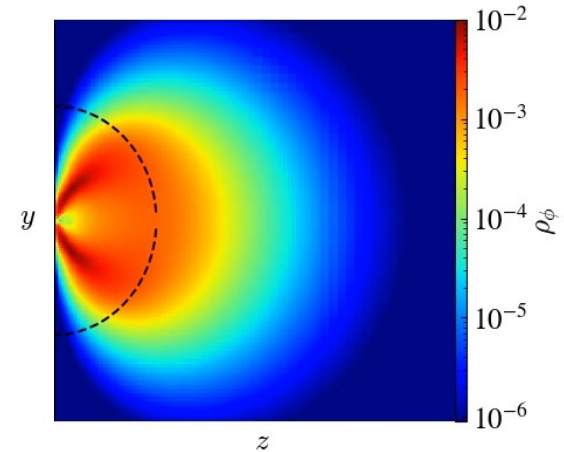


Non-linear dynamics in modified gravity

Llibert Aresté Saló

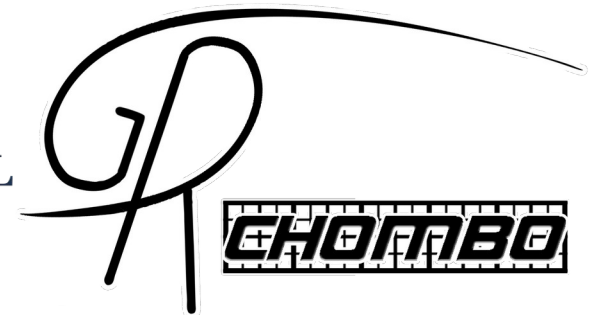
Benás, 10th July 2024



Queen Mary
University of London

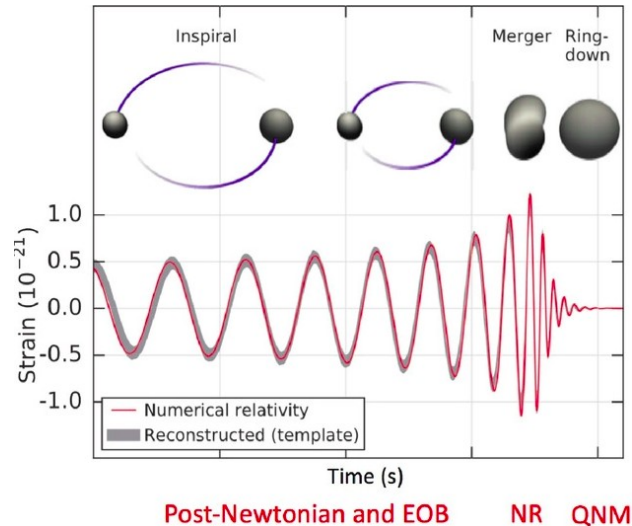


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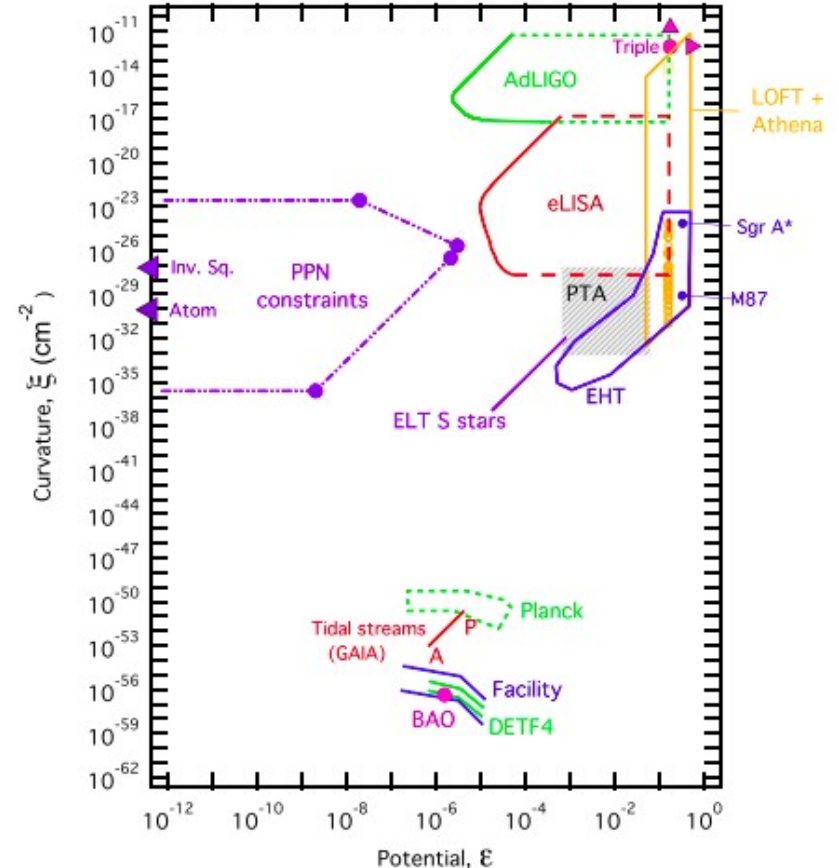


Motivation

- Detection of gravitational waves → Testing of the strong field regime.
- Numerical Relativity enables us to compute those waveforms.



