Quasinormal modes and their excitation beyond general relativity

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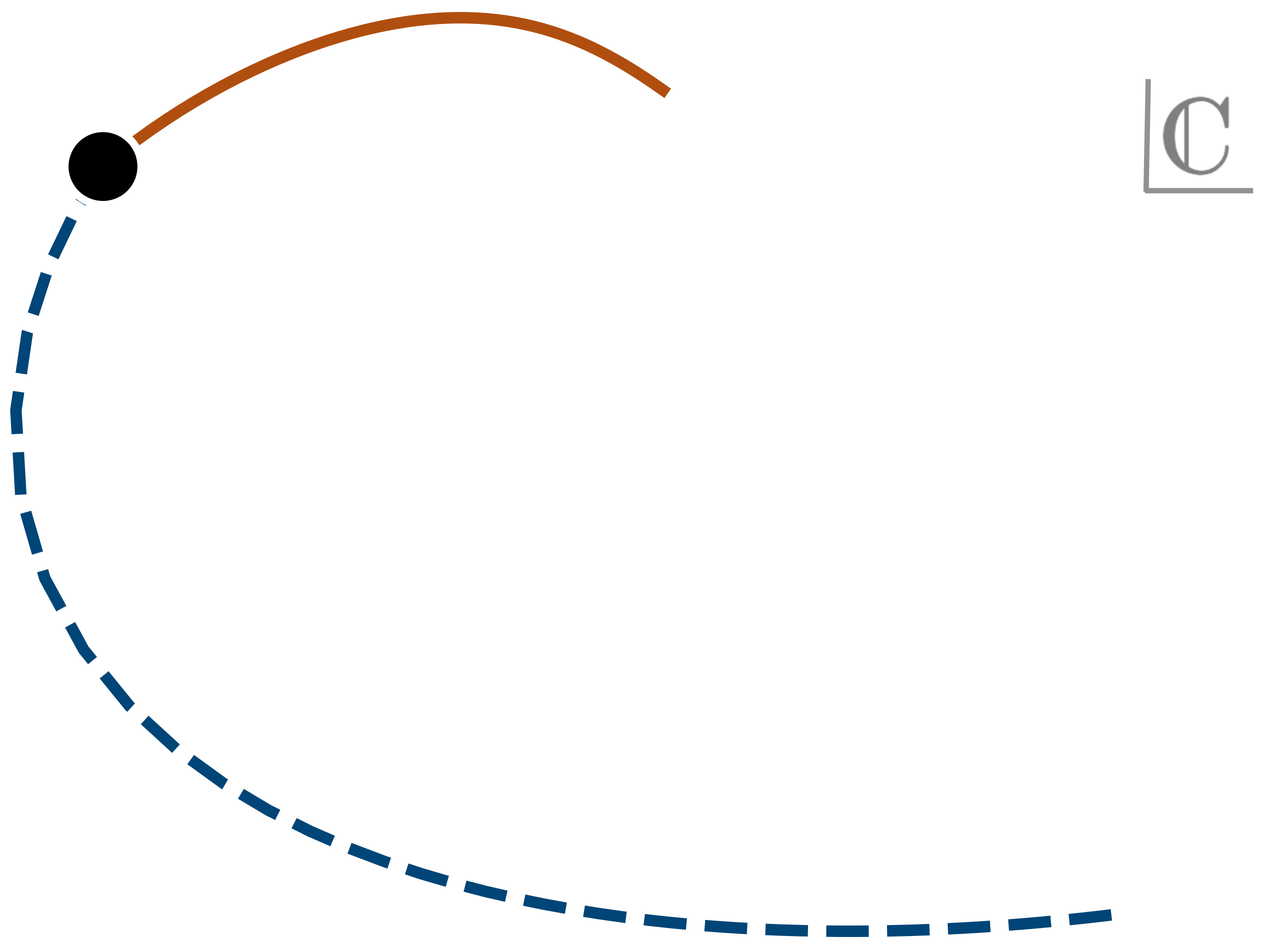

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General relativity and experiment

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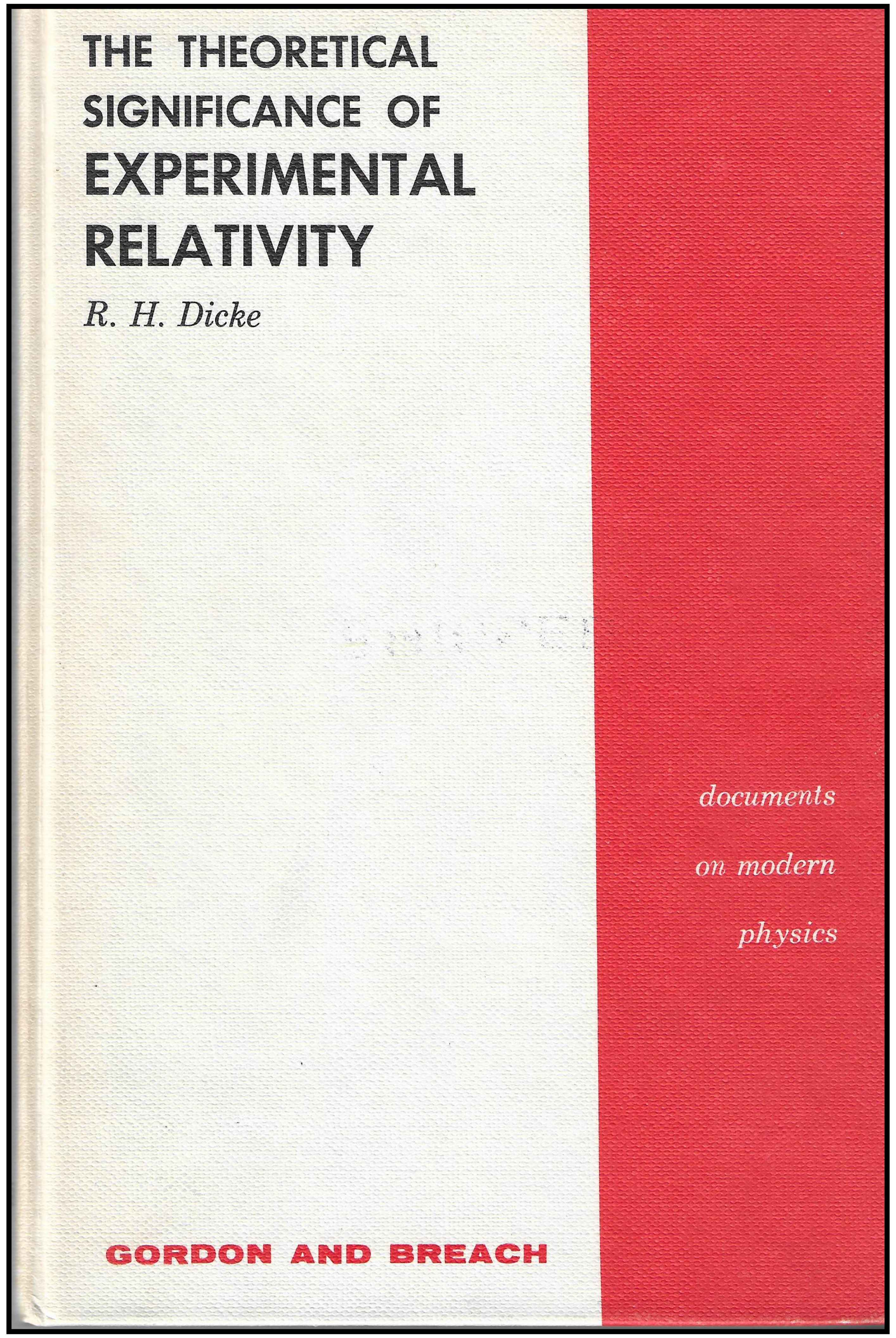

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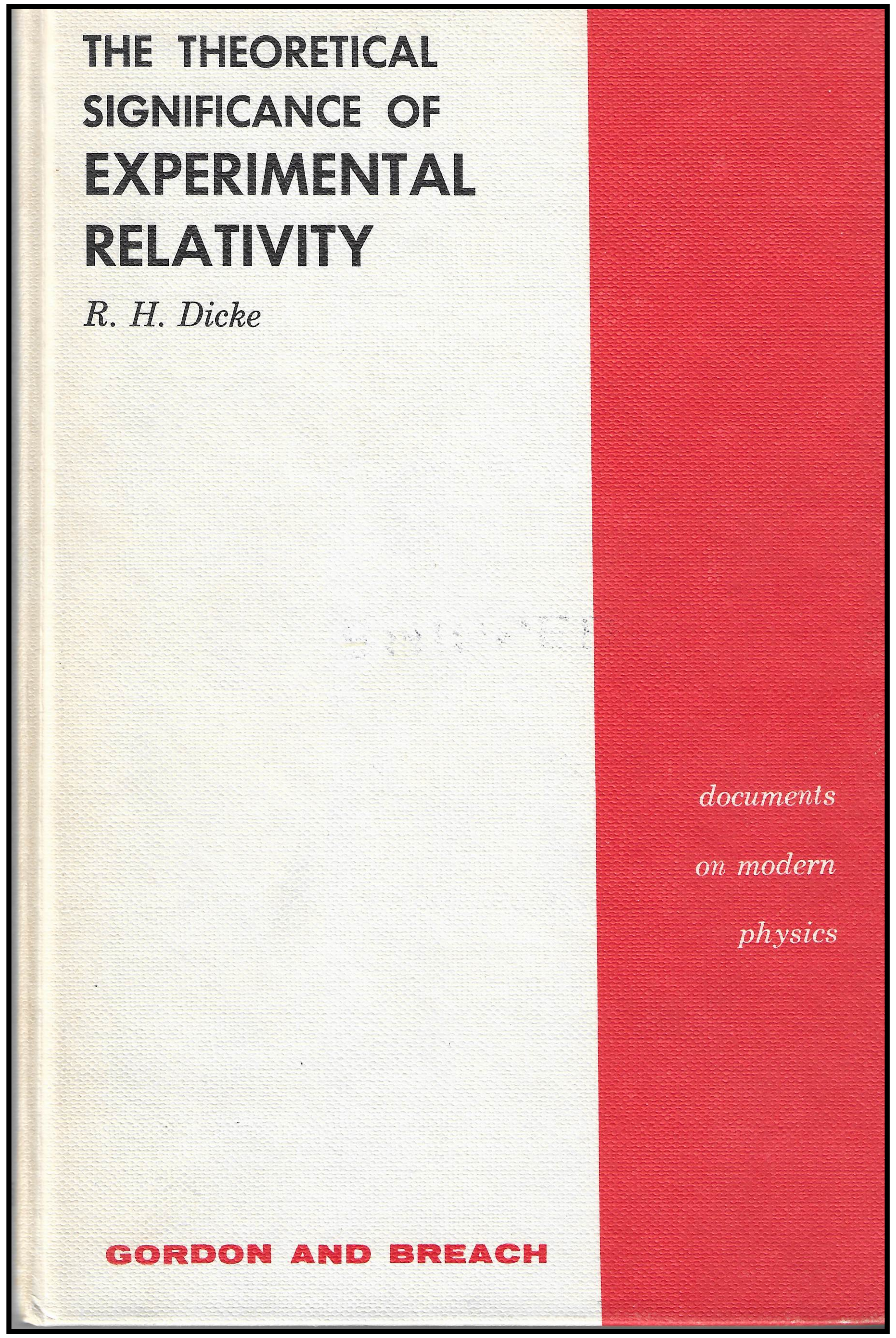

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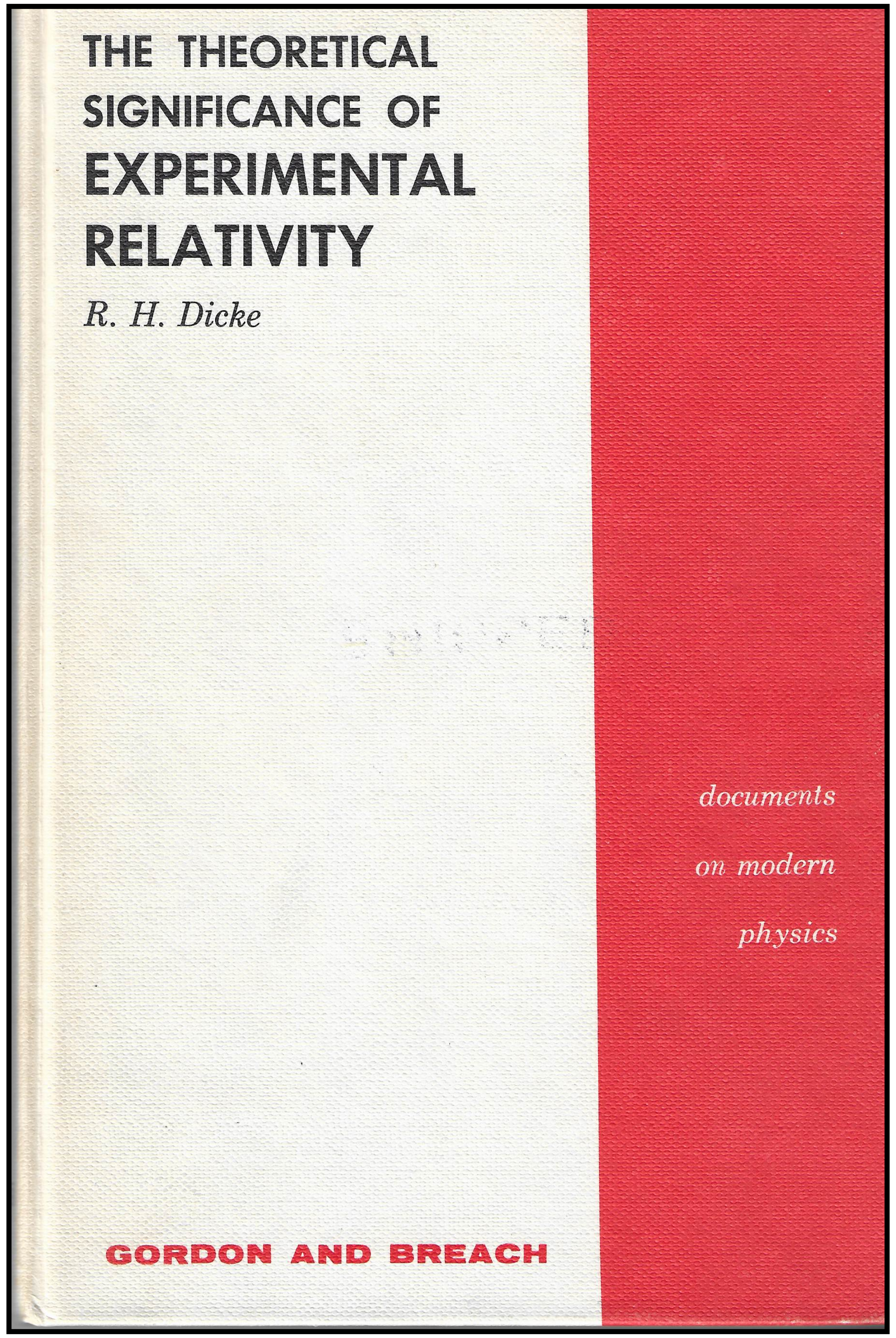

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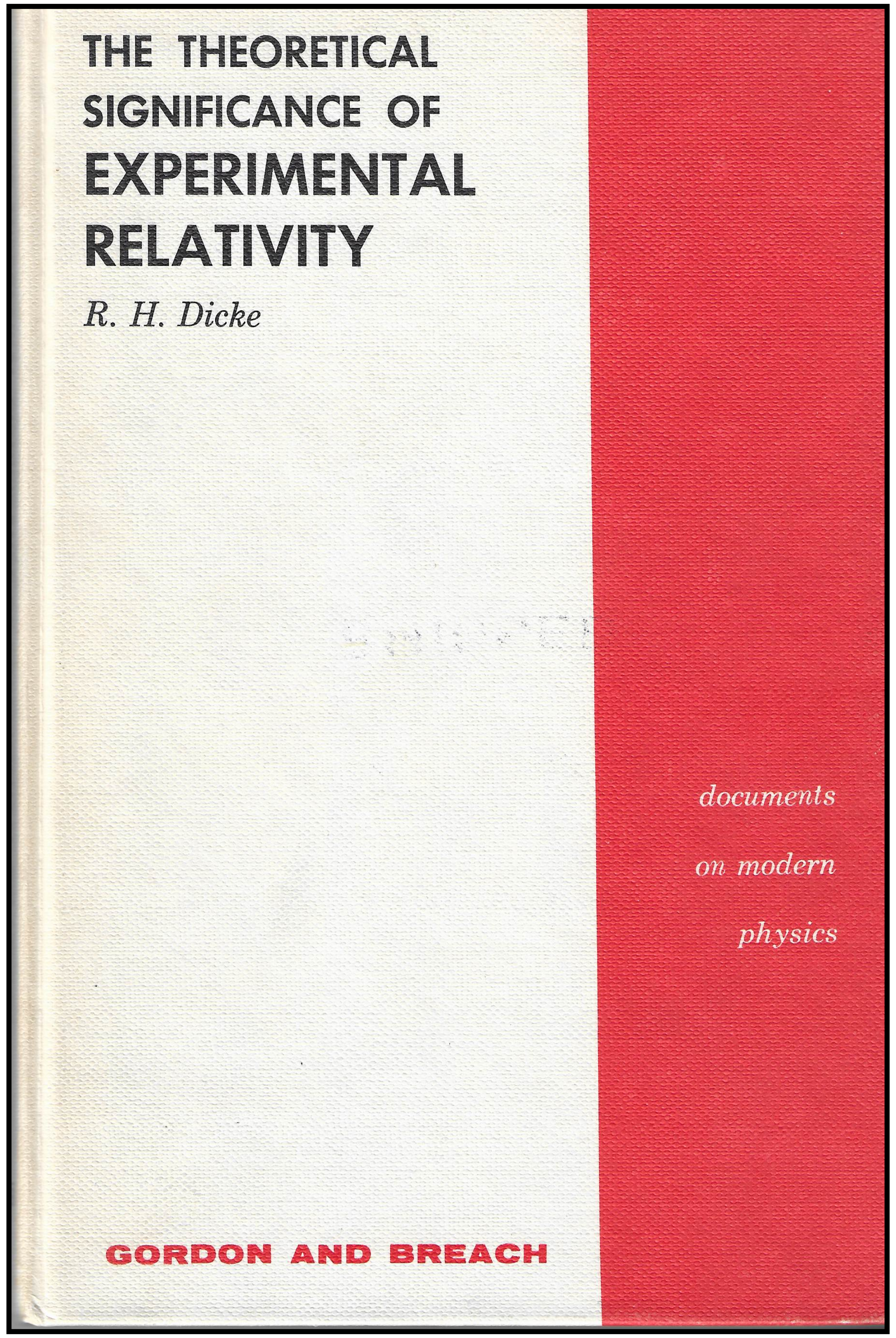

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- "**Experiment and observation can become intertwined in this field of physics** [gravitation], **and will probably become more so in the future** […]. While his chances of finding exactly the experimental setup that he would have liked is rather small, there may be **many** of these 'experiments' that have some bearing on his **particular** problem."

Coalescing binary black holes

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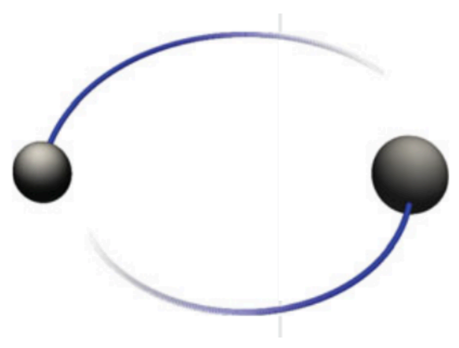

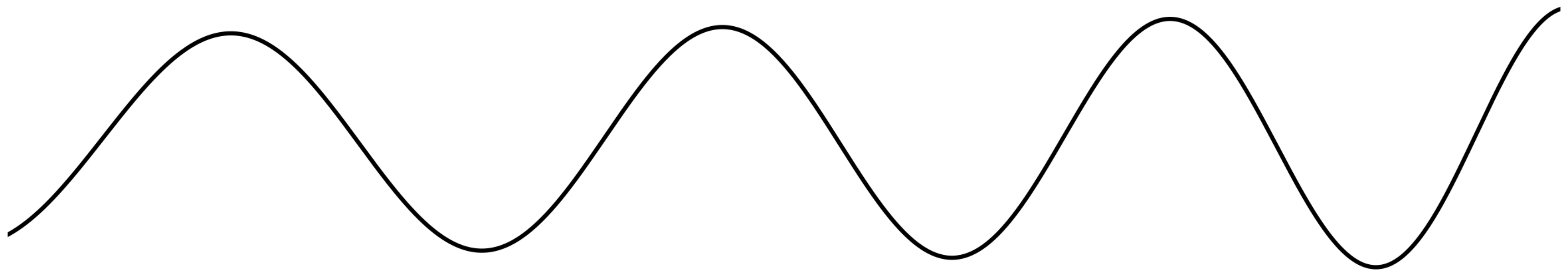

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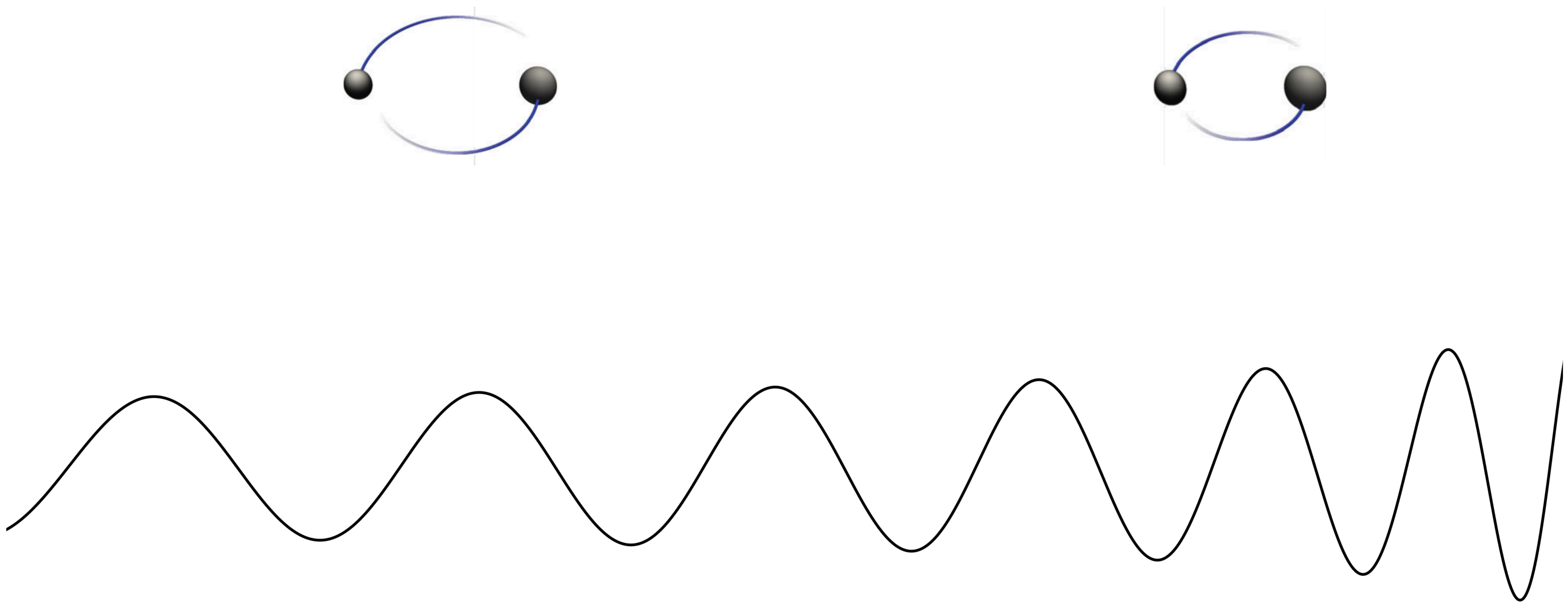
Inspiral

