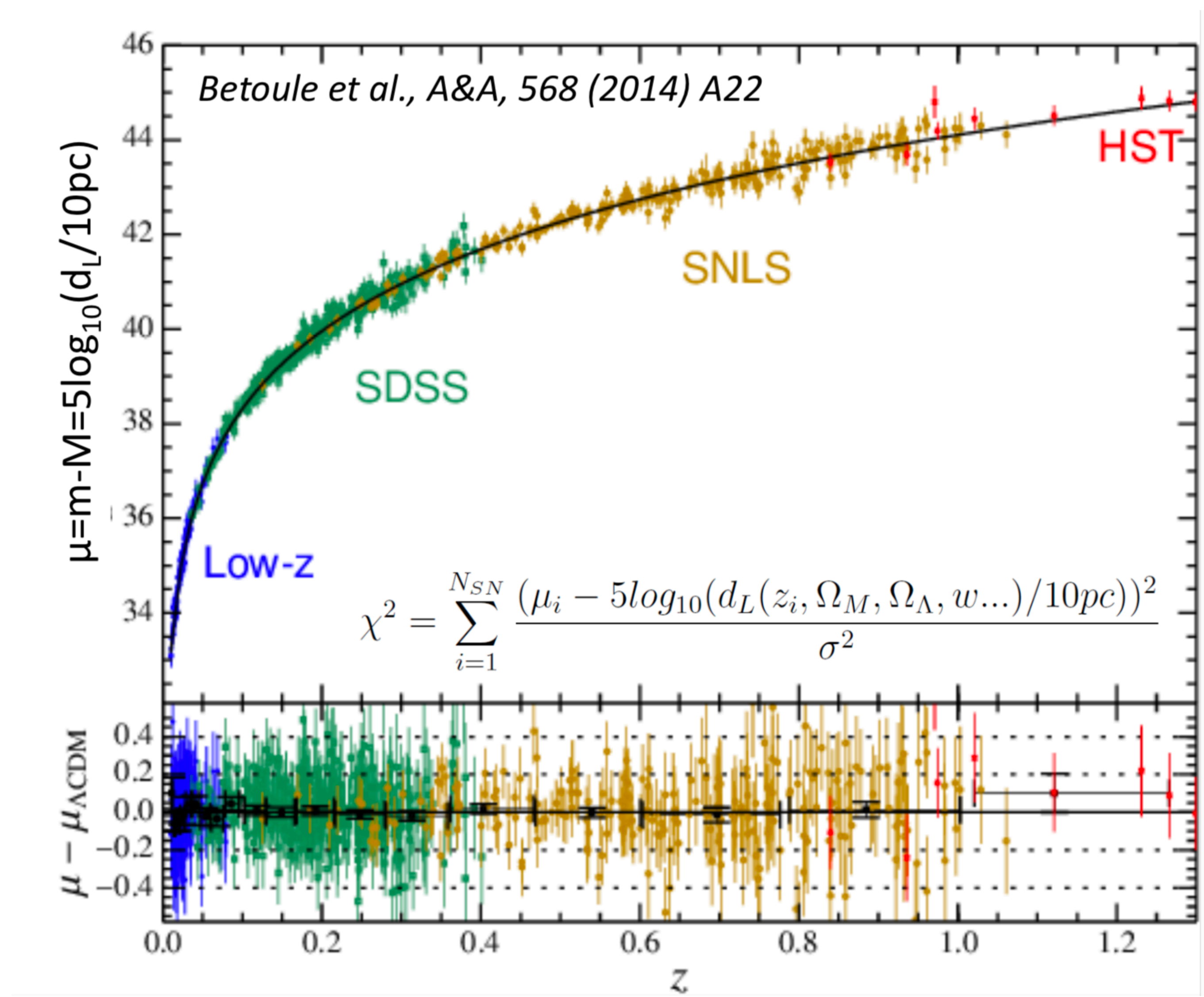


Expansion of the Universe (standard candles)

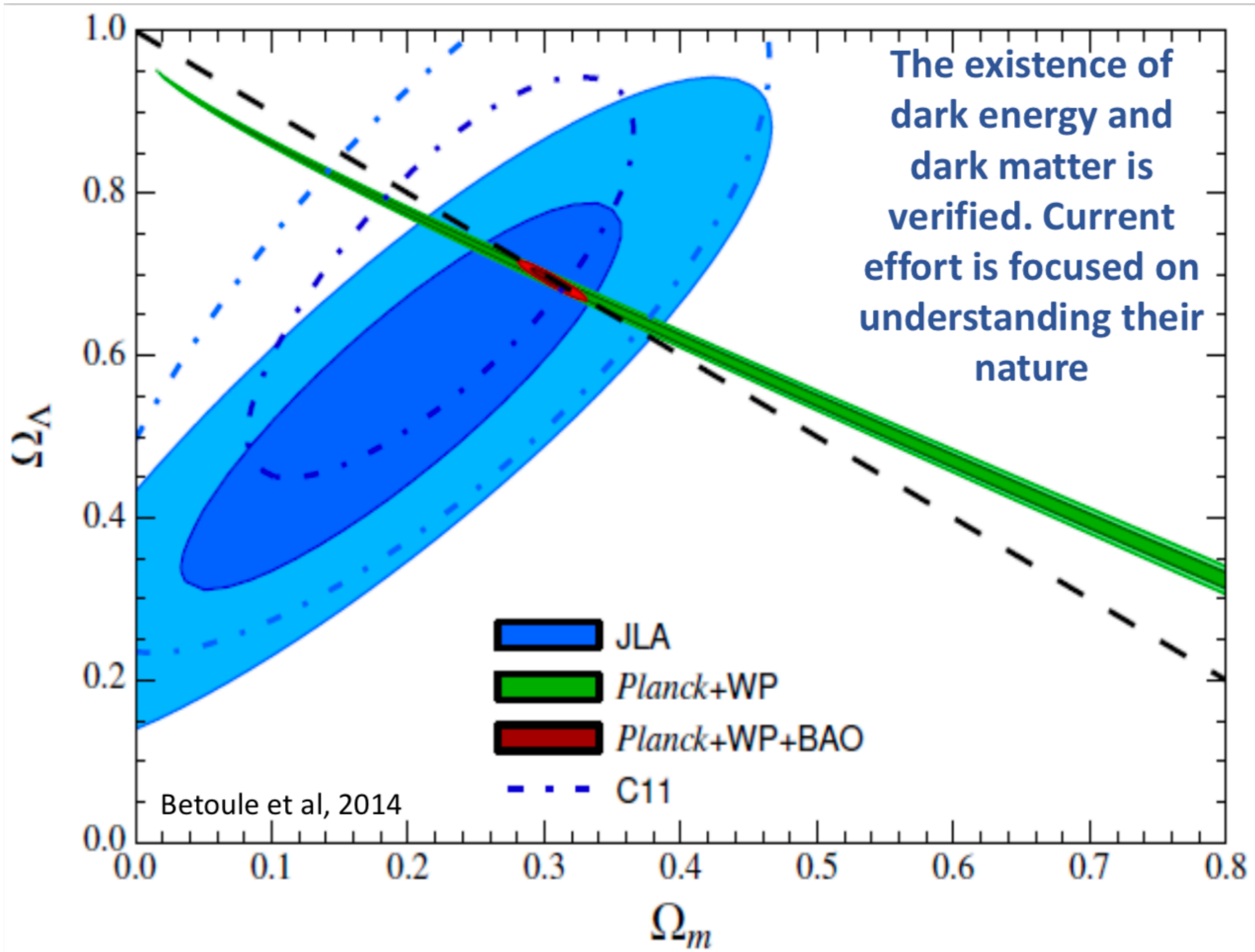
Comparing the distance modulus (measured from light curves) and the redshift, we can determine which geometry of the Universe explains the data better.

$$\mu \equiv m - M = 5 \log D_L(z) + 25$$





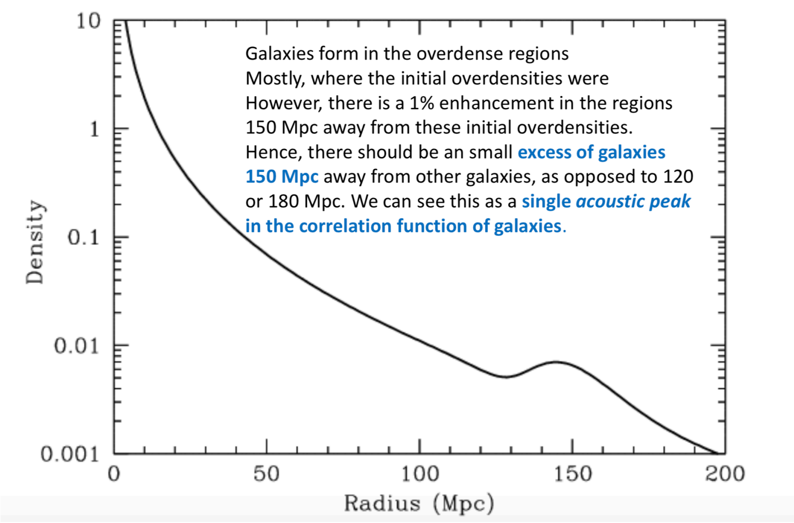
Universe accelerated expansion

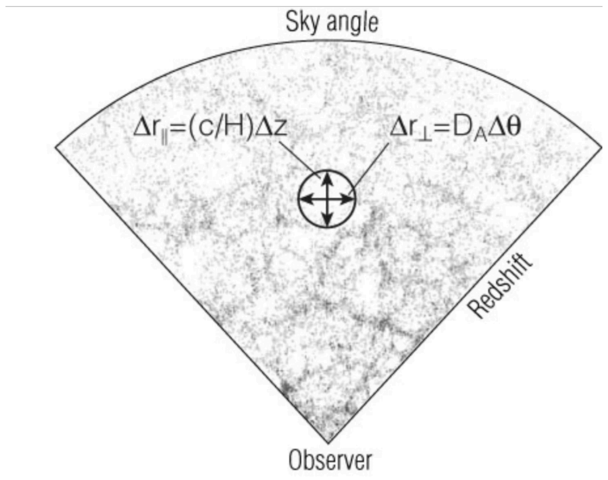


Type Ia SN showed (with 2sigma) that the expansion of the Universe is accelerating. This implies either the Einstein equations have to include the cosmological constant or that we need to modify gravitation theory. If the acceleration is produced by a university fluid, then we denominate it **dark energy.**

Baryonic acoustic oscillations (BAO)

If we measure the BAO over density peak at different times, we can trace the expansion rate.





$$r_s(z_{dec}) = \frac{c}{\sqrt{3}} \int_0^{1/(1+z_{dec})} \frac{da}{a^2 H(a)\sqrt{1+(3\Omega_b/4\Omega_\gamma)a}} \text{ Mpc h}^{-1}$$

CMB o BBN

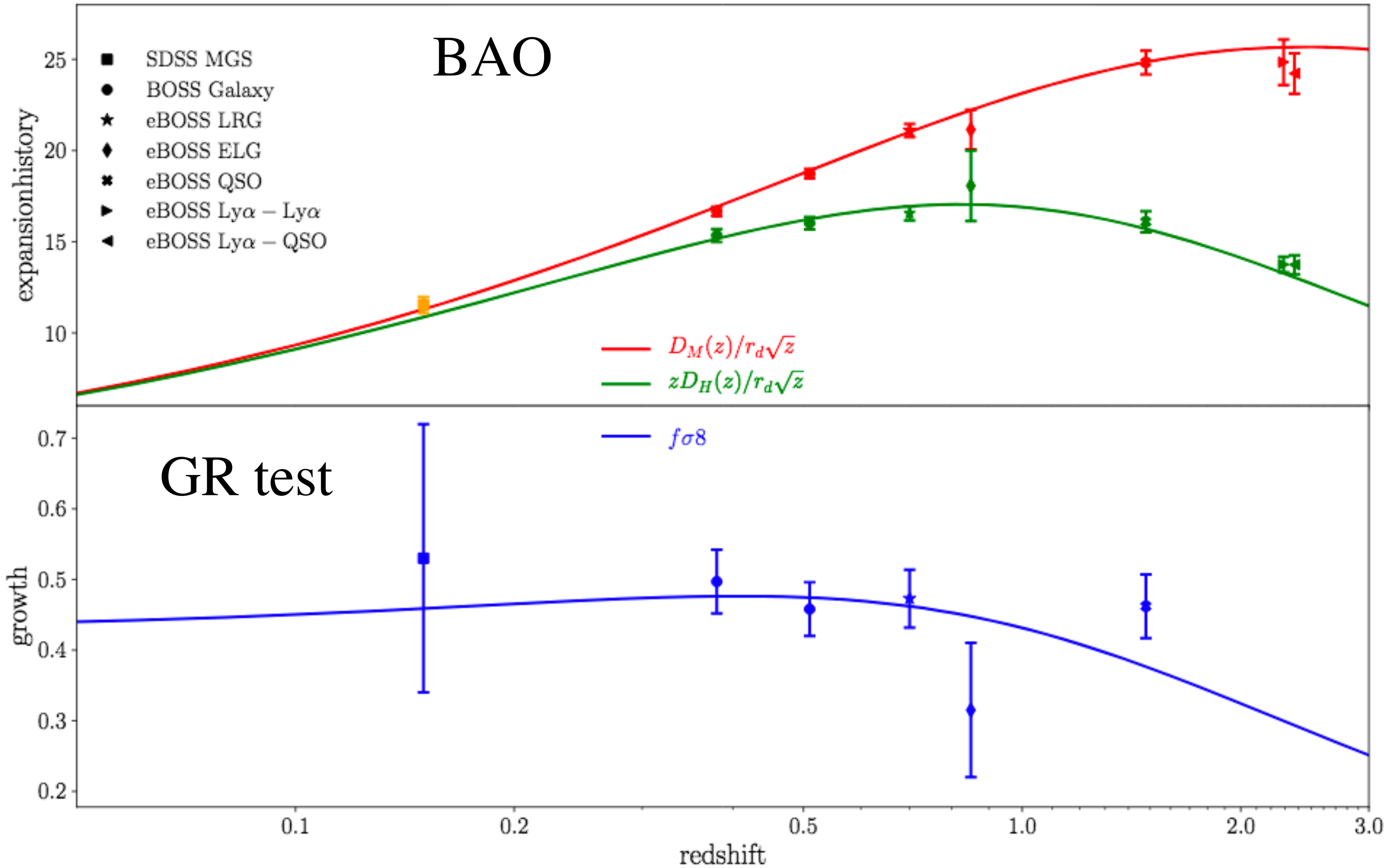
$$\theta_{BAO} = \frac{r_S(\Omega_M, w_0, w_a ...)}{(1+z)\ d_A(z, \Omega_M, w_0, w_a ...)}$$

$$\Delta z_{BAO} = H(z) r_s$$

We can determine r_s either with CMB or BBN and then compare with extragalactic surveys measurements

Expansion of the Universe (standard rulers)

Current results agree quite well with ΛCDM model



eBOSS collaboration 2020

Now we have the new **DESI data** but we leave them for the last part of the lectures.

