A lightning course on Flavor Physics

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TAE 2024

Outline of these lectures and bibliography

- **Flavor physics in the SM**
  - The flavor structure of the SM
  - The Cabibbo-Kobayashi-Maskawa matrix
  - Flavor-changing neutral currents in the SM
- **Elements of flavor physics phenomenology**
  - Effective field theories - EFT lectures
  - Hadronic matrix elements - LQCD lectures
- **Examples of flavor-violating processes**
  - Pion and kaon 2- and 3-body (semi)leptonic decays
  - *B* meson 4-body decays
  - Neutral-meson mixing

- **Bibliography**

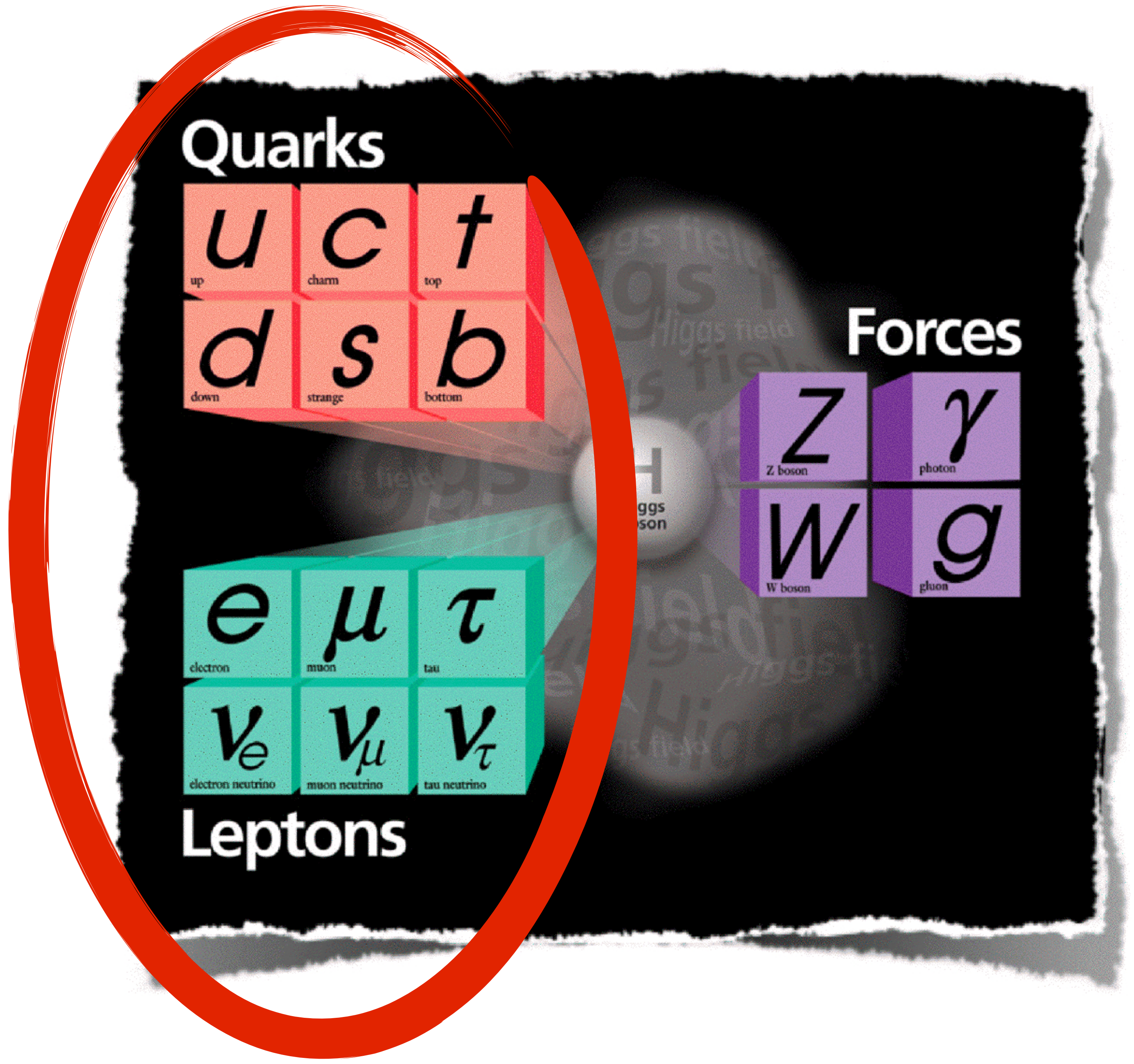
  - **Lecture notes:** Grossman&Tanedo - arXiv: 1711.03624
    Grinstein - arXiv: 1501.05283
  - **Books:** Branco, Lavoura & Silva - "CP violation" - Core reference
    Donoghue, Golowich & Holstein "Dynamics of SM" - Phenomenology
    Buras "Gauge Theory of Weak Decays ..." -Detailed calcs in SM and BSM

Flavor physics in the SM

Flavor universality: gauge interactions

- The SM matter content appears in **3 generations**

$$\mathscr{L}_{\text{gauge}} \subset \bar{\psi}^k \left(i\partial_\mu + g X_\mu^A t_k^A \right) \gamma^\mu \psi^k \qquad k = 1,2 \text{ or } 3$$



Family-independent quantum numbers

$$Q_L^k \sim (3,2)_{1/6} \qquad u_R^k \sim (3,1)_{2/3} \quad d_R^k \sim (3,1)_{-1/3}$$
$$L_L^k \sim (1,2)_{-1/2} \qquad e_R^k \sim (1,1)_{-1}$$

The gauge interactions in the SM are flavor universal

$\mathscr{L}_{\text{gauge}}$ has a **global accidental** $U(3)^5$ flavor symmetry

Flavor breaking: Yukawa interactions

$$\mathscr{L}_{\text{yukawa}} = y_u^{kl} \bar{Q}_L^k \tilde{H} u_R^l + y_d^{kl} \bar{Q}_L^k H d_R^l + y_e^{kl} \bar{L}_L^k H e_R^l + \text{h.c.}$$

Matrices with N^2 complex parameters

- **Mass generation in the SM:** $SU(2)_L \times U(1)_Y \xrightarrow{\text{SSB}} U(1)_{\text{EM}}$

$$m_f^{kl} = v_{\text{ew}}\, y_f^{kl}$$

$$\mathscr{L}_{\text{masses}} = m_u^{kl} \bar{u}_L^k u_R^l + m_d^{kl} \bar{d}_L^k d_R^l + m_e^{kl} \bar{e}_L^k e_R^l + \text{h.c.}$$

- **Diagonalization:** *Linear & unitary* field redefinitions commuting with $U(1)_{\text{EM}}$

Unitary matrix ← $f_L \to L_f f_L$, $f_R \to R_f f_R$

$$m_u \to L_u^\dagger m_u R_u = \text{diag}\left(m_u, m_c, m_t\right)$$
$$m_d \to L_d^\dagger m_d R_d = \text{diag}\left(m_d, m_s, m_b\right)$$
$$m_e \to L_e^\dagger m_e R_e = \text{diag}\left(m_e, m_\mu, m_\tau\right)$$

→ 9 real parameters

Flavor violation in the charged currents (CC)

$$\mathscr{L}_{\text{gauge}} \supset g\bar{\psi}^k_L \left(T^+W^+_\mu + T^-W^-_\mu\right)\gamma^\mu\psi^k_L = g\left(\bar{u}^k_L\gamma^\mu d^k_L + \bar{\nu}^k_L\gamma^\mu e^k_L\right)W^+_\mu + \text{h.c.}$$

$$Q^k_L = (u^k_L, d^k_L)^T \qquad L^k_L = (\nu^k_L, e^k_L)^T$$

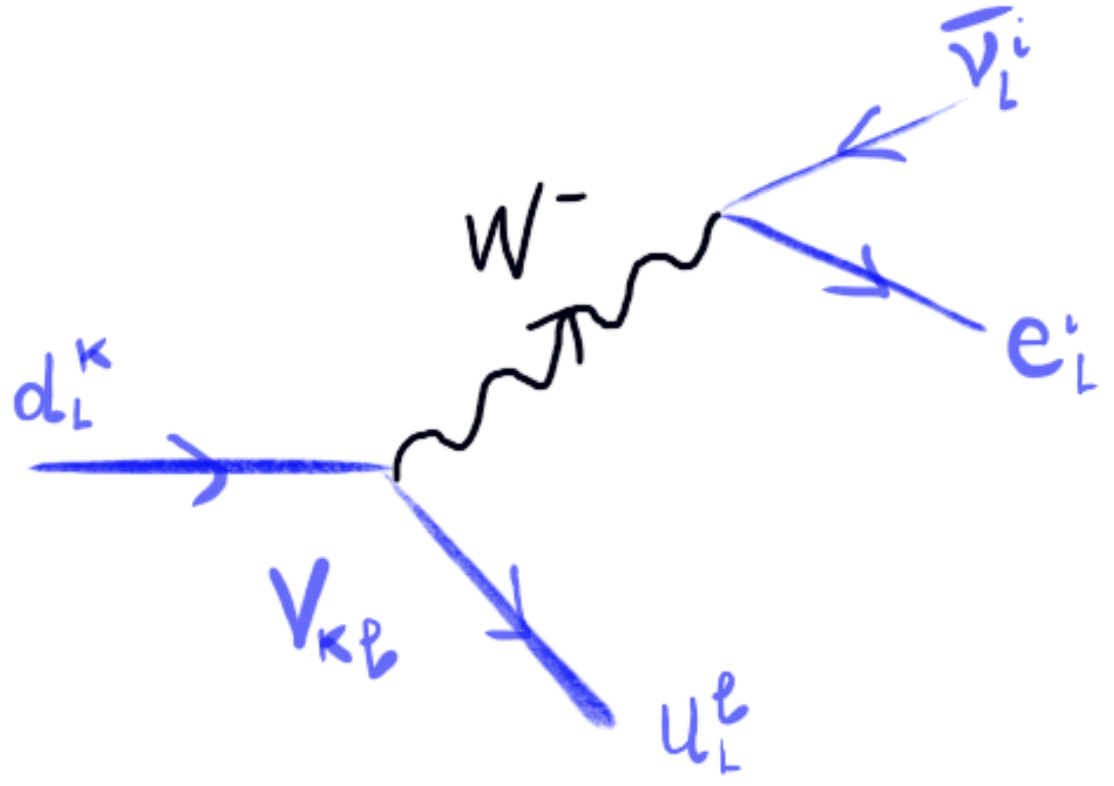
Lepton sector

- Missalignment between gauge and *up* and *down* quark mass matrices

$$\mathscr{L}_{\text{CC}} = g\left(V_{\text{CKM}}\right)_{kl}\bar{u}^k_L\gamma^\mu d^l_L W^+_\mu + g\bar{\nu}^k_L\gamma^\mu e^k_L W^+_\mu + \text{h.c.}$$

- **The Cabibbo-Kobayashi-Maskawa mixing matrix**

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



Flavor violation occurs because we cannot diagonalize simultaneously the gauge and yukawa interactions

- Neutrinos in the SM are massless and flavor mixing can be rotated away

Parameter counting in the CKM matrix

1. V_{CKM} **is a unitary matrix** (it is the product of 2 unitary matrices)

$N \times N$ unitary matrix parametrized by N^2 **real numbers**

$$\#_{\text{angles}} = N(N-1)/2 \qquad \#_{\text{phases}} = N(N+1)/2$$

2. Physics invariant w.r.t. $(2N-1)$ **rephasings** of the quark fields

$$u_L^k \to e^{i\alpha_k}\, u_L^k \qquad d_L^k \to e^{i\beta_k}\, d_L^k$$

The N^{th} dimensional CKM matrix contains …

$$\#'_{\text{angles}} = N(N-1)/2 \qquad \#'_{\text{phases}} = (N-1)(N-2)/2$$

$$N = 3: \qquad \#'_{\text{angles}} = 3 \qquad \#'_{\text{phases}} = 1$$

- **The minimum number of generations needed to generate *CP violation* is 3!**

More about parameter counting and spurions

- **Symmetry argument for parameter counting**
  1. $\mathscr{L}_{\text{gauge}}$ in the SM invariant w.r.t. $U(3)_L \times U(3)_e \Rightarrow$ **18 generators**
  2. $\mathscr{L}_{\text{yukawa}}$ breaks $U(3)_L \times U(3)_e \to U(1)_e \times U(1)_\mu \times U(1)_\tau \Rightarrow$ **3 unbroken generators**
  3. **We can use broken generators to rotate away *unphysical parameters* in** $\mathscr{L}_{\text{yukawa}}$

#physical parameters = #unbroken generators (**3 masses** for leptons)

  5. **For quarks:** $U(3)^3 \to U(1)_B \Rightarrow$ **10** physical parameters (**6 masses, 3 angles, 1 phase**)

- **Spurions:** Pretend yukawa matrices are bifundamentals of the flavor group

  Keep track of flavor violation in the SM and beyond (**Minimal flavor violation**)

A standard parametrization of CKM

- Phase redefinitions of quarks ⇒ **Set** V_{ud} , V_{us} , V_{cb} **and** V_{tb} **real**
- **The "standard" *unitary* parametrization** $(s_{ij} = \sin\theta_{ij} , c_{ij} = \cos\theta_{ij})$

$$V_{\text{CKM}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

The SM is *defined* when the 3 CKM angles and its 1 phase are **determined experimentally** …

$s_{12} = 0.22650(48)$ $s_{23} = 0.04053(71)$ $s_{13} = 0.00361(10)$ $\delta = 68.5(2.6)^\circ$

- The quark mixing matrix is **hierarchical!**

Complex phases and CP violation

- The SM is a **chiral theory** $\Rightarrow$ The SM violates **parity (*P*)** and **charge conjugation** (*C*)
- However the SM **does not necessarily** violate *CP*

The SM violates *CP if* the nontrivial CKM phase is not 0 or π

hermitian conjugate

$$\left.\begin{aligned} \mathscr{L}_{\rm toy} &= y_{ij}\,\bar{\chi}_i\psi_j\,S + y_{ij}^*\,\bar{\psi}_j\chi_i\,S^\dagger \\ (CP)\mathscr{L}_{\rm toy}(CP)^\dagger &= y_{ij}\,\bar{\psi}_j\chi_i\,S^\dagger + y_{ij}^*\,\bar{\chi}_i\psi_j\,S \end{aligned}\right\} \Rightarrow \quad \mathscr{L}_{\rm toy} = (CP)\mathscr{L}_{\rm toy}(CP)^\dagger \iff y_{ij}^* = y_{ij}$$

- Unambiguous (**rephasing invariant**) measure of *CP* violation in the SM:

Jarlskog invariant

$$J = \mathrm{Im}\left(V_{ij}V_{kl}V_{il}^*V_{kj}^*\right)$$

  - In the standard CKM parametrization

$$J = c_{12}s_{12}c_{13}^2s_{13}c_{23}s_{23}\sin\delta$$

All mixing angles must be nonzero for *CP* violation

  - *CP* violation *is in* the SM but *not explained by* the SM

Wolfenstein parametrization

- **Expose the CKM hierarchies explicitly**

Small parameter: **Cabibbo angle**

$$\lambda \equiv s_{12} = 0.22650(48)$$

- **Define ...**

$$s_{23} \equiv A\,\lambda^2$$

$$s_{13}e^{i\delta} \equiv A\lambda^3(\rho + i\eta)$$

... and expand in λ!

First 2 families subspace

$$V_{\rm CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$A = 0.826(12)$ $\rho = 0.152(14)$ $\eta = 0.357(10)$

- The Wolfenstein parametrization is **not exactly unitary**
- Mixing first two families is unitary (and independent of 3rd family) up to $\mathcal{O}(\lambda^2)$

The unitary triangle(s)

Unitary relations

1. Row(column) unitarity: $|V_{i1}|^2 + |V_{i2}|^2 + |V_{i3}|^2 = 1$
2. Off-diagonal unitarity: $V_{i1}V_{j1}^* + V_{i2}V_{j2}^* + V_{i3}V_{j3}^* = 0$

- **2. is a null sum of complex vectors $\Rightarrow$ Unitarity triangles**

  1st and 3rd columns give triangle with all sides of same $\mathcal{O}(\lambda^3)$

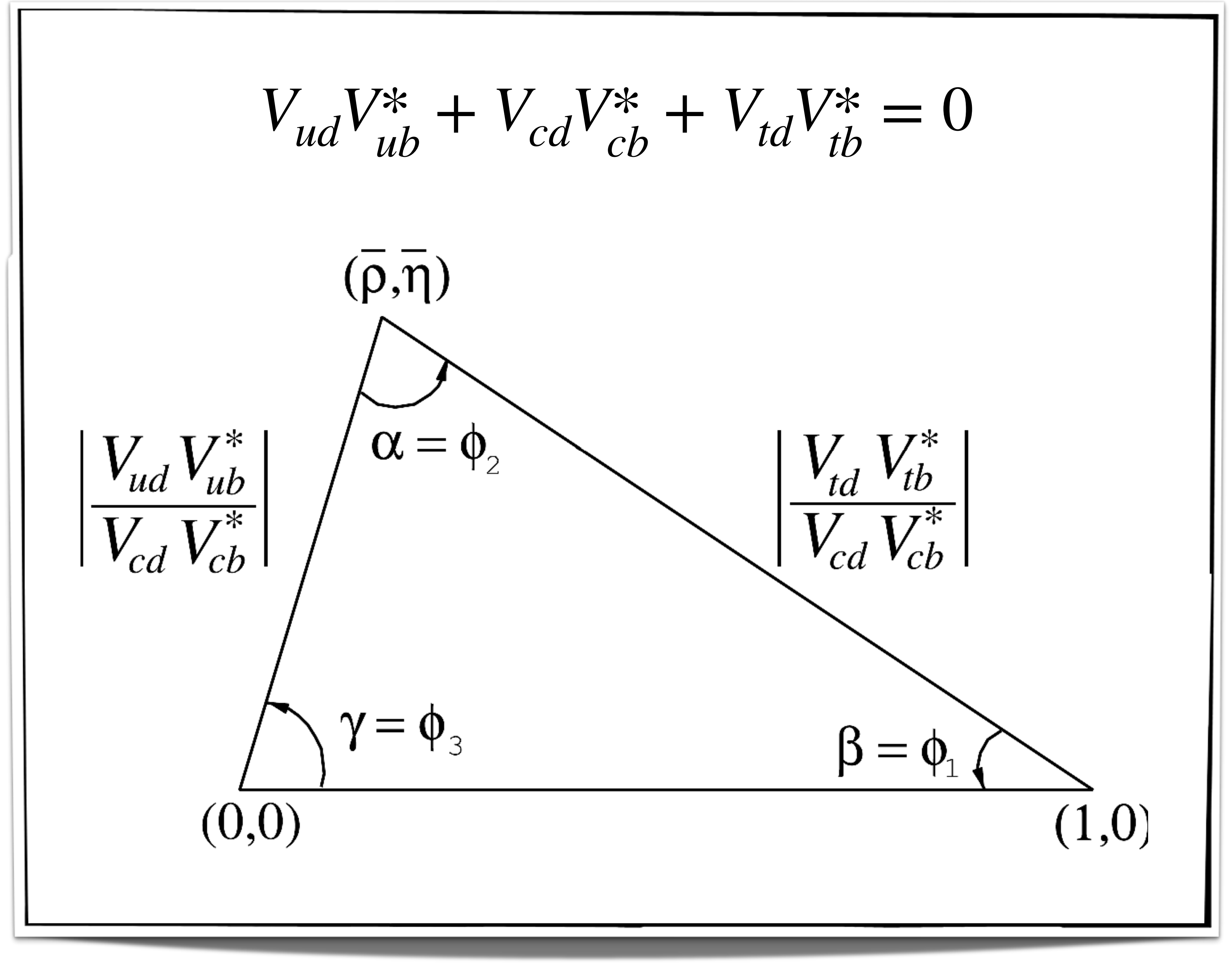
  - Three (**rephasing invariant**) angles (**directly observable!**)

$$\beta = \phi_1 = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right) \qquad \alpha = \phi_2 = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right) \qquad \gamma = \phi_3 = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

  - The apex is fixed by a redefinition:

$$\bar{\rho} + i\bar{\eta} = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}$$

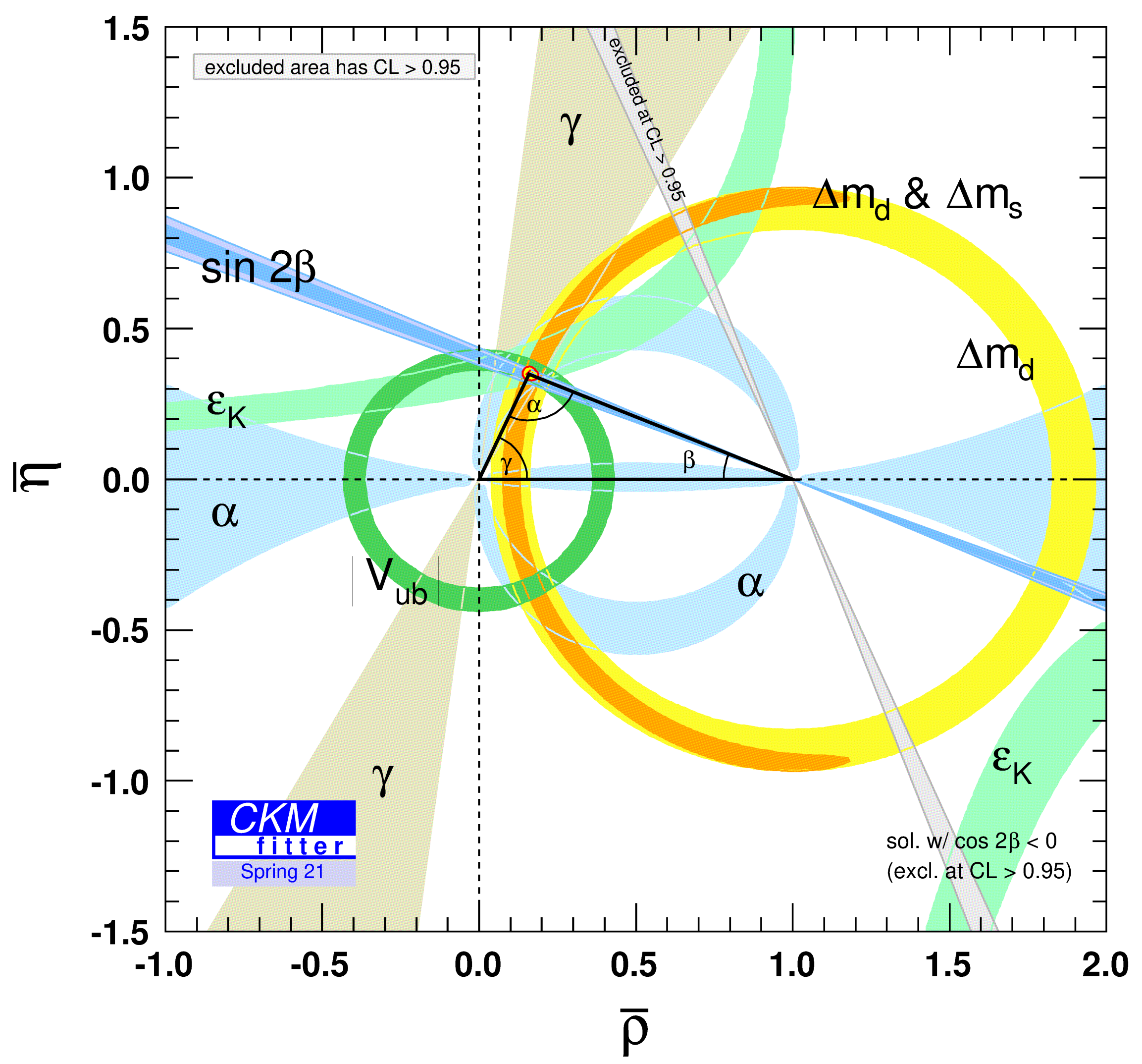
that is **rephasing invariant**



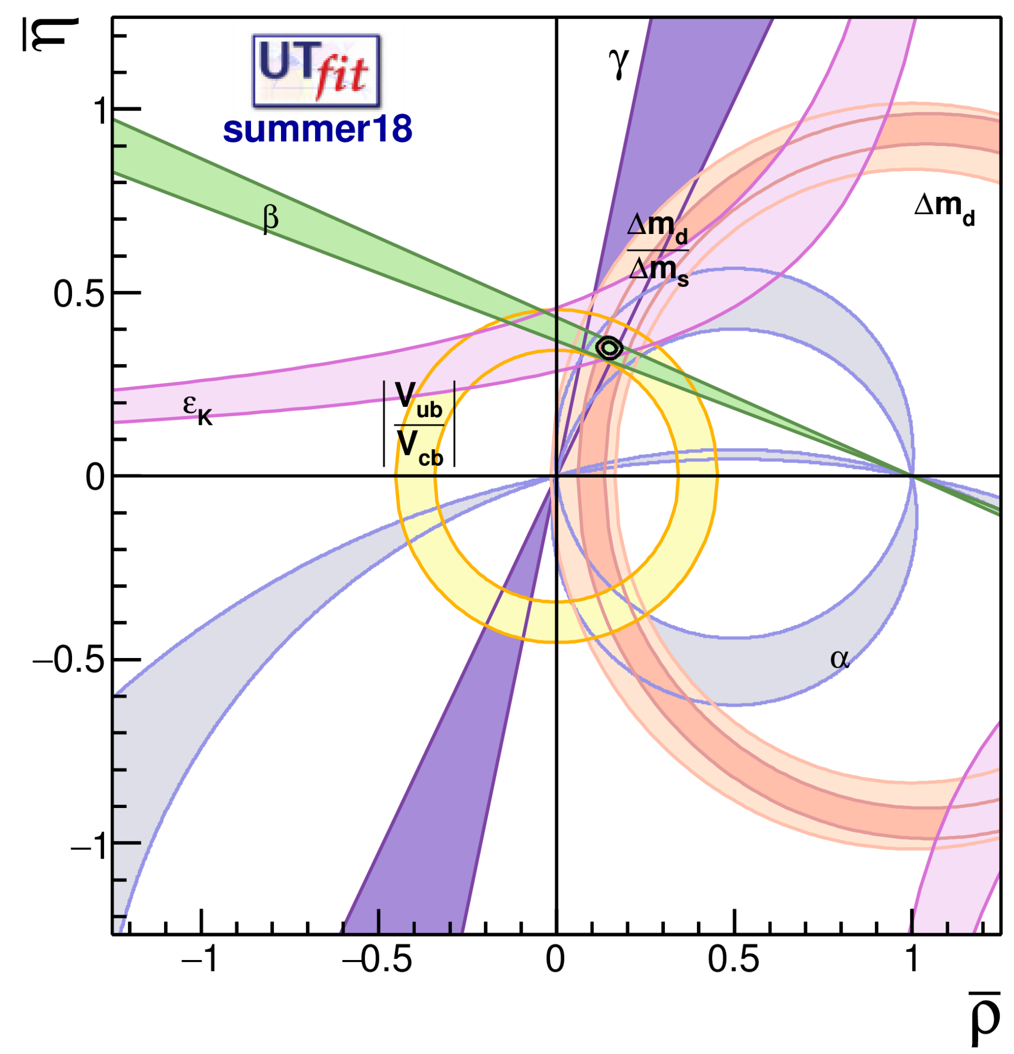
Experimental constraints in the unitary triangle