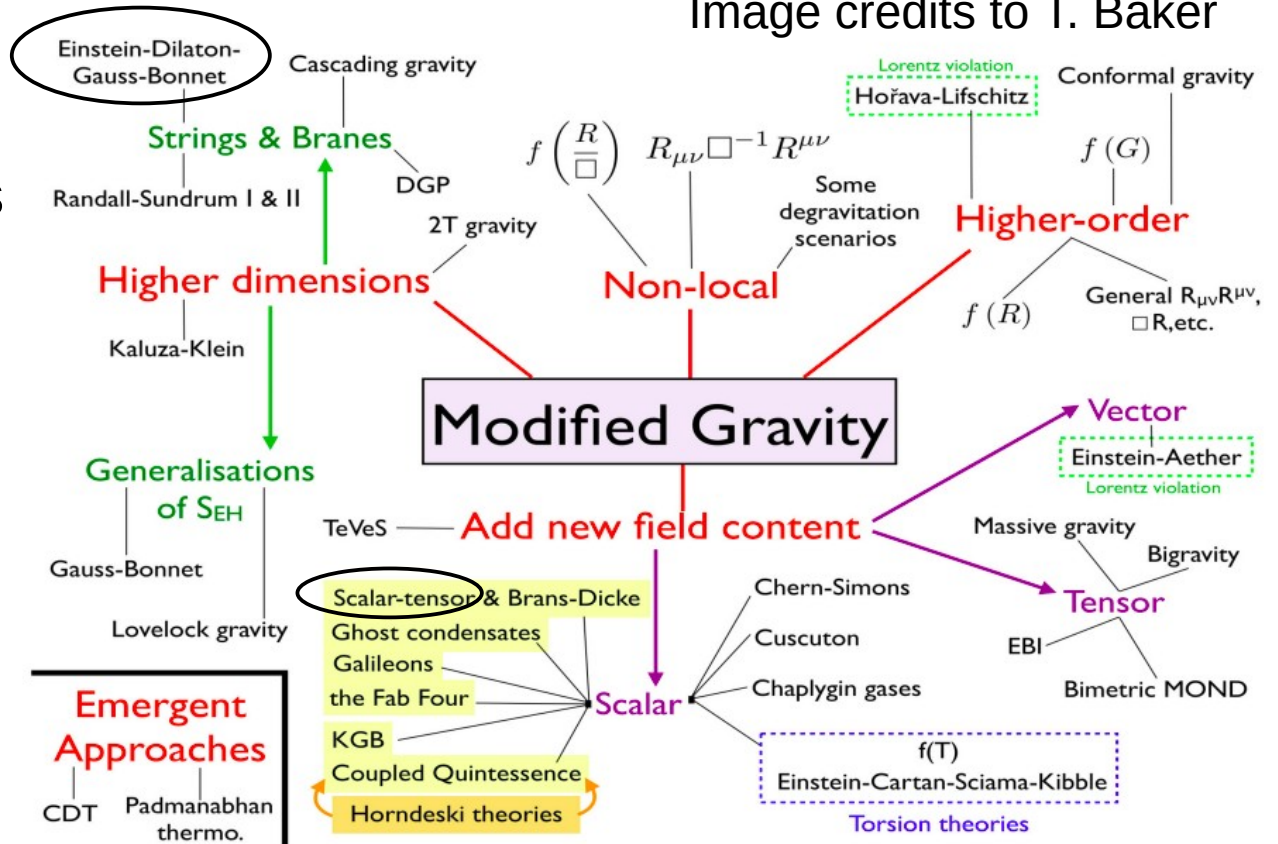
[Baker et al '15]



Modified gravity

Image credits to T. Baker

- Multiple possibilities for modifying gravity.
- Effective field theory.

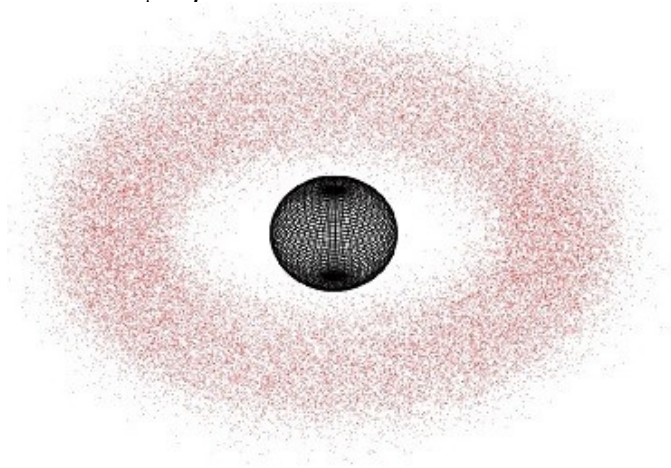


The theory

- We consider the Four-Derivative Scalar Tensor Theory of Gravity:

$$S = \int d^4x \sqrt{-g} (R + X - V(\phi) + g_2(\phi) X^2 + \lambda(\phi) \mathcal{L}^{GB}),$$
$$X = -\frac{1}{2} (\partial_\mu \phi)^2, \mathcal{L}^{GB} = R^2 - 4R_{\mu\nu} R^{\mu\nu} + R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}$$

- It violates the No-Hair Theorem.