3x2pt probes

- Even new probes like weak lensing ones seem to confirm ΛCDM model

Joint constraints

Combining all these data sets we find:

In ΛCDM:

$$S_8 = 0.812^{+0.008}_{-0.008} \quad (0.815)$$

$$\Omega_\mathrm{m} = 0.306^{+0.004}_{-0.005} \quad (0.306)$$

$$\sigma_8 = 0.804^{+0.008}_{-0.008} \quad (0.807)$$

$$h = 0.680^{+0.004}_{-0.003} \quad (0.681)$$

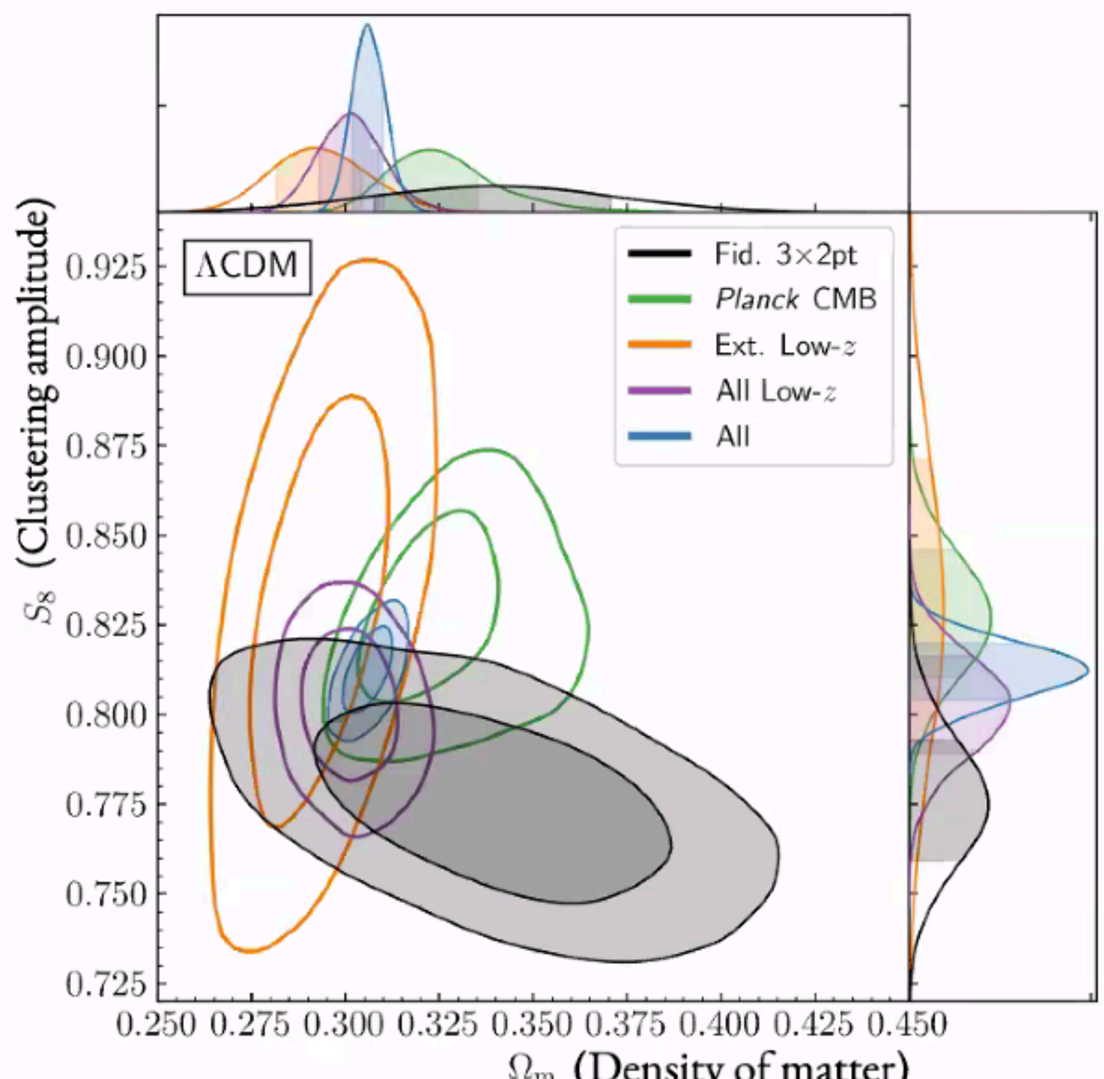
$$\sum m_\nu < 0.13 \text{ eV } (95\% \text{ CL})$$

In wCDM:

$$\sigma_8 = 0.810^{+0.010}_{-0.009} \quad (0.804)$$

$$\Omega_\mathrm{m} = 0.302^{+0.006}_{-0.006} \quad (0.298)$$

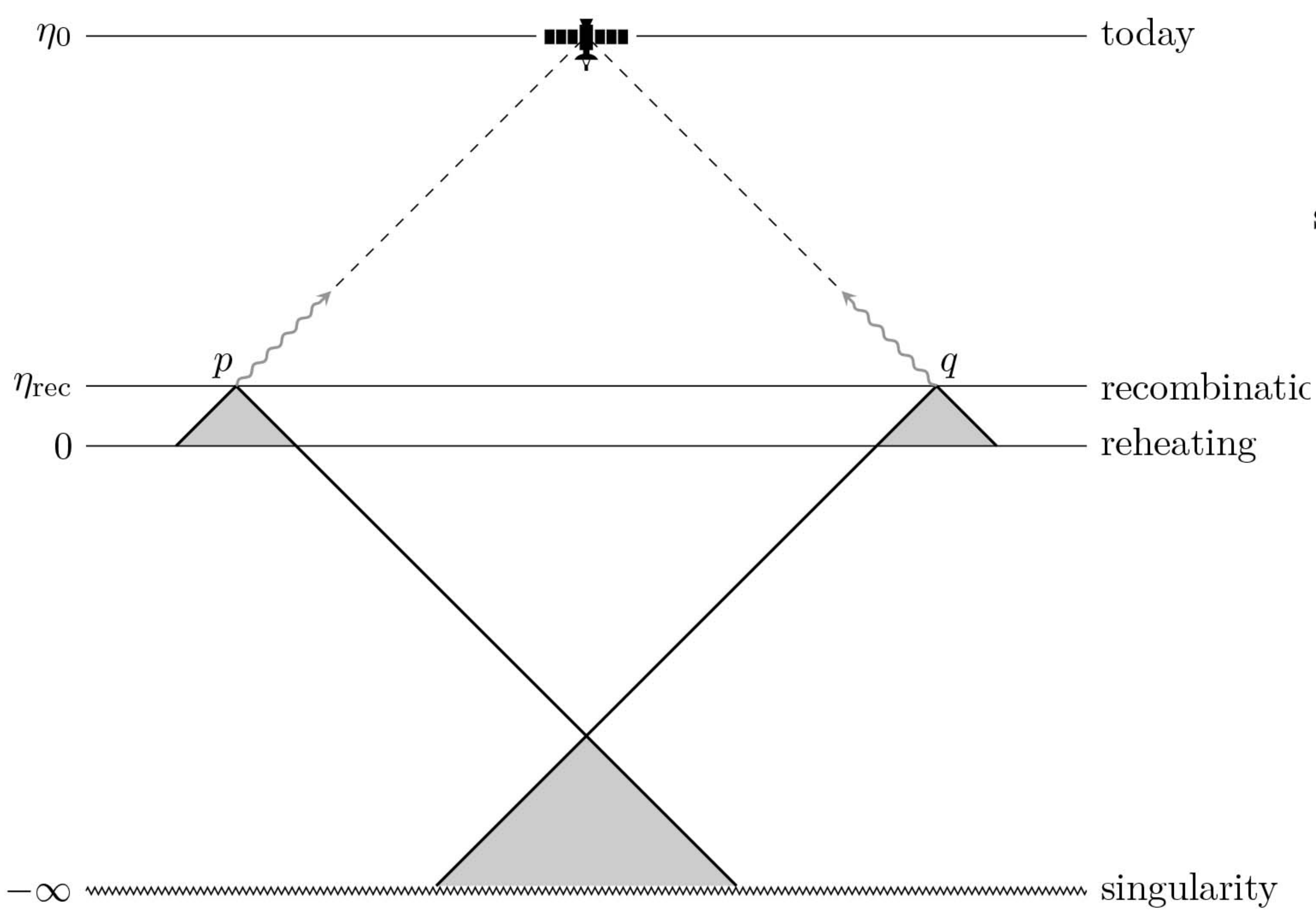
$$w = -1.03^{+0.03}_{-0.03} \quad (-1.00)$$

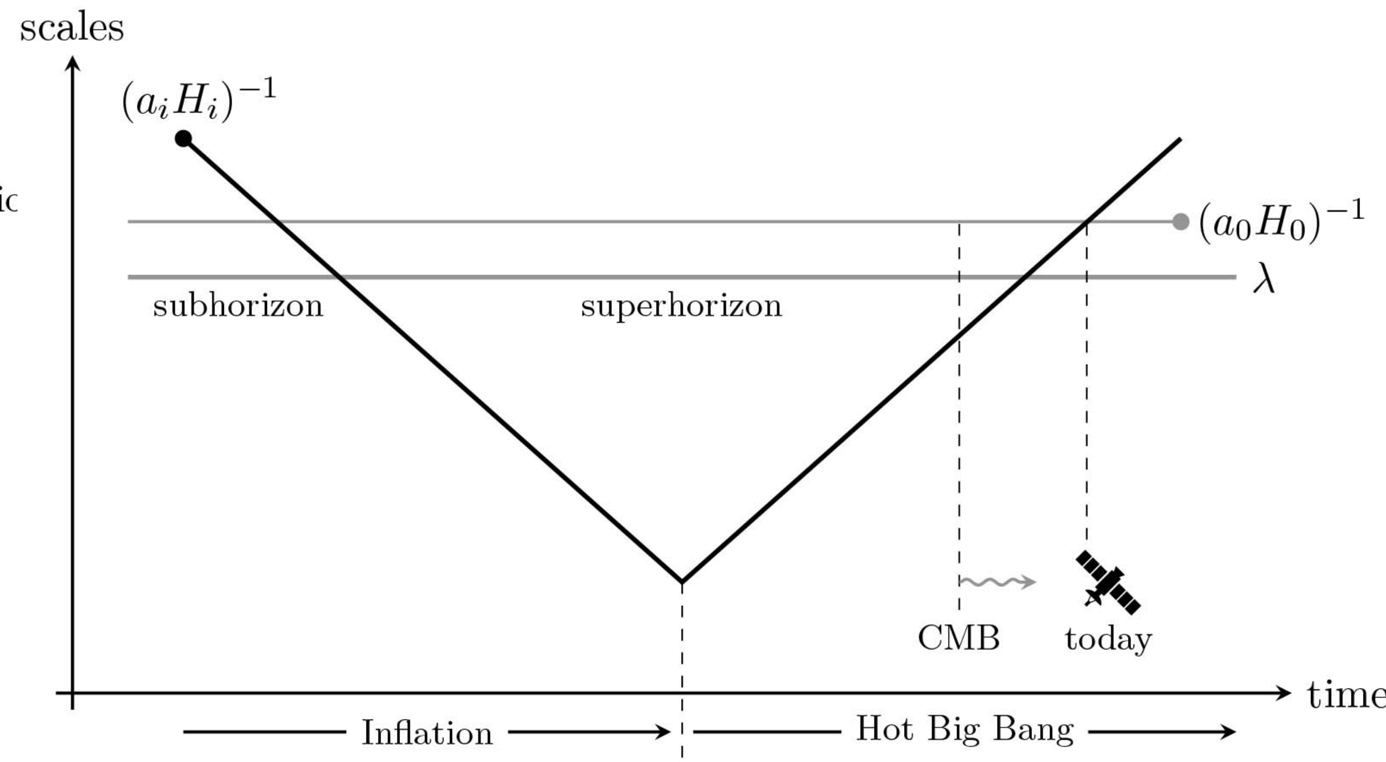


3) Inflation

- Inflation consists on an accelerated phase of the Universe in the early stages. In terms of geometry there are 2 main problems:
    - Flatness problem (without inflation the Universe has to be really well fine tuned)
    - Horizon problem (without inflation it would be impossible for far apart regions to be causally connected)
- Problems are solved with a decreasing Hubble radius in the early Universe

$$\frac{d}{dt}(aH)^{-1} < 0 \quad \text{where} \quad a \propto e^{Ht}, \; H = cnt \quad \text{and} \quad \ddot{a} > 0$$





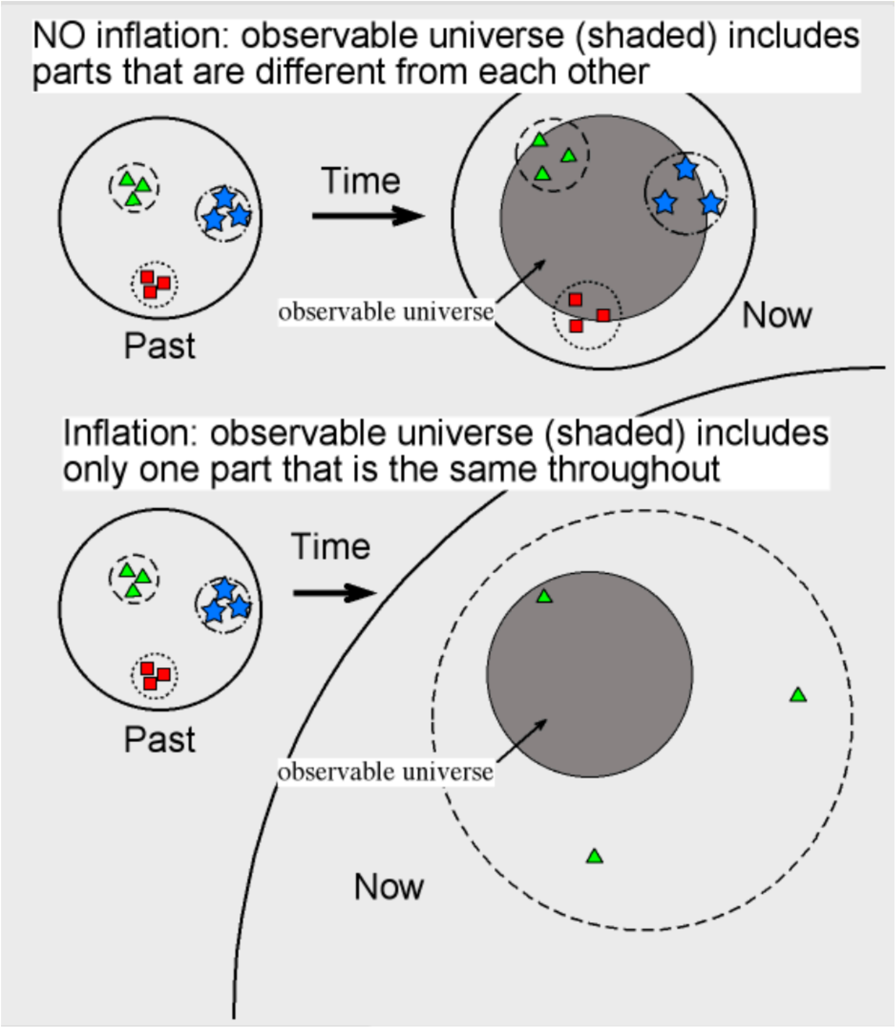
Initial conditions (aka inflation)

Currently, the theory more used by the community to explain the initial conditions is cosmic **Inflation**

It consists on an accelerated phase at the beginning of the Universe which allows us to explain the smoothness of the Universe.