Two collaborations perform updated fits to the CKM parameters

- **CKMfitter** - frequentist analysis
  ckmfitter.in2p3.fr
  - Conservative with uncertainties (*Rfit*)



- **UTfit** - bayesian analysis
  www.utfit.org
  - Includes fits with BSM (EFT) parameters

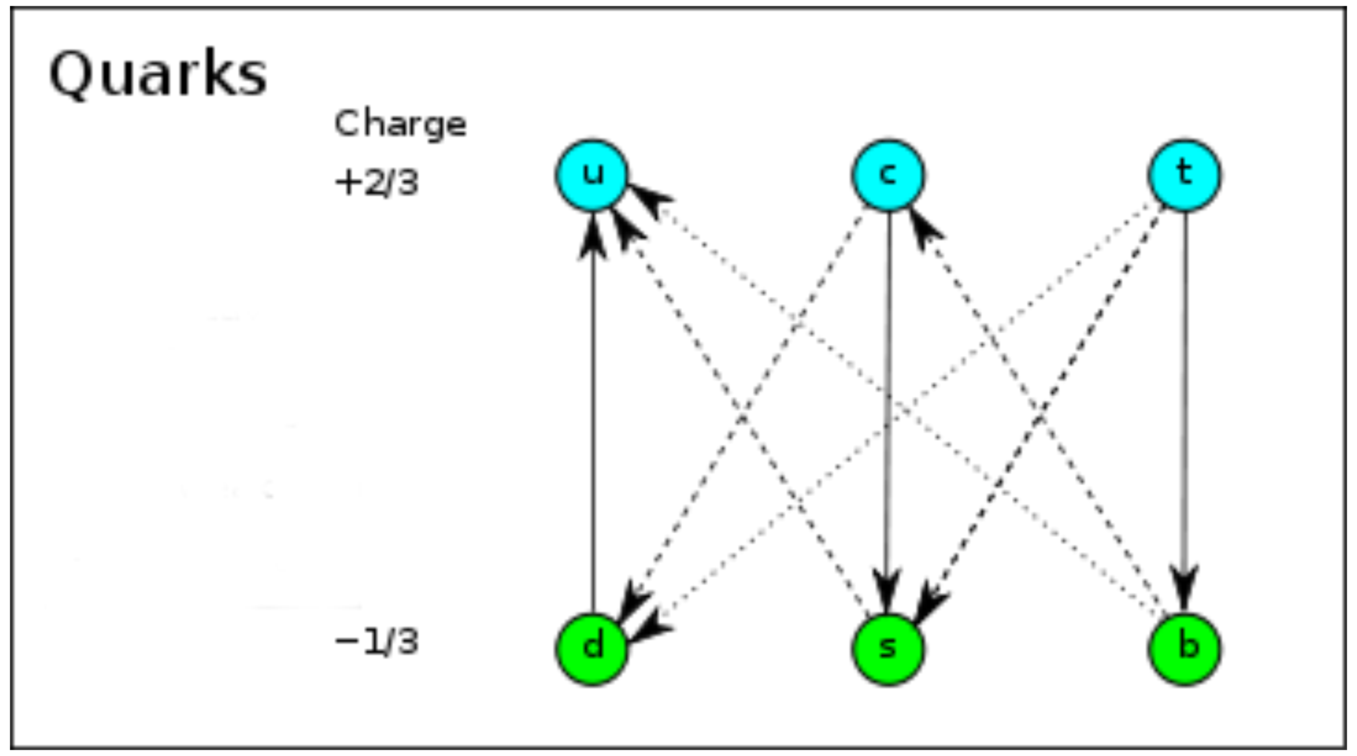


UT triangle and the Jarslkog invariant

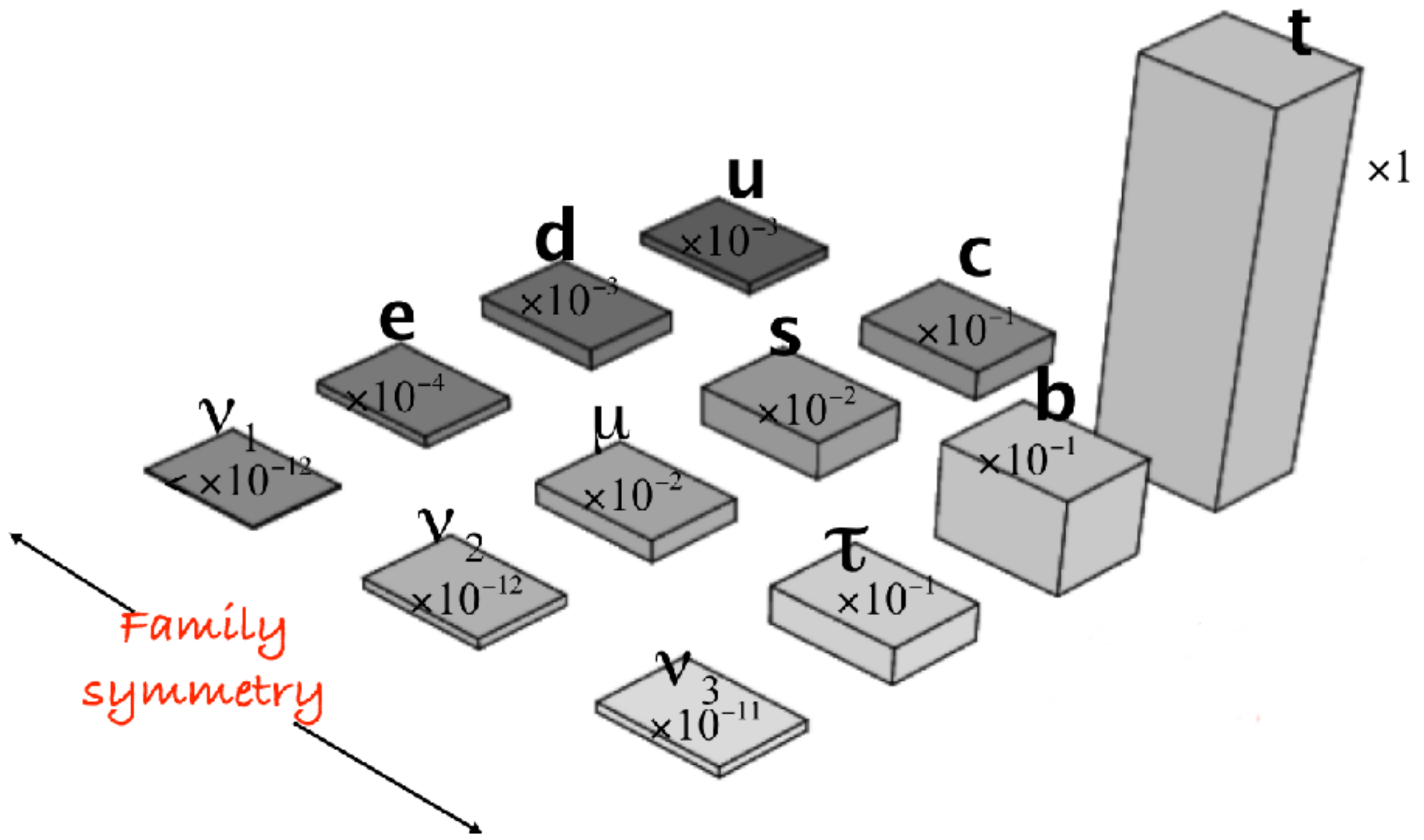
- **Geometric interpretation:** $\text{Area}_{\text{UT}} = J/2$
- ***CP* violation small in SM *because of small mixing*:** $J_{\text{SM}} \approx \lambda^6 A^2 \eta = 3.00(12) \times 10^{-5}$

Flavor hierarchies and the (quark) flavor puzzle

- Flavor transitions



- Masses

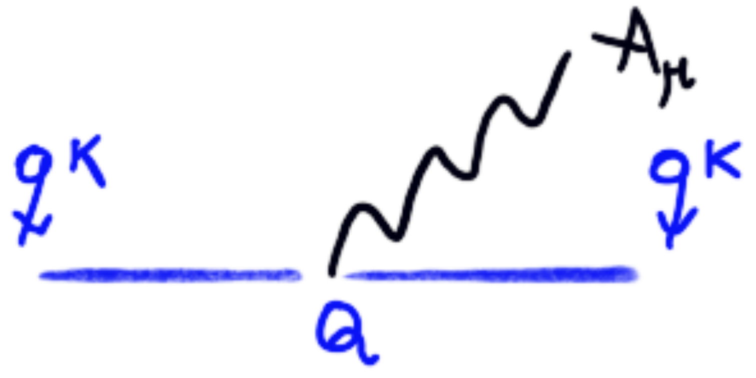


Flavor puzzle: **Origin of patterns and hierarchies in the values of the flavor parameters**

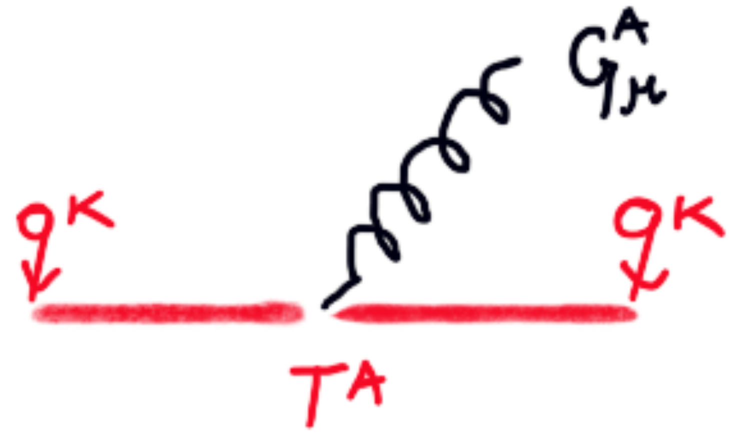
- Portal to BSM physics!
  - Horizontal symmetries (Froggatt-Nielsen), extra dimensions (Randall-Sundrum), tree-loop hierarchies (Weinberg), clockwork mechanism, etc
- **Essential for our existence! - Anthropic principle**
  - Stability of matter (*up* and *down* quark masses) & stability of vacuum (top-quark mass)
- **Origin of *CP* violation? - Connection to baryogenesis**
  - Why 3 families?

Neutral currents at tree level in the SM: Photon, gluon and Higgs

- **QED (photons) and QCD (gluons):** Couplings **diagonal in flavor space** (same charges/reps)

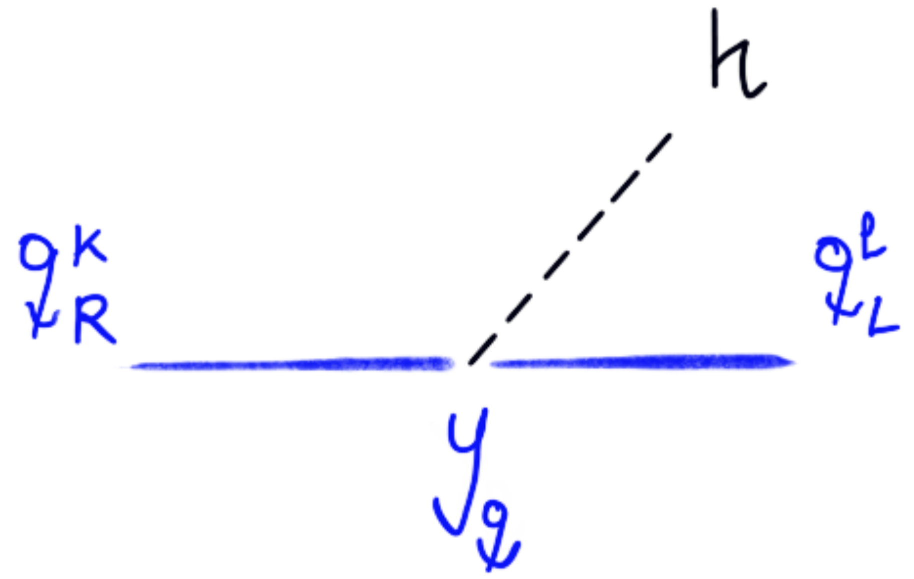


CKM unitarity: $V^\dagger V = \mathbf{1}$



$$J^\mu_{\rm EM} = e\, Q_q\, \bar{q}^k \gamma^\mu\, (\mathbf{1})_{kl}\, q^l \to e\, Q_q\, \bar{q}^k \gamma^\mu\, (V_q^\dagger)_{kj}\, (V_q)_{jl}\, q^l = J^\mu_{\rm EM}$$

- **Yukawa interactions (higgs):** Couplings **aligned** with the mass basis

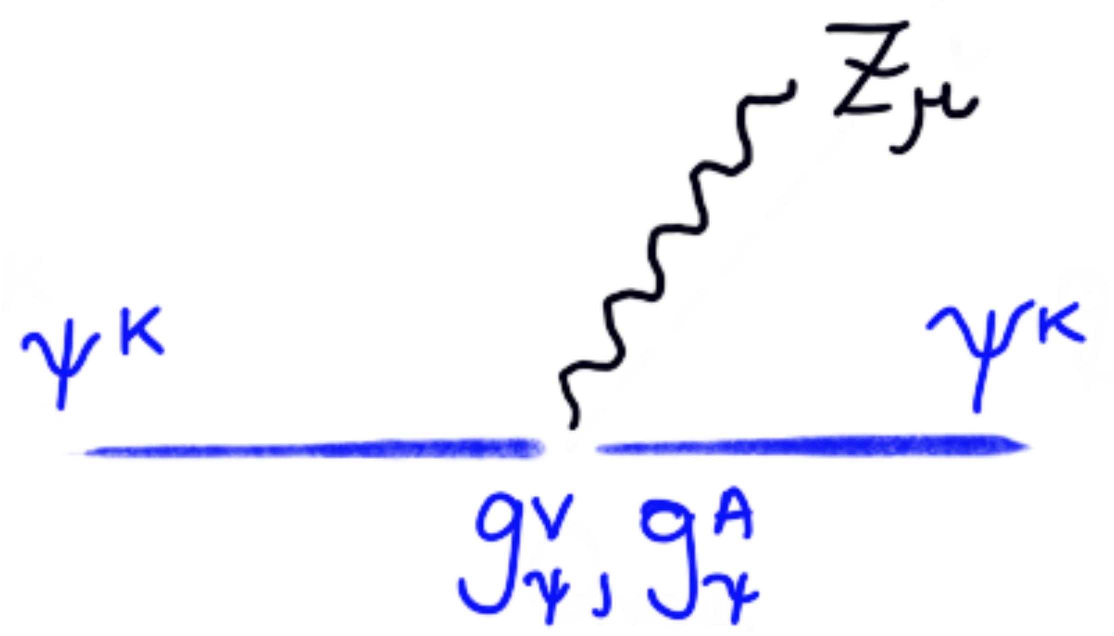


SSB in the SM: $H^T \to \begin{pmatrix} 0 & v + \frac{h}{\sqrt{2}} \end{pmatrix}$

$$\bar{Q}^k_L H\, (y_d)_{kl}\, d^l_R \to \bar{Q}^k_L\, (m_d)_{kl}\, d^l_R \left(1 + \frac{h}{v\sqrt{2}}\right)$$

Neutral currents at tree level in the SM: The Z boson

- **Weak charges:** Couplings of the *Z* also **diagonal in flavor space**



$$J_Z^\mu = -\frac{e}{2s_w^2}\,\bar{\psi}^k\left(g_V^\psi\gamma_\mu + g_A^\psi\gamma^\mu\gamma_5\right)\psi^k$$

$$g_V^{\psi_k} = T_3^{(\psi_k)} - 2s_w^2 Q_\psi \qquad g_A^{\psi_k} = T_3^{(\psi_k)}$$

- What is relevant here is that all *up*-like fermions and all *down*-like fermions have the same **weak isospin**
- **Before 1970 hadrons were thought composed exclusively of *u*, *d* and *s* quark**

  with CC interactions rotated by 2×2 Cabibbo mixing: $J^\mu_{CC} = \bar{u}(1-\gamma_5)\left(\cos\theta_C d + \sin\theta_C s\right)$
- If $(u,d)^T$ is iso-doublet and s isosinglet $\Rightarrow$ **There must be tree-level neutral $\Delta S = 1$ decays**
- PDG (Particle Data Group):

  CC: $\mathrm{Br}(K_L\to\pi^+e^-\bar{\nu}) = 40.55(11)\,\%$

  NC: $\mathrm{Br}(K_L\to\mu^+\mu^-) = 6.84(11)\times\mathbf{10^{-9}}$

Flavor changing neutral currents (FCNC) are suppressed!

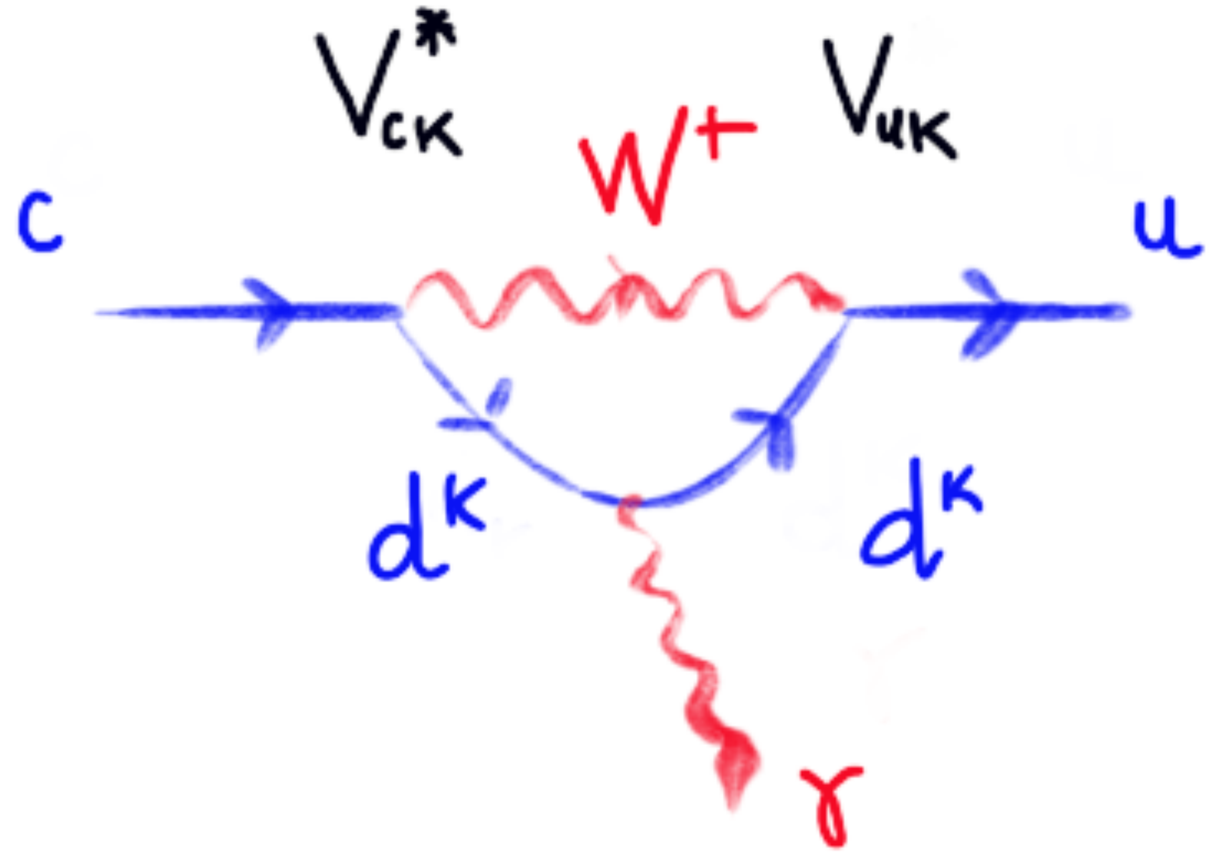
There must be a 4th quark (**charm**)!

Glashow, Iliopoulos & Maiani (GIM) 1970

Flavor-changing neutral currents (FCNC) in the SM

- The GIM mechanism
  - In the SM, FCNCs occur only at **1-loop level!**
  - In addition, they receive a **flavor suppression**

Take the $\Delta C = 1$ **neutral transition** $c \to u\gamma$



$$\text{Amplitude} \approx \frac{e\,g^2}{4\pi^2\,m_W^2}\sum_k V^*_{ck}V_{uk}f(m_k^2/m_W^2)$$

- The loop function can be Taylor expanded

$$f(m_k^2/m_W^2) = a + b m_k^2/m_W^2 + \ldots$$

- CKM unitarity!

$$\text{Amplitude} \approx \frac{e\,g^2}{4\pi^2\,m_W^2}\left(V^*_{cs}V_{us}\frac{m_s^2 - m_d^2}{m_W^2} + V^*_{cb}V_{ub}\frac{m_b^2 - m_d^2}{m_W^2}\right)$$

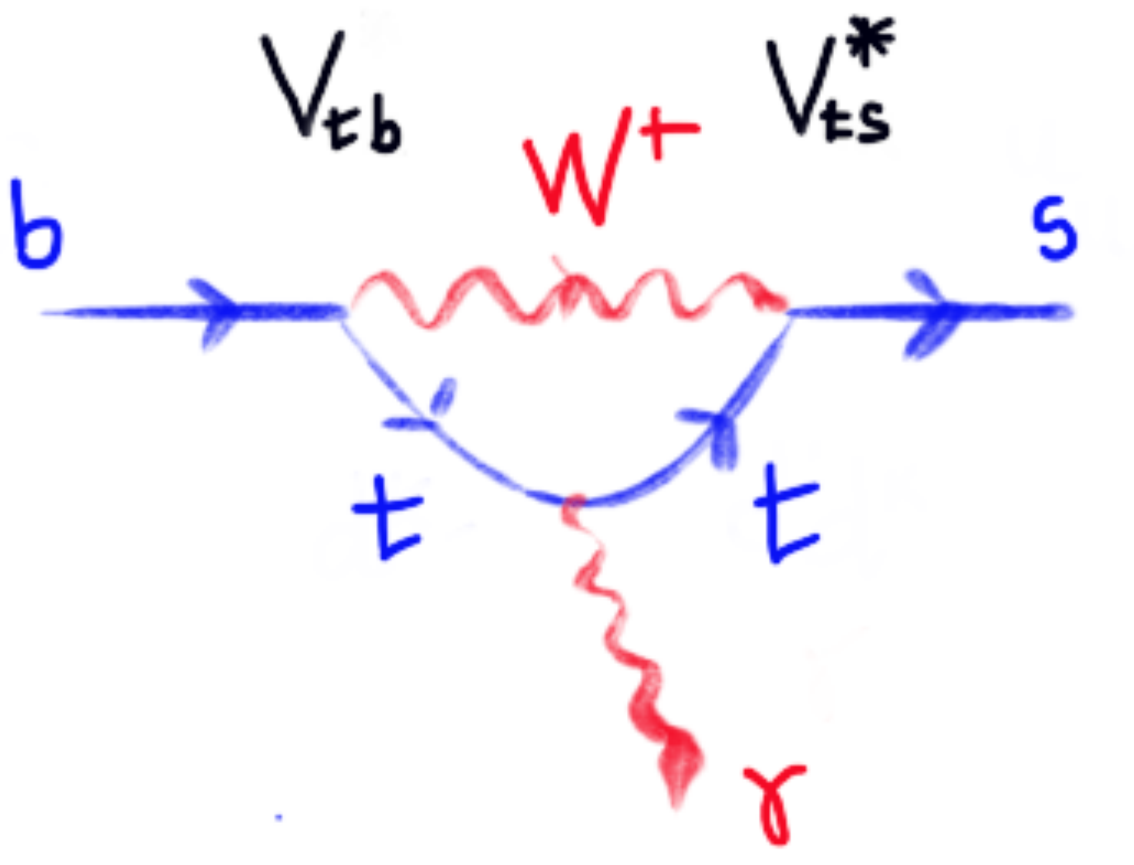
$$\approx \frac{eg^2}{4\pi^2\,m_W^2}\lambda^5 y_b^2$$

- The GIM mechanism is a consequence of CKM unitarity at loop level
- It implies **suppression** of FCNCs by **loop**, **small yukawas** and/or **small mixing angles**

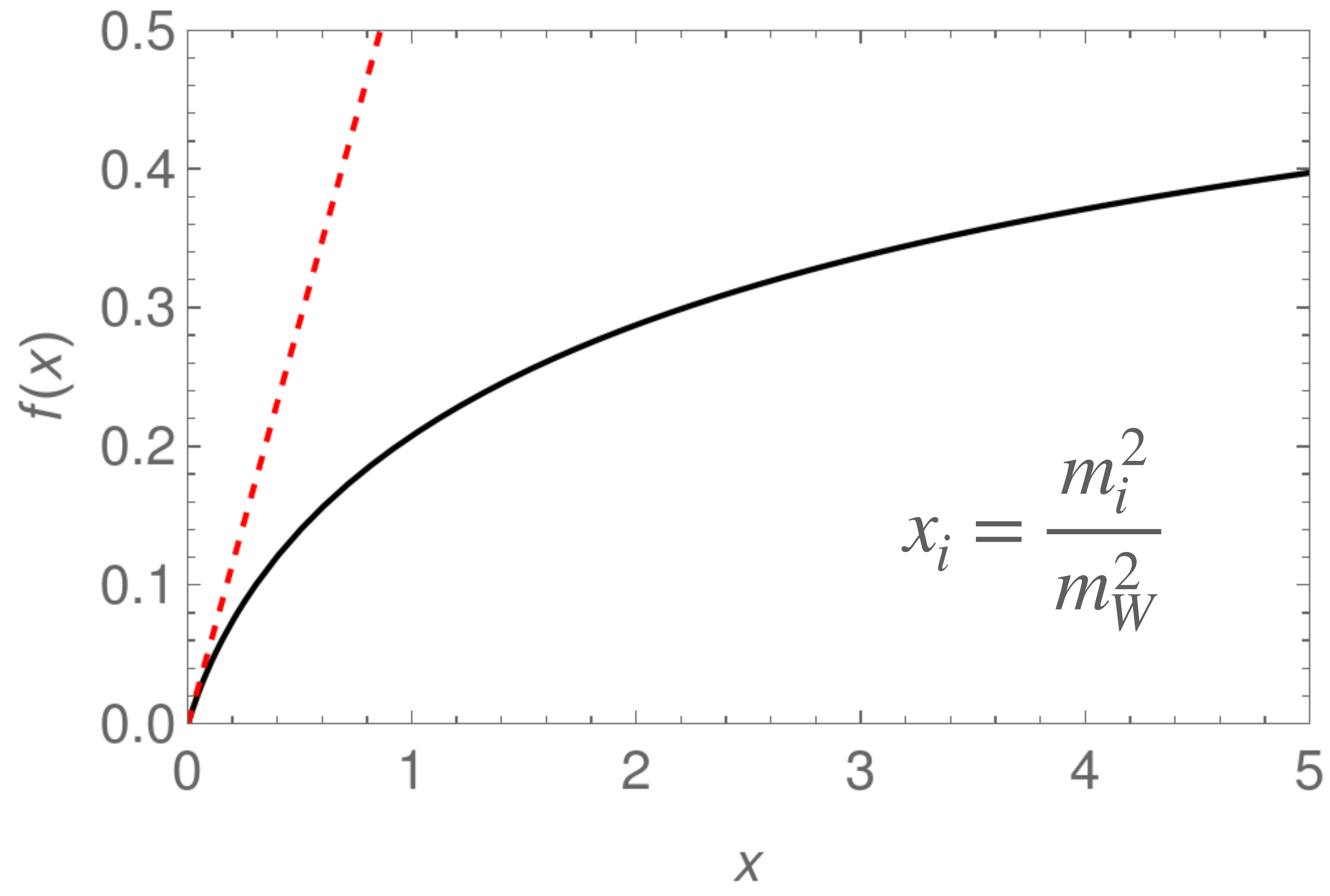
The role of the top-quark in the FCNCs

- FCNCs in the *down*-quark sector
  - Sensitive to *up*-quarks ⇒ **Prominence of top yukawa**
  - $m_W \lesssim m_t$: Suppression to be revisited

