Black hole simulations in axi-dilaton gravity

Chloe Richards; University of Illinois Urbana-Champaign

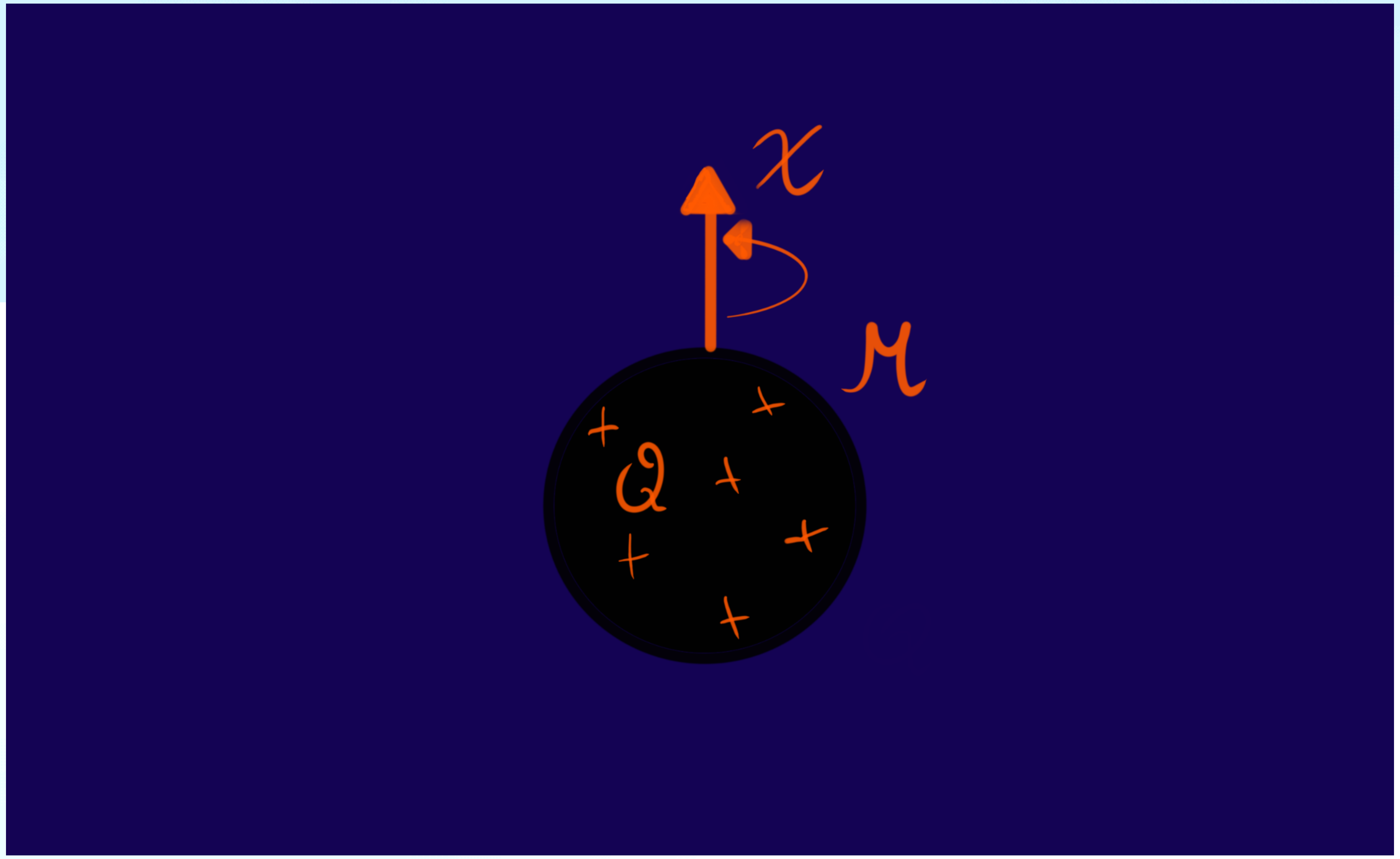
In collaboration with Alexandru Dima, Deborah Ferguson, and Helvi Witek

New Frontiers in Strong Gravity workshop
July 10, 2024

Revisiting General Relativity

A sketch

General Relativity



Kerr-Newman black hole

By Alexandru Dima

Quick review!

Kerr-Newman black hole described by:

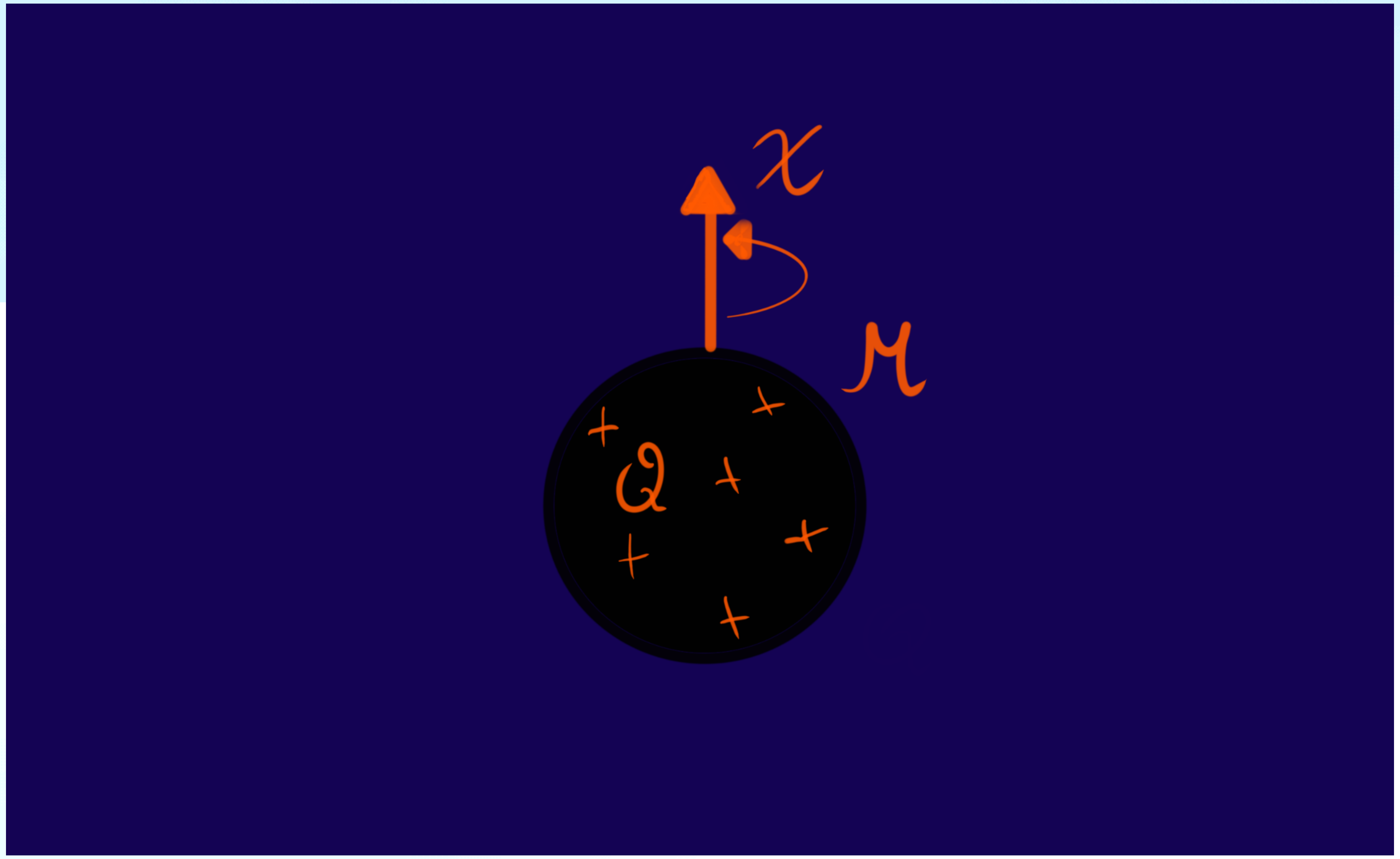
- Mass M
- Spin χ
- EM charge Q

No-hair conjectures

Beyond General Relativity?

A sketch

General Relativity



Kerr-Newman black hole

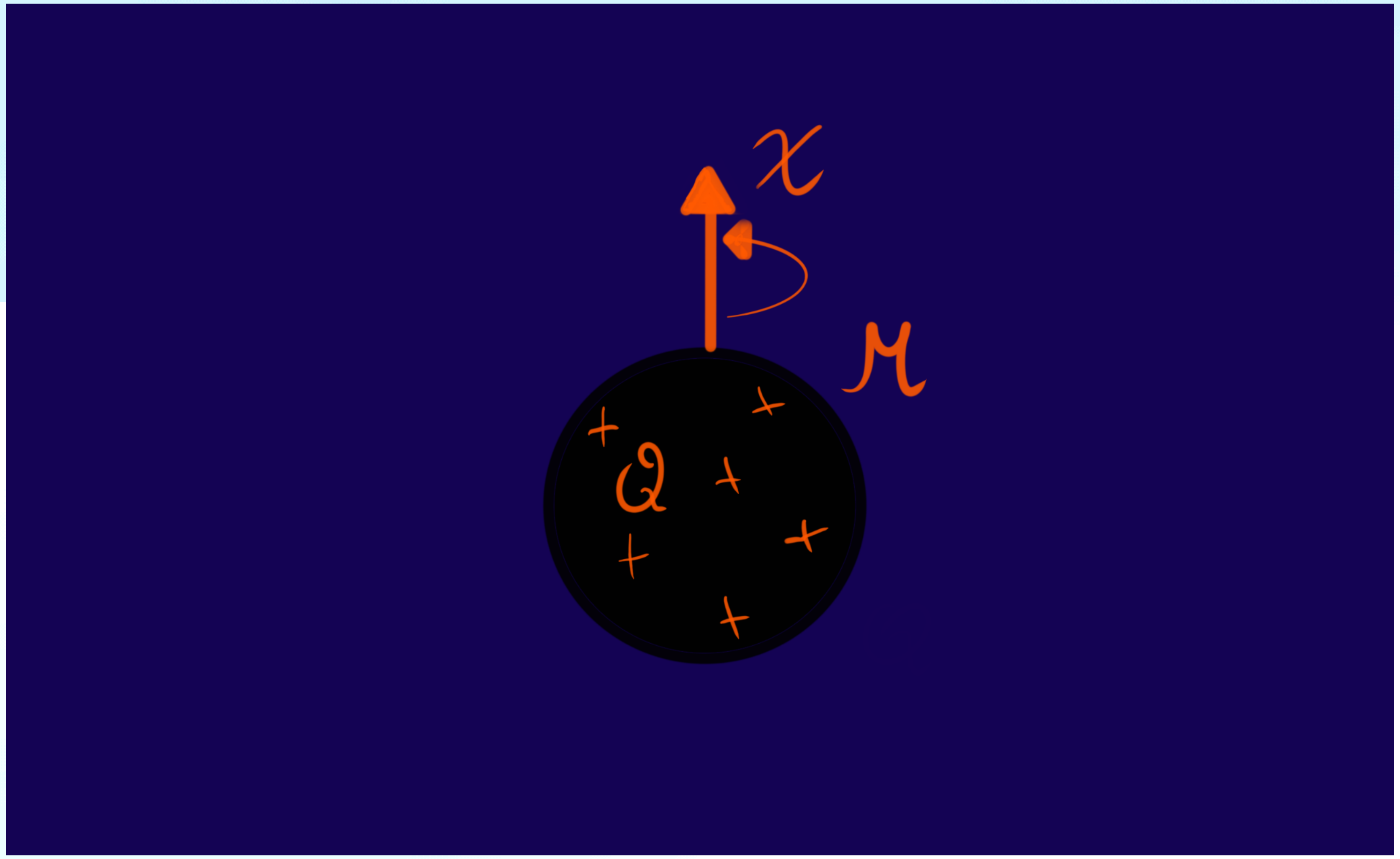
Beyond General Relativity

?