Indirectly, it also provides a prediction for the initial distribution of matter:

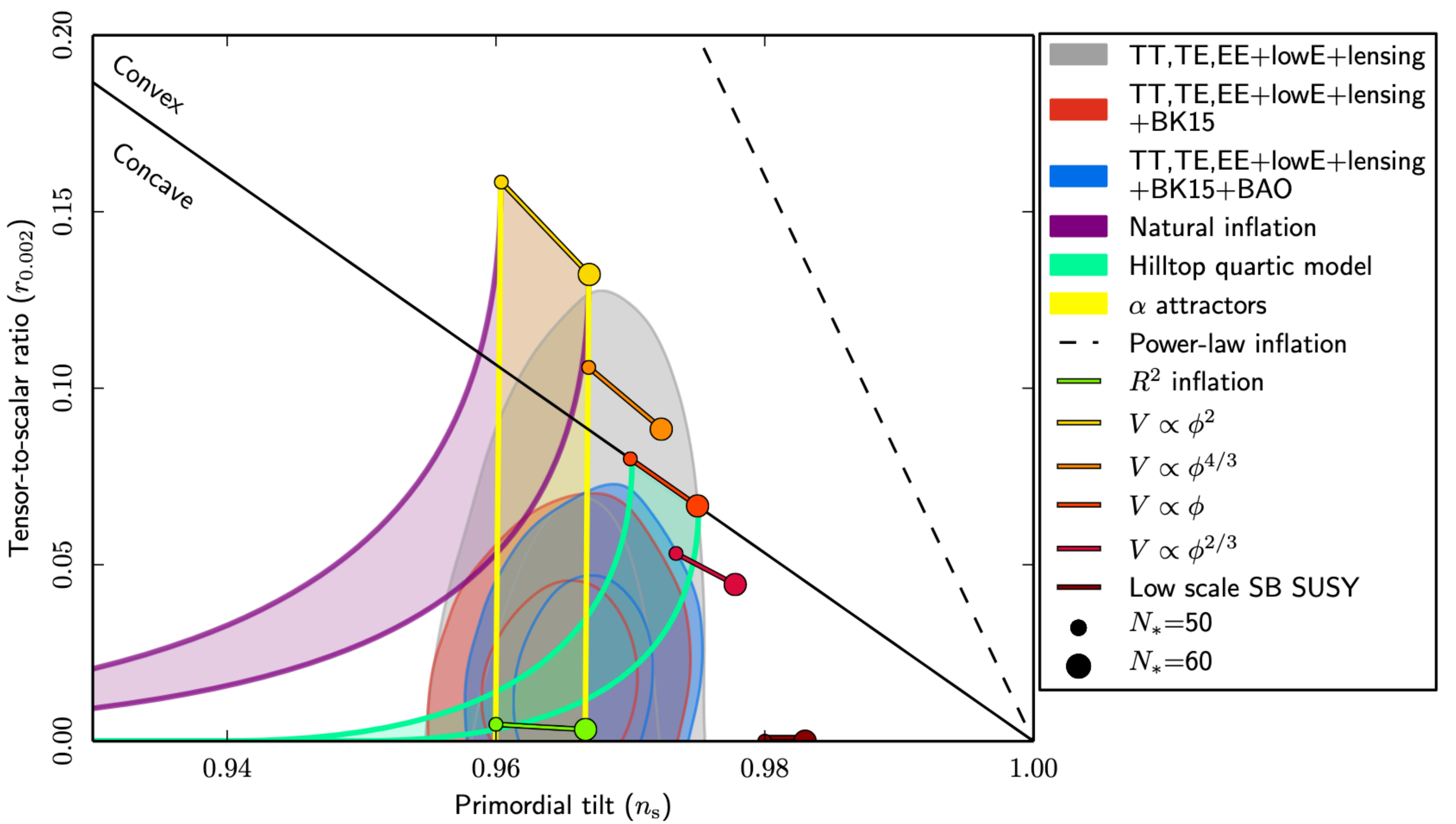
$$\mathcal{P}_\chi(k) = A_s \left(\frac{k}{k_{s0}}\right)^{n_s - 1} .$$



Not directly proven, this spectrum is one of the most successful predictions from inflation. A detection of the primordial B-modes of the CMB polarisation would be a much better detection.

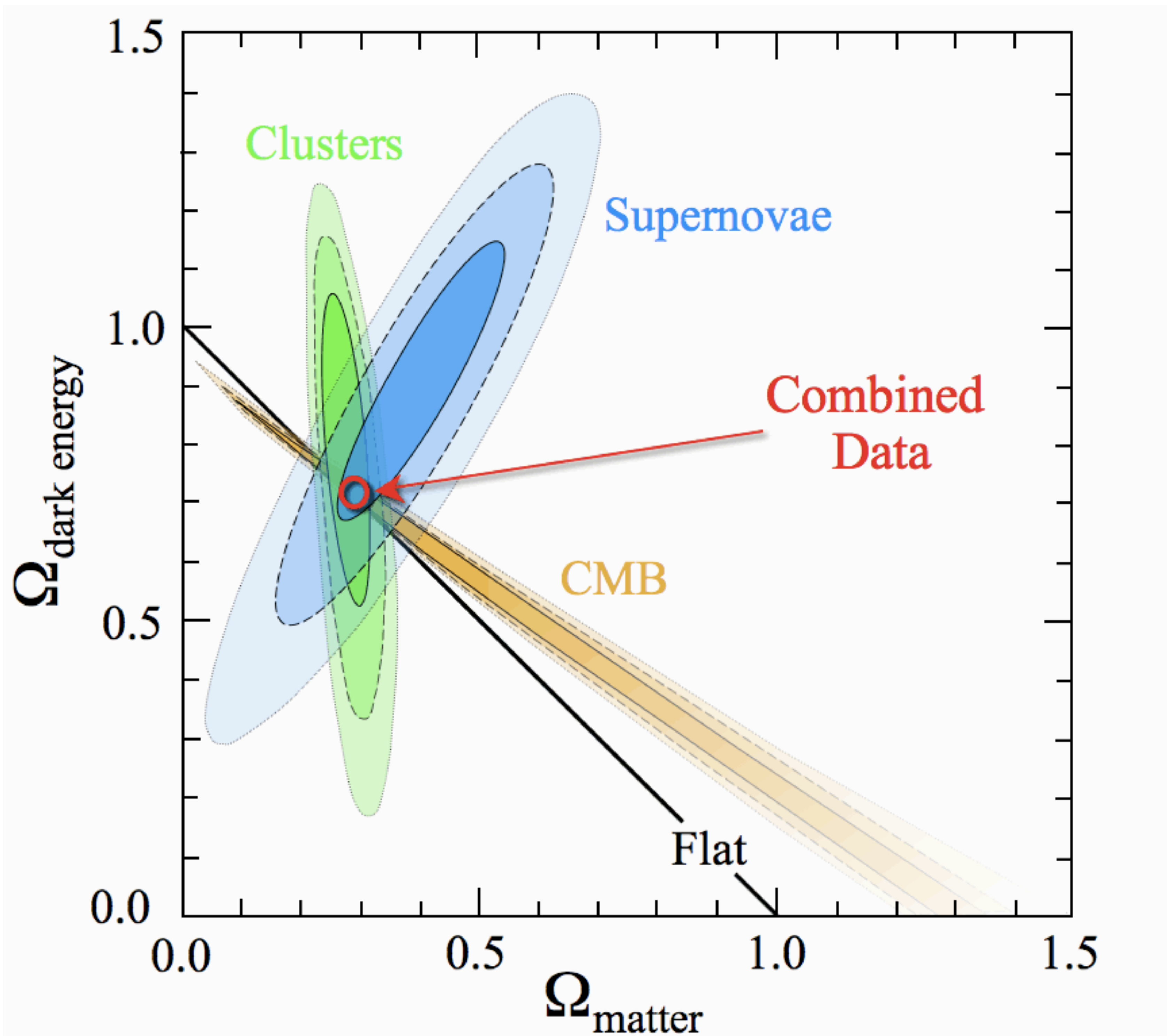
Inflationary predictions

Observations from Planck agree with the almost invariant initial power spectrum predicted by inflation



Concordance model

- The combination of the different datasets gives us a great significance of the model

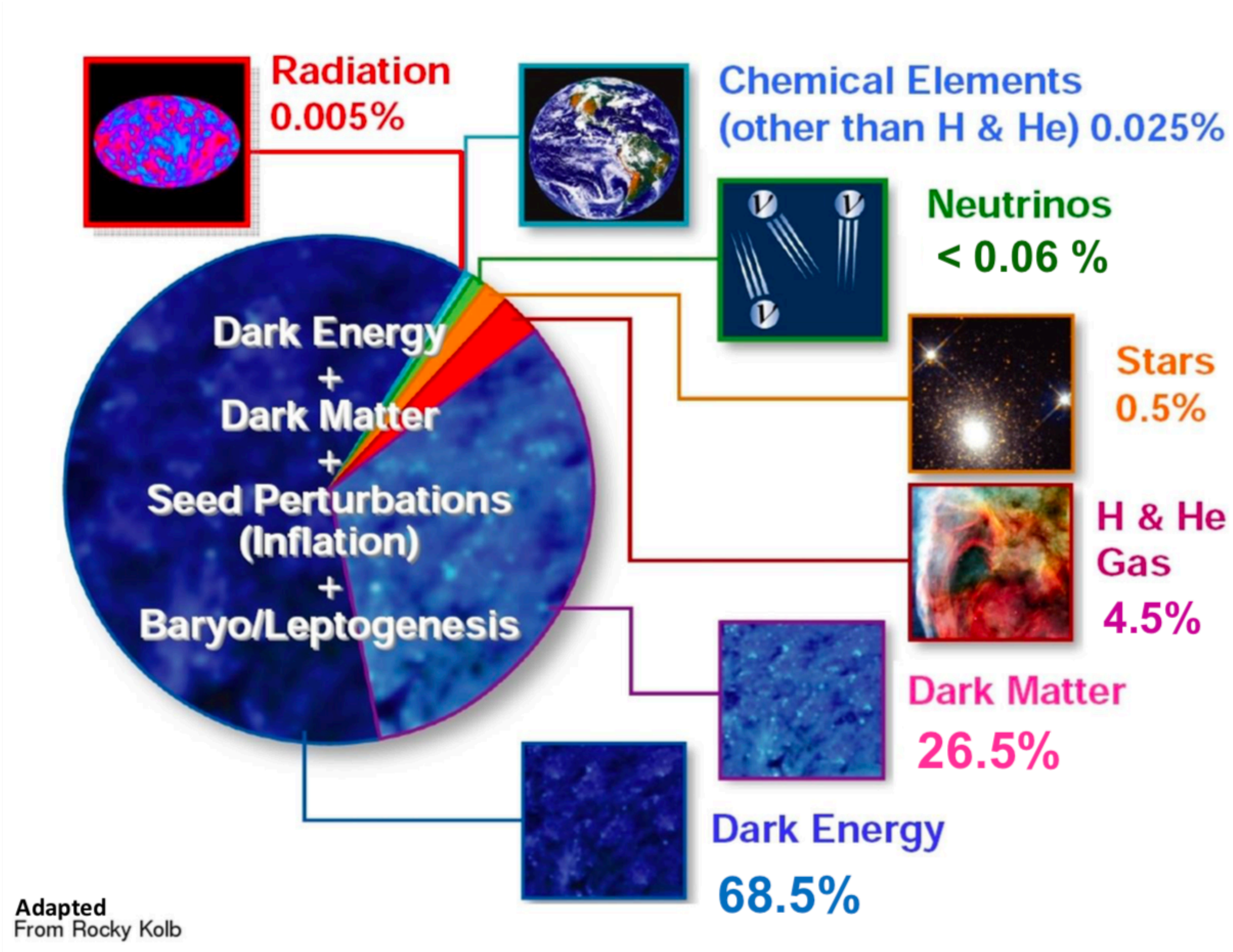


ΛCDM model

We can describe the Universe with only 6 parameters:

$$(\Omega_b, \Omega_m, n_s, A_s, \tau, H_0)$$

At least 2 of these components are still known known unkwowns.



ΛCDM model best fit (Plik_TTTEEE_lowl_lowE_BAO)

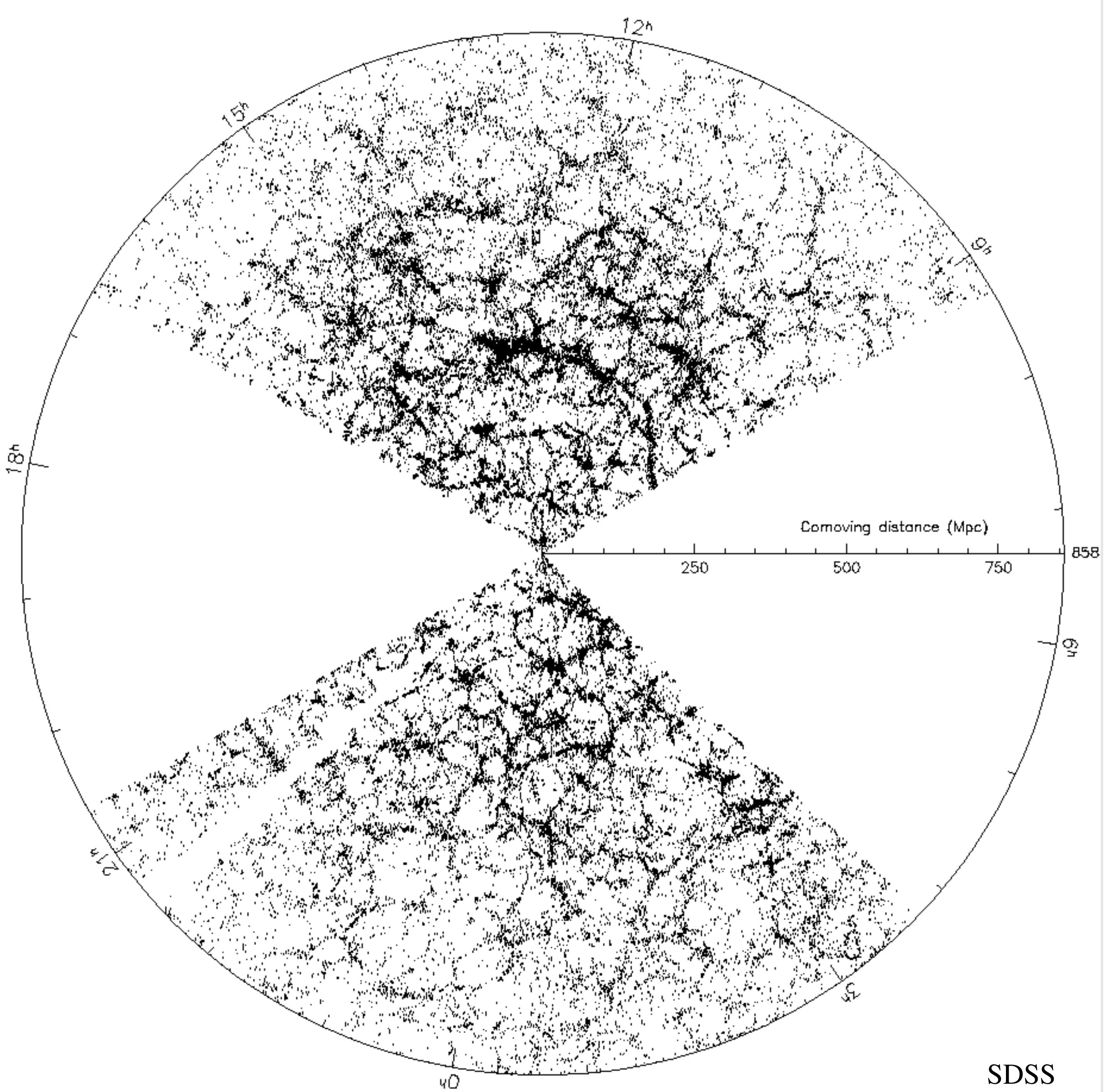
Parameter name	Symbol	Measured value	Fiducial value
Spatial curvature	Ω_k	0.001 ± 0.002	0
Matter density rel. to critical	$\Omega_{\rm M}$	0.310 ± 0.007	0.30
Baryon density	$\Omega_{\rm B}h^2$	0.0224 ± 0.0002	0.0224
Hubble constant	H_0	$(67.9 \pm 0.7)\,\mathrm{km/s/Mpc}$	$67\,\mathrm{km/s/Mpc}^{**}$
$P(k)$ amplitude at $k_{\rm piv} = 0.05$	A_s	$(2.10 \pm 0.03) \times 10^{-9}$	2.1×10^{-9}
Scalar spectral index	n_s	0.966 ± 0.005	0.966
Age of universe	t_0	$(13.76 \pm 0.08)\,\mathrm{Gyr}$	derived
Amplitude of mass fluctuations	σ_8	0.810 ± 0.007	derived
CMB temperature	T_0	$(2.7255 \pm 0.0006)\,\mathrm{K}$	$2.725\,\mathrm{K}$
Photon density	$\Omega_\gamma h^2$	derived from T_0	2.47×10^{-5}
Assumed-massless neutrino density	$\Omega_{\nu,\rm rel}h^2$	derived from T_0	1.68×10^{-5}
Equation of state of dark energy	w	-1.04 ± 0.06	-1

Credit: D. Huterer from Aghanim et al. 2020 (Planck Collaboration)

Contents

Growth of structure (perturbations)

- In order to study the inhomogeneous Universe, we need to study the evolution of perturbations of the metric and the energy distributions + initial conditions



Growth of structure (newtonian perturbations)

- By considering these 3 classical equations:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$ Continuity equation

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{1}{\rho}\nabla p - \nabla \phi$$ Euler equation

$$\nabla^2 \phi = 4\pi G \rho$$ Poisson equation

and the equation of state: $p = p(\rho)$

we can solve the equations to get $\rho, \mathbf{v}, \phi, p$

Growth of structure in a expanding Universe (newtonian perturbations)

- We define the background as the smooth and homogenous Universe, given by:

$$\rho = \rho(t) \qquad \mathbf{r} = a(t)\chi \qquad \mathbf{v} = H(t)\mathbf{r}$$

- To perturbe the equations, we need to define the perturbed quantities:

Perturbed density: $\rho(\chi, t) = \bar{\rho}(t) + \delta\rho(\chi, t) = \bar{\rho}(1 + \delta(\chi, t))$

Perturbed pressure: $p = \bar{p} + \delta p$

Perturbed velocity: $\mathbf{v} = a(t)\chi + \delta\mathbf{v}$

Perturbed potential: $\phi = \phi + \delta\phi(\chi, t)$

Growth of structure in a expanding Universe (newtonian perturbations)

- Introducing the perturbations in the 3 dynamical equations:

$$\rho(\chi, t) = \bar{\rho}(1 + \delta(\chi, t))$$
$$p = \bar{p} + \delta p$$
$$\mathbf{v} = a(t)\chi + \delta\mathbf{v}$$
$$\phi = \phi + \delta\phi(\chi, t)$$

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) = 0$$
$$\frac{\partial\mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\frac{1}{\rho}\nabla p - \nabla\phi$$
$$\nabla^2\phi = 4\pi G\rho$$

we reach (considering only linear terms as the perturbations are small):

$$\frac{\partial\delta\rho}{\partial t} + \bar{\rho}\nabla\delta\mathbf{v} + \nabla(\delta\rho\mathbf{v}) = 0$$
$$\frac{\partial\delta\mathbf{v}}{\partial t} + \mathbf{v}(\nabla)\delta\mathbf{v} + (\delta\mathbf{v}\nabla)\mathbf{v} + \frac{c_s^2}{\rho}\nabla\delta\rho + \nabla\delta\phi = 0$$
$$\nabla^2\delta\phi = 4\pi G\delta\rho$$

Expanding Universe Coordinates: $\nabla_r = \frac{1}{a}\nabla_\chi$ →

$$\left(\frac{\partial\delta}{\partial t}\right) + \frac{1}{a}\nabla\delta\mathbf{v} = 0$$
$$\frac{\partial\delta\mathbf{v}}{\partial t} + H\delta\mathbf{v} + \frac{c_s^2}{\rho}\nabla\delta + \nabla\delta\phi = 0$$
$$\nabla^2\delta\phi = 4\pi G a^2\delta$$

Growth of structure in a expanding Universe (newtonian perturbations)

- Now, combining these equations and also changing to coordinates that move with the expansion we can obtain the growth history for density perturbations in an expanding Universe:

$$\ddot{\delta} + 2H\dot{\delta} - \frac{c_s^2}{a^2}\nabla^2\delta = 4\pi G\bar{\rho}\delta$$

In Fourier space:

$$\delta(\boldsymbol{x}, t) = \sum_{\boldsymbol{k}} \delta_{\boldsymbol{k}}(t)\, e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$$

$$\delta_{\boldsymbol{k}}(t) = \frac{1}{V}\int \delta(\boldsymbol{x}, t)\, e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}\, d^3x$$

$$\ddot{\delta}_k + 2H\dot{\delta}_k = \left(4\pi G\rho_0(t) - \frac{k^2 c_s^2}{a^2}\right)\delta_k.$$ For baryonic matter

$$\ddot{\delta}_k + 2H\dot{\delta}_k - 4\pi G\rho_m(t)\delta_k = 0$$ For dark matter

Some regimes of LSS

The newtonian approximation is fine to describe the non-relativistic evolution of sub-horizon modes. As they are second order ODEs, we define the 2 solutions in terms of the linear growth function $D(t)$:

$$\delta(\mathbf{k},t) = \delta_+(\mathbf{k})D_+(t) + \delta_-(\mathbf{k})D_-(t)$$

Matter fluctuations in a matter-dominated Universe:

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G\bar{\rho}_m\delta_m = 0$$

In matter domination: $D_+(t) \propto t^{2/3} \propto a$ & $D\_(t) \propto t^{-1} \propto a^{-3/2}$. It can be proved the growth function applies to both CDM and baryons. This solution means $\phi \propto cnt$

CDM fluctuations in a radiation-dominated Universe:

$$\ddot{\delta}_c + 2H\dot{\delta}_c - 4\pi G\sum_a \bar{\rho}_a\delta_a = 0 \longrightarrow \ddot{\delta}_c + 2H\dot{\delta}_c - 4\pi G\bar{\rho}_c\delta_c = 0$$

During radiation domination: $D\_ = cnt$ & $D_+(t) \propto \ln t \propto \ln a$. This means that $\phi \propto a^{-2}$

Matter fluctuations in a DE-dominated Universe (H is constant)

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G\bar{\rho}_m\delta_m = 0$$

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Matter fluctuations in a DE-dominated Universe (H is constant)

$$\ddot{\delta}_m + 2H\dot{\delta}_m - \cancel{4\pi G\bar{\rho}_m\delta_m} = 0$$

During DE domination: $D_- = a^{-2}$ & $D_+(t) \propto cnt$. This means that $\phi \propto a^{-1}$

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$$\delta(\mathbf{k},t) = \delta_+(\mathbf{k})D_+(t) + \delta_-(\mathbf{k})D_-(t)$$

Matter fluctuations in a matter-dominated Universe:

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G\bar{\rho}_m\delta_m = 0$$

In matter domination: $D_+(t) \propto t^{2/3} \propto a$ & $D_-(t) \propto t^{-1} \propto a^{-3/2}$. It can be proved the growth function applies to both CDM and baryons. This solution means $\phi \propto cnt$

CDM fluctuations in a radiation-dominated Universe:

$$\ddot{\delta}_c + 2H\dot{\delta}_c - 4\pi G\sum_a \bar{\rho}_a\delta_a = 0 \longrightarrow \ddot{\delta}_c + 2H\dot{\delta}_c - 4\pi G\bar{\rho}_c\delta_c = 0$$

During radiation domination: $D_- = cnt$ & $D_+(t) \propto \ln t \propto \ln a$. This means that $\phi \propto a^{-2}$

Matter fluctuations in a DE-dominated Universe (H is constant)

$$\ddot{\delta}_m + 2H\dot{\delta}_m - \cancel{4\pi G\bar{\rho}_m\delta_m} = 0$$

During DE domination: $D_- = a^{-2}$ & $D_+(t) \propto cnt$. This means that $\phi \propto a^{-1}$

The full treatment (relativistic)

- If we want to do the full treatment, we need to do the perturbations (assumed small) in the frame of General Relativity and the flat FLRW metric:

$$ds^2 = \left[{}^{(0)}g_{\alpha\beta} + \delta g_{\alpha\beta}(x^\gamma)\right] dx^\alpha dx^\beta$$

- This produces a set of scalar, vector and tensor fluctuations. The tensor fluctuations are relevant for gravitational waves and polarisation of the CMB photons but we focus on the scalar one. We choose the Newtonian gauge in which:

$$ds^2 = a^2\left[(1+2\phi_l)d\eta^2 - (1-2\psi_l)\delta_{ij}dx^i dx^j\right]$$

The reason we choose this gauge is that because also the 2 potentials are the same, reducing to the standard Newtonian potential.

The full treatment

- In this gauge, then the equations of scalar perturbations are:

$$\Delta\Phi - 3\mathcal{H}\left(\Phi' + \mathcal{H}\Phi\right) = 4\pi G a^2 \overline{\delta\varepsilon},$$

$$(a\Phi)'_{,i} = 4\pi G a^2 (\varepsilon_0 + p_0)\, \overline{\delta u}_{\parallel i},$$

$$\Phi'' + 3\mathcal{H}\Phi' + \left(2\mathcal{H}' + \mathcal{H}^2\right)\Phi = 4\pi G a^2 \overline{\delta p}.$$

- Combining them we get:

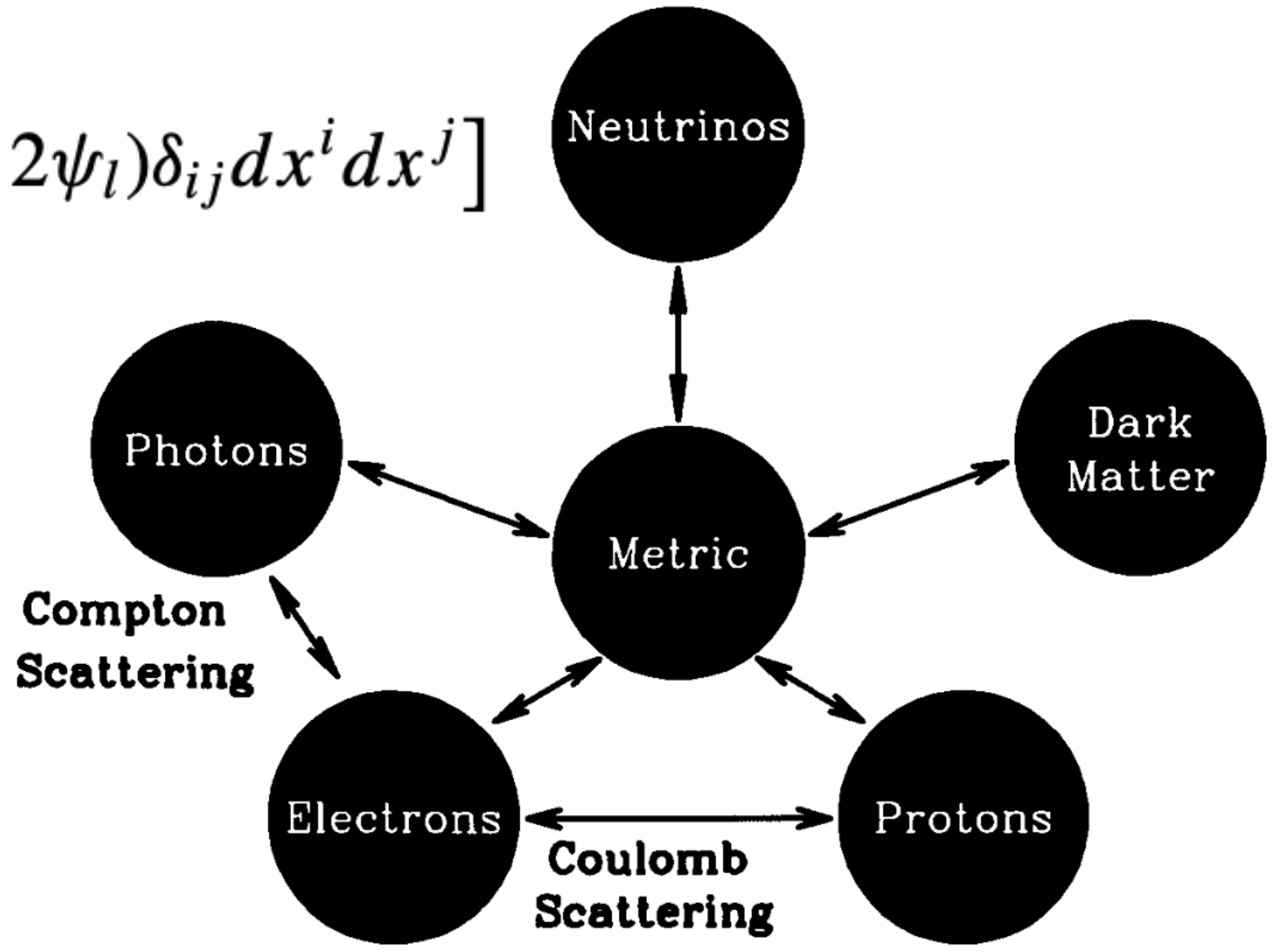
$$\Phi'' + 3\left(1 + c_s^2\right)\mathcal{H}\Phi' - c_s^2 \Delta\Phi + \left(2\mathcal{H}' + \left(1 + 3c_s^2\right)\mathcal{H}^2\right)\Phi = 0$$

Boltzmann equations

- In order to study the cosmic distribution of photons and matter inhomogeneities, we can use the Boltzmann equations in the phase space

$$ds^2 = a^2[(1+2\phi_l)d\eta^2 - (1-2\psi_l)\delta_{ij}dx^i dx^j]$$

$$\frac{df}{dt} = C[f]$$



where C[f] accounts for the collisions, in case there are some.

Boltzmann equations for photons

- We need to solve the Boltzmann equations for the different components of the Universe are for the zero-order and the perturbations on the equilibrium distribution

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x^i}\frac{dx^i}{dt} + \frac{\partial f}{\partial \hat{p}^i}\frac{d\hat{p}^i}{dt} + \frac{\partial f}{\partial p}\frac{dp}{dt}$$

Using the metric perturbations:

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\hat{p}^i}{a}\frac{\partial f}{\partial x^i} + \frac{\partial f}{\partial p}\frac{dp}{dt}$$

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\hat{p}^i}{a}\frac{\partial f}{\partial x^i} - p\frac{\partial f}{\partial p}\left[H + \frac{\partial \phi}{\partial t} + \frac{\hat{p}^i}{a}\frac{\partial \psi}{\partial x^i}\right]$$

Boltzmann equations for photons

- We need to address the photon distribution where the zero-th order is the Bose-Einstein distribution function:

$$f(\vec{x}, p, \vec{p}, t) = \left[exp\left(\frac{p}{T(t)[1+\Theta(\vec{x}, \vec{p}, t)]}\right)\right]^{-1}$$

- Zero- order equation (collision-less):

$$\left.\frac{df}{dt}\right|_0 = \frac{\partial f^{(0)}}{\partial t} - Hp\frac{\partial f^{(0)}}{\partial p} = 0 \longrightarrow T \propto \frac{1}{a}$$

- First- order equation:

(Compton scattering)

$$\frac{\partial \Theta}{\partial t} + \frac{\hat{p}^i}{a}\frac{\partial \Theta}{\partial x^i} + \frac{\partial \phi}{\partial t} + \frac{\hat{p}^i}{a}\frac{\partial \psi}{\partial x^i} = n_e \sigma_T [\Theta_0 - \Theta + \bar{p} \cdot \vec{v}_b]$$

- And in Fourier space and conformal time:

$$\dot{\tilde{\Theta}} + ik\mu\tilde{\Theta} + \dot{\tilde{\Phi}} + ik\mu\tilde{\Psi} = -\dot{\tau}\left[\tilde{\Theta}_0 - \tilde{\Theta} + \mu\tilde{v}_{\rm b}\right]$$

Boltzmann equations for dark matter and baryons

- Similar derivation but for DM no collision term while for baryons there is the Coulomb scattering proton-electron and the Compton scattering of the electron-photons coupling. For both components, the zero-order equation is just the same one as the density of both fluids in the background model

$$\dot{\Theta} + ik\mu\Theta = -\dot{\Phi} - ik\mu\Psi - \dot{\tau}\left[\Theta_0 - \Theta + \mu v_b - \frac{1}{2}\mathcal{P}_2(\mu)\Pi\right]$$

$$\Pi = \Theta_2 + \Theta_{P2} + \Theta_{P0}$$

Photons

$$\dot{\Theta}_P + ik\mu\Theta_P = -\dot{\tau}\left[-\Theta_P + \frac{1}{2}(1 - \mathcal{P}_2(\mu))\Pi\right]$$

Final set of eqs is:

$$\dot{\delta} + ikv = -3\dot{\Phi}$$

$$\dot{v} + \frac{\dot{a}}{a}v = -ik\Psi$$

DM

$$\dot{\delta}_b + ikv_b = -3\dot{\Phi}$$

$$\dot{v}_b + \frac{\dot{a}}{a}v_b = -ik\Psi + \frac{\dot{\tau}}{R}\left[v_b + 3i\Theta_1\right]$$

Baryons

Initial conditions are set in the early Universe (adiabatic modes)

Linearized Boltzmann - Einstein equations

- Similar derivation for DM but no collision term while for baryons there is the Coulomb scattering pro electron and the Compton scattering of the electron-photons coupling. For both components, the zero-o equation is just the same one as the density of both fluids in the background model.