Take now the **neutral *down* quark transition** $b \to s\gamma$



$$\text{Amplitude} \approx \frac{e\, g^2}{4\pi^2 m_W^2} \overbrace{V_{tb} V_{ts}^*}^{\lambda^2} f\Big(\frac{m_t^2}{m_W^2}\Big)$$



Inami-Lin function(s)

$$f(x) = -\frac{8x^3 + 5x^2 - 7x}{12(1-x)^3} + \frac{x^2(2-3x)}{2(1-x)^4} \log x$$

$$\approx \frac{7}{12} x + \mathcal{O}(x^2)$$

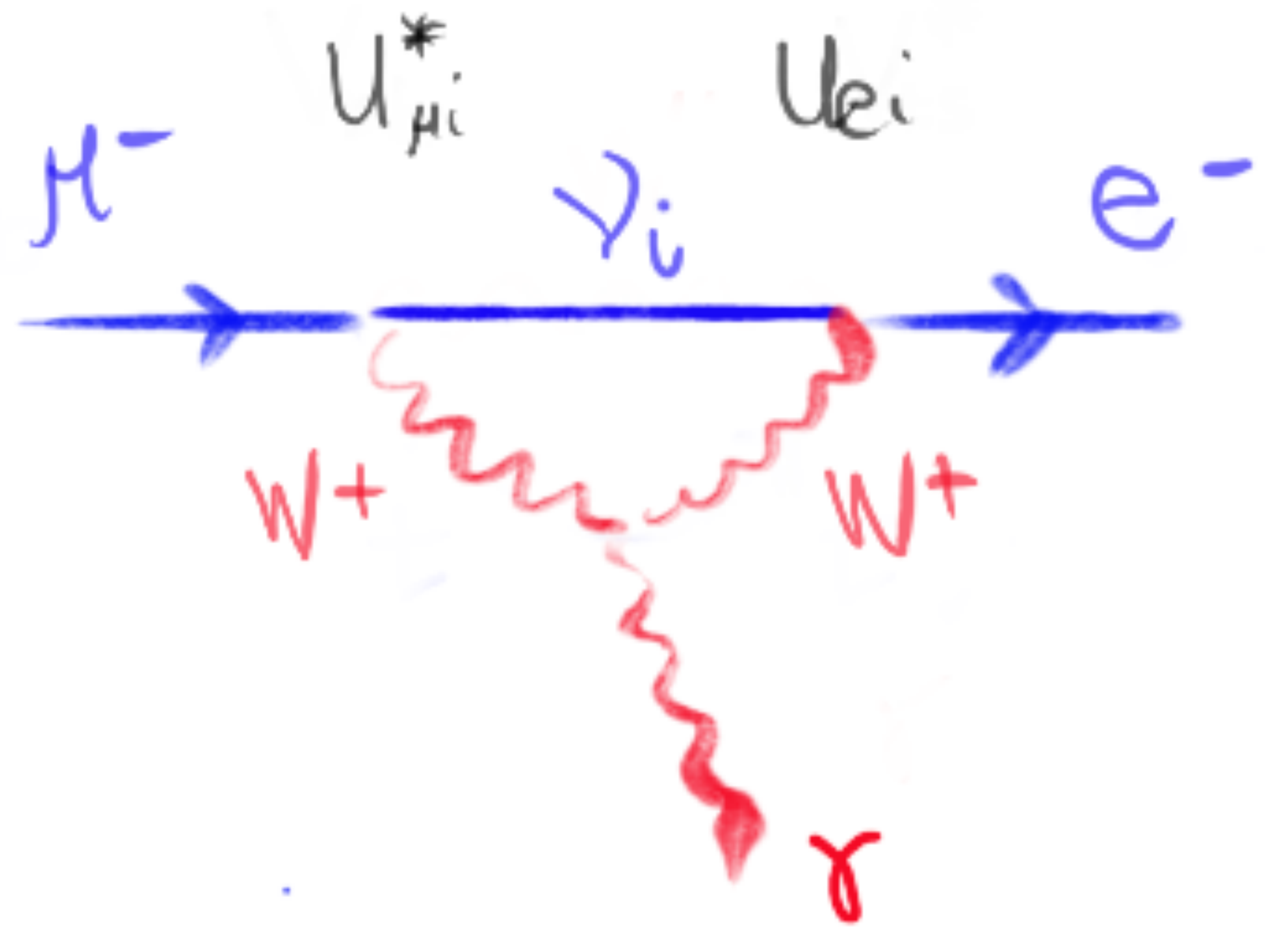
- $f(x)$ linear in x close to 0 and $\mathcal{O}(1)$ for x_t
- $f(x) \to 2/3$ at $x \to \infty$

The case of the charged leptons

- FCNCs in the charged *lepton sector*
  - $U(1)_\tau \times U(1)_\mu \times U(1)_e$ accidental symmetry in the SM $\Rightarrow$ No charged-lepton flavor violation (CLFV)
  - Symmetry broken by **neutrino masses!**

Take now the **neutral charged-lepton transition** $\mu \to e\gamma$

- In the simplest case with **Dirac Neutrinos**

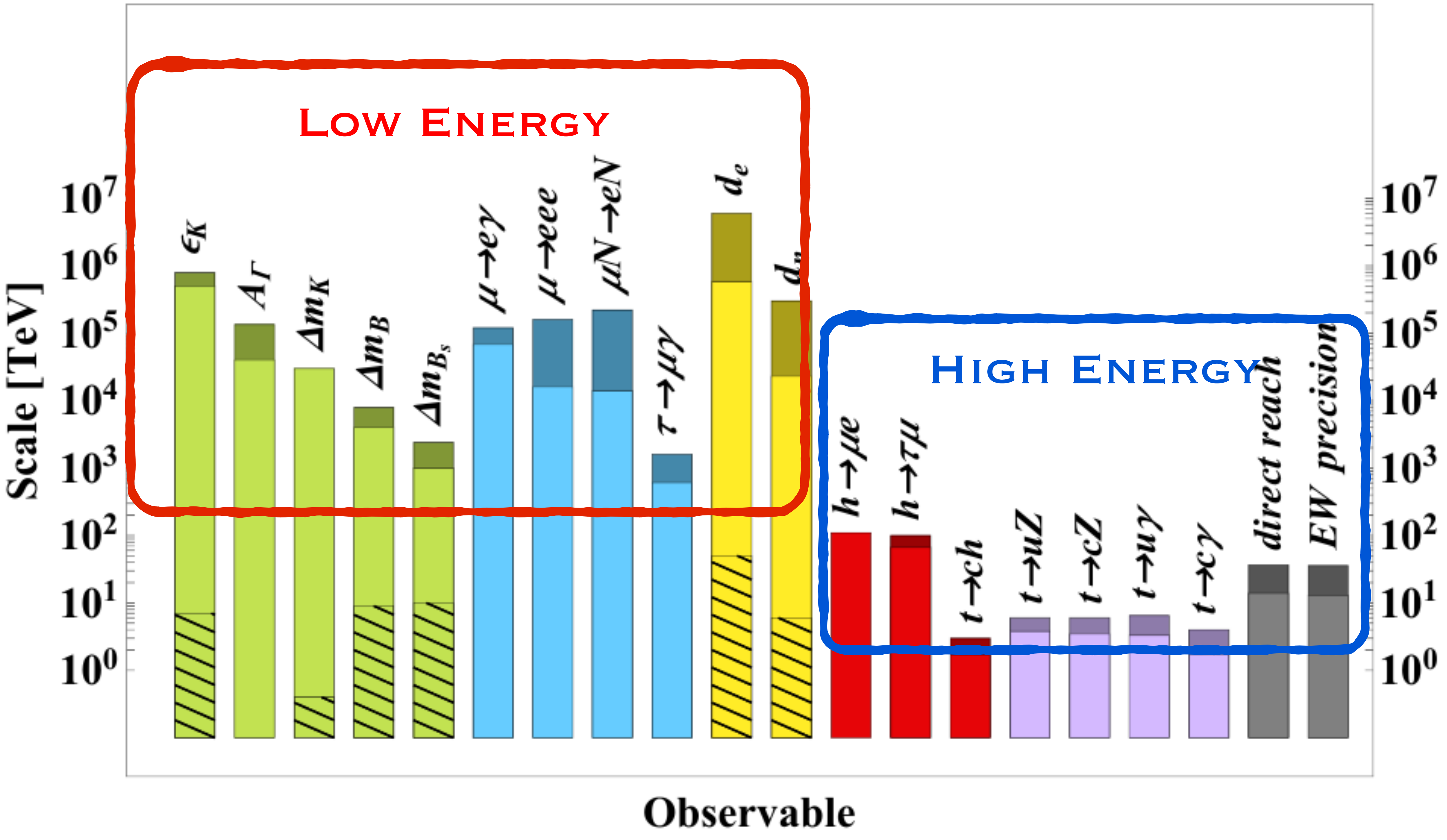


$$\text{Amplitude} \approx \frac{e\,g^2}{4\pi^2\, m_W^2} \sum_i V_{\mu i} V^*_{ei} \frac{m^2_{\nu_i}}{m_W^2}$$

- Cosmological bound $\sum m_{\nu_i} \lesssim 0.1$ eV
  - **CLFV is suppressed by** $\approx 10^{-22}$ **compared to quark sector!**
  - Similar conclusions for Majorana fermions

Flavor physics are sensitive probes of BSM

- Flavor violation is very sensitive to BSM with *non-standard* gauge or flavor structure
  - Searching for FCNCs in experiment could herald the discovery of New Physics
  - Null searches are typically expressed as **lower-bounds on mass scales of the putative BSM**



- Flavor physics **is mostly low-E endeavour**
- Flavor is sensitive to **BSM scales orders of magnitude higher** than direct searches or EW precision tests

Flavor NP puzzle: BSM at TeV scales requires non-trivial flavor structure

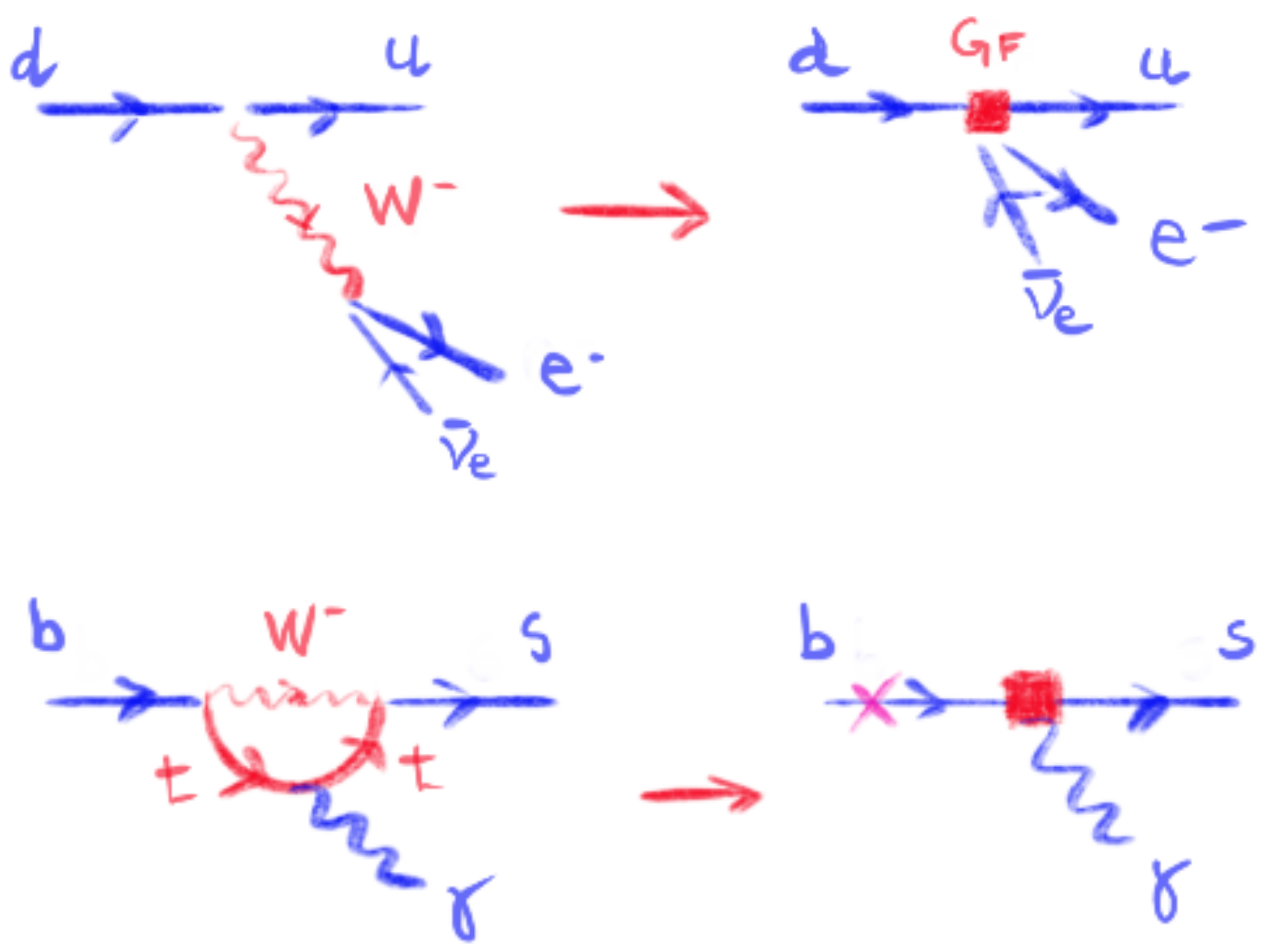
Elements of flavor physics phenomenology

The theorist's tool kit: Effective field theories

EFT lectures tomorrow

- **Energies involved in hadron decays $m_h \ll m_W$**

Rigorous and **systematic expansion** in the small parameter $\epsilon \approx m_h/m_W$ within the **Effective Field Theory (EFT)**



- Modern subnuclear extension of Fermi Theory
  - Neutron β decay

$$\mathcal{M}_\beta \approx G_F\, C_\beta \left(\bar{u}\gamma^\mu P_L d\right) \left(\bar{e}\gamma_\mu P_L \nu\right)$$

- Extended also to FCNCs
  - Radiative B-meson decays (e.g. $B^0 \to K^*\gamma$)

$$\mathcal{M}_\gamma \approx \frac{e\, m_b}{4\pi^2} G_F C_\gamma\, \bar{s}\sigma^{\mu\nu} P_R\, b\, F_{\mu\nu}$$

- **Low-energy effective Lagrangians:**
  - **Dimensionful constant:** Scale of dynamics that have been *integrated out* - $G_F \approx 1/m_W^2$
  - **Wilson coefficient:** Structure and constants of UV theory - $C_\beta \approx V_{ud}$, $C_\gamma = V_{tb}V_{ts}^* f(x_t)$
  - **Non-renormalizable operators:** with $d \geq 5$ and composed of dynamical fields at $E \ll m_W$

EFT for BSM: Low energies

1. List **fields** that can be made *on-shell* at the energies of interest
2. List **gauge symmetries** manifest at the energies of interest
3. Construct all **gauge invariant operators** with these fields up to a given dimension d

Power counting: Ordering of the ∞ operators according to power n in $(E/\Lambda_{\rm BSM})^n$

- Only a **finite number of operators** needed for a given precision!

- **Example CCs:** Leading (dim-6) weak Lagrangian at $\mu \approx E_{\rm low}$

$$\mathscr{L}_{\rm CC} = \frac{4G_F}{\sqrt{2}} \sum_{ij,\alpha\beta} \Big(C_{LL}^{ij,\alpha\beta} (\bar{u}_i \gamma^\mu P_L d_j)(\bar{e}_\alpha \gamma_\mu P_L \nu_\beta) + C_{RL}^{ij,\alpha\beta} (\bar{u}_i \gamma^\mu P_R d_j)(\bar{e}_\alpha \gamma_\mu P_L \nu_\beta)$$
$$+ C_{S_L S_L}^{ij,\alpha\beta} (\bar{u}_i P_L d_j)(\bar{e}_\alpha P_L \nu_\beta) + C_{S_R S_L}^{ij,\alpha\beta} (\bar{u}_i P_R d_j)(\bar{e}_\alpha P_L \nu_\beta) + C_{T_L T_L}^{ij,\alpha\beta} (\bar{u}_i \sigma^{\mu\nu} P_L d_j)(\bar{e}_\alpha \sigma_{\mu\nu} P_L \nu_\beta) \Big)$$

- SM is recovered for $C_{LL}^{ij,\alpha\alpha} = 1$ and all other WCs=0
- Most general BSM with SM d.o.f.

Imposing a flavor ansatz in the EFT: Minimal Flavor violation

- **Minimal Flavor Violation (MFV):** *All* the flavor violation in SM+BSM stems from *just* the SM Yukawas
  - One can implement MFV in the EFT using the **spurion analysis**

Impose an additional global symmetry $\mathcal{G} = U(3)_Q \times U(3)_u \times U(3)_d$

$$Q_L \sim (3,1,1), \quad u_R \sim (1,3,1), \quad d_R \sim (1,1,3), \quad y_u \sim (3,\bar{3},1), \quad y_d \sim (3,1,\bar{3})$$

  - MFV is useful because it transfers the flavor component of the GIM suppression to BSM
  - **Note:** Works only in the EFT defined in terms of the SM fields/symmetries (SMEFT)

Example: **Contribution to the FCNC** $b \to s\gamma$

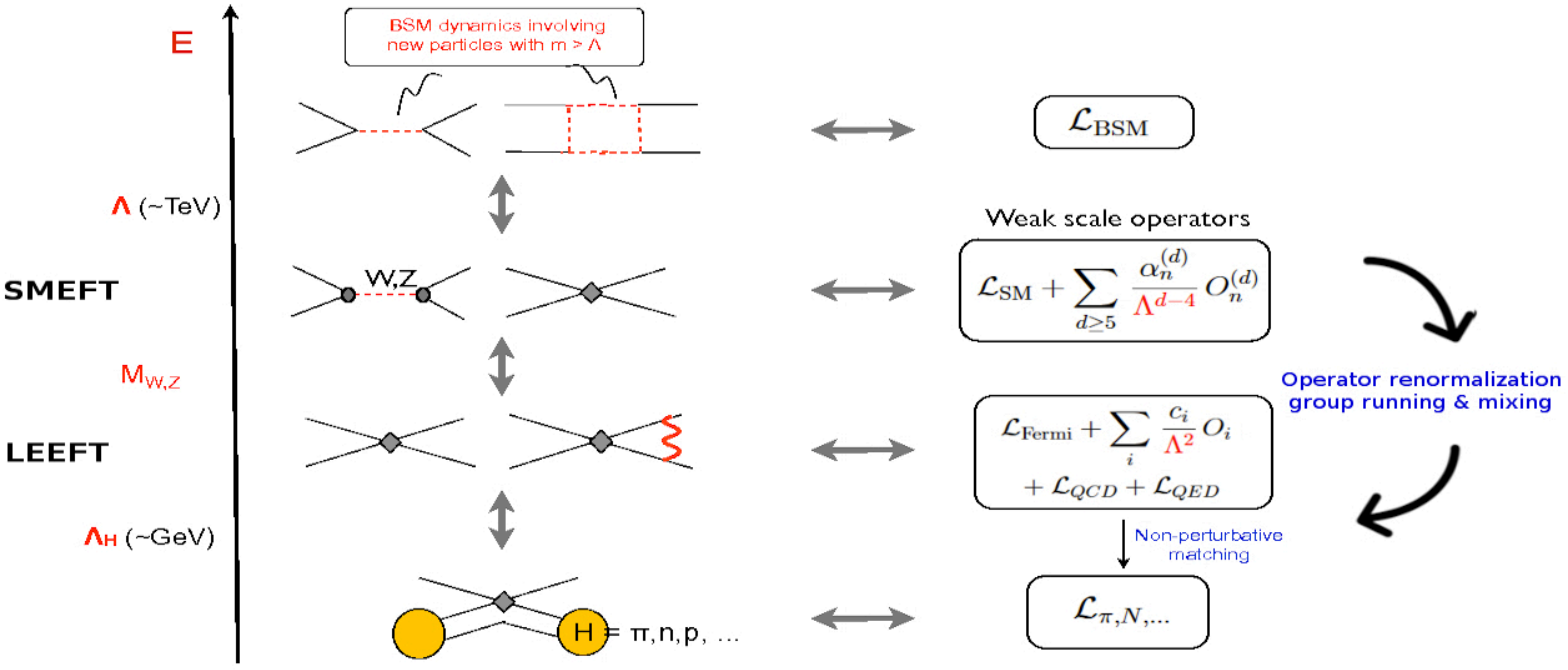
$$\frac{e\,\bar{c}}{\Lambda_{\rm NP}^2} F_{\mu\nu} \bar{Q}_L \sigma^{\mu\nu} \underbrace{y_u y_u^\dagger y_d}\, b_R \quad \Rightarrow \quad \frac{e\,\bar{c}}{\Lambda_{\rm NP}^2} F_{\mu\nu} \begin{pmatrix} \bar{U}_L \\ \bar{D}_L V^\dagger \end{pmatrix} \sigma^{\mu\nu} m_u^2\, V\, m_d\, b_R$$

y_d alone does not produce FCNC

Same yukawa suppression as in the SM!

$$C_\gamma = \frac{e\,\bar{c}}{\Lambda_{\rm NP}^2} m_b\, y_t^2\, V_{ts}^* V_{tb}$$

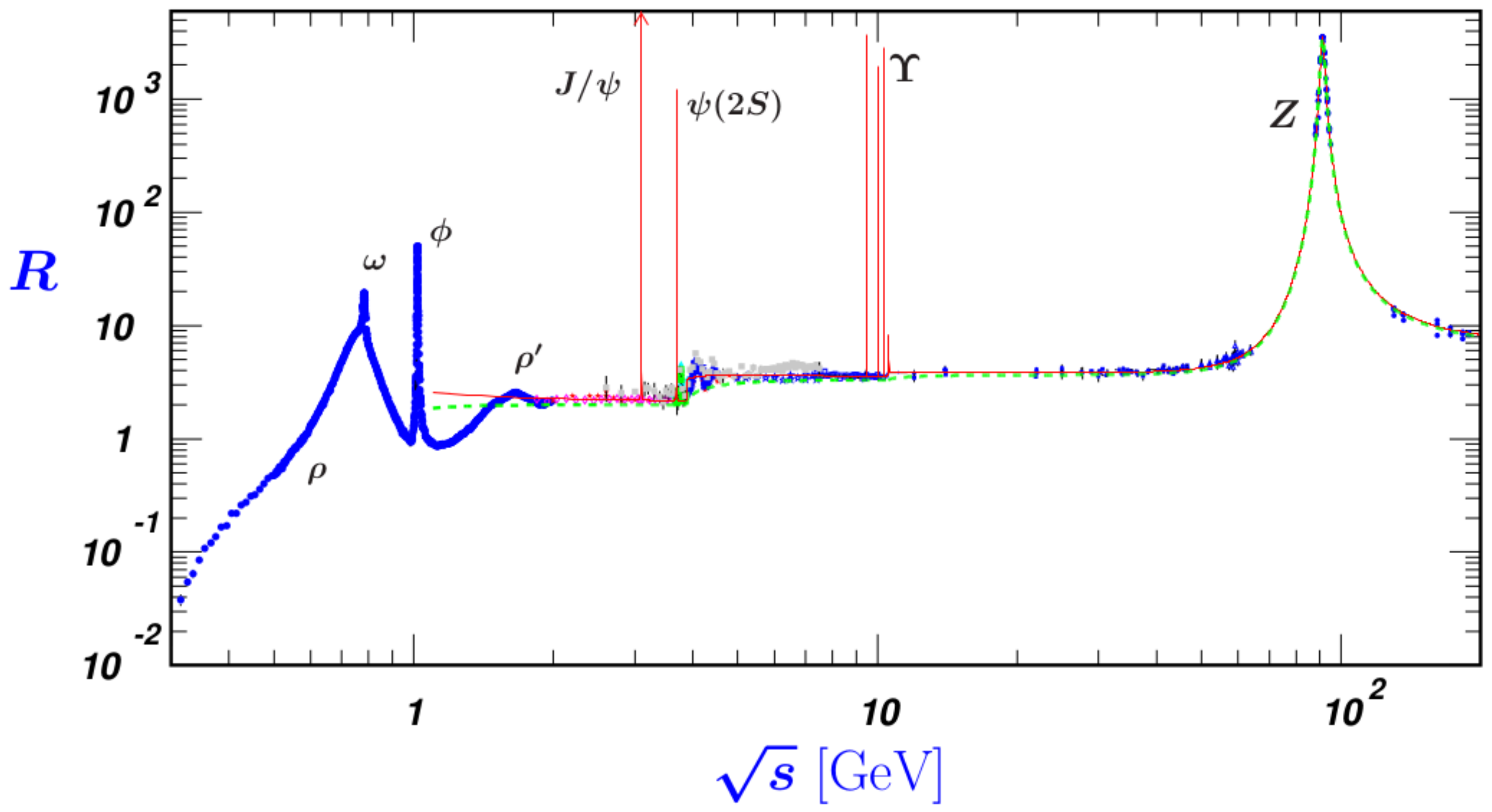
Summary of the EFT procedure



Cirigliano and Mussolf Prog.Part.Nucl.Phys. 71 (2013) 2-20

Low-energy: The realm of the hadrons

- QCD confines around and below energies ~ $\Lambda_{\mathrm{QCD}} \approx$ 200 MeV



Baryons qqq and Antibaryons $\bar{q}\bar{q}\bar{q}$ Baryons are fermionic hadrons. There are about 120 types of baryons.					
Symbol	Name	Quark content	Electric charge	Mass GeV/c^2	Spin
p	proton	uud	1	0.938	1/2
$\bar{p}$	anti-proton	$\bar{u}\bar{u}\bar{d}$	–1	0.938	1/2
n	neutron	udd	0	0.940	1/2
Λ	lambda	uds	0	1.116	1/2
Ω^-	omega	sss	–1	1.672	3/2

Mesons $q\bar{q}$ Mesons are bosonic hadrons. There are about 140 types of mesons.					
Symbol	Name	Quark content	Electric charge	Mass GeV/c^2	Spin
π^+	pion	$u\bar{d}$	+1	0.140	0
K^-	kaon	$s\bar{u}$	–1	0.494	0
ρ^+	rho	$u\bar{d}$	+1	0.770	1
B^0	B-zero	$d\bar{b}$	0	5.279	0
η_c	eta-c	$c\bar{c}$	0	2 .980	0

- Only the proton is (almost) really stable!