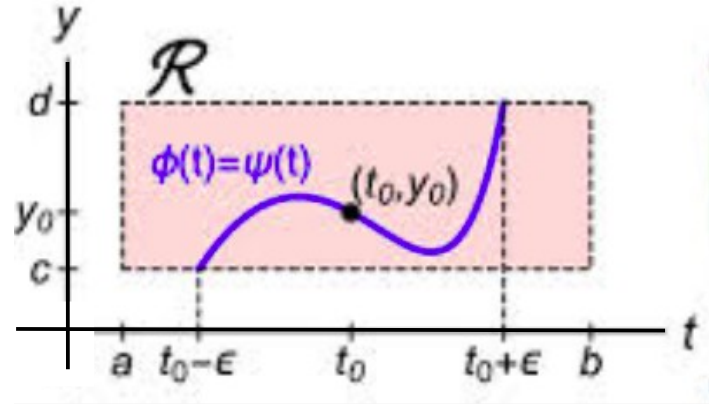


Well-posedness

- An initial value (or Cauchy) problem

$$\partial_t u = F(x^i, u, \partial_i u, \dots, \partial_{i_1} \dots \partial_{i_m} u, \dots)$$

$$u|_{t=0} = f(x^i)$$



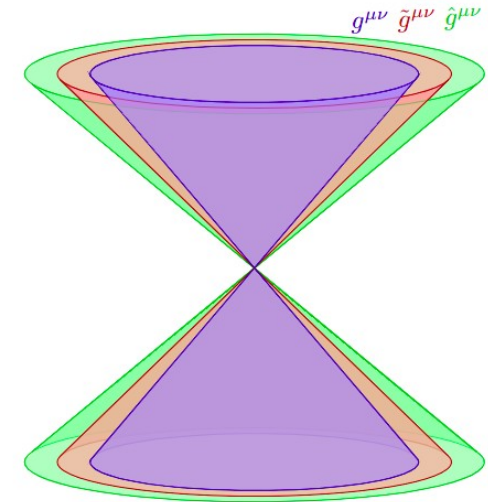
is (locally) well-posed if there exists a unique (local) solution for $t \in [0, T]$ for a given $T > 0$ which depends smoothly on the initial data.

- Only well-posed initial value problems can lead to stable Numerical Relativity simulations.

Modified Harmonic Gauge

- Proposed in [Kovács and Reall '20].
- Well-posed formulation of weakly coupled Lovelock and Horndeski theories of gravity.
- Different propagation speeds for the unphysical modes.
- Implemented in EsGB [East and Ripley '21, Corman, Ripley and East '23].

[Kovács and Reall '20]



$$\begin{aligned}\tilde{g}^{\mu\nu} &= g^{\mu\nu} - a(x)n^\mu n^\nu \\ \hat{g}^{\mu\nu} &= g^{\mu\nu} - b(x)n^\mu n^\nu \\ 0 &< a(x) < b(x)\end{aligned}$$

Our formulation

- Moving puncture gauge (singularity-avoiding coordinates).
- 1+log slicing for the lapse and Gamma driver for the shift:

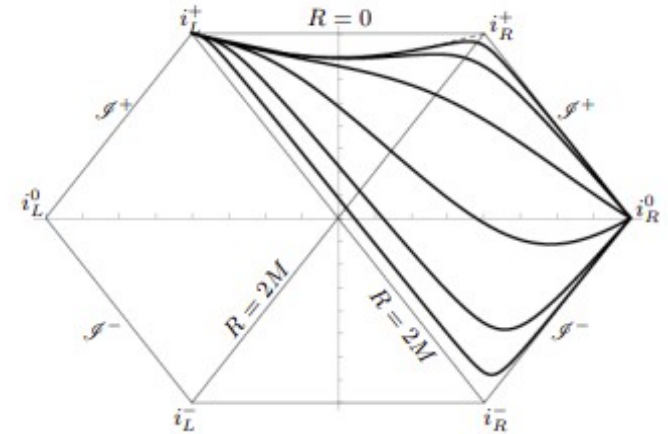
$$\partial_t \alpha = \beta^i \partial_i \alpha - 2\alpha(K - 2\Theta),$$

$$\partial_t \beta^i = \beta^j \partial_j \beta^i + \frac{d}{2(d-1)} \hat{\Gamma}^i,$$

which yield in the modified approach as

$$\partial_t \alpha = \beta^i \partial_i \alpha - \frac{2\alpha}{1+a(x)} (K - 2\Theta),$$

$$\partial_t \beta^i = \beta^j \partial_j \beta^i + \frac{d}{2(d-1)} \frac{\hat{\Gamma}^i}{1+a(x)} - \frac{a(x)\alpha D^i \alpha}{1+a(x)}.$$