- Here is the full set of coupled ODEs with the initial conditions set in the early times.

$$\begin{cases} \dot{\Theta} + ik\mu\Theta = -\dot{\Phi} - ik\mu\Psi - \dot{\tau}\left[\Theta_0 - \Theta + \mu v_b - \frac{1}{2}\mathcal{P}_2(\mu)\Pi\right] & [\dot{\tau} \equiv -n_e\sigma_T a] \\ \Pi = \Theta_2 + \Theta_{P2} + \Theta_{P0} \\ \dot{\Theta}_P + ik\mu\Theta_P = -\dot{\tau}\left[-\Theta_P + \frac{1}{2}\left(1 - \mathcal{P}_2(\mu)\right)\right] \\ \dot{\delta} + ikv = -3\dot{\Phi} \\ \dot{v} + \frac{\dot{a}}{a}v = -ik\Psi \\ \dot{\delta}_b + ikv_b = -3\dot{\Phi} \\ \dot{v}_b + \frac{\dot{a}}{a}v_b = -ik\Psi + \frac{\dot{\tau}}{R}\left(v_b + 3i\Theta_1\right) & [R \equiv 3\rho_b/4\rho_\gamma] \\ \dot{\mathcal{N}} + ik\mu\mathcal{N} = -\dot{\Phi} - ik\mu\Psi \\ k^2\Phi + 3\frac{\dot{a}}{a}\left(\dot{\Phi} - \frac{\dot{a}}{a}\Psi\right) = 4\pi G a^2\left(\rho_c\delta + \rho_b\delta_b + 4\rho_\gamma\Theta_0 + 4\rho_\nu\mathcal{N}_0\right) \\ k^2\left(\Phi + \Psi\right) = -32\pi G a^2\left(\rho_\gamma\Theta_2 + \rho_\nu\mathcal{N}_2\right) \\ \ddot{h}_\alpha + 2\frac{\dot{a}}{a}\dot{h}_\alpha + k^2 h_\alpha = 0 & [\alpha = +, x] \end{cases}$$

Initial conditions

$$\begin{cases} \Phi(k,\eta_i) = -\Psi(k,\eta_i) = 2\Theta_0(k,\eta_i) = 2\mathcal{N}_0(k,\eta_i) = \Phi_p(k) \\ \delta(k,\eta_i) = \delta_b(k,\eta_i) = \frac{3}{2}\Phi_p(k) \\ \Theta_1(k,\eta_i) = \mathcal{N}_1(k,\eta_i) = \frac{iv(k,\eta_i)}{3} = \frac{iv_b(k,\eta_i)}{3} = -\frac{k\Phi_p}{6aH} \end{cases}$$

Note: $4\Theta_0$ "$\sim$" δ_γ, $4\mathcal{N}_0$ "$\sim$" δ_ν, $-3i\Theta_1$ "$\sim$" v_γ, $-3i\mathcal{N}_1$ "$\sim$" v_ν

Credit: S. Vangozzi & Modern Cosmology (Dodelson)

Numerical solutions

- For the current model, the system of Einstein-Boltzmann equations have to be solved numerically.

- Most used public software:

    - CAMB: https://camb.info/ (Fortran)

      Python wrapper: https://camb.readthedocs.io/en/latest/

      Fast and widely used but difficult to modify from theoretical point of view

    - CLASS: https://lesgourg.github.io/class_public/class.html

      C++ code (also with python wrapper).

      Fast, modular and with several theoretical model implementations.

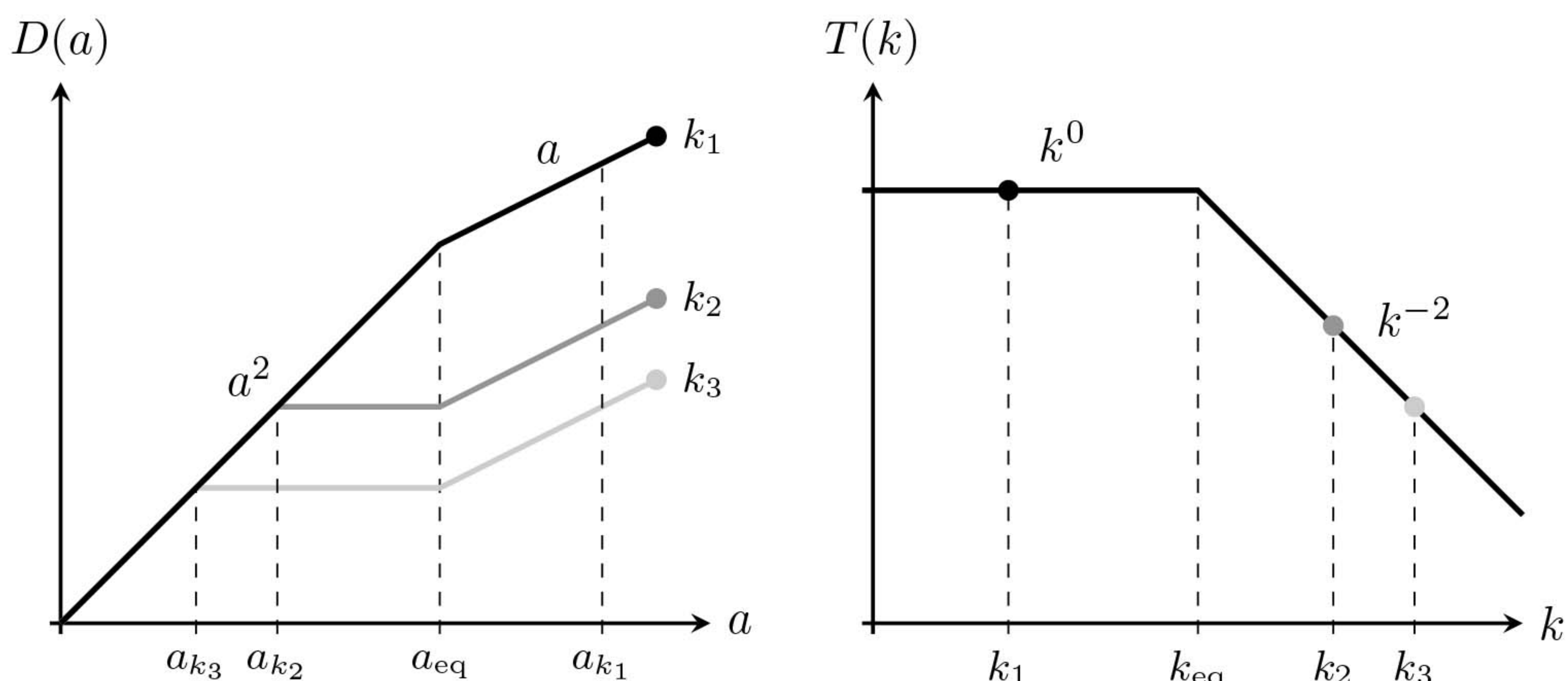
Transfer function

- The transfer function describes the impact on the growth of denisity perturbations during the transition from a radiation dominated Universe to a matter dominated Universe.

- The primary effect is a turnover around $k = 0.02hMpc^{-1}$. The perturbations are both inversely proportational to the particle horizon[2].

$$\delta_k \propto \frac{1}{(aH)^2} \propto \begin{cases} a^2 & \text{radiation era} \\ a & \text{matter era} \end{cases} \qquad \delta_k(t) \propto \begin{cases} \left(\frac{a_{eq}}{a_i}\right)^2 \frac{a}{a_{eq}} \delta_k(t_i) = 1 & \text{for } k < k_{eq} \\ \left(\frac{a_k}{a_{eq}}\right)^2 \left[\left(\frac{a_{eq}}{a_i}\right)^2 \frac{a}{a_{eq}}\right] \delta_k(t_i) & \text{for } k > k_{eq} \end{cases}$$

Transfer function: $$T(k) = \begin{cases} 1 & \text{for } k < k_{eq} \\ (k_{eq}/k)^2 & \text{for } k > k_{eq} \end{cases}$$



Matter power spectrum

- The theoretical cosmological function we mostly use for the analysis of large-scale structure is the power spectrum:

$$P(k,t) = D(t)^2 T(k)^2 P_{ini}(t_i)$$

Asymptotically:

$$P(k) = \begin{cases} k^{n_s} & \text{for } k < k_{eq} \\ k^{n_s - 4} & \text{for } k > k_{eq} \end{cases}$$

- We need to obtain the transfer function through the Boltzmann - Einstein equations. For scales that cross the horizon at matter dominated time, there is an overall decrease in the potential but for the modes that enter during radiation dominated phase, the potential changes because of the interaction with the radiation (photons or neutrinos)

- The growth during matter domination is decoupled from this and grows only depending on the scale factor

Growth of structure

For scales larger than 10 Mpc, we can assume linear theory and estimate the growth for different times

Universe dominated by **matter**:

$$\delta = A(x)t^{2/3} + B(x)t^{-1}$$

Grows with scale factor $a(t) \sim t^{2/3}$.

Universe dominated by **radiation**

$$\delta_{\boldsymbol{k}}(t) = A + B \ln t$$

Suppressed growth

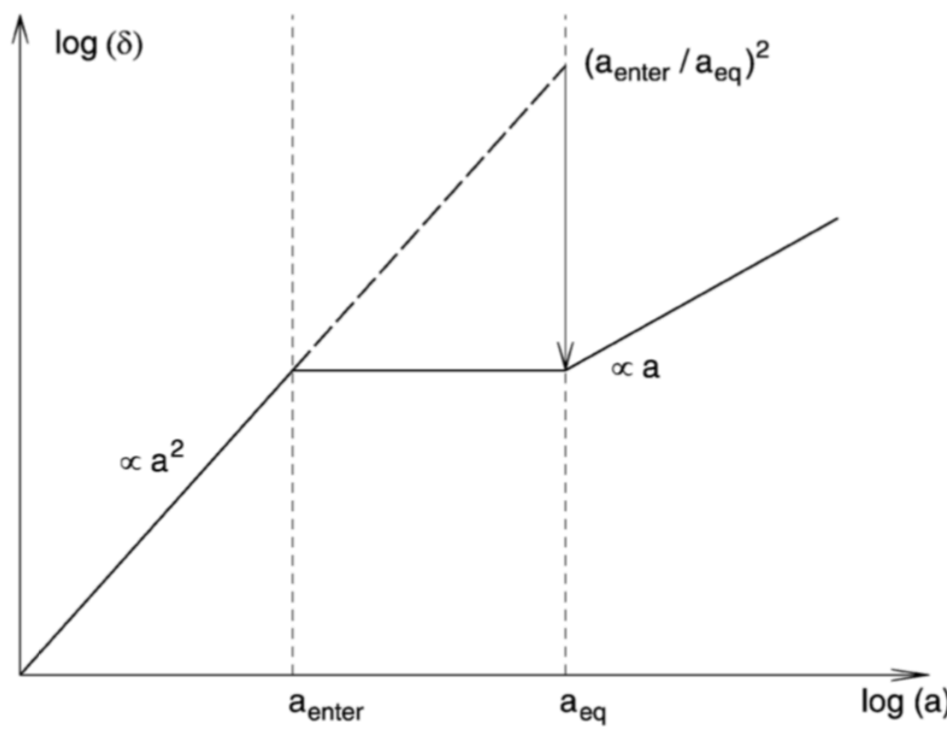


Fig. 7.5. A density perturbation that enters the horizon during the radiation-dominated epoch of the Universe ceases to grow until matter starts to dominate the energy content of the Universe. In comparison to a perturbation that enters the horizon later, during the matter-dominated epoch, the amplitude of the smaller perturbation is suppressed by a factor $(a_{\rm eq}/a_{\rm enter})^2$, which explains the qualitative behavior (7.29) of the transfer function

We can only predict the statistical properties of the distribution -> power spectrum P(k)

$$\left\langle \hat{\delta}(\vec{k})\hat{\delta}^*(\vec{k}') \right\rangle \equiv (2\pi)^3 P(k)\delta_{\rm D}\left(\vec{k}-\vec{k}'\right)$$

We model P(k) and growth of structure with the transfer function T(k)

$$P_0(k) = \underline{A\,k^{n_s}}\;T^2(k)$$

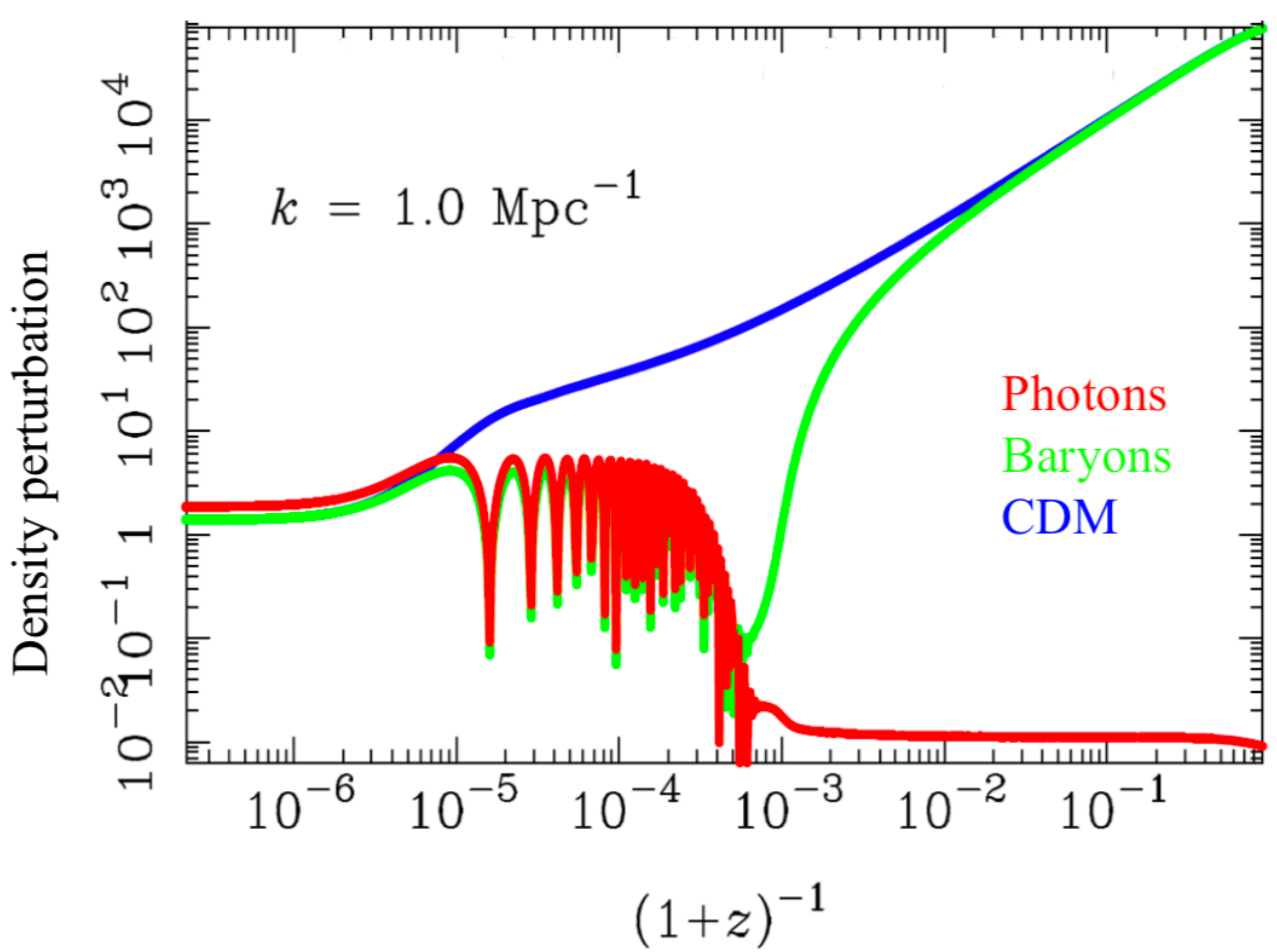
Espectro inicial

Role of DM

If $\delta \sim t^{2/3}$, and decoupling fluctuations size (z~1100) are of the order of 10^{-5}, we couldn't reach the current amplitude. We need DM