Plunge



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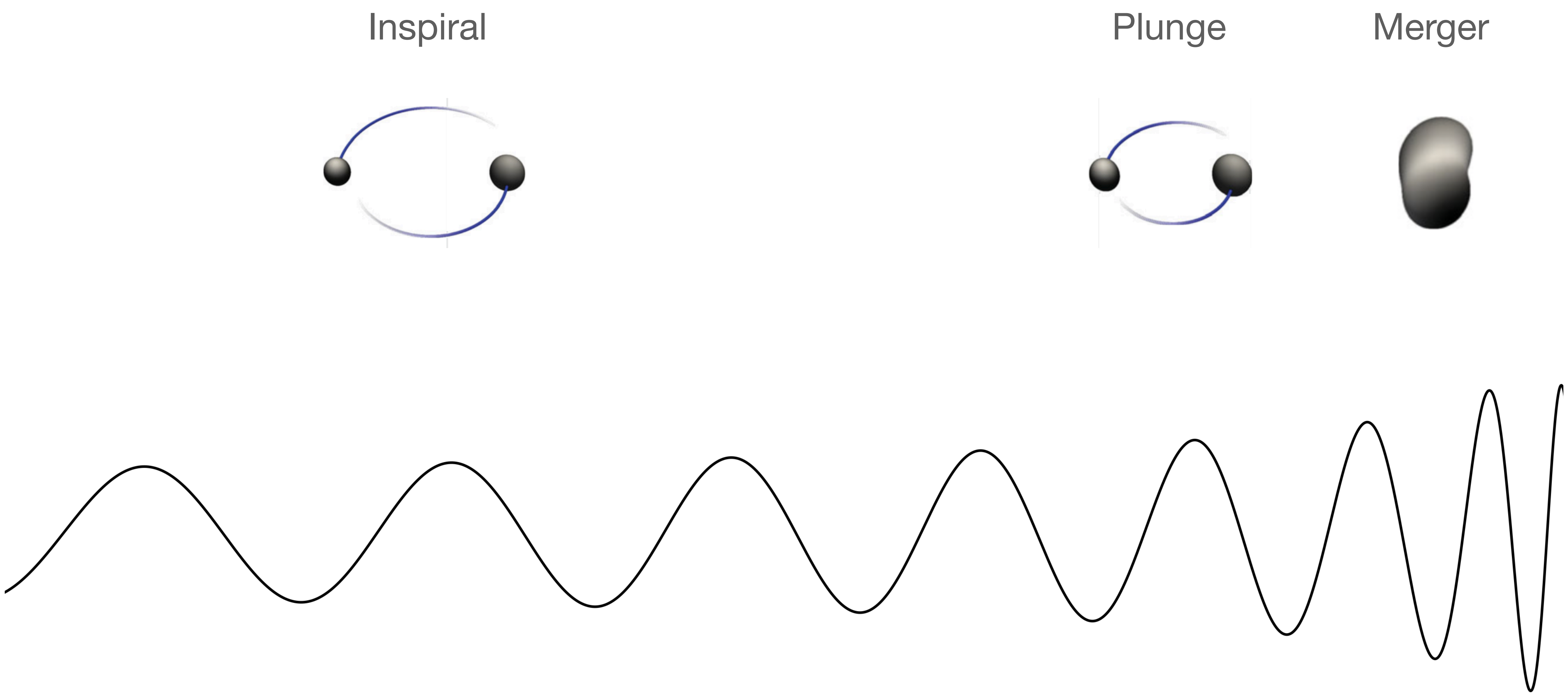

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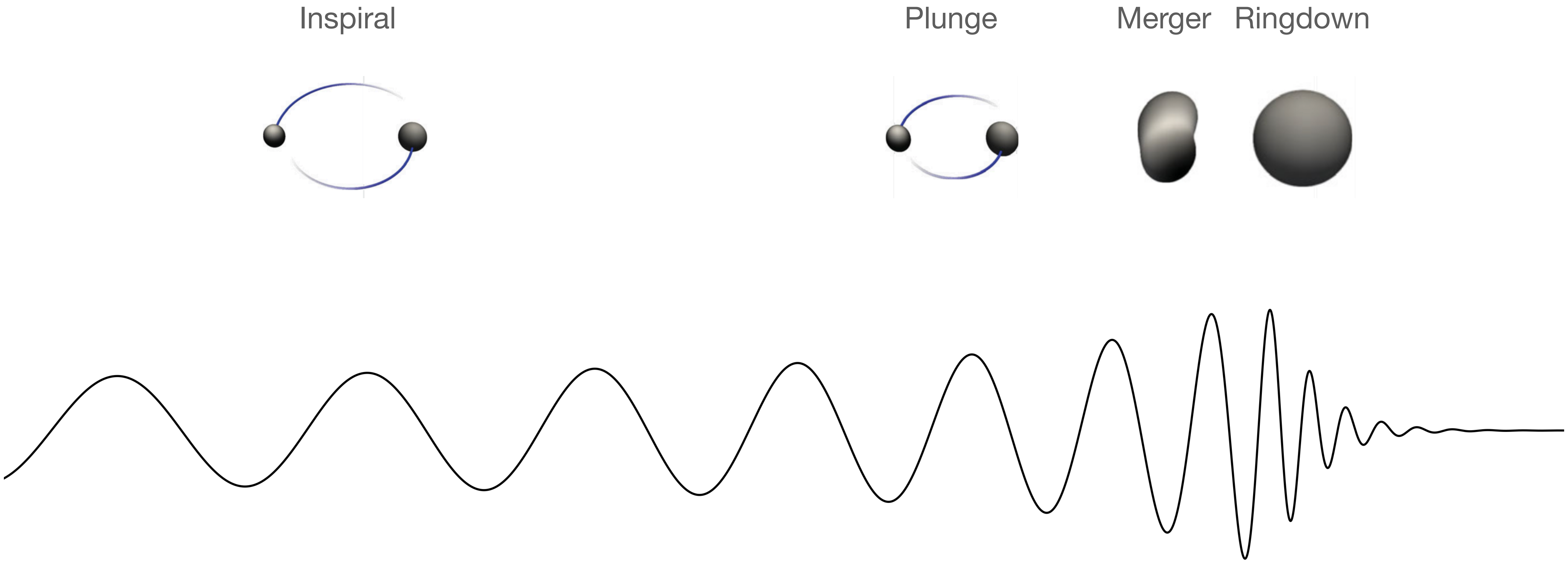

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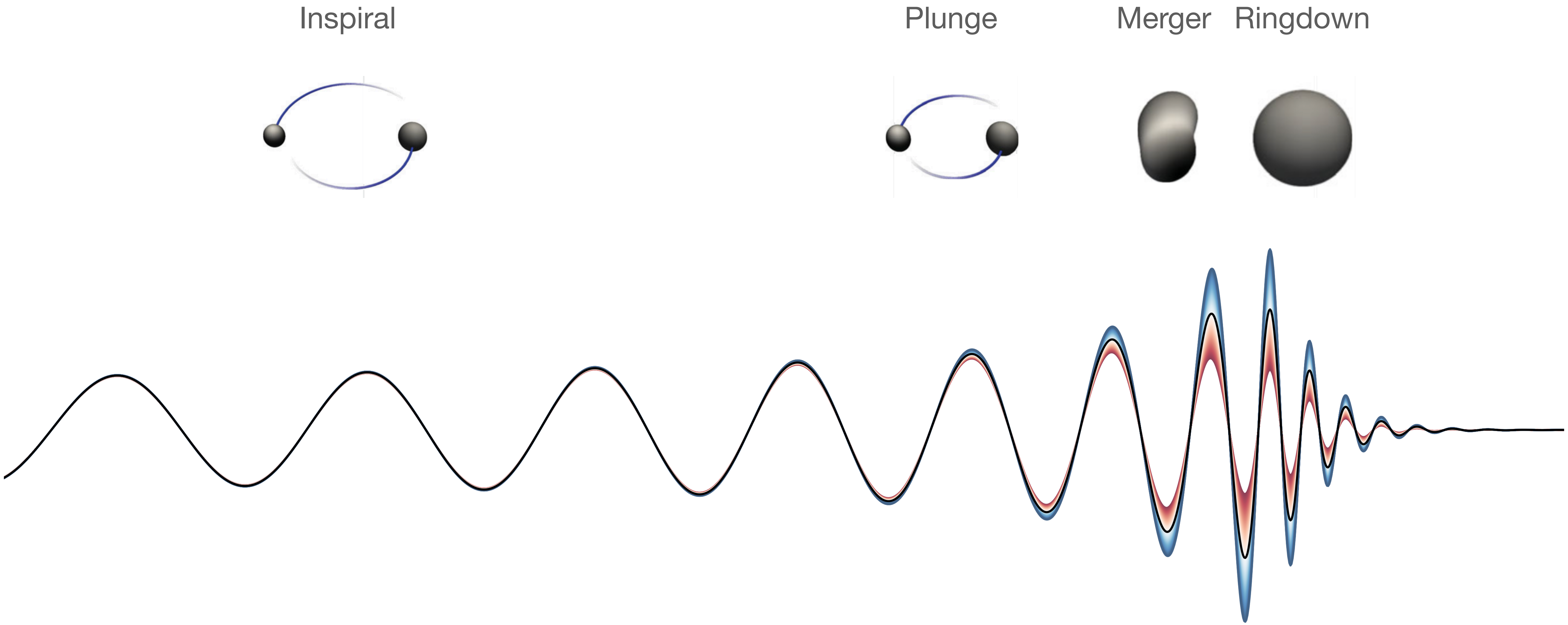

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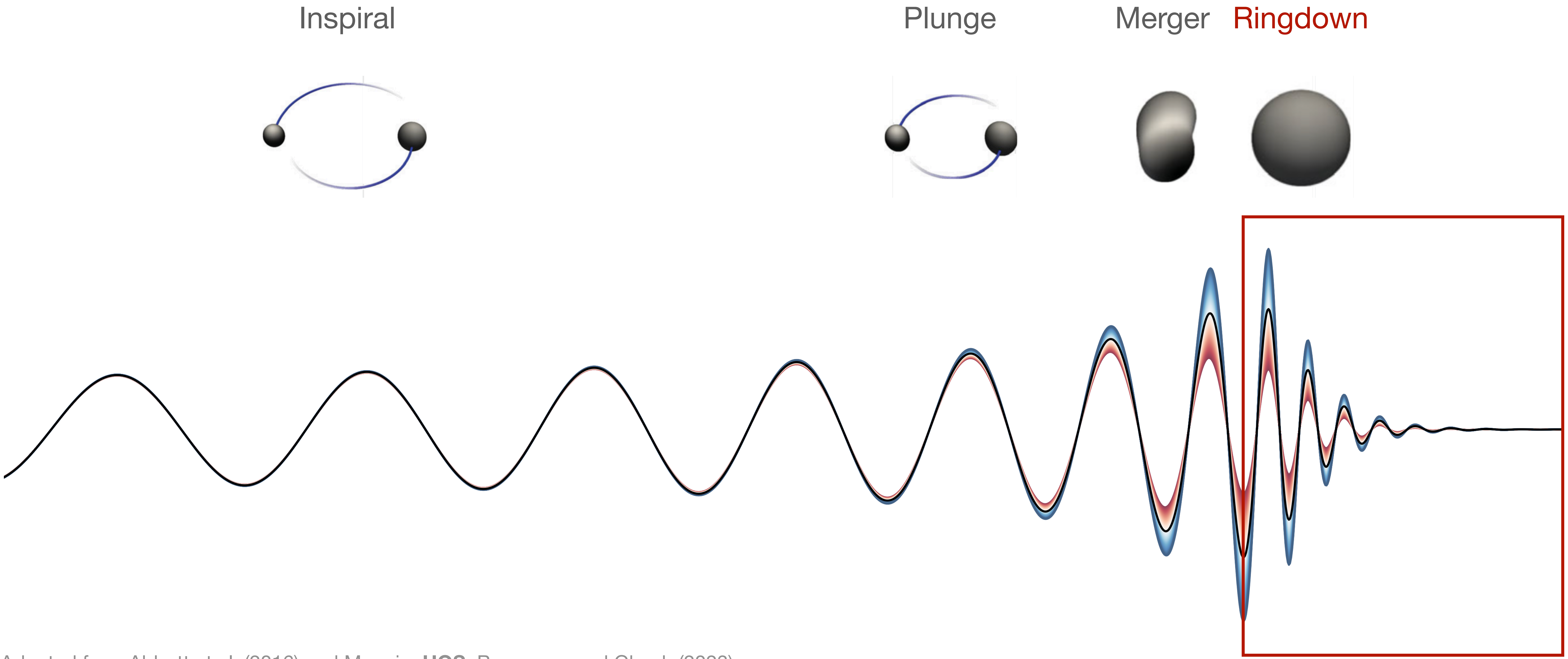

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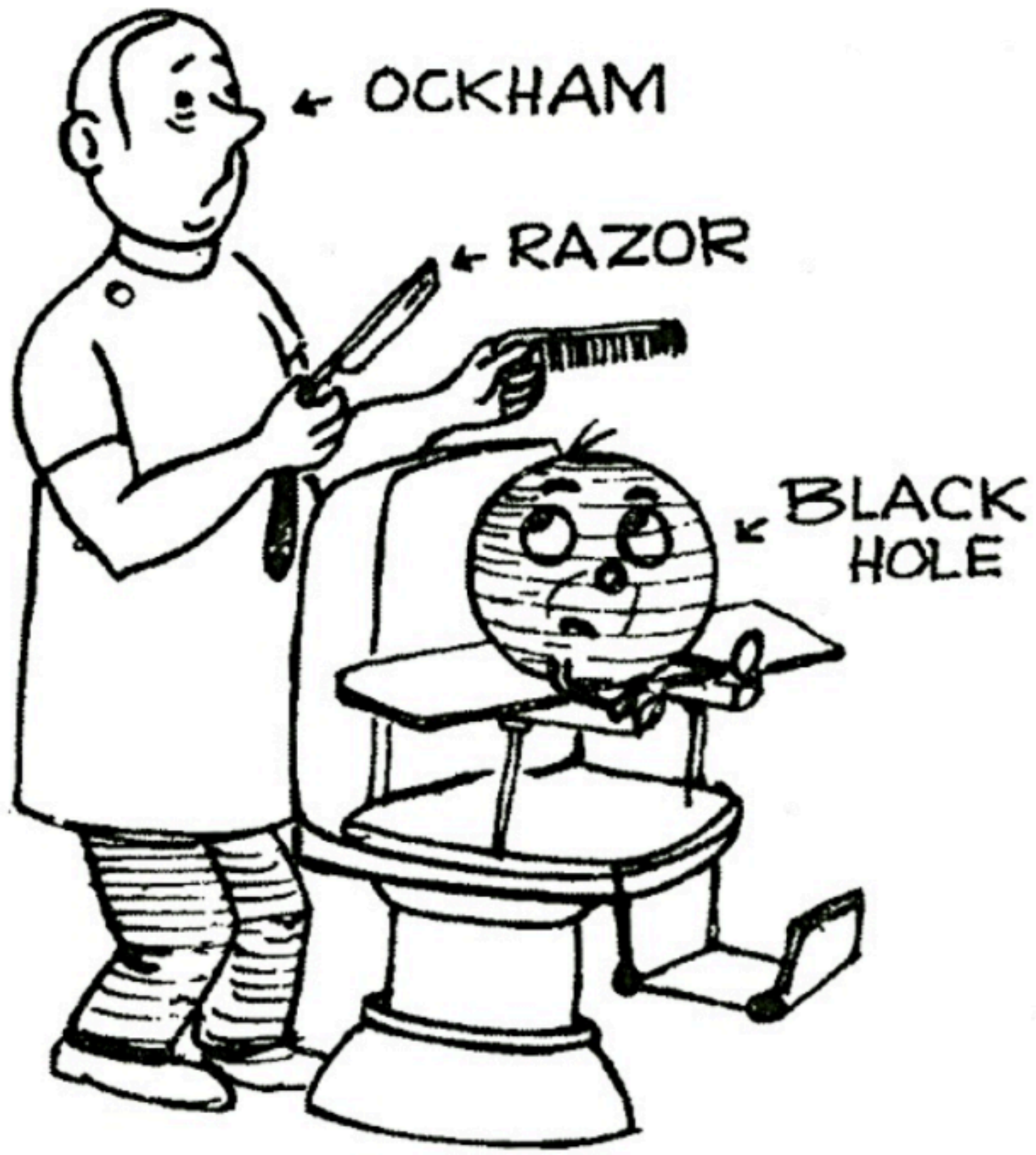

Cartoon by C. V. Vishveshwara

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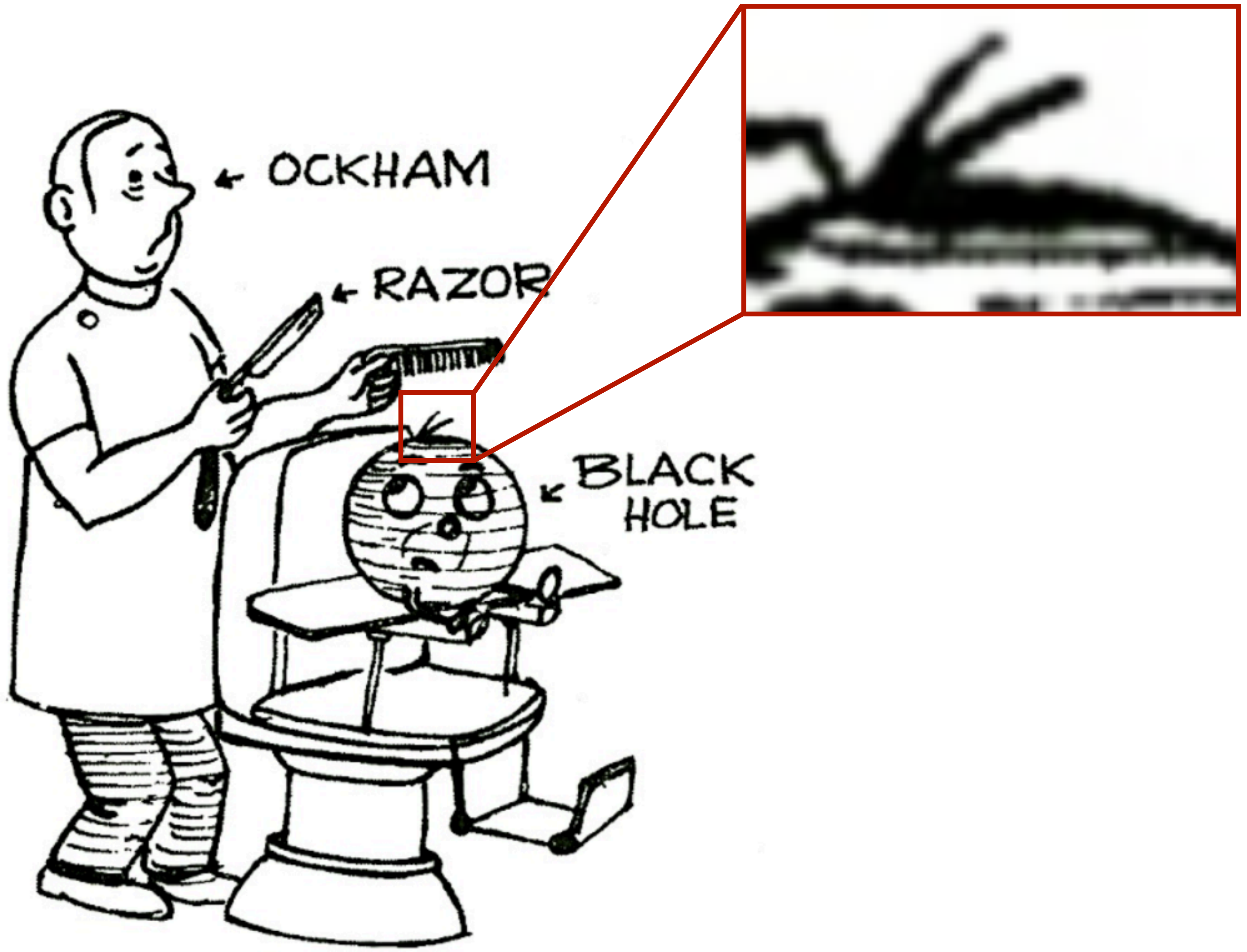

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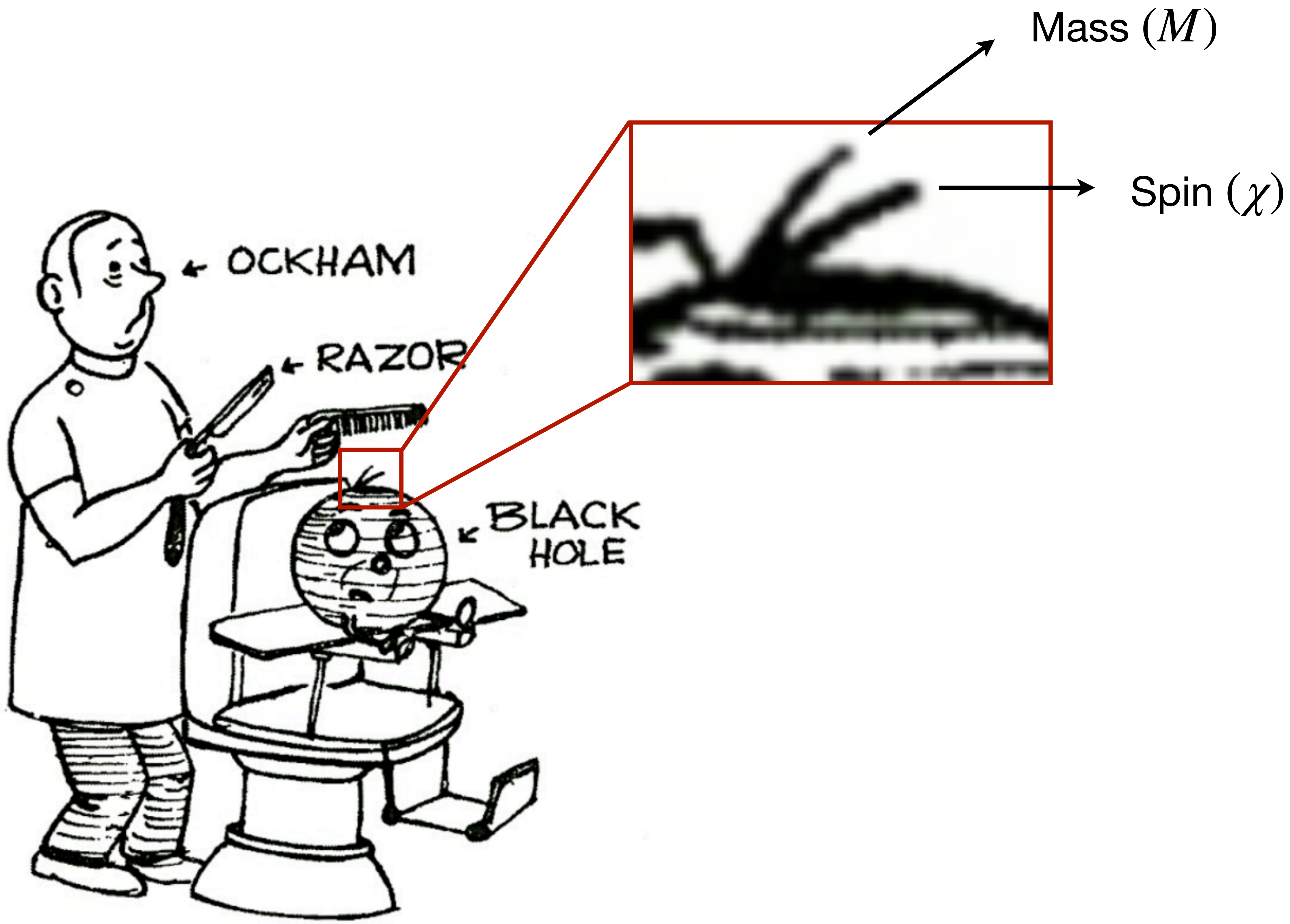