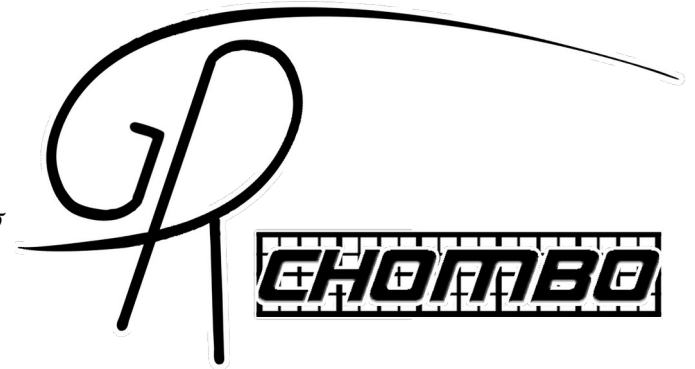


[Hannam et al '08]

Results

$$S = \int d^4x \sqrt{-g} (R + X + g_2(\phi) X^2 + \lambda(\phi) \mathcal{L}^{GB})$$

$$X = -\frac{1}{2} (\partial_\mu \phi)^2, \mathcal{L}^{GB} = R^2 - 4 R_{\mu\nu} R^{\mu\nu} + R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}$$

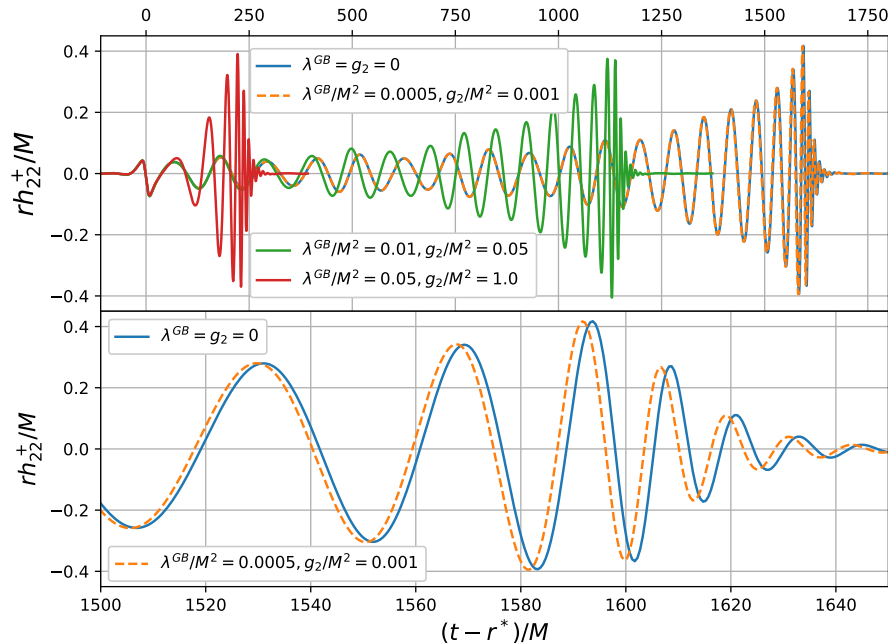


- Well-posed in our modified CCZ4 formulation in the weakly coupled regime.
- Implemented in its full non-linear form in **GRFolres** [LAS et al '23], an extension of **GRChombo**, <https://github.com/GRTLCollaboration/GRFolres>.

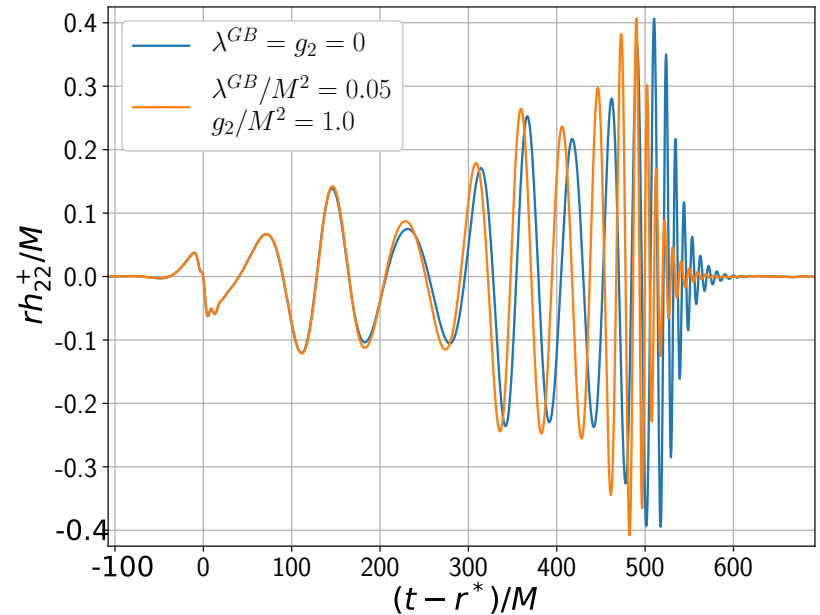
Shift-symmetric 4dST theory

- All Black Hole solutions are hairy.

