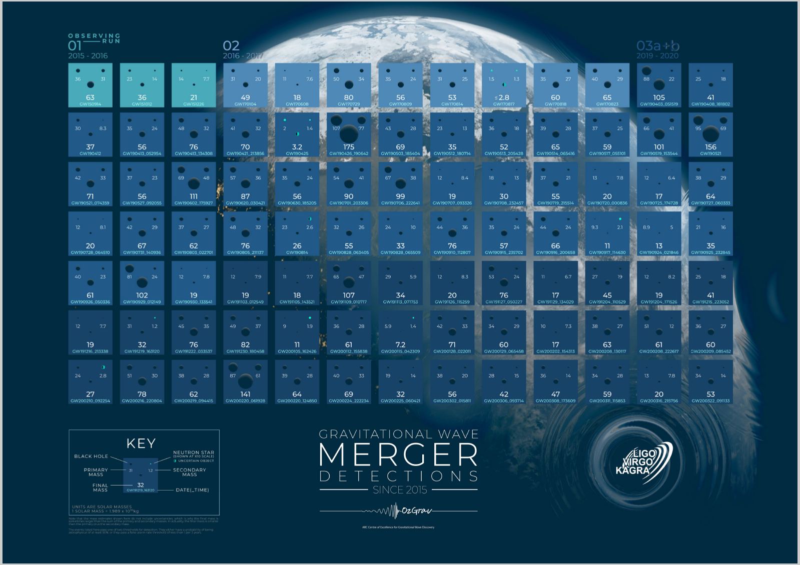
Nonlinear dynamics of compact object mergers in beyond General Relativity.

Maxence Corman

July 15, 2024

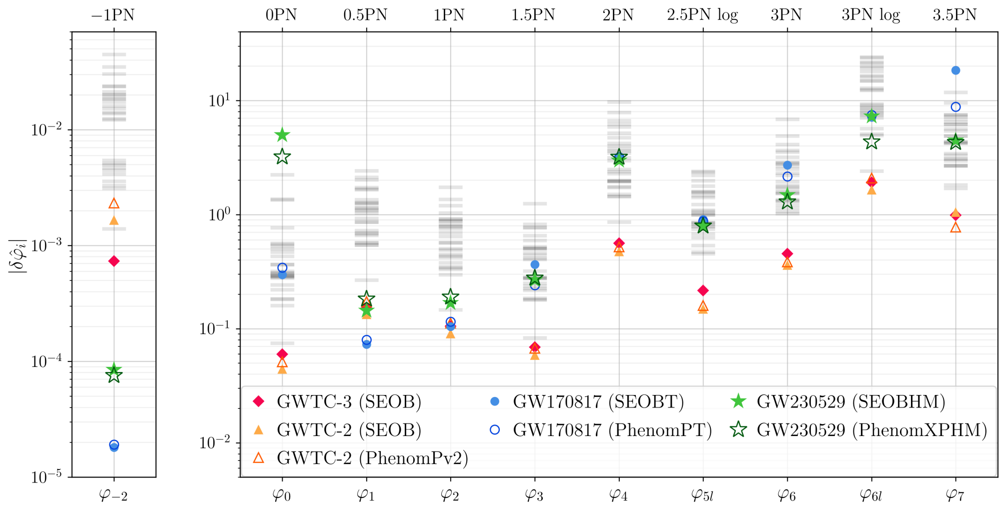
Max Planck Institute for Gravitational Physics

Motivation



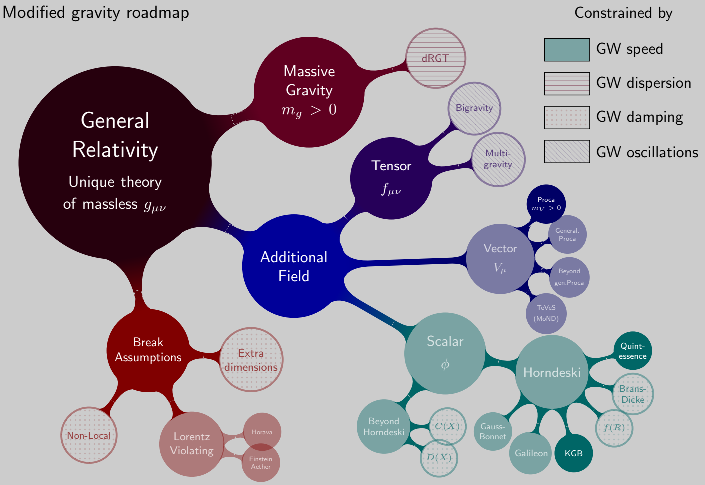
OBSERVING RUN
O1
2015 - 2016
O2
2016 - 2017
O3a+b
2019 - 2020
KEY
BLACK HOLE
NEUTRON STAR (SHOWN AT X10 SCALE)
UNCERTAIN OBJECT
PRIMARY MASS
SECONDARY MASS
FINAL MASS
DATE(_TIME)
UNITS ARE SOLAR MASSES
GRAVITATIONAL WAVE
MERGER
DETECTIONS
SINCE 2015
OzGrav
LIGO
VIRGO
KAGRA

Motivation



Parameterized test of GR during inspiral from Sanger+ 2024, arXiv:2406.03568

Motivation



Modified gravity roadmap summarizing the possible extensions of GR from Ezquiaga and Zumalacarregui, arXiv:1807.09241

Approaches to studying modifications to general relativity

Full solution: Requires well-posed initial value problem formulation

- Same principal part as GR: Scalar-Tensor theories Damour, Esposito-Farese → Barausse+,Shibata+, Quadratic Gravity at weak coupling Noakes ⇒ Held+,East+
- Only scalar part modified: Cubic Horndeski Figueras+, Screening theories Bezares+
- Horndeski theories: Modified Generalized Harmonic formulation Kovacs and Reall → East+,Corman+ or modified CCZ4 formulation Salo+