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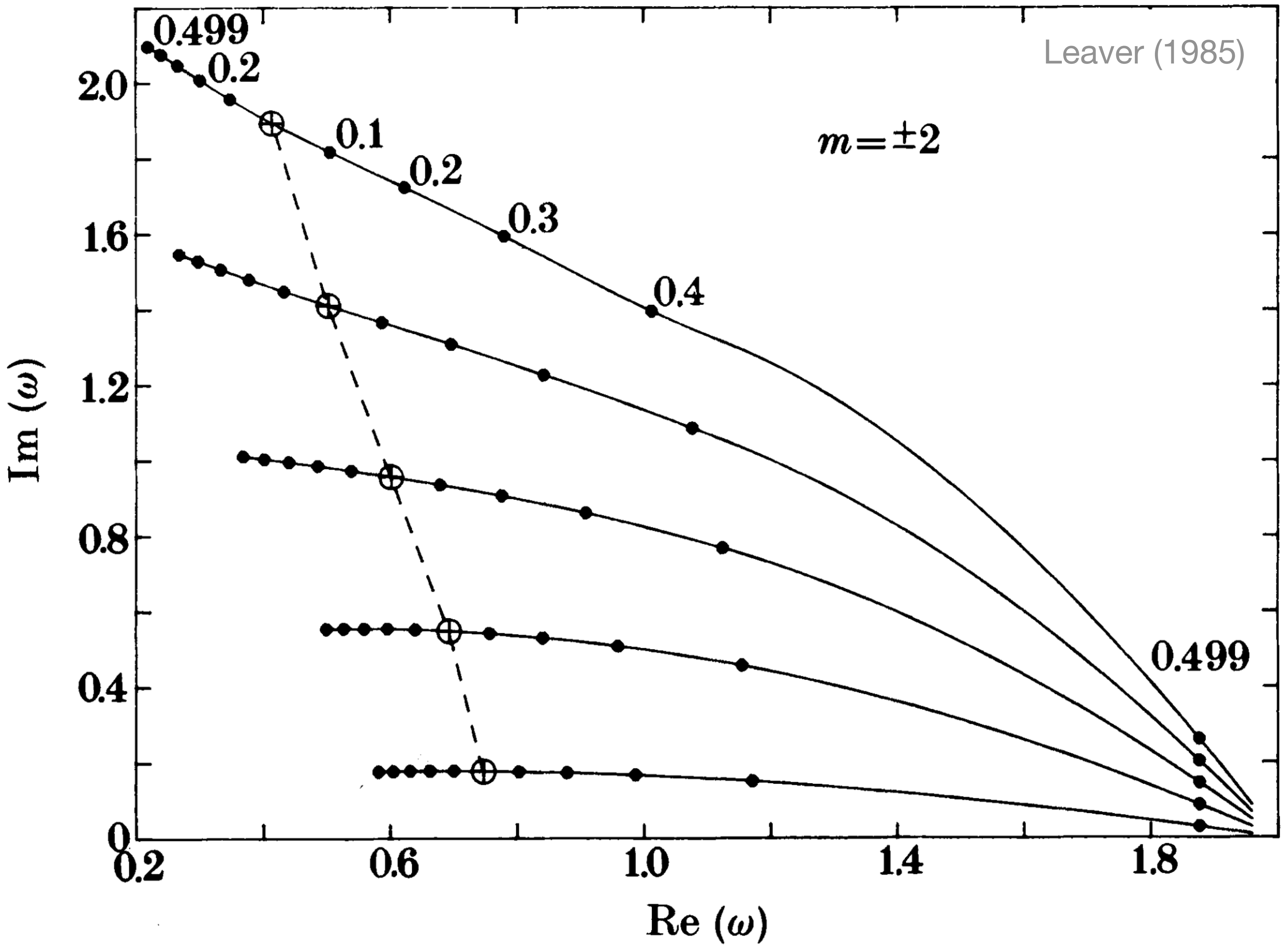

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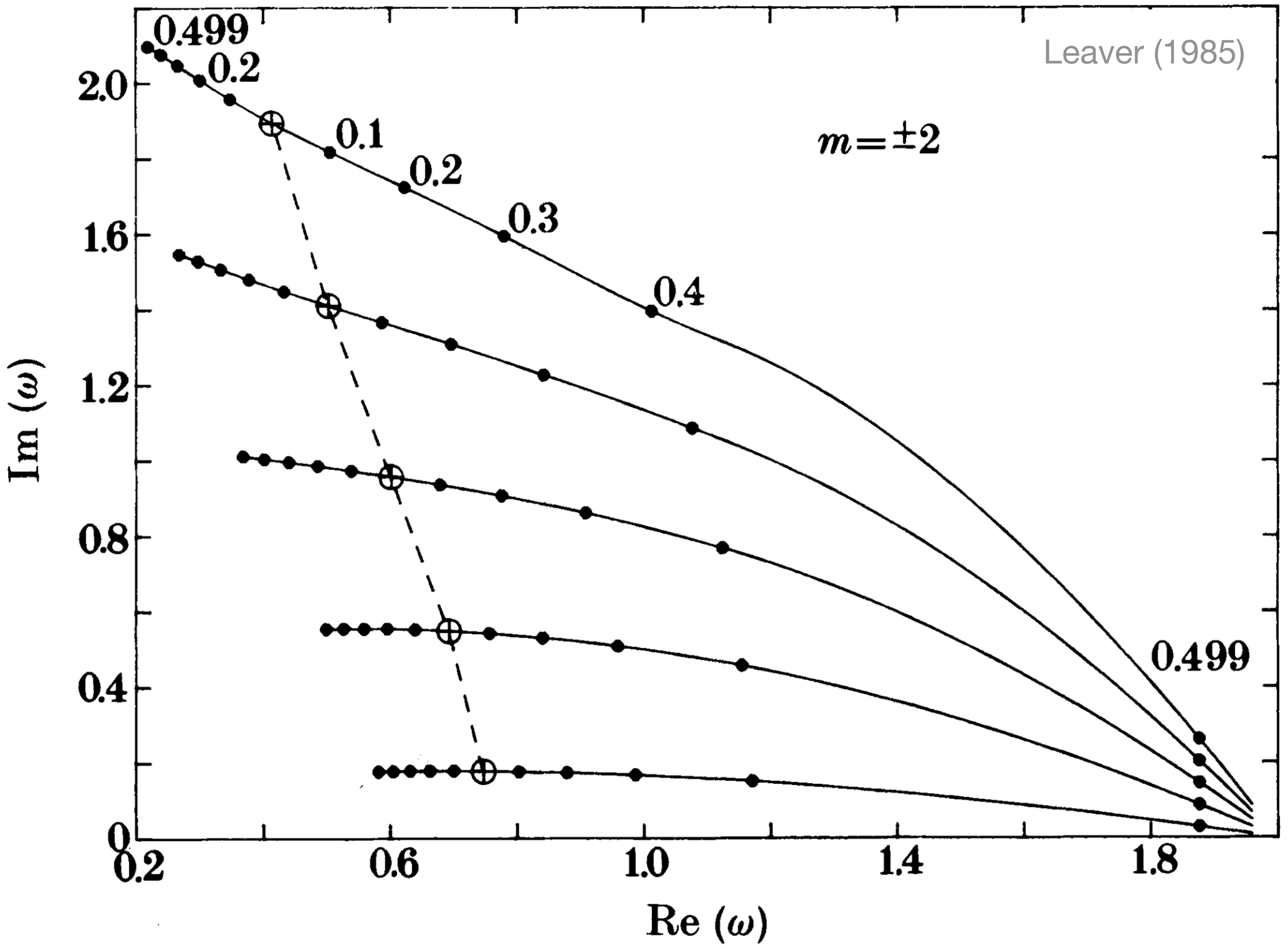

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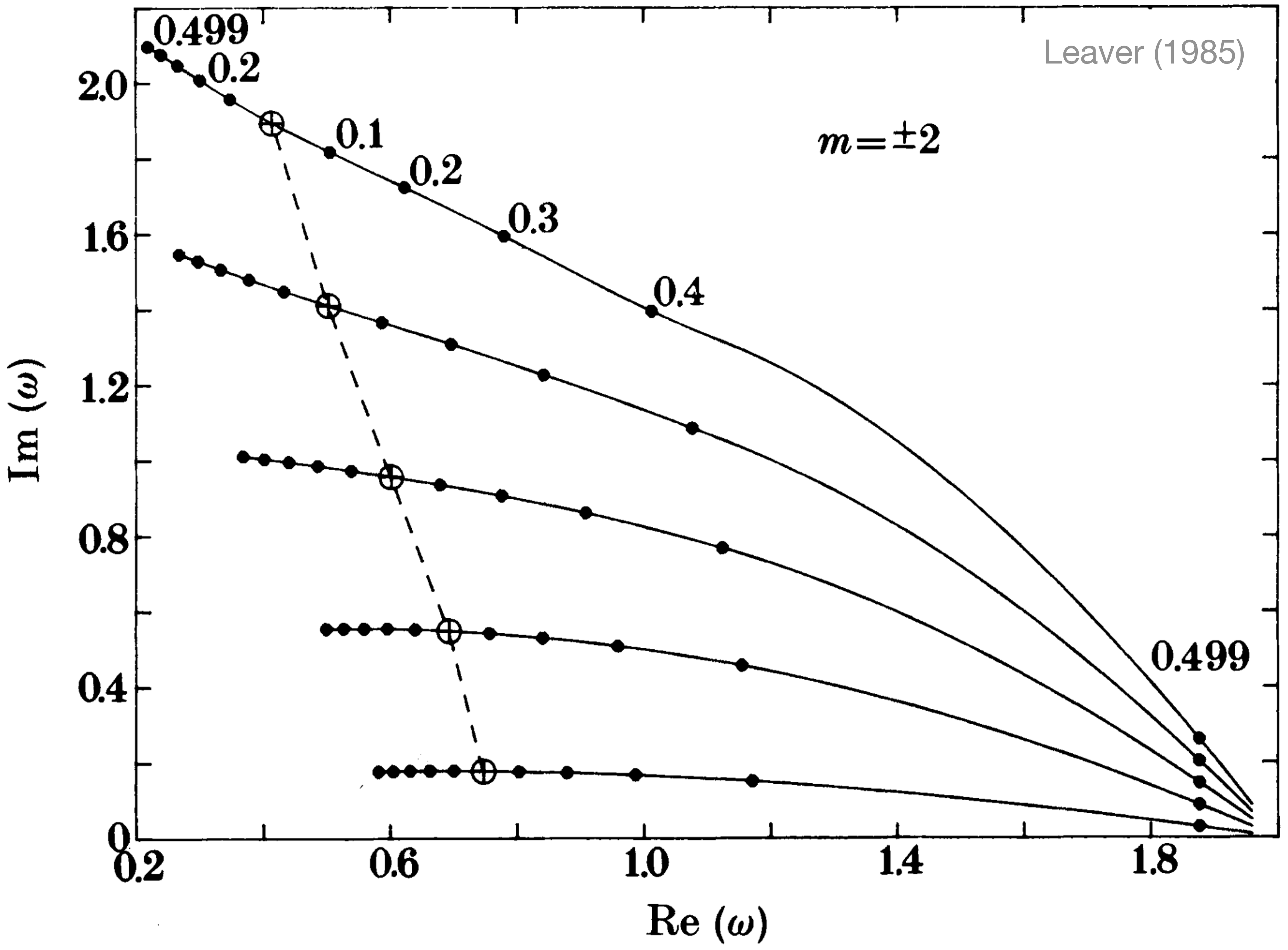

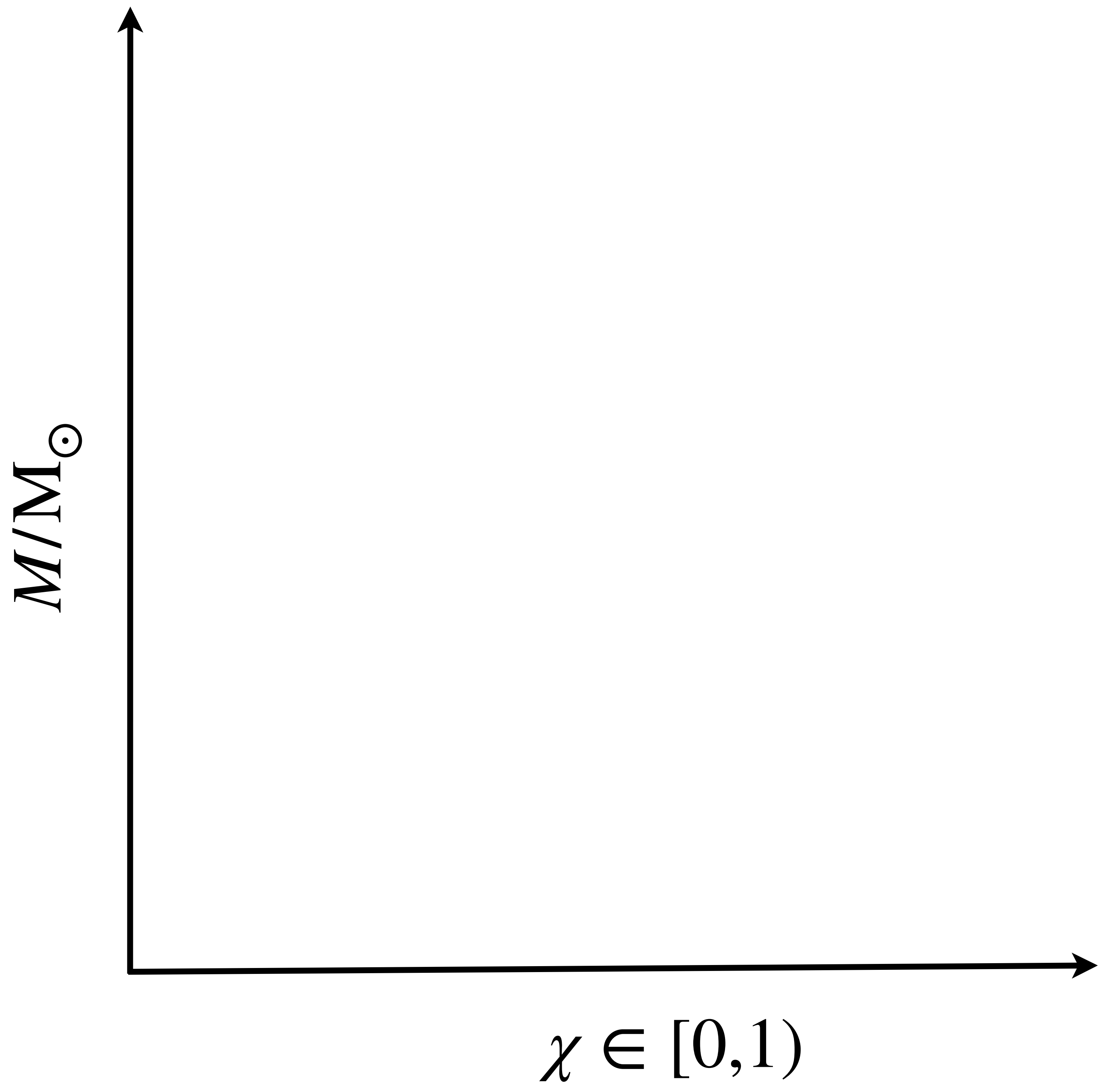

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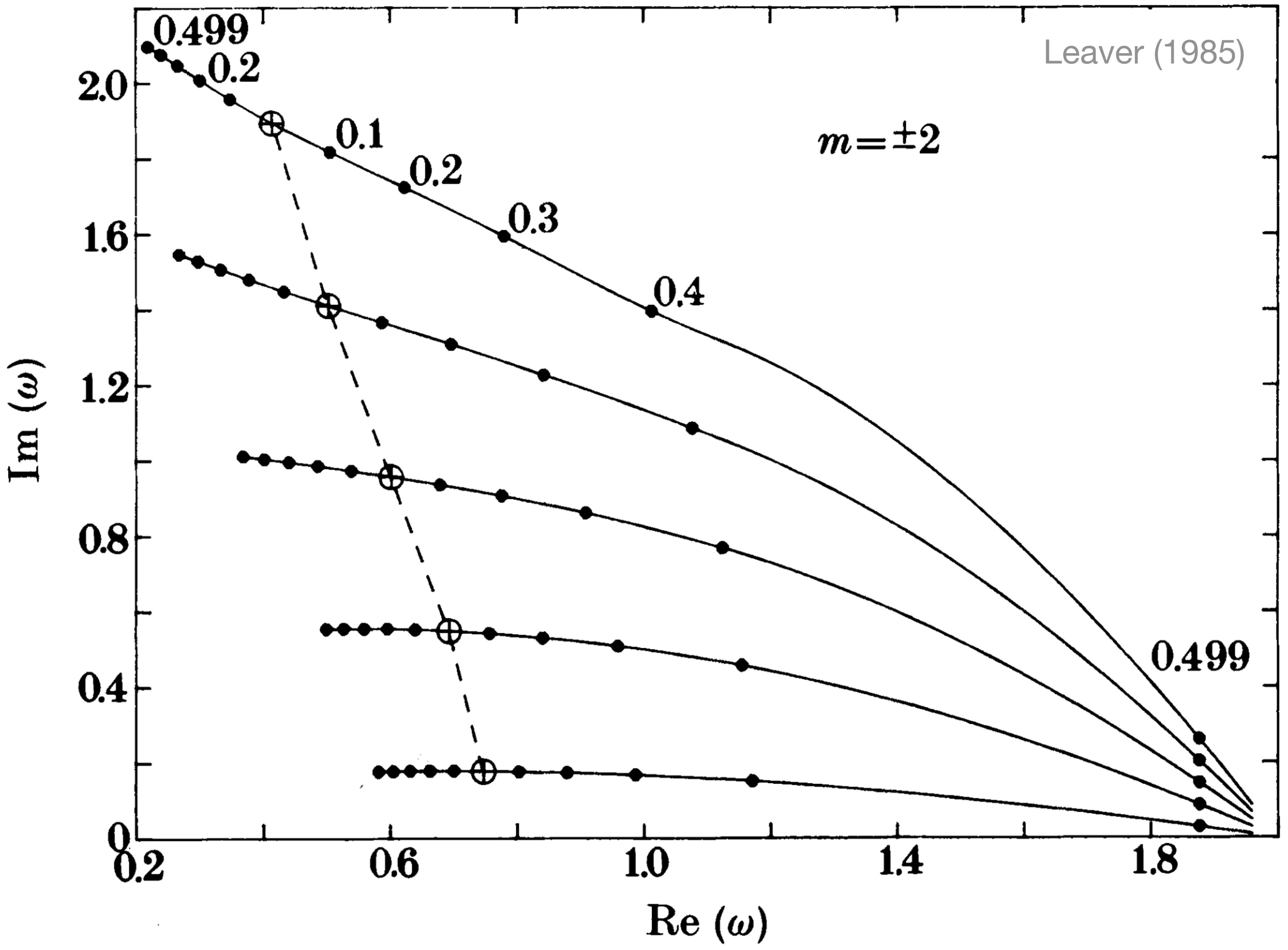

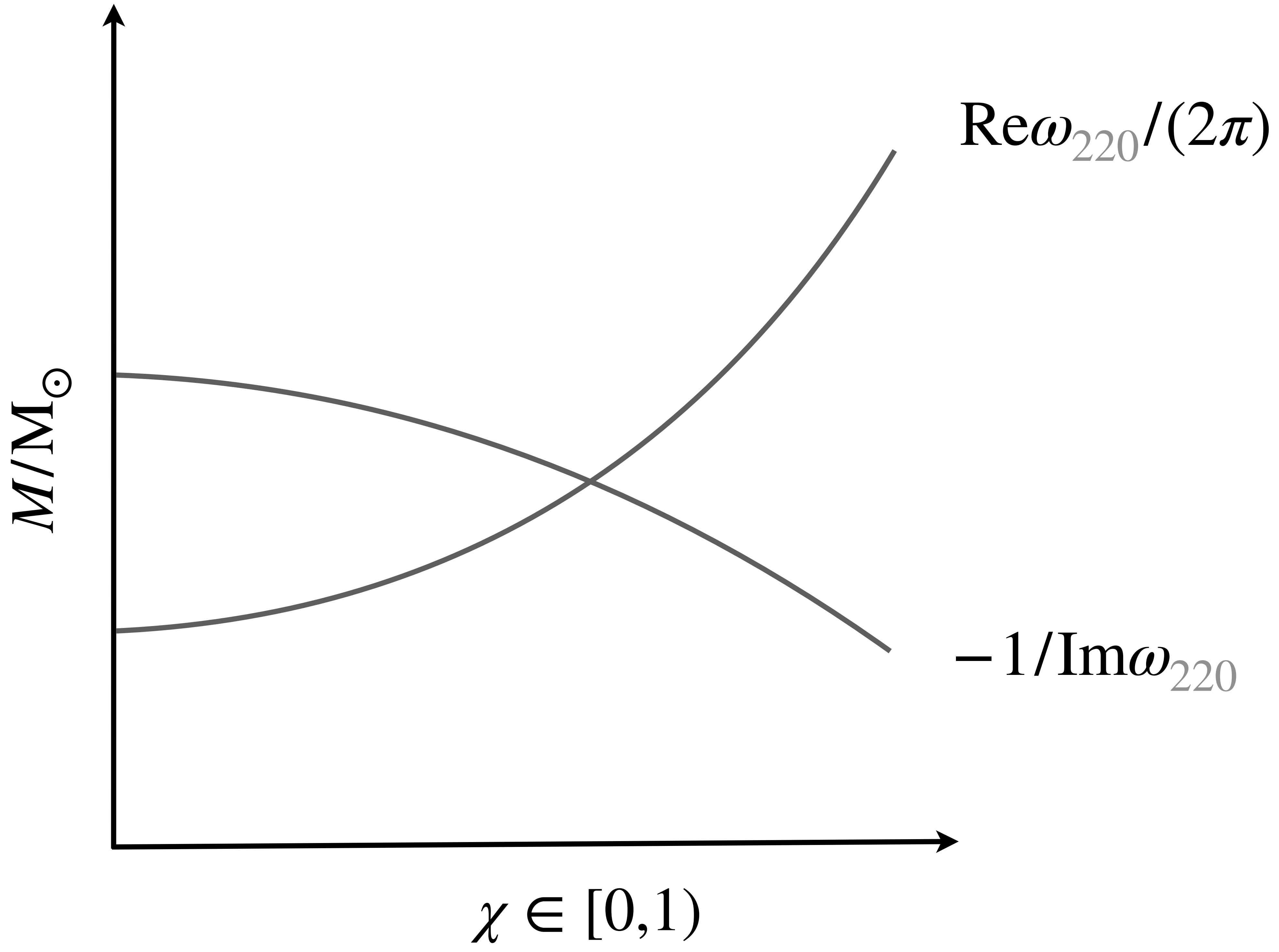

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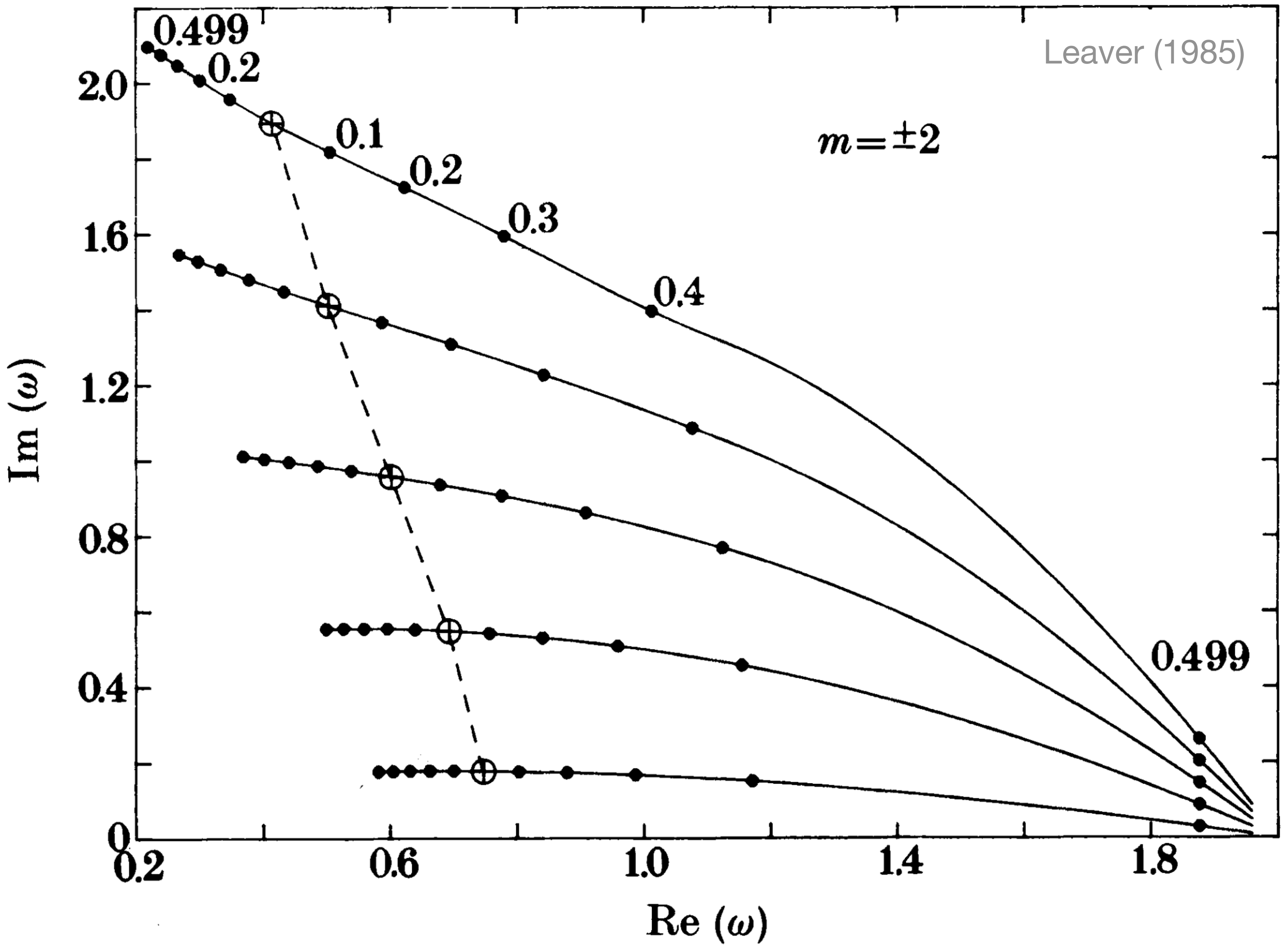