$$\lambda(\phi) = \frac{1}{4} \lambda^{GB} \phi \quad g_2(\phi) = g_2$$



Non-spinning Black Hole binaries
[LAS, Clough and Figueras '22]

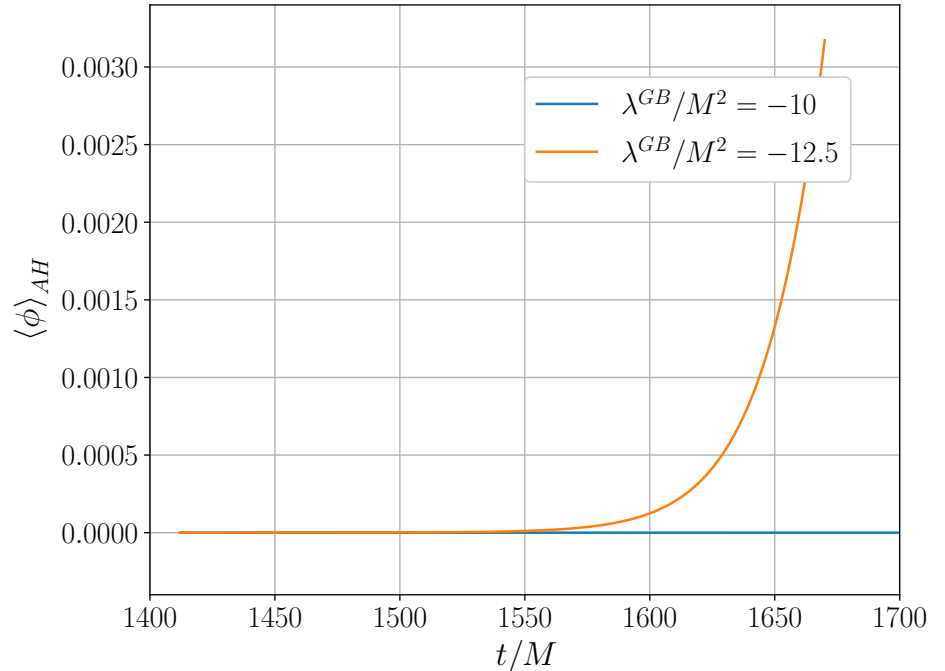


Initially spinning Black Hole binaries
[LAS, Clough and Figueras '23]

Quadratic EsGB

$$\lambda(\phi) = \frac{1}{4} \lambda^{GB} \phi^2$$

- Studied in [Silva et al '21, Elley et al '22] without backreaction.
- Hairy and non-hairy Black Holes.
- Spin-induced tachyonic instability that triggers spontaneous scalarisation [Silva, Sakstein, Gualtieri, Sotiriou and Berti '18].

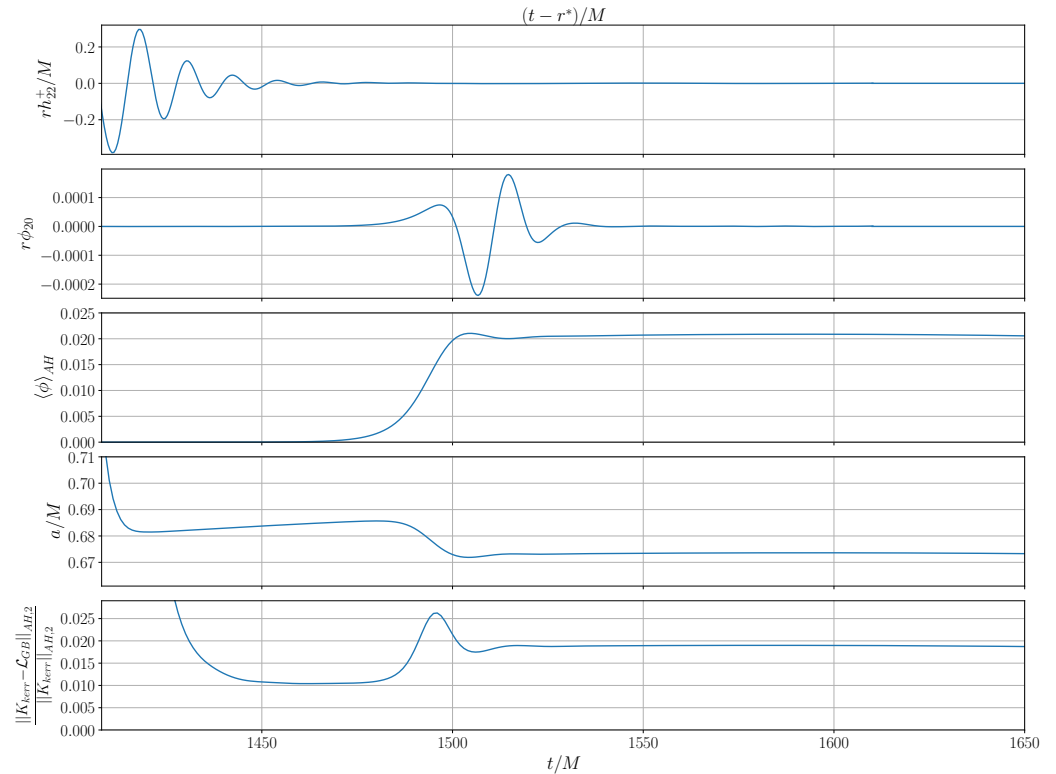


[LAS, Clough and Figueras '23]