Approaches to studying modifications to general relativity

Order-by-order

- Solve the equations *perturbatively*
- Pros: same principal part as GR, easy to implement and flexible
- Cons: secular effects
- Applications: EdGB and dCS

  Okounkova+,Stein+

Fixing

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Okounkova+,Stein+

$$G(g) = \lambda S$$

- $\lambda^0 : G(g^0) = 0$
- $\lambda^1 : G(g^1) = \lambda S(g^0)$

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- Fix evolution equations below some short lengthscale
- Add new dynamical fields with driver equations

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$$G(g) = \lambda S$$

$\rightarrow G(g) = \Pi$

and $\quad \tau\partial_t\Pi = -\Pi + \lambda S$

Approaches to studying modifications to general relativity

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Fixing

- Inspired by Israel-Stewart fixing of relativistic hydrodynamics
- Fix evolution equations below some short lengthscale
- Add new dynamical fields with driver equations
- Pros: Corrections fully backreact
- Cons: Computationally expensive
- Applications: EsGB, Higher derivative theories

Cayuso+,Lehner+,Bezares+,Lara+,Franchini+

Einstein scalar Gauss Bonnet gravity

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[R - (\nabla\phi)^2 + \beta(\phi)\mathcal{G} \right]$$

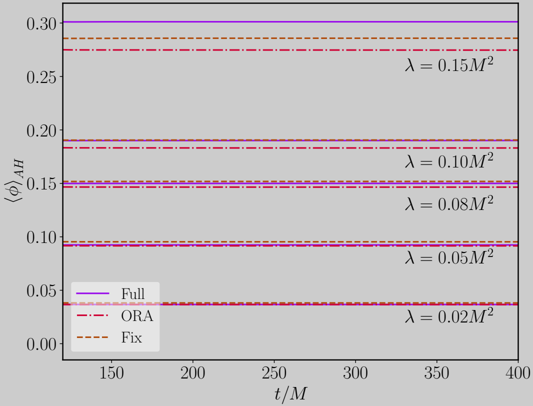
with $\mathcal{G} \equiv R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd}$.

- Horndeski theory $\Rightarrow$ second order equations of motion
- Shift-symmetric $\Rightarrow \beta(\phi) = 2\lambda\phi$
- Black hole solutions with scalar hair $\sim \lambda/m^2$ (Sotiriou & Zhou) $\Rightarrow$ energy loss through scalar radiation, -1PN (at leading order) dephasing in GW signal (Yagi)
- Well-posed initial value formulation (Kovacs and Reall)

To what extent can predictions from approximate treatments such as the *order-by-order* and *fixing* approach be confronted with gravitational wave observations?

MC,Lehner,East and Dideron,2024

Black hole scalarization and saturation



Single, non-spinning black hole in axisymmetry

Order-by-order approach

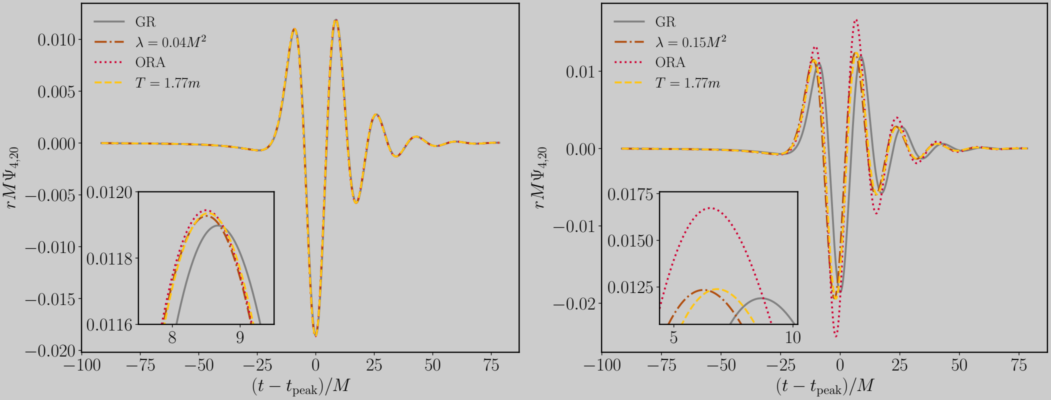
$$\lambda^0: \left(g_{ab}^{(0)}, \phi^{(0)}\right) = \left(g_{ab}^{\mathrm{GR}}, 0\right)$$

$$\lambda^1: \left(g_{ab}^{(1)}, \phi^{(1)}\right) = \left(0, \frac{\lambda}{M^2}\Phi\right)$$

Fixing approach

- Driver equation:
  $\sigma g^{ab}\partial_a\partial_b\mathbf{P} = \partial_0\mathbf{P} + \kappa\left(\mathbf{P} - \mathbf{S}\right)$
  $\mathbf{P} \to \mathbf{S}$ on timescales $T_s(\kappa, \sigma)$ and $T_f(\kappa, \sigma)$
- Solutions obtained by extrapolating $T_s \to 0$

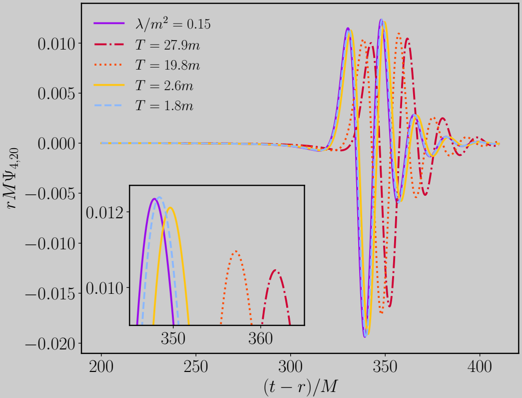
Head on collisions of equal-mass scalarized black holes