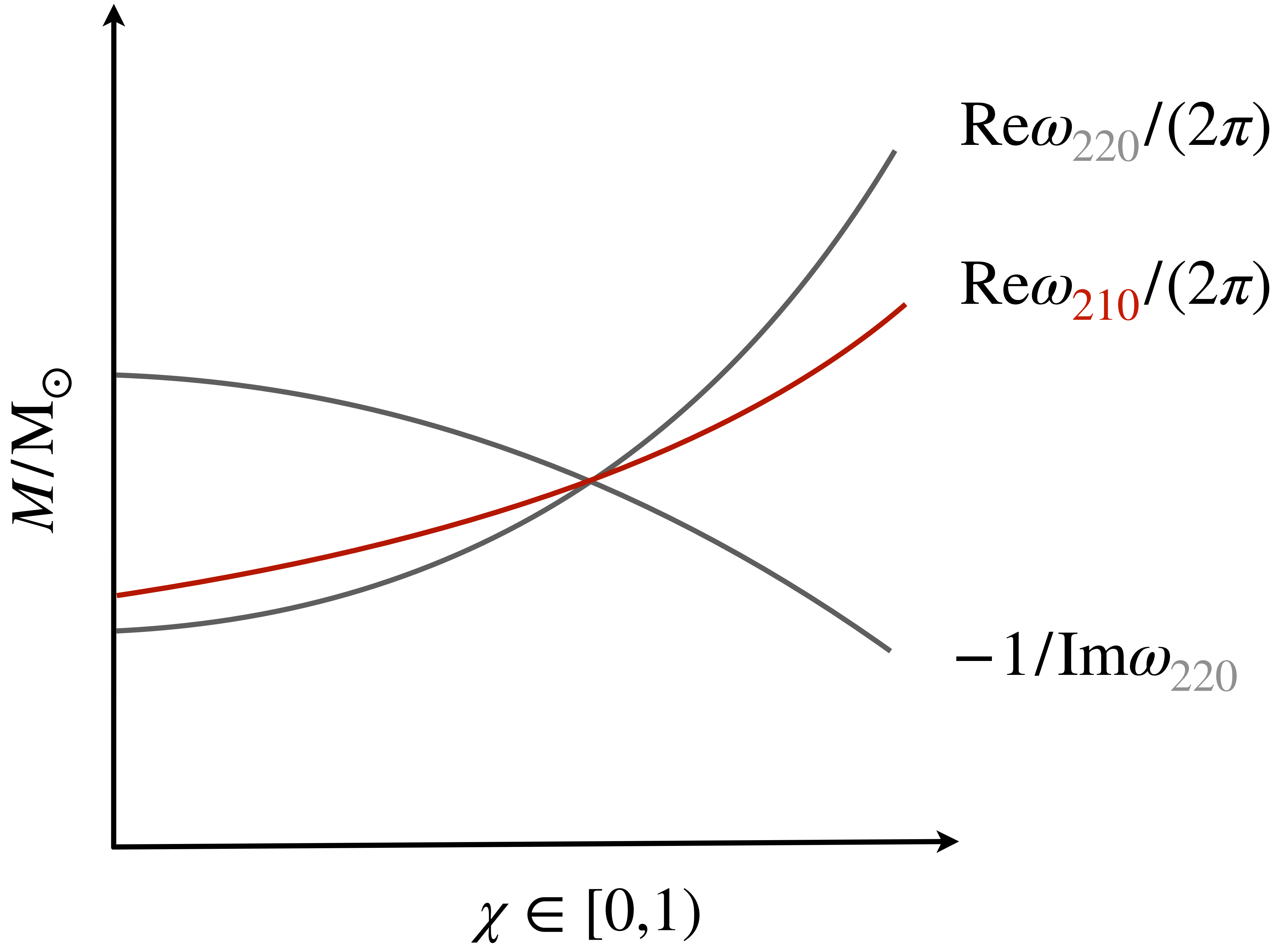

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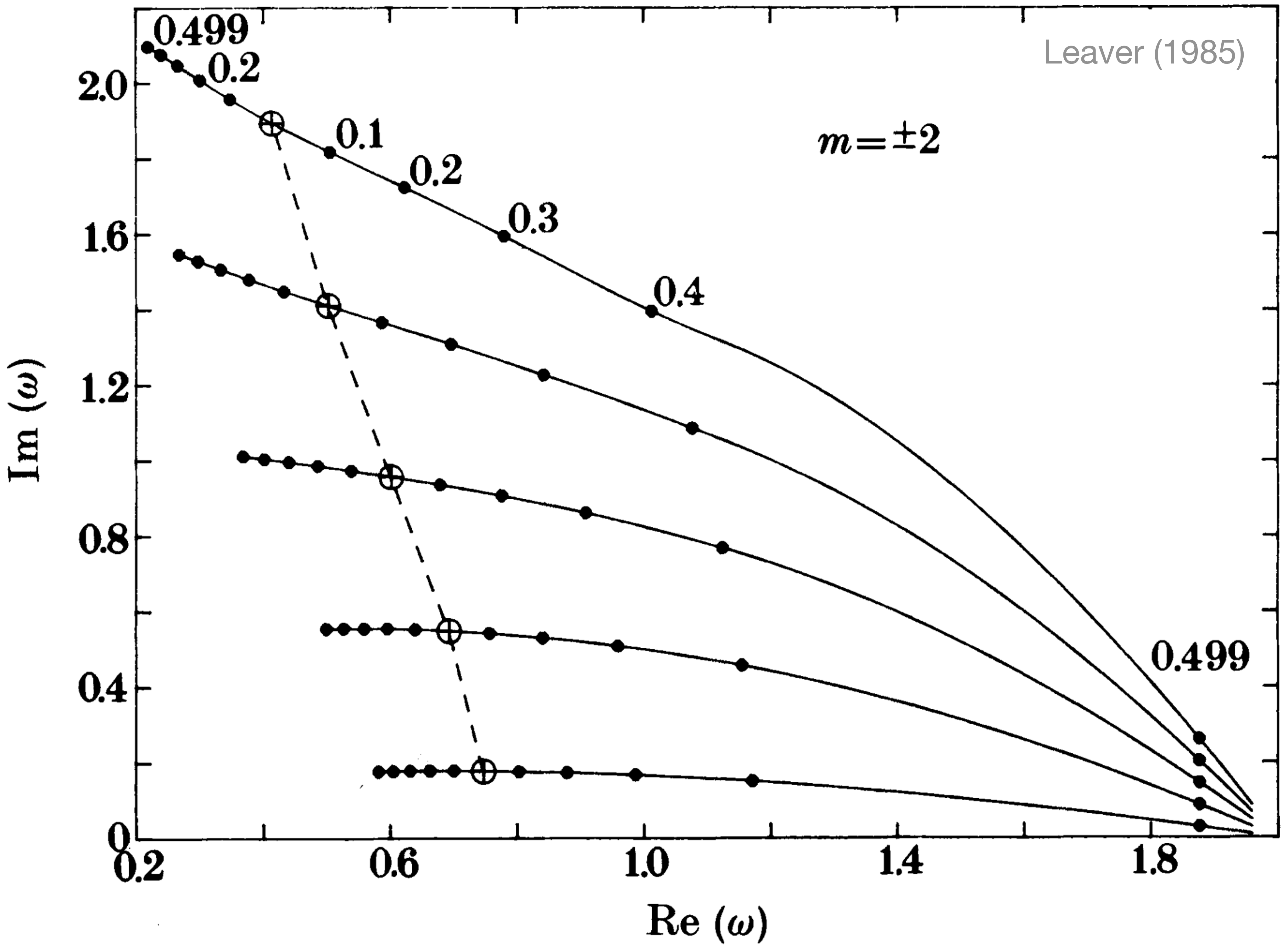

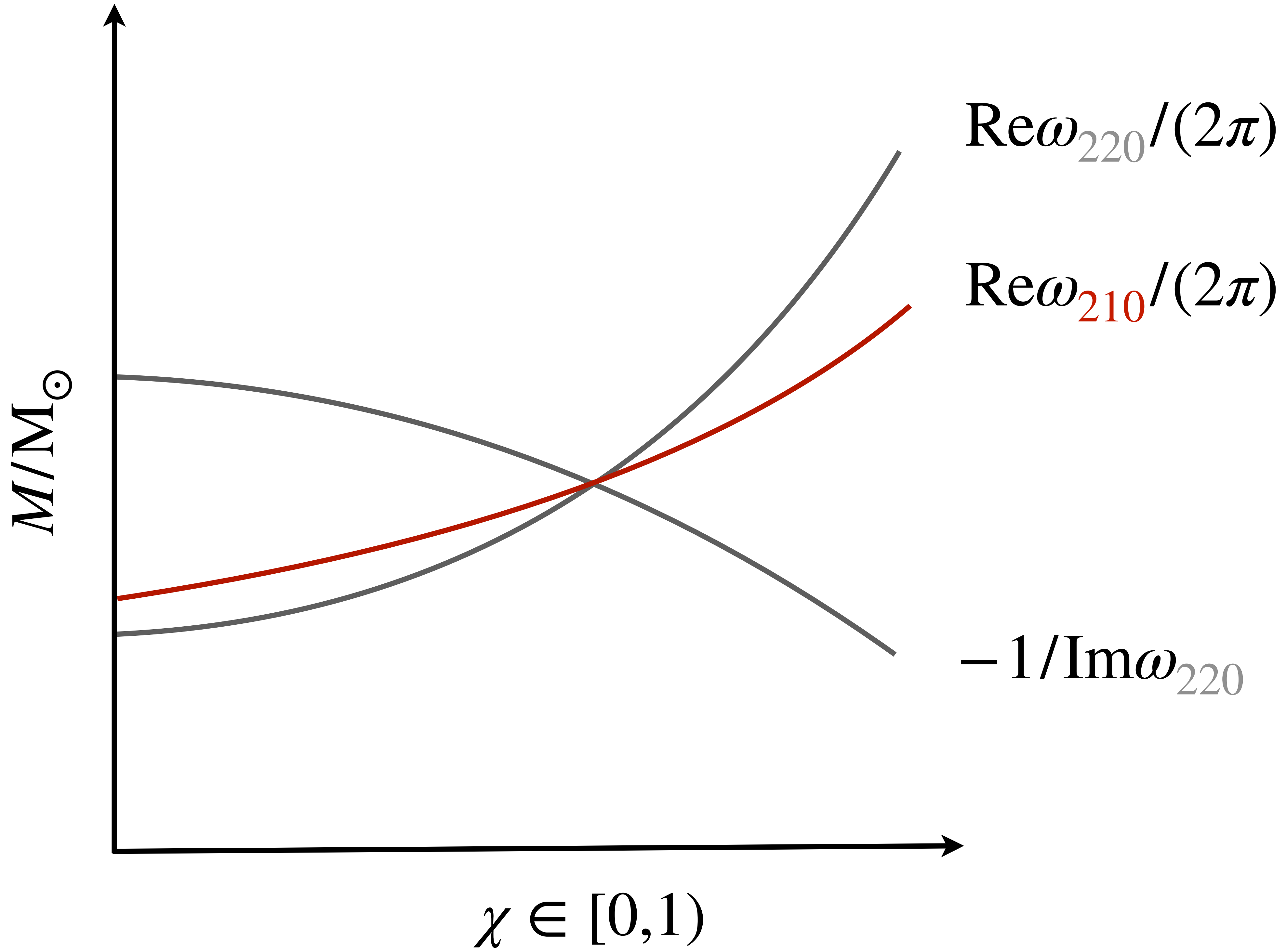

"[…] direct evidence of black holes with the same certainty as, say, the 21 cm line identifies interstellar hydrogen."
Detweiler (1980)

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Today: partial answer to some of these questions based on 2404.11110 (to appear in PRD), with G. Tambalo, K. Glampedakis, K. Yagi, and J. Steinhoff.

Outline

1. Effective-field-theory of general relativity
2. Quasinormal modes: their frequencies
3. Quasinormal modes: their amplitudes
4. Conclusions and some conjectures

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Gorbenko et al., de Rham de et al., Cano et al., Cardoso et al., Sennett et al., Horowitz et al., Cayuso et al., …

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- Analytical (rotating) black hole solutions.

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Mathematically, these assumptions translate into studying differential equations of the form:

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}x^2} + Q_{\ell m}(x, \omega, \vartheta)\right] \psi_{\ell m}(x) = s_{\ell m}(\omega, x)$$

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- x : coordinate that maps the domain $r\in[r_{\mathrm{h}},\infty)$ to $x\in(-\infty,\infty)$.

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Complex-valued poles of the scattering matrix: a boundary-value problem.

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Zerilli (1970): $$Q_\ell^{(+)} = \omega^2 - \left(1 - \frac{2M}{r}\right)\frac{1}{r^2\lambda_\ell^2}\left[2\lambda_\ell^2(\Lambda_\ell + 1) + \frac{18M^2}{r^2}\left(\lambda_\ell + \frac{M}{r}\right)\right]$$

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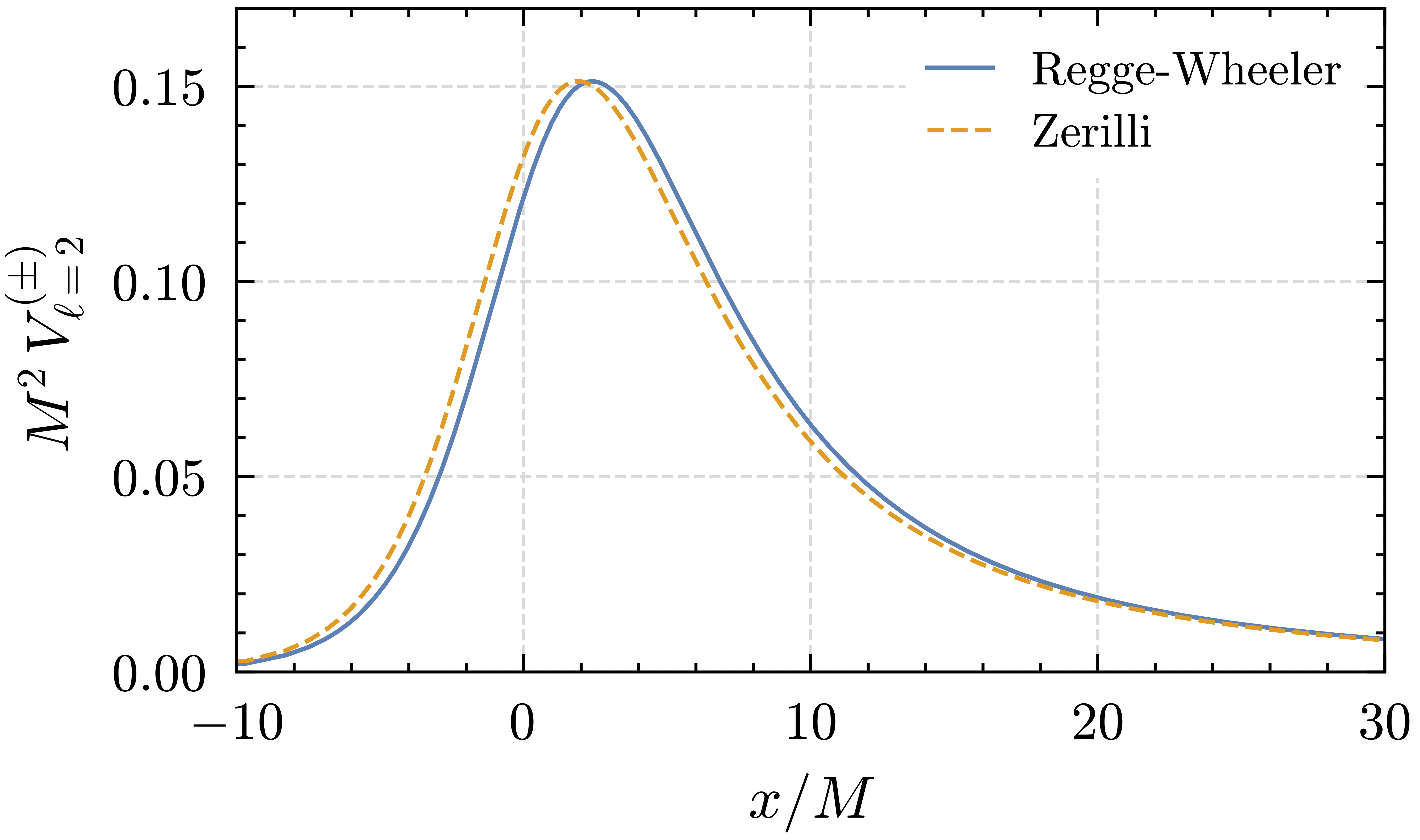
Zerilli (1970):

$$Q_\ell^{(+)} = \omega^2 - \left(1 - \frac{2M}{r}\right)\frac{1}{r^2\lambda_\ell^2}\left[2\lambda_\ell^2(\Lambda_\ell + 1) + \frac{18M^2}{r^2}\left(\lambda_\ell + \frac{M}{r}\right)\right]$$

where $\lambda_\ell = (\ell+2)(\ell-1)/2$ and $\Lambda_\ell = \lambda_\ell + 3M/r$. Hereafter, $V_{\ell m}^{(\pm)} = \omega^2 - Q_{\ell m}^{(\pm)}$ (the "effective potential").

Schwarzschild perturbations — II



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This implies that the quasinormal-frequency spectrum is the same, $\omega^{(+)}_{\ell mn} = \omega^{(-)}_{\ell mn}$.

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de Rham de et al. and Cano et al.