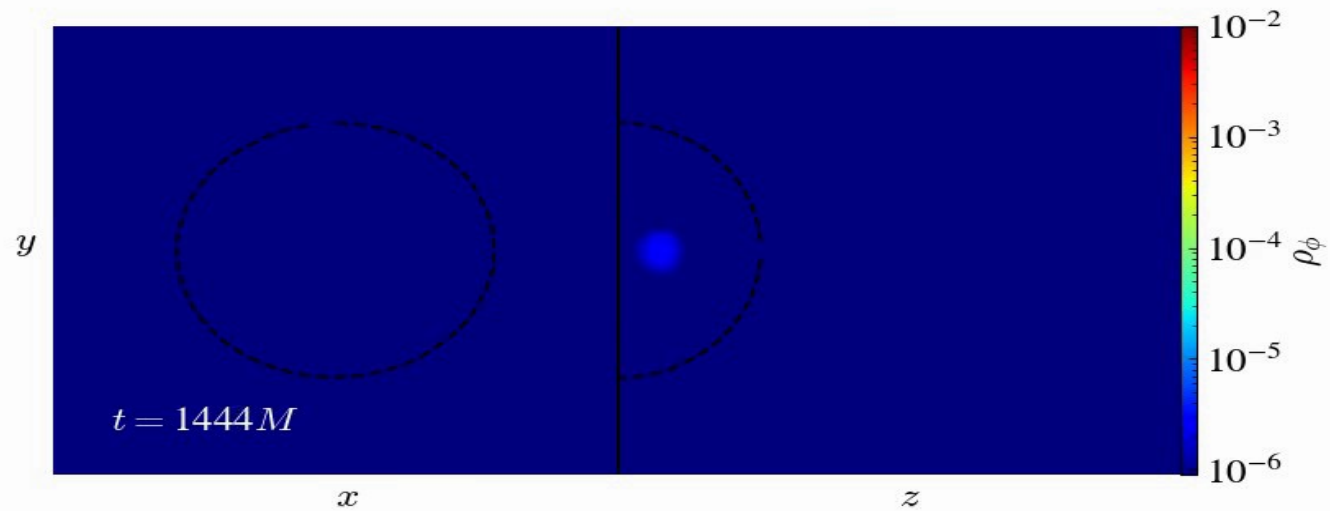
Exponential quadratic EsGB

$$\lambda(\phi) = \frac{1}{4\sigma} \lambda^{GB} (1 - e^{-\sigma\phi^2}) \quad \text{Grav. strain}$$

- Proposed in [Doneva et al '22].
 - Spin-induced scalarisation.
 - Stable hairy Black Hole Merger.
- Scalar radiation
- Exp. value of ϕ at the Merger's AH
- Spin
- Deviation from Kerr's Kretschmann

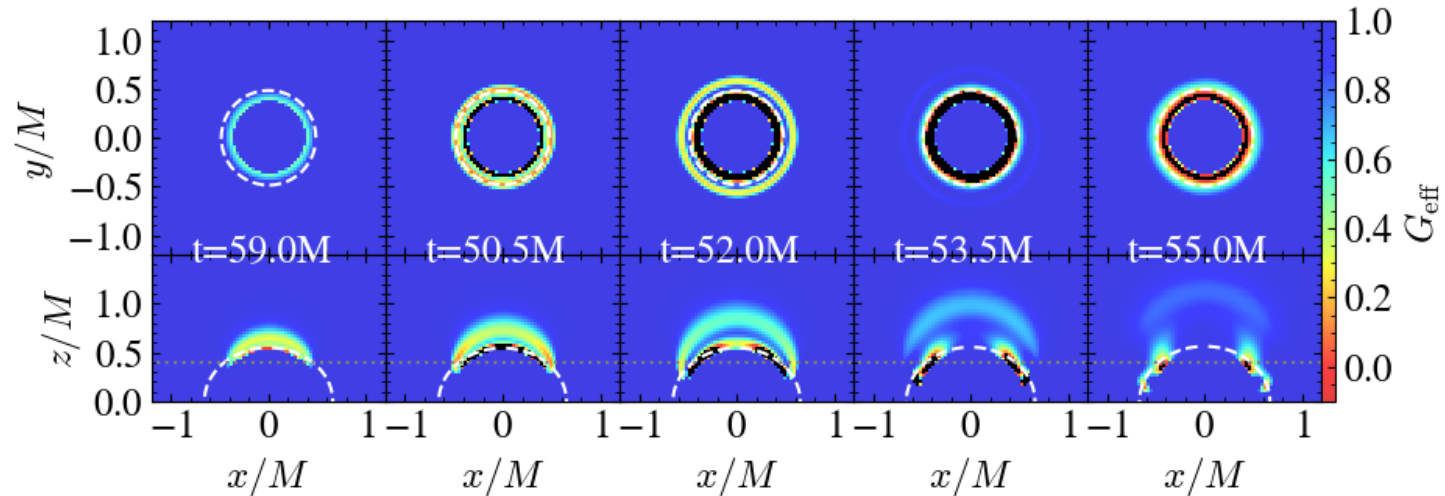


[LAS, Clough and Figueras '23]



Hyperbolicity loss

- Some of the physical modes lie on the null cone of an effective metric.
- The change of sign of its determinant determines the transition from a hyperbolic system to an elliptic system.