The thousands of different decay modes of these hundreds of particles are a **precious source of information**

The PDG is phenomenologist's 1st best friend!

- **Branching fraction** of a decay channel i of a hadron h

$$\mathrm{Br}_i = \Gamma_i/\Gamma_h = \tau_h \Gamma_i$$

- Only hadrons whose main decay channel is *weak*
  1. **Flavor violations !**
  2. **Sensitivity to** $E \gtrsim m_W$ **!**

Connecting to the observables of the hadronic world

- **Our Lagrangians are written in terms of quarks and our observables in terms of hadrons!**

Interactions: $\mathscr{L}(u, d, s, c, b, e, \nu, G, F)$ *Asymptotic* **states:** $|\pi^{\pm}, \pi^{0}, K^{\pm}, D^{\pm}, B^{\pm}, p, n, \Lambda, \ldots\rangle$

- By *asymptotic* we mean hadrons with **long life times (** $\tau_{\text{weak}} \approx \overbrace{10^{-8}}^{\text{Kaons}} - \overbrace{10^{-12}}^{B-\text{mesons}}$ s $vs.$ $\tau_{\text{EM}} \approx \overbrace{10^{-17}}^{\pi^0}$ s OR $\tau_{\text{strong}} \approx \overbrace{10^{-24}\text{ s}}^{\rho-\text{resonance}}$ **)**
- **Observables** defined in terms of matrix elements

$$\mathscr{M} \sim \langle e', \nu', \ldots; H_1', H_2', \ldots | \overbrace{\mathcal{O}_\ell \times \mathcal{O}_q}^{\mathscr{L}} | e, \nu, \ldots; H_1', H_2', \ldots \rangle$$

with $\text{Observables} \sim |\mathscr{M}|^2$

- **Factorization:** Wick's theorem *typically* leads to factorization of matrix element

$$\mathscr{M} \sim \langle e', \nu', \ldots | \mathcal{O}_\ell | e, \nu, \ldots \rangle \times \langle H_1', H_2', \ldots | \mathcal{O}_q | H_1', H_2', \ldots \rangle$$

Perturbative matrix element ↙ ↘ Hadronic matrix element

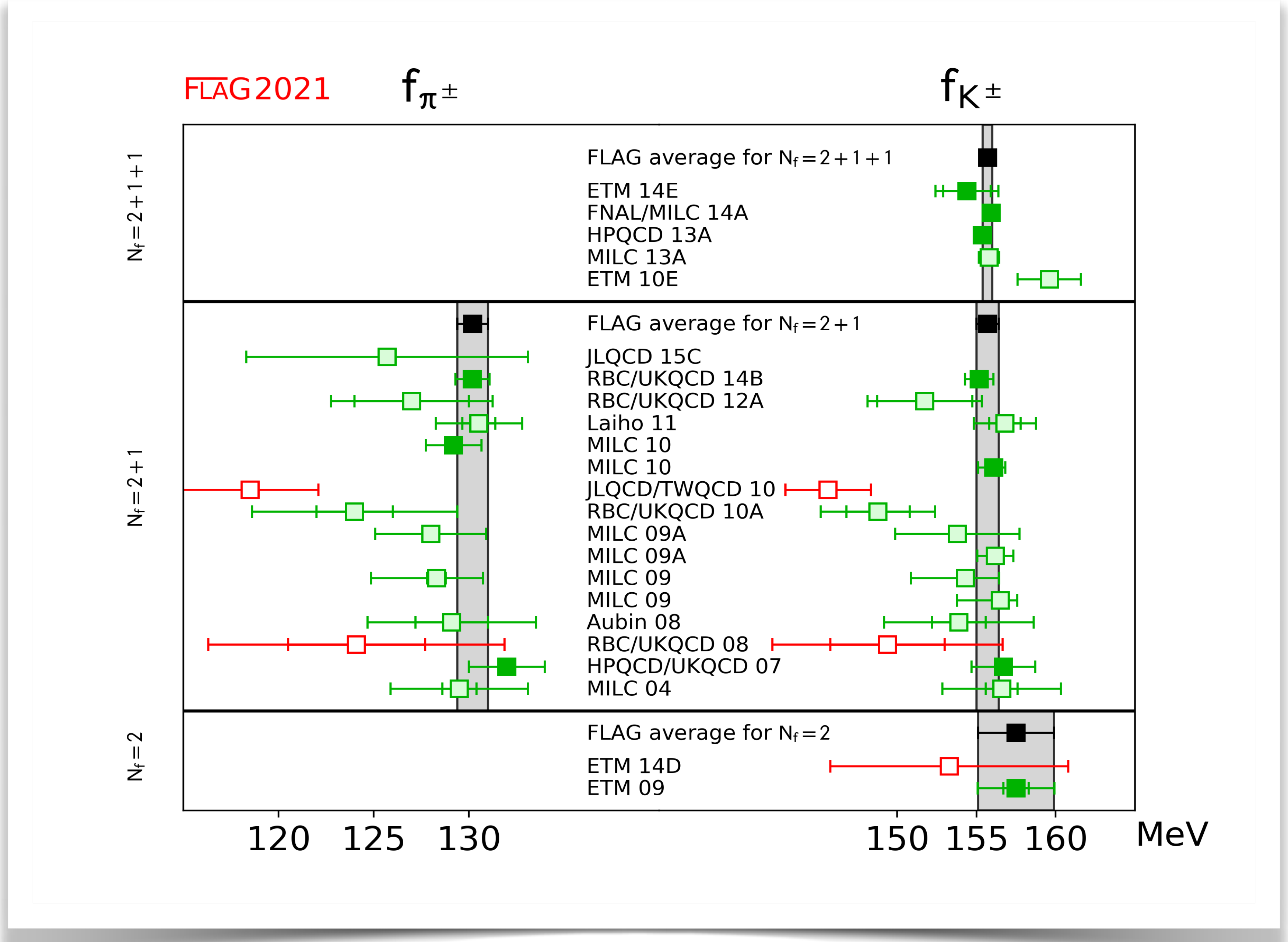
- **Hadronic matrix elements:** Encapsulate all the nonperturbative-QCD information of the transition

Very difficult to compute! They limit our capacity to learn about short distances

Determinations of the hadronic brown muck

- **General strategy:**
  1. **Parametrize** the matrix element (discrete and Lorentz symmetries)
  2. **EFTs of QCD** in perturbative expansions
     - Isospin ($m_d \approx m_u$) and $SU(3)_F$ ($m_u \approx m_d \approx m_s$) in light quarks - **Chiral Perturbation Theory**
     - Heavy-quark symmetry ($m_{c,b} \gg \Lambda_{\mathrm{QCD}}$) - **Heavy quark effective theory**
  3. **Calculate** hadronic matrix elements
     - **Lattice QCD** - systematic approximation from discrete and finite space-time
     - QCD sum rules, quark models, Ads/CFT, etc ...

FLAG (Lattice "PDG")



Example: **Leptonic pion decay $\pi^- \to e^- \bar{\nu}$**

- **Parity invariance:** Vector & Scalar are 0!
- **Lorentz invariance:** Tensor is 0!
- f_π is the pion decay constant $f_\pi = 130.2(0.8)$ MeV

Flavor processes

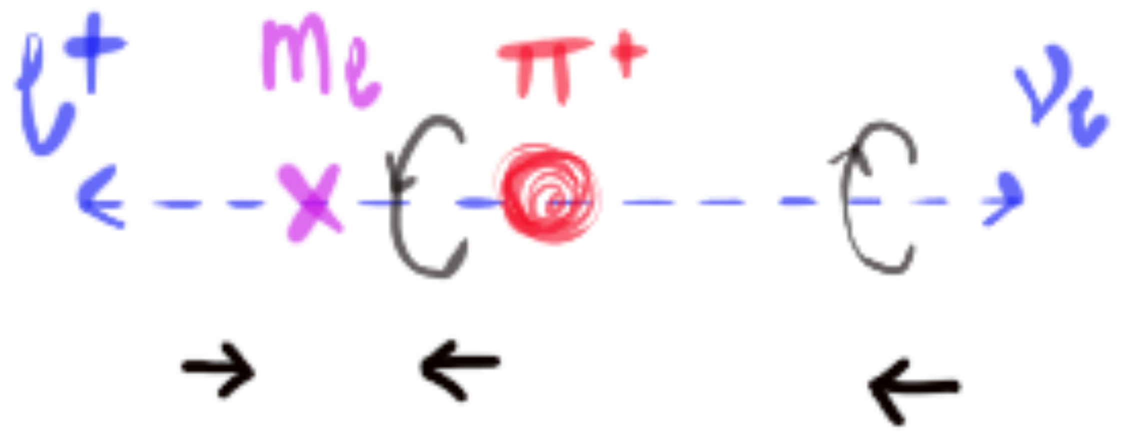
The CC leptonic (2-body) pion decay ($\pi_{\ell 2}$)

LQCD $f_\pi = 130.2(8)$

$$\mathcal{M} = \langle \ell^+ \nu_\ell | \mathcal{L}_{SM} | \pi^+ \rangle = \frac{4 G_F V_{ud}}{\sqrt{2}} \langle \ell^+ \nu_\ell | \bar{\nu}_L \gamma_\mu P_L \ell | 0 \rangle \langle 0 | \bar{d} \gamma^\mu P_L u | \pi^+ \rangle = \frac{G_F f_\pi V_{ud}}{\sqrt{2}} m_\ell \, \bar{\nu}_\ell P_R e$$

chiral suppression

- **Chiral suppression:** In the chiral limit $m_\ell \to 0$ the amplitude vanishes!



Phase space

$$\Gamma_{\ell 2} = \frac{G_F |V_{ud}|^2 f_\pi^2}{8\pi} m_\pi m_\ell^2 \left(1 - \frac{m_\ell^2}{m_\pi^2}\right)^2 \left| C_{LL} - C_{RL} - \frac{m_\pi^2}{m_e(m_d + m_s)} \left(C_{S_R S_L} - C_{S_L S_L} \right) \right|^2$$

- **Pseudoscalar operator:** Contribution of $\bar{d}\gamma_5 u$?
  - Pseudoscalar operator is chirally flipping $\Rightarrow$ **Not chirally suppressed!**

Experimental data

$\mathrm{Br}(\pi^+ \to \mu^+ \nu_\mu) = 99.98770(4)\,\%$

$\mathrm{Br}(\pi^+ \to e^+ \nu_e) = 1.230(4) \times 10^{-4}$

Discovered at CERN (G. Fidecaro) - 1958

Physical results

The SM is a "current-current" interaction

Weinberg's "*V-A* was the key" - 2009

BSM-Vector: $\Lambda_{LL} \approx$ **1 TeV**

The CC semileptonic (3-body) decays ($K_{\ell 3}$)

- **Hadronic form factors:** Functions of $q^2 = (p' - p)^2$

$$\langle \pi^0(p') | \bar{s}\gamma_\mu d | K^+(p) \rangle = f_+(q^2) P_\mu + q_\mu \frac{m_{K^+}^2 - m_{\pi^0}^2}{q^2} \left(f_+(q^2) - f_0(q^2) \right)$$

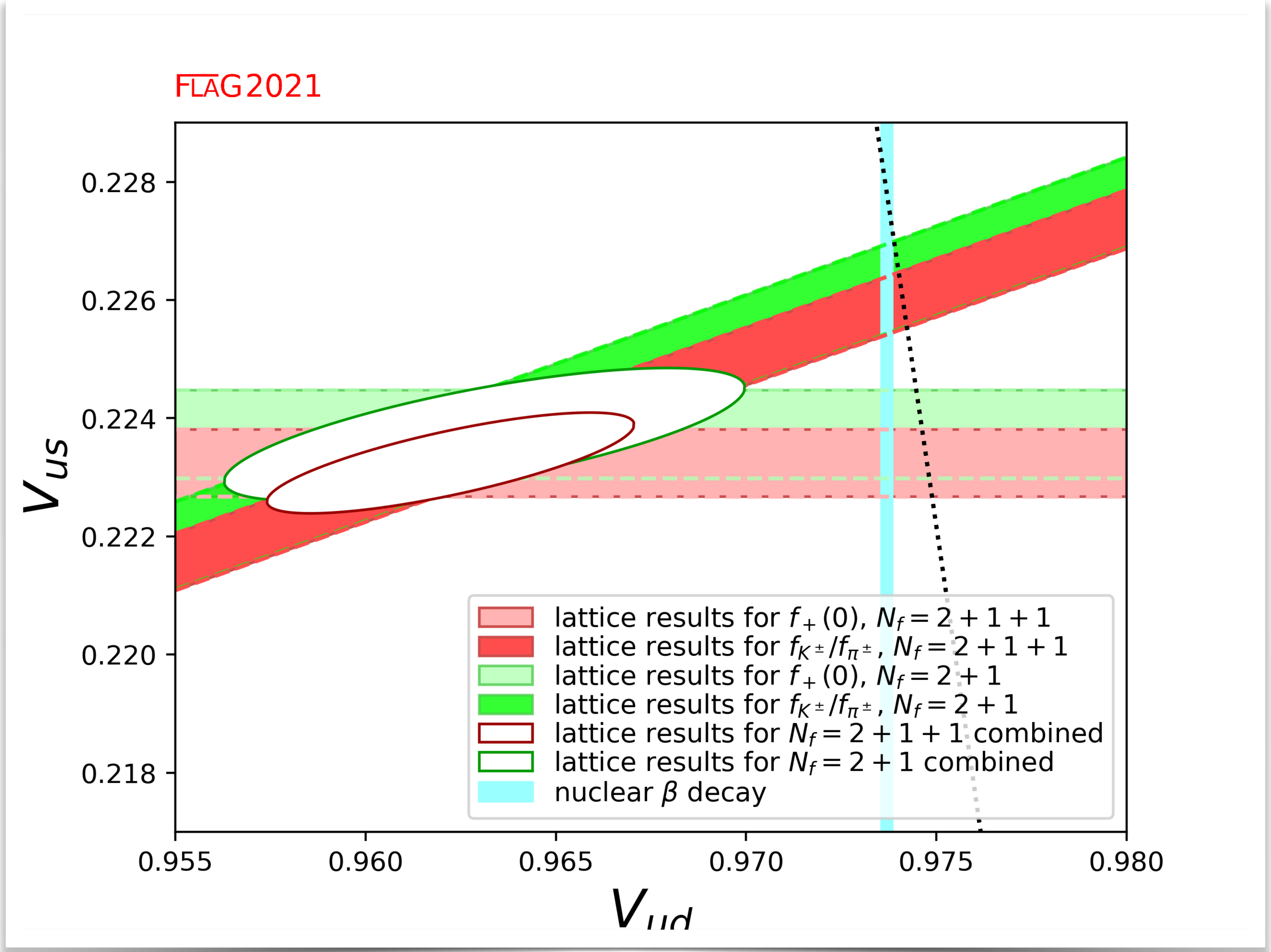
  - Parity and charge invariance ⇒ No pseudoscalar/axial form factors

$$\Gamma(K_{\ell 3(\gamma)}) = \frac{G_\mu^2 m_K^5}{192\pi^3} S_{\rm ew} \overbrace{|\tilde{V}_{us}|^2 f_+(0)^2}^{\text{Norm}} \overbrace{I_K^\ell(\lambda'_{+,0}, \underbrace{C_{S_L S_L} + C_{S_R S_R}}_{\text{Scalar}}, C_{T_L T_L})}^{\text{Phase-space Int.}} \underbrace{\left(1 + \delta^c + \delta_{\rm em}^{c\ell}\right)^2}_{\text{Isospin corr.}}$$

  - Form factors obtained from LQCD ⇒ e.g. $f_+(0) = 0.9698(17)$
  - Normalization (and spectrum) sensitive to BSM ⇒ $\tilde{V}_{us} \approx (1 + C_{LL} + C_{RL} - C_{LL}^\mu) V_{us}^{\rm SM} + \mathcal{O}\left(\frac{m_K^4}{\Lambda^4}\right)$

Testing CKM unitarity

- **Disentangle BSM from CKM:** Unitarity relation $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$



- **Tensions in the $V_{ud} - V_{us}$ plane**
  - Use $K_{\ell 3}$ and ratio $\frac{K_{\ell 2}}{\pi_{\ell 2}}$ (to determine $\frac{\tilde{V}_{us}}{\tilde{V}_{ud}}$)

Lattice results ($N_f = 2+1+1$)

$$|\tilde{V}_{ud}|^2 + |\tilde{V}_{us}|^2 = 0.9816(64)$$

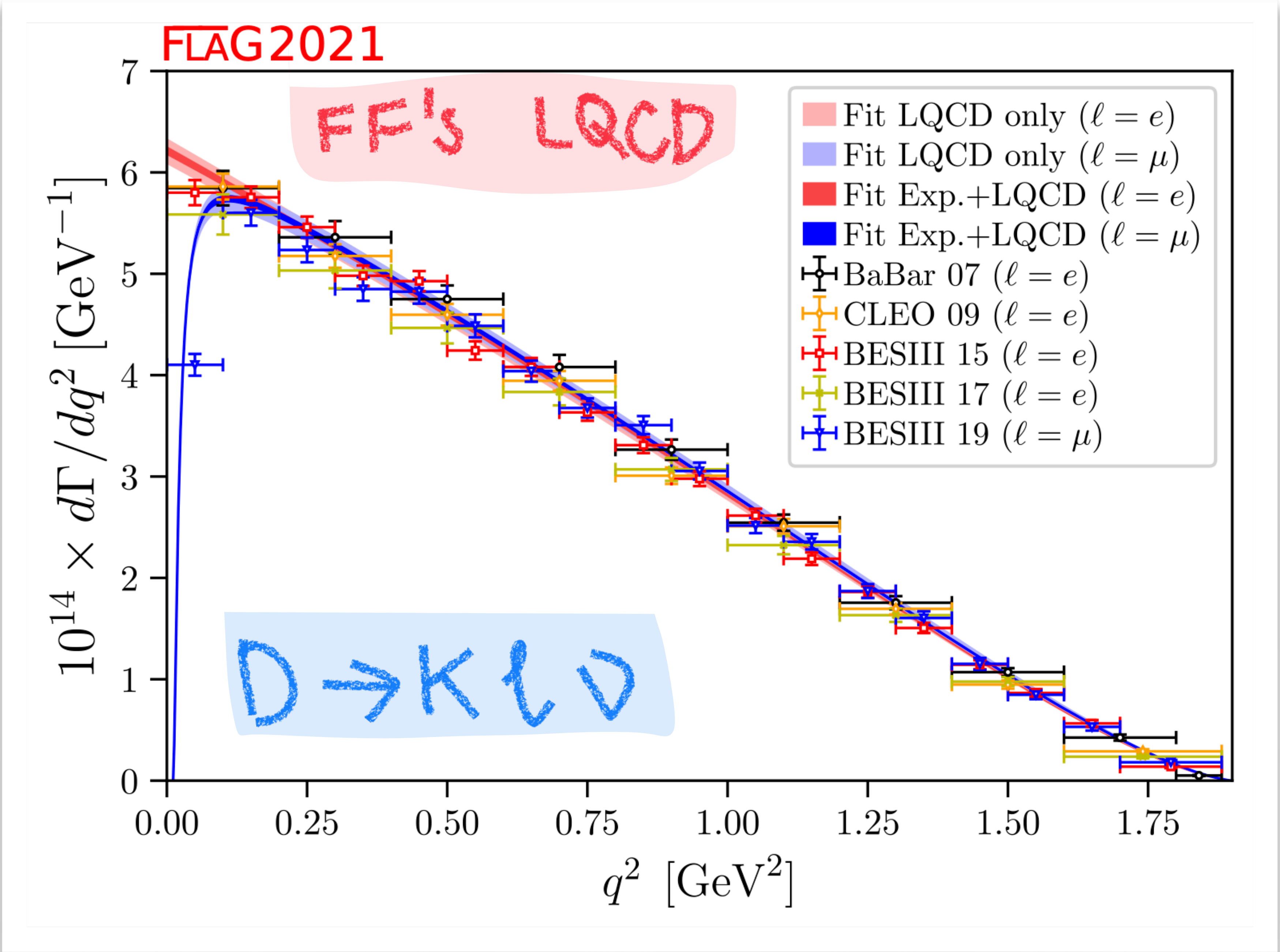
Tension at $\sim 3\sigma$

  - Tension increases with β - decays
  - BSM or uncontrolled EM/isospin corrections?

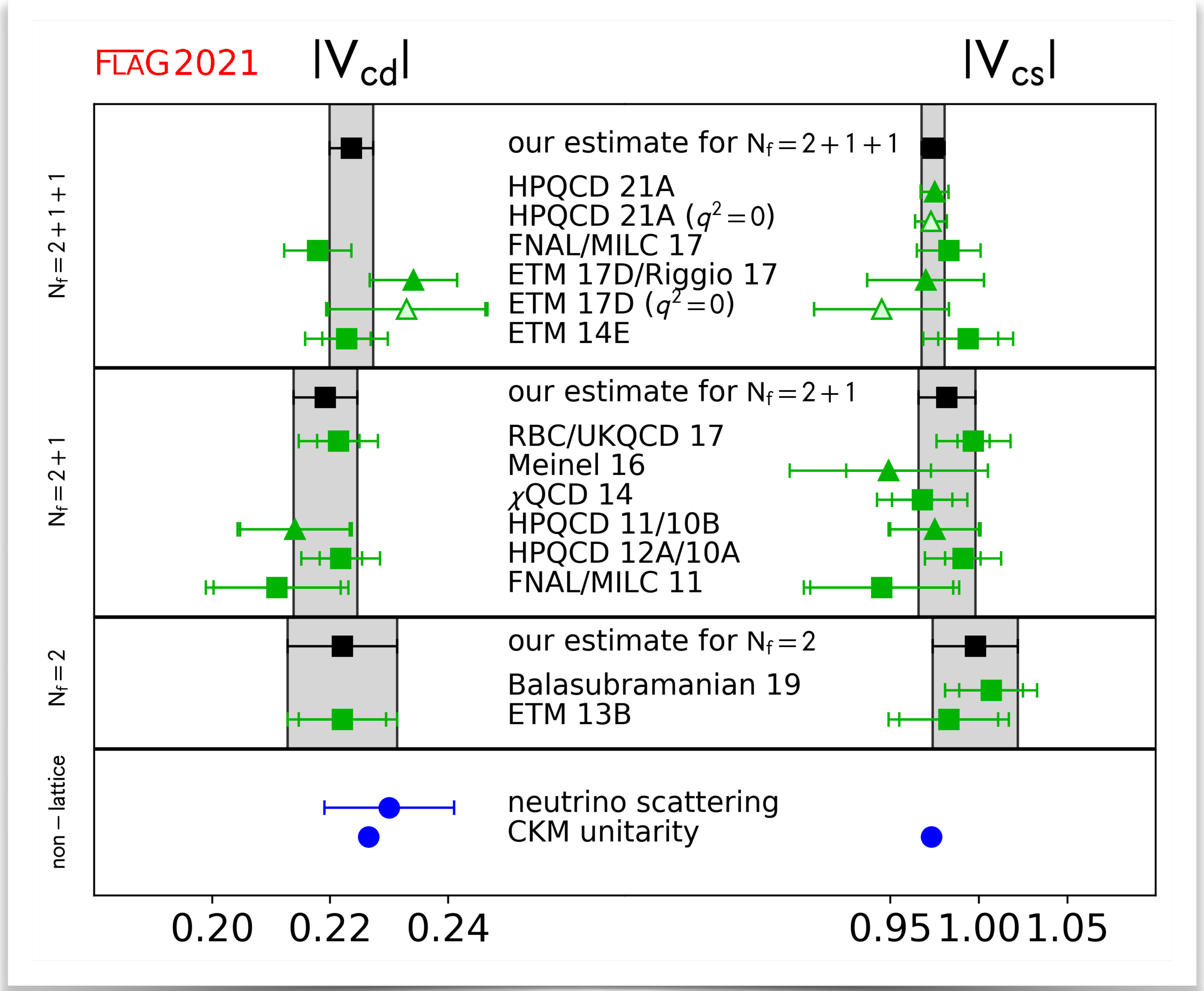
Charmed-meson CC decays: the unitarity test

π or K

- **Same strategy as with kaon decays:** Use leptonic $D_{(s)} \to \ell^+ \nu$ and semileptonic $D \to P \ell \bar{\nu}$



- **2nd-row unitarity**

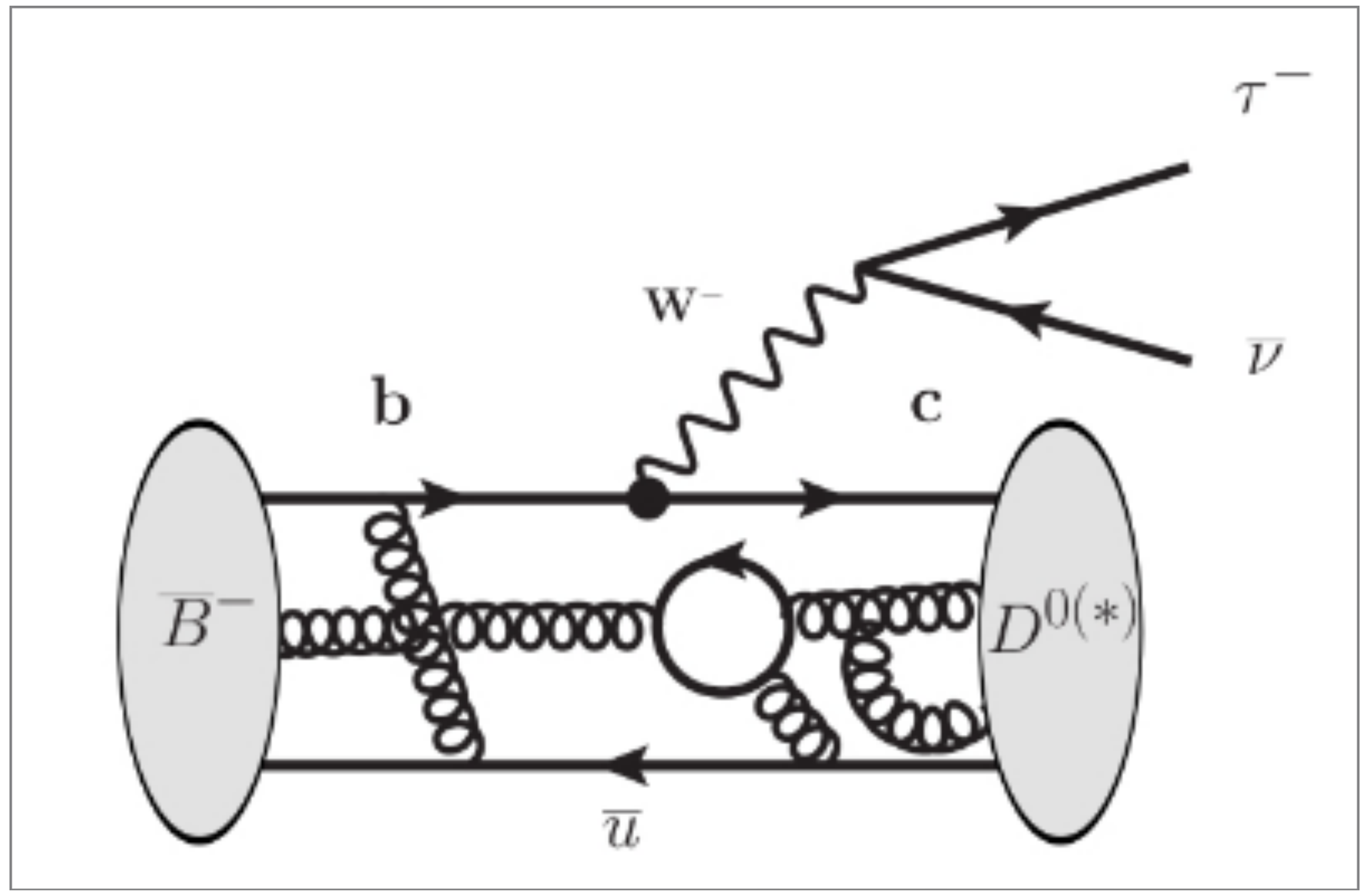


- **Phase space:** Many decay modes potentially available!