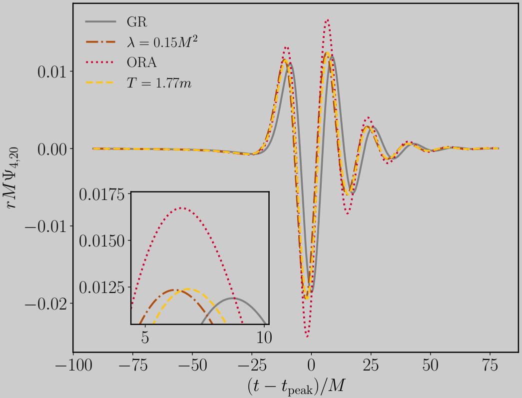
All agree reasonably well but differences are small.

Amplitude order-by-order solution increases by 40% compared to 3.7% for full solution, while error in peak time remain small.

Head on collisions of scalarized black holes



Different combination of fixing parameters.



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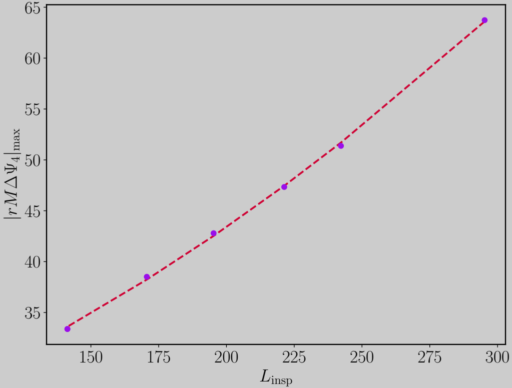
Quasi-circular inspirals of scalarized black holes



Secular effects reflected in amplitude waveform at merger, $\Psi_4^{(2)} = \left(\frac{\lambda}{M^2}\right)^2 \Delta\Psi_4$.

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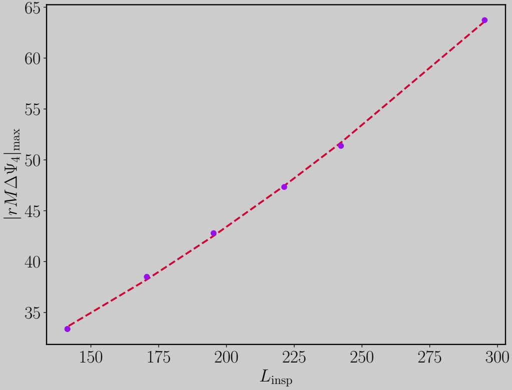
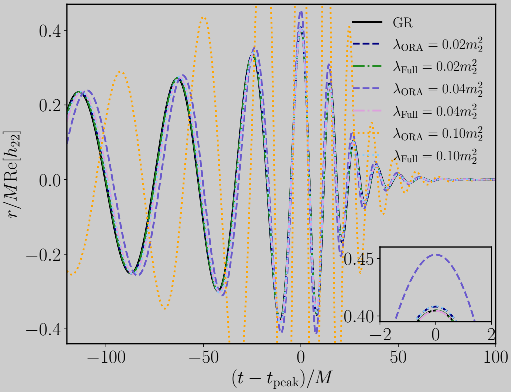
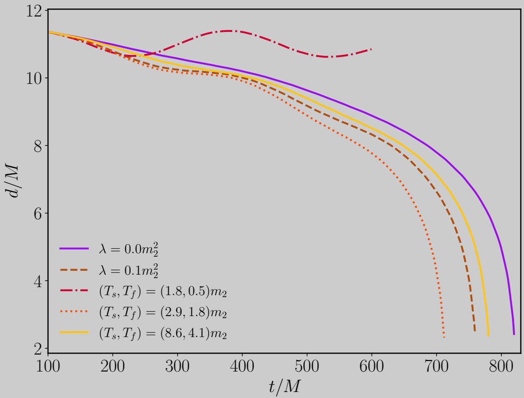




Secular effects reflected in amplitude waveform at merger, $\Psi_4^{(2)} = \left(\frac{\lambda}{M^2}\right)^2 \Delta\Psi_4$.

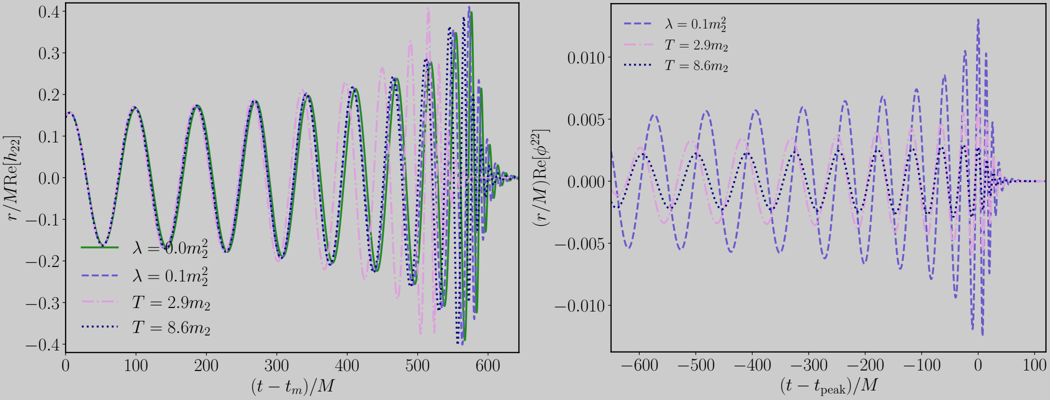
Weak dependence of amplitude at merger for full solution. Order-by-order overshoots full solution.