Beyond General Relativity

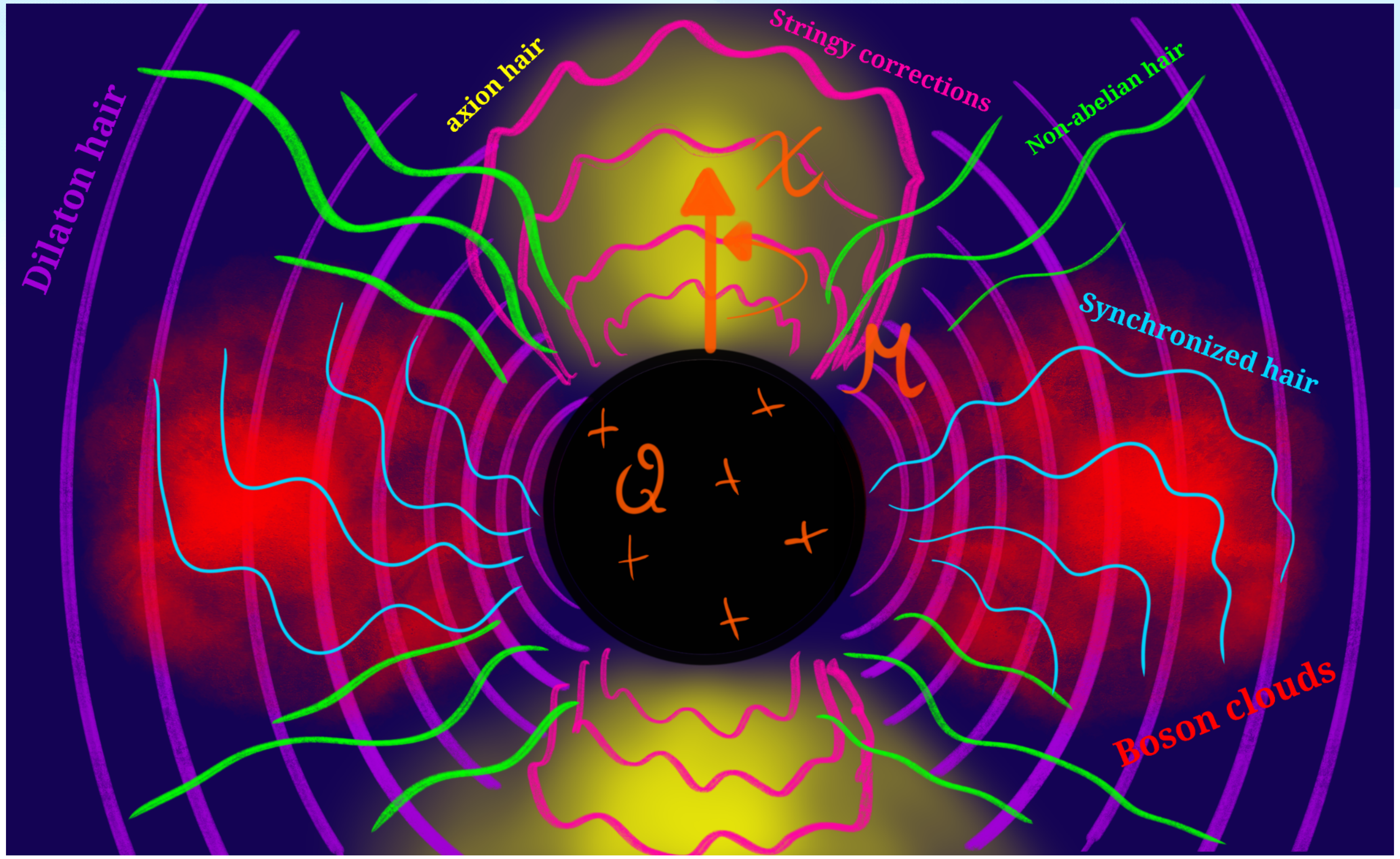
A sketch

General Relativity



Kerr-Newman black hole

Beyond General Relativity



Kerr-Newman black hole + …

- “Exotic” matter
- Modified gravity

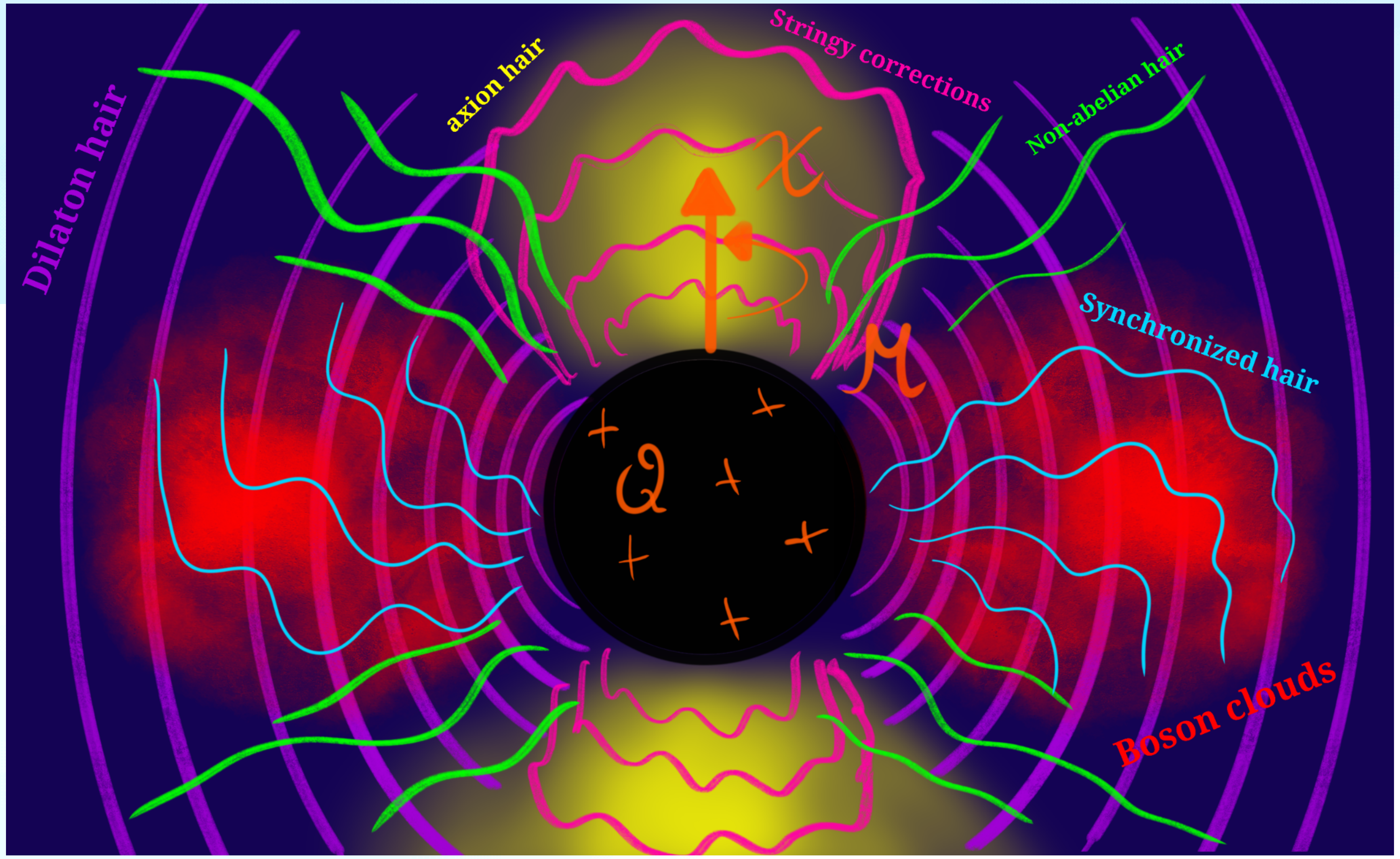
By Alexandru Dima

Introducing axi-dilaton gravity I

Theoretical background

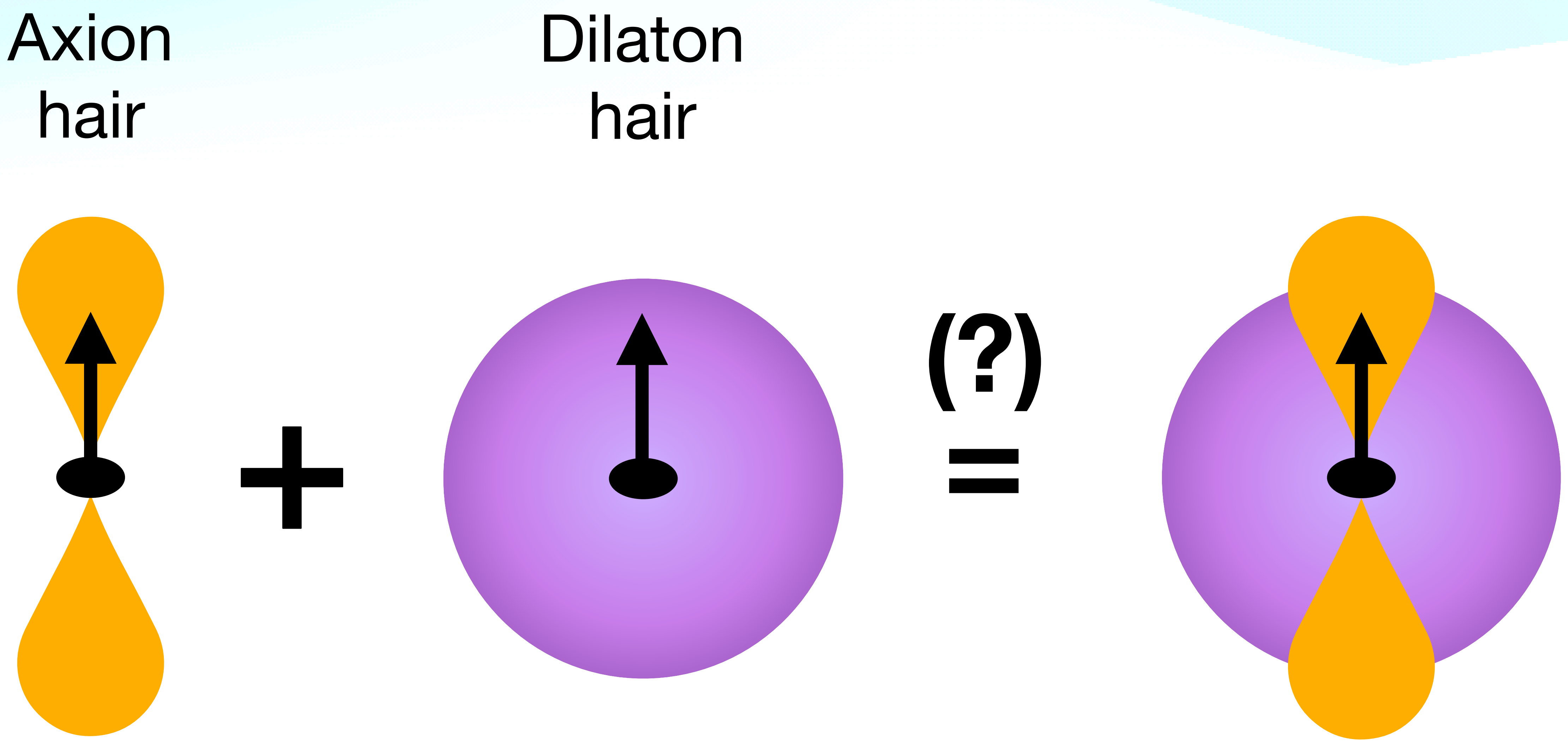
Beyond General Relativity →

Axi-dilaton gravity
focus on the growth and formation
of axion and dilaton hair



Kerr-Newman black hole + …

- “Exotic” matter
- Modified gravity



Axion hair + Dilaton hair (?) =

Introducing axi-dilaton gravity II

Action

$$S=\int d^4x\frac{\sqrt{-g}}{16\pi}\left[\underbrace{R}_{\textbf{Einstein-Hilbert}}+\underbrace{\frac{\alpha_{GB}}{4}f(\overset{\substack{\text{Dilaton}\\ \downarrow}}{\Phi})\mathcal{G}-\frac{1}{2}(\nabla\Phi)^2-V(\Phi)}_{\textbf{Scalar Gauss-Bonnet (sGB)}}-g^2(\Phi)\underbrace{\left(\frac{1}{2}(\nabla\Theta)^2+V(\Theta)\right)+\frac{\alpha_{CS}}{4}h(\overset{\substack{\text{Axion}\\ \downarrow}}{\Theta}){}^*RR}_{\textbf{Dynamical Chern-Simons (dCS)}}\right]$$