DM fluctuations start growing before decoupling at: z~3300

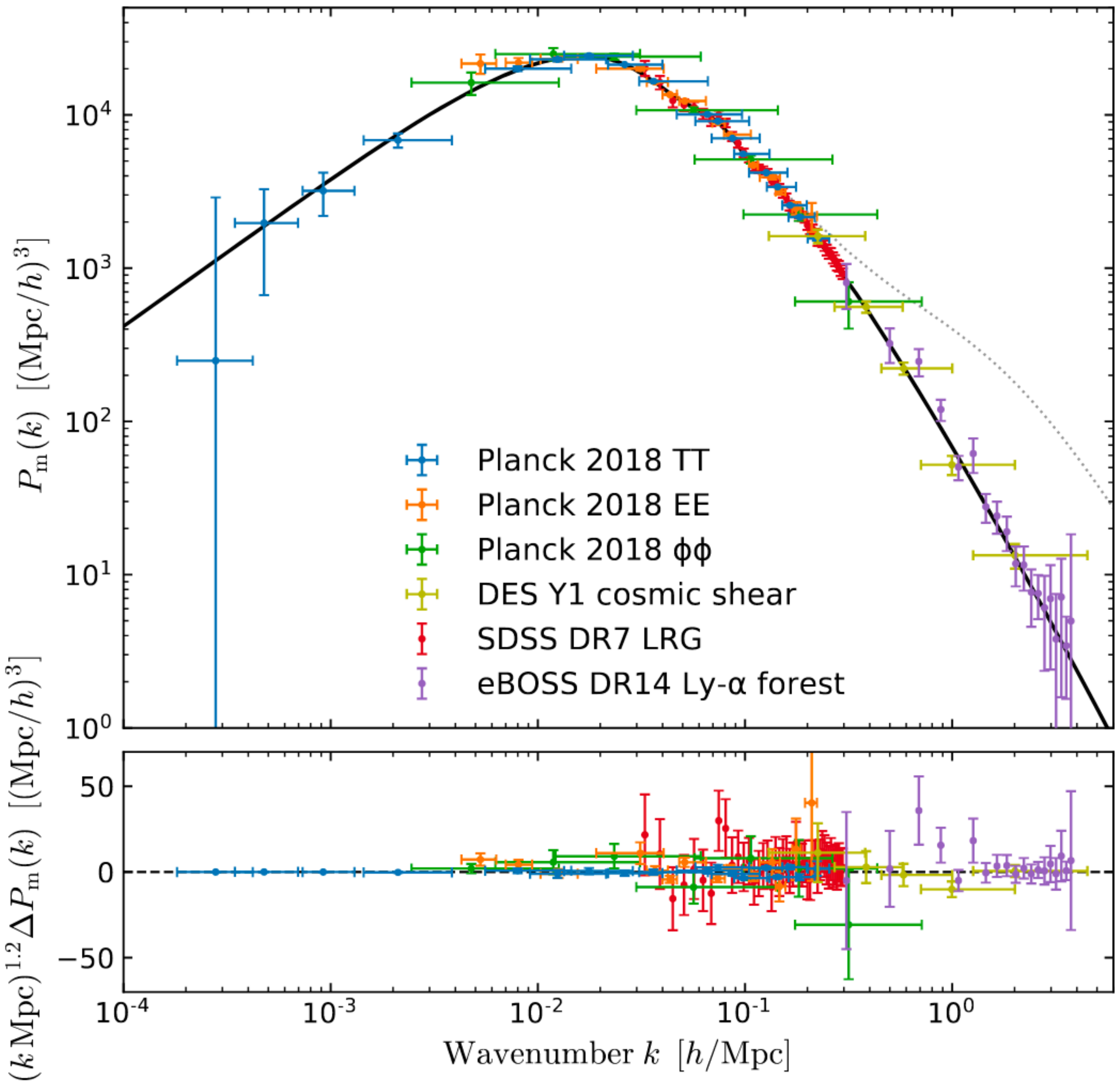
After decoupling, the baryons start following the DM fluctuations.



This is another evidence for the existence of DM

Large-scale structure

- Universe filled with density fluctuations
- Structure only only visible through galaxies (distribution) and photons (weak lensing)
- Galaxies and photons here are functioning as test particles - tracing out the gravitational field
- Most low-redshift surveys have measured the transfer function.
- Need very large volumes to measure primordial power spectrum and determine initial conditions (independently from CMB)



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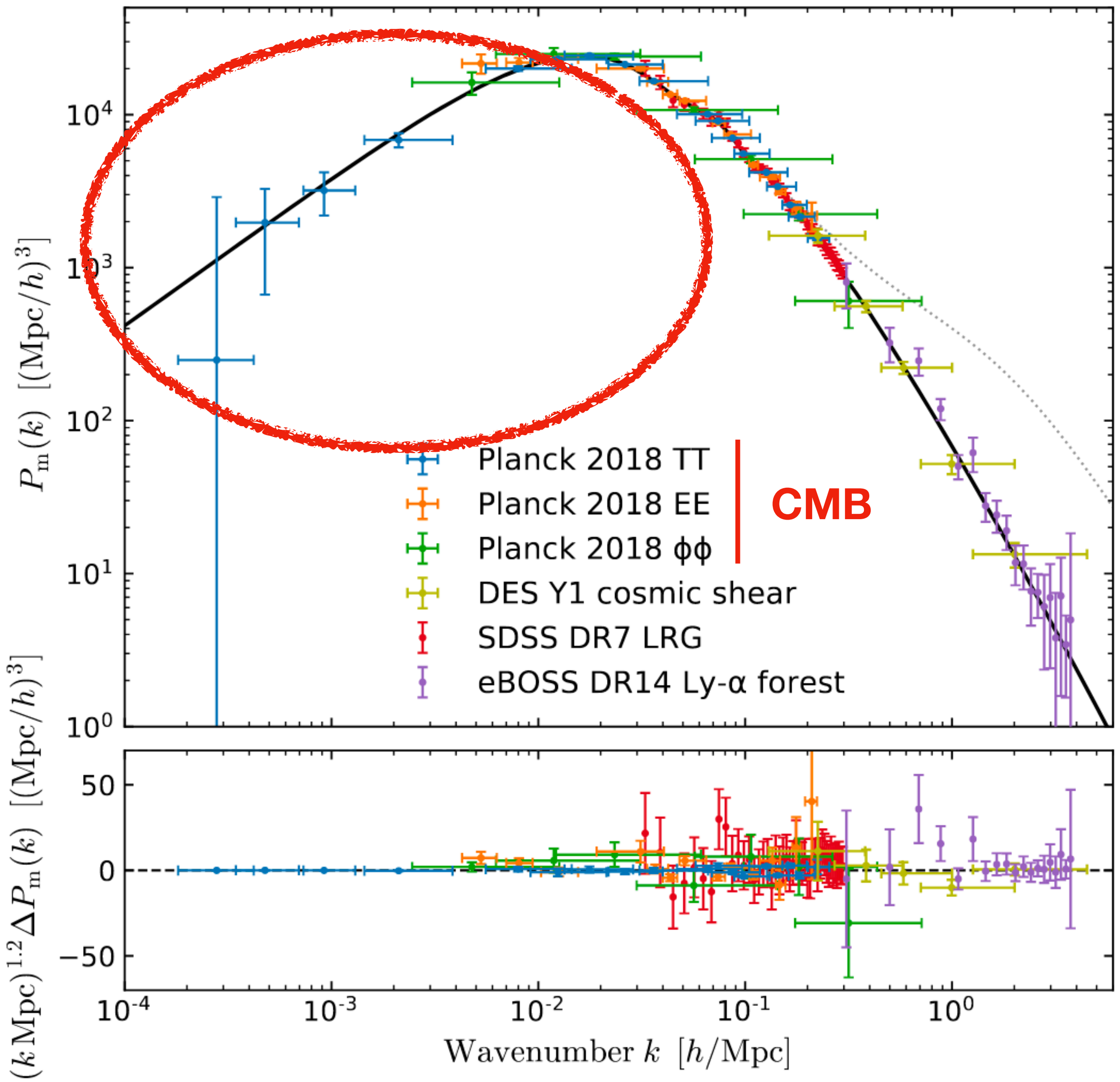

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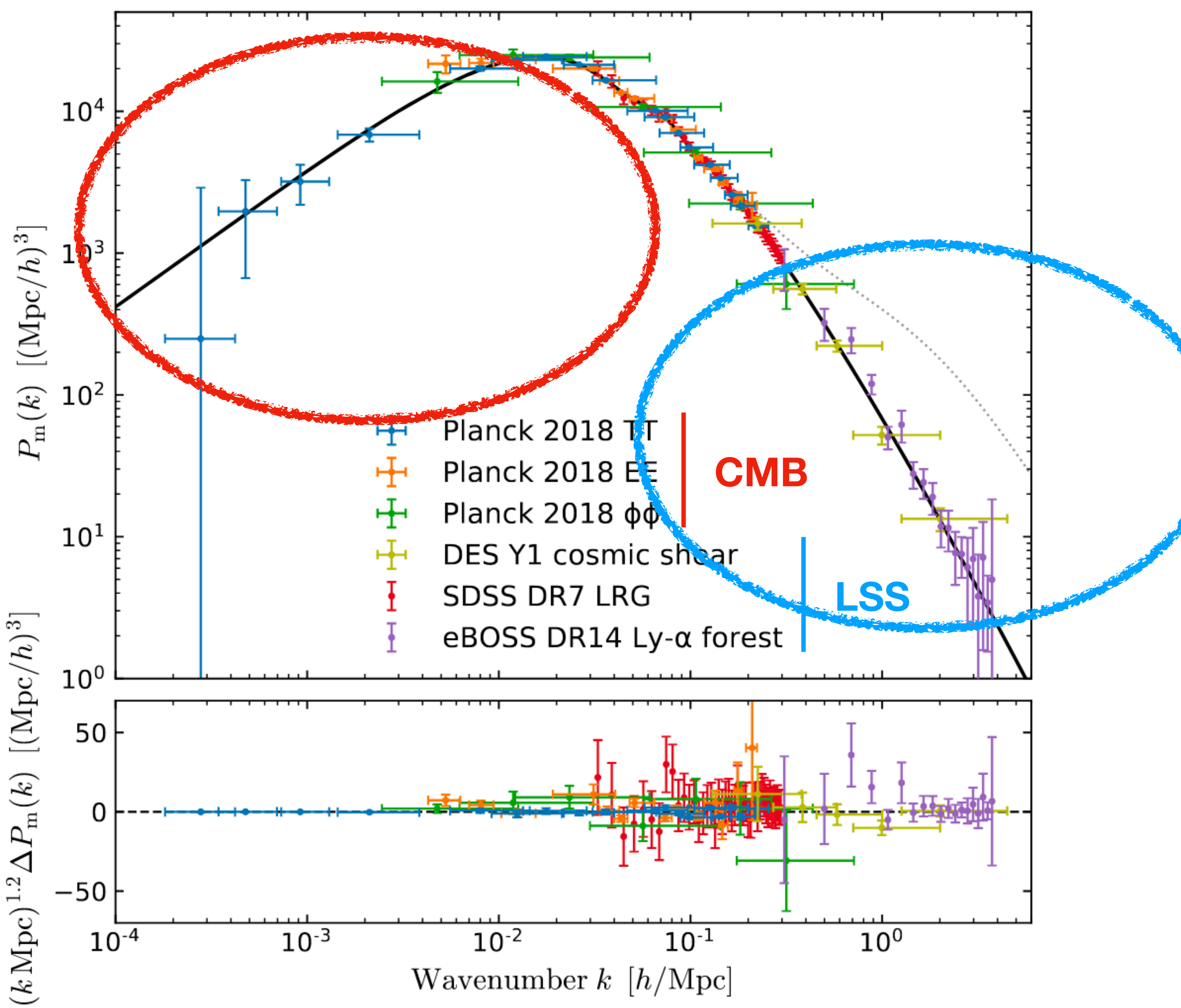


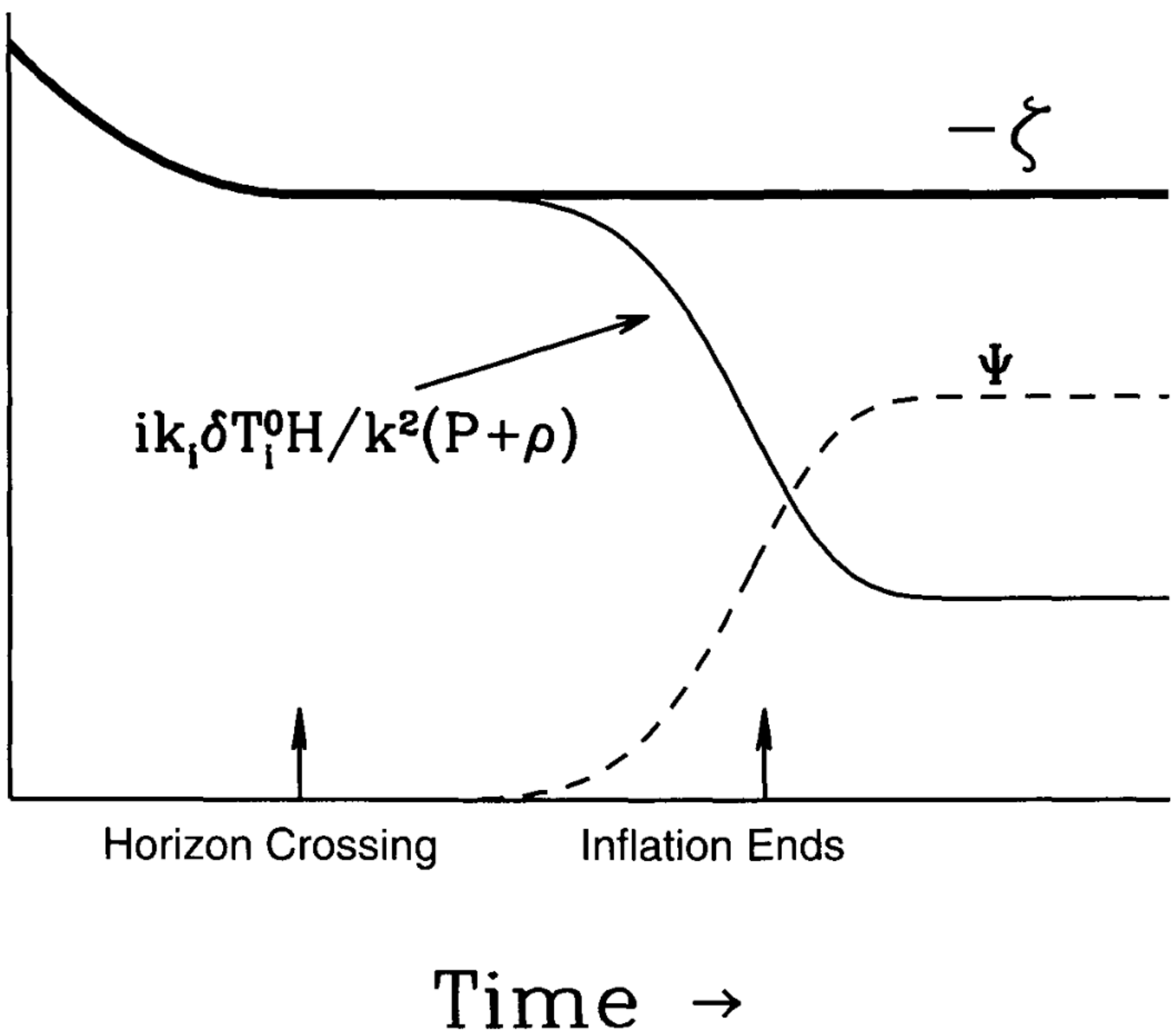
Initial conditions

- Besides the equations that describe the evolution of metric perturbations and energy perturbations, we need to set the initial conditions.

- The initial distribution of scalar perturbations is almost scale invariant and inflationary models tend to predict some deviations from the pure scale-free spectrum.

- These modes leave the horizon and then they enter the Universe later on in radiation and matter dominated phases depending on the scale.