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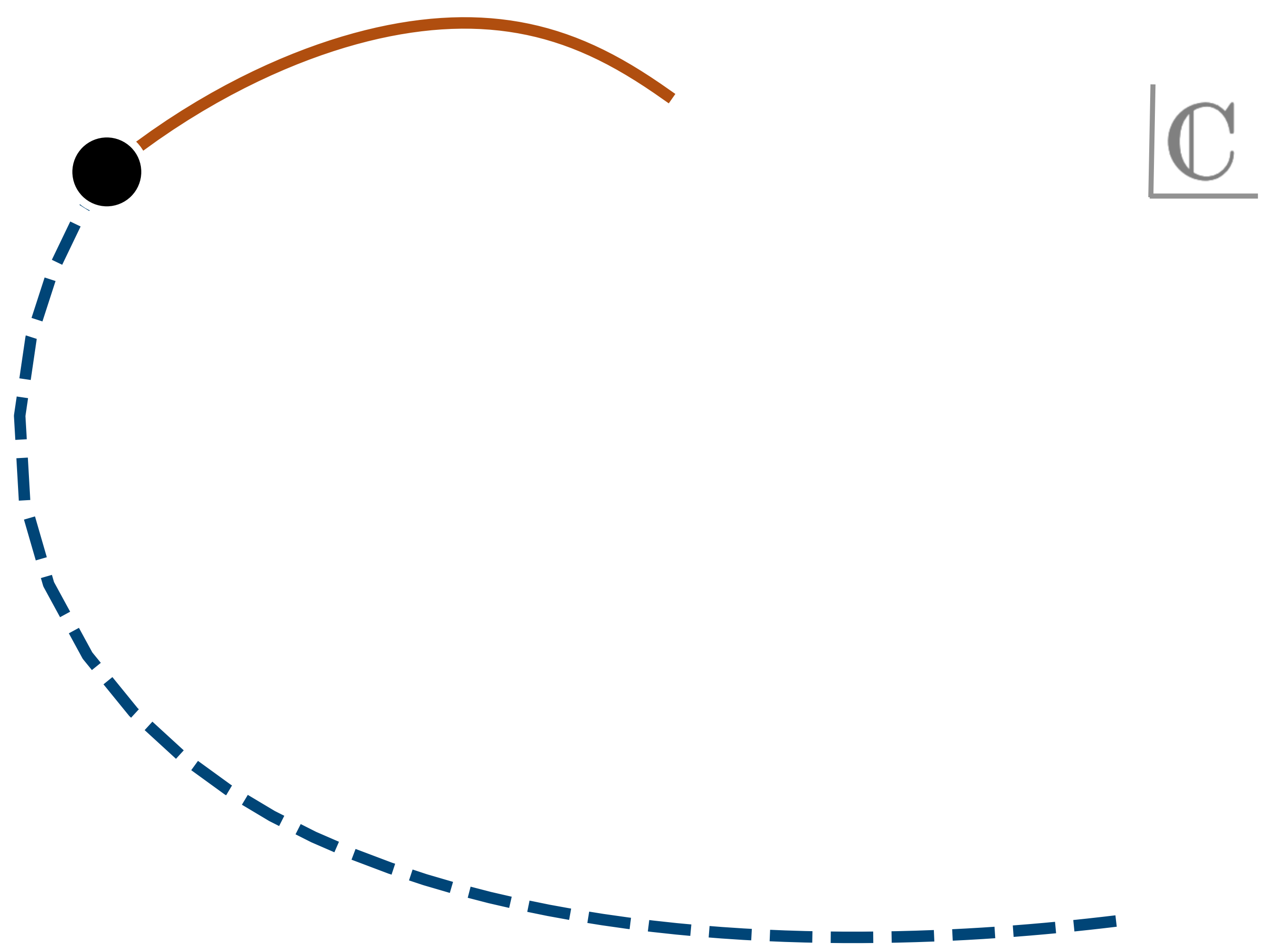

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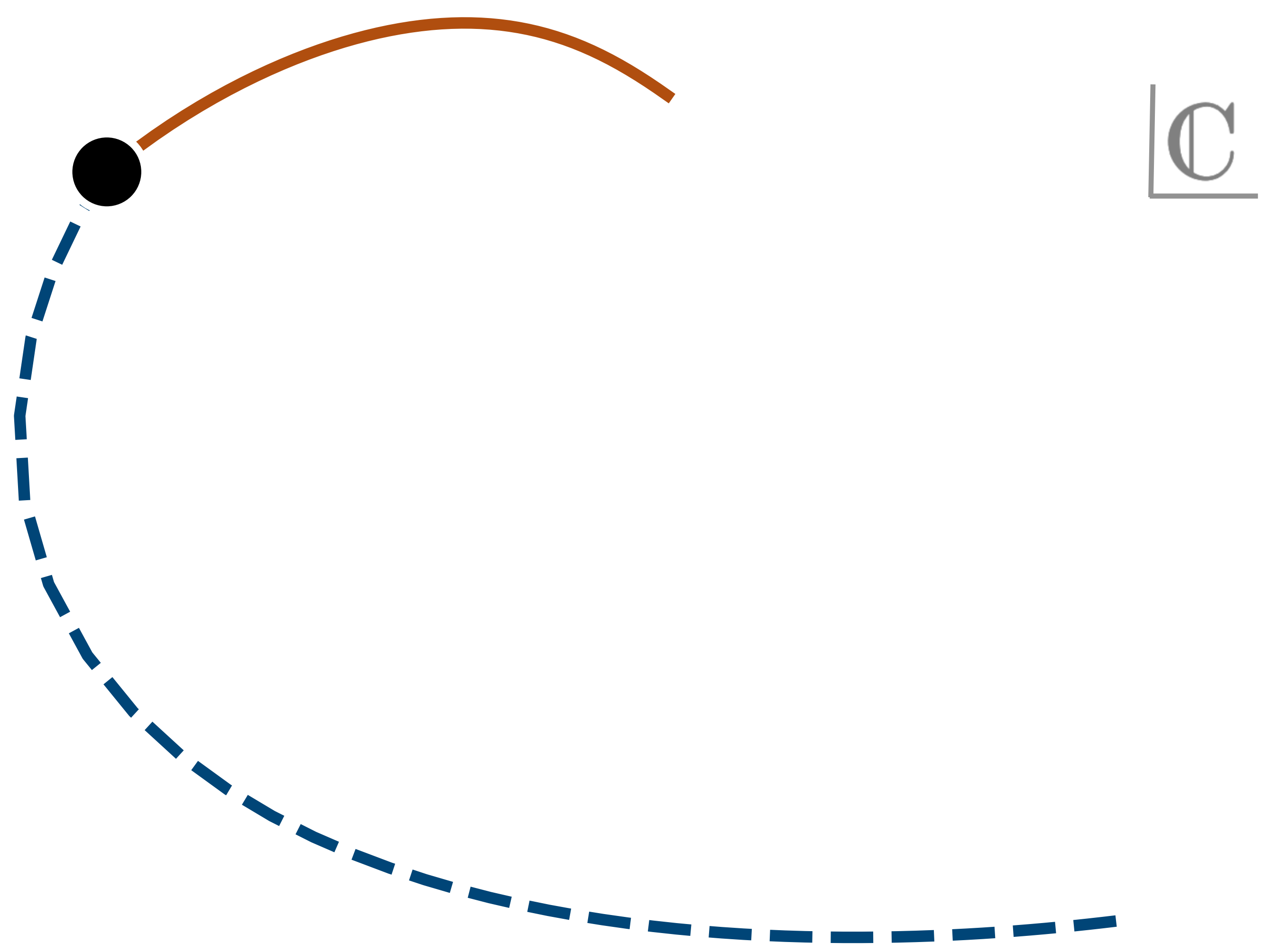

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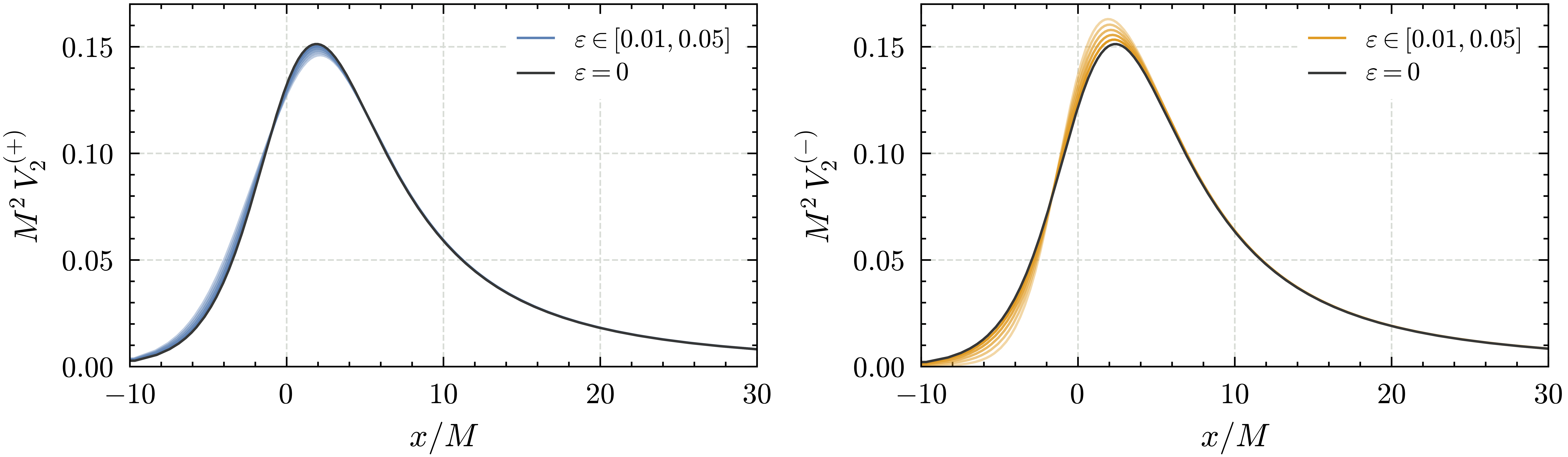
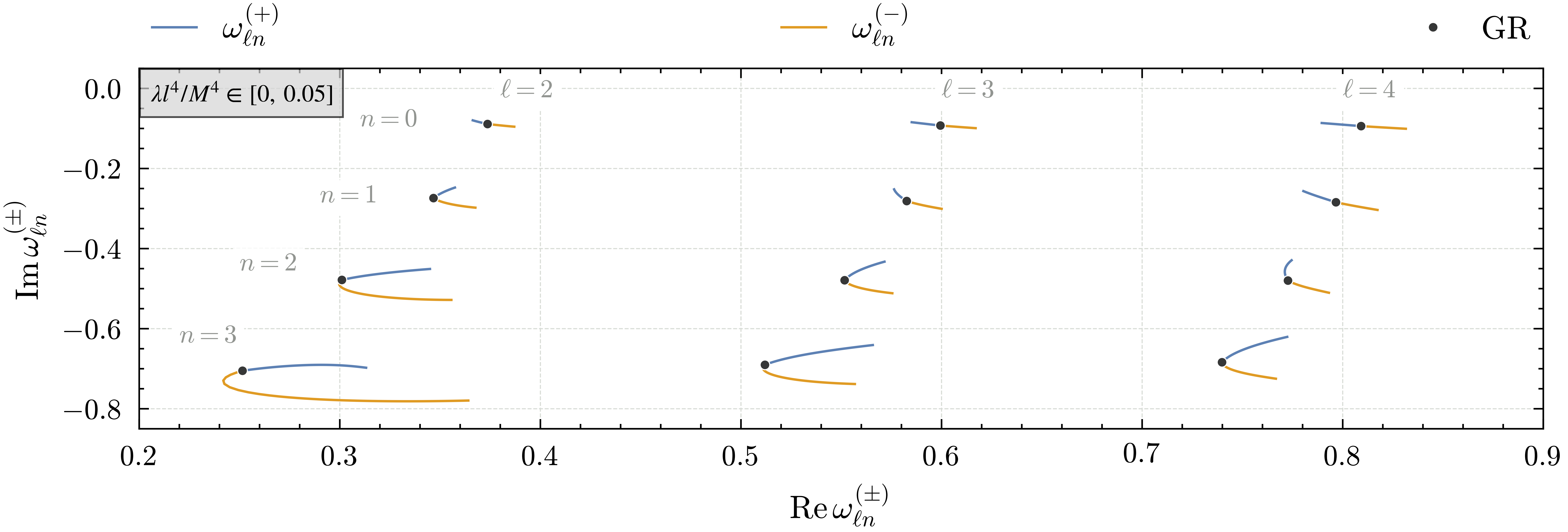
- Perturbations have a position-dependent propagation speed c_s^2.

EFT perturbations — II



Quasinormal frequency spectra



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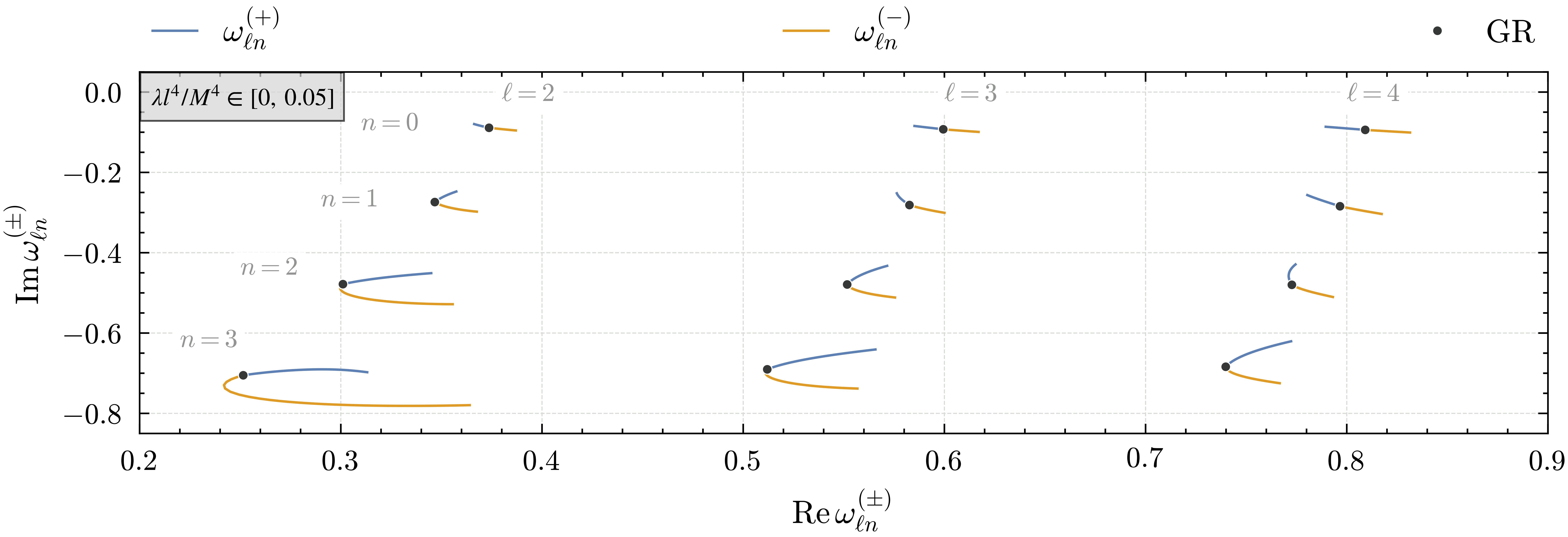

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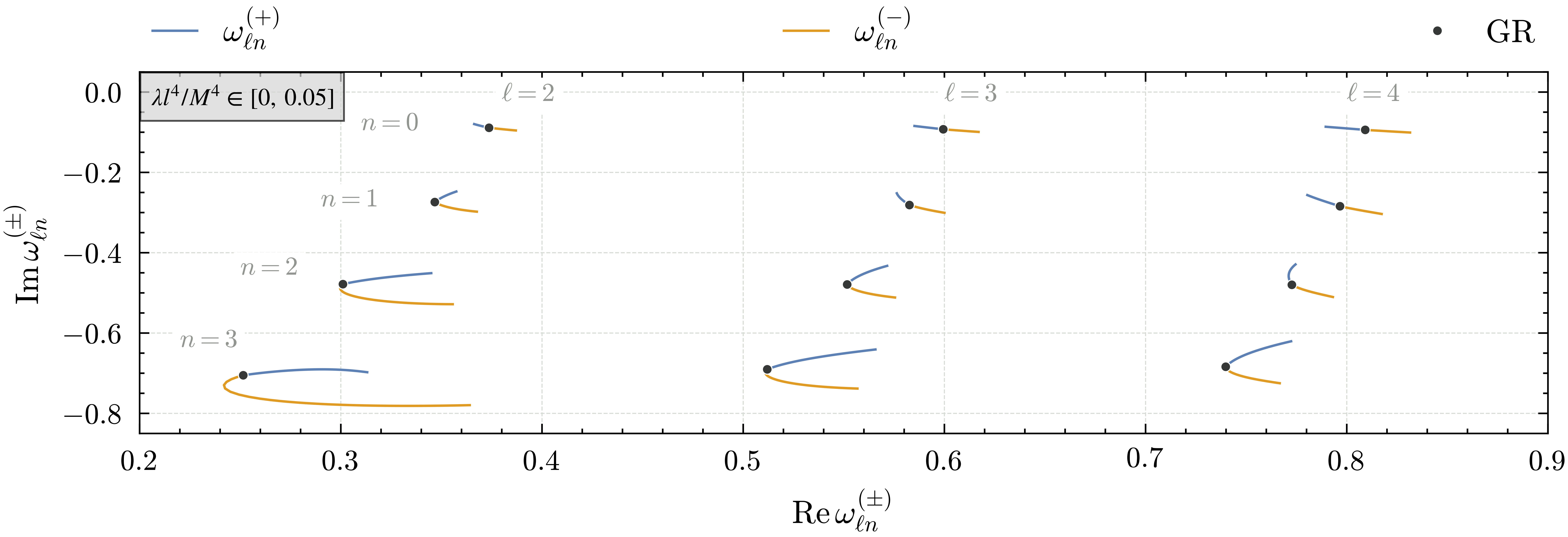

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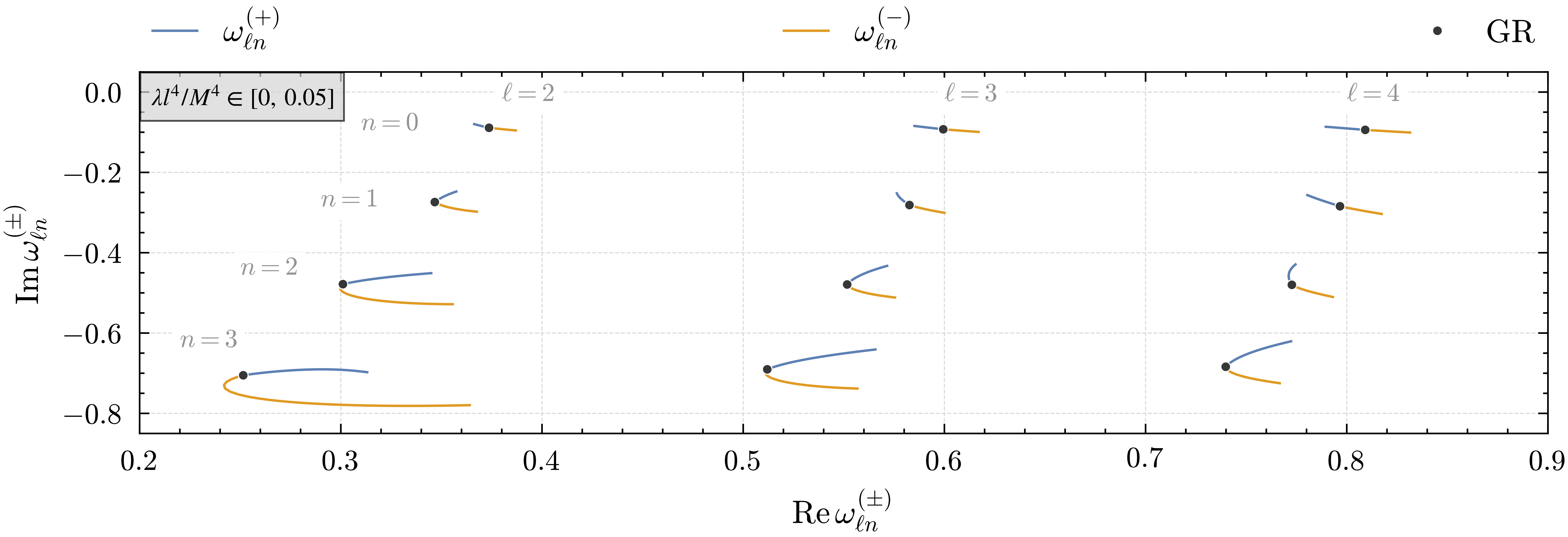

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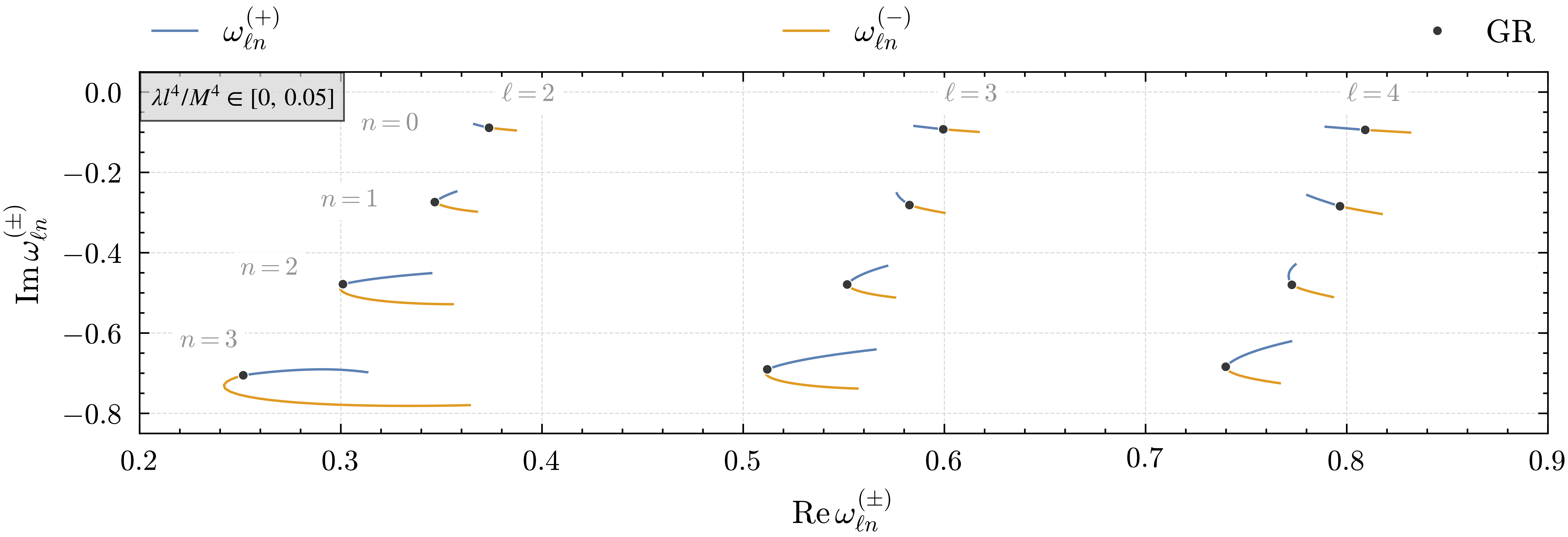

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- Overtones are more sensitive to the lengthscale l.
- Scaling with $\varepsilon = \lambda l^4/M^4$ is nonlinear and depends on ℓ and n.