Scalar Gauss-Bonnet (sGB)

$$(h(\Theta)=g(\Phi)=0)$$

with

$$\mathcal{G}=R^2-4R_{ab}R^{ab}+R_{abcd}R^{abcd}$$

Dynamical Chern-Simons (dCS)

$$(f(\Phi)=0,\ g(\Phi)=1)$$

with

$${}^*RR=-\frac{1}{2}\epsilon^{cd}{}_{ef}R^{abef}R_{abcd}$$

Axi-dilaton gravity [1], [2]: $f(\Phi)=g(\Phi)^{-1}=e^{-\Phi}$ and $h(\Theta)=\Theta$

---

[1] P. Kanti et al. (1995)

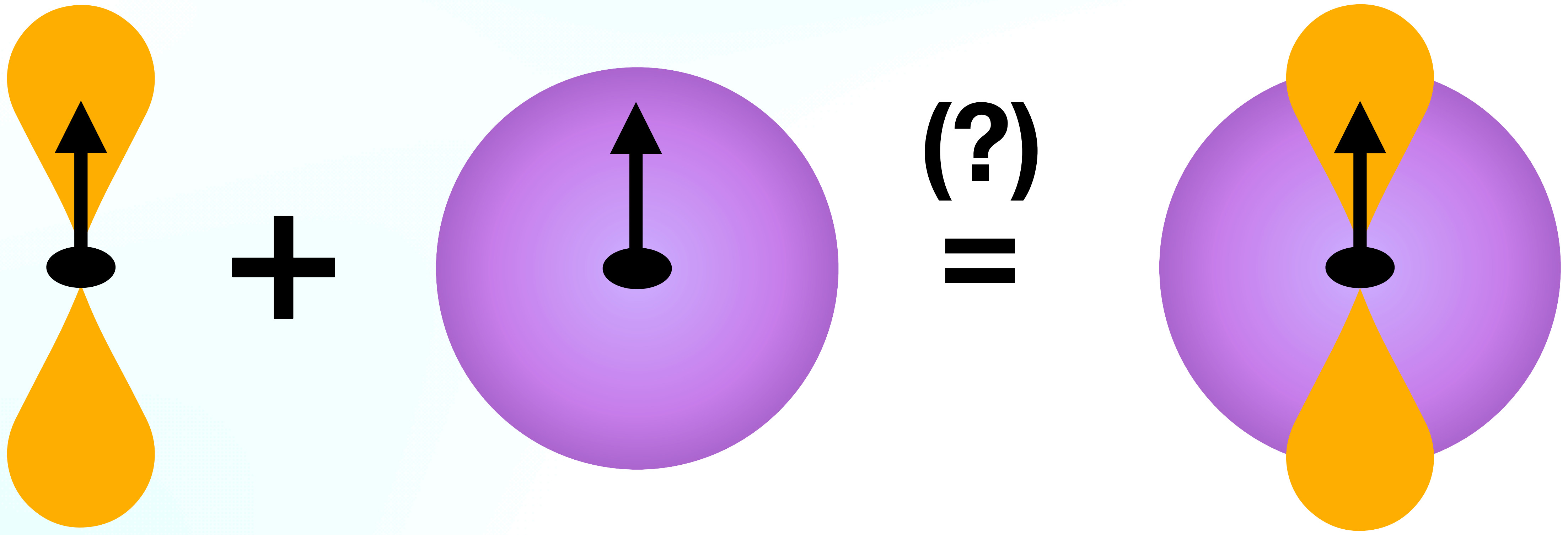
[2] P. A. Cano et al. (2022)

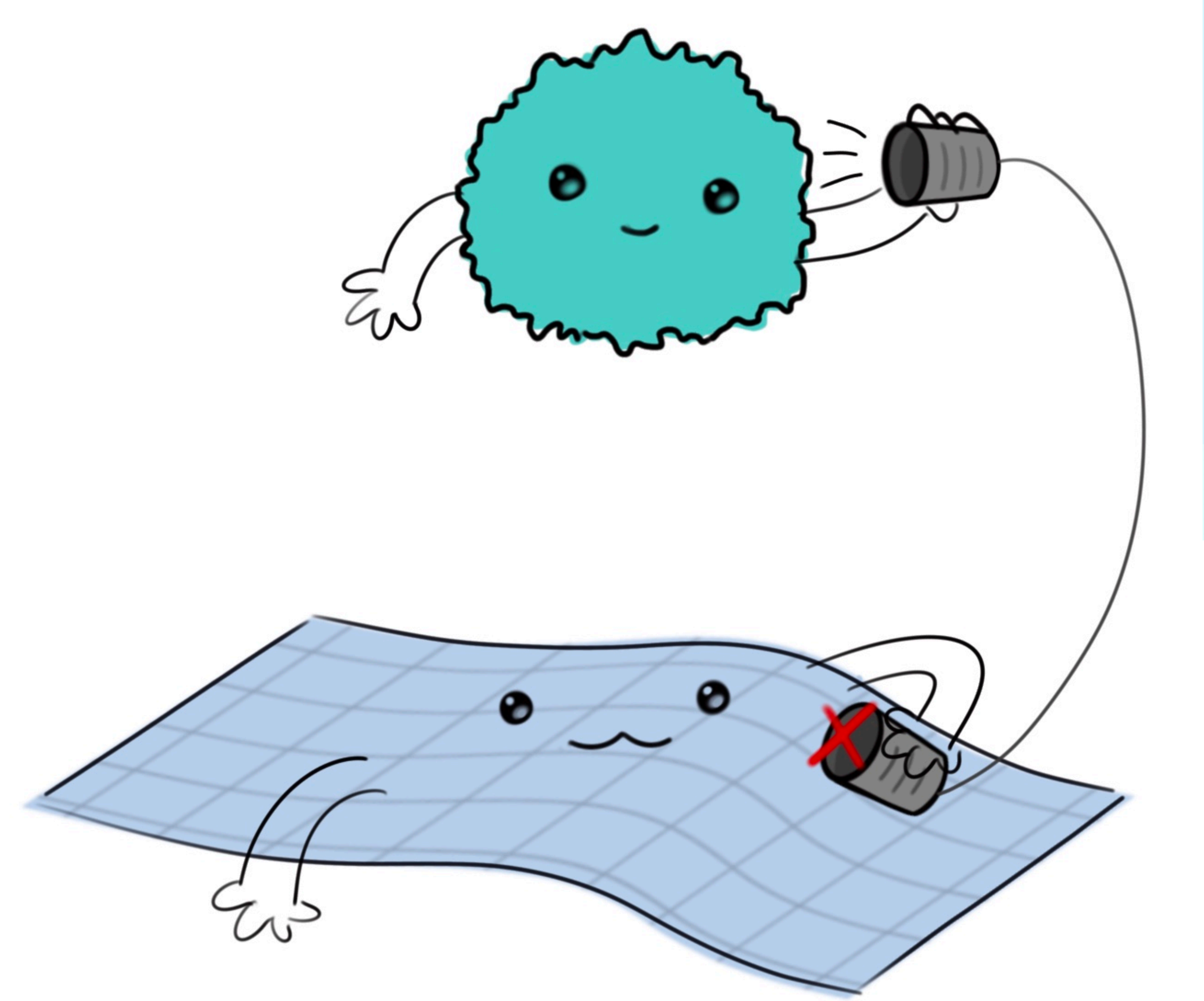
Introducing axi-dilaton gravity III

Field equations in decoupling limit

$$\Box\Theta - \dot{V}(\Theta) + \frac{\alpha_{CS}}{4}\frac{\dot{h}(\Theta)}{g(\Phi)^2}{}^{*}RR + 2\frac{g'(\Phi)}{g(\Phi)}\nabla_\mu\Phi\,\nabla^\mu\Theta = 0$$

$$\Box\Phi - V'(\Phi) + \frac{\alpha_{GB}}{4}f'(\Phi)\mathscr{G} - g'(\Phi)g(\Phi)\left[(\nabla\Theta)^2 + 2V(\Theta)\right] = 0$$





By Noora Ghadiri

---

[3] M. Okounkova et al. (2017) **[5]** W. East et al. (2021) **[7]** M. Corman et al. (2023) **[9] C. Richards** et al. (2023)

[4] H. Witek et al. (2019) **[6]** L. Aresté Saló et al. (2022) **[8]** D. Doneva et al. (2023)