$$D_{(s)} \to V \ell \bar{\nu},\ D_s \to \tau \nu \ldots$$

$$|\tilde{V}_{cd}|^2 + |V_{cs}|^2 + |\tilde{V}_{cb}|^2 = 0.999(8)$$

B meson CC decays into tau leptons



- **Semi-tauonic charged-current decay**
  - Governed by the weak amplitude $G_F V_{cb}$
  - Two main **hadronic** channels studied

$$B \to D \text{ with } J^P(D) = 0^- \\ B \to D^* \text{ with } J^P(D^*) = 1^+$$

- **Hadronic form factors**
  - Heavy-quark EFT with data light leptons and/or LQCD
  - Define **Lepton Universality** ratio to cancel uncertainties

HFLAV collaboration

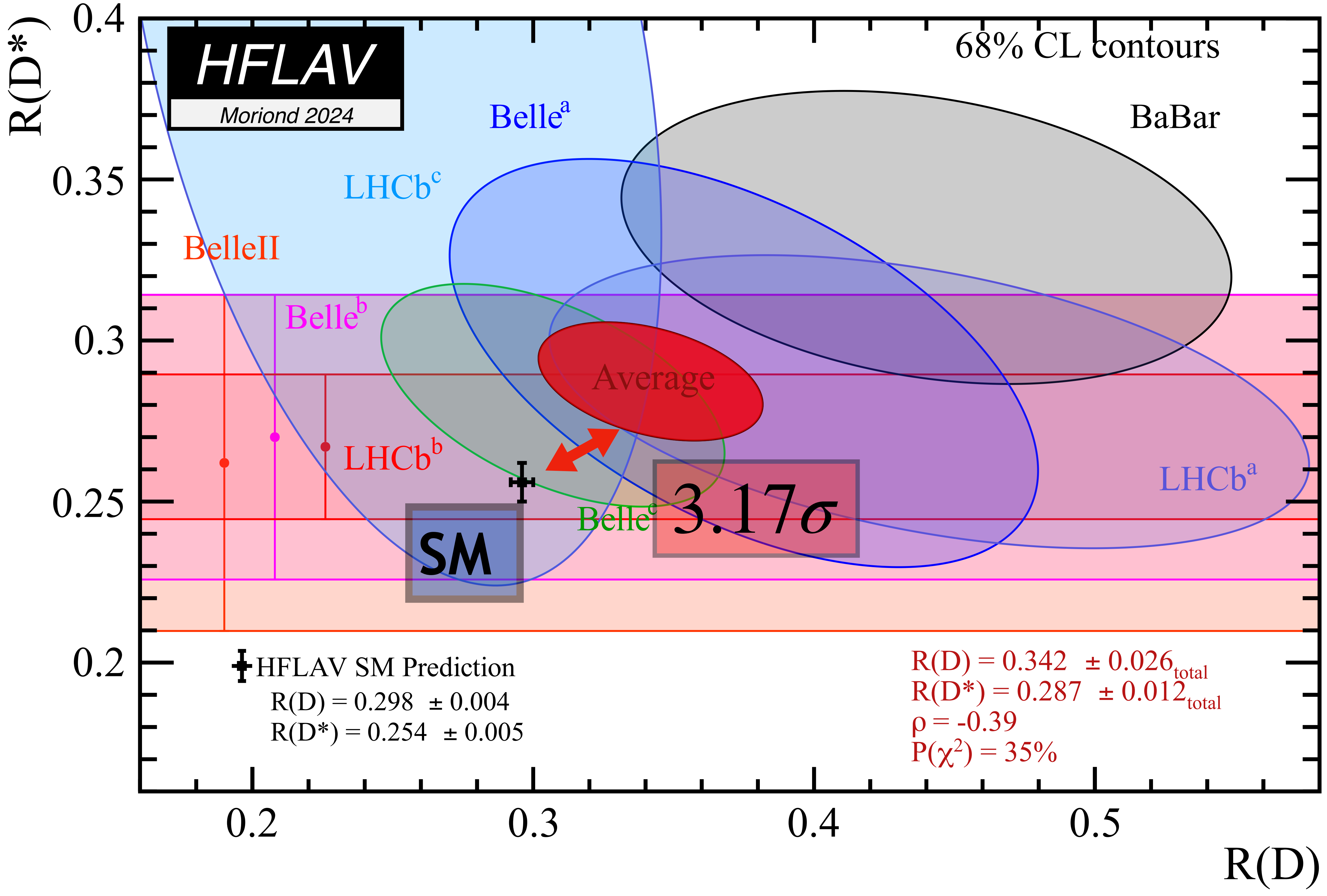
$$R_{D^{(*)}} = \frac{\mathrm{Br}(B \to D^{(*)}\tau\nu)}{\mathrm{Br}(B \to D^{(*)}\ell\nu)}$$

$$R_D = 0.298 \pm 0.004 \\ R_{D^*} = 0.254 \pm 0.005$$

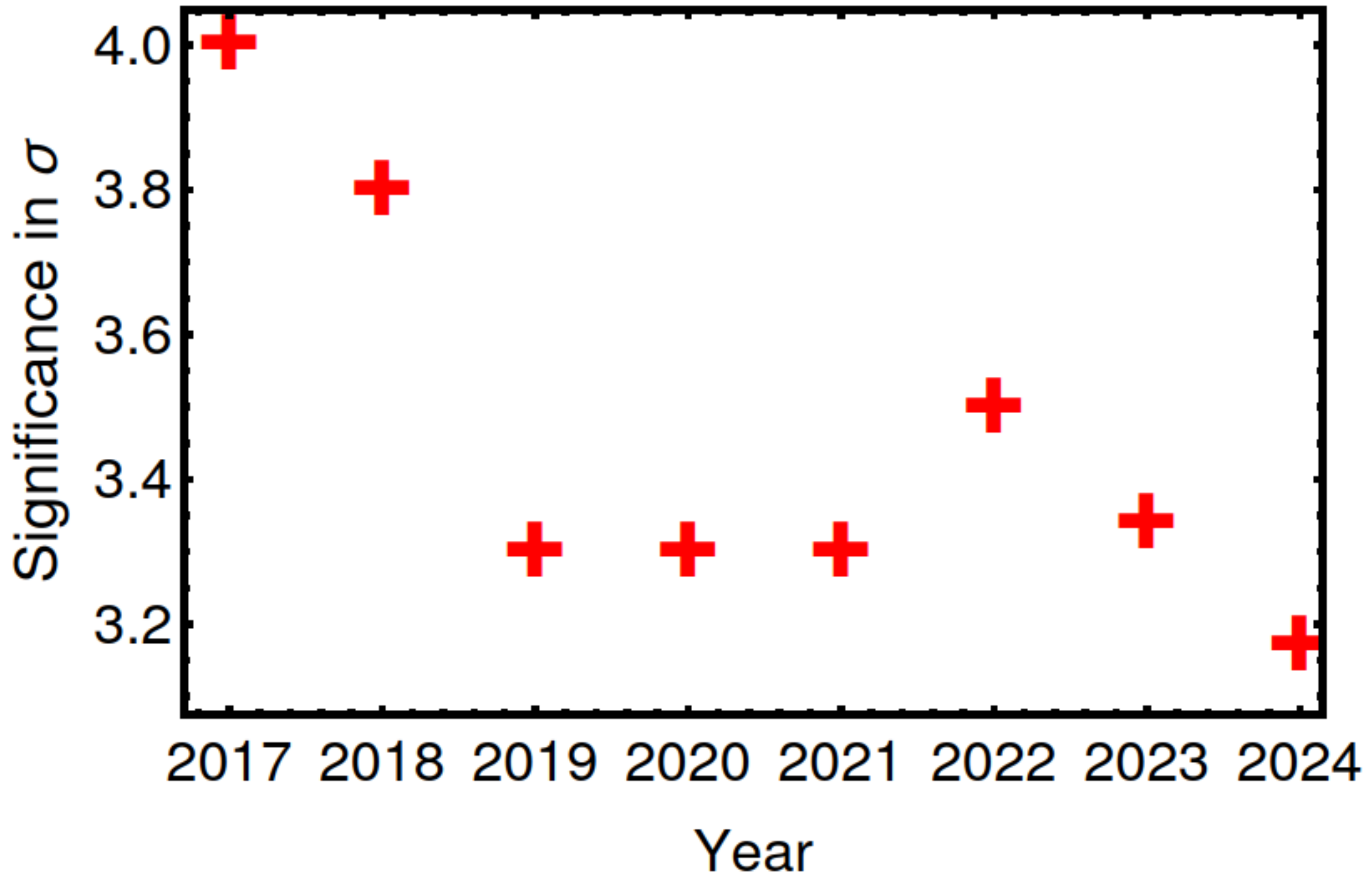
Theoretical errors well controlled at the 3 - 6% level

B-meson decays into tau leptons

- Situation in 2024



- New measurements with 1st by **Belle II**
- Gradually descending to SM: Excess 3.17σ



- **BaBar** outlier? ⇒Down to ~ 2σ

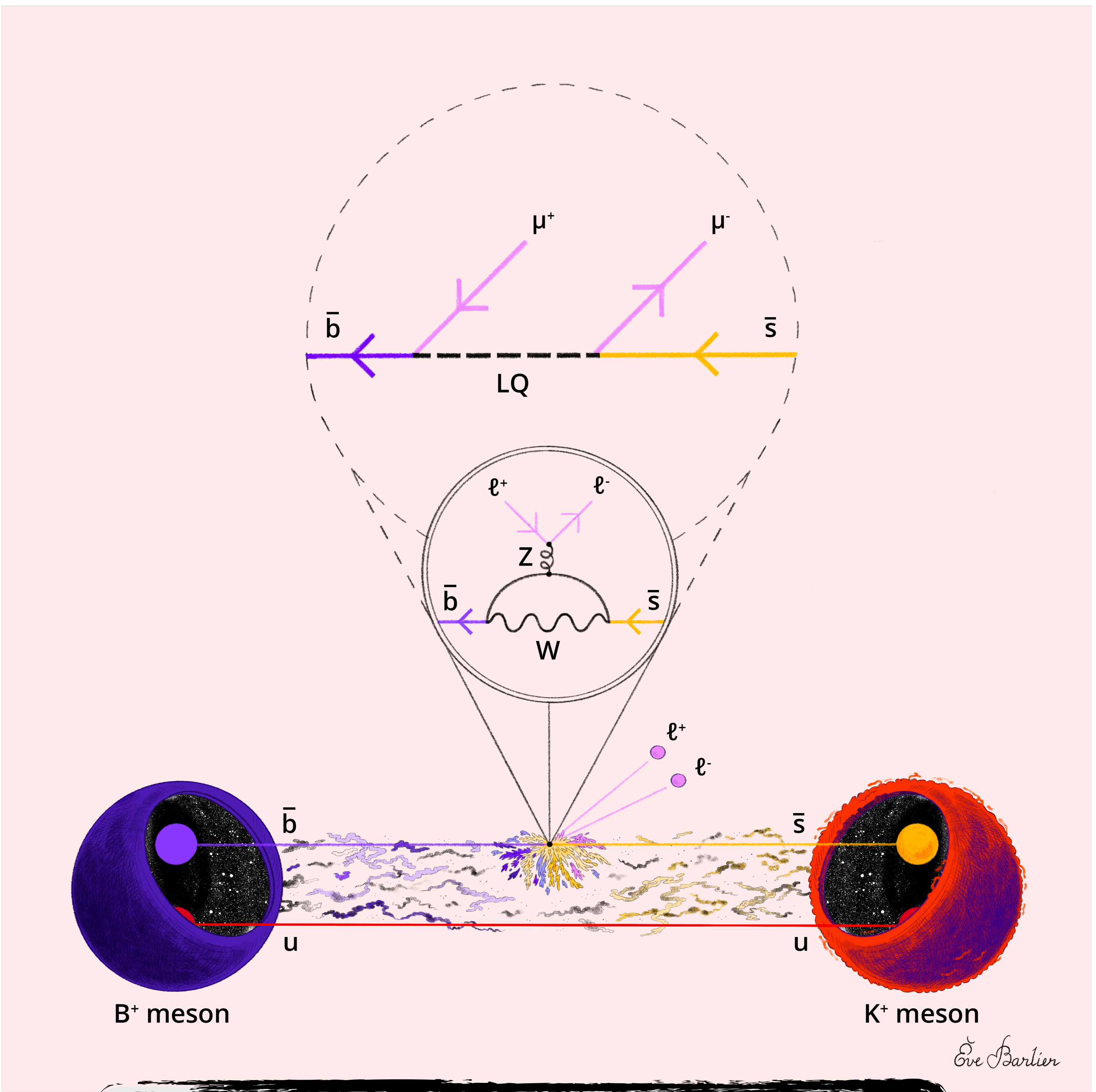
Picture is not clear ⇒ More data needed!

Semileptonic rare B decays

- FCNC decays of B mesons into kaons and leptons

$$\mathscr{H}_{\rm had} = \frac{4G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \Big[\underbrace{\left(C_1^p \mathcal{O}_1^p + C_2^p \mathcal{O}_2^p\right)}_{\text{Current-current}} + \underbrace{\sum_{i=3,6} C_i \mathcal{O}_i}_{\text{QCD penguins}} + \underbrace{C_{8g}\mathcal{O}_{8g}}_{\text{Chromo}} \Big]$$

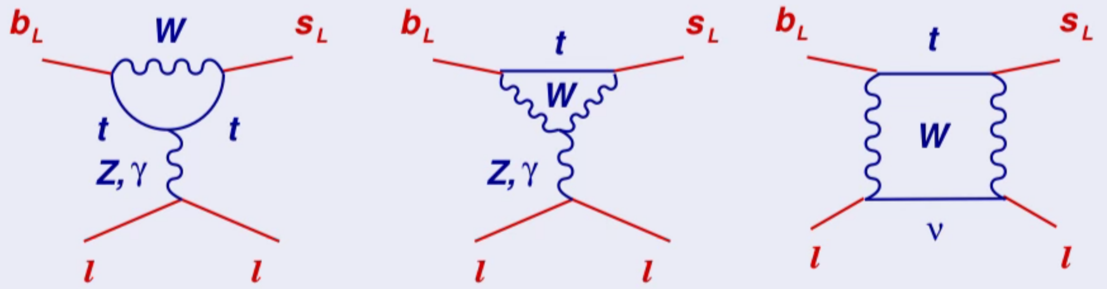
$$\mathscr{H}_{\rm sl} = -\frac{4G_F}{\sqrt{2}} \lambda_t \Big[\underbrace{C_{7\gamma}\mathcal{O}_{7\gamma}}_{\text{EM}} + \underbrace{C_9 \mathcal{O}_9 + C_{10}\mathcal{O}_{10}}_{\text{Semileptonic}} + \underbrace{\sum_\ell C_{\nu_\ell}\mathcal{O}_{\nu_\ell}}_{\text{neutrino}} \Big]$$



Sensitive to $\Lambda \gtrsim 50$ TeV

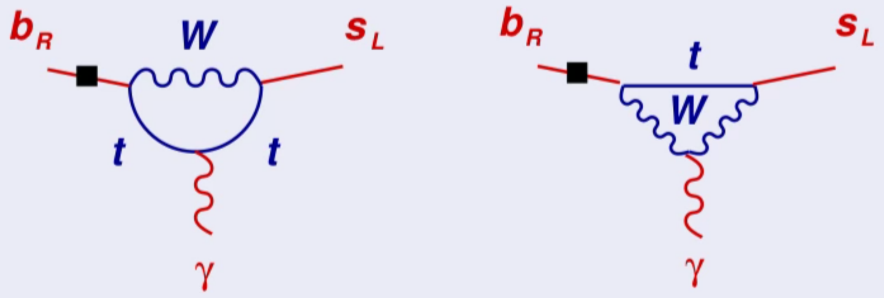
The $b \to s\ell\ell$ transition in the SM

- **Semileptonic operators**: $\mathcal{O}_9$ ($L + V$), $\mathcal{O}_{10}$ ($L + A$)



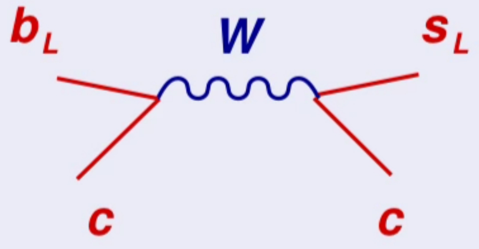
$$\frac{\alpha}{4\pi}\, C_{9(10)}\, \bar{s}_L \gamma^\mu b_L\, \bar{\ell}\gamma_\mu(\gamma_5)\ell$$

- **Electromagnetic penguin**: $\mathcal{O}_7$



$$\frac{e}{4\pi^2}\, m_b\, C_7\, \bar{s}_L \sigma_{\mu\nu} b_R\, F^{\mu\nu}$$

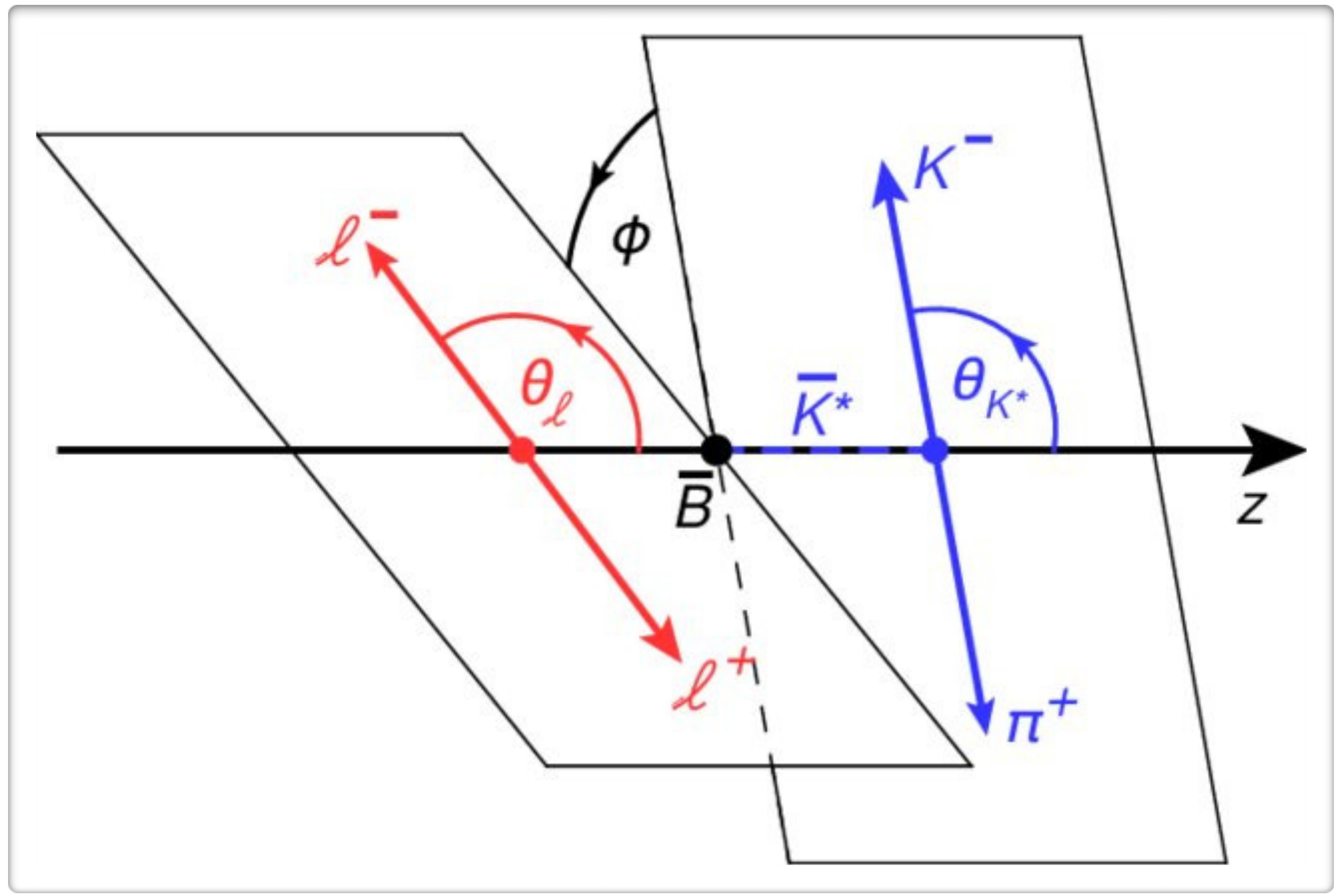
- **CC @ 1 loop**



$$C_2\, \bar{c}_L \gamma^\mu b_L\, \bar{s}_L \gamma_\mu c_L$$

The rare semileptonic (4-body) decay $B \to K^*(\to K\pi)\ell\ell$

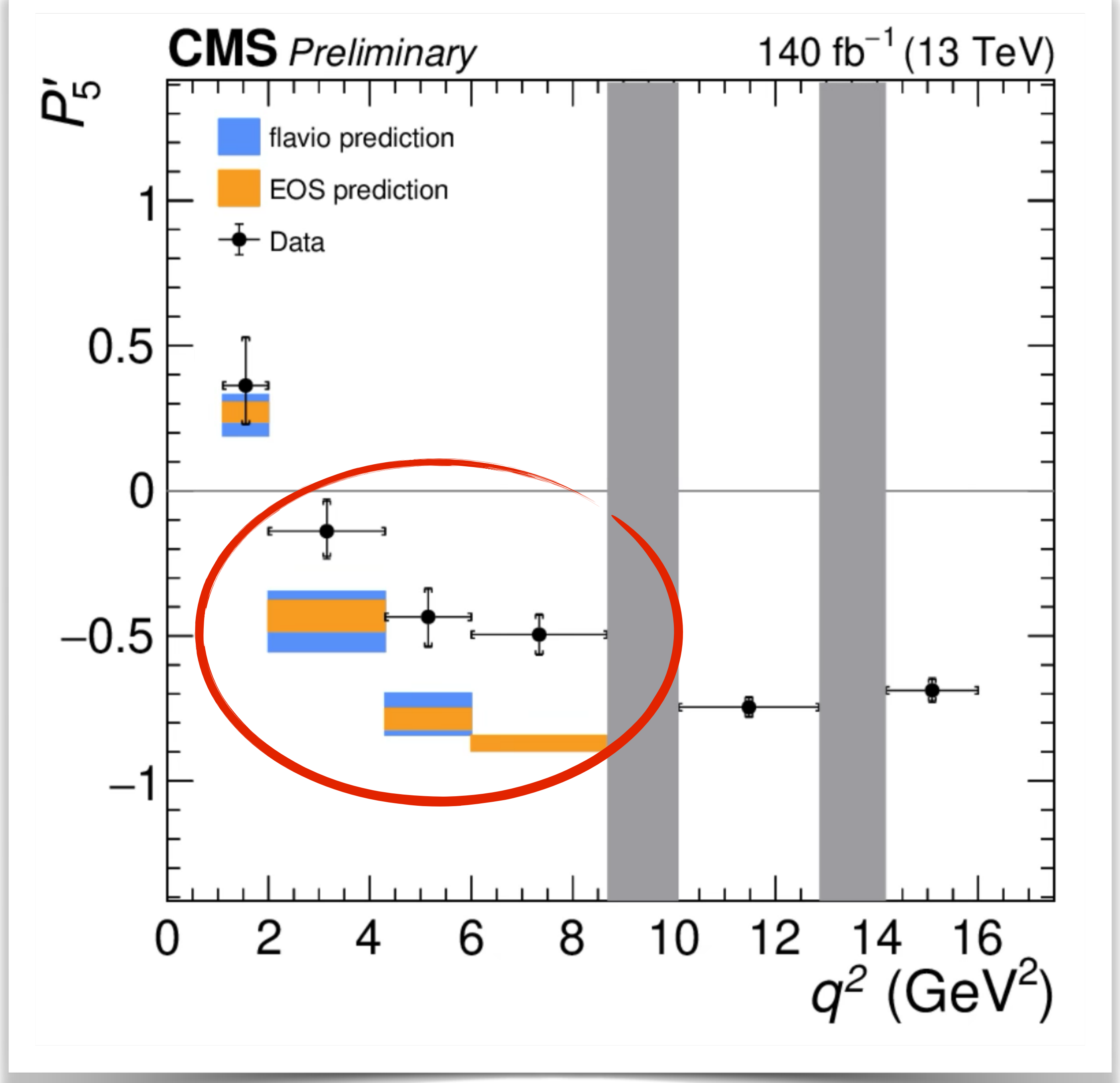
- Kinematic variables: $(p_B - p_{K^*})^2 = q^2$, $\cos\theta_\ell$, $\cos\theta_K$, ϕ



$$\frac{d^{(4)}\Gamma}{dq^2\, d(\cos\theta_l) d\cos\theta_k) d\phi} = \frac{9}{32\pi}\Big[I_1^s \sin^2\theta_k + I_1^c \cos^2\theta_k + (I_2^s \sin^2\theta_k + I_2^c \cos^2\theta_k)\cos 2\theta_\ell$$
$$+ I_3 \sin^2\theta_k \sin^2\theta_\ell \cos 2\phi + I_4 \sin 2\theta_k \sin 2\theta_\ell \cos\phi$$
$$+ I_5 \sin 2\theta_k \sin\theta_\ell \cos\phi + I_6 \sin^2\theta_k \cos\theta_\ell$$
$$+ I_7 \sin 2\theta_k \sin\theta_\ell \sin\phi + I_8 \sin 2\theta_k \sin 2\theta_l \sin\phi$$
$$+ I_9 \sin^2\theta_k \sin^2\theta_\ell \sin 2\phi\Big]$$

- **4-body decay:** Very rich phenomenology
  - Each coefficient $I_i(q^2)$ is a q^2-dependent observable
  - **The P_5' anomaly** (related to the coefficient I_5)