Quasi-circular inspirals of scalarized black holes



Lack of resolution to resolve T_s and/or T_f?

Quasi-circular inspirals of scalarized black holes



The smaller T_s the closer scalar charge to full solution and faster inspiral. Can we extrapolate to $T \to 0$?

Take aways comparison study

To what extent can predictions from approximate treatments such as the *order-by-order* and *fixing* approach be confronted with gravitational wave observations?

- Order-by-order approach cannot faithfully track the solutions when the corrections to general relativity are non-negligible.
- Fixing approach can provide consistent solutions, provided the ad-hoc timescale over which the dynamical fields are driven to their target values is made short compared to the physical timescales $\rightarrow$ computationally feasable?

Black hole-neutron star mergers in EsGB gravity

Motivation for black hole-neutron star mergers in EsGB gravity

Observation of gravitational waves from two neutron star–black hole coalescences

THE LIGO SCIENTIFIC COLLABORATION, THE VIRGO COLLABORATION, AND THE KAGRA COLLABORATION

(Dated: June 30, 2021)

ABSTRACT

We report the observation of gravitational waves from two compact binary coalescences in LIGO's and Virgo's third observing run with properties consistent with neutron star–black hole (NSBH) binaries. The two events are named GW200105_162426 and GW200115_042309, abbreviated as GW200105 and GW200115; the first was observed by LIGO Livingston and Virgo, and the second by all three LIGO–Virgo detectors. The source of GW200105 has component masses $8.9^{+1.2}_{-1.5}\,M_\odot$ and $1.9^{+0.3}_{-0.2}\,M_\odot$, whereas the source of GW200115 has component masses $5.7^{+1.8}_{-2.1}\,M_\odot$ and $1.5^{+0.7}_{-0.3}\,M_\odot$ (all measurements quoted at the 90% credible level). The probability that the secondary's mass is below the maximal mass of a neutron star is 89%–96% and 87%–98%, respectively, for GW200105 and GW200115, with the ranges arising from different astrophysical assumptions. The source luminosity distances are 280^{+110}_{-110} Mpc and 300^{+150}_{-100} Mpc, respectively. The magnitude of the primary spin of GW200105 is less than 0.23 at the 90% credible level, and its orientation is unconstrained. For GW200115, the primary spin has a negative spin projection onto the orbital angular momentum at 88% probability. We are unable to constrain the spin or tidal deformation of the secondary component for either event. We infer an NSBH merger rate density of $45^{+75}_{-33}\ \mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1}$ when assuming that GW200105 and GW200115 are representative of the NSBH population, or $130^{+112}_{-69}\ \mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1}$ under the assumption of a broader distribution of component masses.

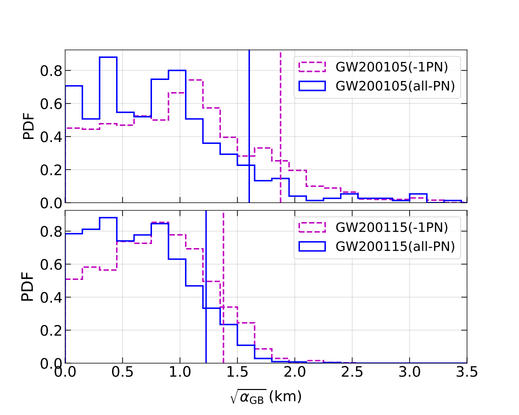
Observation of Gravitational Waves from the Coalescence of a 2.5–4.5 $M_\odot$ Compact Object and a Neutron Star

THE LIGO SCIENTIFIC COLLABORATION, THE VIRGO COLLABORATION, AND THE KAGRA COLLABORATION

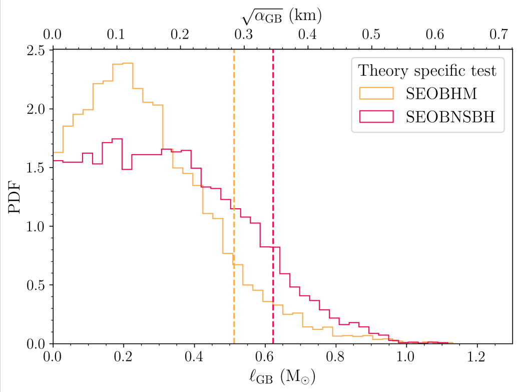
ABSTRACT

We report the observation of a coalescing compact binary with component masses 2.5–4.5 $M_\odot$ and 1.2–2.0 $M_\odot$ (all measurements quoted at the 90% credible level). The gravitational-wave signal GW230529_181500 was observed during the fourth observing run of the LIGO–Virgo–KAGRA detector network on 2023 May 29 by the LIGO Livingston observatory. The primary component of the source has a mass less than 5 $M_\odot$ at 99% credibility. We cannot definitively determine from gravitational-wave data alone whether either component of the source is a neutron star or a black hole. However, given existing estimates of the maximum neutron star mass, we find the most probable interpretation of the source to be the coalescence of a neutron star with a black hole that has a mass between the most massive neutron stars and the least massive black holes observed in the Galaxy. We estimate a merger rate density of $55^{+127}_{-47}\ \mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1}$ for compact binary coalescences with properties similar to the source of GW230529_181500; assuming that the source is a neutron star–black hole merger, GW230529_181500-like sources constitute about 60% of the total merger rate inferred for neutron star–black hole coalescences. The discovery of this system implies an increase in the expected rate of neutron star–black hole mergers with electromagnetic counterparts and provides further evidence for compact objects existing within the purported lower mass gap.