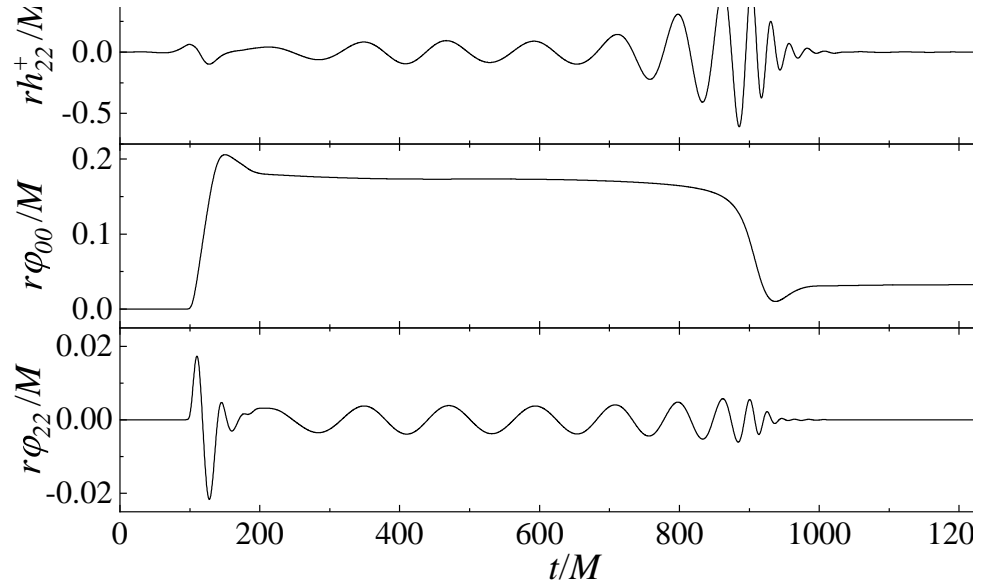


[Doneva, LAS, Clough, Figueras and Yazadjev '23]

See also [Ripley and Pretorius '19]

Non-equal mass binaries

- We have been able to evolve non-equal mass binaries with mass ratios 1:2 and 1:3

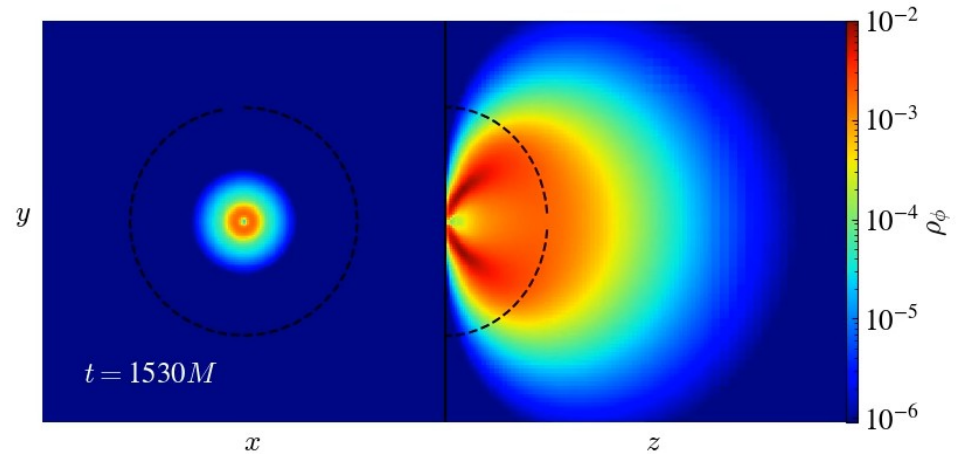


[LAS, Doneva, Clough, Figueras and Yazadjev '24] (in preparation)

See also [Corman, Ripley and East '23]

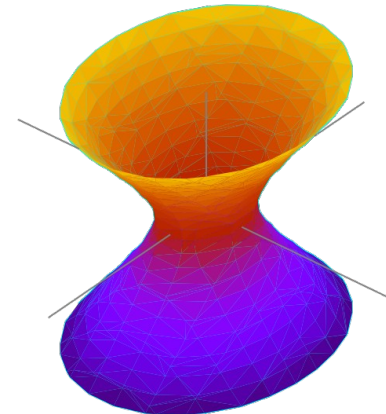
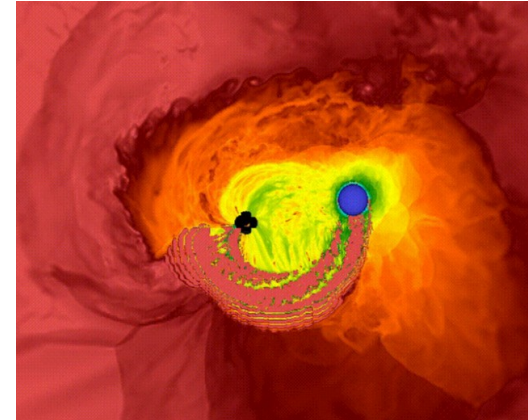
Summary

- Well-posed formulation in singularity-avoiding coordinates of the 4∂ ST theory.
- Equal and non-equal mass binaries simulations in the full non-linear theory.
- Non-trivial dynamics for the scalar field.