Overtones and the limits of the EFT description

Maggiore (2008)

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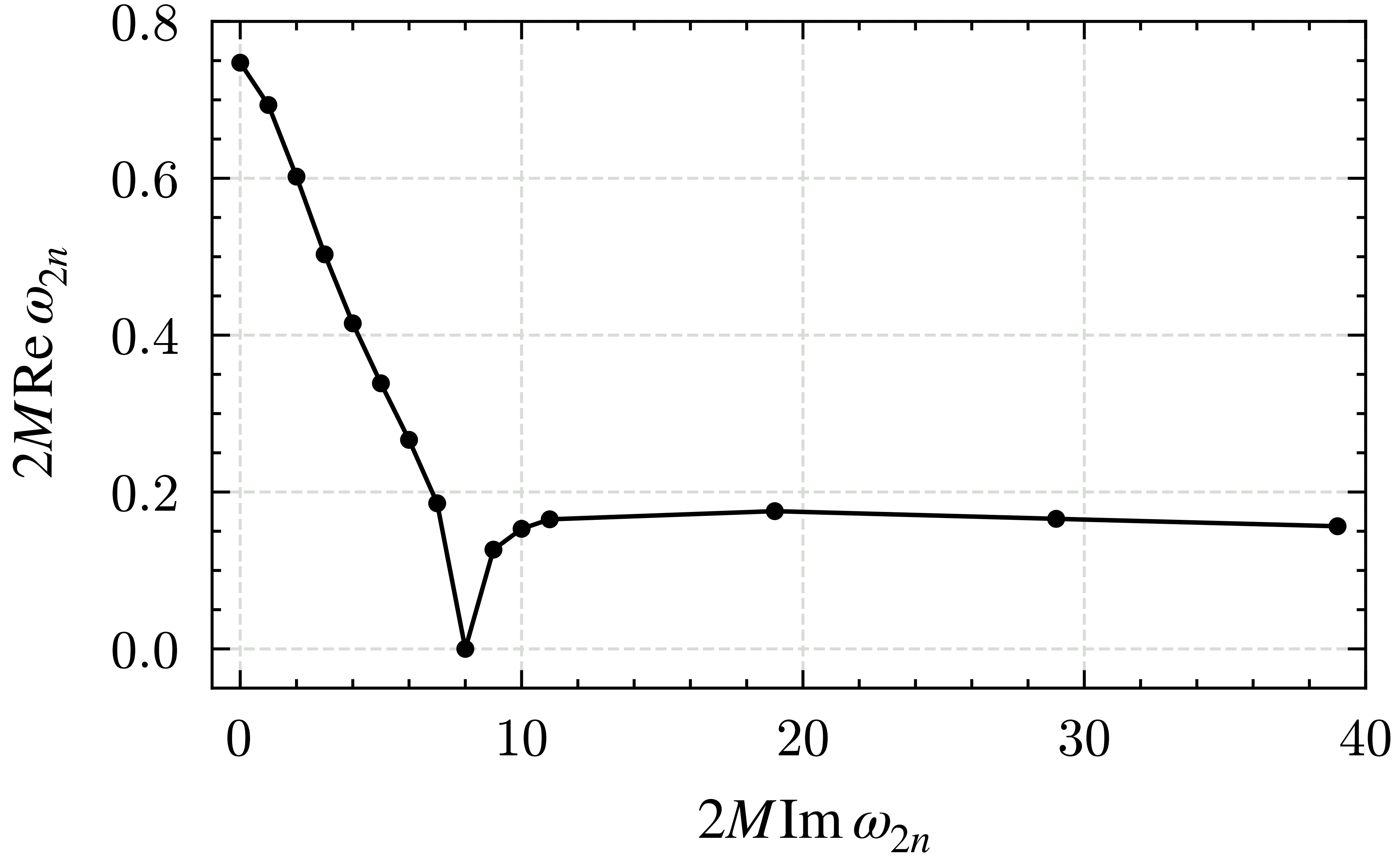

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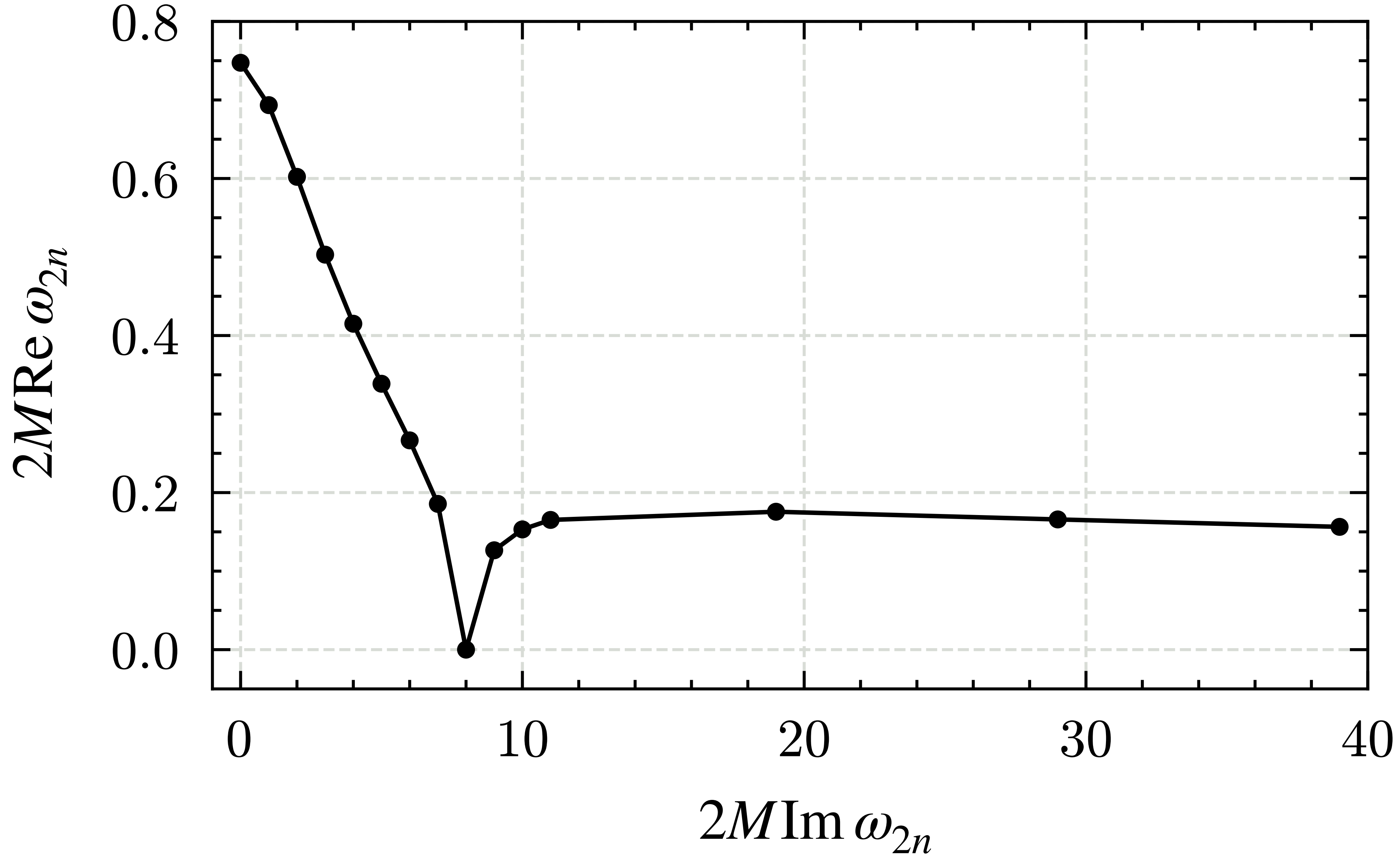
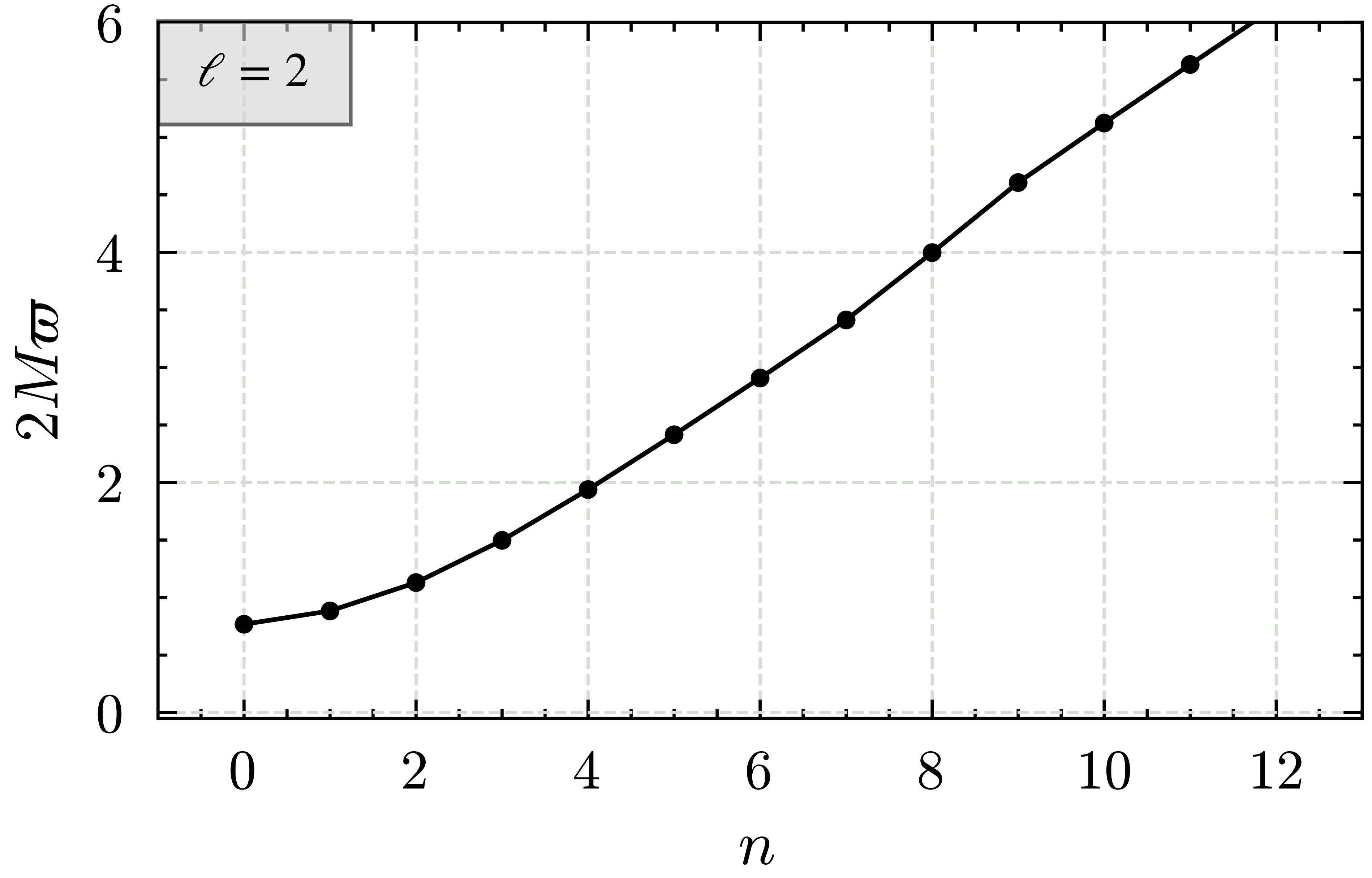
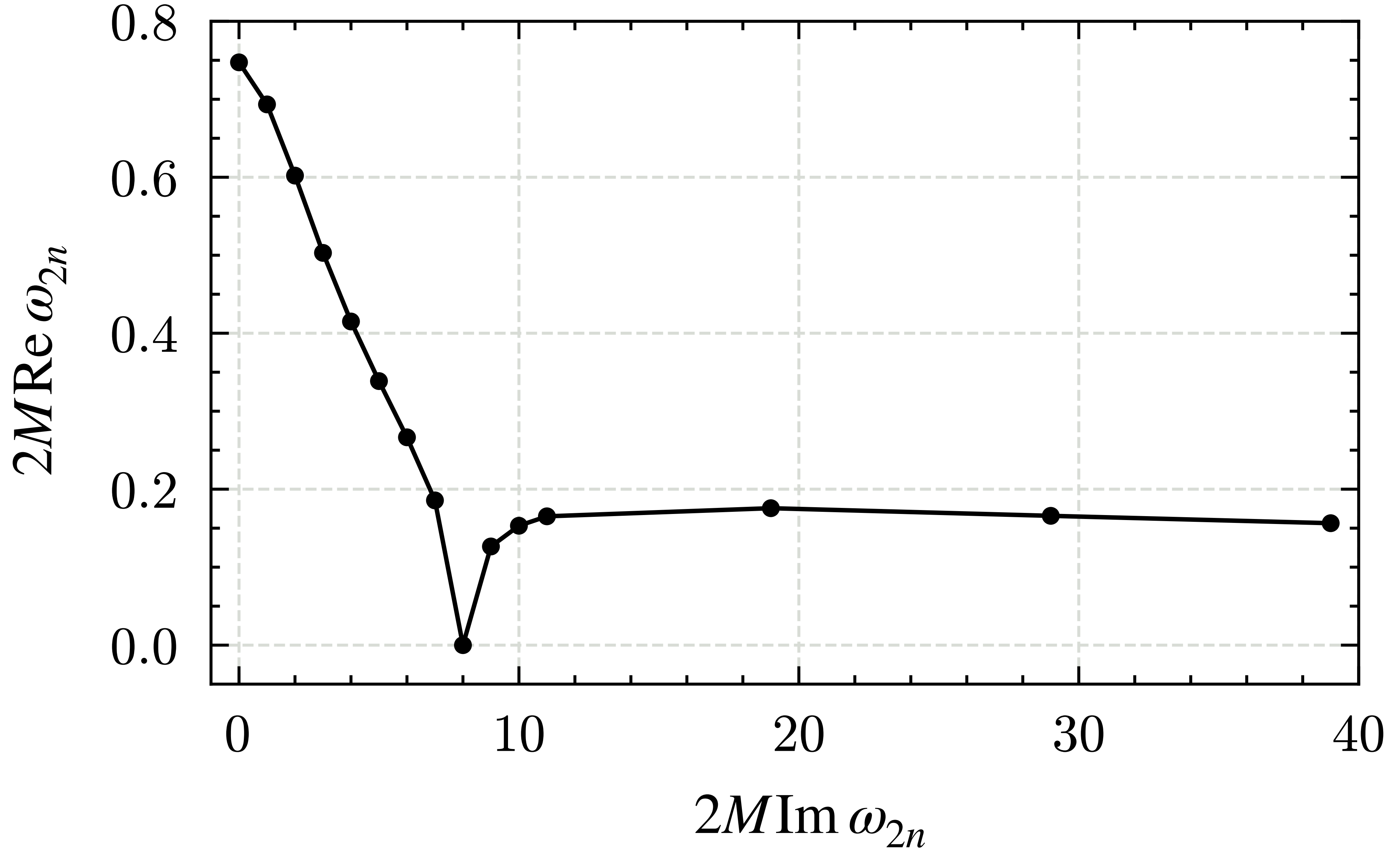
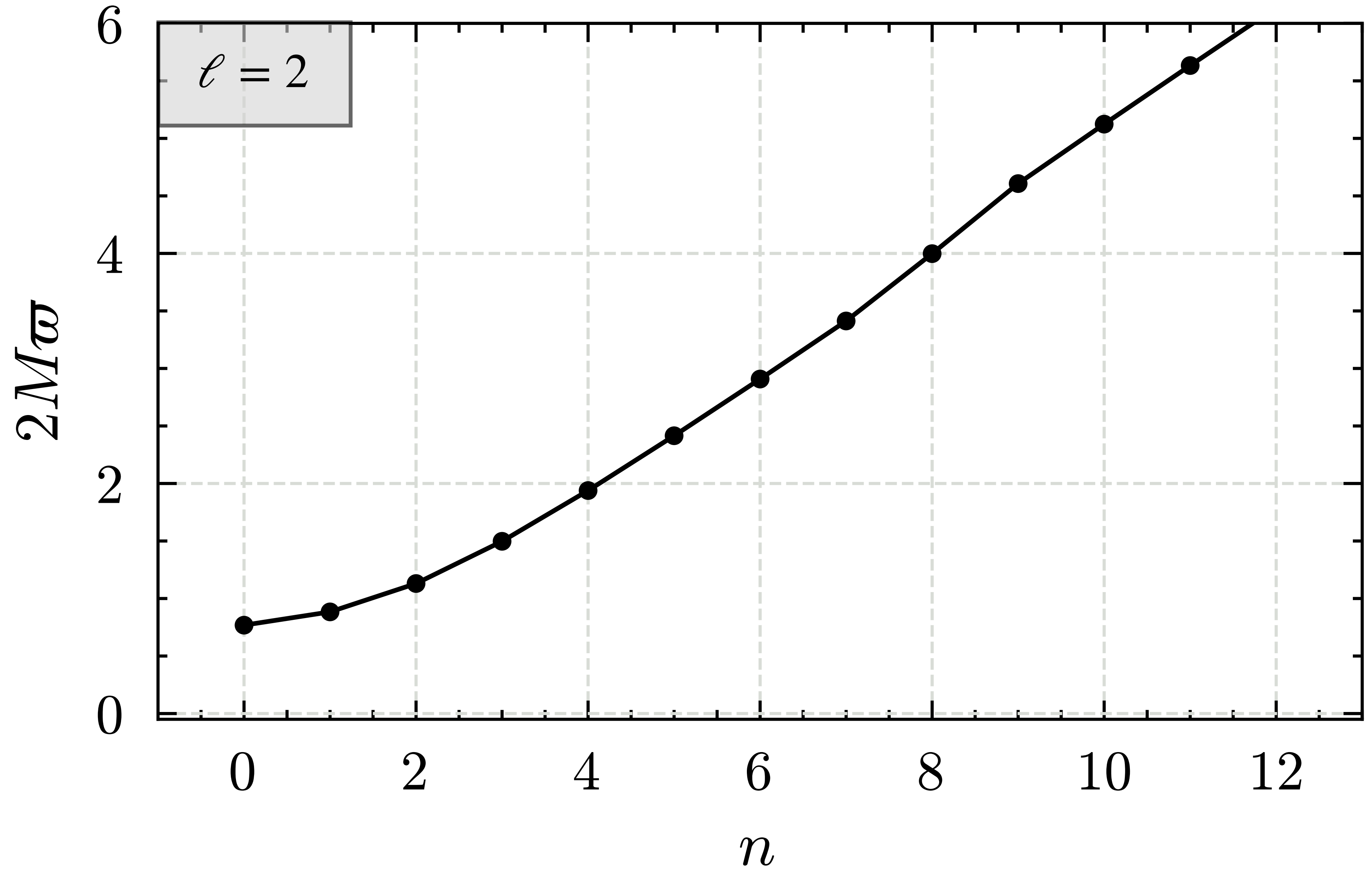




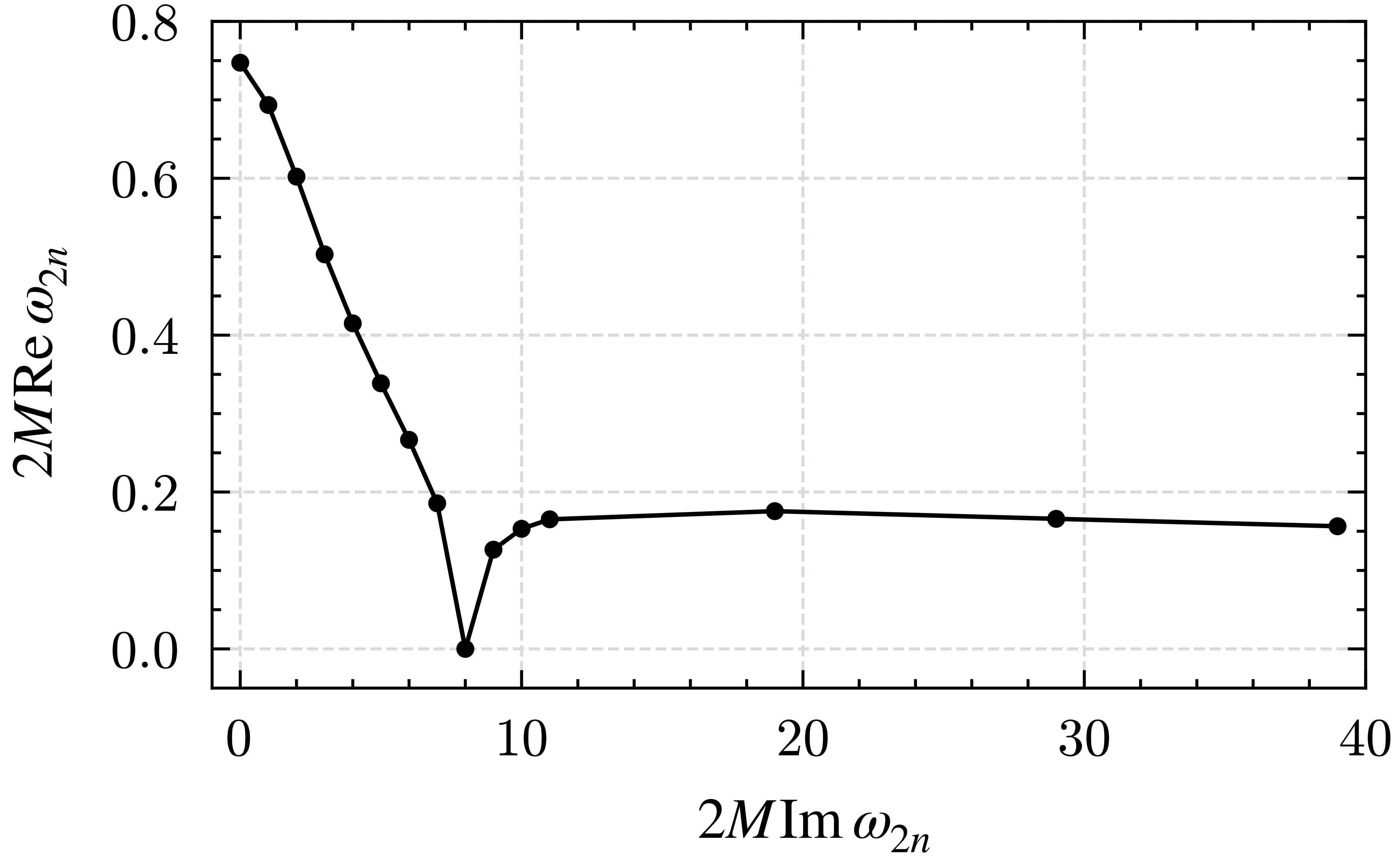
Overtones and the limits of the EFT description — II

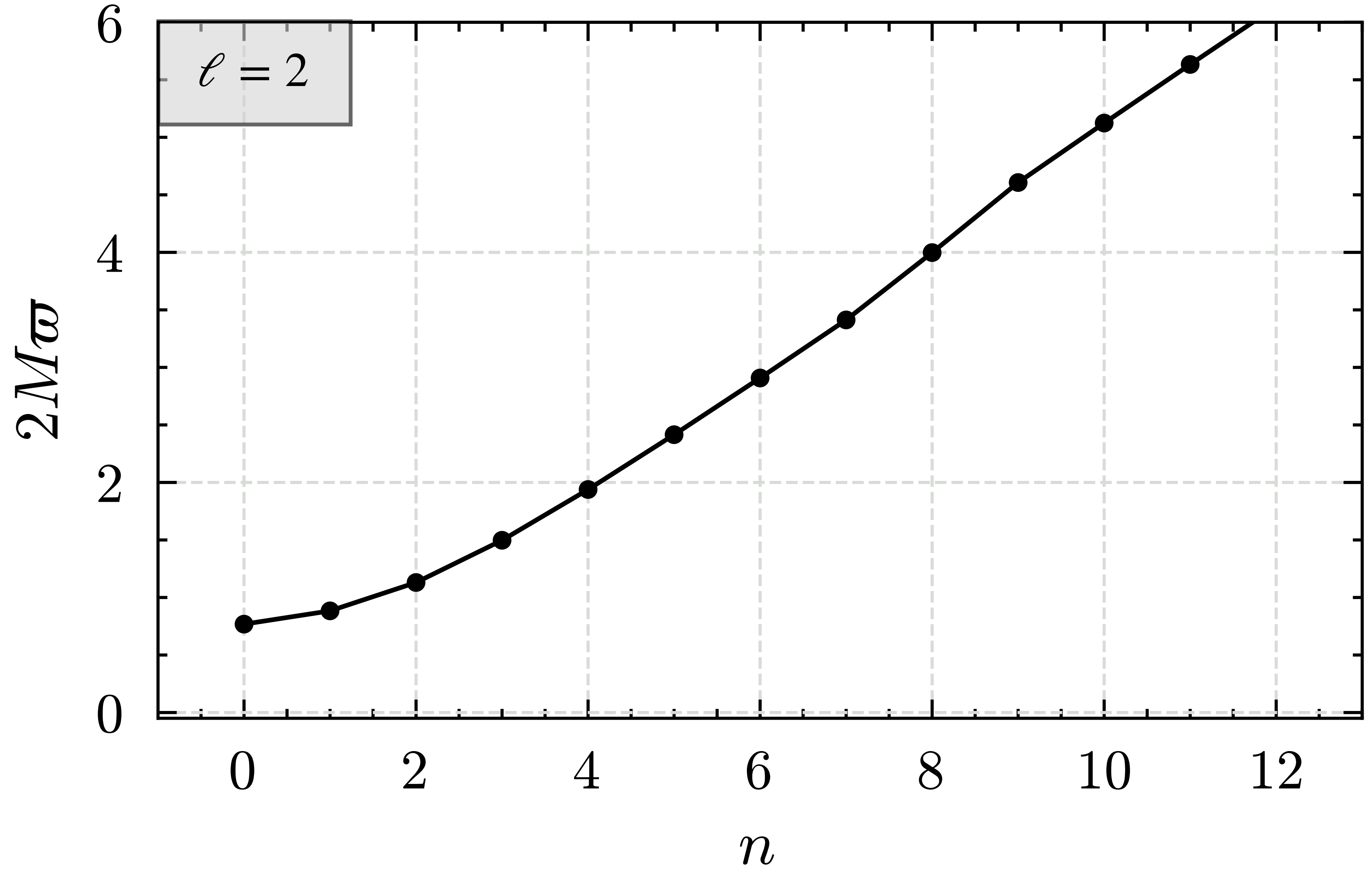




Overtones and the limits of the EFT description — II

- Lengthscale associated to the wavelength of the modes





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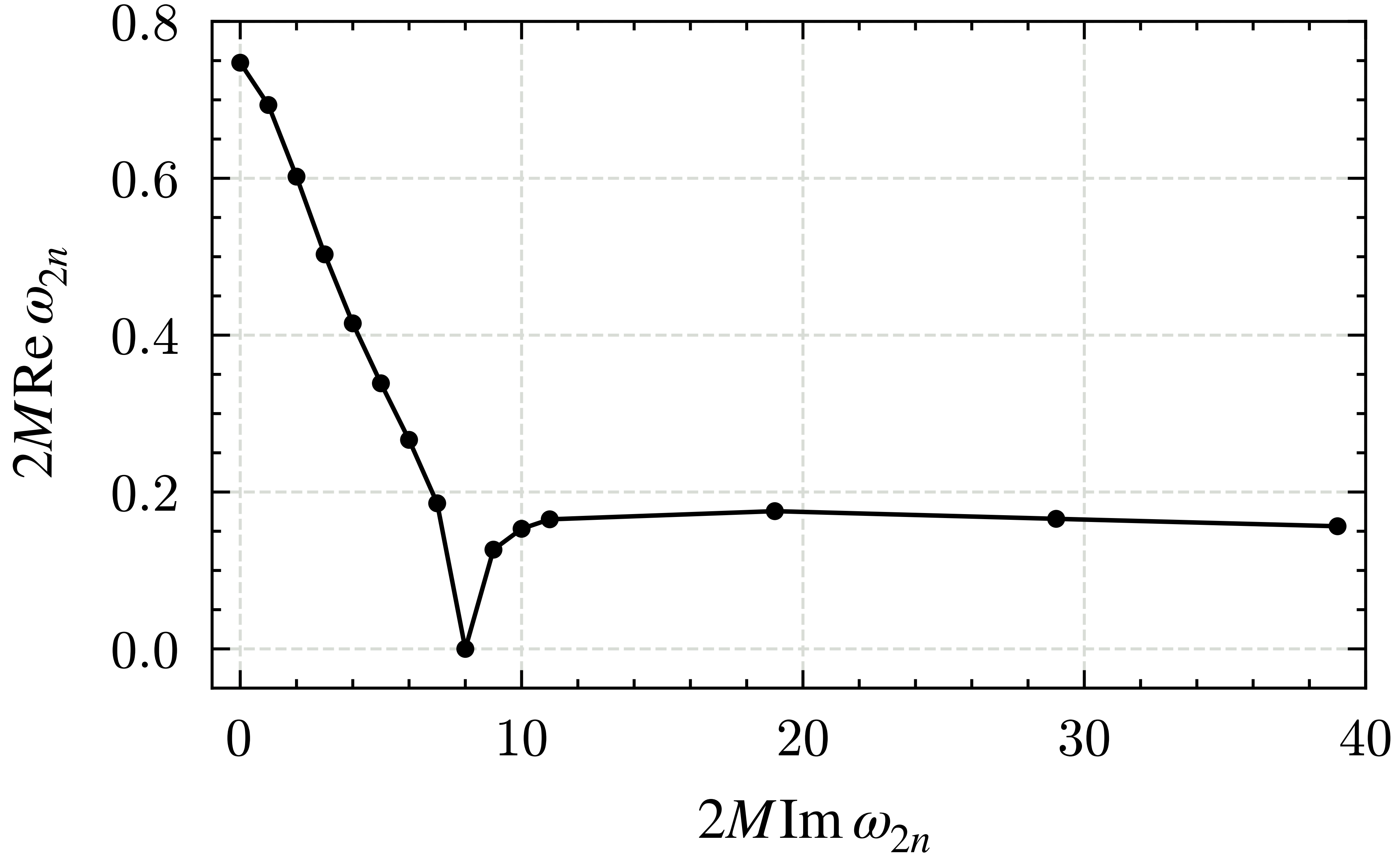