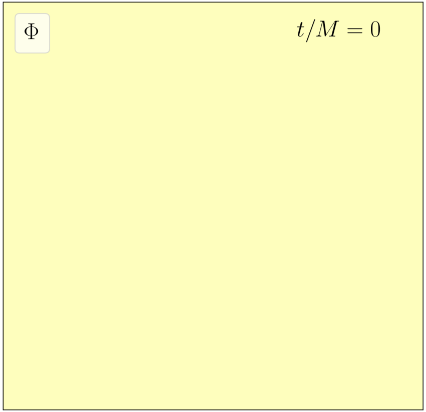
Motivation for black hole-neutron star mergers in EsGB gravity



Posteriors on $\sqrt{\alpha_{\rm GB}}$ from the leading -1PN correction and those including higher PN corrections (up to 2PN). Taken from Lyu+2022.



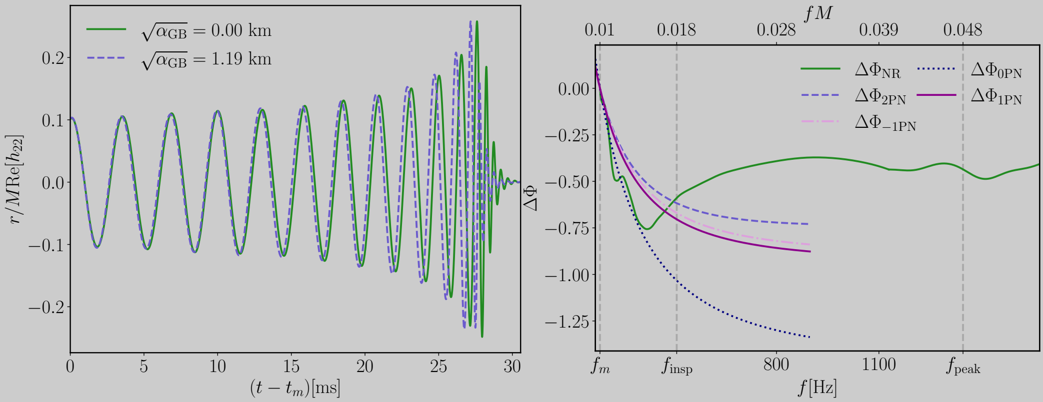
Posteriors on $\sqrt{\alpha_{\rm GB}}$ from the theory-specific test of FTI framework. Taken from Sanger+2024.


Φ
t/M = 0

Questions

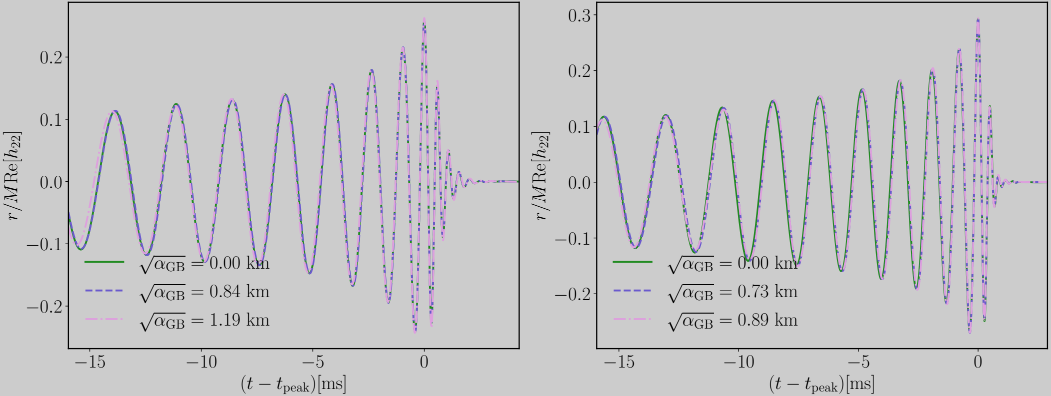
1. Are PN predictions accurate enough to model inspiral?
2. What does the GW signal look like in non-linear regime?
3. Can we comment on the ringdown signal?

Are PN predictions accurate enough to model inspiral? (MC and East 2024)



PN predictions taken from Sennet+2016 and mapped to Einstein frame using recipe outlined in Julie+2022 or more recently Julie+2024.

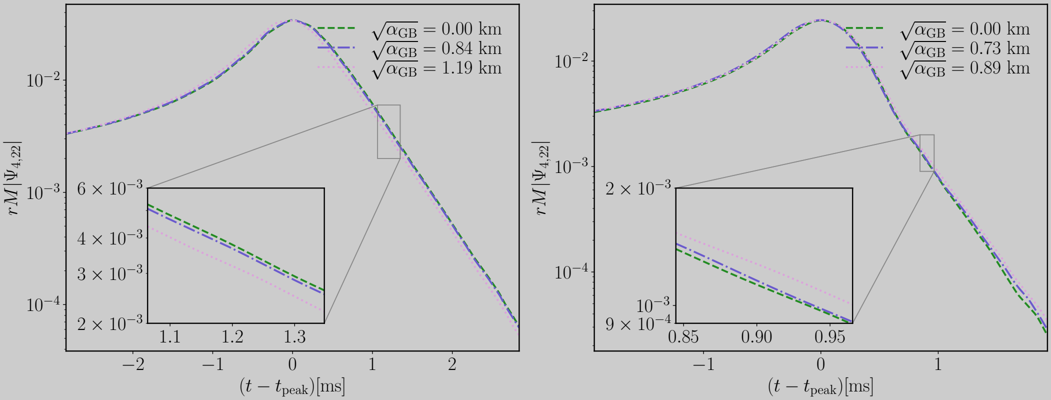
What does the GW signal look like in non-linear regime? (MC and East 2024)