Introducing axi-dilaton gravity III

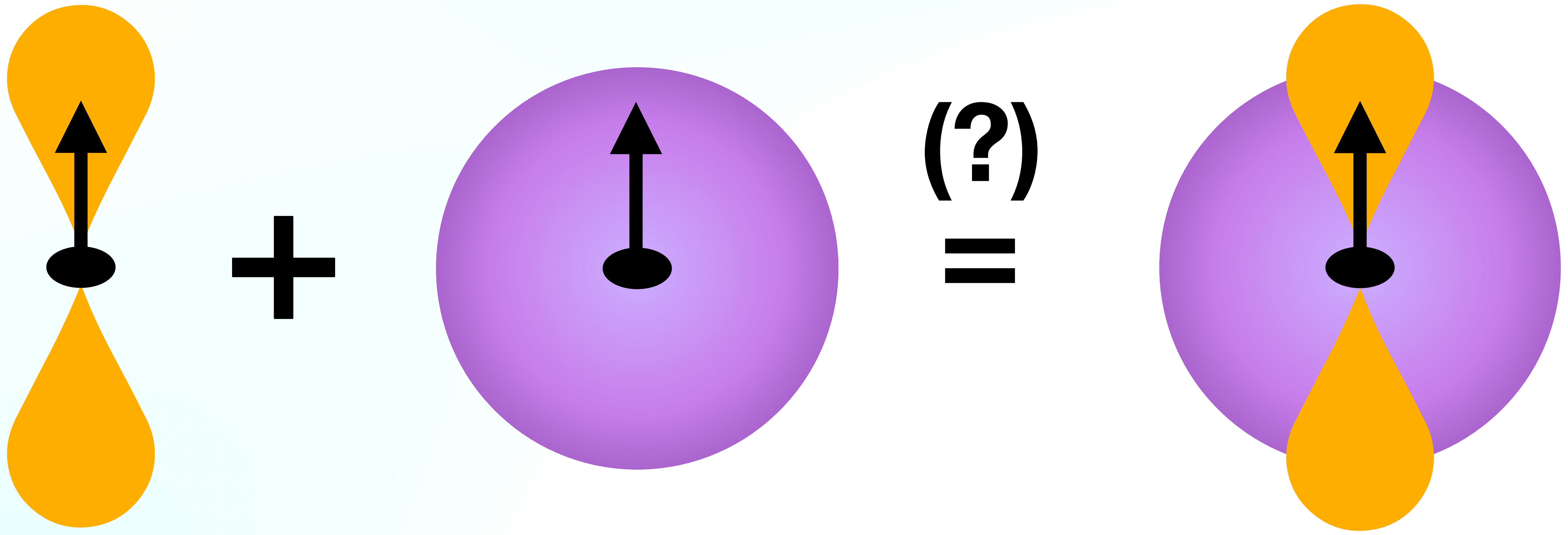
Field equations in decoupling limit

$$\Box\Theta - \dot{V}(\Theta) + \frac{\alpha_{CS}}{4}\frac{\dot{h}(\Theta)}{g(\Phi)^2}{}^*RR + 2\frac{g'(\Phi)}{g(\Phi)}\nabla_\mu\Phi\,\nabla^\mu\Theta = 0$$

$$\Box\Phi - V'(\Phi) + \frac{\alpha_{GB}}{4}f'(\Phi)\mathcal{G} - g'(\Phi)g(\Phi)\left[(\nabla\Theta)^2 + 2V(\Theta)\right] = 0$$

Model selection:
Axi-dilaton gravity

$$\begin{bmatrix} h(\Theta) = \Theta \\ f(\Phi) = e^{\lambda\Phi} \\ g(\Phi) = f(\Phi)^{-1} = e^{-\lambda\Phi} \end{bmatrix}$$



---

[3] M. Okounkova et al. (2017) **[5]** W. East et al. (2021) **[7]** M. Corman et al. (2023) **[9] C. Richards** et al. (2023)
[4] H. Witek et al. (2019) **[6]** L. Aresté Saló et al. (2022) **[8]** D. Doneva et al. (2023)

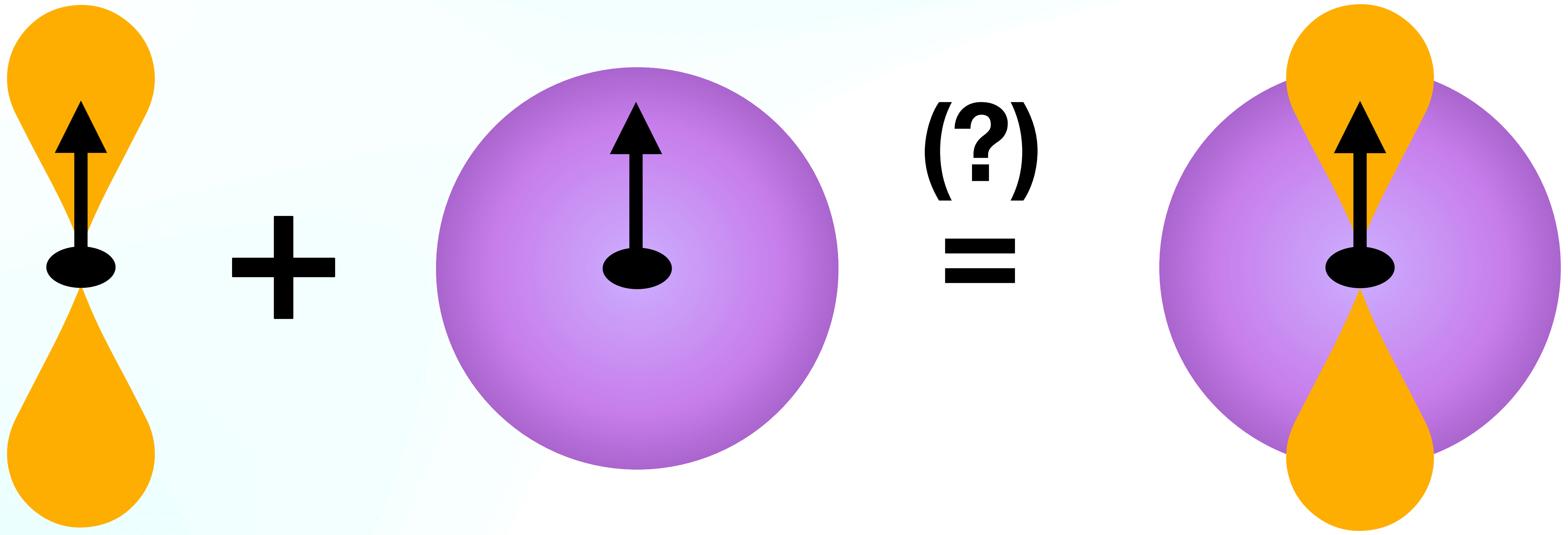
Introducing axi-dilaton gravity III

Field equations in decoupling limit

$$\Box\Theta - \dot{V}(\Theta) + \frac{\alpha_{CS}}{4}\frac{\dot{h}(\Theta)}{g(\Phi)^2}{}^*RR + 2\frac{g'(\Phi)}{g(\Phi)}\nabla_\mu\Phi\,\nabla^\mu\Theta = 0$$

$$\Box\Phi - V'(\Phi) + \frac{\alpha_{GB}}{4}f'(\Phi)\mathcal{G} - g'(\Phi)g(\Phi)\left[(\nabla\Theta)^2 + 2V(\Theta)\right] = 0$$

Q: what happens to the axion and dilaton hair with the additional coupling $g(\Phi)$ between the fields?



---

[3] M. Okounkova et al. (2017)
[4] H. Witek et al. (2019)
[5] W. East et al. (2021)
[6] L. Aresté Saló et al. (2022)
[7] M. Corman et al. (2023)
[8] D. Doneva et al. (2023)
[9] **C. Richards** et al. (2023)

Side quest: numerical relativity in a nutshell

3+1 formulation

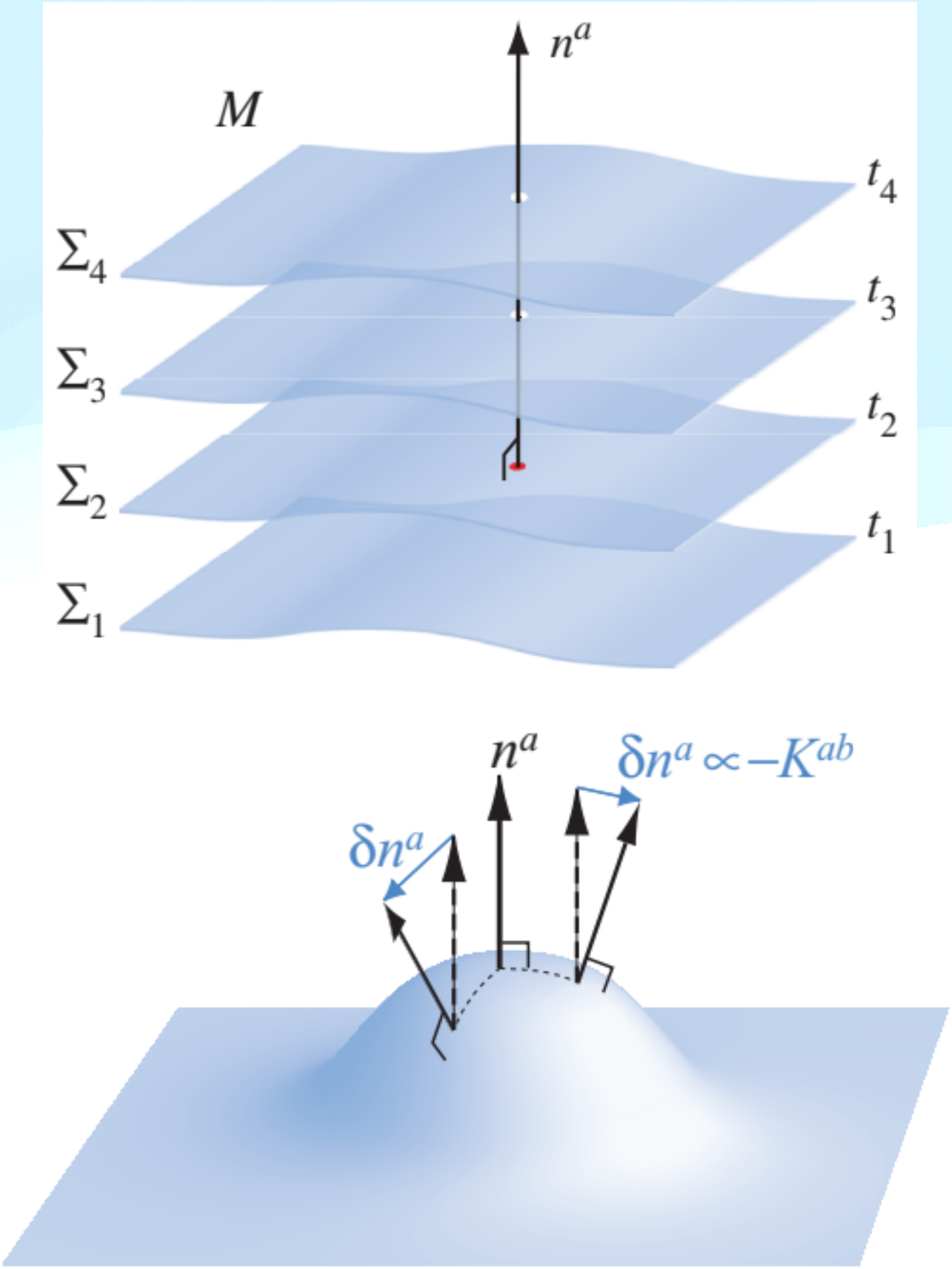
- Metric evolution given by extrinsic curvature
  - Think of as “momentum of metric”

$$K_{ij} \propto \mathscr{L}_n \gamma_{ij}$$

- Similarly, define momentum of field

$$K_\Theta \propto \mathscr{L}_n \Theta$$

- Implement 3+1 scalar equations of motion in the decoupling approximation



Figs. 2.1 and 2.2 **[10]**

---

[10] T. Baumgarte et al. (2010)