Further directions

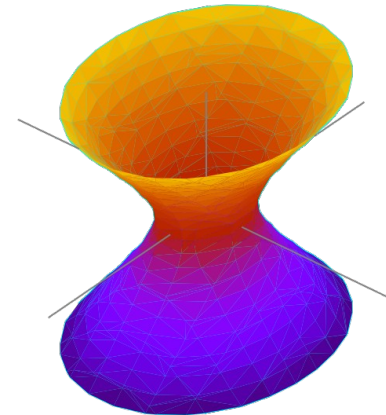
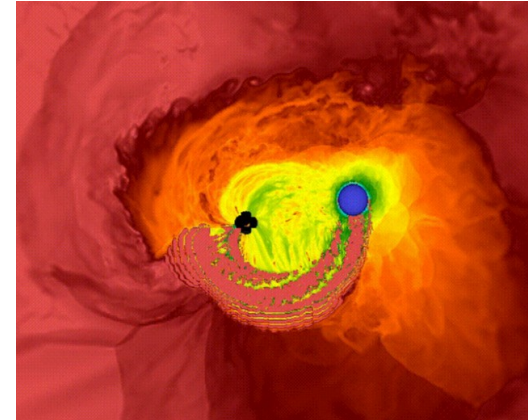
- MHD EsGB in MHDuet (in collaboration with Miguel Bezares).
- Hydrodynamics in GRChombo (in collaboration with Dina Traykova and Josu Aurrekoetxea).
- AdS spacetimes – work in progress with Pau Figueras.



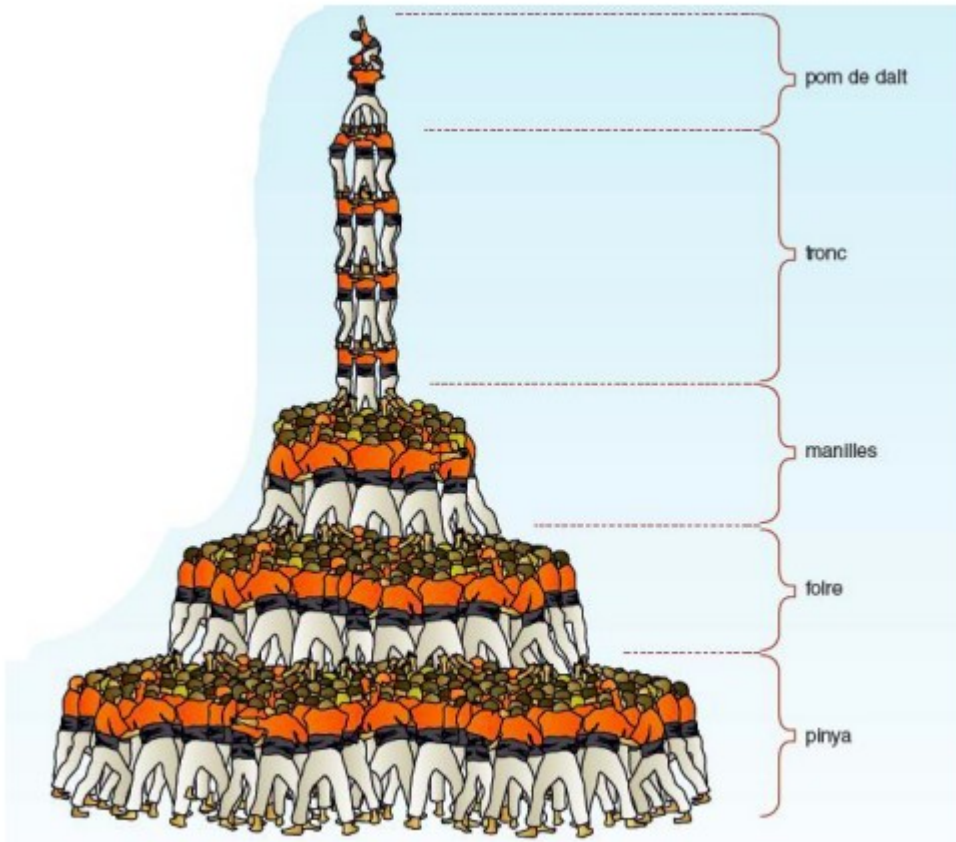
THANK YOU FOR YOUR ATTENTION!

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GRFolres



GRAVITY

...

Higher derivatives (GRManilles??)

4∂ ST (GRFolres)

GR (GRChombo / GRTeclyn)