GW200115, $\{M_{\rm BH}, M_{\rm NS}\} = \{5.7, 1.5\}\, M_\odot$

GW230529, $\{M_{\rm BH}, M_{\rm NS}\} = \{3.5, 1.4\}\, M_\odot$

Can we comment on the ringdown signal? (MC and East 2024)



GW200115, $\{M_{\rm BH}, M_{\rm NS}\} = \{5.7, 1.5\}$

GW230529, $\{M_{\rm BH}, M_{\rm NS}\} = \{3.5, 1.4\}$

Conclusion

1. Are PN predictions accurate enough to model inspiral?
   We find reasonable agreement up to the end of inspiral.
2. What does the GW signal look like in non-linear regime? *We find weak dependence of amplitude GW signal on coupling at merger.*
3. Can we comment on the ringdown signal? *Sign of shift in frequencies consistent with perturbation theory but main effect is on amplitude GW signal.*

Order by order approach in EsGB

$$\Box\phi + \lambda\mathcal{G} = 0, \quad R_{ab} - \frac{1}{2}g_{ab}R - \nabla_a\phi\nabla_b\phi + \frac{1}{2}\left(\nabla\phi\right)^2 g_{ab} + 2\lambda\delta^{efcd}_{ijg(a}g_{b)d}R^{ij}{}_{ef}\nabla^g\nabla_c\phi = 0$$

$$g_{ab} = g^{(0)}_{ab} + \sum_{k=1}^{\infty}\epsilon^k h^{(k)}_{ab},$$

$$\phi = \sum_{k=0}^{\infty}\epsilon^k\phi^{(k)}$$

0^{th} order: $G^{(0)}_{ab}\left[g^{(0)}_{ab}\right] = 0$, $\Box^{(0)}\phi^{(0)} = 0$

1^{st} order: $G^{(1)}_{ab}\left[g^{(0)}_{ab}, h^{(1)}_{ab}\right] = 0, \quad \Box^{(0)}\phi^{(1)} = -\lambda\mathcal{G}^{(0)}$

2^{nd} order:

$$G^{(0)}_{ab}\left[h^{(2)}_{ab}\right] = 8\pi T^{(2)}_{ab} - 2\lambda\delta^{efcd}_{ijg(a}g^{(0)}_{b)d}R^{(0)ij}{}_{ef}\nabla^{(0)g}\nabla^{(0)}_c\phi^{(1)},$$

$$\Box^{(0)}\phi^{(2)} = 0$$

Fixing approach in EsGB

$$\Box\phi + \lambda\mathcal{G} = 0, \quad R_{ab} - \frac{1}{2}g_{ab}R - \nabla_a\phi\nabla_b\phi + \frac{1}{2}\left(\nabla\phi\right)^2 g_{ab} + 2\lambda\delta^{efcd}_{ijg(a}g_{b)d}R^{ij}{}_{ef}\nabla^g\nabla_c\phi = 0$$

$$\Box\phi + \Pi^{(\phi)} = 0, \quad R_{ab} - \frac{1}{2}g_{ab}R - \nabla_a\phi\nabla_b\phi + \frac{1}{2}\left(\nabla\phi\right)^2 g_{ab} + \Pi^{(g)}_{ab} = F_{ab},$$

$$\sigma g^{ab}\partial_a\partial_b\mathbf{P} = \partial_0\mathbf{P} + \kappa\left(\mathbf{P} - \mathbf{S}\right)$$

Modified generalized harmonic formulation (Kovacs & Reall, 2020)

- Well-posed initial value problem $\Leftrightarrow$ strongly hyperbolic system
  $\Rightarrow$ Certain matrix $M^i\xi_i$ to be **diagonizable** with **real** eigenvalues
- Harmonic gauge in GR: $M^i\xi_i$ is diagonizable but eigenvalues are **degenerate**
- Weakly coupled four-derivative scalar tensor theory can be viewed as a small deformation of GR
  $\Rightarrow$ In harmonic gauge, $M^i\xi_i$ is not diagonizable in **generic** weakly coupled background (Papallo & Reall,2017)
- Problem: mixing between pure gauge and gauge-condition violating solutions
  $\Rightarrow$ Solution: give modes different characteristic speeds

Modified generalized harmonic formulation (Kovacs & Reall, 2020)

- Introduce two auxiliary Lorentzian metrics: $\tilde{g}^{ab}$ and $\hat{g}^{ab}$
- Gauge condition:

$$C^a \equiv H^a - \tilde{g}^{ab}\nabla_a \nabla_b x^c = 0$$

- Gauge-fixed equation

$$E^{ab} - \frac{1}{2}\left(\delta^a{}_b \hat{g}^{ab} + \delta^b{}_d \hat{g}^{ac} - \delta^c{}_d \hat{g}^{ab}\right)\nabla_c C^d = 0$$

so constraint propagates as

$$0 = \hat{g}^{ab}\nabla_a \nabla_b C^c + \ldots$$

- Gauge-condition violating solution propagate along $\hat{g}^{ab}$, pure gauge solutions along $\tilde{g}^{ab}$ and physical solutions along g^{ab}.
- If null cones do not intersect $\Rightarrow$ Horndeski theories are strongly hyperbolic in **weakly coupled regime** $\Rightarrow \lambda << L^2$

Loss of hyperbolicity?