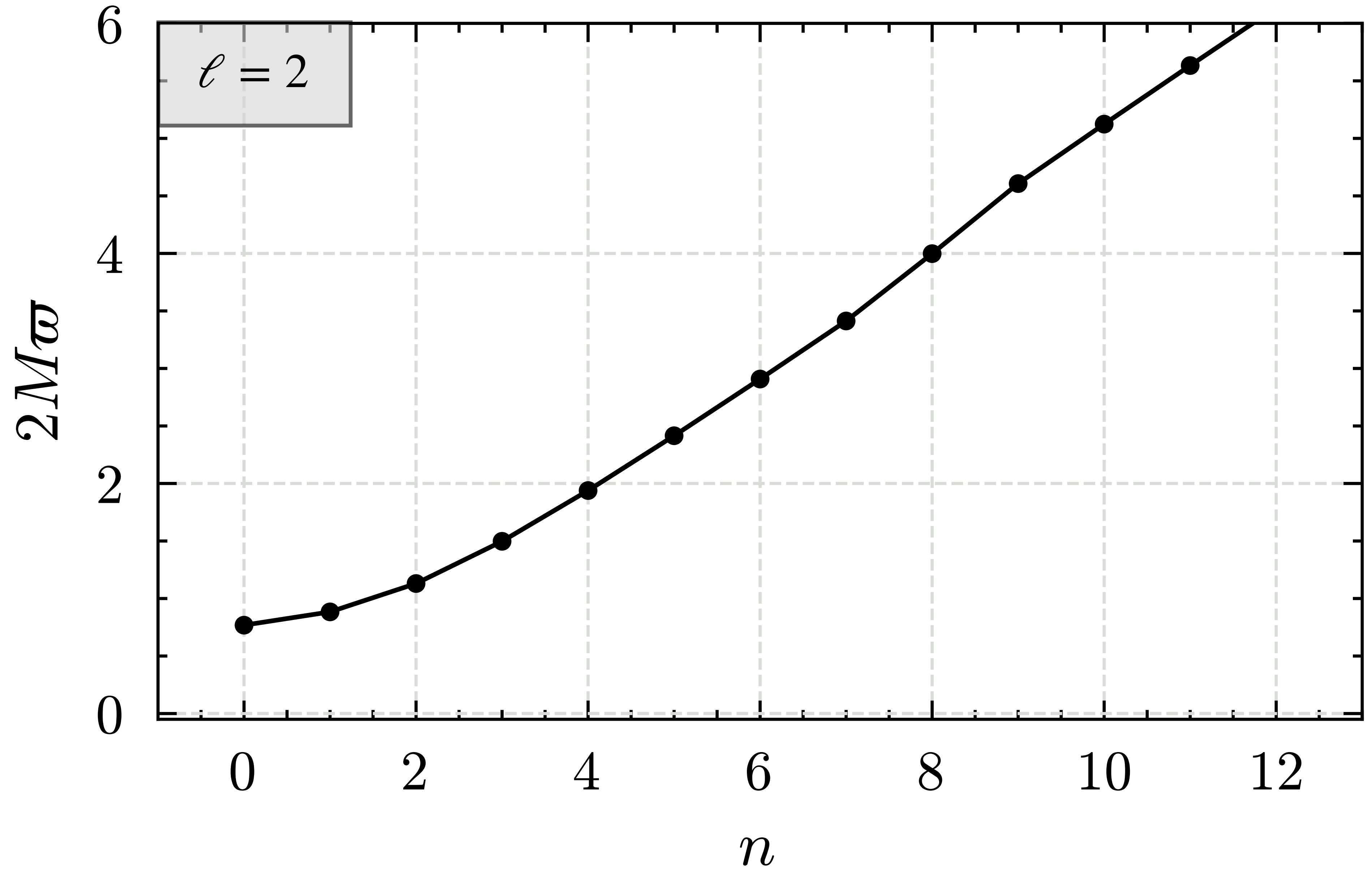

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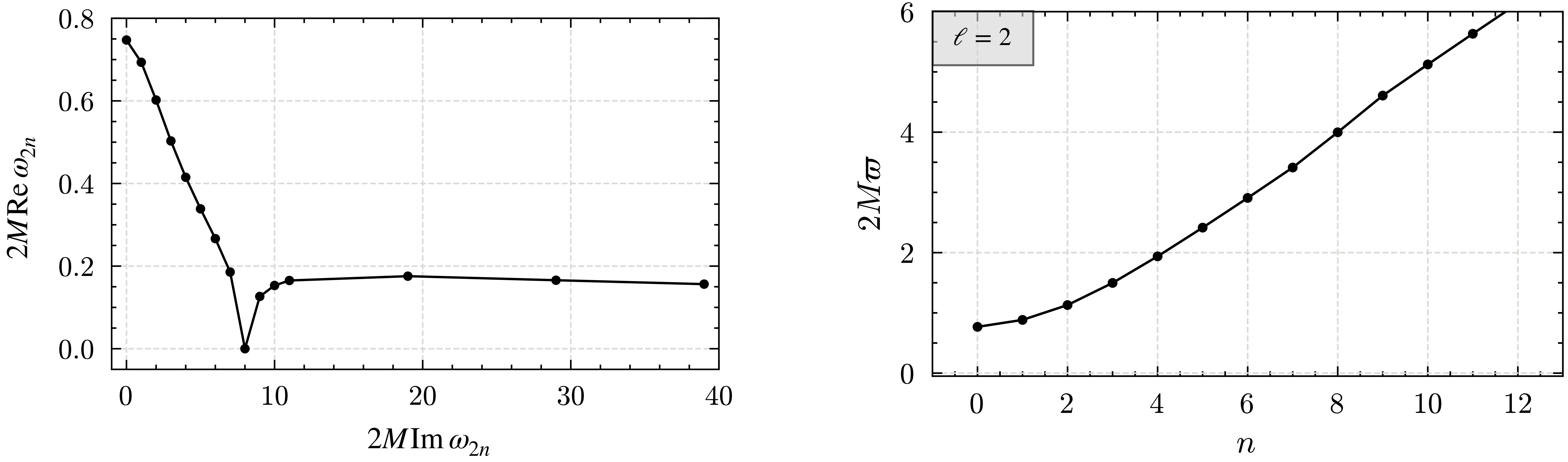
- High-frequency waves probe deeper the potential barrier, hence the near-horizon region of the spacetime.



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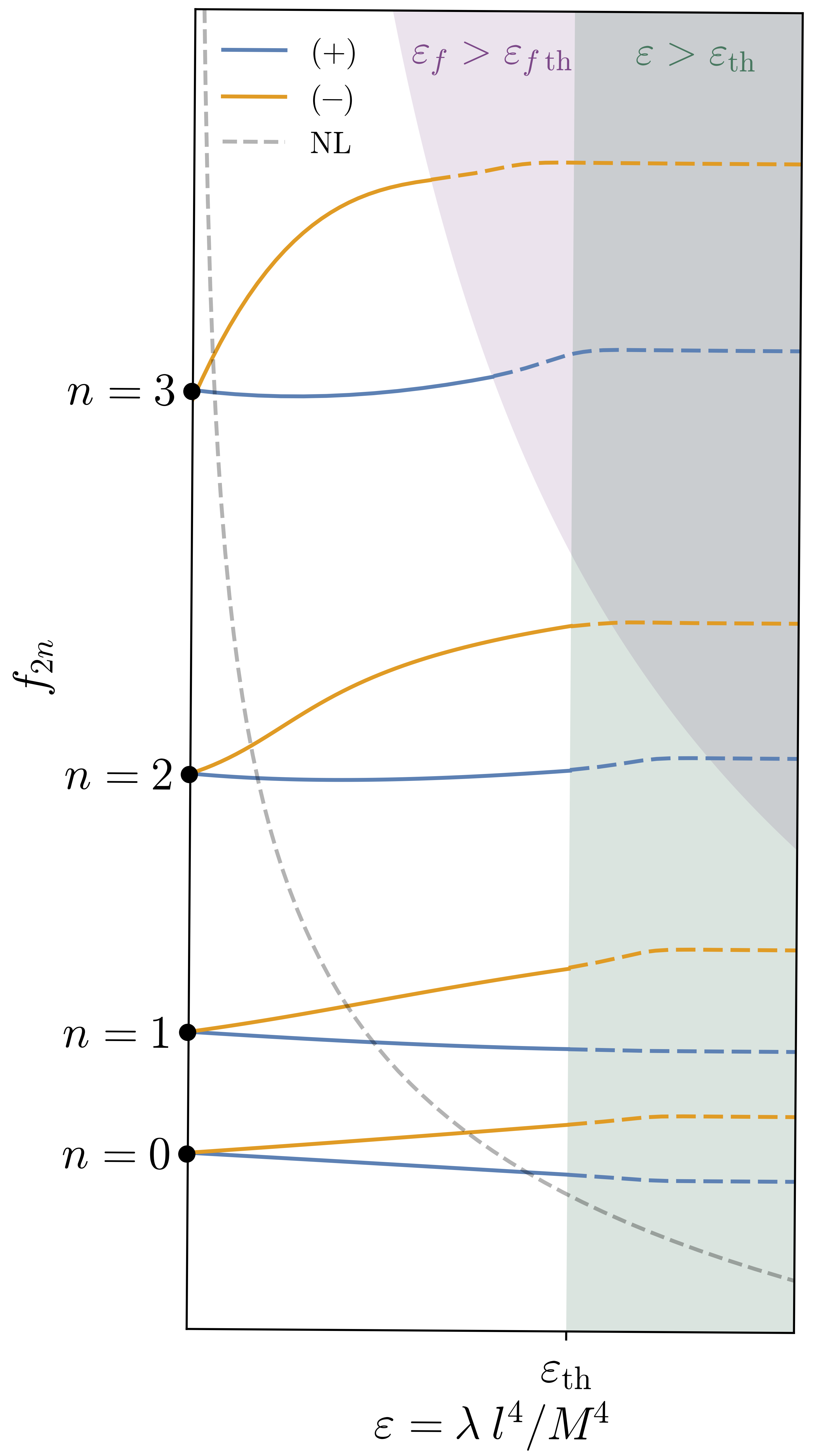

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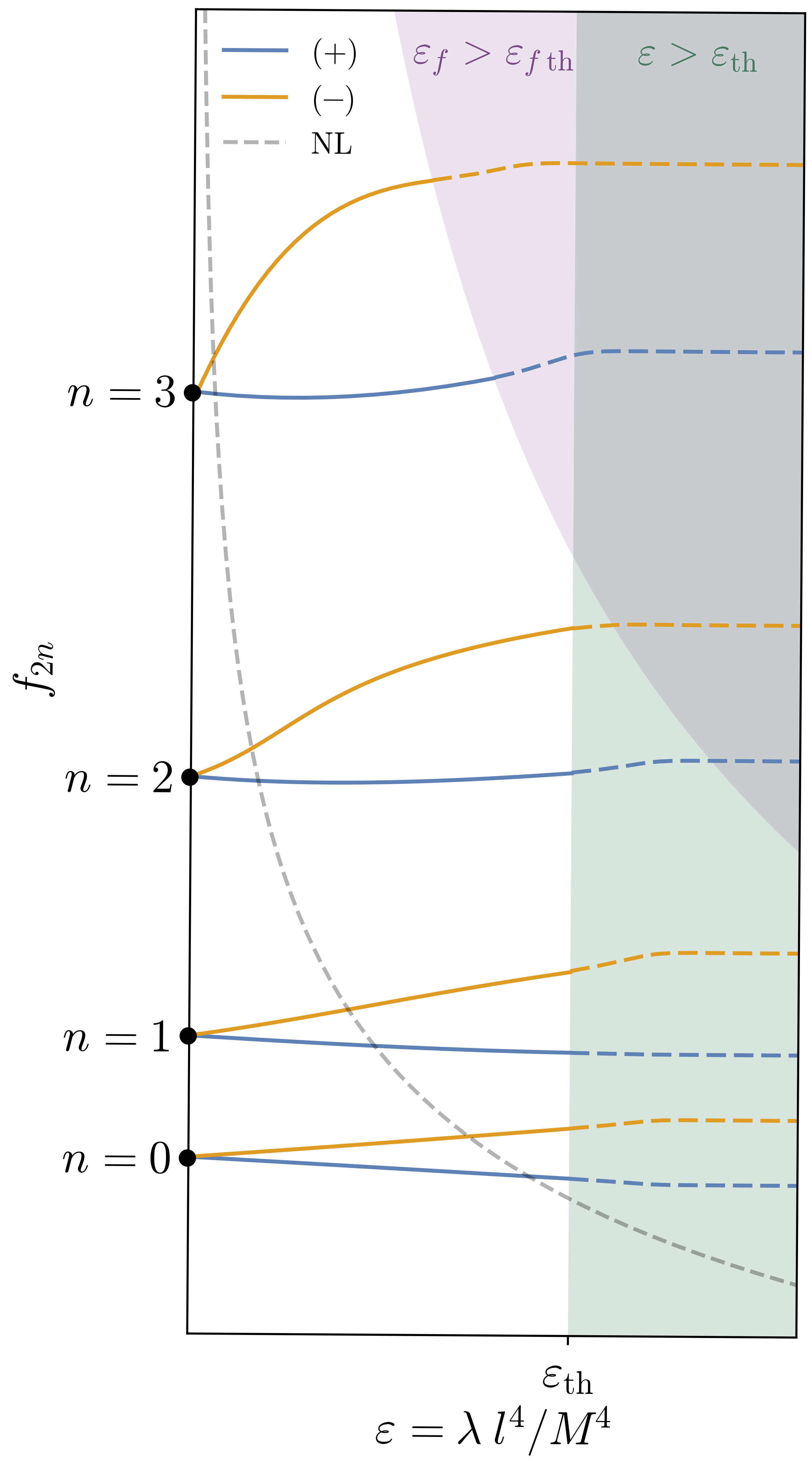

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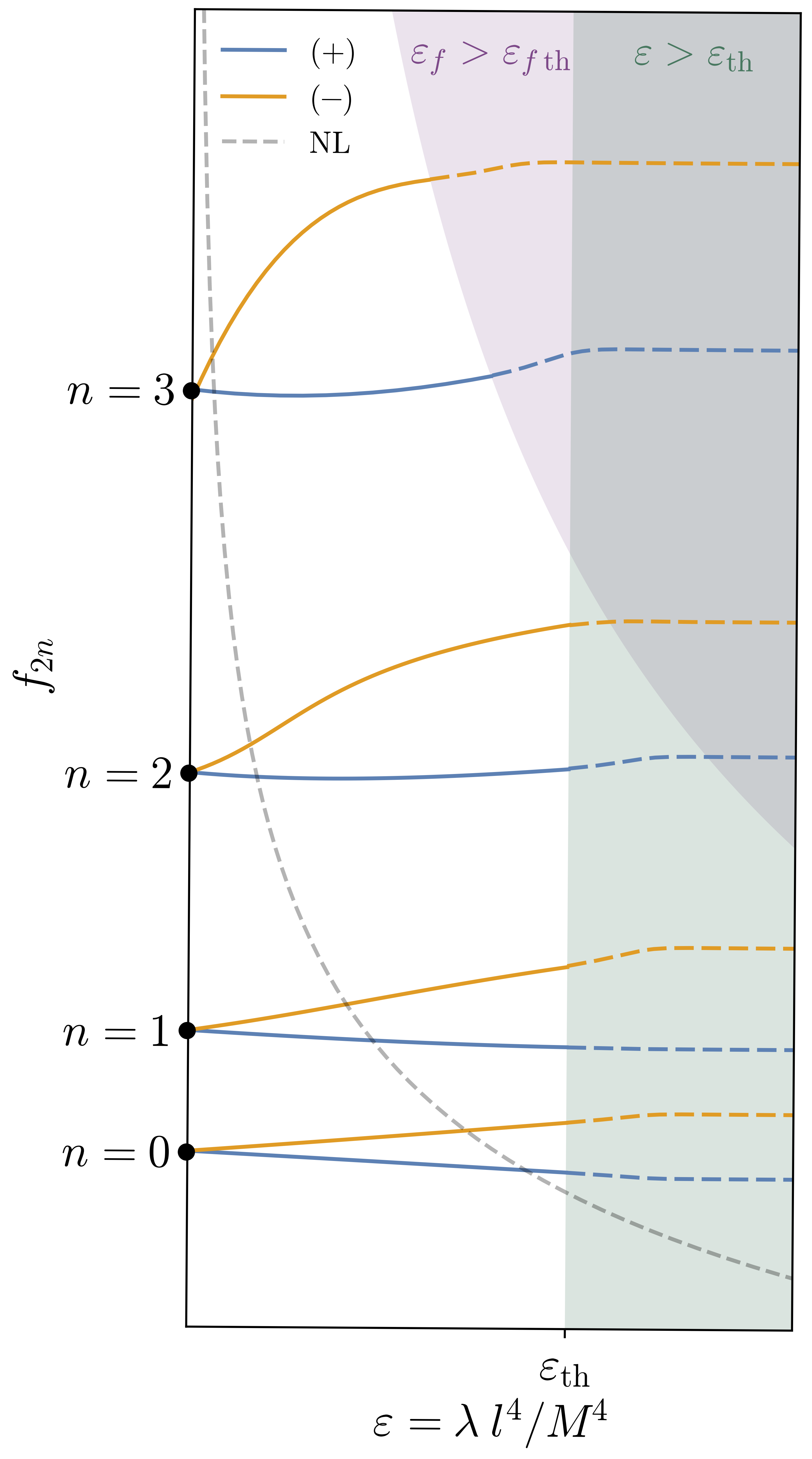

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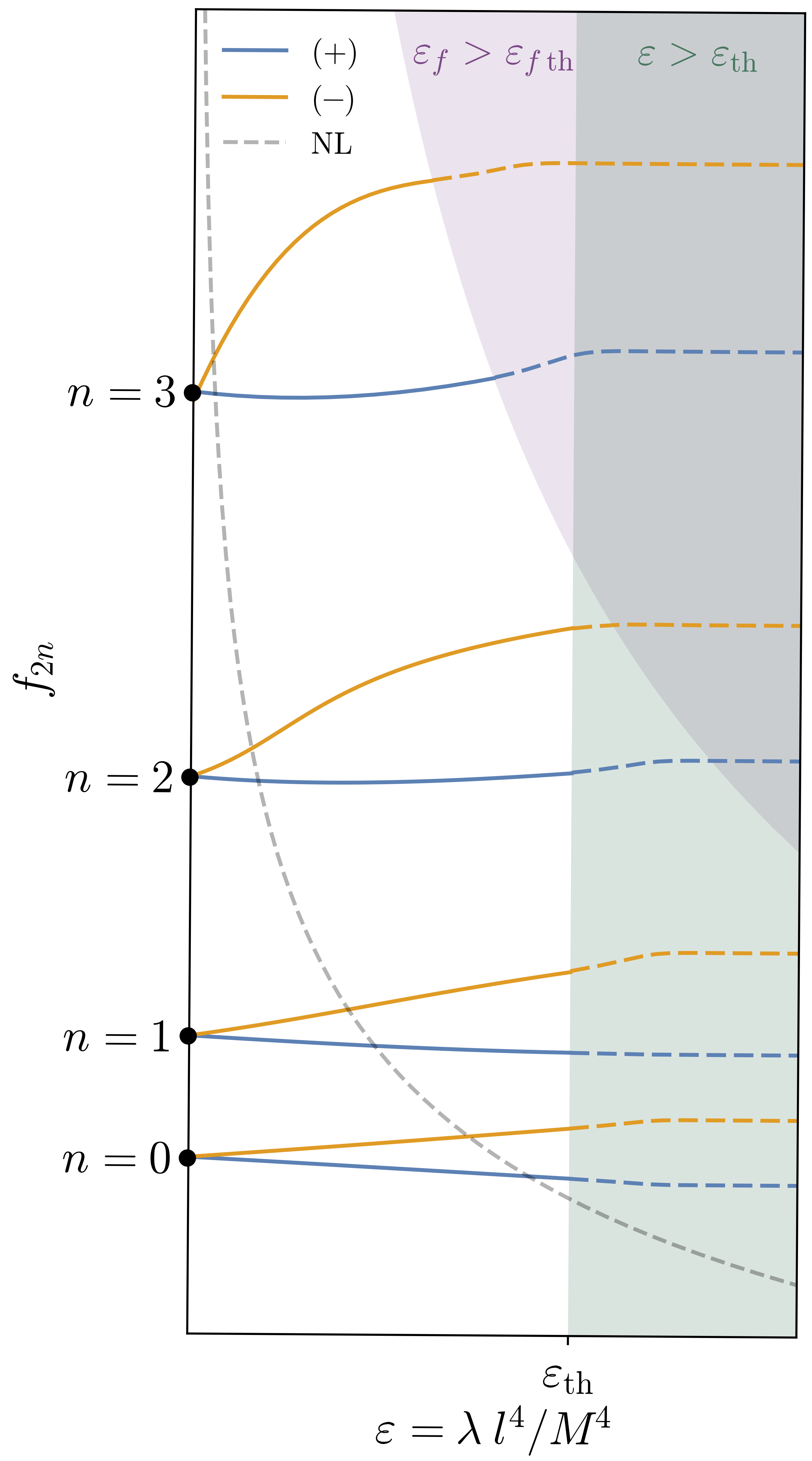