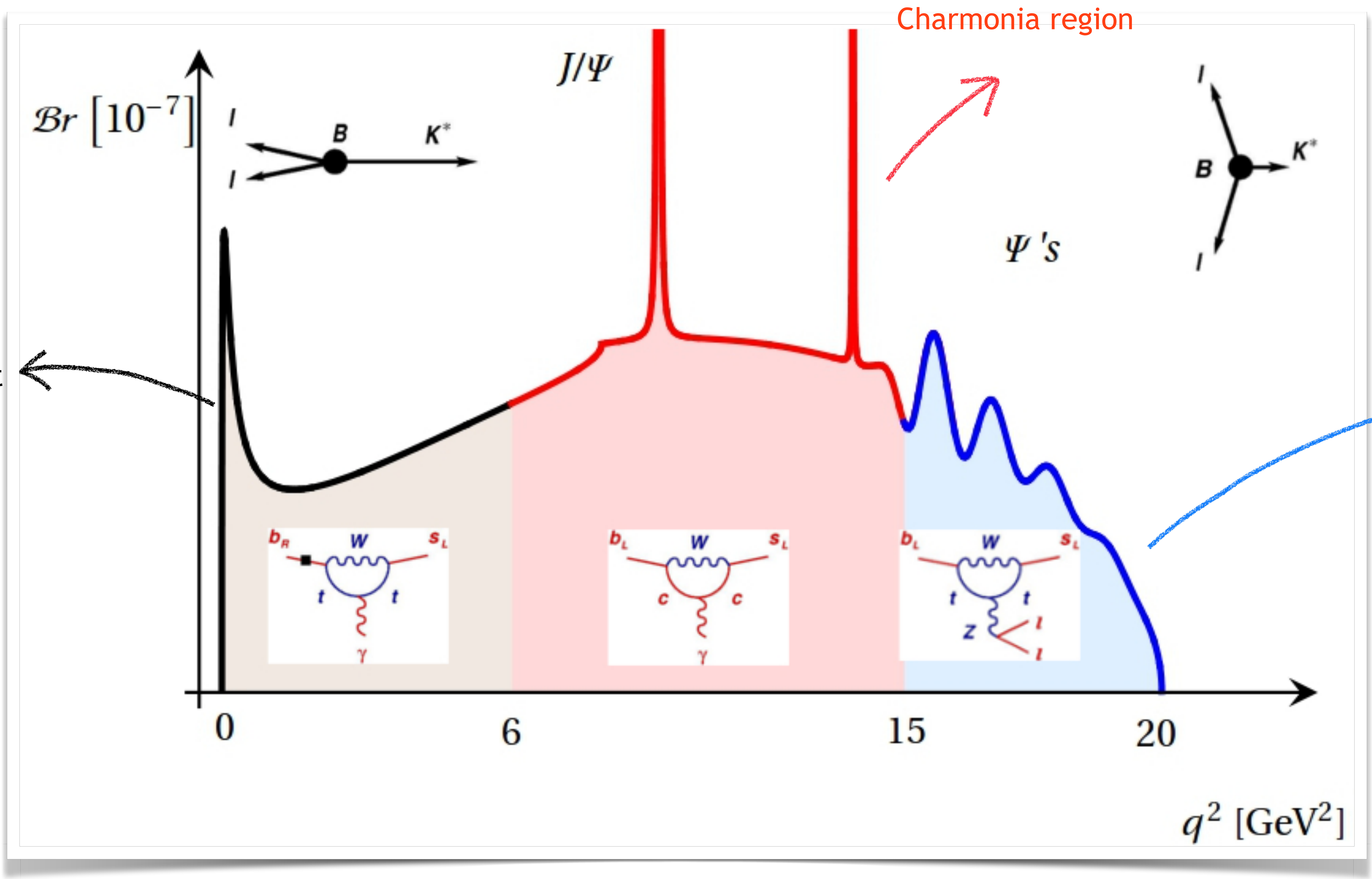
New Physics hypothesis: $C_9^{NP} \simeq -1$ ($-25\,\%$ w.r.t. SM)



Descotes-Genon et al.,PRD88 (2013) 074002

Kinematic regions in the $B \to K^*\ell\ell$ decay

Low q^2
High-recoil limit



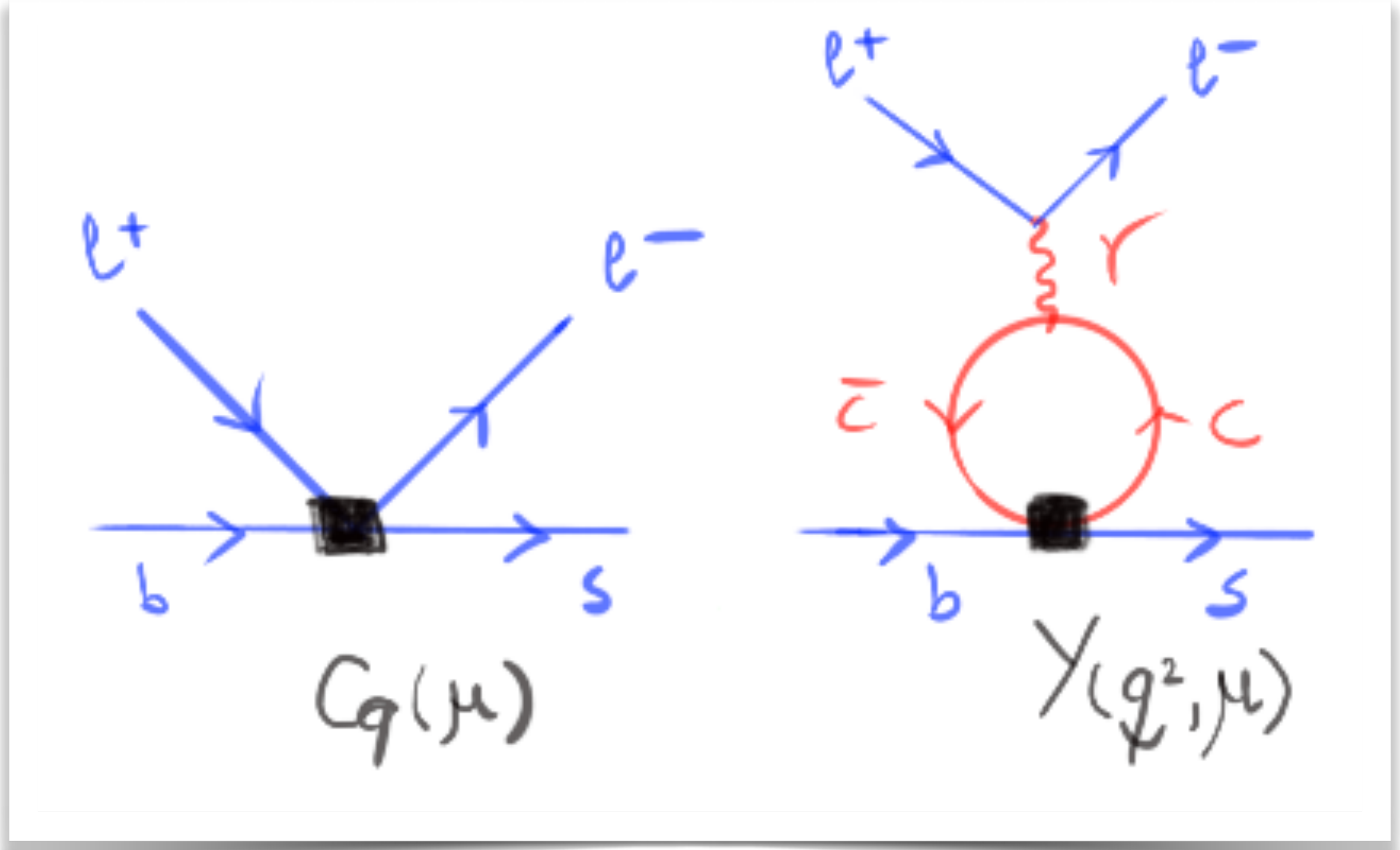
High q^2
Low-recoil limit

Anatomy of the vectorial $B \to K^{(*)}\ell\ell$ amplitude

- **Helicity amplitudes**

$$H_V(\lambda) = -iN\left\{ \overbrace{\left[C_9 \tilde{V}_{L\lambda} + \frac{m_B^2}{q^2} h_\lambda \right]}^{C_9^{\text{eff}}} - \frac{\hat{m}_b m_B}{q^2} C_7 \tilde{T}_{L\lambda} \right\}$$

  - 7 (local) form factors (independent) and 3 non-local form factors
  - **Vector amplitude!** ⇒ Sensitive to the charm contributions!



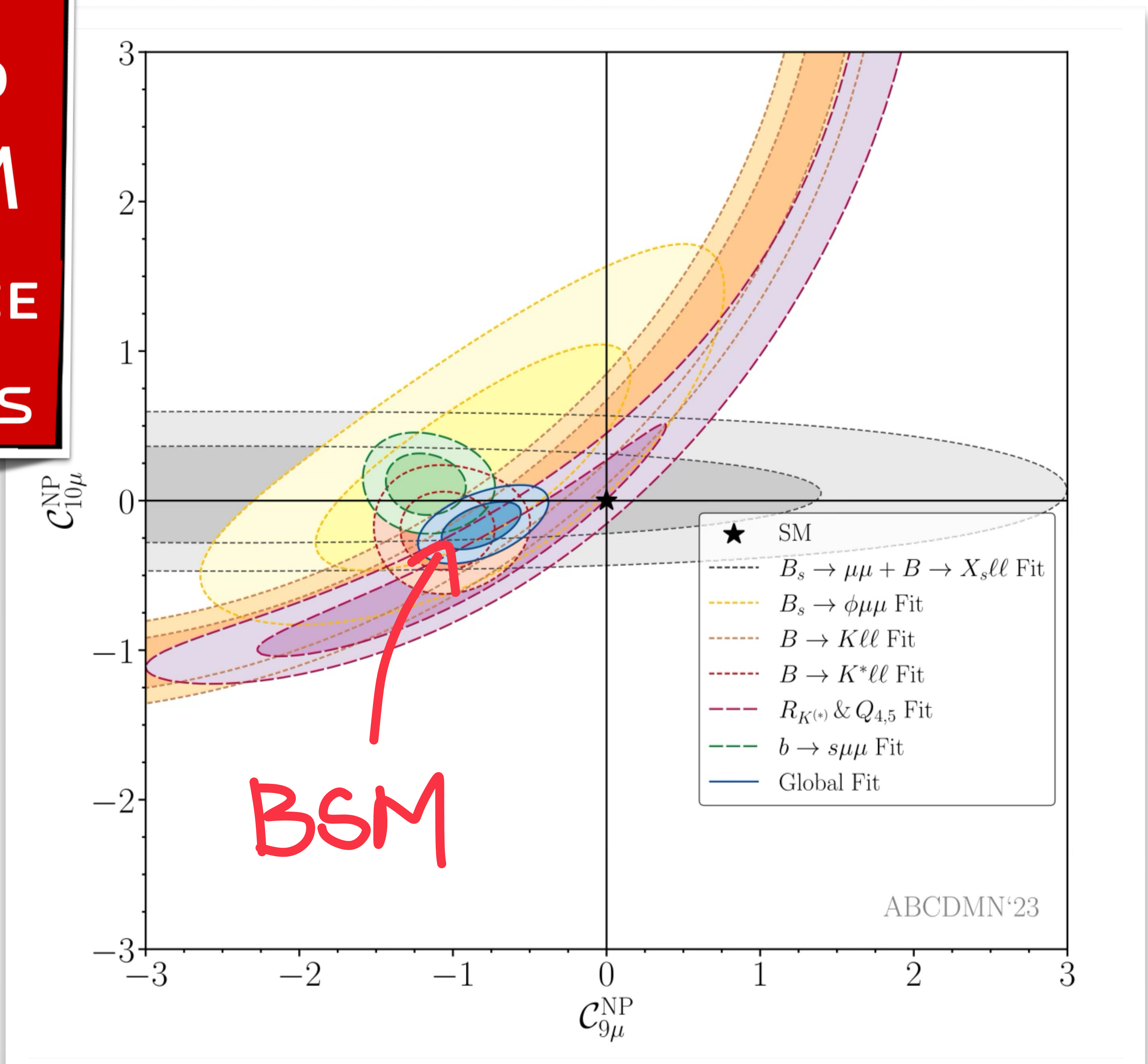
- **At leading order** $C_9^{\text{eff}} = C_9(\mu) + Y(q^2, \mu)$
  - In fact C_9^{eff} is observable ⇒ **Scale independent**
  - **One cannot disentangle** C_9 **from** C_9^{eff} **without** h_λ

The $b \to s\ell\ell$ anomalies: two approaches to life

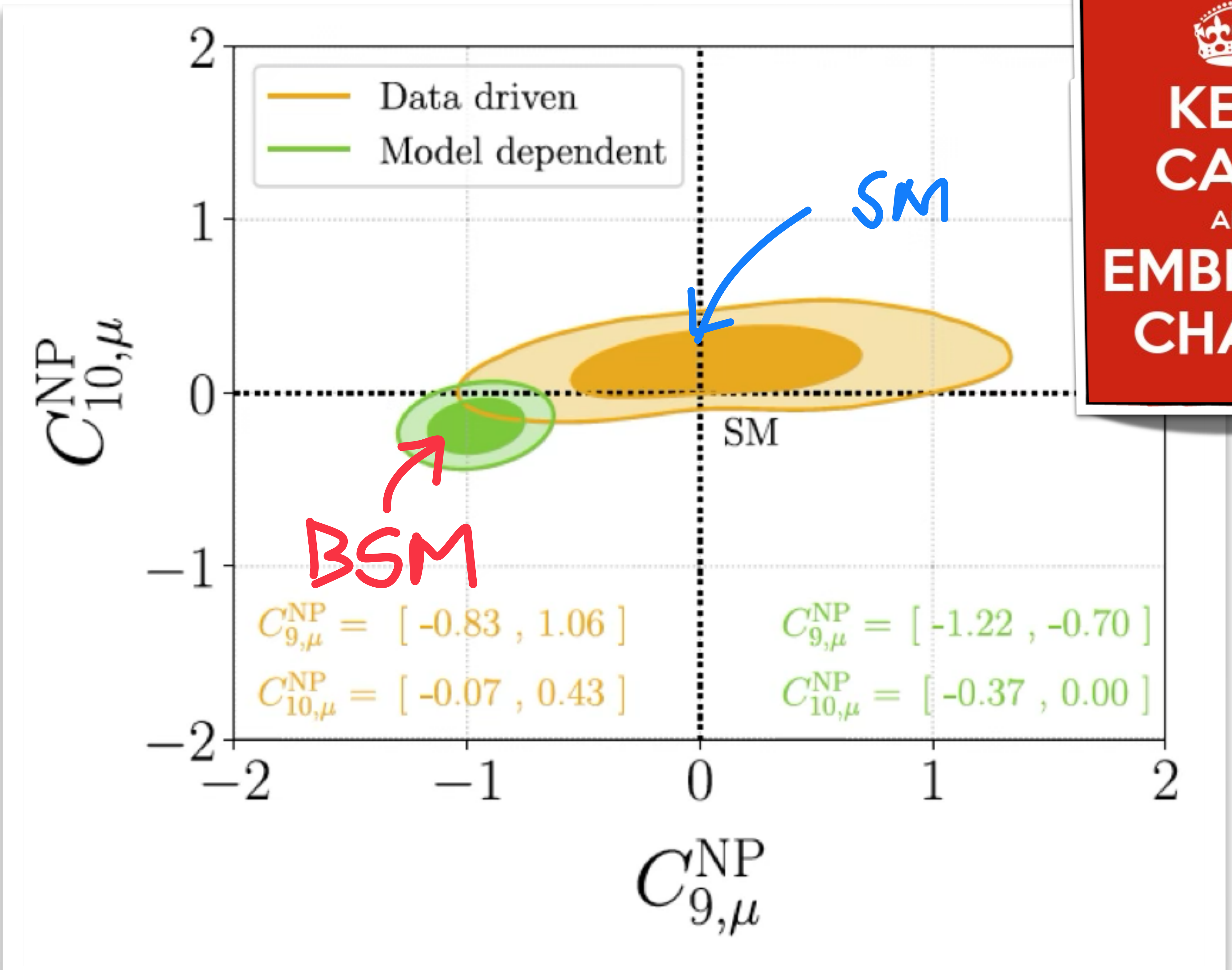
- Interpretation of data depends on **prior beliefs** about "charm"

Algueró et al., EPJ.C(2023)83:648

Ciuchini et al., PRD107 (2023) 5, 055036



Tension with SM at $> 6\sigma$



Consistent within 1σ

Generalities about neutral meson mixing

- $U(1)$ ***flavor symmetries*** **(e.g. *strangeness*)**

S=+1: $K^0 = d\bar{s},\ K^+ = u\bar{s}$	**S= -1:** $\bar{K}^0 = s\bar{d},\ K^- = s\bar{u}$
C=+1: $D^0 = c\bar{u},\ D^+ = c\bar{d}$	**C= -1:** $\bar{D}^0 = u\bar{c},\ D^- = d\bar{c}$
B=+1, S=-1: $B_s^0 = s\bar{b}$	**B= -1, S=+1:** $\bar{B}_s^0 = b\bar{s}$

  - Charges conserved by strong and EM

Weak interactions $\Rightarrow$ Flavor (symmetry) violations

  - **Neutral meson mixing**

Flavor eigenstates $\neq$ Mass eigenstates

Oscillates

$$H_{eff} \equiv \mathbf{R} = \mathbf{M} - i\frac{\boldsymbol{\Gamma}}{2}$$

Decays

Neutral meson mixing in QM

Weisskopf-Wigner QM formalism

- Γ is definite positive!
- *CPT*: $M_{11} = M_{22} \equiv m_K \,,\, \Gamma_{11} = \Gamma_{22} \equiv \Gamma$

$$H_{eff} \equiv \mathbf{R} = \mathbf{M} - i\frac{\mathbf{\Gamma}}{2} = \begin{pmatrix} m_K & M_{12} \\ M_{12}^* & m_K \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma \end{pmatrix}$$

- *CP* conservation (?)
  - Eigenstates:

$$K_{L,S} = \frac{1}{\sqrt{2}}\left(K^0 \mp \bar{K}^0\right) \quad \Rightarrow \quad K_{L,S} = K^0_{\mp}$$

  - Eigenstates of *CP* too: $CP\,|\,K^0_{\pm}\rangle = \pm\,|\,K^0_{\pm}\rangle$ with $CP\,|\,K^0\rangle = -\,|\,\bar{K}^0\rangle$

- Eigenvalues: mass and width differences (observables)

$$\Delta m = 2\,|\,M_{12}\,| \qquad \Delta\Gamma = 2\,|\,\Gamma_{12}\,|$$

Evidence of CP violation in kaons

- *CP* violation discovered in Kaon decays

$$CP\,|\pi^+\pi^-\rangle = +\,|\pi^+\pi^-\rangle \quad (CP\text{-even})$$

IF *CP* is conserved
THEN $K_L \to \pi^+\pi^-$ is forbidden

VOLUME 13, NUMBER 4 PHYSICAL REVIEW LETTERS 27 JULY 1964

EVIDENCE FOR THE 2π DECAY OF THE K_2^0 MESON*†

J. H. Christenson, J. W. Cronin,‡ V. L. Fitch,‡ and R. Turlay§
Princeton University, Princeton, New Jersey
(Received 10 July 1964)

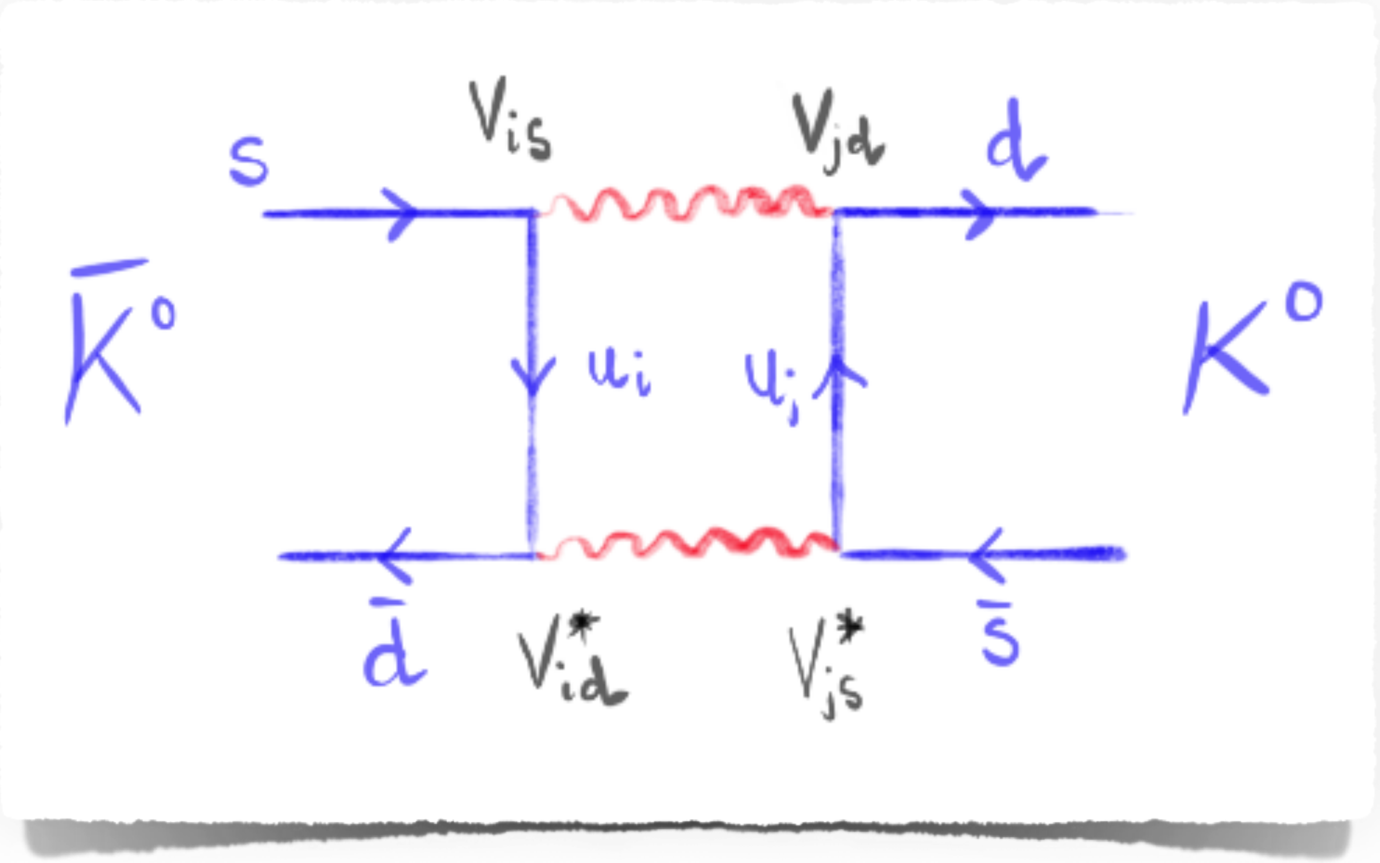
PDG

$$\mathrm{BR}(K_S \to \pi\pi) = 99.89(10)\,\%$$
$$\mathrm{BR}(K_L \to \pi\pi\pi) = 32.06(17)\,\%$$
$$\mathrm{BR}(K_L \to \pi\pi) = 0.2831(16)\,\%$$

$K_L \to \pi^+\pi^-$ is observed
THEN *CP* is violated in Kaon decays!

Neutral-kaon mixing in the SM and mass difference

- **FCNC:** Box diagram



- **Low-energy EFT**

SM: $\mathscr{H}_{\text{eff}} = \dfrac{G_F^2}{4\pi^2} C(\mu) \left(\bar{d}\gamma^\mu P_L s\right) \left(\bar{d}\gamma_\mu P_L s\right) + \text{h.c.}$

$$M_{12} = \frac{1}{2m_K} \langle K^0 | \mathscr{H}_{\text{eff}} | \bar{K}^0 \rangle$$

- **Perturbative calculation**

$\lambda_i = V_{id}^* V_{is}$

$x_i = \dfrac{m_i^2}{m_W^2}$

  - Wilson coefficient: $C(\mu) = b(\mu)\left(\lambda_c^2\, S_0(x_c)\eta_1 + \lambda_t^2\, S_0(x_t)\eta_2 + 2\lambda_c\lambda_t S_0(x_c, x_t)\eta_3\right)$
    - Higher-order QCD corrections: $b(\mu)$, η_i
    - GIM hidden in Inami-Lim functions: e.g.
    - **Charm** $\approx \lambda^2 x_c$ vs. **top** $\approx \lambda^{10} x_t$

$$S_0(x) = \frac{x}{(1-x)^2}\left(1 - \frac{11x}{4} + \frac{x^2}{4} - \frac{3x^2 \log x}{2(1-x)}\right)$$

CP Violation - Branco, Lavoura & Silva, Appendix B

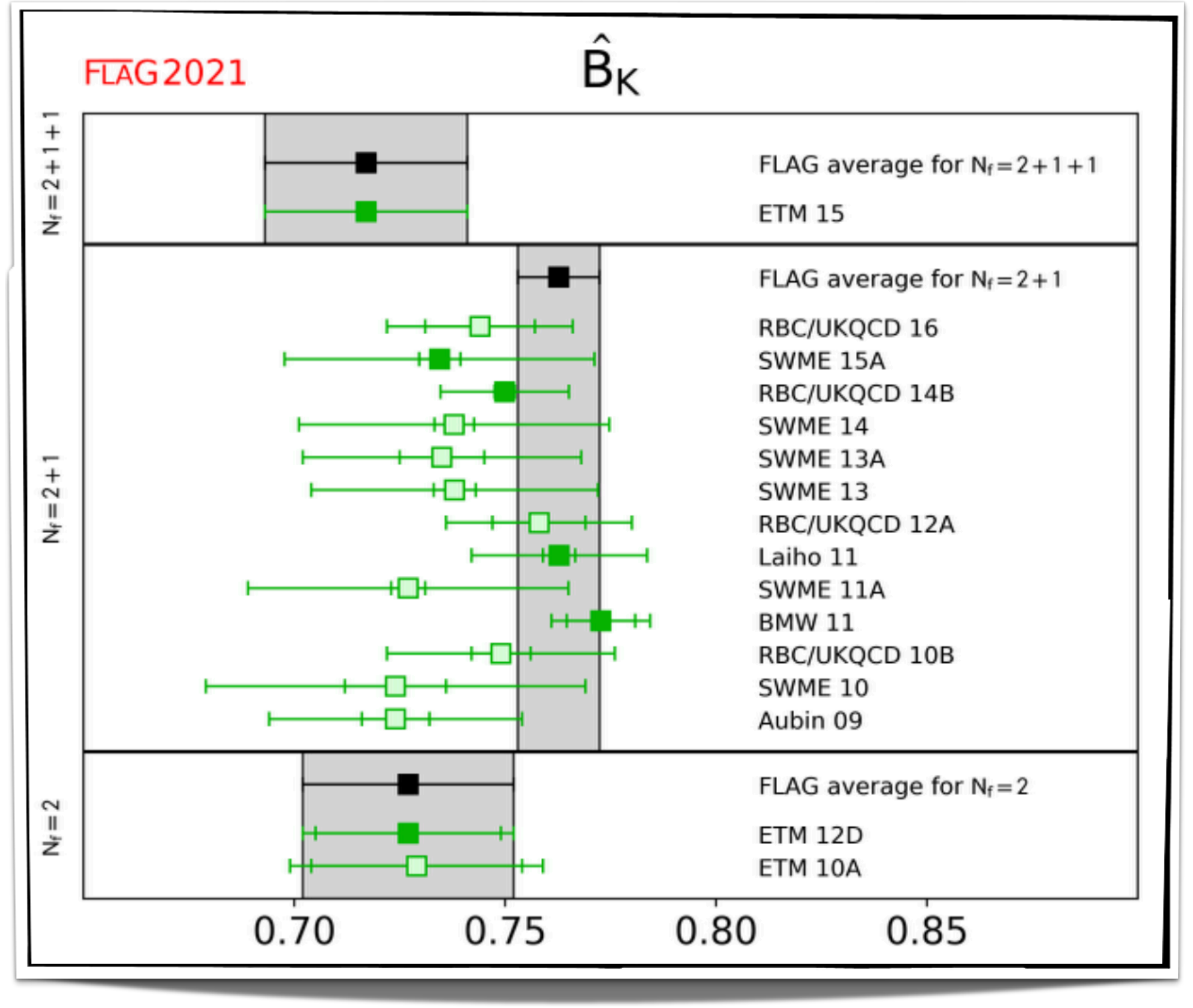
Hadronic matrix element for kaon mixing

- To make predictions we need a hadronic matrix element

$$\langle K^0 | \left(\bar{d}\gamma^\mu P_L s\right) \left(\bar{d}\gamma_\mu P_L s\right) | \bar{K}^0 \rangle = \frac{2}{3} m_K^2 f_K^2 B_K(\mu)$$