Main idea

Separation of causal cones of different types of mode solutions

(i) "pure gauge" modes propagate along the null cone of $\tilde{g}^{\mu\nu}$

(ii) "gauge condition violating" modes propagate along the null cone of $\hat{g}^{\mu\nu}$

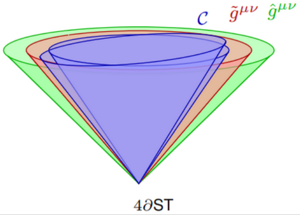
(iii) "physical" polarizations

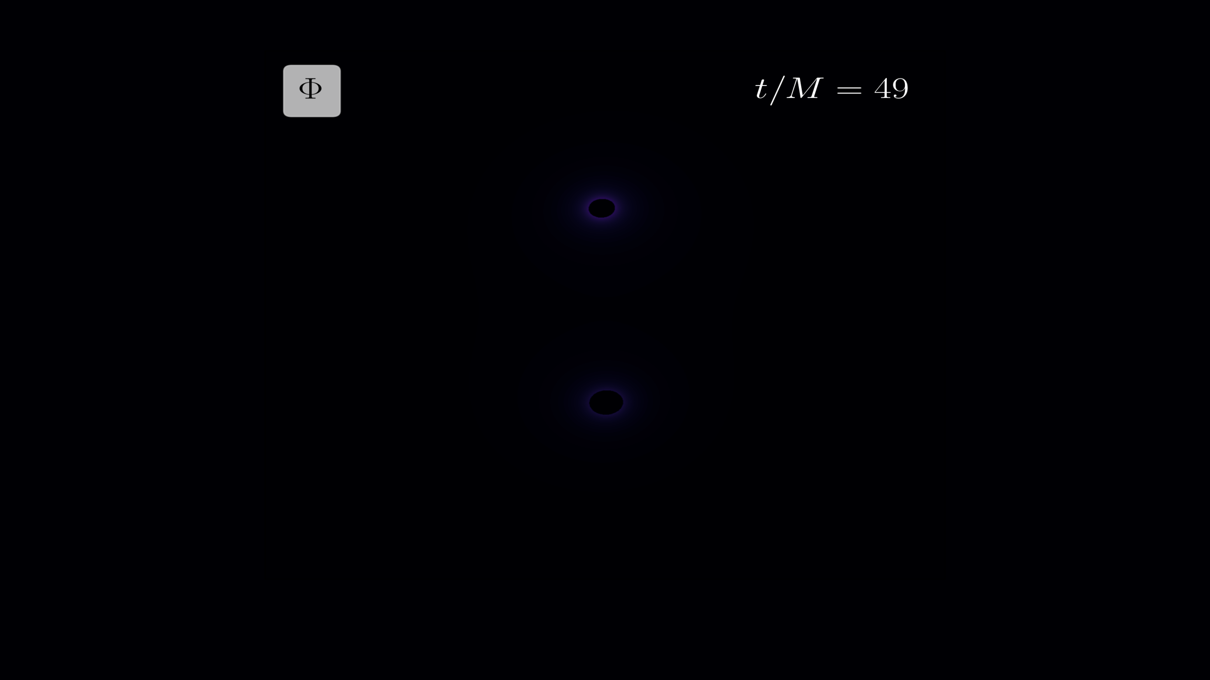
- propagate along the null cone of $g^{\mu\nu}$ is Einstein-scalar-field theory
- propagate along characteristic hypersurfaces that are "almost null" in weakly coupled 4∂ST and are gauge-invariant [Reall (2021)]

If characteristic polynomial for physical d.o.f is hyperbolic then the theory admits a well-posed formulation (e.g. in a suitable choice of the modified harmonic gauge condition and gauge fixing), otherwise the theory breaks down independently of gauge choice. (See also [Hegade, Ripley, Yunes (2023)])

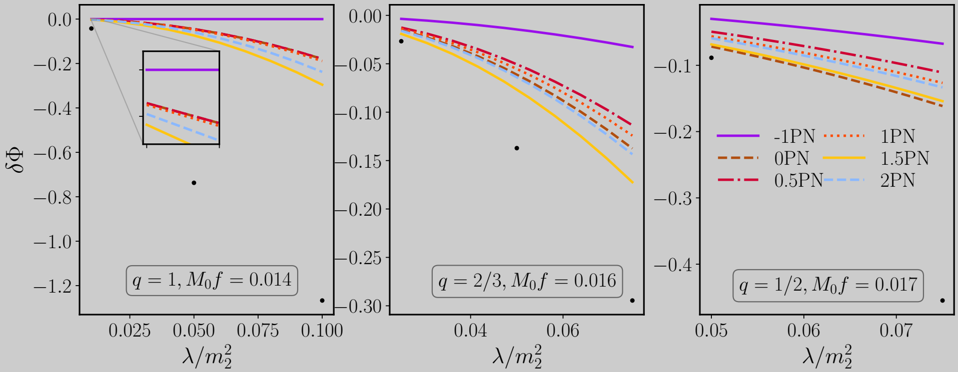
$$p(\xi) = (C^{-1})^{\mu\nu}\xi_\mu\xi_\nu Q(\xi)$$

$$C_{\mu\nu} = g_{\mu\nu} - \beta'(\Phi)\nabla_\mu\nabla_\nu\Phi - \beta''(\Phi)\nabla_\mu\Phi\nabla_\nu\Phi$$




Φ
$t/M = 49$

Are PN predictions accurate enough to model inspiral? (MC,Ripley and East 2022)