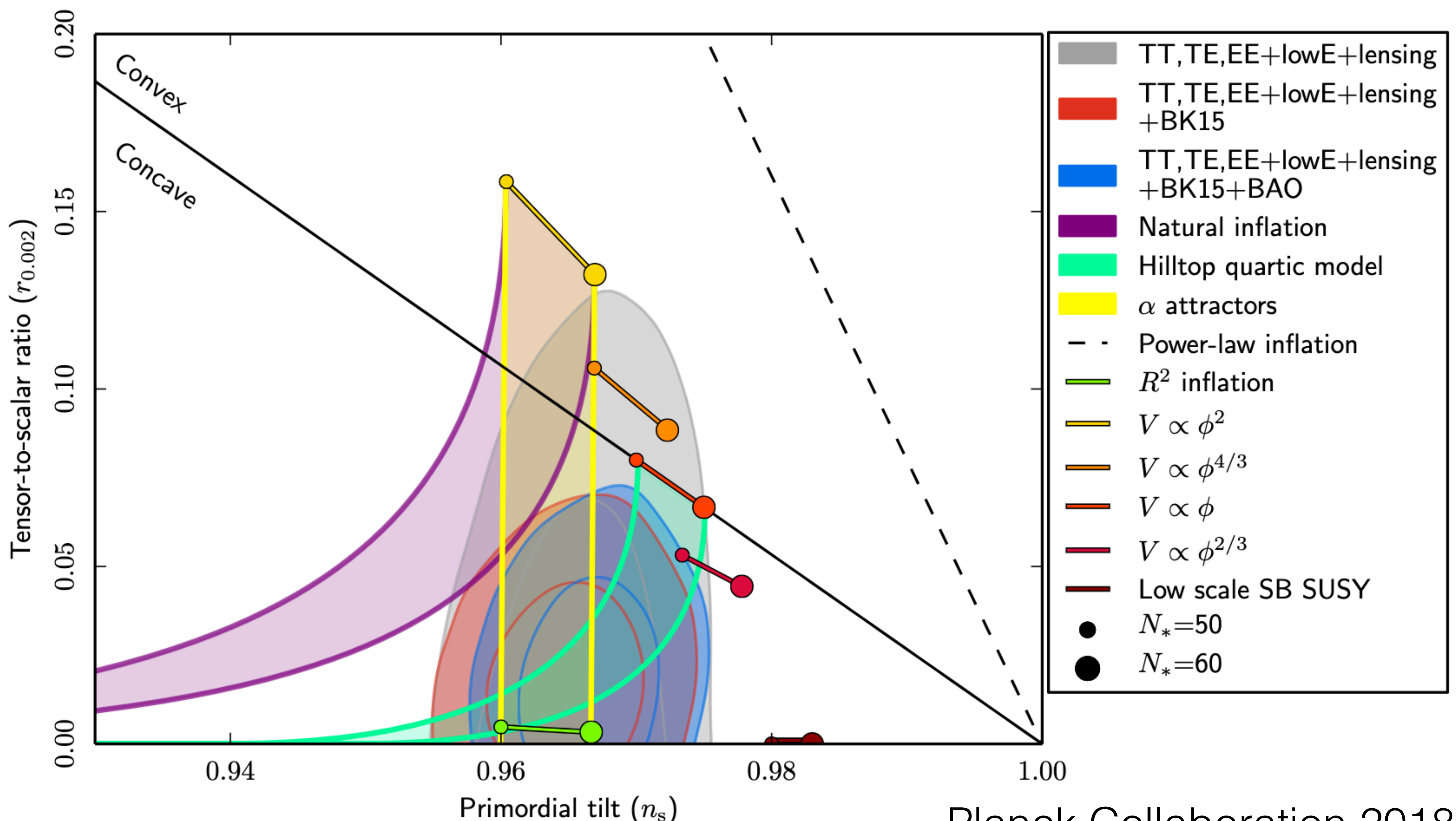
$$\mathcal{P}_\chi(k) = A_s \left(\frac{k}{k_{s0}}\right)^{n_s - 1}.$$



Inflationary constraints from CMB

- n_S=1 has been ruled out by Planck.

- Inflation also predicts an initial spectrum for tensor modes of the metric perturbations -> GW imprint and primordial polarisation of CMB would be a direct probe of inflation.



Planck Collaboration 2018

Correlation function

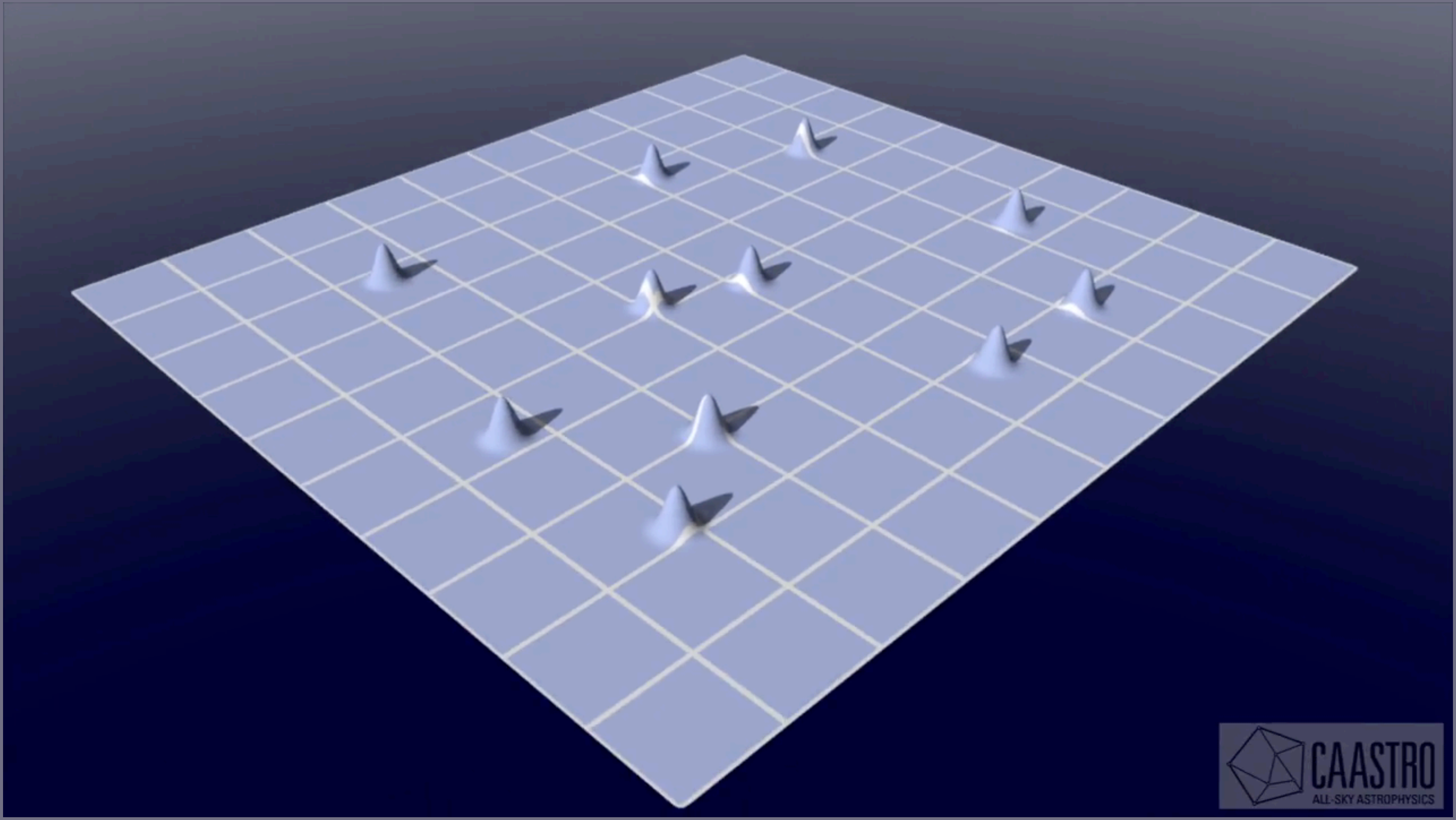
- The Fourier transform of the power spectrum is given by the correlation function:

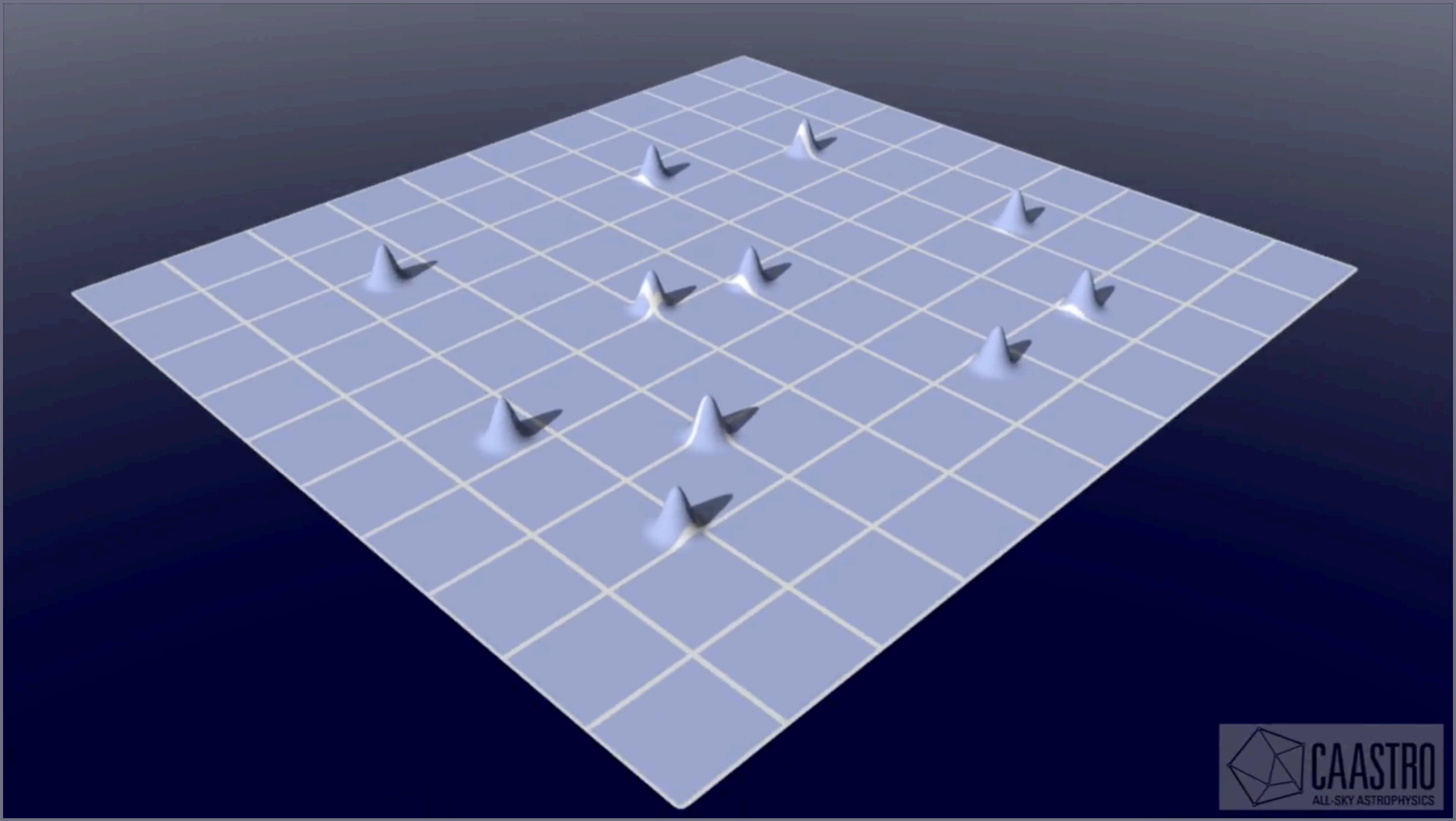
$$\xi(\mathbf{r}) = \frac{V}{(2\pi)^3} \int |\delta_k|^2 e^{-i\mathbf{k}\cdot\mathbf{r}} d^3k$$

- For an isotropic Universe this is:

$$\xi(\mathbf{r}) = \frac{V}{(2\pi)^3} \int P(k) \frac{\sin kr}{kr} 4\pi k^2 dk$$

- The physical meaning is that it measures the excess with respect to a uniform distribution.


CAASTRO
ALL-SKY ASTROPHYSICS


CAASTRO
ALL-SKY ASTROPHYSICS

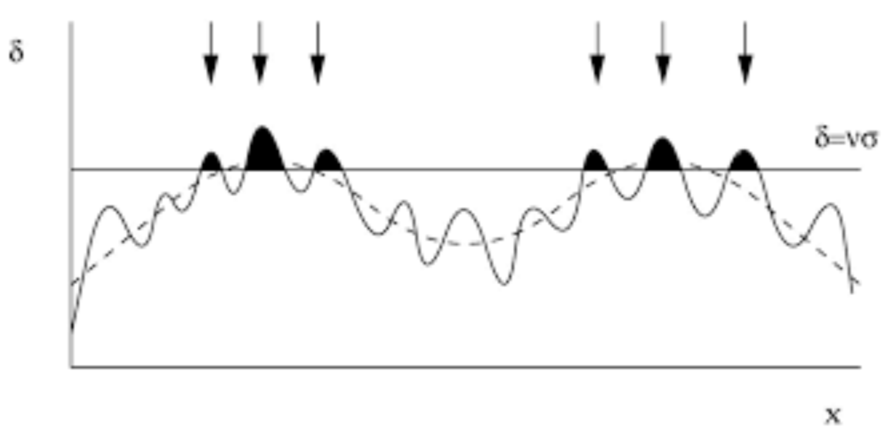
Galaxy bias

A particular problem is that we observe galaxies as tracers of the matter field, but the distribution of baryonic matter is biased with respect to the total matter field (dominated by dark matter). Galaxies grow in the peaks of the density field.

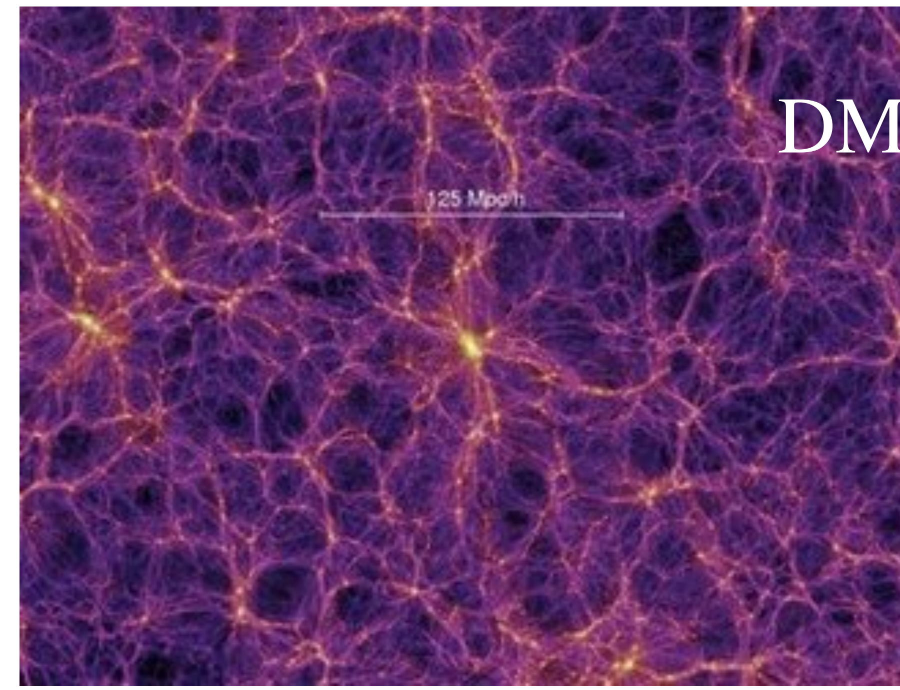
We parametrize linearly the bias with:

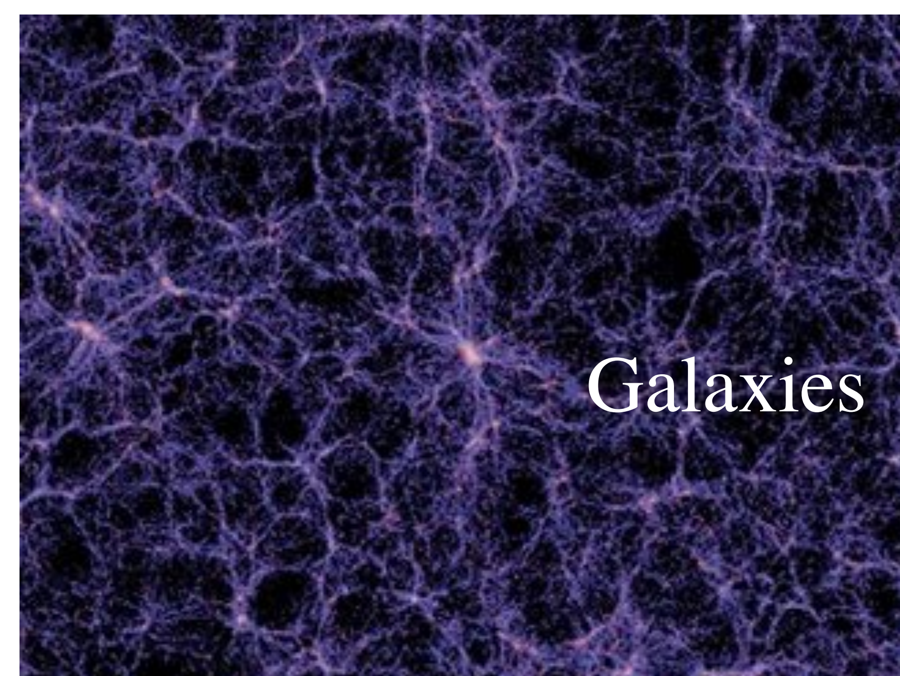
$$\delta_g(k,z) = b(k,z)D(z)\delta(k)$$

Bias is degenerated with the growth factor D(z).