- **Bag parameter:** B_K dimensionless parameter
  - Parametrization inspired by "vacuum approximation" $(B_K = 1)$ *CP Violation* - Branco *et al.*, Appendix C
- **Scale & Scheme independent:** $\hat{B}_K = b(\mu) B_K(\mu)$
- Standard calculation in LQCD today

$$N_f = 2 + 1$$
$$\hat{B}_K = 0.763(10)$$

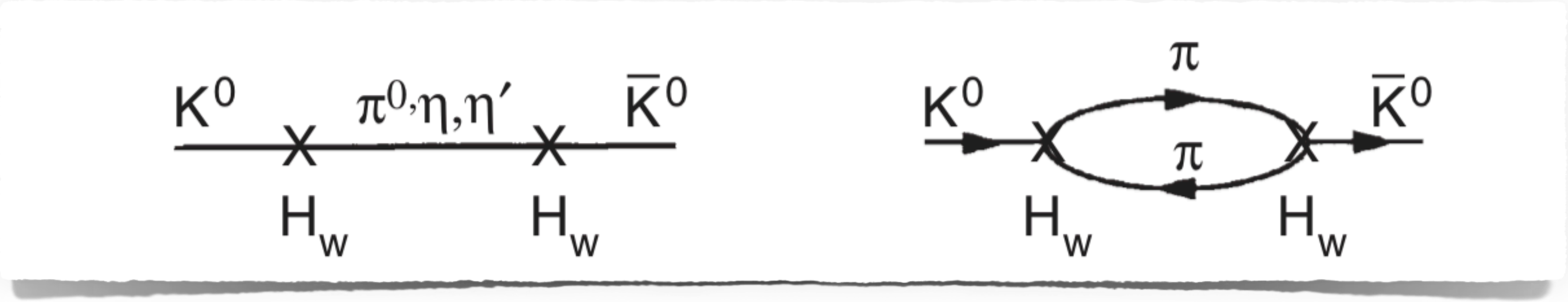


The kaon-mass difference in the SM

- **Kaon mass difference:** $\Delta m_K \approx 2\text{Re}(M_{12})$
  - The charm-quark contribution dominates:

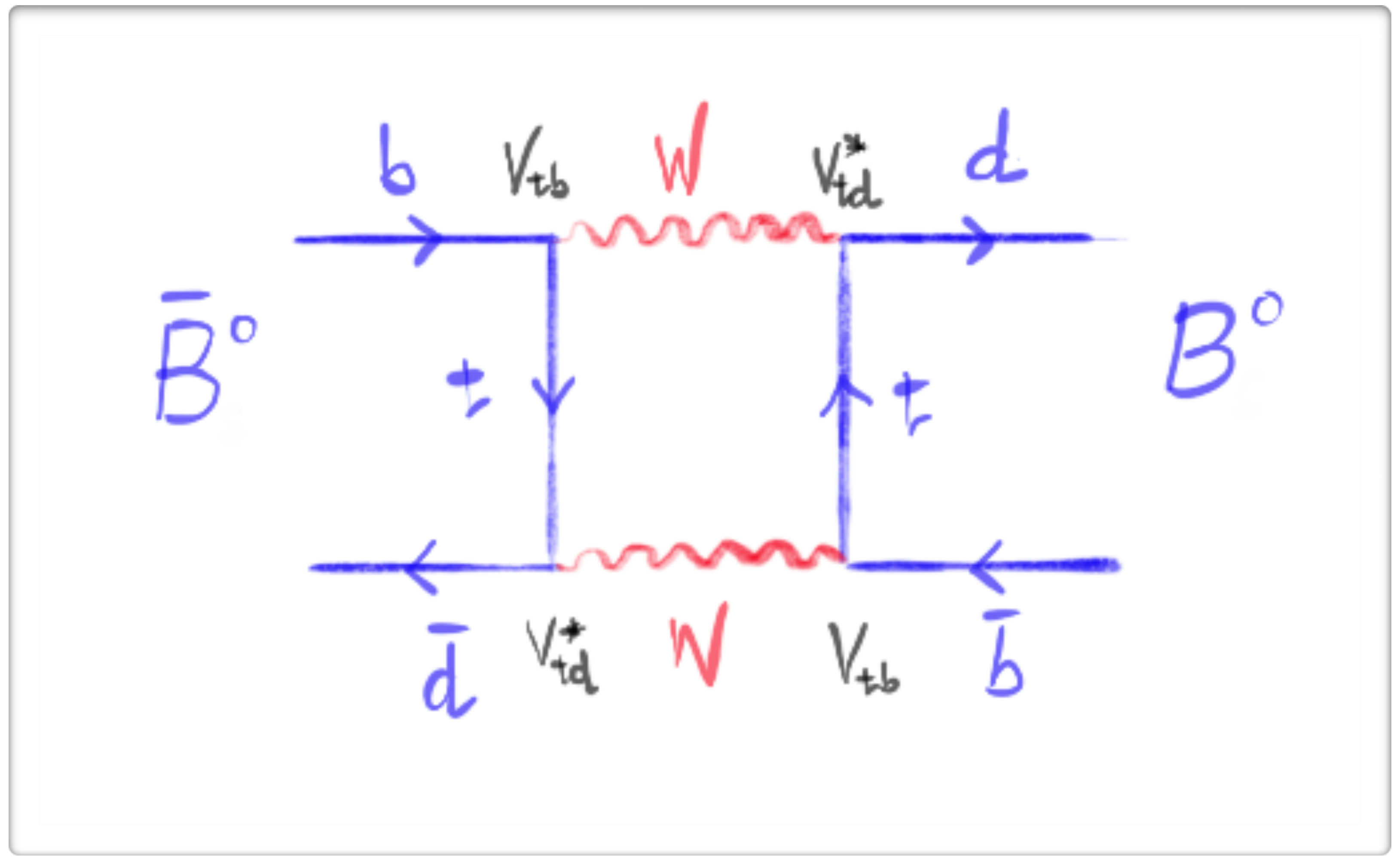
$$\Delta m_K^{\text{SD}} \approx \frac{G_F^2}{24\pi^2} m_c^2 \Re(V_{cd}^* V_{cs}) f_K^2 m_K \hat{B}_K \approx 10^{-15} \text{ GeV}$$

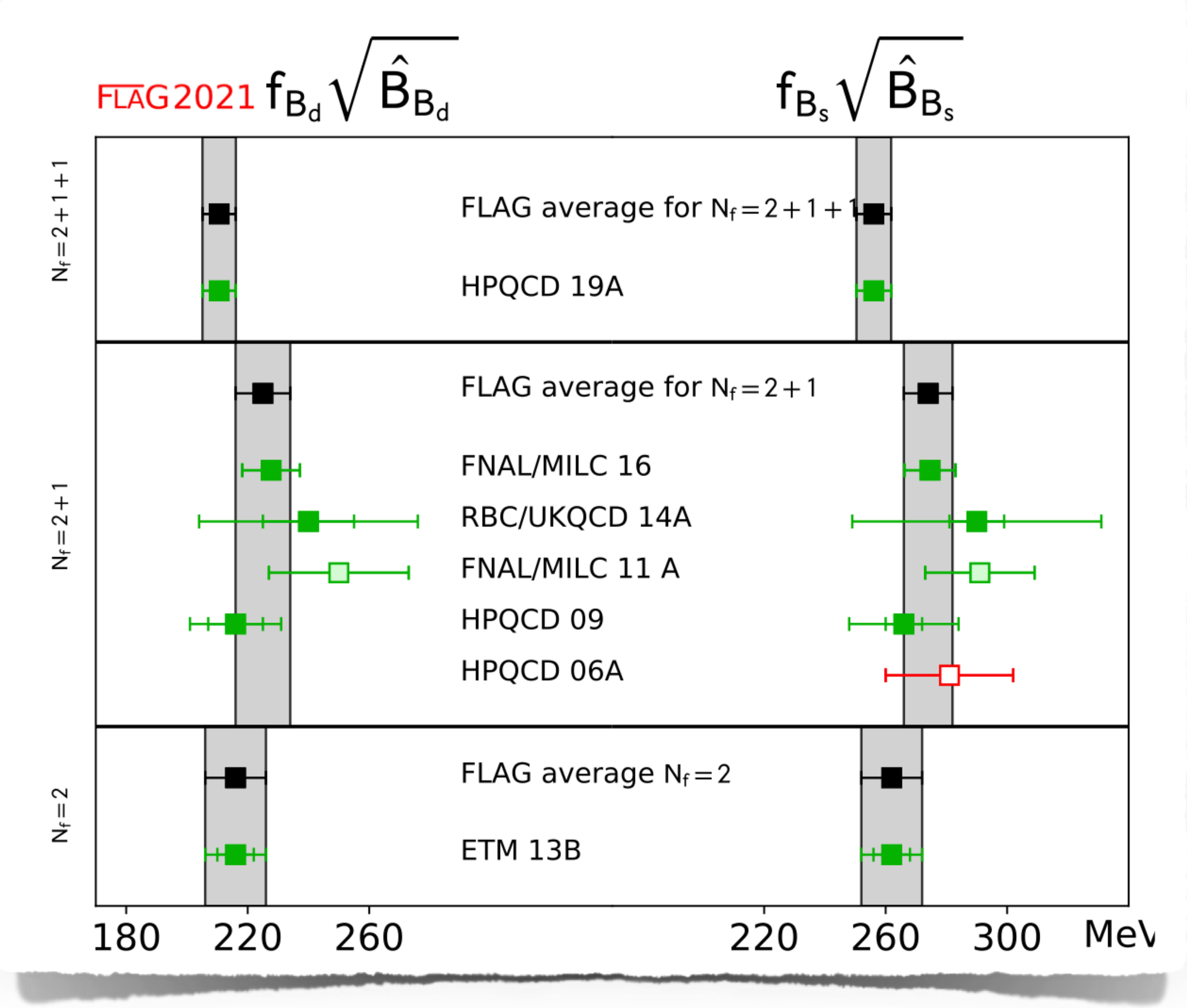
  - Same ballpark as experiment! $\Delta m_K^{\text{expt}} = 3.484(6) \times 10^{-15}$ GeV
- **Problem:** Uncontrolled **long-distance** contributions
  - Exchange of pions and other hadrons at $d \approx 1/\Lambda_{\text{QCD}}$



  - Δm_K is not used to test the SM but taken as experimental fact in kaon mixing

SM predictions for heavy meson mixing: $B^0 - \bar{B}^0$





- B^0**-meson mixing dominated by top loop!**

GIM

$$M_{12} = \frac{G_F^2}{12\pi^2} f_B^2 m_B \hat{B}_{B_d} (V_{td}^* V_{tb})^2 S_0(x_t) \equiv |M_{12}| e^{i\phi}$$

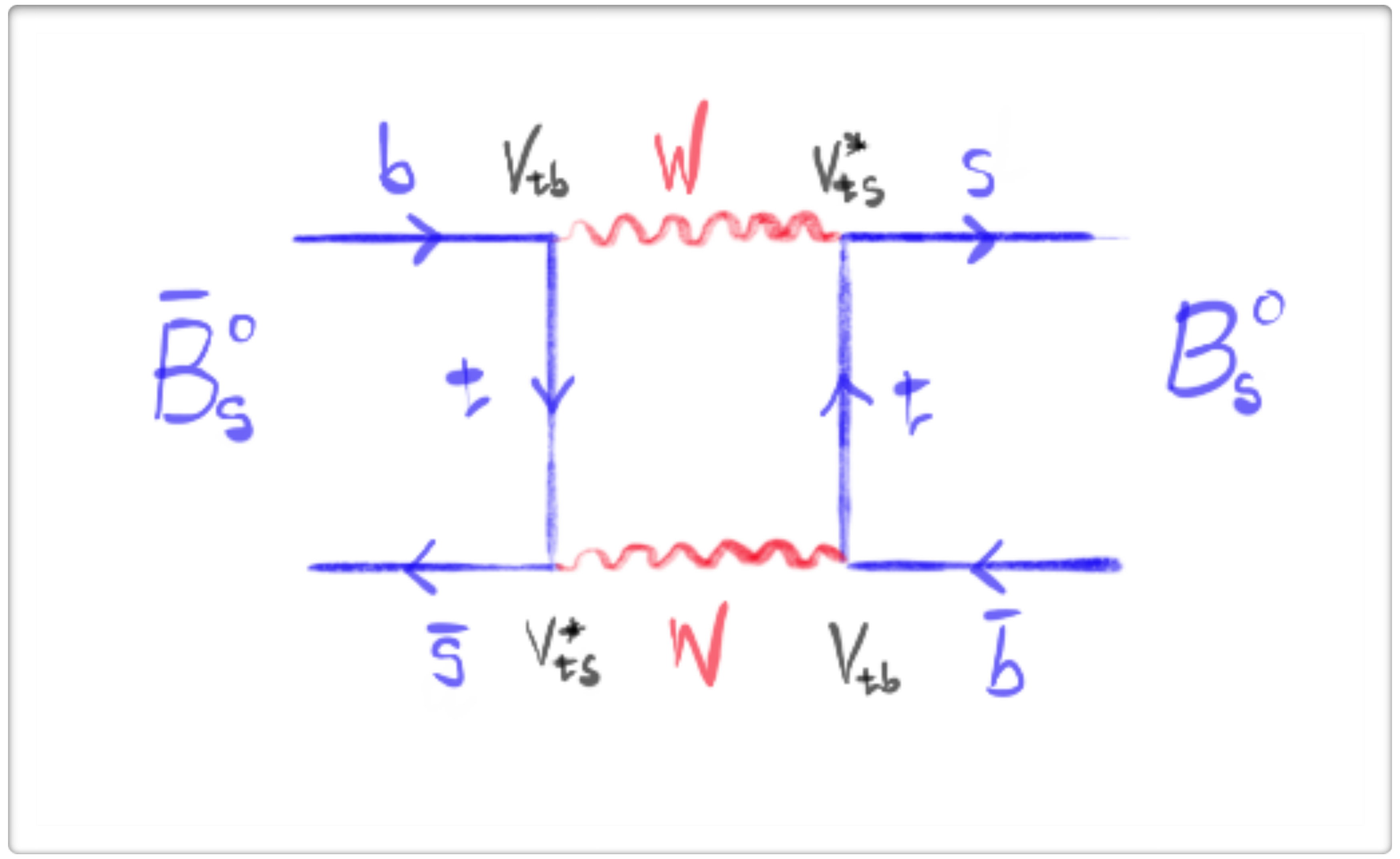
$$\langle 0|A^\mu|B_q(p)\rangle = ip_B^\mu f_{B_q}$$

$$B_{B_q}(\mu) = \frac{\langle \bar{B}_q^0 | Q_R^q(\mu) | B_q^0 \rangle}{\frac{8}{3} f_{B_q}^2 m_B^2}$$

- **Predictions in the SM**

$$\Delta m_d^{SM} = 0.555(50)\ \text{ps}^{-1}$$
$$\Delta m_d^{expt} = 0.5065(19)\ \text{ps}^{-1}$$
$$\phi = \arg(V_{td}^* V_{tb}) \approx \beta$$

$B_s^0 - \bar{B}_s^0$ and ratios with the B^0 system



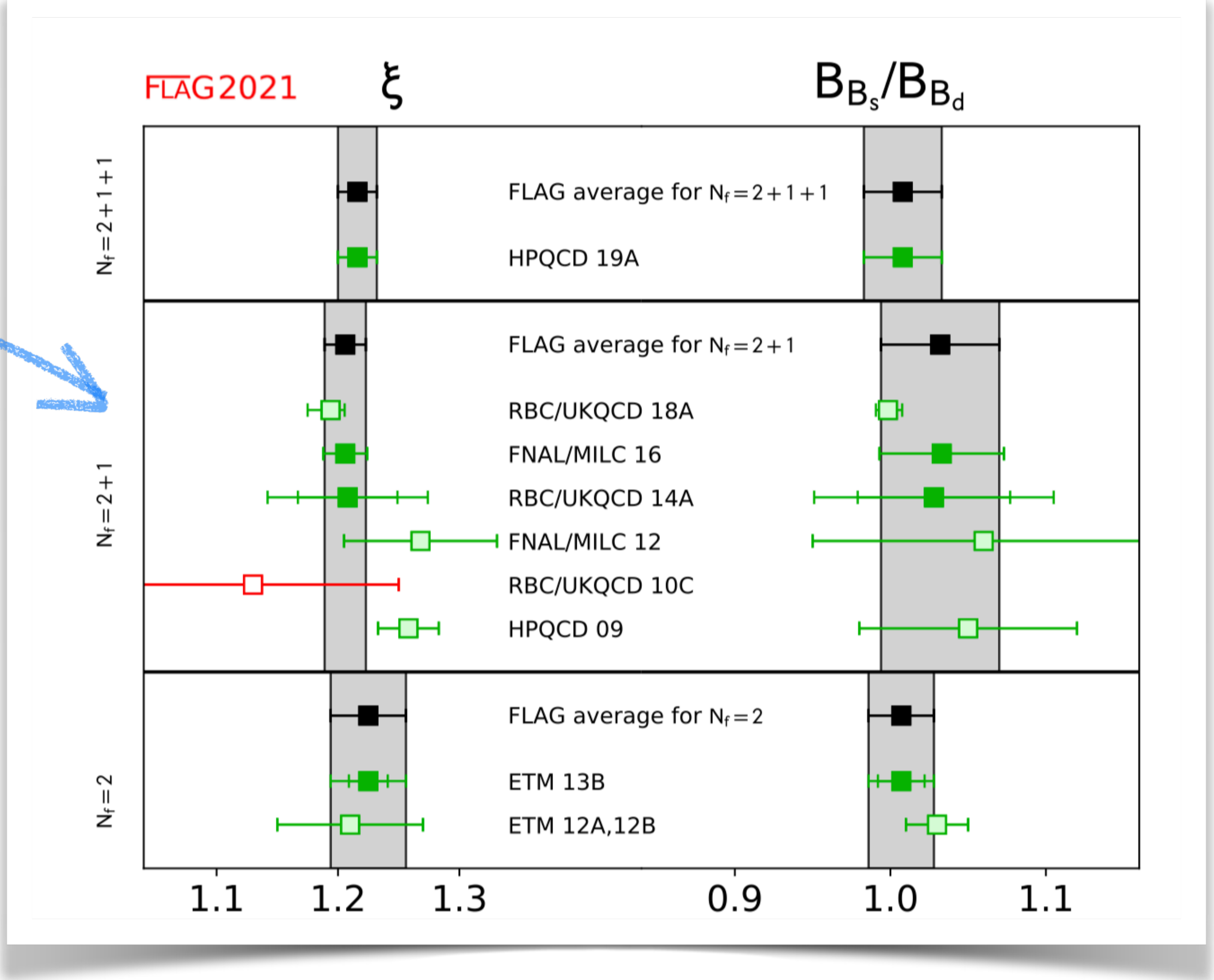
- Identical to B^0 replacing $d \to s$

$$\Delta m_s^{\text{SM}} = 17.6(1.0)\ \text{ps}^{-1}$$
$$\Delta m_s^{\text{expt}} = 17.7656(57)\ \text{ps}^{-1}$$
$$\phi = \arg(V_{ts}^* V_{tb}) \approx \beta_s$$

- Difference B^0 and B_s^0 ? $\Rightarrow$ CKM!

$$\frac{\Delta m_s}{\Delta m_d} = \overbrace{\left[\frac{f_{B_s}^2 \hat{B}_s}{f_{B_d}^2 \hat{B}_d}\right]}^{\xi^2} \left|\frac{V_{ts}}{V_{td}}\right|^2 \Rightarrow \left|\frac{V_{td}}{V_{ts}}\right| = 0.2071(27)$$

Determined better



Phenomenology of neutral-meson oscillations

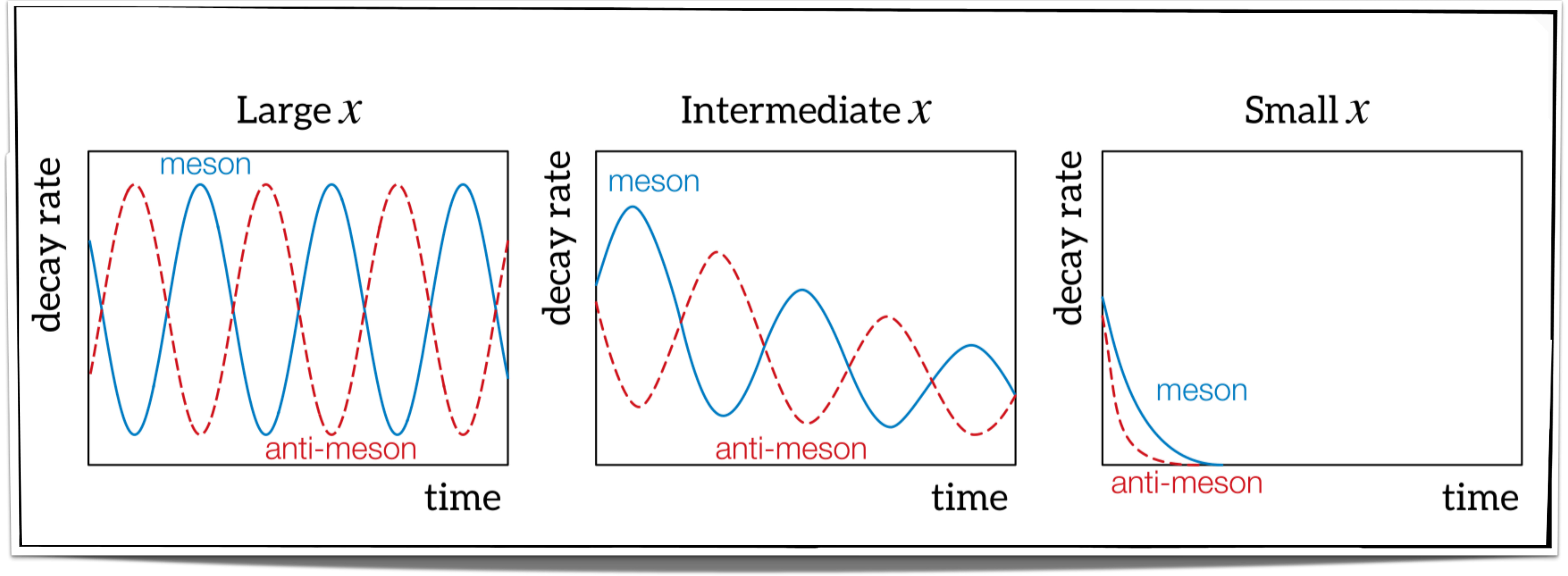
- **Define:**

$$x = \frac{\Delta m}{\Gamma} \qquad y = \frac{\Delta\Gamma}{2\Gamma}$$

- **Observable:**

$$P(t) = |\langle X_0(t)|X_0\rangle|^2 = |f_+(t)|^2 = \frac{e^{-\Gamma t}}{2}\left(\cosh(y\Gamma t) + \cos(\Gamma x t)\right)$$

  - We can use QM to measure small **mass differences** Δm (x)



Approximate values

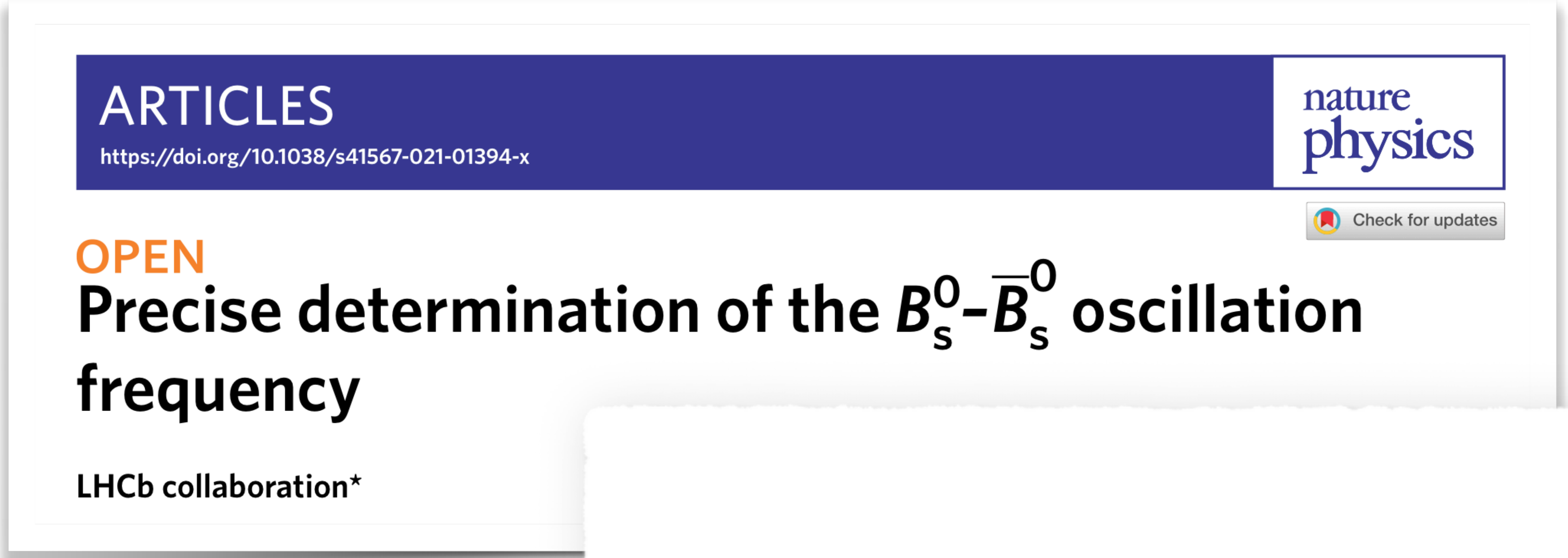
	x	y
K	1	1
B	1	10^{-2} (*)
B_s	10	10^{-1} (*)
D	10^{-2}	10^{-2}

Flavor tagging with heavy mesons

- **Use leptonic decays as tags!**
  - ***B factories:*** Entangled $\Upsilon \to B\bar{B}$ pairs $\Rightarrow$ **Same-sign leptons** is a smoking gun!