Axi-dilaton code description

Parameterized numerical relativity code for theories of quadratic gravity

SCAN FOR dCS CODE

SCAN FOR dCS THORNLIST

- Implement axi-dilaton code in the decoupling limit with open-source Canuda software **[11]** in the Einstein Toolkit **[12]**
- **Background:** Kerr in quasi isotropic coordinates
- **Axion and dilaton initial data:** approximate analytical solution **[2]**

$$\Theta|_{t=0} \sim \frac{\alpha_{CS}}{M^2}\frac{a}{M}\cos\theta\frac{M^2}{r^2} \qquad \& \qquad \Phi|_{t=0} \sim \frac{\alpha_{GB}}{M^2}\frac{M}{r}$$

$$K_{\Theta}|_{t=0} = 0 \qquad K_{\Phi}|_{t=0} = 0$$

- Evolve axion and dilaton with BSSN formulation **[13], [14]**

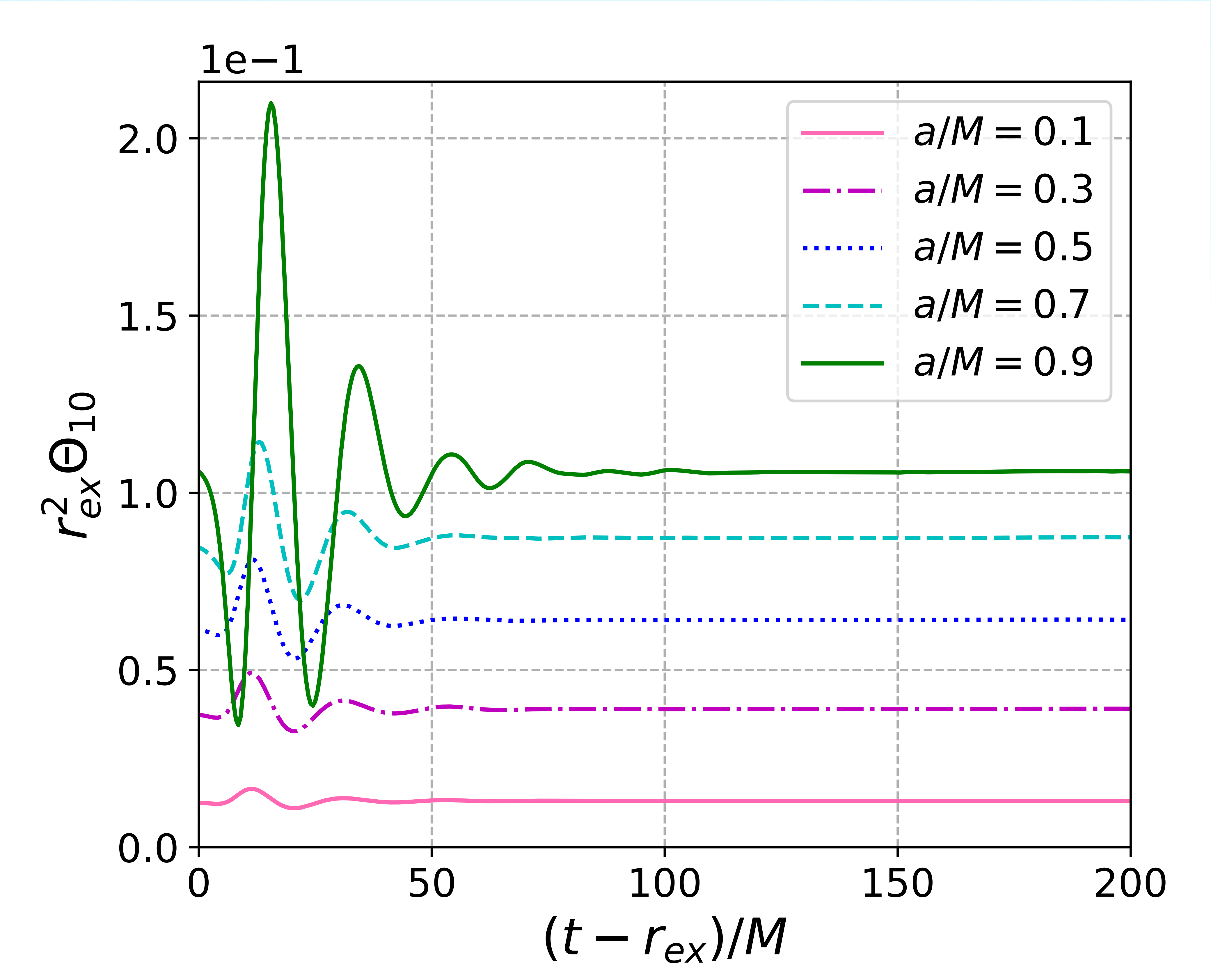
---

[2] P. A. Cano et al. (2022)
[11] H. Witek et al. (2023)
[12] L. Werneck et al. (2023)
[13] Shibata et al. (1995)
[14] Baumgarte et al. (1998)

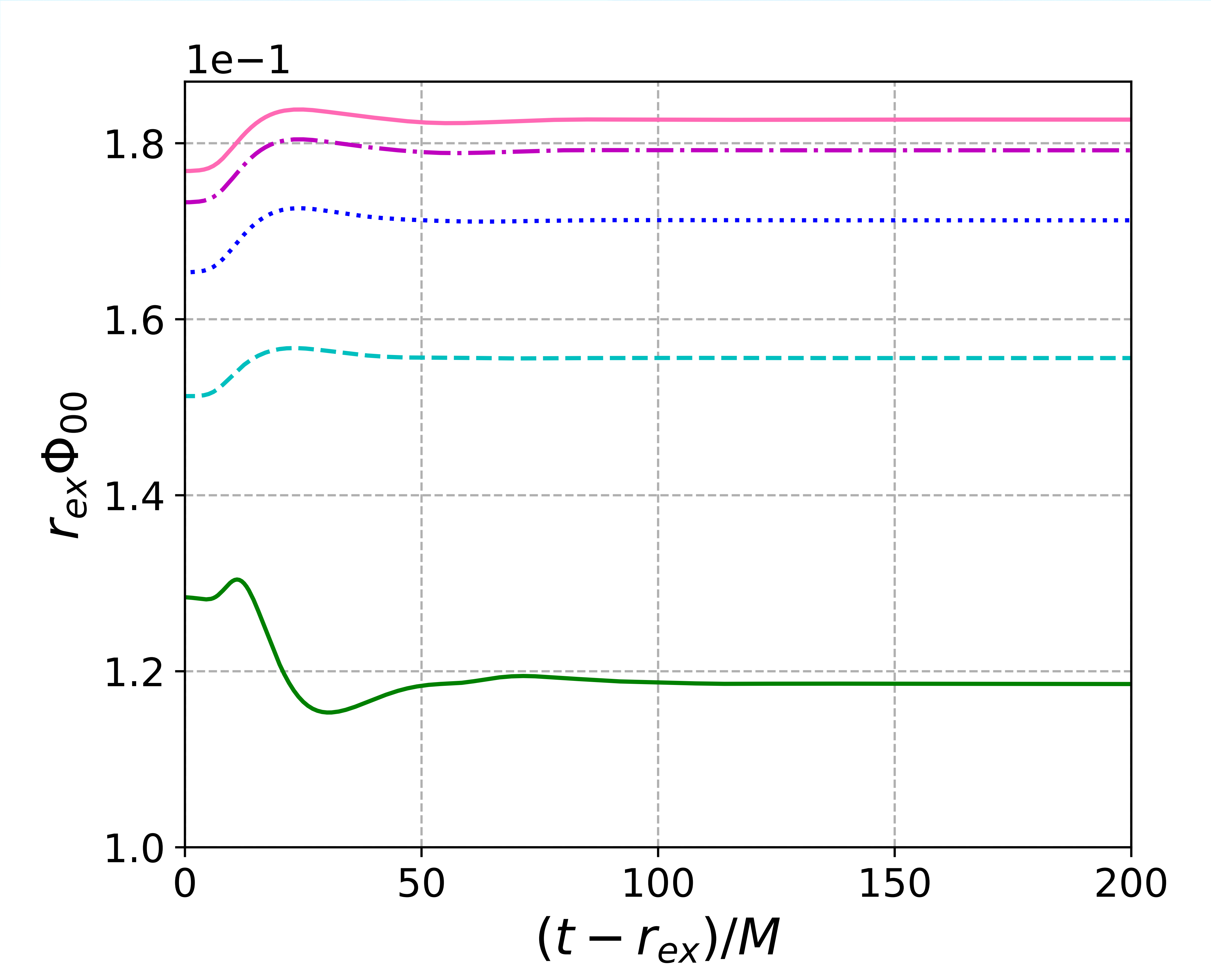
Numerical results

Evolution of axion and dilaton hair; $\hat{\alpha}_{CS} = \hat{\alpha}_{GB} = 0.1$, $r_{ex} = 20M$

Evolution of axion hair

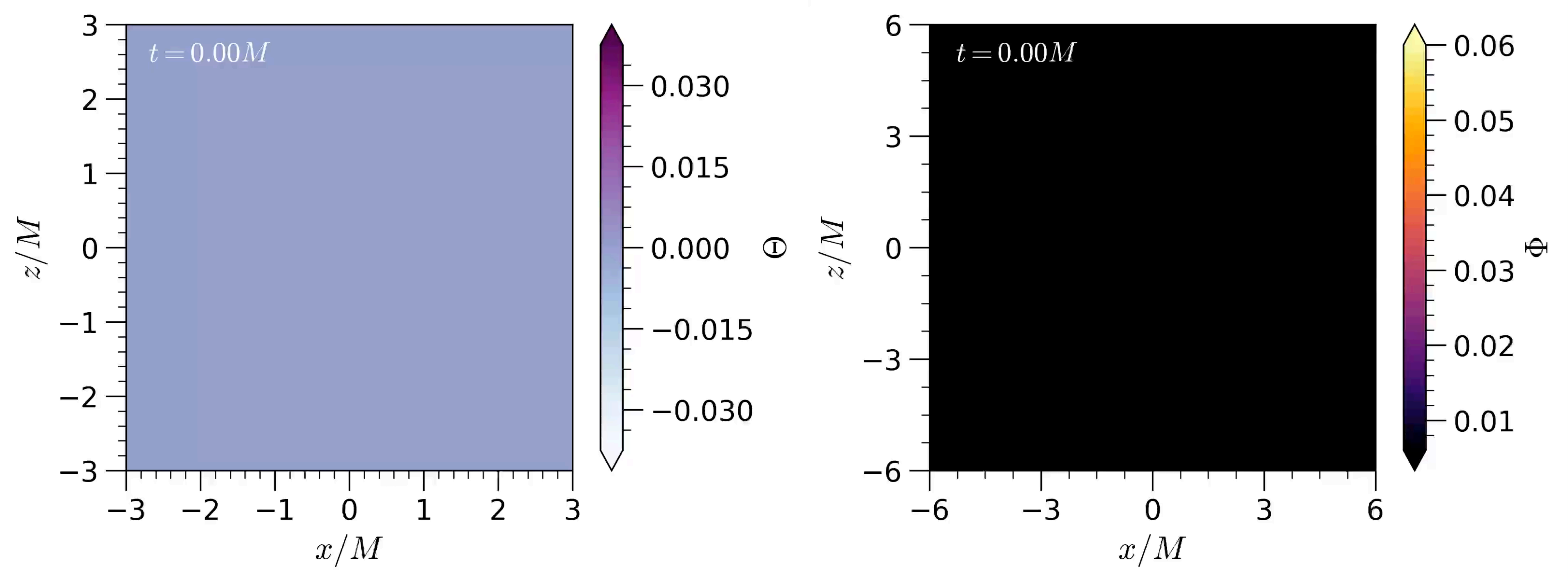


Evolution of dilaton hair



Numerical Results III

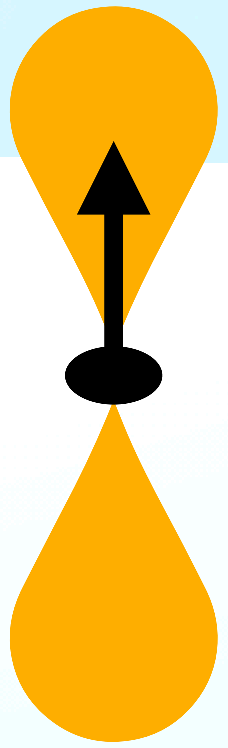
2D evolution of axion and dilaton fields; $\hat{\alpha}_{CS} = \hat{\alpha}_{GB} = 0.1$, $a/M = 0.9$