Millennium Simulation





Linear redshift space distortions

- 3D maps of the Universe are in redshift space where galaxy redshift positions differ from the real space positions due to their peculiar velocities.

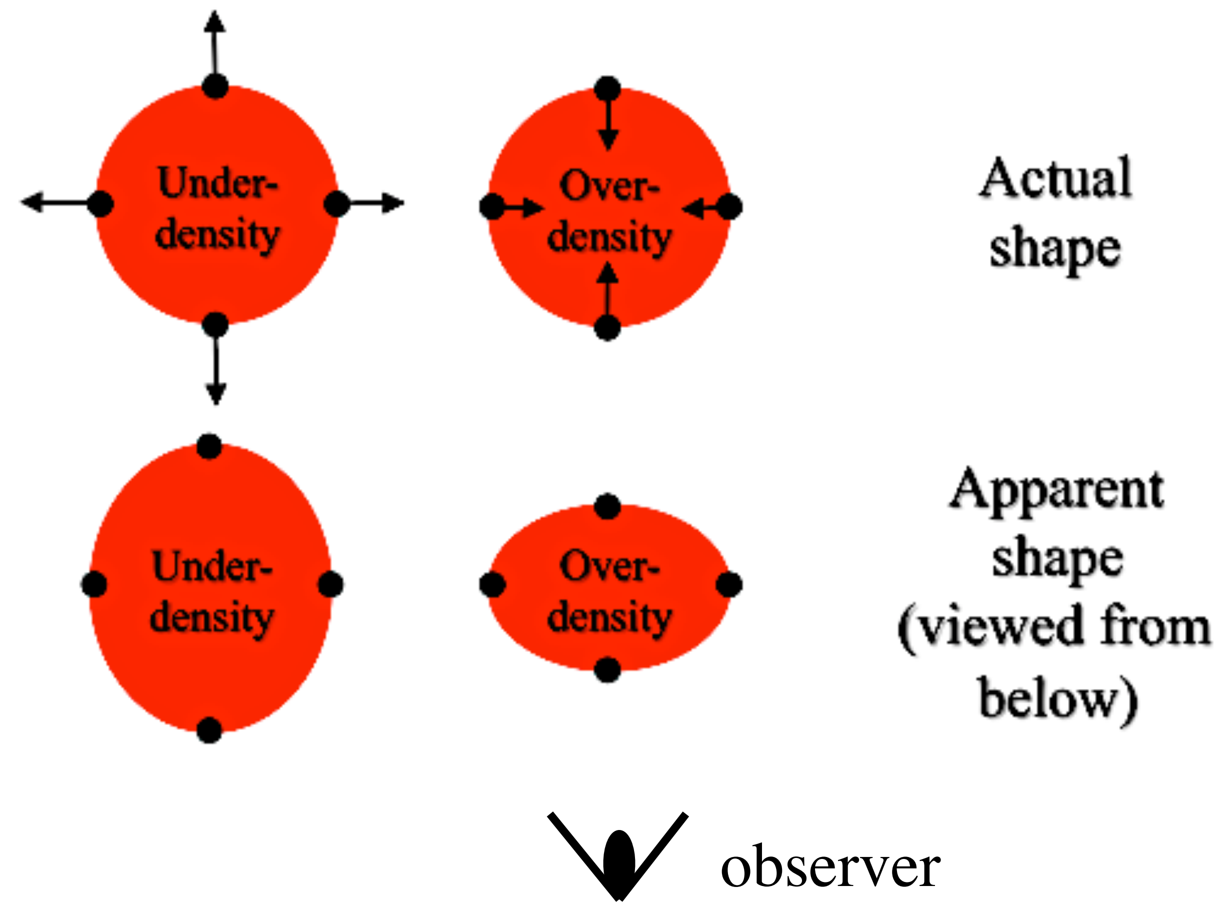
Kaiser 1987

$$\delta^s_{gal}(k,\mu) = \underbrace{b\,\delta_{mass}(k)}_{\delta^r_{gal}} + \mu^2 \underbrace{\theta_{mass}(k)}_{\theta_{gal}}$$

$$\theta = -\nabla \cdot \mathbf{v}/\mathcal{H}$$

$$\frac{\partial \delta}{\partial \tau} + \nabla \cdot \mathbf{v} = 0$$

$$\theta_{mass} = f(z)\delta_{mass}$$



RSD break degeneracy between growth and bias

RSD is a test of Growth History : how does structure form and grow within the background evolution (Modified Grav. vs DE models (GR))

$$\delta^s(k,\mu) = (b + \mu^2 f)\delta_{mass}$$

Anisotropic clustering

Linear Growth factor: $D(z) = \delta(z)/\delta(z=0)$

Linear Growth rate: $f(z) = \frac{\partial \ln D(z)}{\partial \ln a}$

$$f(z) = \Omega_m(z)^\gamma$$

γ growth rate index

Linder 2005

Linear redshift space distortions

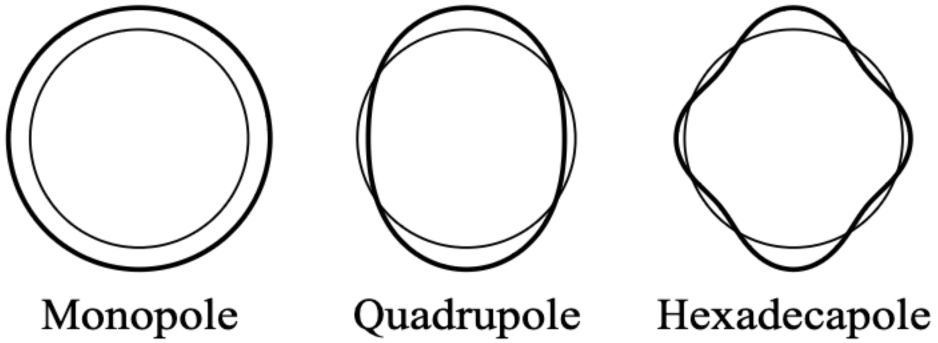
- RSD introduce an anisotropy we should include in the power spectrum or correlation function:

$$\xi(\sigma,\pi) = \xi_0(s)P_0(\mu) + \xi_2(s)P_2(\mu) + \xi_4(s)P_4(\mu)$$

where
$$\xi_\ell(s) = \frac{2\ell+1}{2}\int_{-1}^{+1} \xi(\pi,\sigma)P_\ell(\mu)d\mu.$$

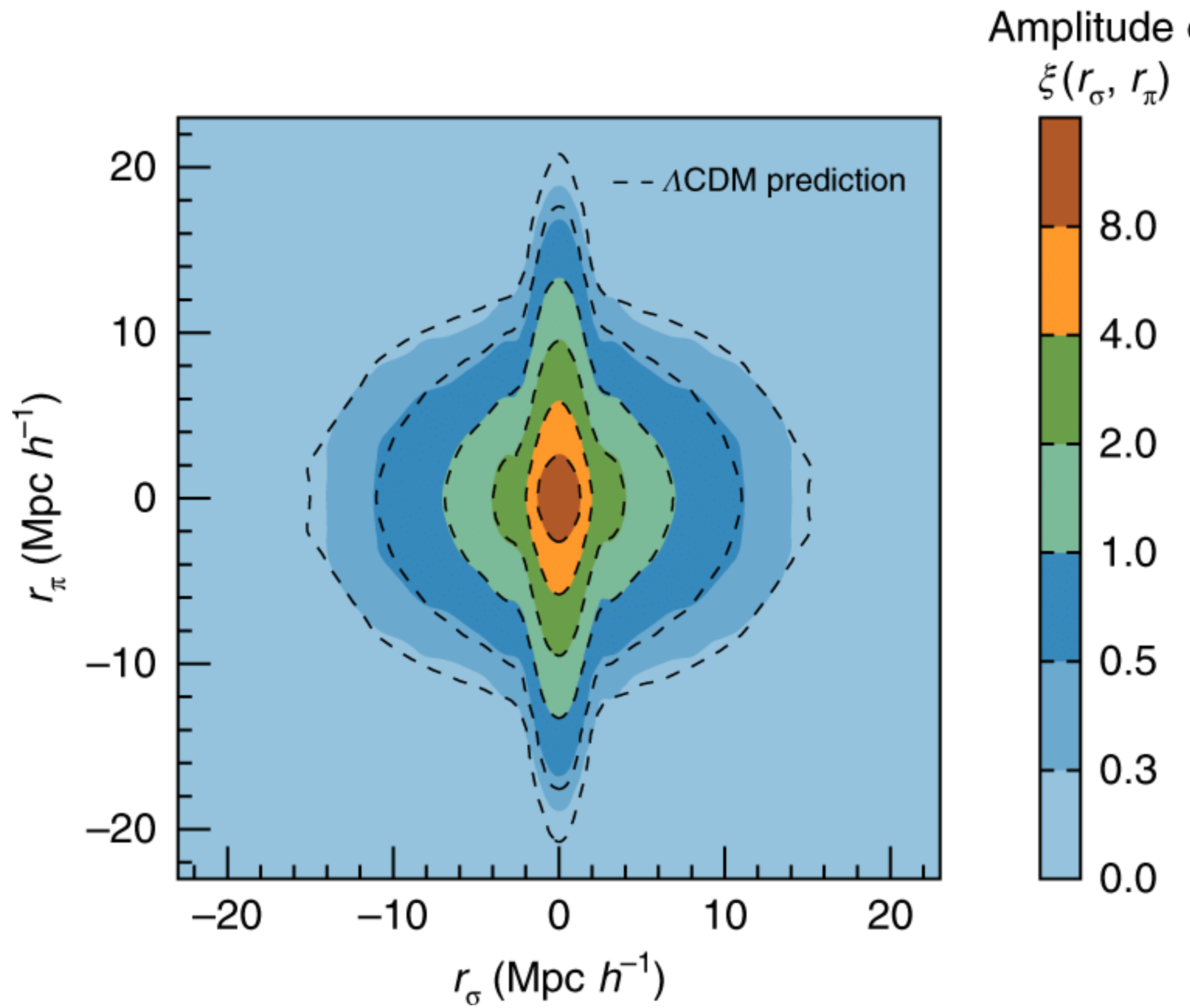
$$\begin{aligned}
\xi_0(s) &= b^2\left(1 + \frac{2\beta}{3} + \frac{\beta^2}{5}\right)\xi(s) \\
\xi_2(s) &= b^2\left(\frac{4\beta}{3} + \frac{4\beta^2}{7}\right)\left[\xi(s) - \bar{\xi}(s)\right] \\
\xi_4(s) &= b^2\frac{8\beta^2}{35}\left[\xi(s) + \frac{5}{2}\bar{\xi}(s) - \frac{7}{2}\bar{\bar{\xi}}(s)\right]
\end{aligned}$$

$$\begin{aligned}
\bar{\xi}(r) &= \frac{3}{r^3}\int_0^r \xi(r')r'^2dr', \\
\bar{\bar{\xi}}(r) &= \frac{5}{r^5}\int_0^r \xi(r')r'^4dr'.
\end{aligned}$$



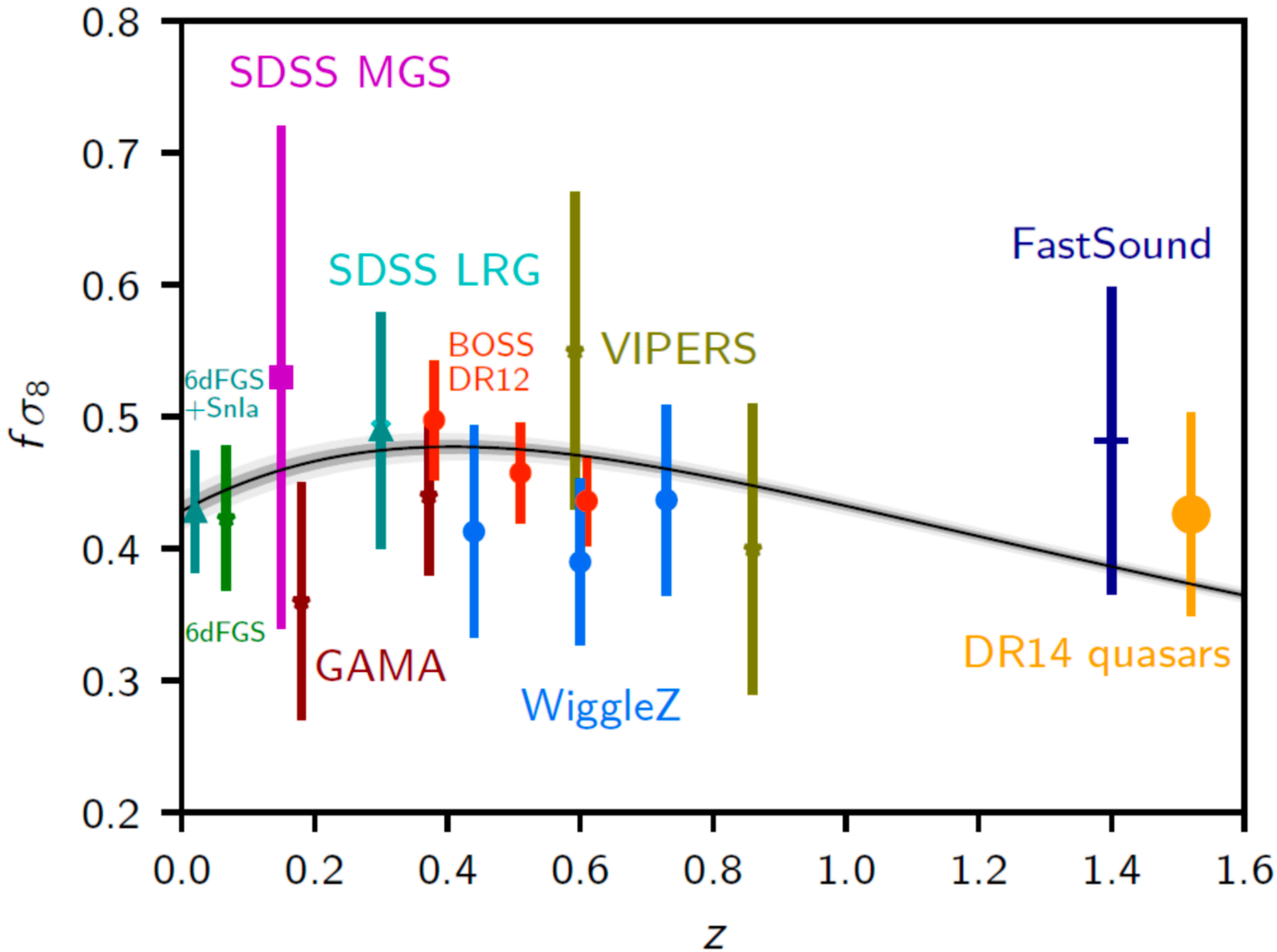
Linear redshift space distortions

- We can decompose the radial and transverse directions in order to measure the redshift space spectrum



Linear redshift space distortions

- RSD offer us a great GR test as we can measure the growth rate of structure for several populations.



Non-linear evolution

- When using the information from small scales, we need to include the information from non-linear evolution of the growth of structures.

- This can be done with non-perturbative methods but usually done with N-body simulations.

- Once the simulation is done, we can try to produce fitting formulas to include in our theory (e.g, Halofit).

- Also, ensemble of simulations for different cosmological and astrophysical parameters allows us to create emulators as we sample the space of simulations

- Simulations + Artificial intelligence can allow us to determine the best model without the need of fitting -> likelihood free inference