1987: Discovery of $B^0 - \bar{B}^0$ mixing! (ARGUS)

- **Exquisite $B_s^0 - \bar{B}_s^0$ oscillations at LHCb**
  - Tag final flavor state with hadronic decays


ARTICLES
https://doi.org/10.1038/s41567-021-01394-x
nature physics
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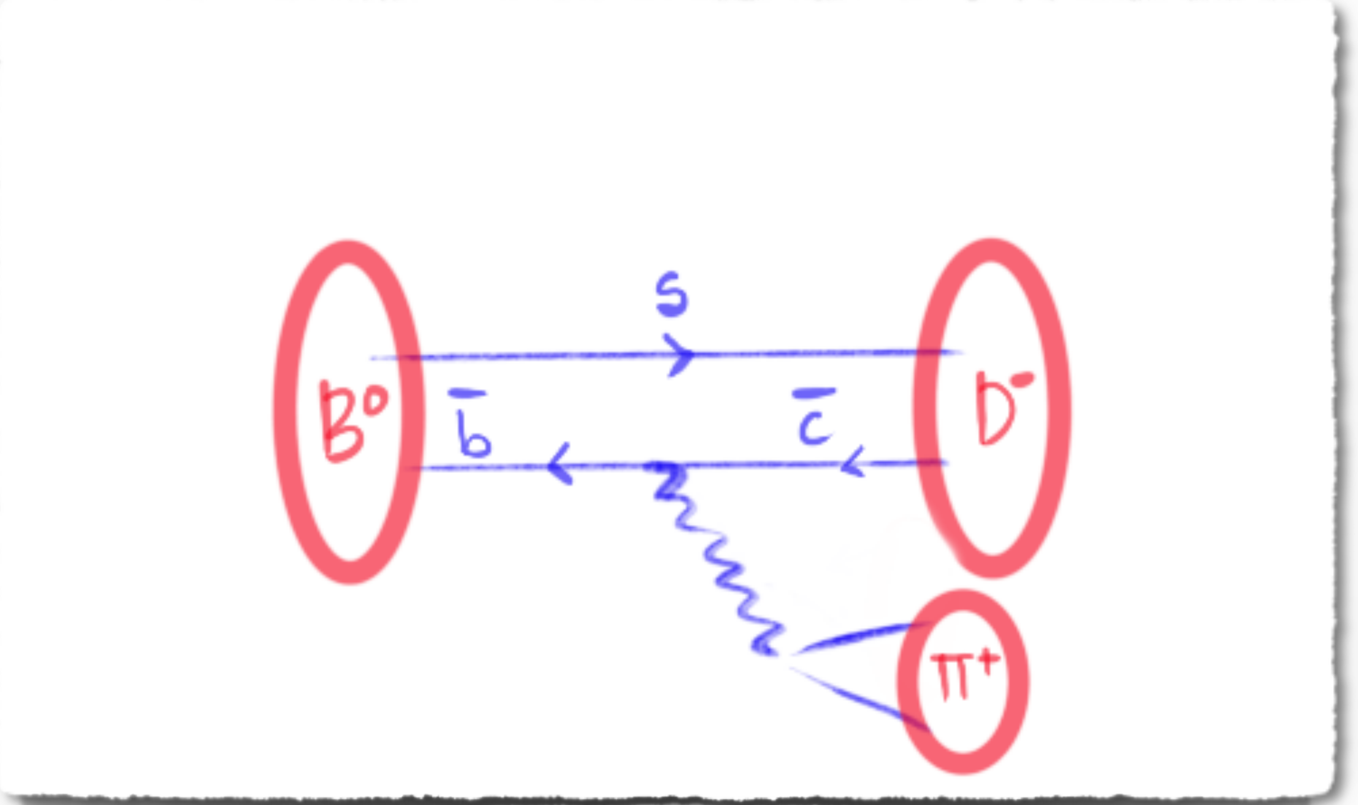
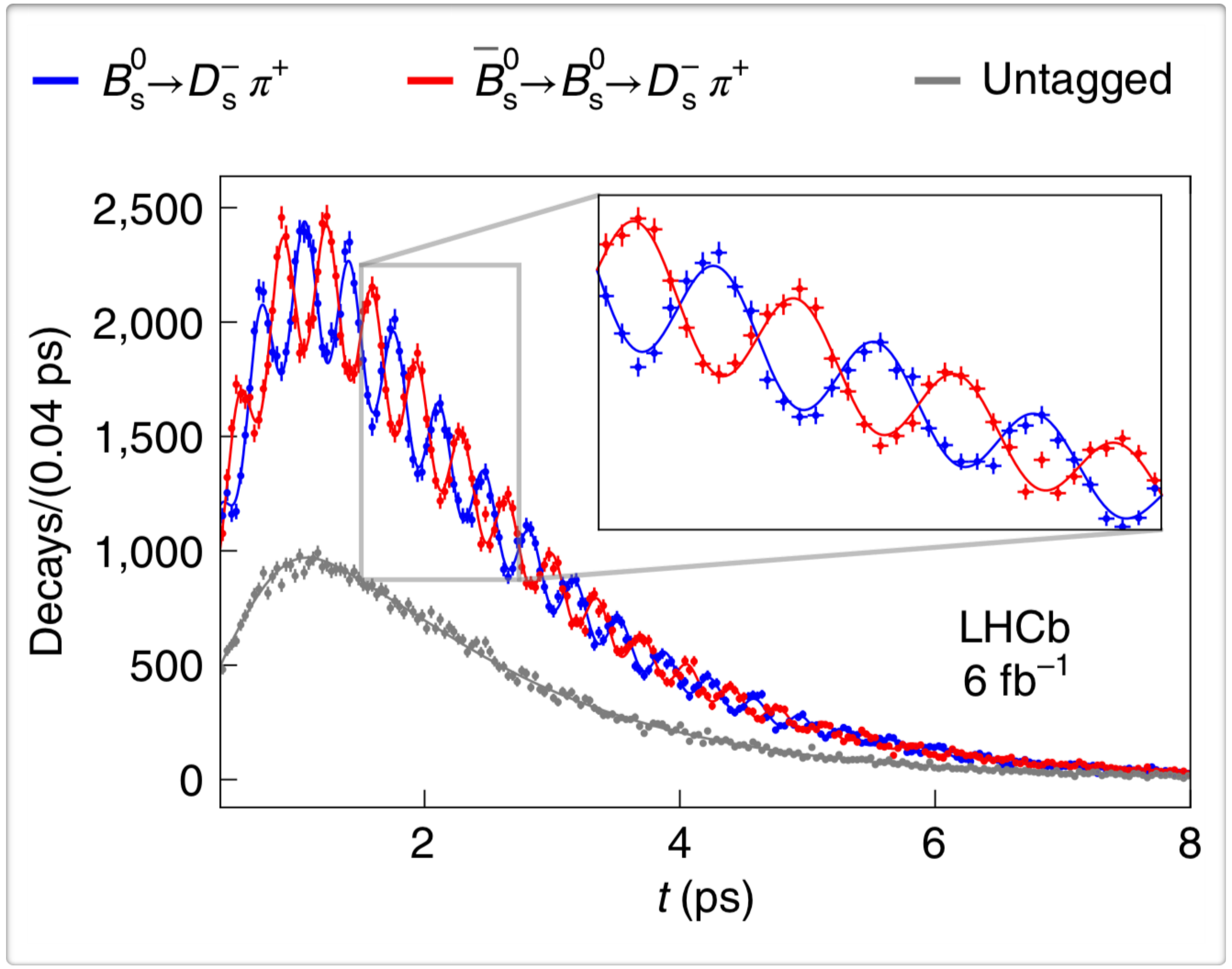
Precise determination of the B_s^0–$\bar{B}_s^0$ oscillation frequency
LHCb collaboration*



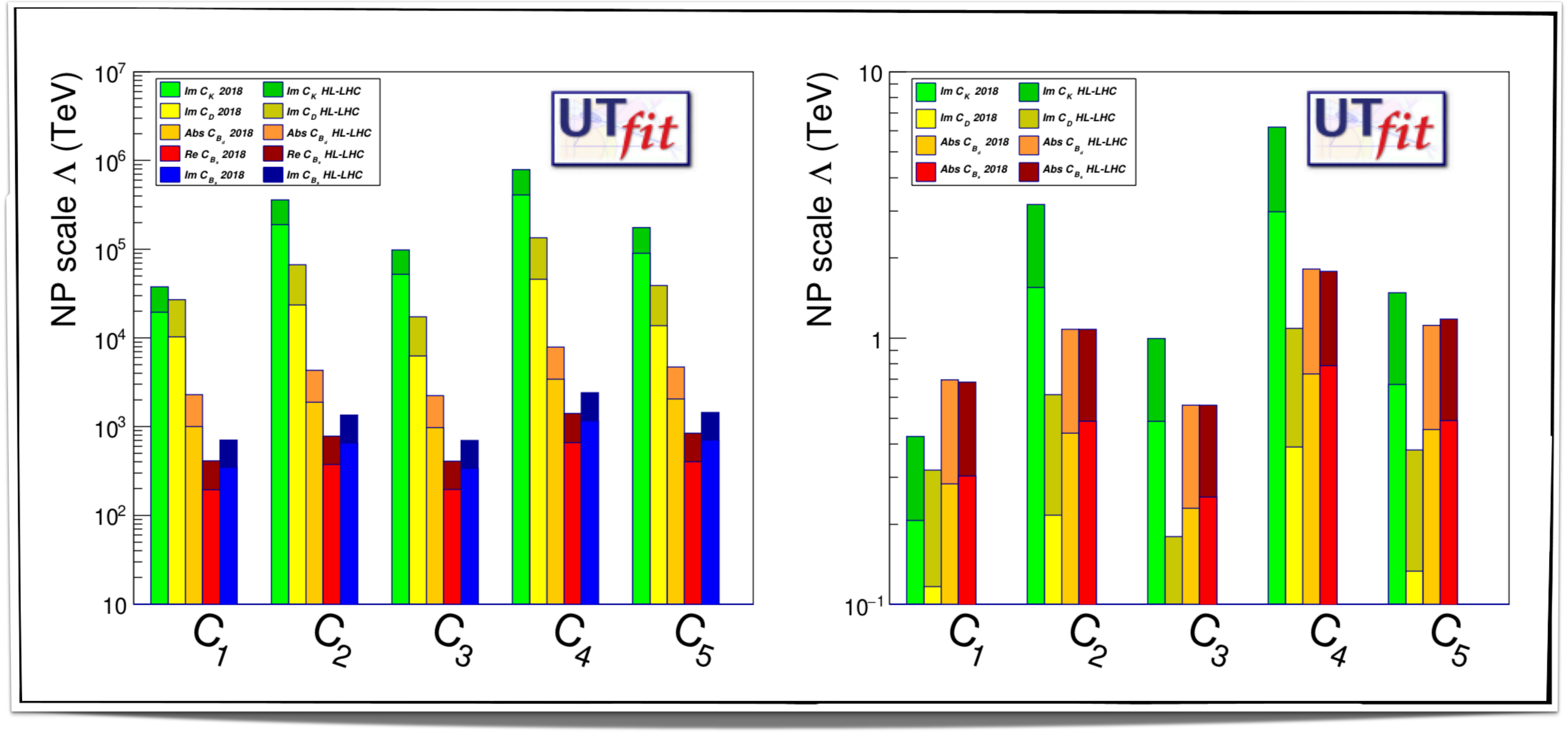


BSM bounds from neutral-meson mixing

- **Neutral-meson mixing leads to very strong bounds on BSM physics**
  - They need to be taken into accoount by almost any flavor model building
  - Very sensitive to SM flavor structure ⇒ **Only MFV survives at low scales!**

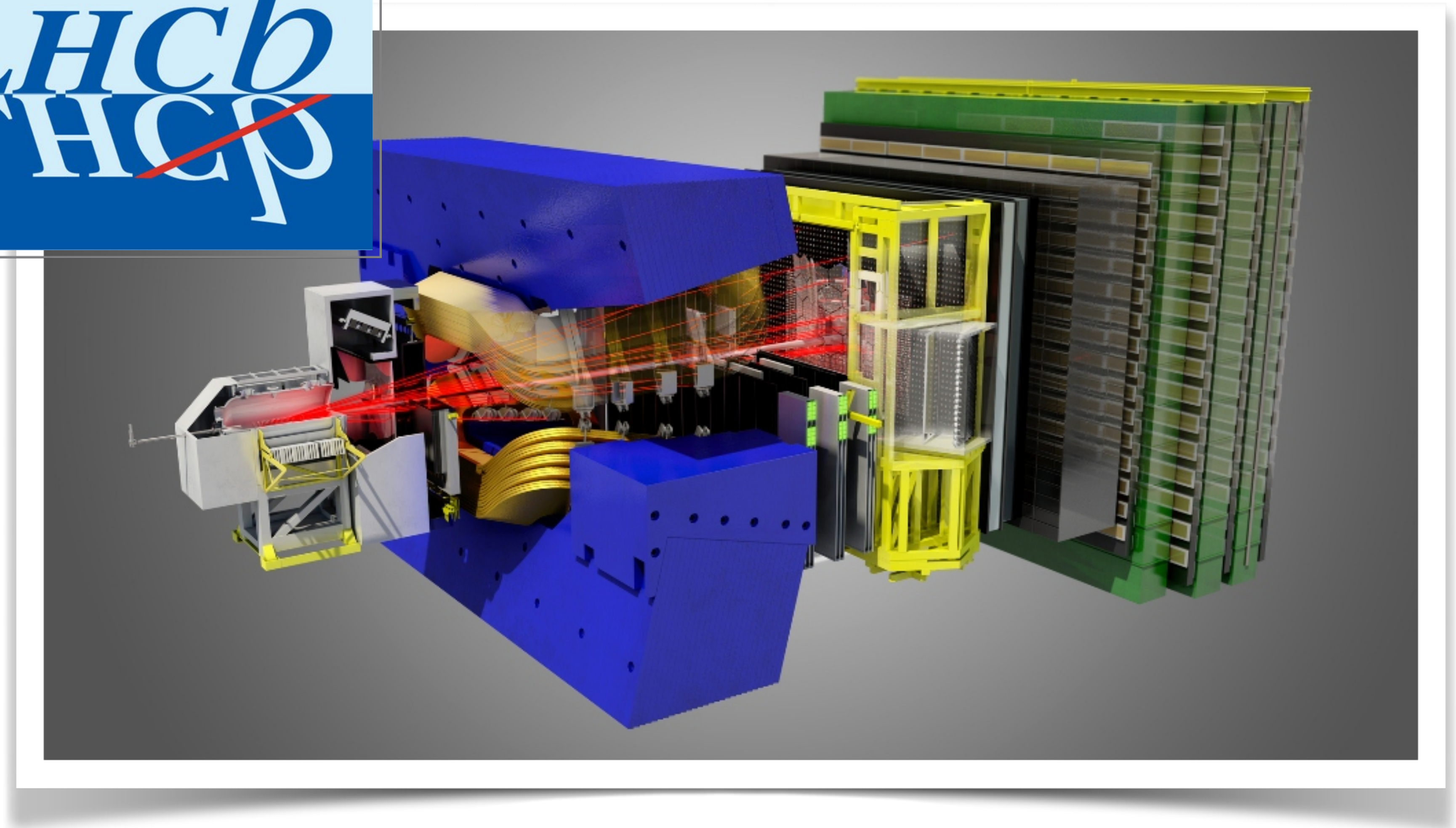
$$H_{\text{eff}}^{\Delta F=2} = \frac{G_F^2}{16\pi^2} M_W^2 \sum_i V_{\text{CKM}}^i C_i(\mu) Q_i$$

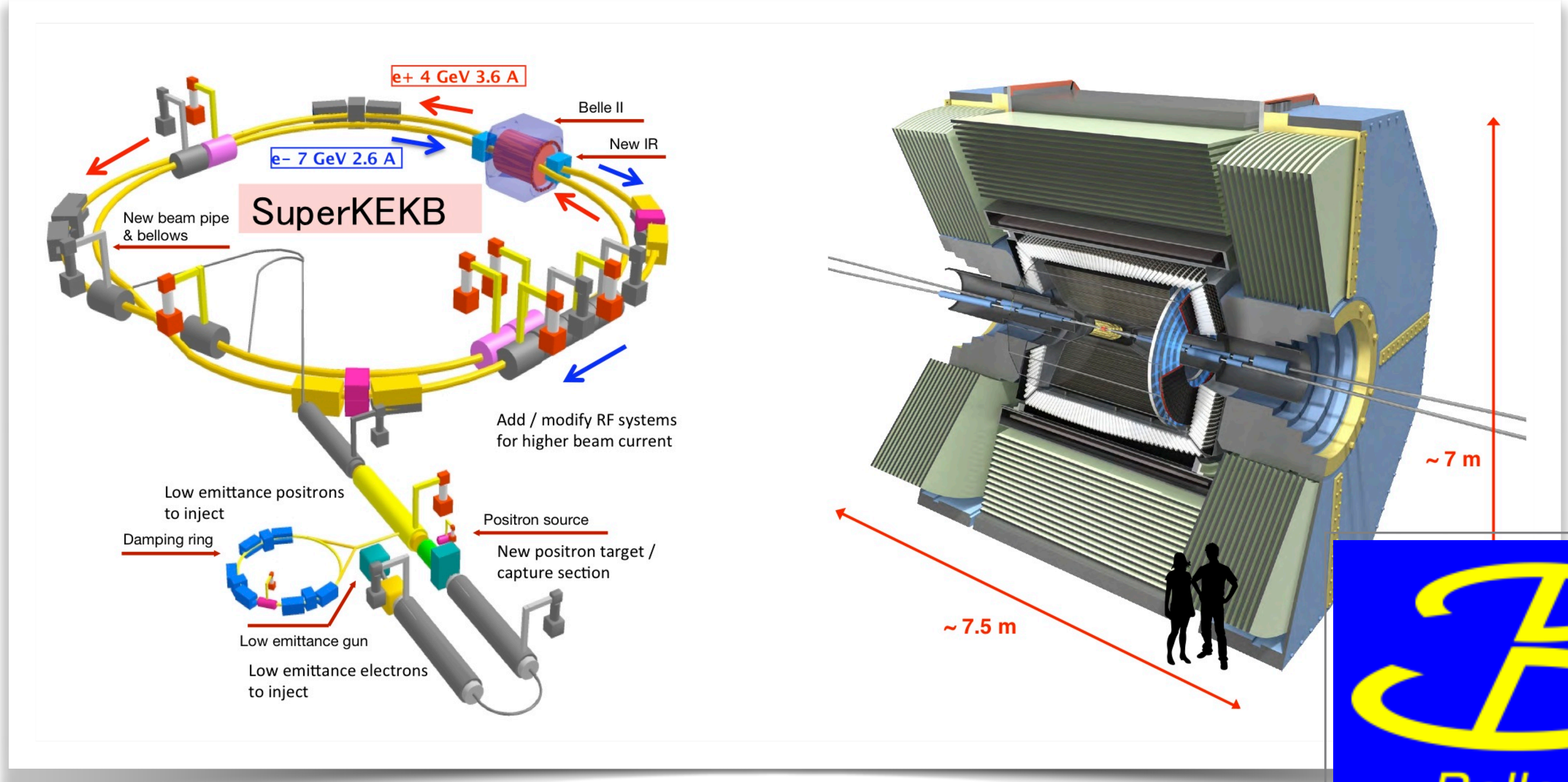
$$\begin{aligned}
&Q_1^{\text{VLL}} = (\bar{s}^\alpha \gamma_\mu P_L d^\alpha)(\bar{s}^\beta \gamma^\mu P_L d^\beta), \\
&Q_1^{\text{LR}} = (\bar{s}^\alpha \gamma_\mu P_L d^\alpha)(\bar{s}^\beta \gamma^\mu P_R d^\beta), \qquad Q_2^{\text{LR}} = (\bar{s}^\alpha P_L d^\alpha)(\bar{s}^\beta P_R d^\beta), \\
&Q_1^{\text{SLL}} = (\bar{s}^\alpha P_L d^\alpha)(\bar{s}^\beta P_L d^\beta), \qquad Q_2^{\text{SLL}} = (\bar{s}^\alpha \sigma_{\mu\nu} P_L d^\alpha)(\bar{s}^\beta \sigma^{\mu\nu} P_L d^\beta),
\end{aligned}$$



Concluding: experimental golden era

- "Multi-purpose" *B*-meson factories





- Many more flavor experiments at different scales

Kaons and muons

Taus, hyperons, charm

TeV scale

Concluding: probe to physics beyond the SM

Flavor Physics spearheaded the discovery of the SM when the SM was the New Physics!

- **Nuclear β decay:** Discovery of weak interactions and the neutrinos
- **Rare kaon decays:** Discovery of charm quark
- **Kaon decays:** Discovery of *CP* violation → Discovery of 3 generations

PROPOSAL FOR K^o_2 DECAY AND INTERACTION EXPERIMENT

J. W. Cronin, V. L. Fitch, R. Turlay

(April 10, 1963)

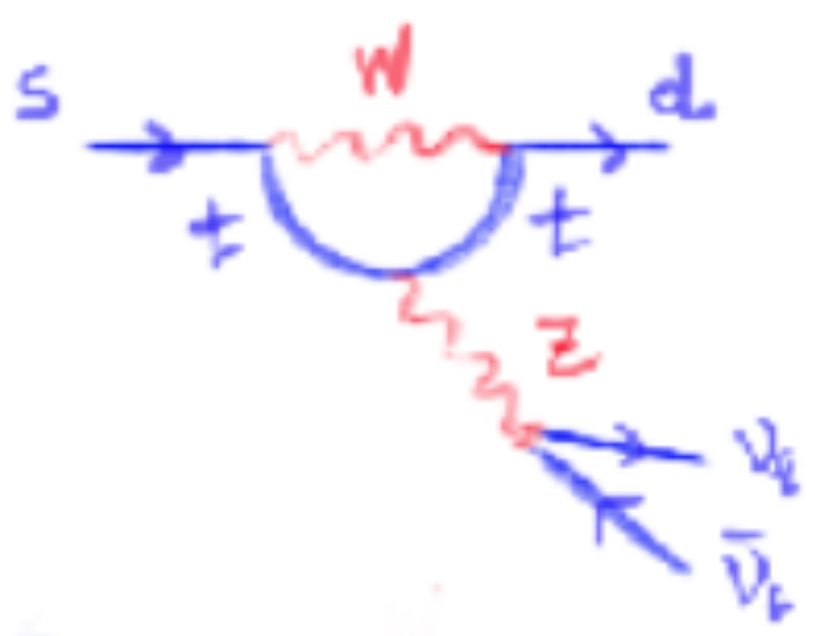
I. INTRODUCTION

The present proposal was largely stimulated by the recent anomalous results of Adair et al., on the coherent regeneration of K^o_1 mesons. It is the purpose of this experiment to check these results with a precision far transcending that attained in the previous experiment. Other results to be obtained will be a new and much better limit for the partial rate of $K^o_2 \to \pi^+ + \pi^-$, a new limit for the presence (or absence) of neutral currents as observed through $K_2 \to \mu^+ + \mu^-$.

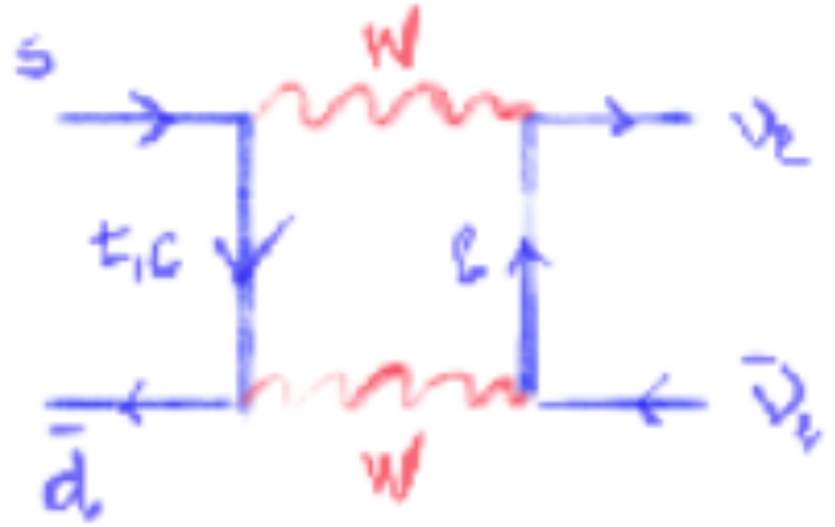
Hands - on workshop: $K^+ \to \pi^+ \nu \bar{\nu}$

- Prototypical very-rare kaon decay: $\Delta S = 1$ FCNC

Penguin diagram



Box diagram



- Effective Lagrangian

$$\mathscr{L}_{\rm SM} = -\frac{4G_F}{\sqrt{2}} V_{ts} V_{td}^* \frac{\alpha}{2\pi} \sum_\ell C_{\nu_\ell} (\bar{d}\gamma^\mu P_L d)(\bar{\nu}_\ell \gamma_\mu \nu_\ell)$$

Wilson Coefficient: $C_{\nu_\ell} = \frac{1}{s_w^2}\left(\frac{V_{cs}V_{cd}^*}{V_{ts}V_{td}^*} X_c^\ell + X_t\right)$

$$C_{\nu_\ell}^{\rm SM} \simeq 9$$

- Relevant form factors related by isospin to CCs

$$\mathrm{Br}(K^+ \to \pi^+ \nu \bar{\nu})^{\rm SM} = \frac{\alpha^2 |V_{ts}V_{td}^*|^2 \mathrm{Br}(\mathrm{K}^+ \to \pi^0 \mathrm{e}^+ \nu_{\rm e})}{2\pi^2 |V_{us}|^2} \sum_\ell \left|C_{\nu_\ell}\right|^2$$

$$\simeq \frac{\alpha^2}{2\pi^2} \mathrm{A}^4 \lambda^8\, \mathrm{Br}(\mathrm{K}^+ \to \pi^0 \mathrm{e}^+ \nu_{\rm e}) \sum_\ell \left|C_{\nu_\ell}\right|^2$$

$$\mathrm{Br}(K^+ \to \pi^+ \nu \bar{\nu})_{\rm SM} = 8.55(4) \times 10^{-11}$$

$$\mathrm{Br}(K^+ \to \pi^+ \nu \bar{\nu})_{\rm expt} = 1.14(36) \times 10^{-10}$$

Hands - on workshop: $K^+ \to \pi^+\nu\bar{\nu}$

- Let's take a Z' boson of mass $m_{Z'}$ that is coupled to the SM via

$$\mathscr{L} \supset \left(g^Q_{ij}\bar{Q}^i_L\gamma^\mu Q^j_L + g^L\bar{L}^\alpha_L\gamma^\mu L^\alpha_L \right) Z'_\mu$$

g^Q is a matrix in general real matrix in flavor space and g_L a universal coupling for leptons.

Exercise

1. Calculate BR in the SM using (approximate) formula
2. Match the UV model to the LEEFT
3. *Estimate* the lower bound on $m_{Z'}$ given by $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})_{\mathrm{expt}}$ assuming $\mathcal{O}(1)$ couplings
4. How does this bound change if we impose MFV?