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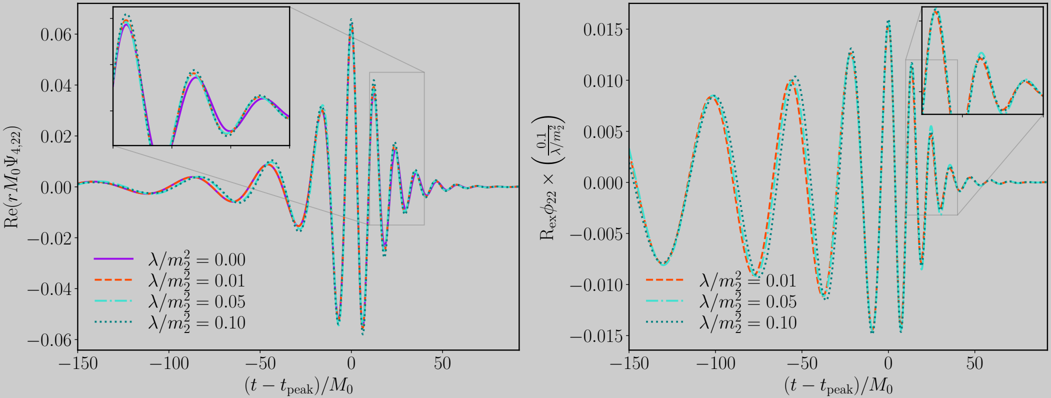
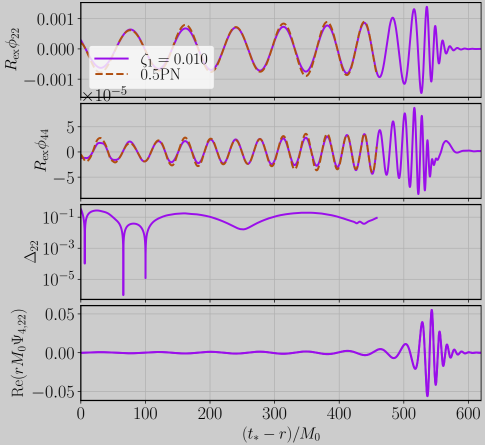
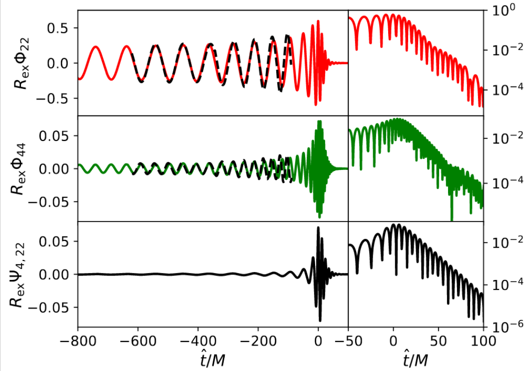


Comparison scalar waveforms to PN theory (Witek+2018)





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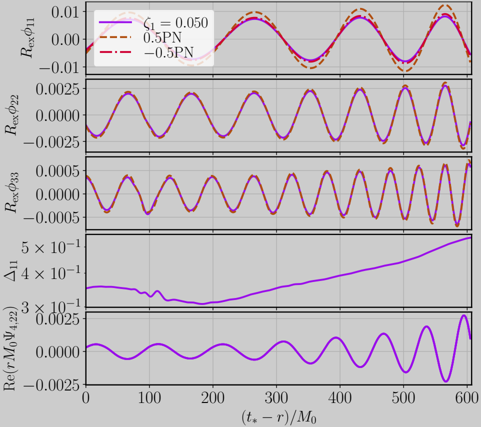
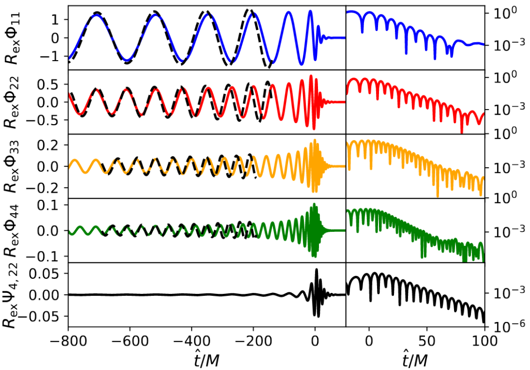
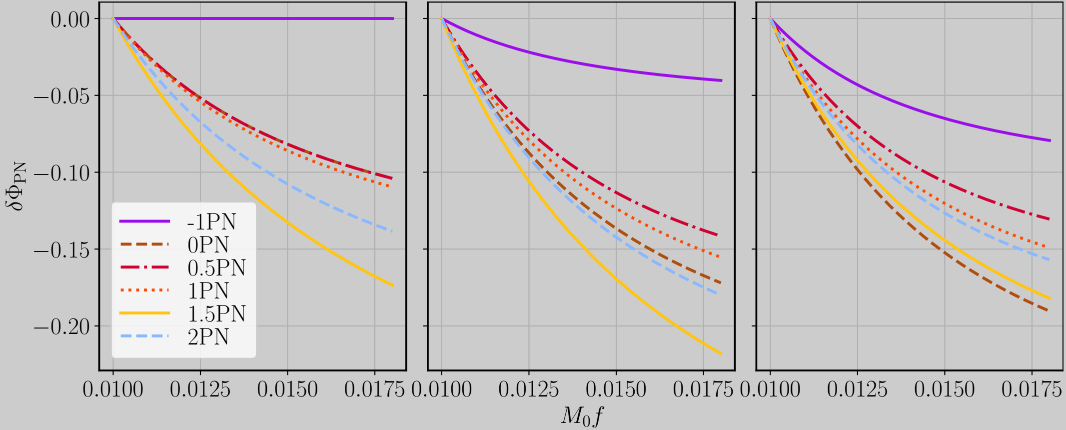




Orbital phase in PN theory (Sennet+2016)



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