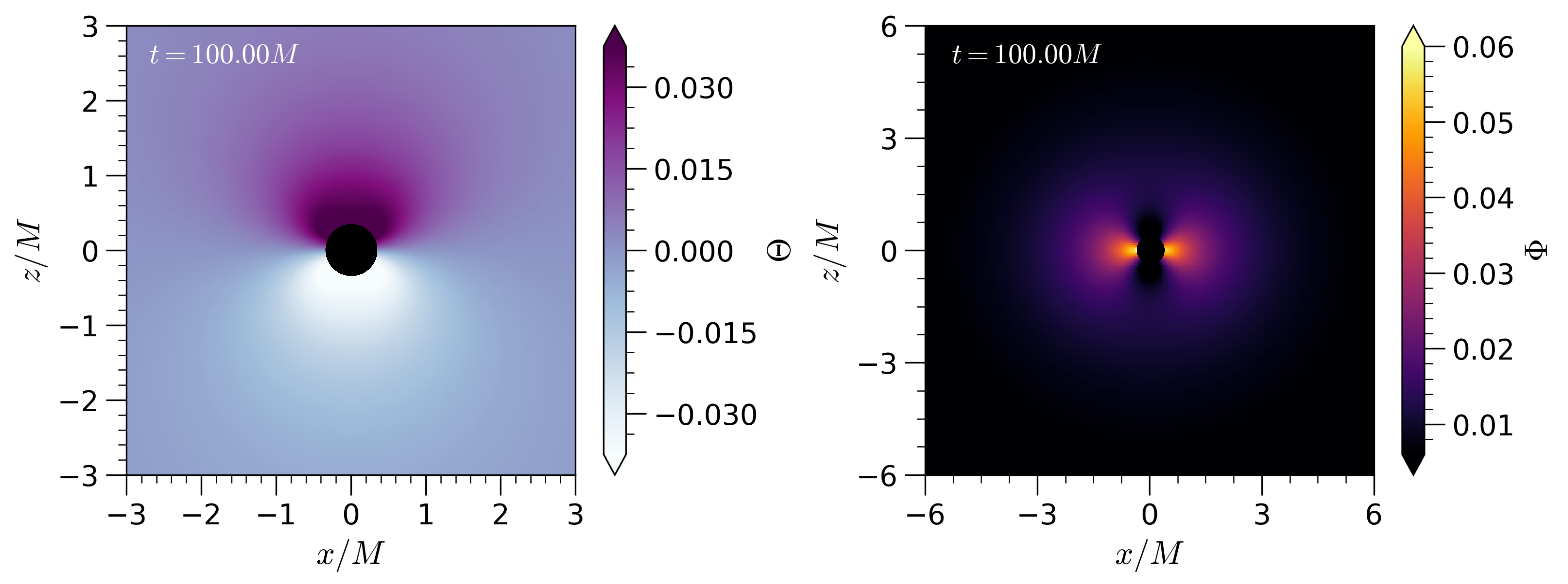


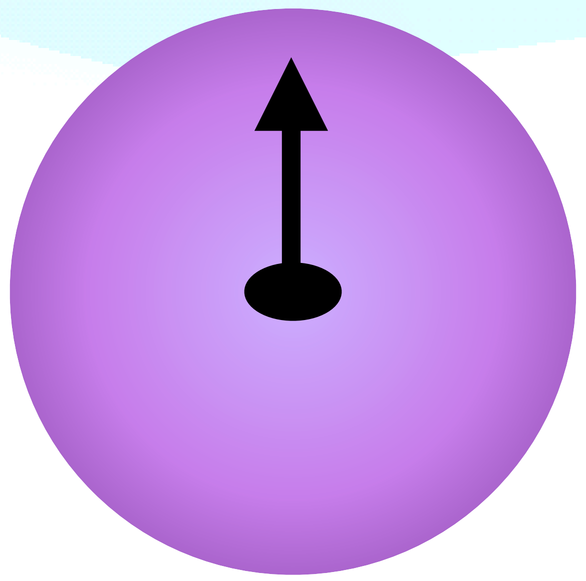
Numerical Results II

2D evolution of axion and dilaton fields; $\hat{\alpha}_{CS} = \hat{\alpha}_{GB} = 0.1$, $a/M = 0.9$

Axion hair



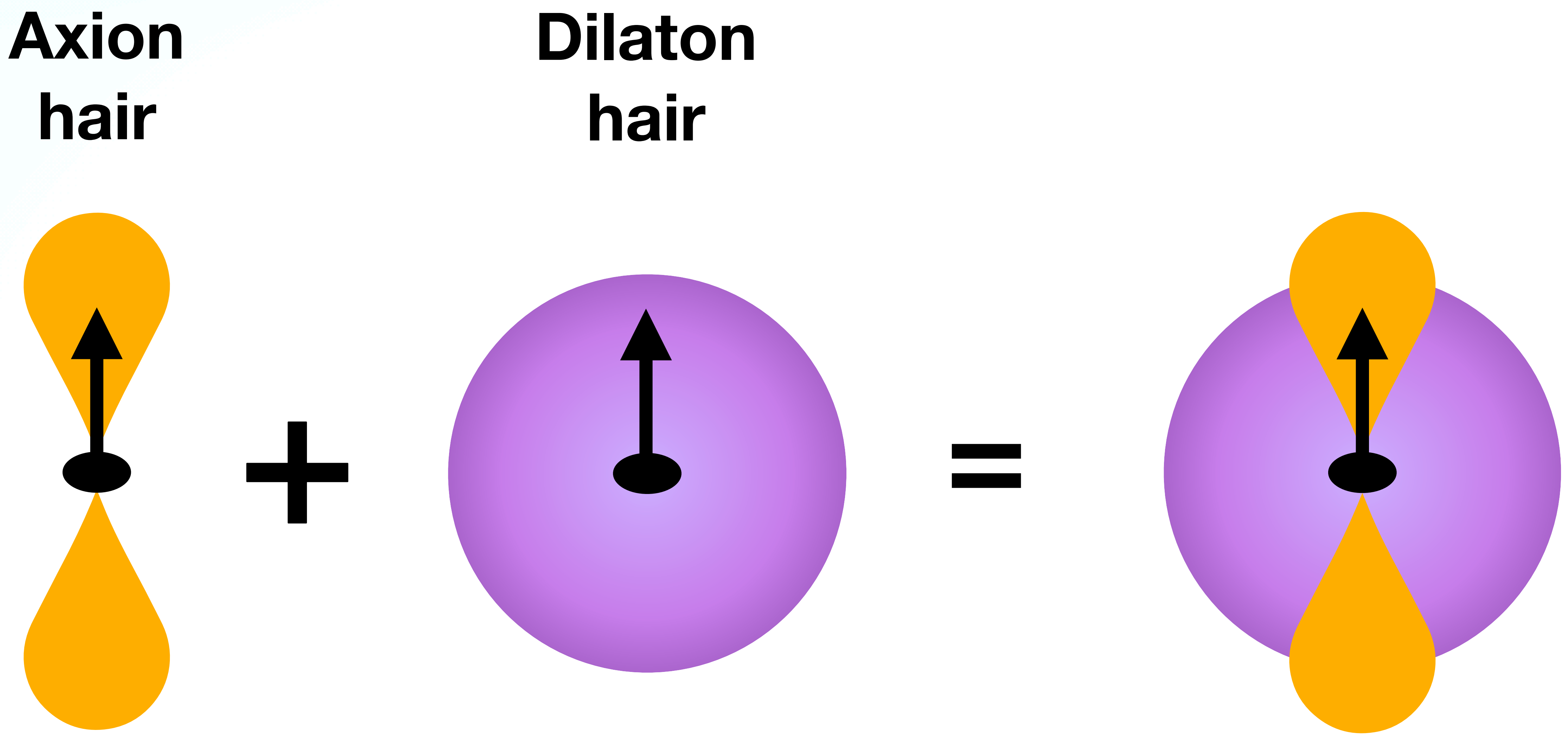




Dilaton hair

Brief Summary

- Introduced parameterized numerical relativity code for quadratic theories of gravity
- Find agreement with Einstein-dilaton-Gauss-Bonnet and dynamical-Chern-Simons codes
- Explore axion and dilaton hair growth for single black holes evolving in axi-dilaton gravity
- Consider effect on axion and dilaton hair evolution due to coupling between the fields
- Future outlook: continue binary black hole simulations



Thank you!

Questions or comments?

This work acknowledges support from the ICASU-Physics Fellowship

Appendix

Introducing axi-dilaton gravity

Field equations

③ $R_{ab} - \frac{1}{2}g_{ab}R - \frac{1}{2}T_{ab}^{\text{eff}} = 0$ with effective energy momentum tensor

$$T_{ab}^{eff} = -2\left(\alpha_{CS}C_{ab}^{CS} + \alpha_{GB}C_{ab}^{GB}\right) + \nabla_a\Phi\,\nabla_b\Phi - \frac{1}{2}g_{ab}\left((\nabla\Phi)^2 + 2V(\Phi)\right) + g(\Phi)^2\left(\nabla_a\Theta\,\nabla_b\Theta - \frac{1}{2}g_{ab}\left((\nabla\Theta)^2 + 2V(\Theta)\right)\right)$$

Introducing axi-dilaton gravity

Field equations in decoupling limit

③ $R_{ab} - \frac{1}{2}g_{ab}R - \cancel{\frac{1}{2}T^{\text{eff}}_{ab}} = 0$ with effective energy momentum tensor

$$T^{eff}_{ab} = -2\left(\alpha_{CS}C^{CS}_{ab} + \alpha_{GB}C^{GB}_{ab}\right) + \nabla_a\Phi\,\nabla_b\Phi - \frac{1}{2}g_{ab}\left((\nabla\Phi)^2 + 2V(\Phi)\right) + g(\Phi)^2\left(\nabla_a\Theta\,\nabla_b\Theta - \frac{1}{2}g_{ab}\left((\nabla\Theta)^2 + 2V(\Theta)\right)\right)$$

Introducing axi-dilaton gravity

Field equations in decoupling limit

③ $R_{ab} - \frac{1}{2} g_{ab} R = 0$ with effective energy momentum tensor

$$T_{ab}^{eff} = -2\left(\alpha_{CS} C_{ab}^{CS} + \alpha_{GB} C_{ab}^{GB}\right) + \nabla_a \Phi \nabla_b \Phi - \frac{1}{2} g_{ab}\left((\nabla \Phi)^2 + 2V(\Phi)\right) + g(\Phi)^2 \left(\nabla_a \Theta \nabla_b \Theta - \frac{1}{2} g_{ab}\left((\nabla \Theta)^2 + 2V(\Theta)\right)\right)$$

Q2: how do the additional (pseudo-)scalar fields Θ and Φ impact the gravitational wave signal produced by a binary black hole coalescence?