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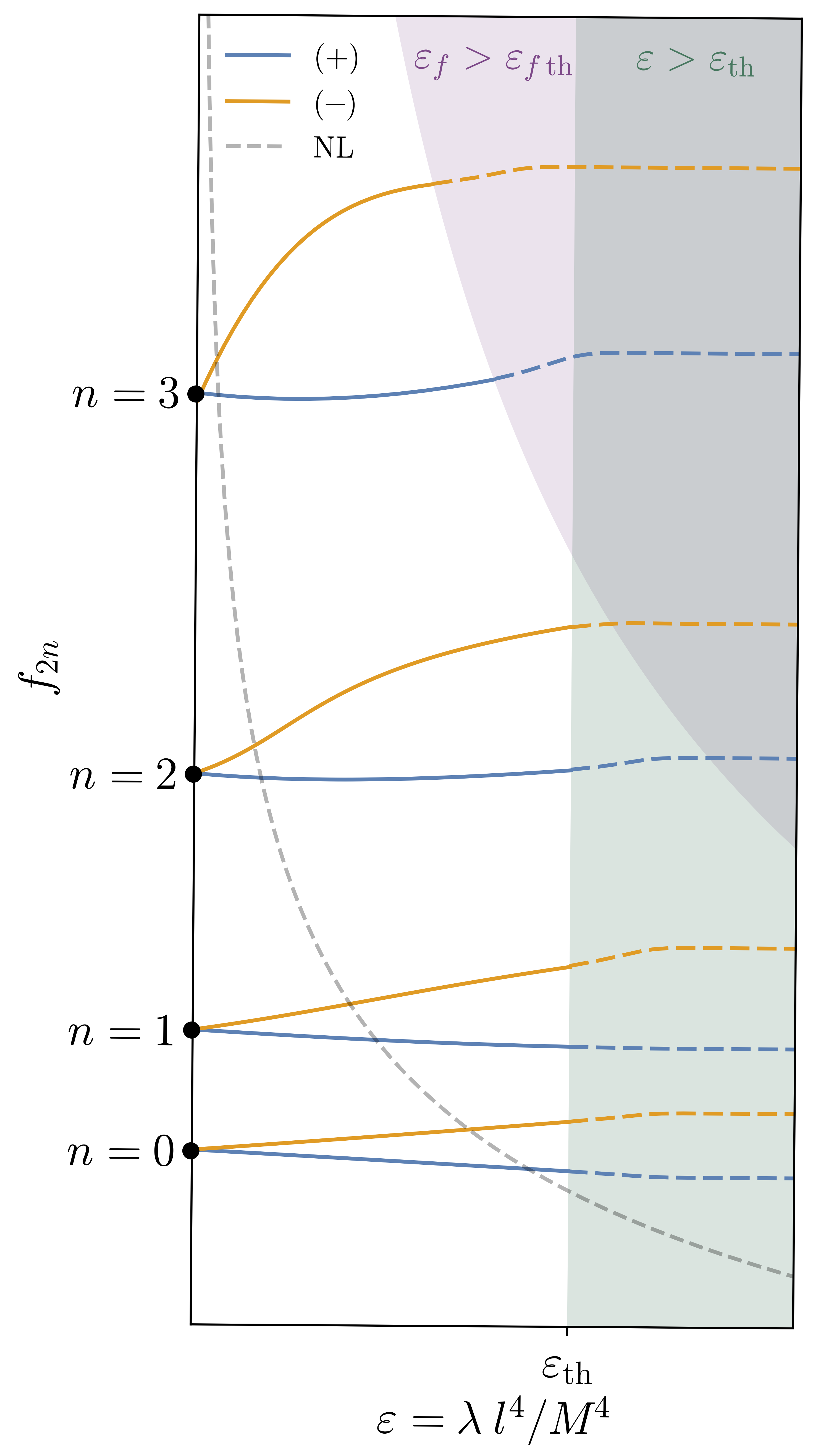

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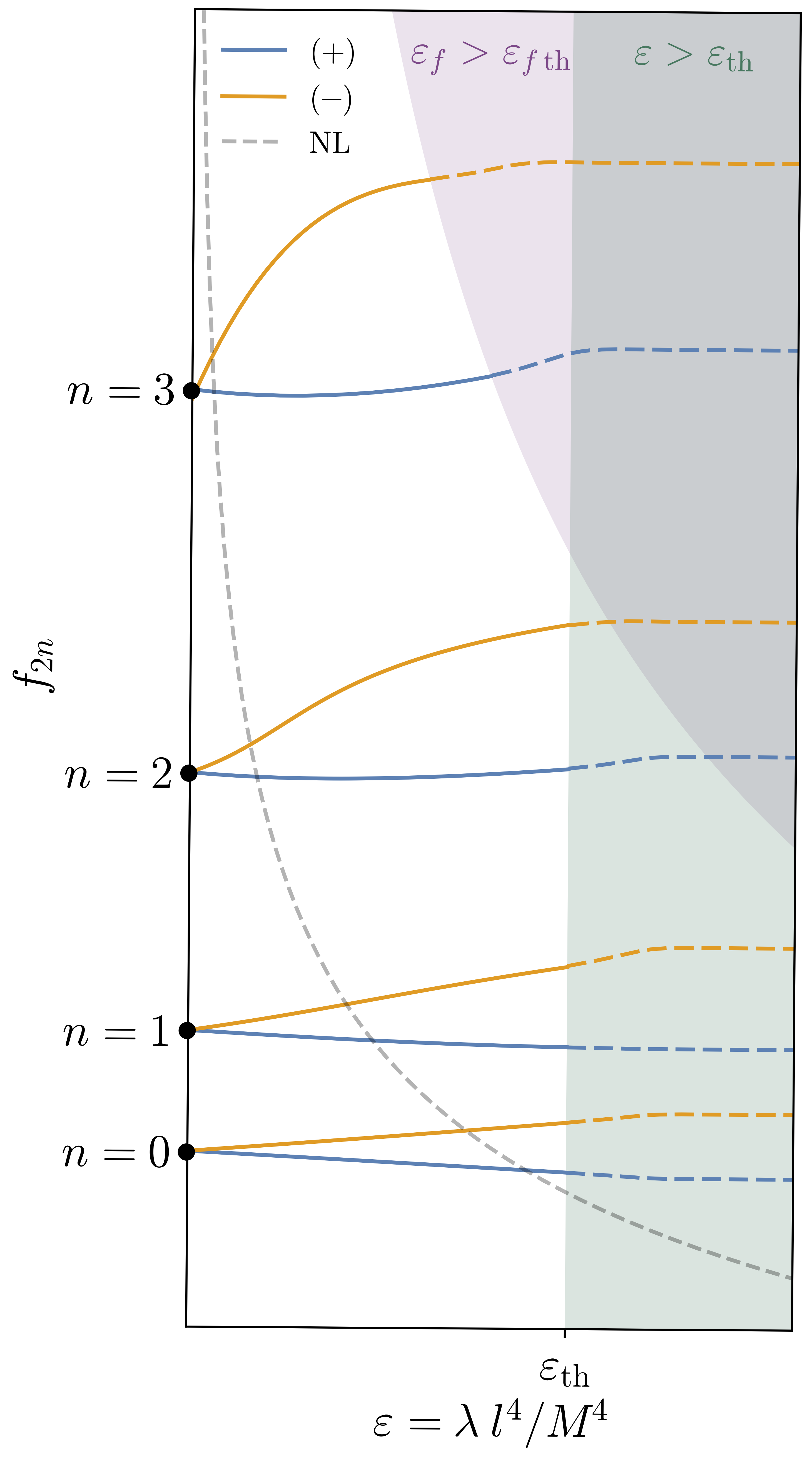

Excitation of quasinormal modes

Leaver (1986), Andersson (1995)

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}x^2} + Q_{\ell m}(x, \omega, \vartheta)\right] \psi_{\ell m}(x) = s_{\ell m}(x)$$

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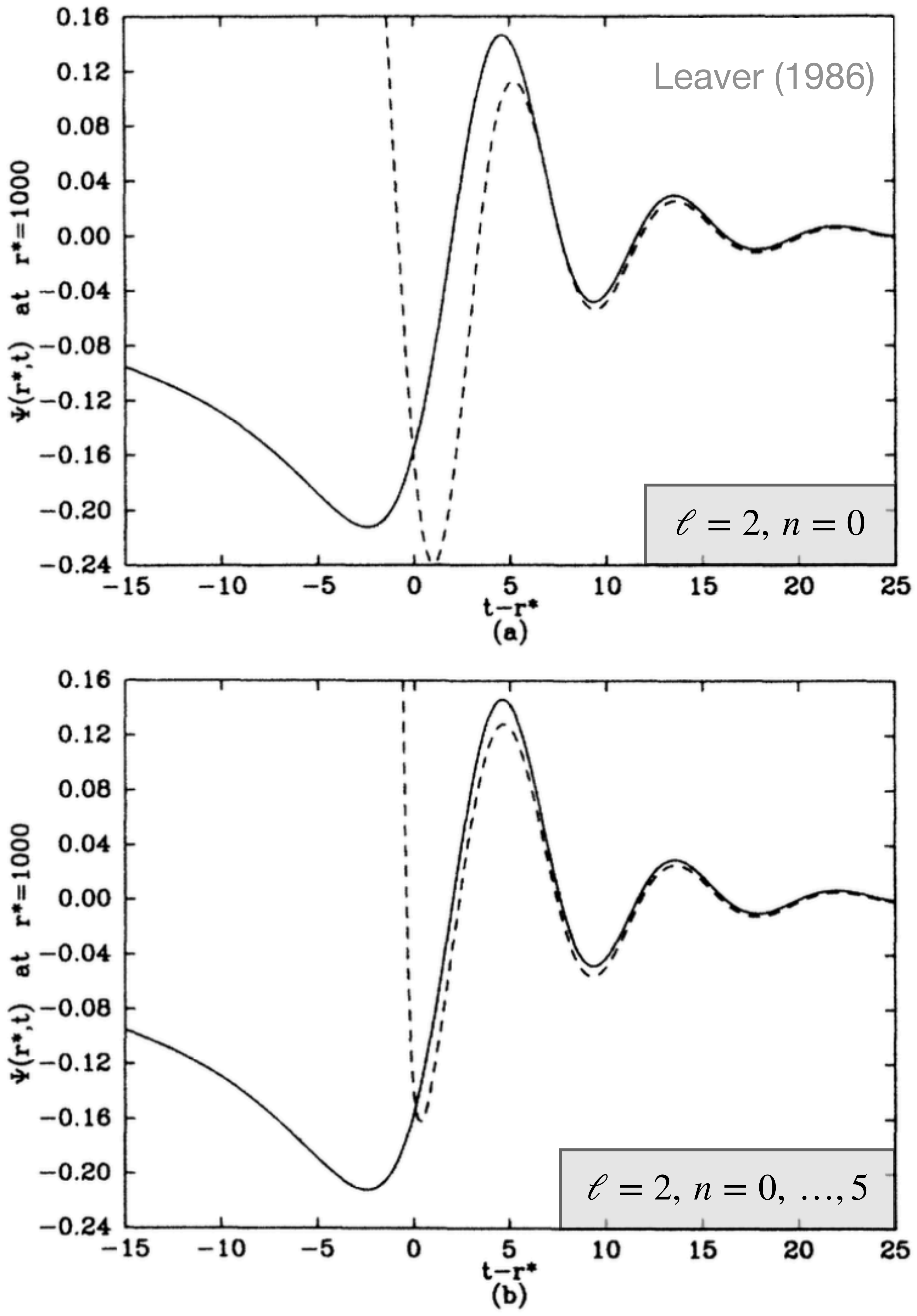

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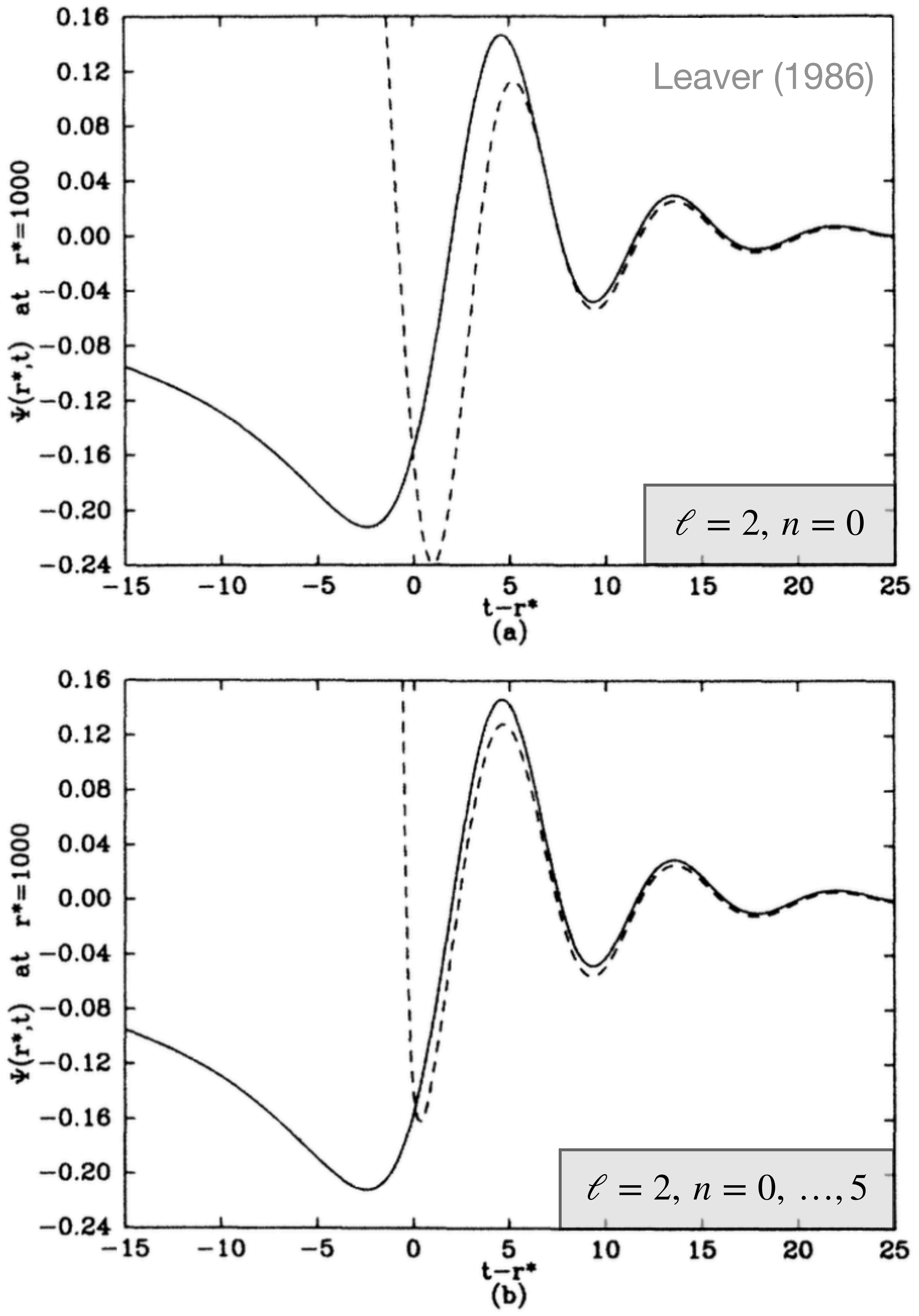
Neither have been computed in any beyond-general-relativity spacetime. I don’t know why!



Quadrupole excitation factors

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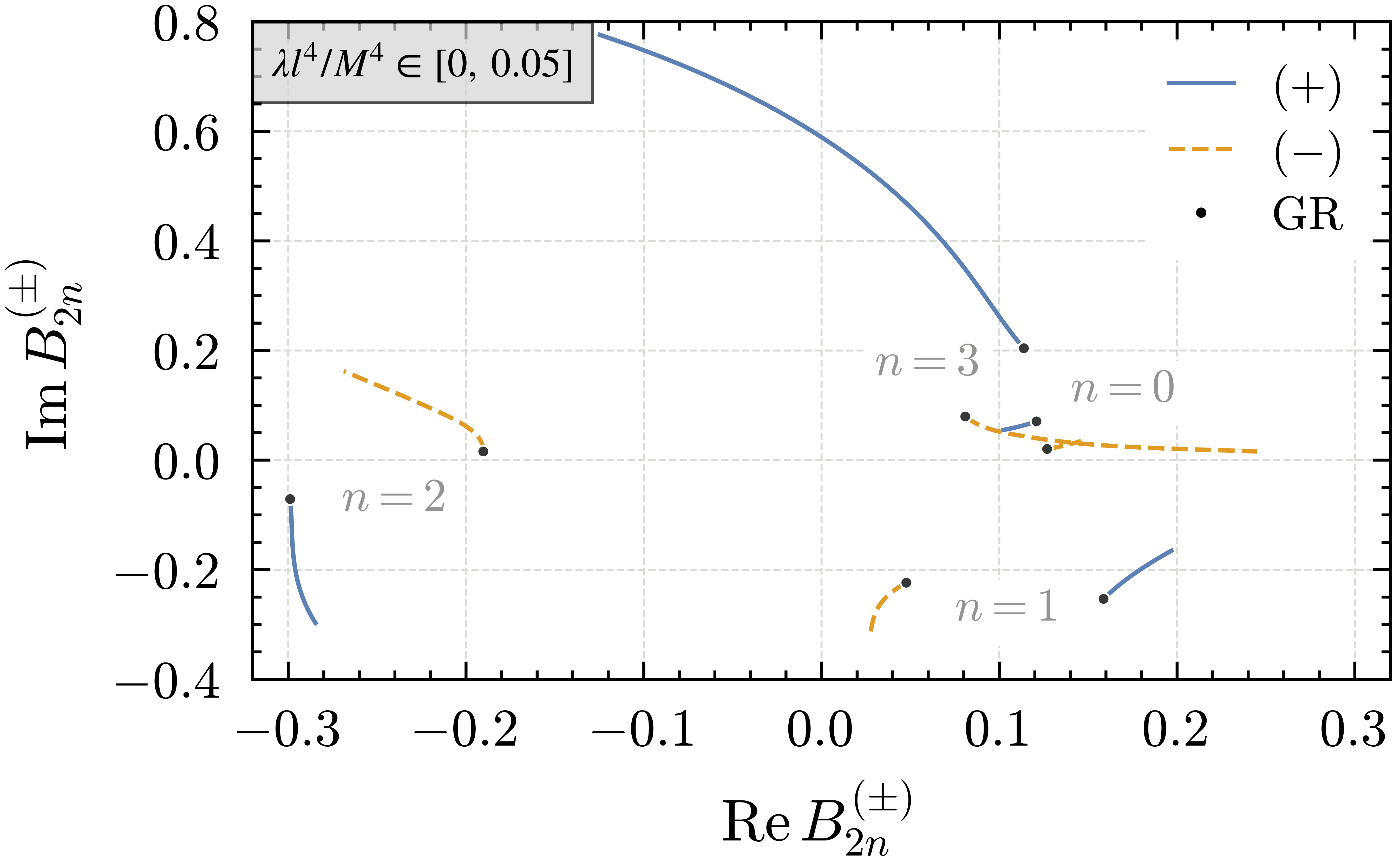
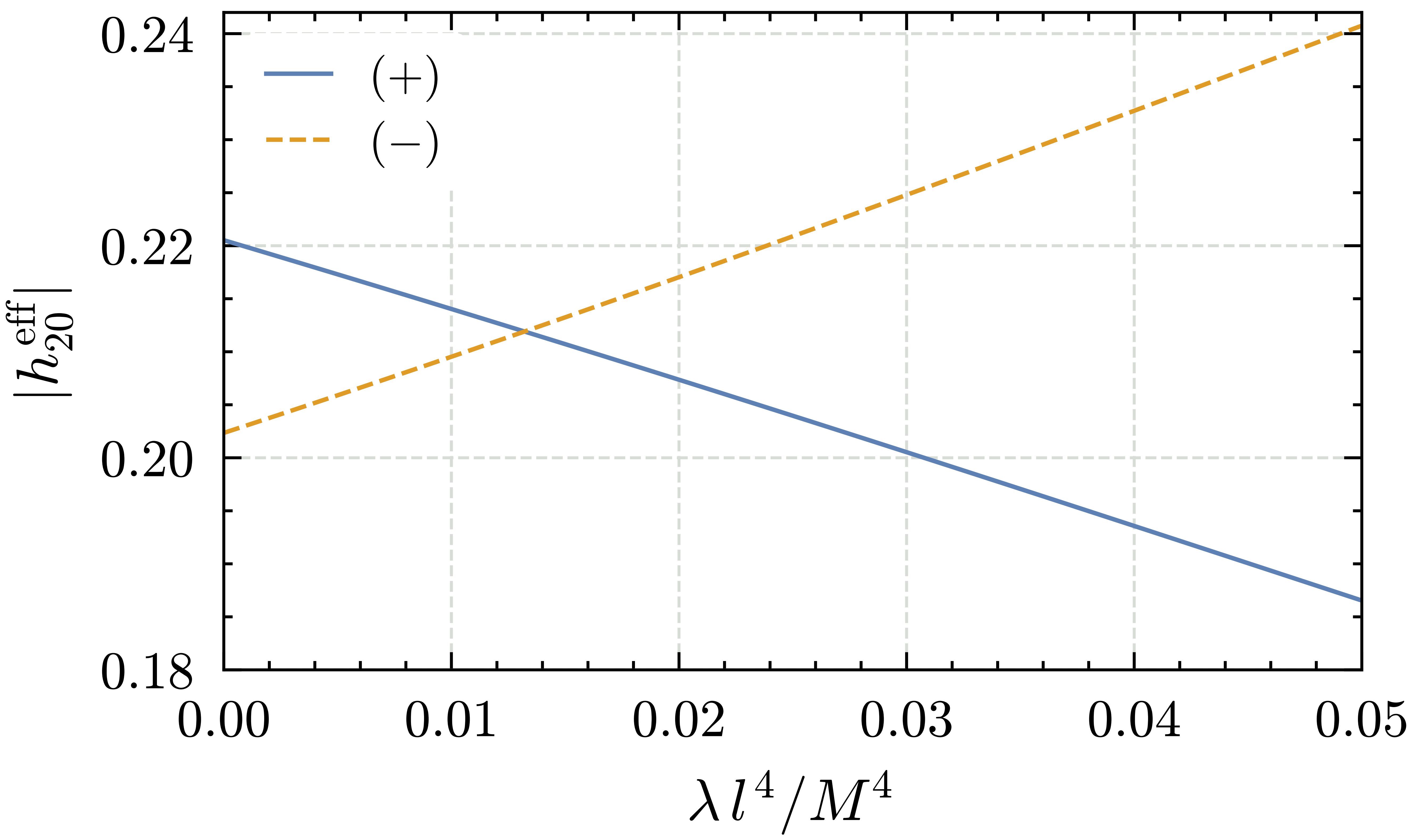


Effective gravitational wave amplitudes

Glampedakis and Andersson (2001, 2003)



Conclusions

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- A lot to explore!