→ Calculate the scalar radiation and emitted energy flux **[3], [4]**

---

[3] M. Okounkova et al, Phys. Rev. D 96, 044020, (2017)
[4] H. Witek et al, Phys. Rev. D 99, 064035, (2019)

Axi-dilaton code description

★ Parameterized numerical relativity code for theories of quadratic gravity ★

Implement axi-dilaton code in the decoupling limit with open-source Canuda software **[8]** in the Einstein Toolkit **[9]**

SCAN FOR dCS CODE

AxiDil_Base
- Set up grid functions for scalars
- Define parameters for model selection

SCAN FOR dCS THORNLIST

AxiDil_Init
- Gives initial data profiles for scalars
- BBH ID: set scalar profiles around both BHs

AxiDil_Evol
- Evolve scalar fields in axi-dilaton gravity in vacuum GR background

---

[9] H. Witek et al, Zenodo, (2023)
[10] L. Werneck et al, Zenodo, (2023)

Axi-dilaton code description

Implementation

Recall evolution equations:

$$\Box\Phi - V'(\Phi) + \frac{\alpha_{GB}}{4} f'(\Phi)\mathcal{G} - g'(\Phi)g(\Phi)\left[(\nabla\Theta)^2 + 2V(\Theta)\right] = 0$$

$$\Box\Theta - \dot{V}(\Theta) + \frac{\alpha_{CS}}{4}\frac{\dot{h}(\Theta)}{g(\Phi)^2}{}^*RR + 2\frac{g'(\Phi)}{g(\Phi)}\nabla_\mu\Phi\,\nabla^\mu\Theta = 0$$

AxiDil_Evol

Evolution equations in BSSN variables:

$$d_t\Phi = -\alpha K(\Phi)$$

$$d_tK(\Phi) = -\alpha D^iD_i\Phi - D^i\alpha D_i\Phi + \alpha\left(K\ K(\Phi) + V'(\Phi) - \frac{\alpha_{GB}}{4} f'(\Phi)\mathcal{G}\right) - \alpha g'(\Phi)g(\Phi)\left(K(\Theta)^2 - 2V(\Theta) - D_i\Theta D^i\Theta\right)$$

$$d_t\Theta = -\alpha K(\Theta)$$

$$d_tK(\Theta) = -\alpha D^iD_i\Theta - D^i\alpha D_i\Theta + \alpha\left(K\ K(\Theta) + \dot{V}(\Theta) - \frac{\alpha_{CS}}{4}\frac{\dot{h}(\Theta)}{g(\Phi)^2}{}^*RR\right) + 2\alpha\frac{g'(\Phi)}{g(\Phi)}\left(K(\Theta)K(\Phi) - D_i\Theta D^i\Phi\right)$$

Axi-dilaton code description

Users guide (brief)

For the simulations presented, we adopt the following parameter choices:

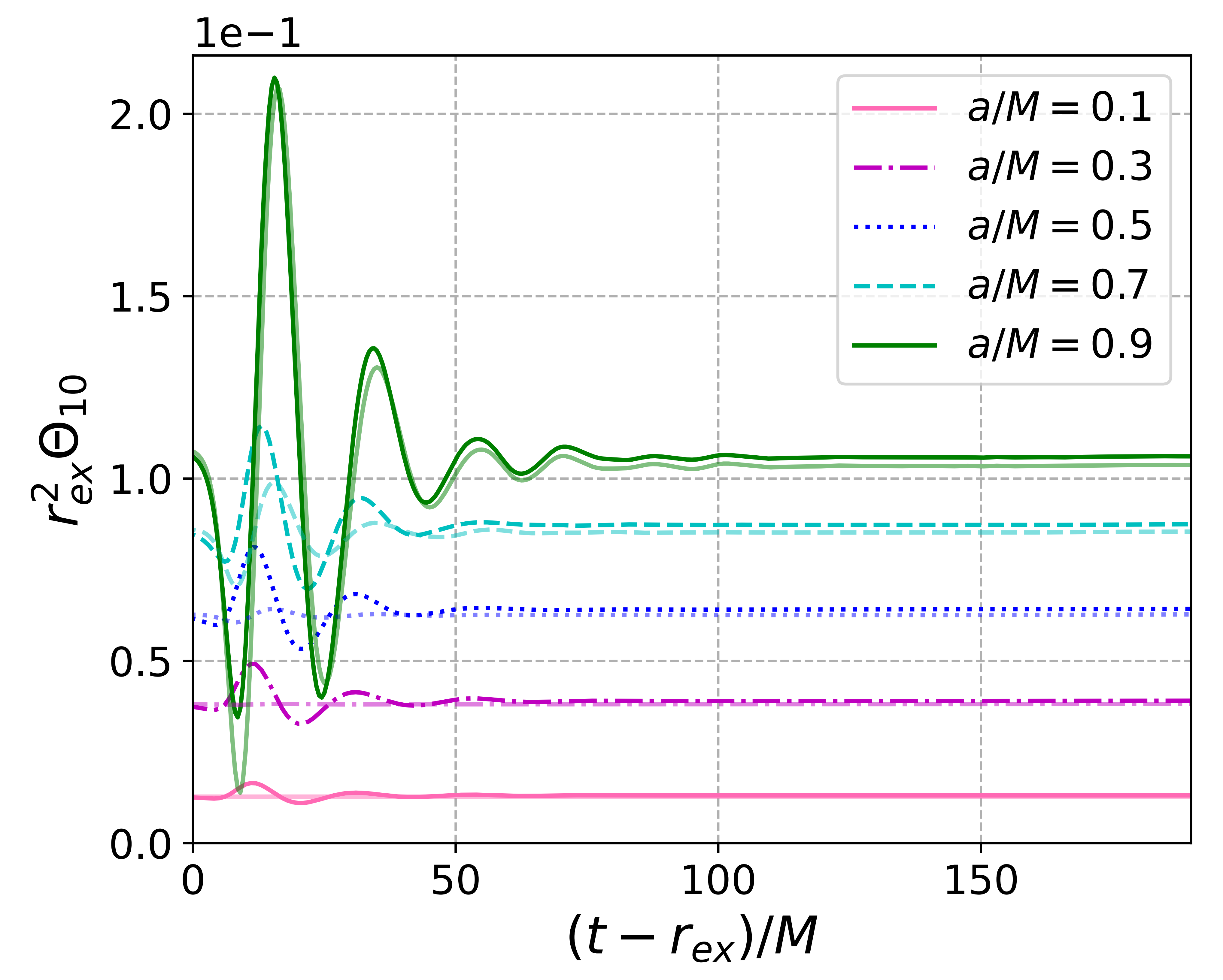
- dil_coupling = exponential (dilaton coupling function $f(\Phi) = e^{\lambda\Phi}$)
- dil_potential = zero (dilaton potential $V(\Phi) = 0$)
- dil_lambda = 1 (coefficient λ in exponential coupling function $f(\Phi)$)
- AD_coupling = exponential (coupling between dilaton and axion $g(\Phi) = f(\Phi)^{-1} = e^{-\lambda\Phi}$)
- AD_lambda = 1 (dil_lambda = AD_lambda so that $g(\Phi) = f(\Phi)^{-1}$)
- axi_coupling = linear (axion coupling function $h(\Theta) = \Theta$)
- axi_potential = zero (axion potential $V(\Theta) = 0$)

Numerical Results II

Effect of coupling between axion and dilaton on final hair; $\hat{\alpha}_{CS} = \hat{\alpha}_{GB} = 0.1$, $r_{ex} = 20M$

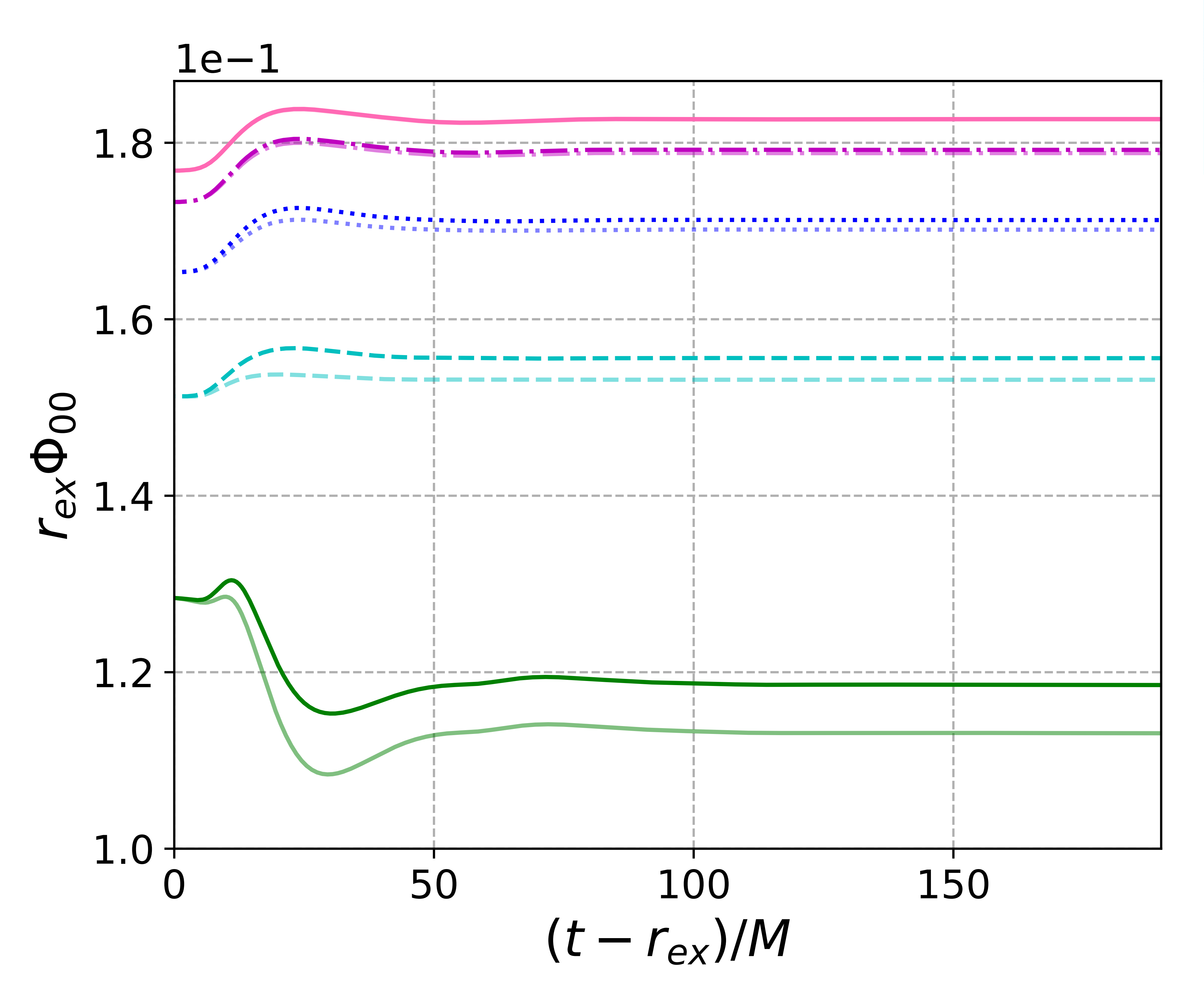
Evolution of axion hair:

Axi-dilaton vs dCS



Evolution of dilaton hair:

Axi-dilaton vs EdGB

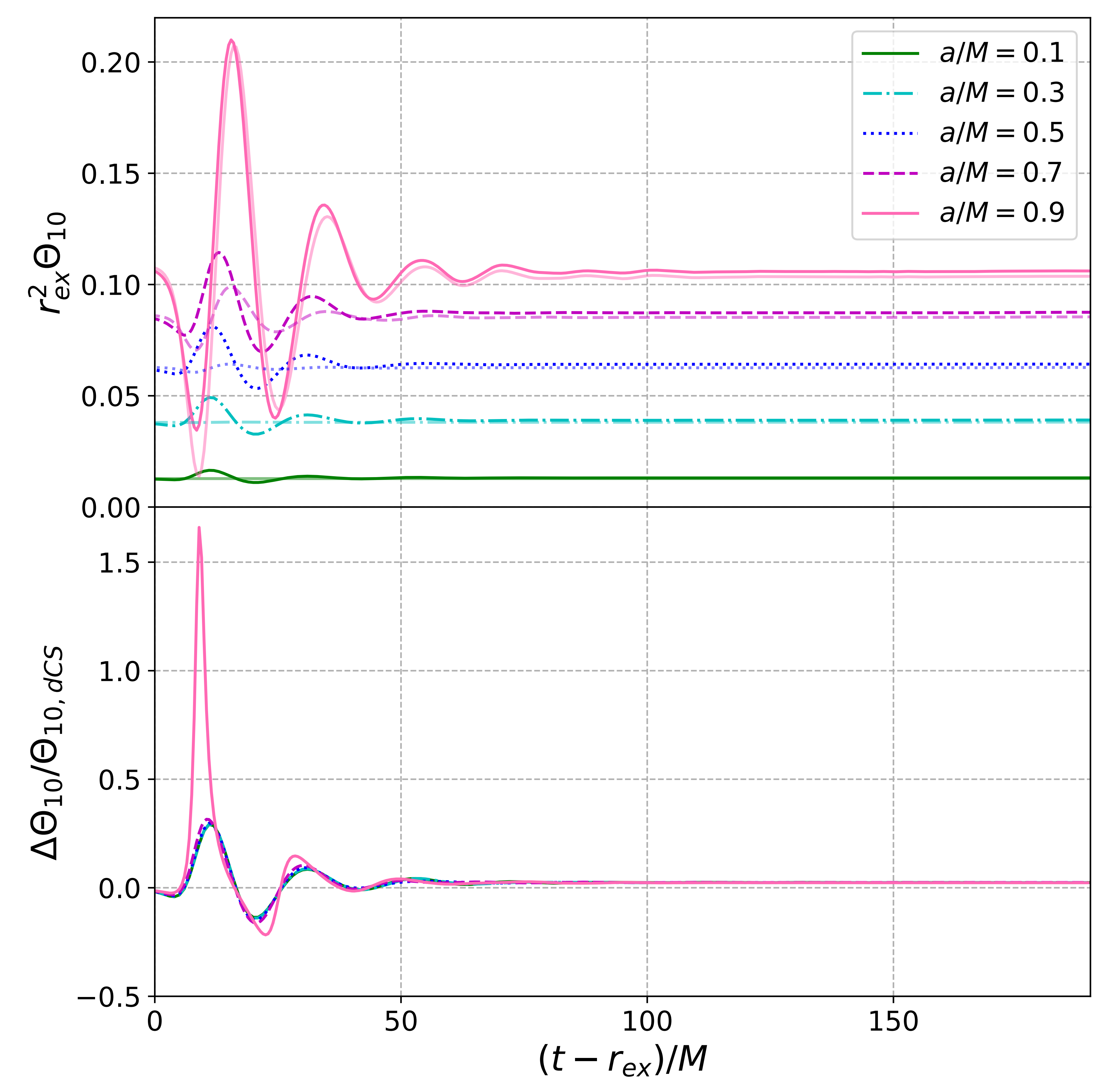


Numerical Results II

Effect of coupling between axion and dilaton on final hair

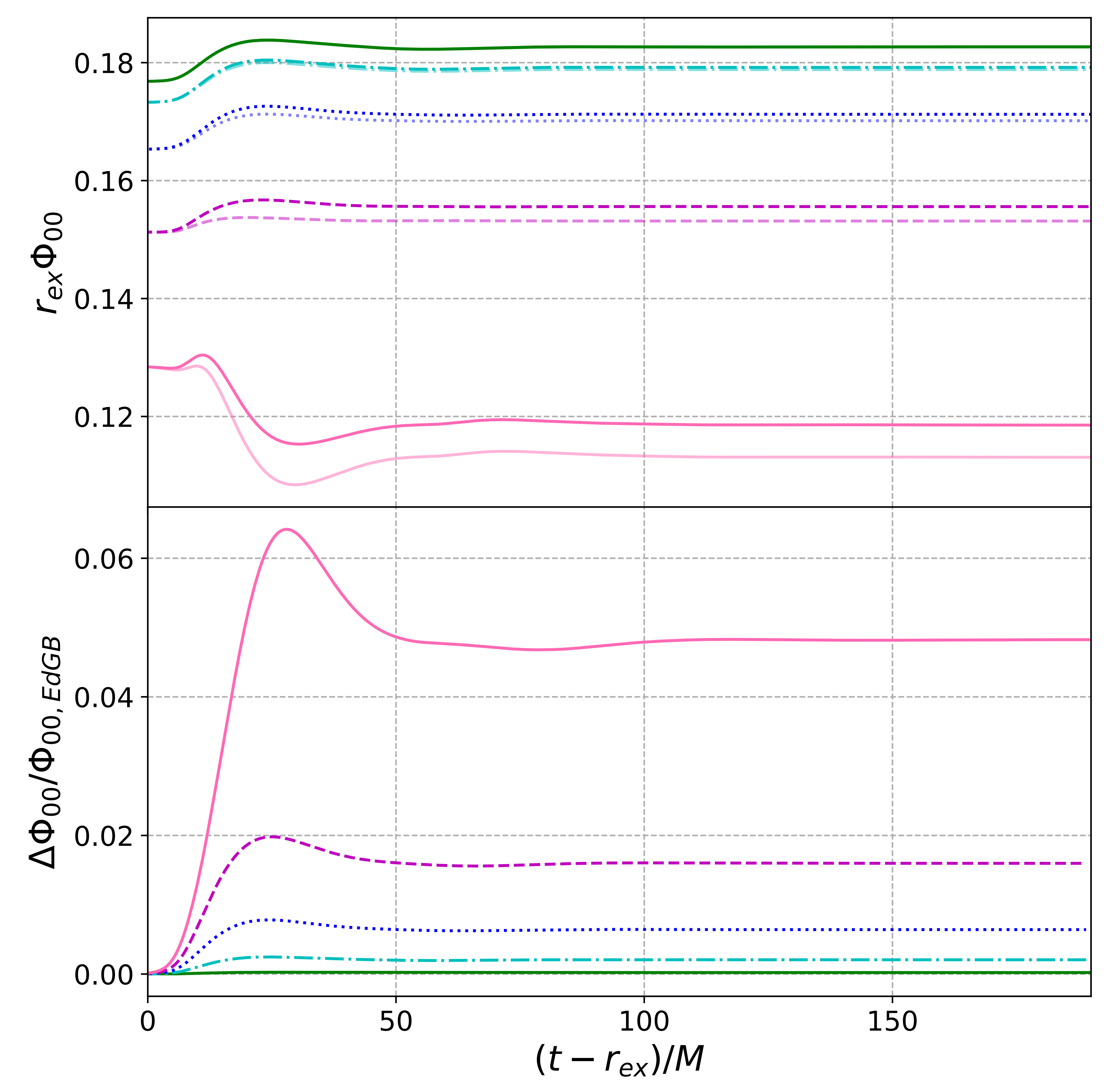
Evolution of axion hair:

Axi-dilaton vs dCS



Evolution of dilaton hair:

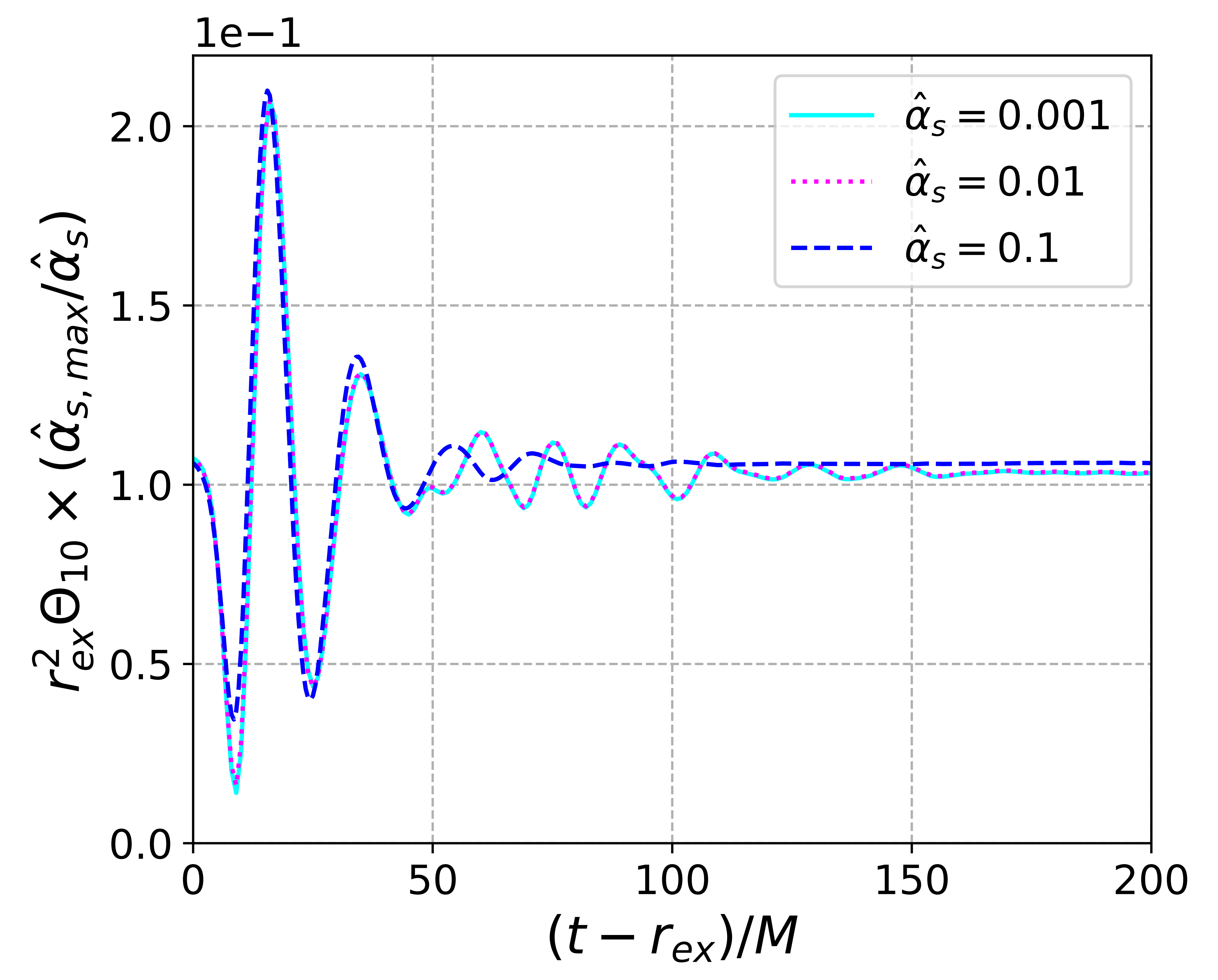
Axi-dilaton vs EdGB



Numerical Results III

Effect of coupling on axion and dilaton on final hair; $a/M = 0.9, r_{ex} = 20M$

Evolution of axion hair:



Evolution of dilaton hair: