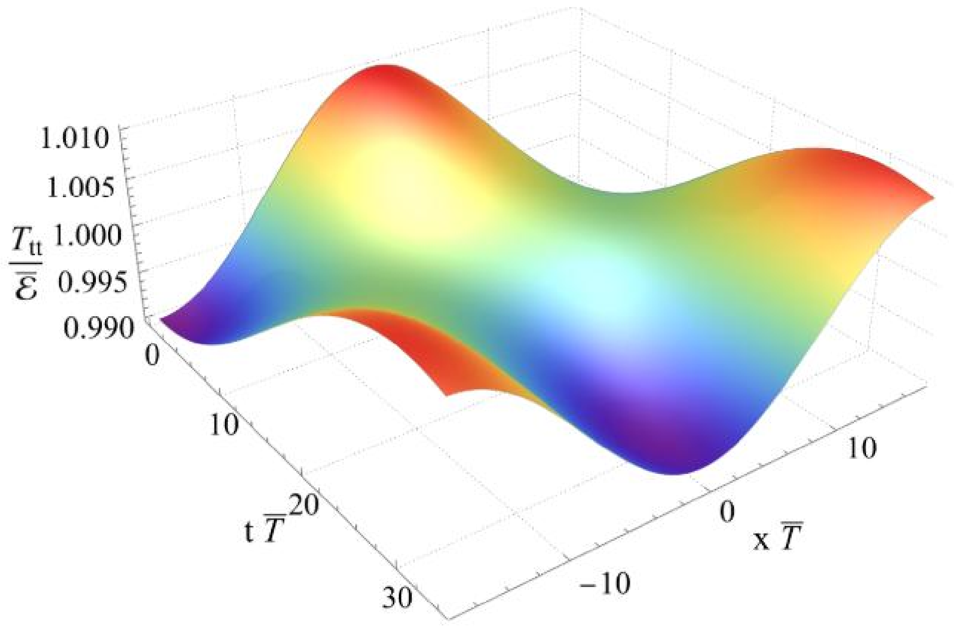
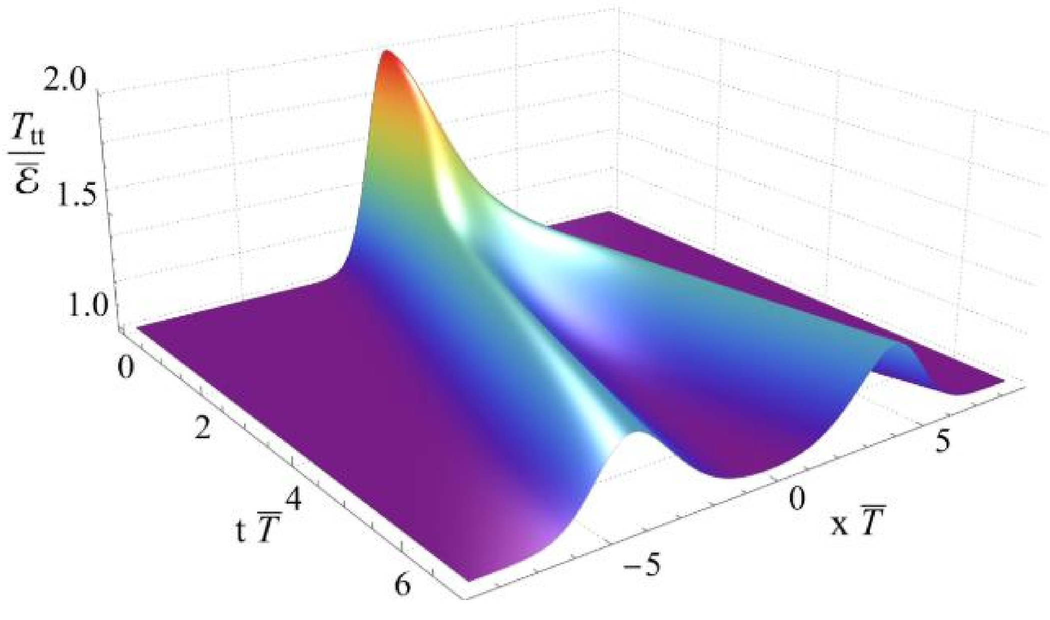
Relativistic Navier-Stokes: recent developments

Yago Bea

University of Barcelona





Motivation

Why hydrodynamics? ⟶ It describes interesting phenomena:

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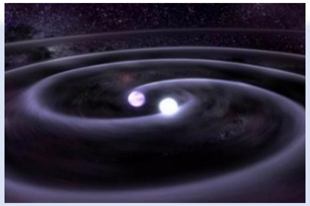
Neutron star mergers



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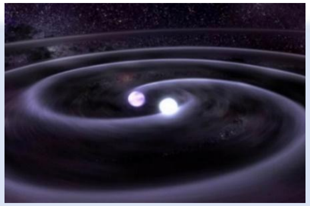
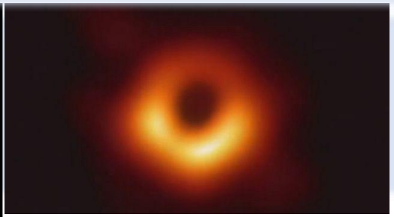


Black hole accretion disks



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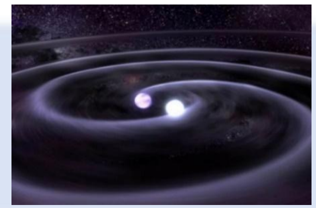

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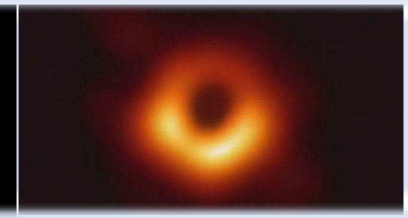
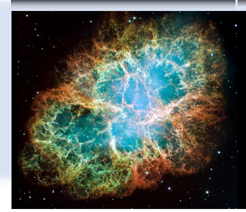


Supernovae



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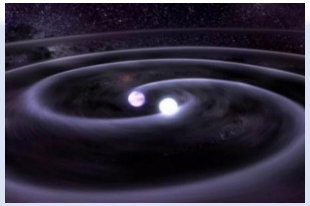

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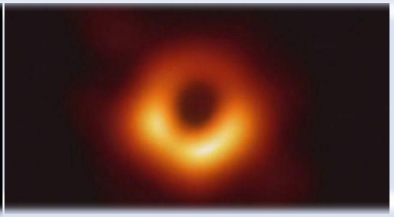

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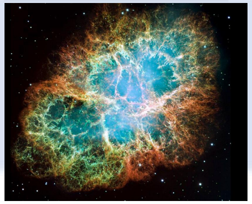
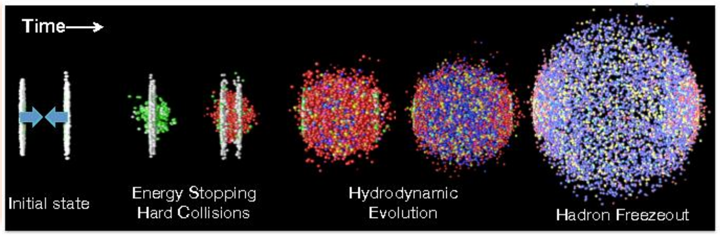


Quark-Gluon Plasma



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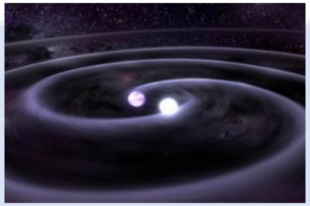

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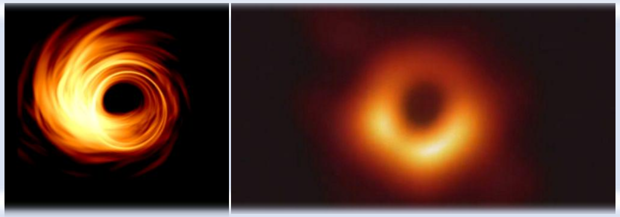

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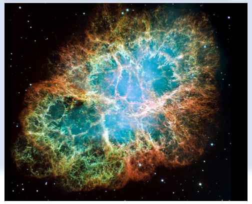

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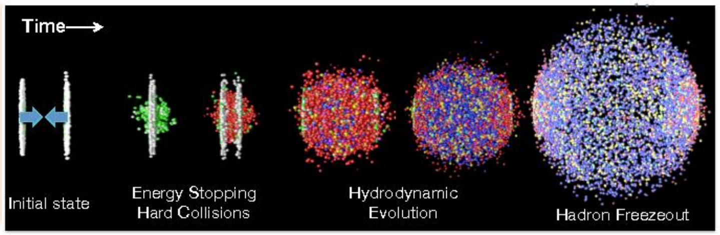
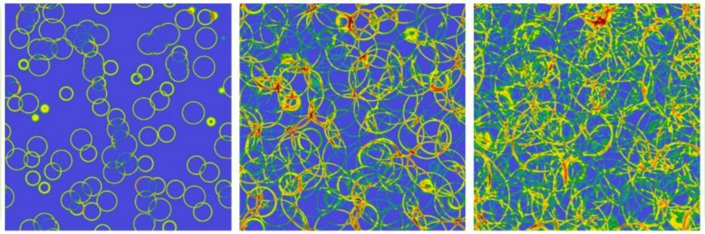


Early universe



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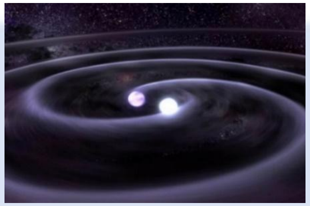

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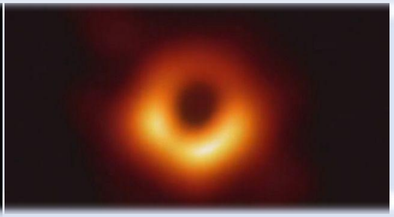

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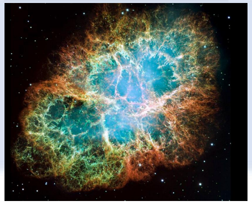

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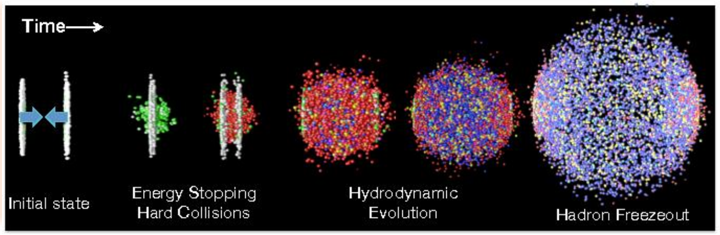

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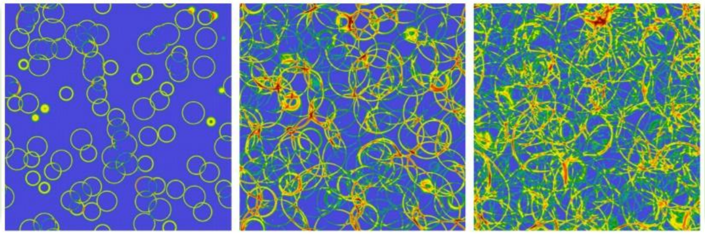

⟶ Relevant for groundbreaking research!

Hydrodynamics

What is hydrodynamics? ⟶ **Effective theory**

Hydrodynamics

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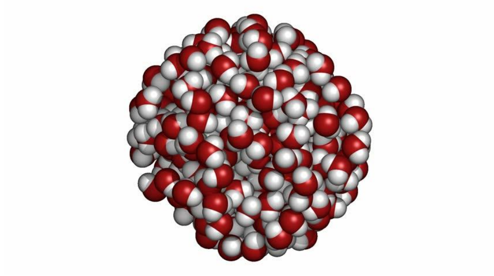
Water



Complicated molecular dynamics

Collective description: hydrodynamics

Hydrodynamics

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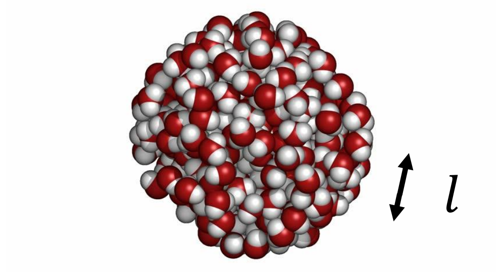

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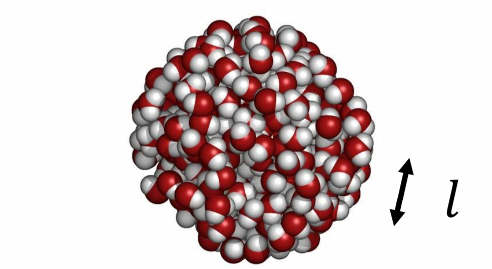

↕ l

↕ L

Complicated molecular dynamics

Collective description: hydrodynamics

⟶ Two scales well separated: $l \ll L$

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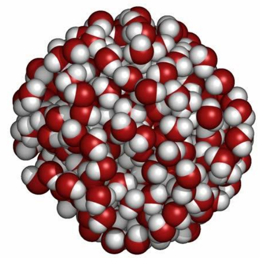

l

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L

Collective description: <u>hydrodynamics</u>

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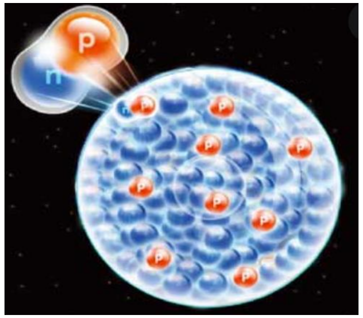
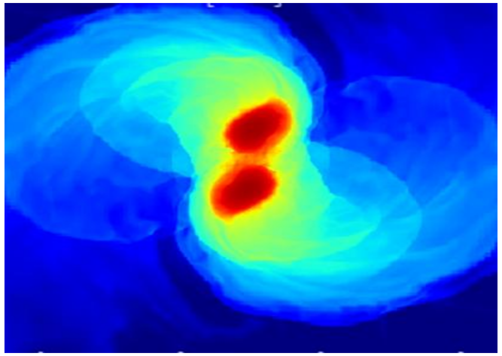
Neutron star merger

Complicated QCD dynamics



<u>Hydrodynamics</u>



Hydrodynamics: viscosity

Effective field theory

$$T^{\mu\nu} = T^{\mu\nu}_{ideal} + \partial + \partial^2 + \ldots$$

0th order 1st 2nd

$$\frac{l}{L} << 1$$

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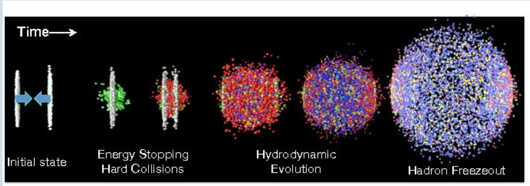
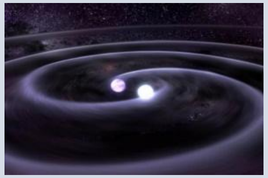
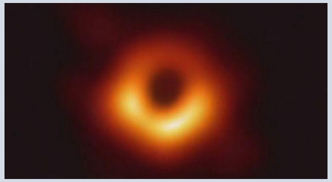
Dynamical equations

Real-time evolutions are required!!







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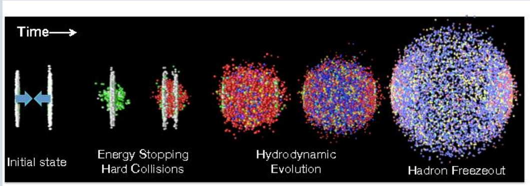

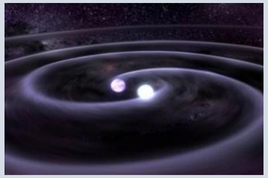

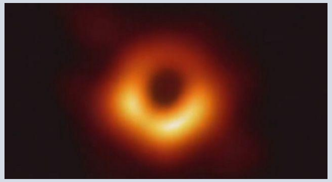

Ideal hydro → Well posed

Viscous hydro → Difficulties...

Hydrodynamics: viscosity

Effective field theory

$$T^{\mu\upsilon} = T^{\mu\upsilon}_{ideal} + \partial + \partial^2 + \ldots \qquad \frac{l}{L} << 1$$

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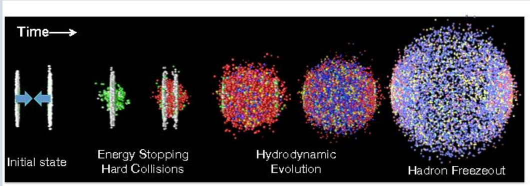
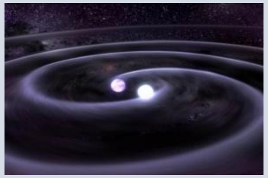
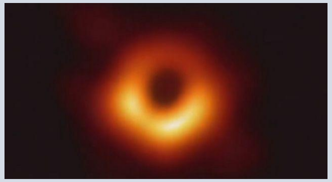






Ideal hydro ⟶ Well posed

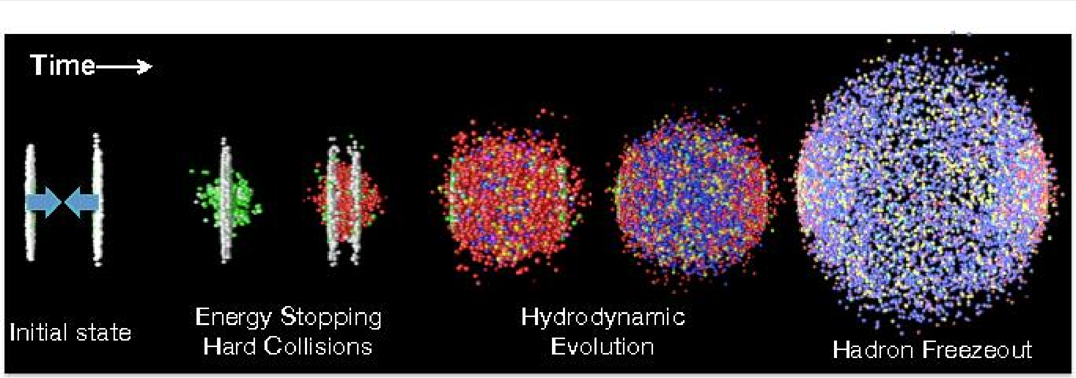
Viscous hydro ⟶ Difficulties...

But... is viscosity relevant? ⟶ Yes!

Quark-gluon plasma: viscosity

Viscosity is expected to be relevant in the physics of the quark-gluon plasma because the scale of the system and the microscopic scale of QCD are comparable

$$l \lesssim L$$



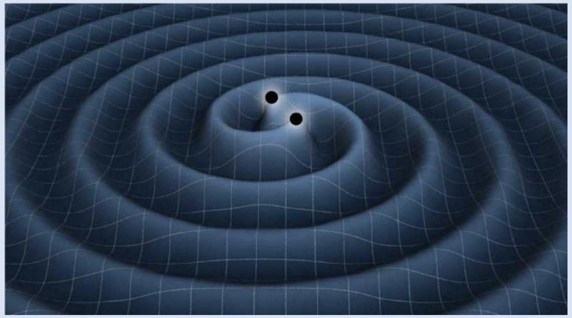
This is confirmed by experiments:

⟶ when including viscosity, better fits to the experimental data

Neutron star mergers: viscosity

Black hole mergers

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa T_{\mu\nu} \Rightarrow R_{\mu\nu} = 0$$



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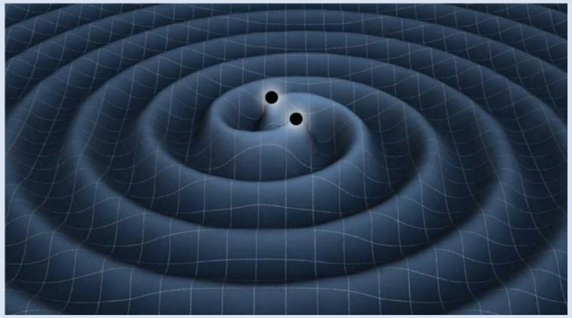
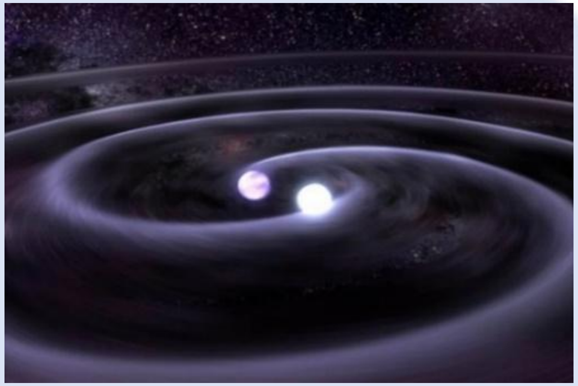


Neutron star mergers

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa T_{\mu\nu}$$

Matter must be specified



→ Gravity coupled to QCD

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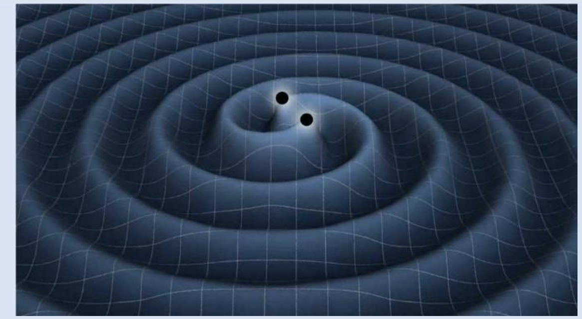

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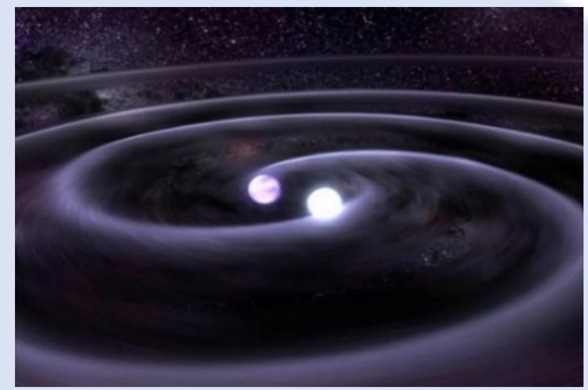

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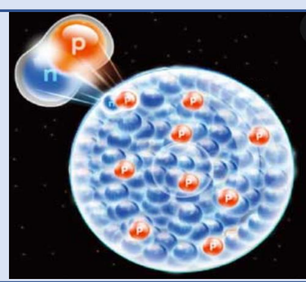
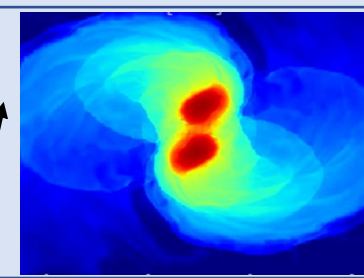


$l \sim$ fm $\ll$ L $\sim$ km

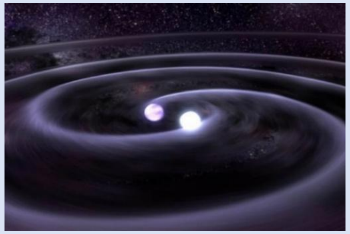


➡ Hydrodynamics provides a good description

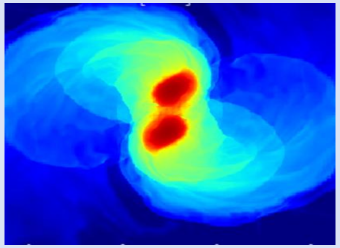
Neutron star mergers: viscosity

Picture from simulations:

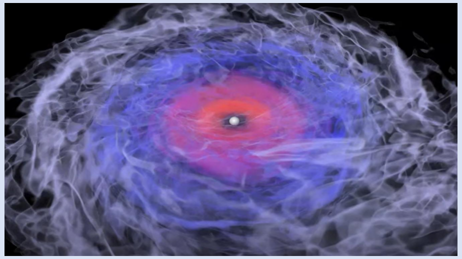
NS + NS



HMNS



BH+torus



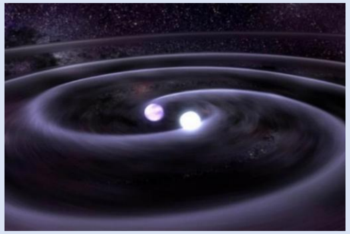
Highly dynamical post merger region

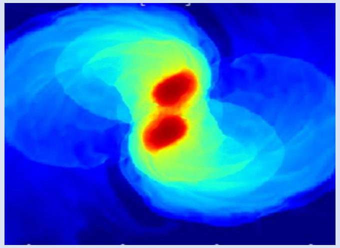
⟶ We must include all the **relevant physics**

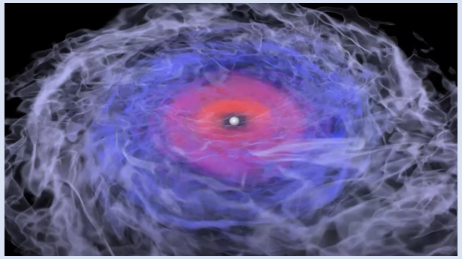
Neutron star mergers: viscosity

Picture from simulations:

NS + NS **HMNS** **BH+torus**







Highly dynamical post merger region

→ We must include all the **relevant physics**

Weak processes operate in timescales that are comparable!

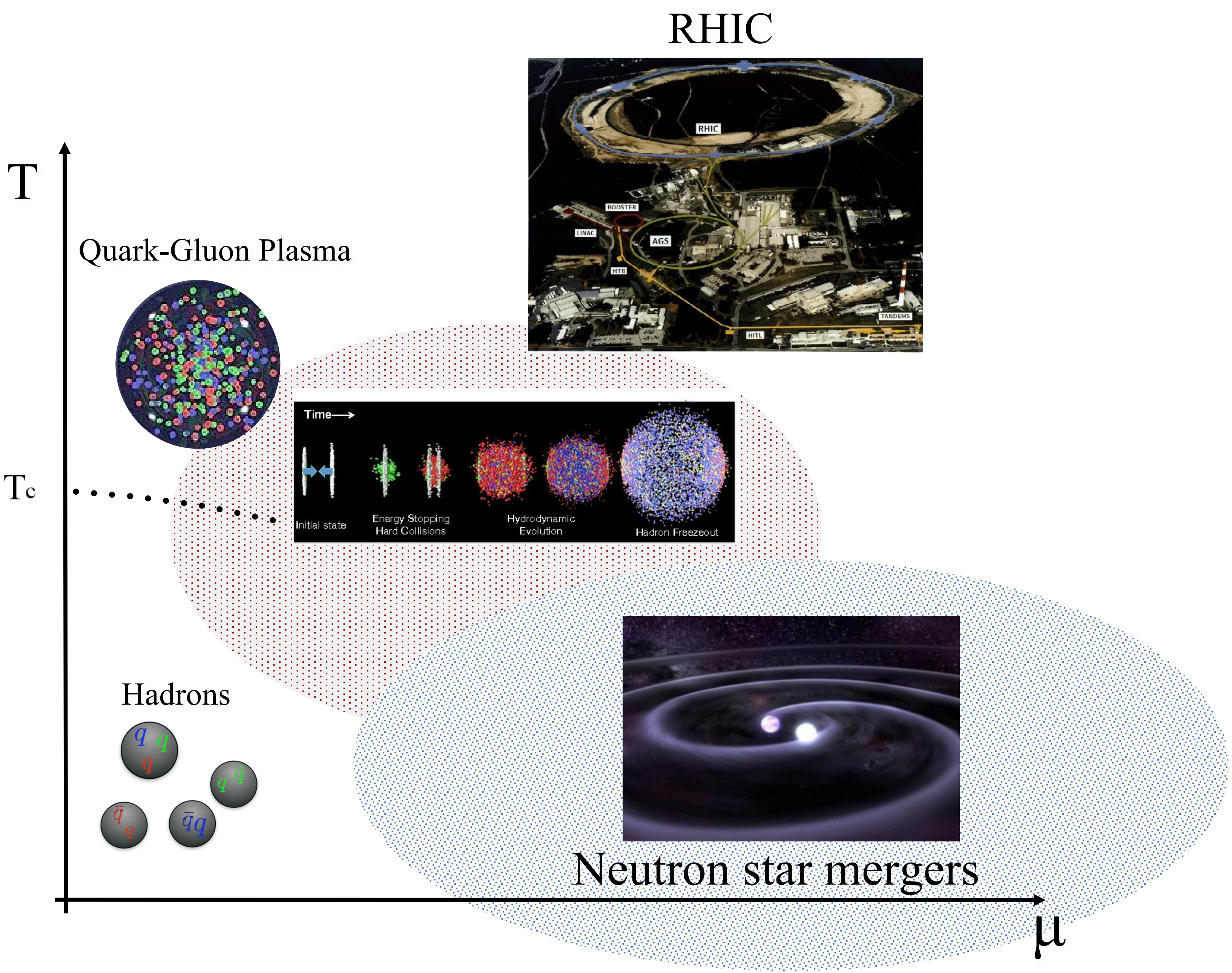
$$n \to p + e^- + \bar{\nu}_e$$

$$p + e^- \to n + \nu_e$$

→ **Effective bulk viscosity**!

M. Alford, A. Harutyunyan, A. Sedrakian '22
E. R. Most, A. Haber, S. P. Harris, Z. Zhang, M. G. Alford, J. Noronha'22
Alford, Haber, Harris, Zhang'21
Alford, Harutyunyan, Sedrakian '21
Most, Harris, Plumberg, Alford, Noronha, Noronha-Hostler, Pretorius, Witek, Yunes'21
....

QCD phase diagram



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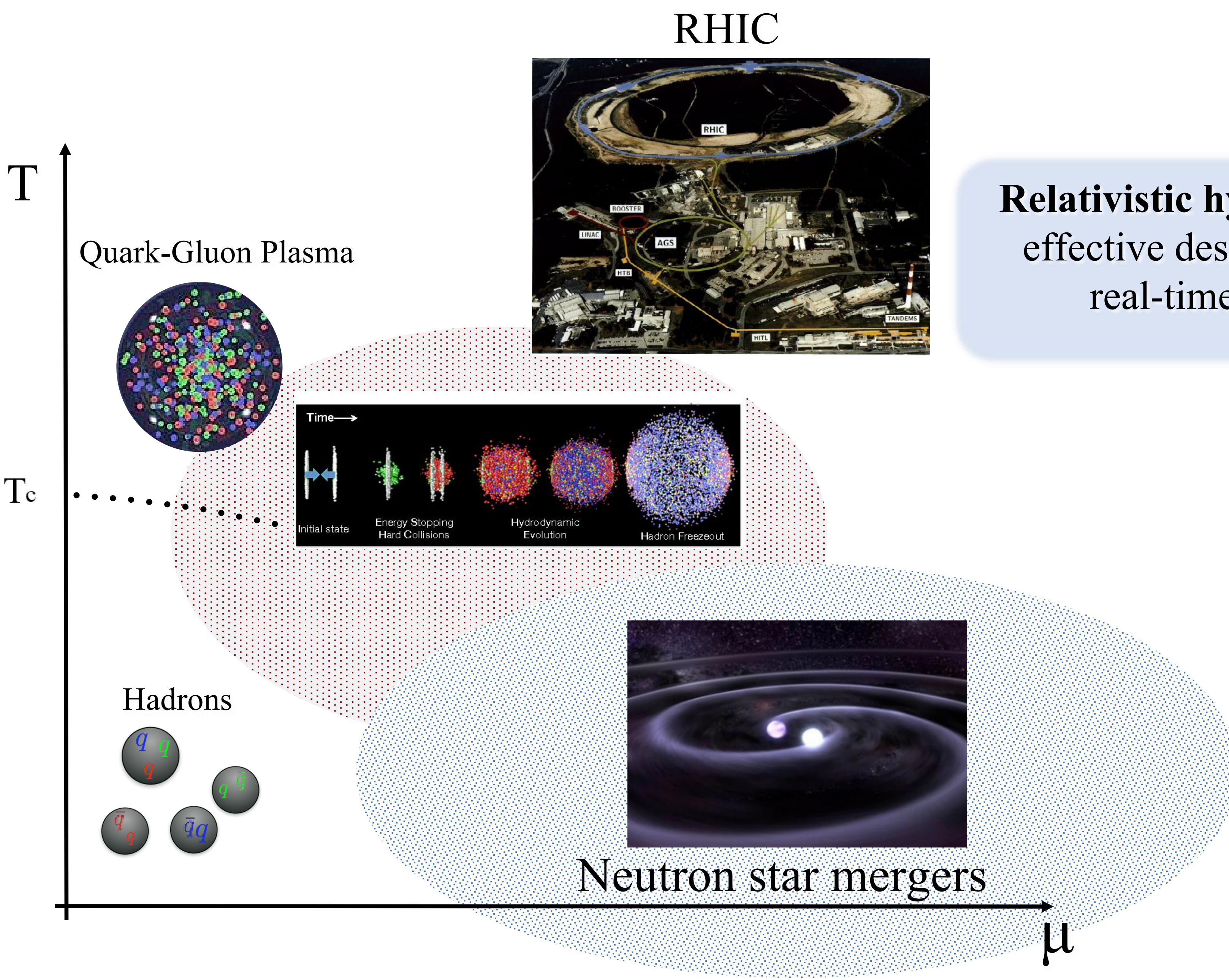
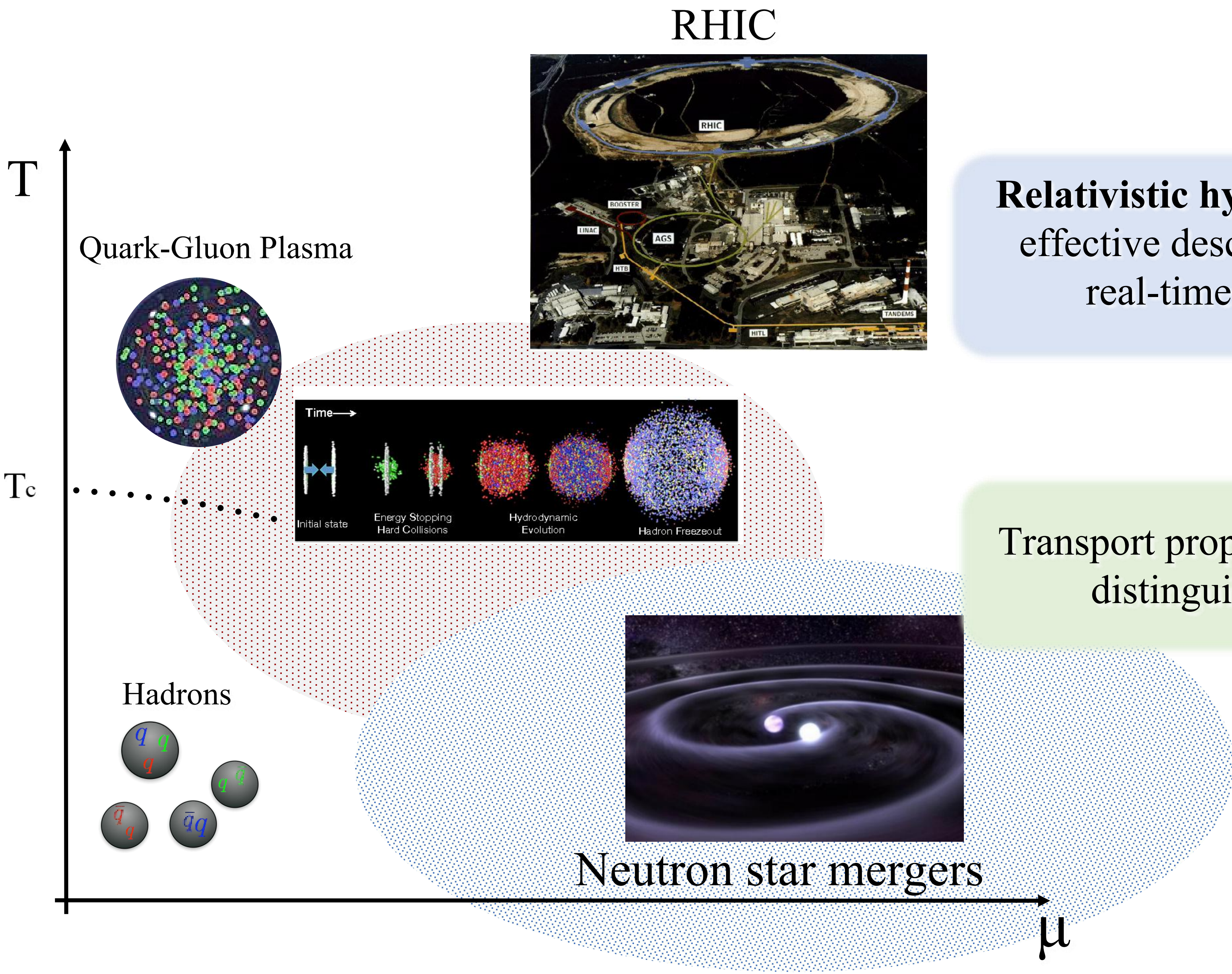


Relativistic hydrodynamics: effective description of the real-time dynamics

QCD phase diagram

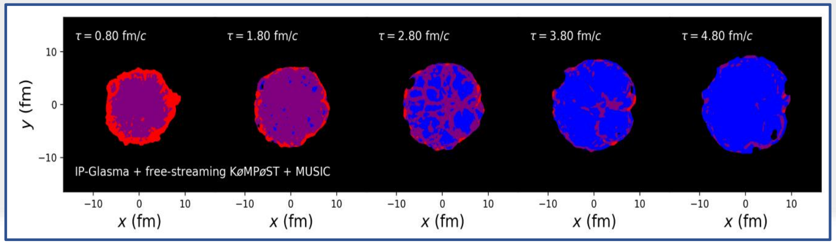


Relativistic hydrodynamics: effective description of the real-time dynamics

Transport properties: allow to distinguish phases

Relativistic Navier-Stokes: Plan

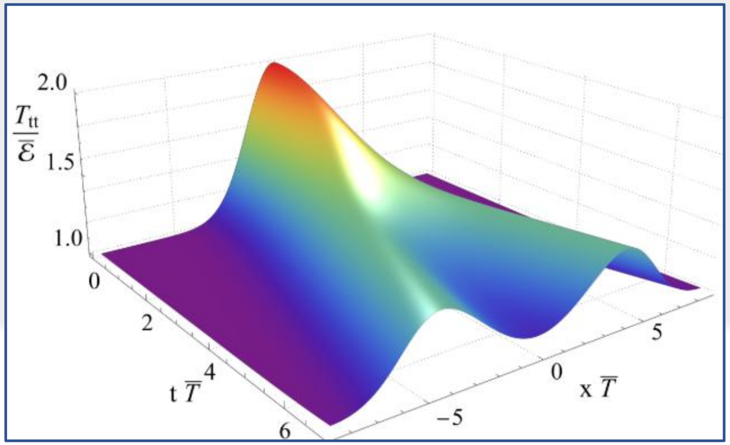
1 - Historical perspective, well posedness and alternative theories



2 - The equations

$$T^{\mu\nu} = (\epsilon + \mathcal{A})\left(u^\mu\, u^\nu + \frac{1}{3}\Delta^{\mu\nu}\right) + \mathcal{Q}^\mu\, u^\nu + u^\mu\, \mathcal{Q}^\nu - \eta\, \sigma^{\mu\nu}$$

3 - Real-time evolutions



Relativistic Navier-Stokes

Historical perspective, well-posedness and alternative theories

Relativistic Navier-Stokes

The relativistic version of Navier-Stokes equations was originally formulated by Eckart (1940) and Landau&Lifshitz (1959)

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Hiscock, Lindblom ‘85

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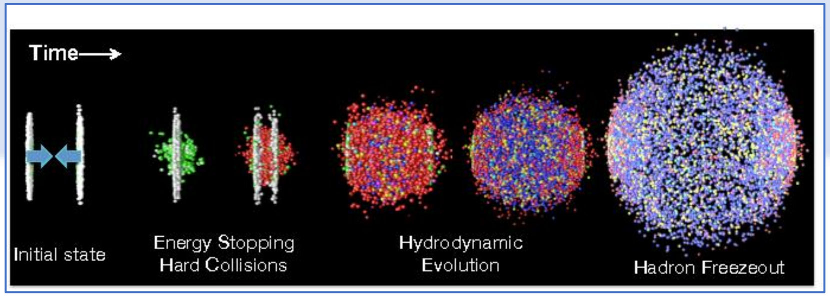
From a modern perspective:

→ Eckart and Landau frames related by field redefinitions

Alternative: MIS theories

Meanwhile the experimental analysis of the quark-gluon plasma required of some viscous hydrodynamical description



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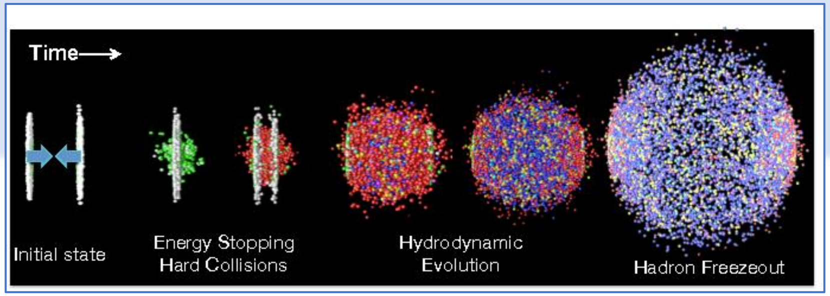

An approach that provides such a description by Müller, Israel and Stewart (MIS)

Muller '67
Israel '76
Israel, Stewart '79

Different variants:
- -BRSSS
- -DNMR
- -Divergence type
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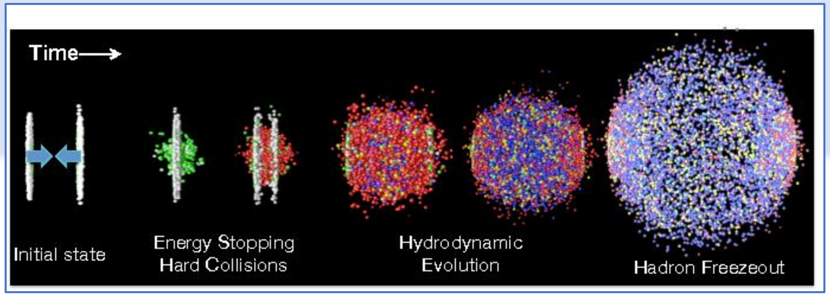

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MIS-type theories

Problems alleviated ⟶ allows to describe the experimental data!

(Still, lack of a well-posedness proof...)

Well-behaved relativistic Navier-Stokes

The problem with relativistic version of Navier Stokes remained unsolved...

In recent years a well-behaved version of relativistic Navier-Stokes has been proposed

Bemfica, Disconzi, Noronha ‘17
Bemfica, Disconzi, Noronha ‘19
Kovtun ‘19

The key insight was to realize that by performing specific field redefinitions, good properties can be restored.

Nomenclature

BDNK = Relativistic first-order viscous hydrodynamics = Relativistic Navier-Stokes

Well-posedness

Well posedness well established for many physically relevant equations:

- Maxwell equations
- Einstein equations
- Ideal hydrodynamics
- ....

⟶ However, limited results in relativistic viscous hydrodynamics

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Relativistic Navier-Stokes

Proofs of well-posedness obtained in different levels of generality in recent years ⇒

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Local well-posedness of the initial value problem (Cauchy problem) for initial data in Sobolev spaces, in non-conformal theories in the presence of charge.

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→ Existence and uniqueness of solutions

Respect the principles of relativity: characteristics not faster than speed of light

→ Sufficiently good properties for applications in systems of interest like neutron star mergers and quark-gluon plasma.

Navier-Stokes vs MIS

If MIS theories provide a good description of experimental data:

→ Why do we need another formulation of viscous hydrodynamics?

→ Is one theory 'better' than the other?

Navier-Stokes vs MIS

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→ Why do we need another formulation of viscous hydrodynamics?

→ Is one theory 'better' than the other?

Three relevant arguments:

1 - Well-posedness

2 - Characteristic velocities

3 - Strong shockwaves

Navier-Stokes vs MIS

- Well posedness of MIS unknown until 2020.

Nonlinear Constraints on Relativistic Fluids Far From Equilibrium

Fábio S. Bemfica,[1] Marcelo M. Disconzi,[2] Vu Hoang,[3] Jorge Noronha,[4] and Maria Radosz[3]

[1] *Escola de Ciências e Tecnologia, Universidade Federal do Rio Grande do Norte, 59072-970, Natal, RN, Brazil**
[2] *Department of Mathematics, Vanderbilt University, Nashville, TN, USA†*
[3] *Department of Mathematics, The University of Texas at San Antonio, One UTSA Circle, San Antonio, TX 78249, USA‡*
[4] *Department of Physics, University of Illinois, 1110 W. Green St., Urbana IL 61801-3080, USA§*

(Dated: May 26, 2020)

$$(2\eta + \lambda_{\pi\Pi}\Pi) - \frac{1}{2}\tau_{\pi\pi}|\Lambda_1| \geq 0$$

$$\varepsilon + P + \Pi - \frac{1}{2\tau_\pi}(2\eta + \lambda_{\pi\Pi}\Pi) - \frac{\tau_{\pi\pi}}{4\tau_\pi}\Lambda_3 \geq 0,$$

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  Depend on the state! ⟶ Must be checked pointwise in spacetime for every evolution!!

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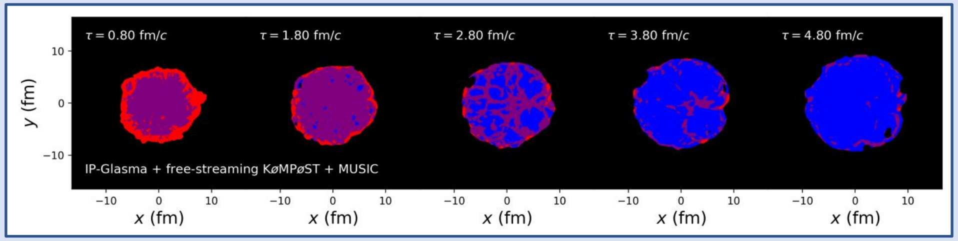
- Check in realistic simulations:

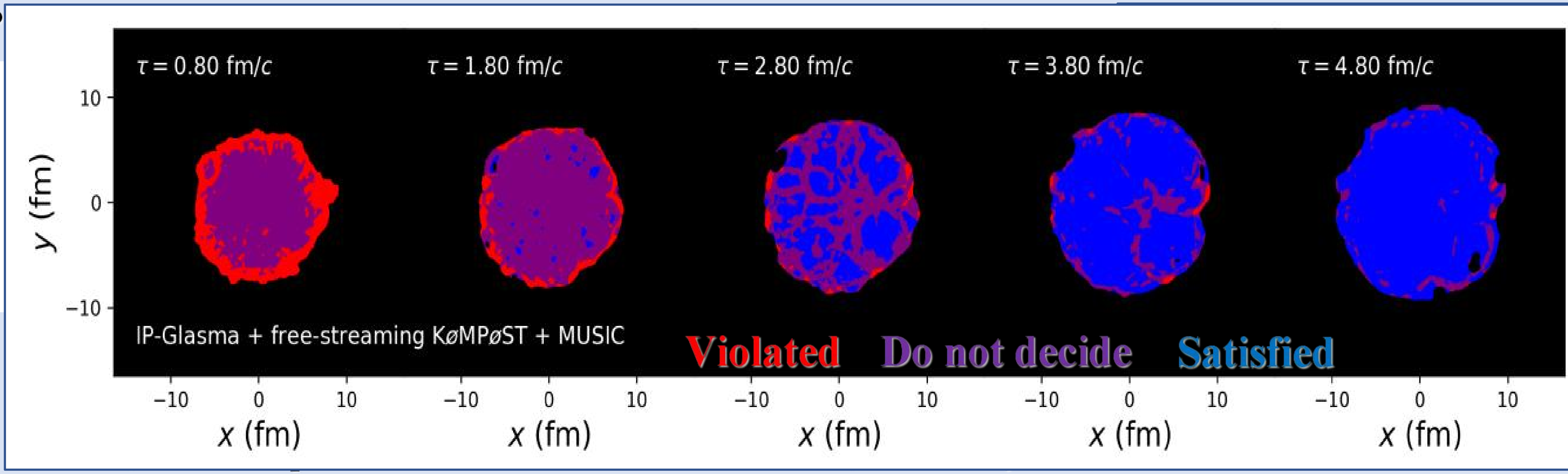
Causality violations in realistic simulations of heavy-ion collisions

Christopher Plumberg,[1] Dekrayat Almaalol,[2] Travis Dore,[1] Jorge Noronha,[1] and Jacquelyn Noronha-Hostler[1]
[1]*Illinois Center for Advanced Studies of the Universe, Department of Physics, University of Illinois at Urbana-Champaign, Urbana, IL 61801, USA*
[2]*Department of Physics, Kent State University, Kent, OH 44242, USA*
(Dated: March 31, 2021)

⟶ **Significant violations!**



Navier-Stokes vs MIS



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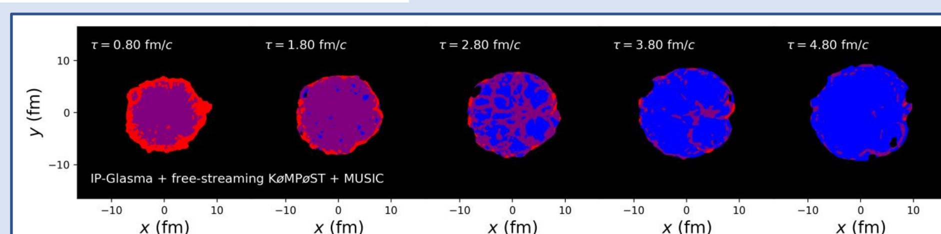

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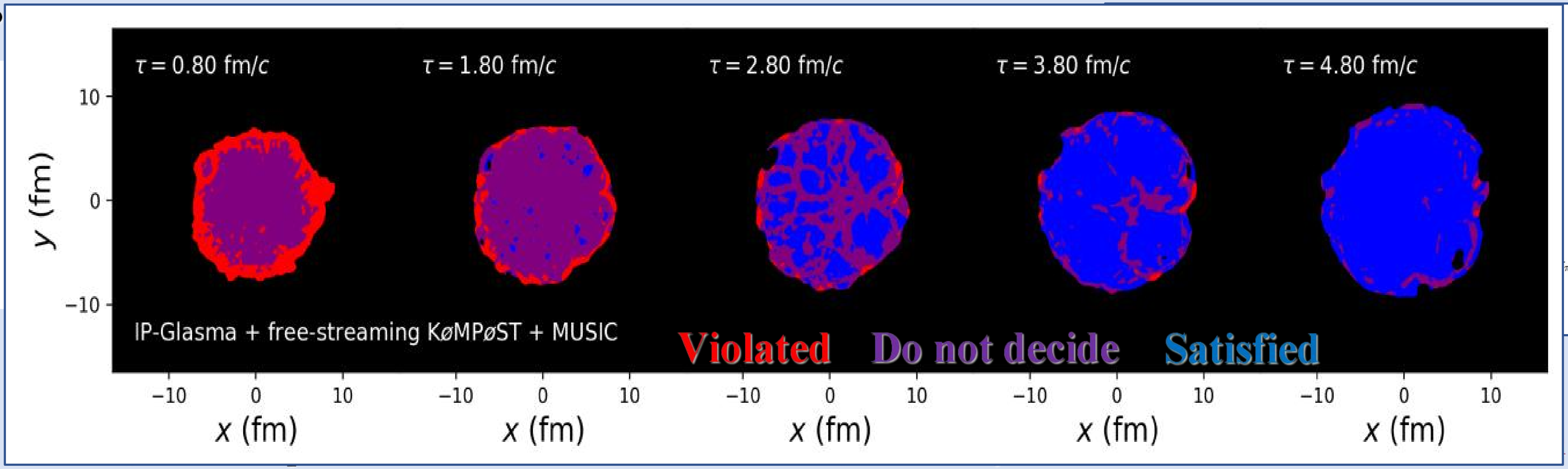
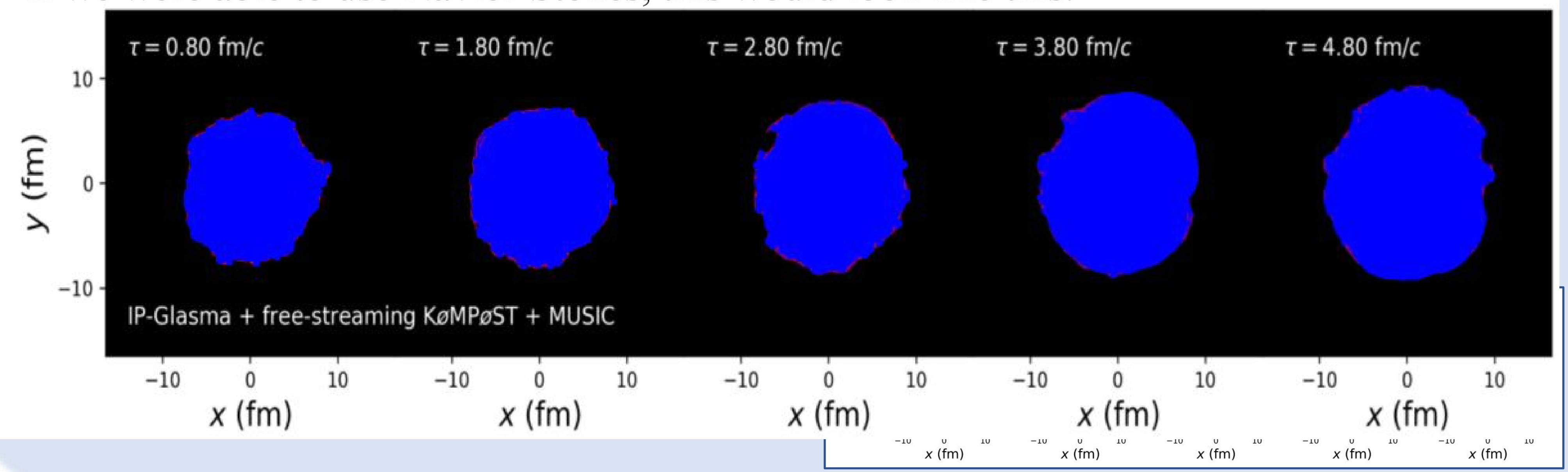


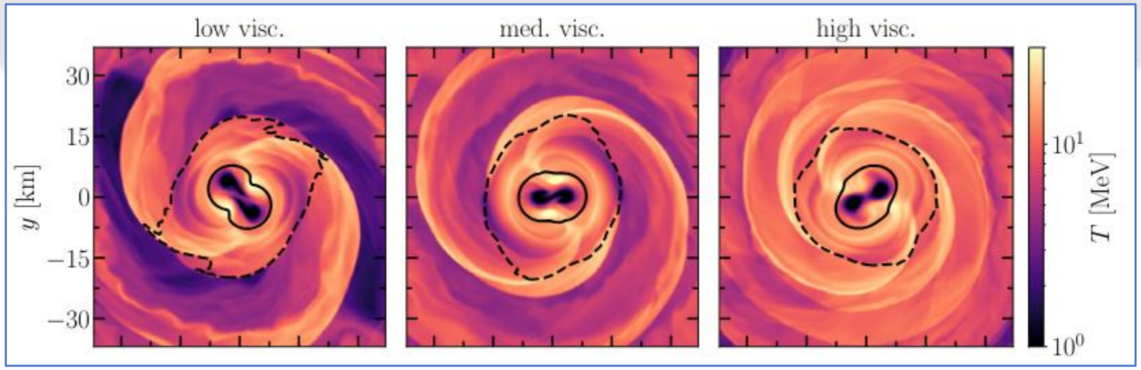
If we were able to use Navier-Stokes, this would look like this:



Navier-Stokes vs MIS: neutron star mergers

Only very recently the first viscous neutron star mergers were constructed

Chabanov, Rezzolla ‘23 (a)
Chabanov, Rezzolla ‘23 (b)



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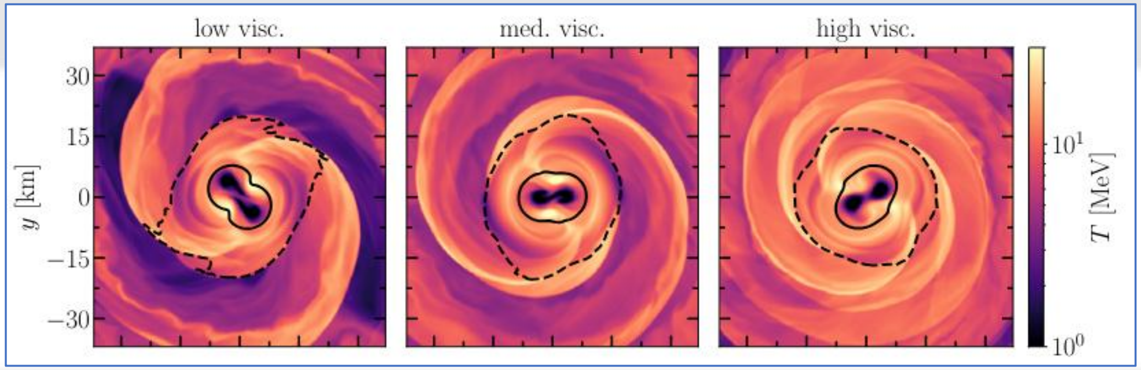

They use MIS theory: suffer same issues!

They have to change, by hand, the viscosity in regions of spacetime where the conditions are violated.

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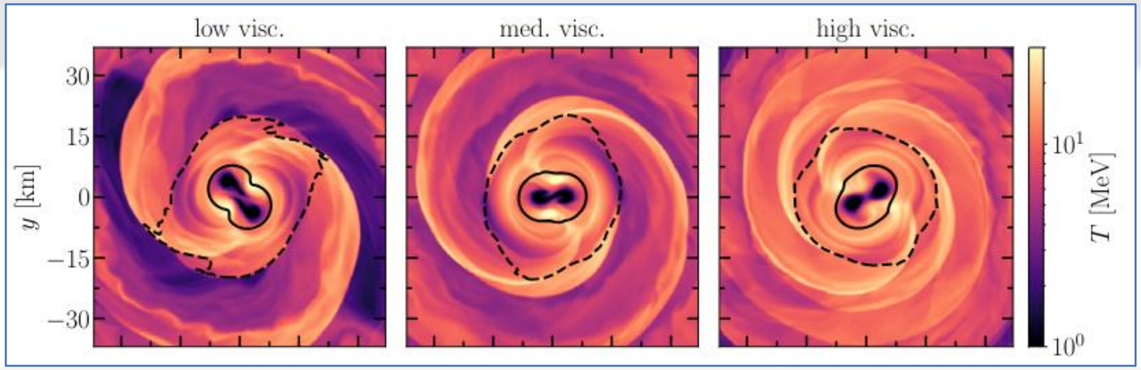

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Relativistic Navier-Stokes ⟶ Well-posedness ensured!!

Promising alternative!

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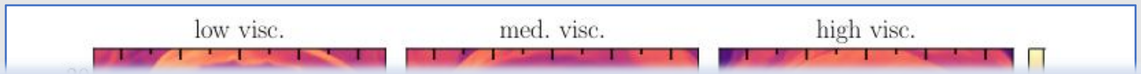

But even in the case that for all those examples the conditions were satiesfied

We still would have to check those conditions pointwise in **every future evolution**

For Navier Stokes this is ensured for every evolution

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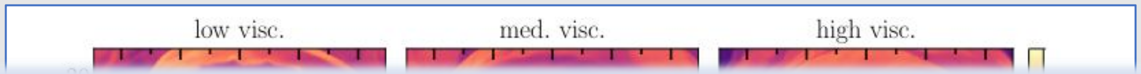

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Thus, **at a fundamental level, Navier-Stokes promising alternative to MIS**

(In this specific sense the theory is ‘better’)

Relativistic Navier-Stokes:
The equations

Navier-Stokes equations

→ Conformal theory

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- Ideal hydro $$T^{\mu\nu} = \epsilon \left(u^\mu u^\nu + \frac{1}{3} \Delta^{\mu\nu} \right)$$ ⇒ $\nabla_\mu T^{\mu\nu} = 0$ Well posed!!

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$$\epsilon \to \epsilon + \mathcal{A} \,,$$

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Real numbers specifying the frame (a_1, a_2)

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Well-posed!!

$$a_1 \geq 4 \,, \qquad a_2 \geq \frac{3a_1}{a_1 - 1}$$

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Bemfica, Disconzi, Noronha '17'19
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$$T^{\mu\nu} = \epsilon \left(u^\mu u^\nu + \frac{1}{3} \Delta^{\mu\nu} \right) - \eta$$

→ What is the significance of these terms?

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- First order hydro: **general frame**

$$T^{\mu\nu} = (\epsilon + \mathcal{A}) \left(u^\mu u^\nu + \frac{1}{3} \Delta^{\mu\nu} \right) + \mathcal{Q}^\mu u^\nu + u^\mu \mathcal{Q}^\nu - \eta\, \sigma^{\mu\nu}$$

→ $\nabla_\mu T^{\mu\nu} = 0$

Well-posed!!

$$a_1 \geq 4 \,, \qquad a_2 \geq \frac{3a_1}{a_1 - 1}$$

Real numbers specifying the frame

$\{a_1, a_2\} = \{0, 0\}$ Landau frame

Bemfica, Disconzi, Noronha '17'19
Kovtun '19

Navier-Stokes equations

→ Conformal theory

- Ideal hydro $$T^{\mu\nu} = \epsilon\left(u^\mu u^\nu + \frac{1}{3}\Delta^{\mu\nu}\right)$$ ⇒ $\nabla_\mu T^{\mu\nu} = 0$ Well posed!!

$$\frac{3}{4}\frac{\dot\epsilon}{\epsilon} + \nabla\cdot u = 0 \qquad \dot u^\mu + \frac{1}{4}\frac{\nabla_\perp^\mu \epsilon}{\epsilon} = 0$$

- First order hydro **Landau frame** $$T^{\mu\nu} = \epsilon\left(u^\mu u^\nu + \frac{1}{3}\Delta^{\mu\nu}\right) - \eta\,\sigma^{\mu\nu}$$

→ What is the significance of these terms?

In the spirit of effective field theory:

$$\epsilon \to \epsilon + \mathcal{A}\,, \qquad u^\mu \to u^\mu + \frac{\mathcal{Q}^\mu}{\epsilon + p}$$

$$\mathcal{A} := a_1\eta\left(\frac{3}{4}\frac{\dot\epsilon}{\epsilon} + \nabla\cdot u\right) \qquad \mathcal{Q}^\mu := a_2\eta\left(\dot u^\mu + \frac{1}{4}\frac{\nabla_\perp^\mu\epsilon}{\epsilon}\right)$$

- First order hydro: **general frame**

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⇒ $\nabla_\mu T^{\mu\nu} = 0$ Well posed!!

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- First order hydro **Landau frame**

$$T^{\mu\nu} = \epsilon \left(u^\mu u^\nu + \frac{1}{3} \Delta^{\mu\nu} \right) - \eta$$

→ What is the significance of these terms?

→ On-shell are of second order!

→ should not to affect the physics to first order

In the spirit of effective field theory:

$$\epsilon \to \epsilon + \mathcal{A} \,, \qquad u^\mu \to u^\mu + \frac{\mathcal{Q}^\mu}{\epsilon + p}$$

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→ What is the significance of these terms?

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→ TYPE I versus TYPE II frames

In the spirit of effective field theory:

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→ What is the significance of these terms?

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→ TYPE I versus TYPE II frames

→ Formal statement versus finite gradients: Numerical evolutions for assessment

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$$\mathcal{A} := a_1\eta\left(\frac{3}{4}\frac{\dot\epsilon}{\epsilon} + \nabla\cdot u\right) \qquad \mathcal{Q}^\mu := a_2\eta\left(\dot u^\mu + \frac{1}{4}\frac{\nabla_\perp^\mu \epsilon}{\epsilon}\right)$$

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Real numbers specifying the frame

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Bemfica, Disconzi, Noronha '17'19
Kovtun '19

Relativistic Navier-Stokes:

Real-time evolutions

Evolving relativistic Navier-Stokes

Mathematical results have been established, but... do these equations admit solutions?

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Studies of real-time evolutions:

Bea, Figueras '23
Bantilan, Bea, Figueras '22
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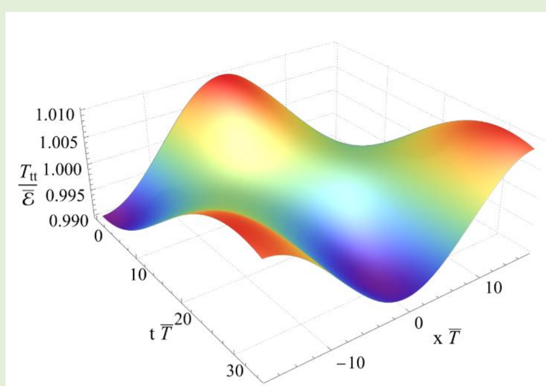
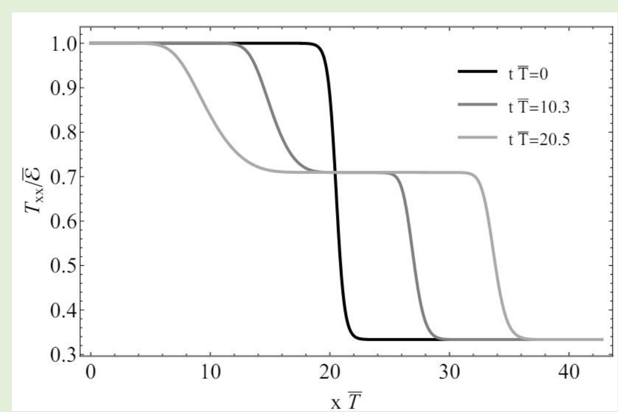
First conclusion ⟶ They admit physically sensible solutions!

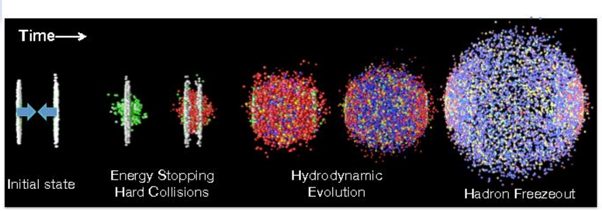
Sound waves, Riemann problem, shockwaves, etc.

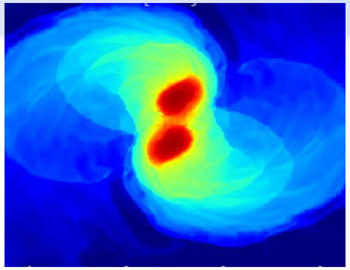




Frame independence

If we want to implement Navier-Stokes in these physical systems of interest,
we first need to understand the effect of using different frames





$$a_1 \geq 4\,, \qquad a_2 \geq \frac{3a_1}{a_1 - 1}$$

We make precise and provide evidence for the statement:

The arbitrarily chosen frame does not affect the physics up to first order, as long as the system is in the effective field theory regime

Bea, Figueras '23

To make this precise we define 3 criteria: A, B and C

Criteria

$$T^{\mu\nu} = T^{\mu\nu}_{ideal} + \boldsymbol{T^{(1)}_{\mu\nu}} + \boldsymbol{T^{(2)}_{\mu\nu}} + \ \dots$$

$$T^{\mu\nu} = (\epsilon + \mathcal{A}) \left(u^\mu \, u^\nu + \frac{1}{3} \Delta^{\mu\nu} \right) + \mathcal{Q}^\mu \, u^\nu + u^\mu \, \mathcal{Q}^\nu - \eta \, \sigma^{\mu\nu}$$

Criterion A

Motivated by effective field theory:

If hierarchy $\boldsymbol{T^{ideal}_{\mu\nu}} \gg \boldsymbol{T^{(1)}_{\mu\nu}} \gg \boldsymbol{T^{(2)}_{\mu\nu}}$ ⇒ Solution is in the effective field theory regime

(Notice: solutions might not be in the hydrodynamic regime)

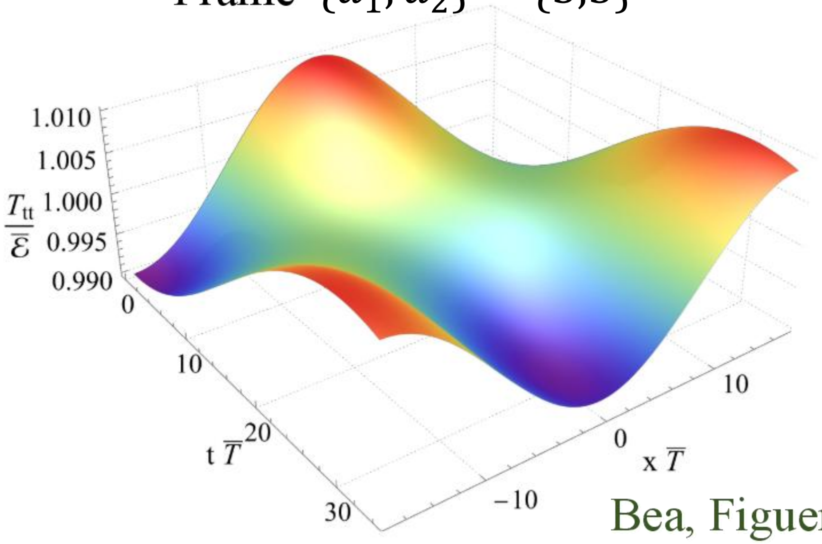
Criterion B

If $\boldsymbol{\eta\sigma^{\mu\nu}} \gg \boldsymbol{\mathcal{A}}, \mathcal{Q}$ ⇒ The physics to first order is independent on the arbitrarily chosen frame

(if we change $\boldsymbol{\mathcal{A}}$, $\mathcal{Q}$ by a factor of 2, this is still much smaller than first order physics, namely $\boldsymbol{\eta\sigma^{\mu\nu}}$)

Sound wave

Frame $\{a_1, a_2\} = \{5,5\}$



Bea, Figueras ‘23

Sound wave

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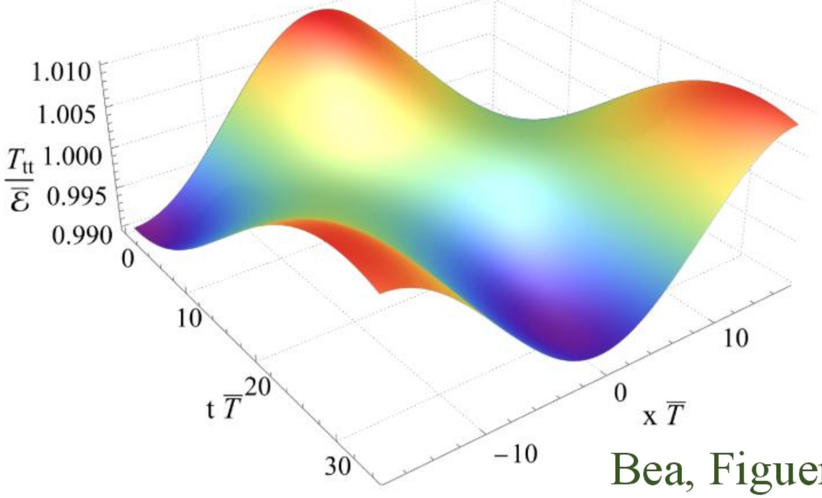


Bea, Figueras ‘23

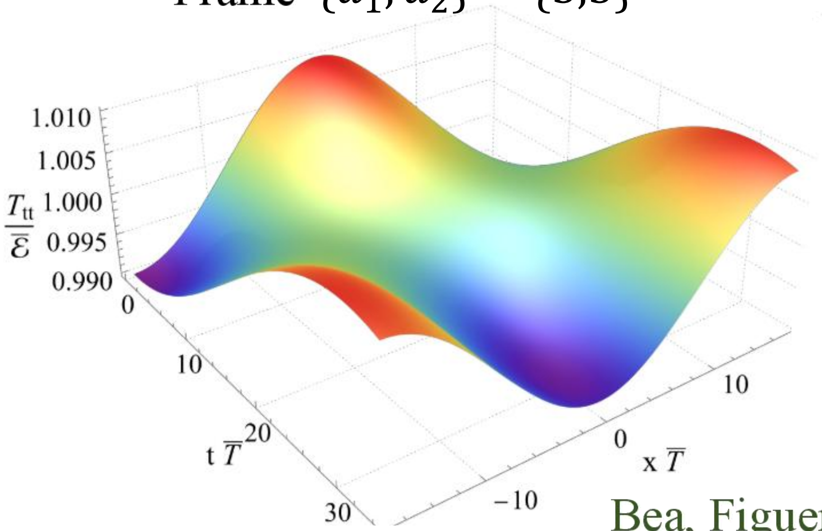
$$T^{\mu\nu} = (\epsilon + \mathcal{A})\left(u^\mu u^\nu + \frac{1}{3}\Delta^{\mu\nu}\right) + \mathcal{Q}^\mu u^\nu + u^\mu \mathcal{Q}^\nu - \eta\,\sigma^{\mu\nu}$$

$$\mathcal{A} := a_1\,\eta\left(\frac{3}{4}\frac{\dot\epsilon}{\epsilon} + \nabla\cdot u\right)$$

$$\mathcal{Q}^\mu := a_2\eta\left(\dot u^\mu + \frac{1}{4}\frac{\nabla_\perp^\mu \epsilon}{\epsilon}\right)$$

Sound wave

Frame $\{a_1, a_2\} = \{5{,}5\}$



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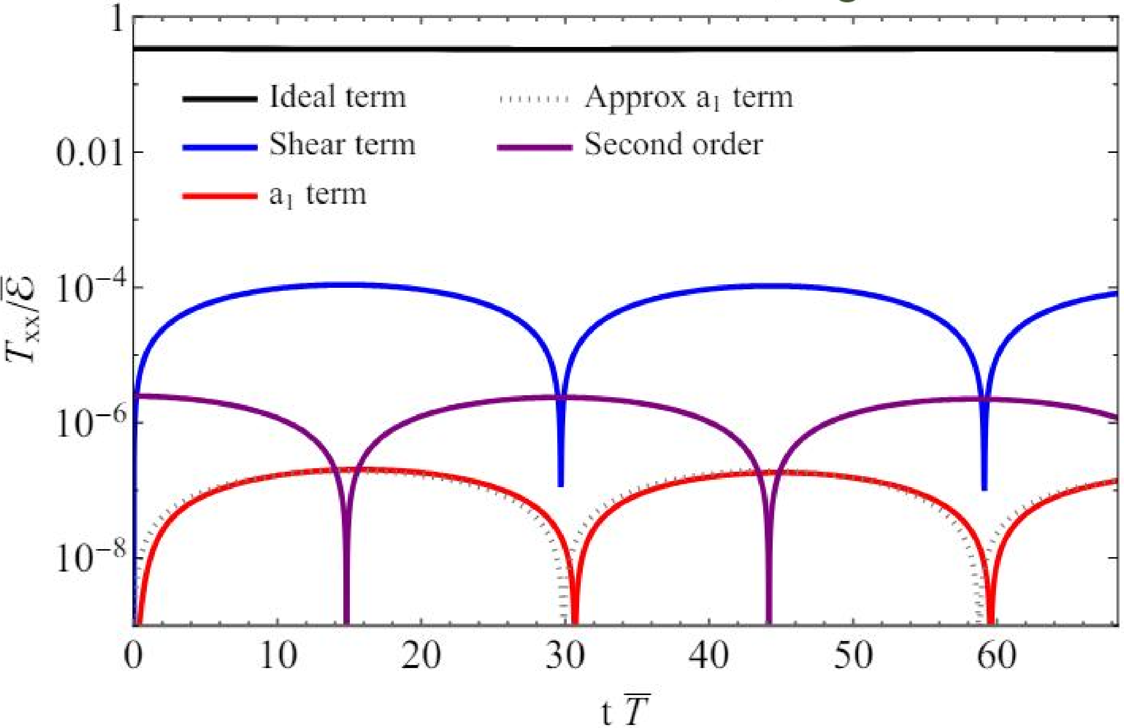

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Sound wave

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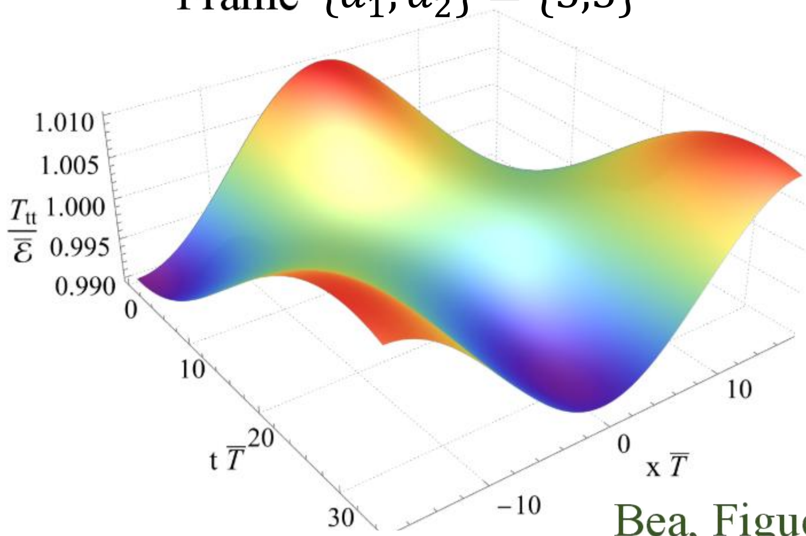
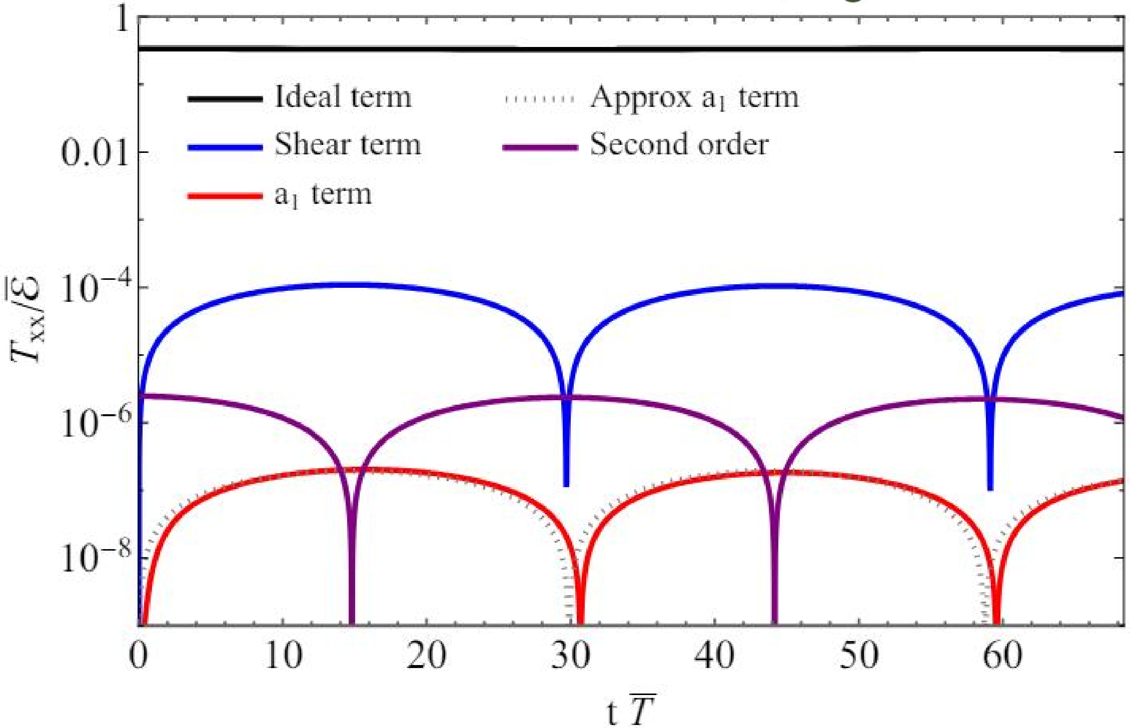


Bea, Figueras '23



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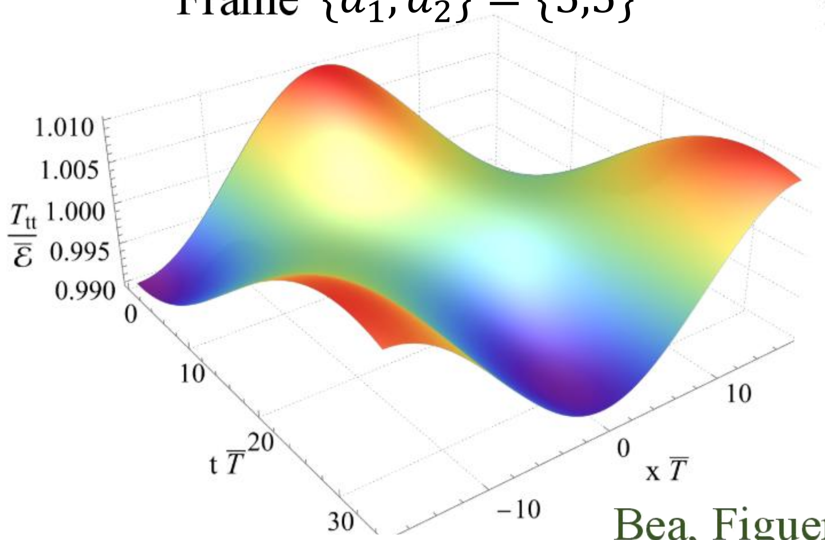
Criterion A satisfied

$$T_{\mu\nu}^{ideal} \gg T_{\mu\nu}^{(1)} \gg T_{\mu\nu}^{(2)}$$

Solution is in the effective field theory regime

Sound wave

Frame $\{a_1, a_2\} = \{5,5\}$



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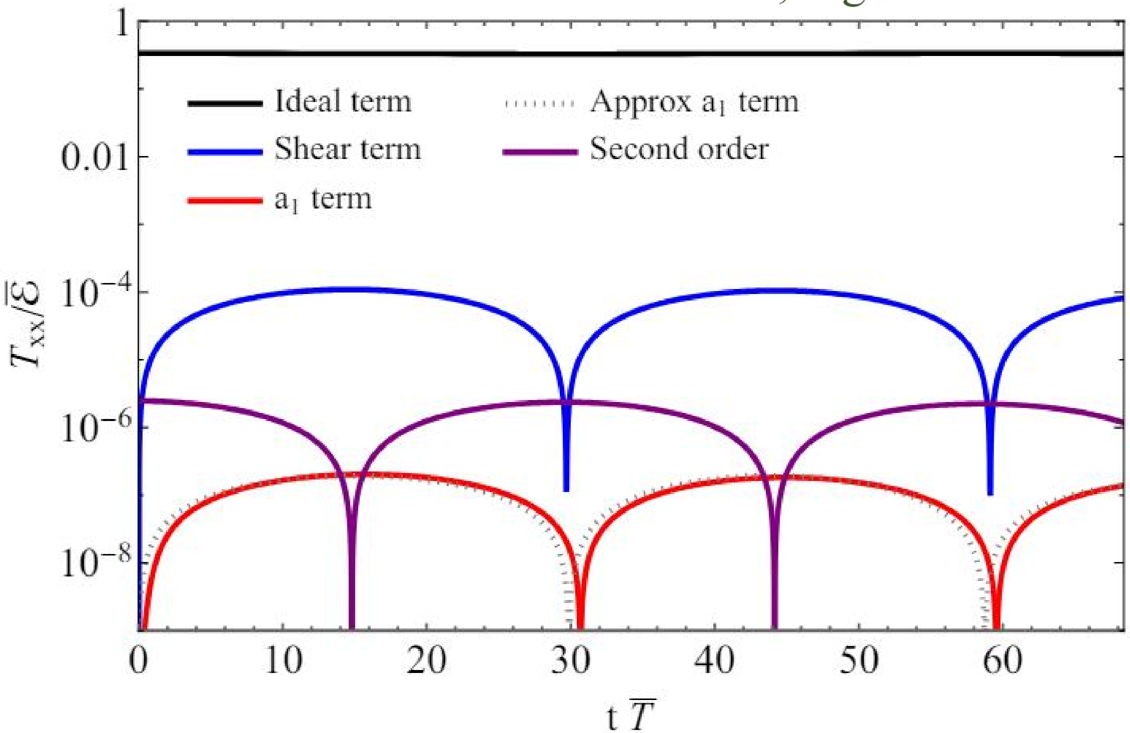

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Criterion A satisfied

$$T_{\mu\nu}^{ideal} >> T_{\mu\nu}^{(1)} >> T_{\mu\nu}^{(2)}$$

Solution is in the effective field theory regime

Criterion B satisfied

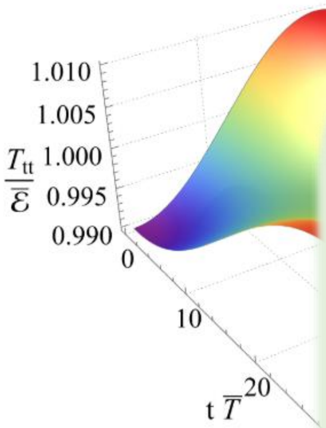
$$\eta\sigma^{\mu\nu} >> \mathcal{A}\,,\,\mathcal{Q}$$

The physics to first order is independent of the arbitrarily chosen frame

Sound wave

Frame $\{a_1, a_2\} = \{5,5\}$

$$T^{\mu\nu} = (\epsilon + \mathcal{A})\left(u^\mu u^\nu + \frac{1}{3}\Delta^{\mu\nu}\right) + \mathcal{Q}^\mu u^\nu + u^\mu \mathcal{Q}^\nu - \eta\,\sigma^{\mu\nu}$$



After analyzing a variety solutions, we conclude:

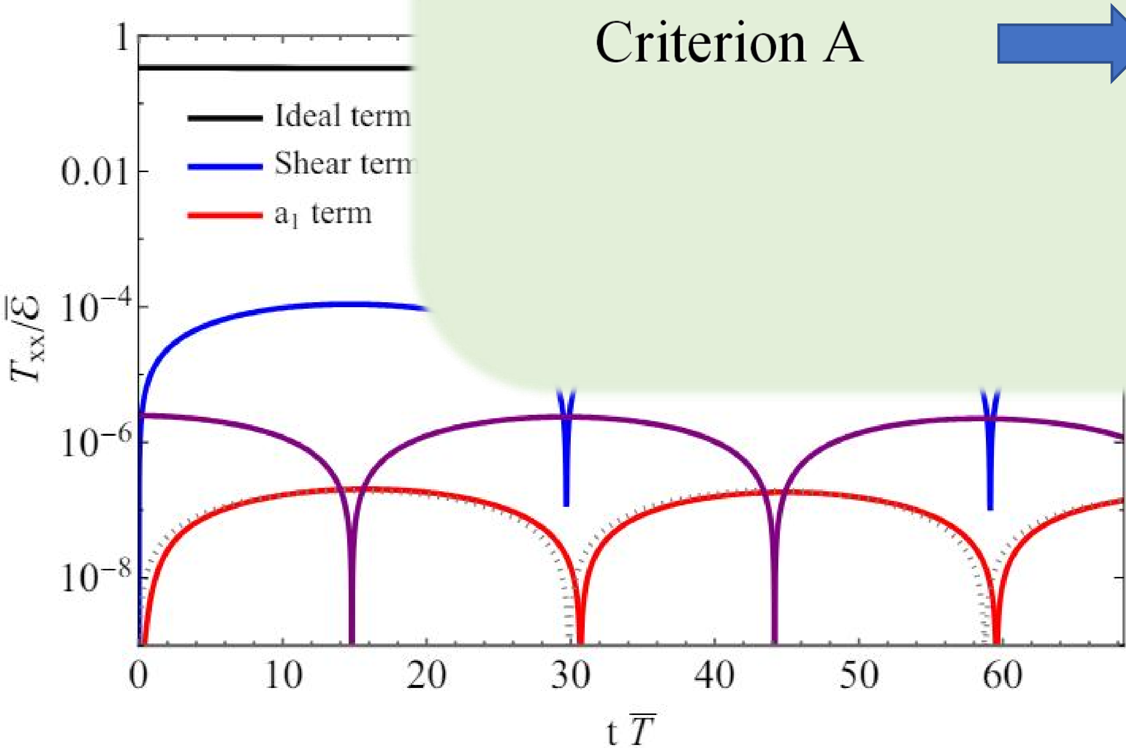
Criterion A → Criterion B

> $T^{(2)}_{\mu\nu}$

ld theory regime

$\mathcal{A}, \mathcal{Q}$

The physics to first order is independent of the arbitrarily chosen frame



Sound wave

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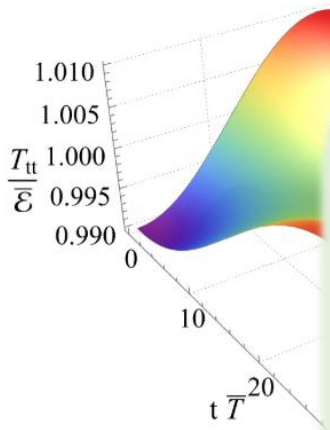
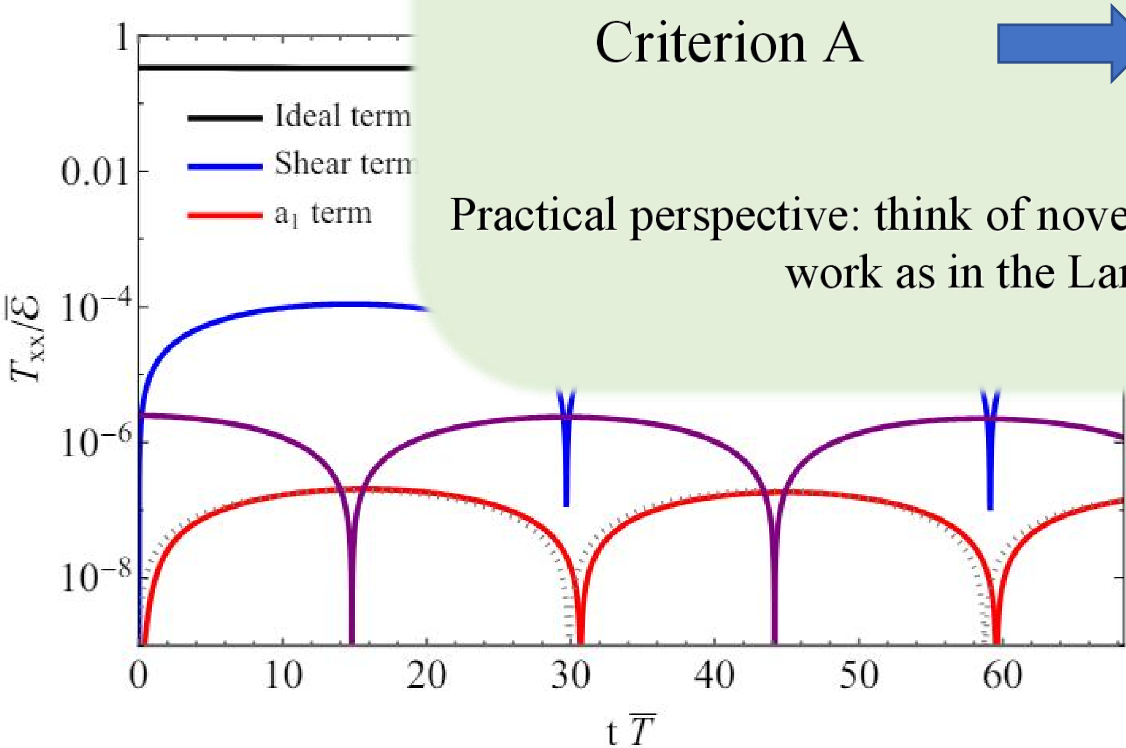




After analyzing a variety solutions, we conclude:

Criterion A → Criterion B

Practical perspective: think of novel terms as 'mere regulators', work as in the Landau frame.

> $T^{(2)}_{\mu\nu}$

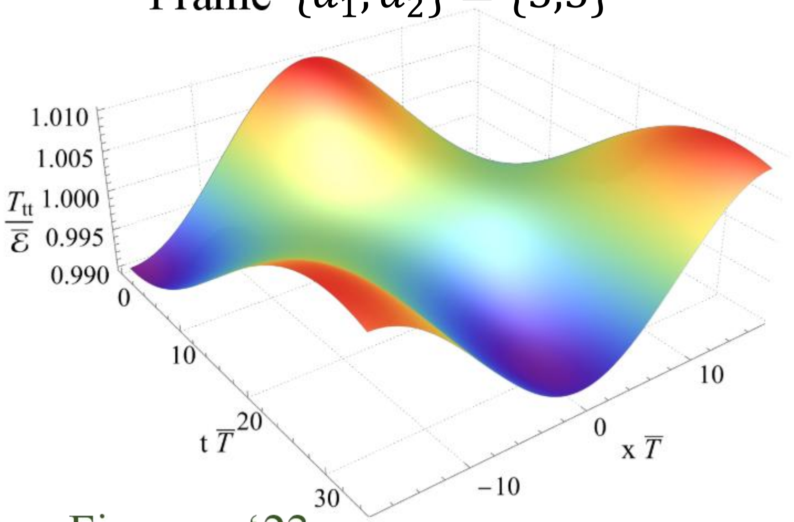
ld theory regime

$\mathcal{A}$, $\mathcal{Q}$

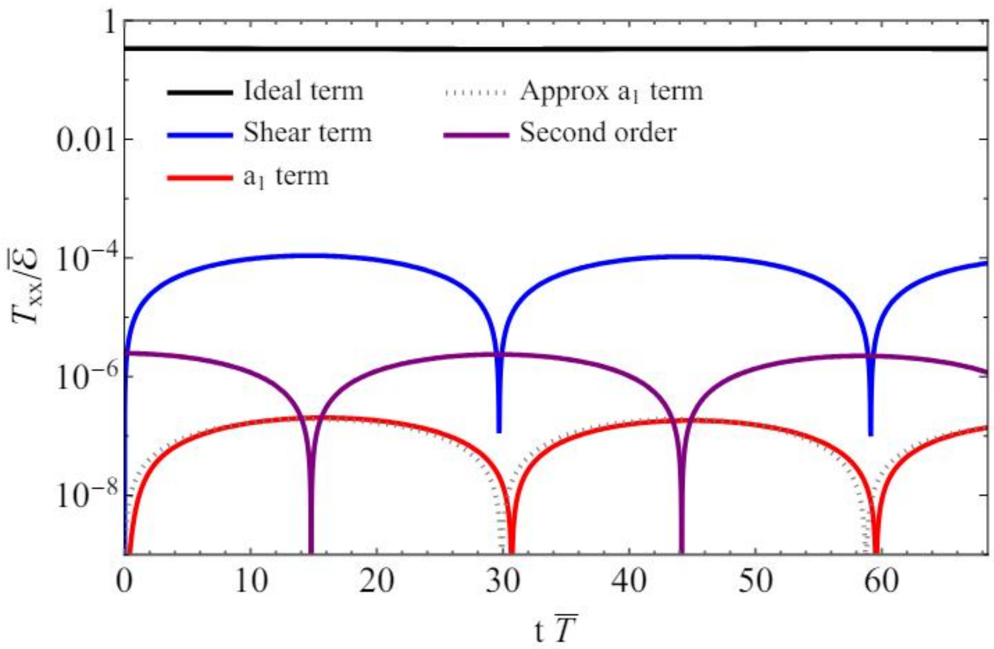
The physics to first order is independent of the arbitrarily chosen frame

Evolutions in different frames (criterion C)

Frame $\{a_1, a_2\} = \{5,5\}$



Bea, Figueras '23



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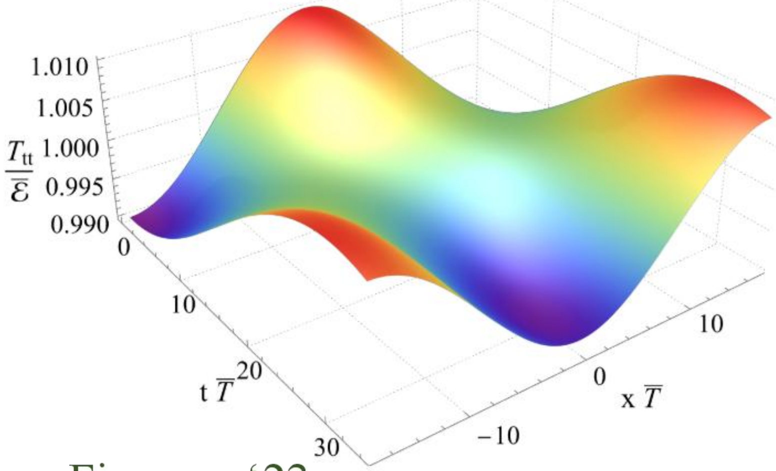
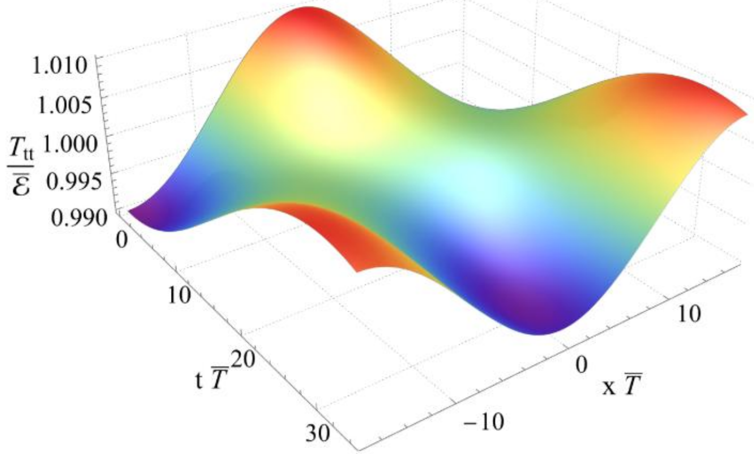


Frame $\{a_1, a_2\} = \{10,10\}$



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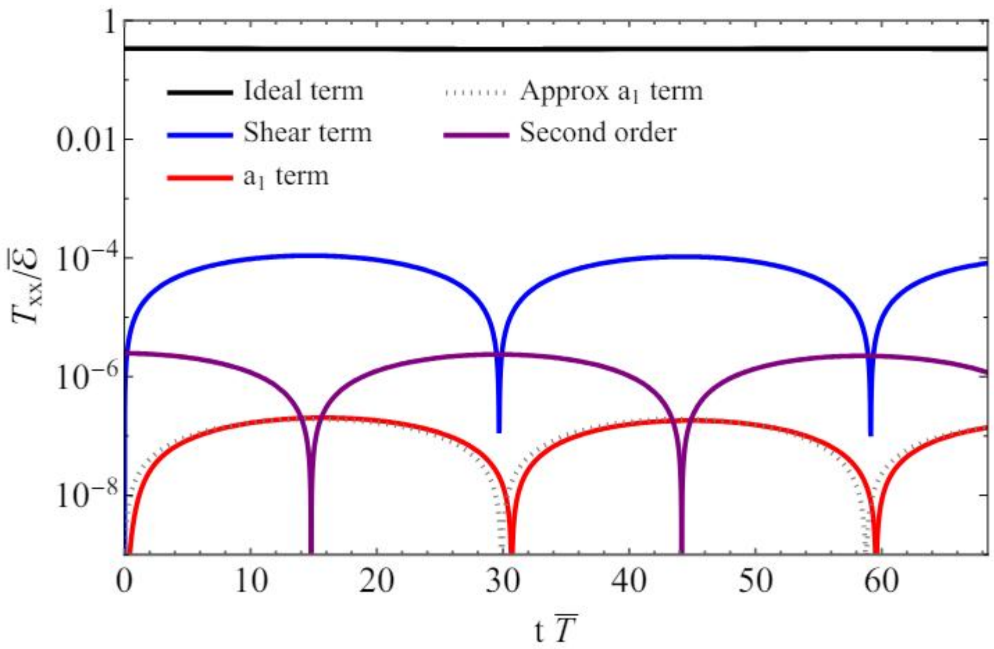

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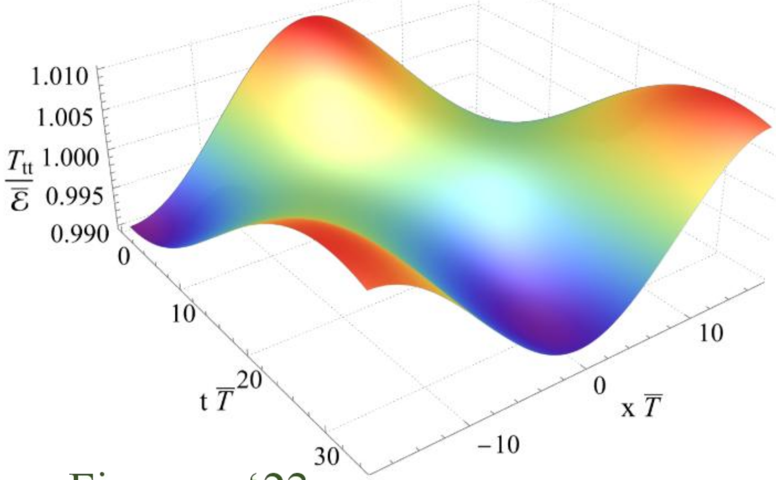

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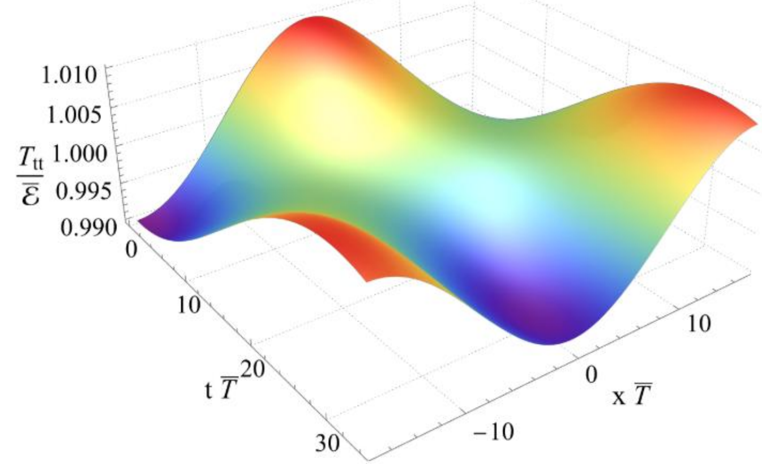

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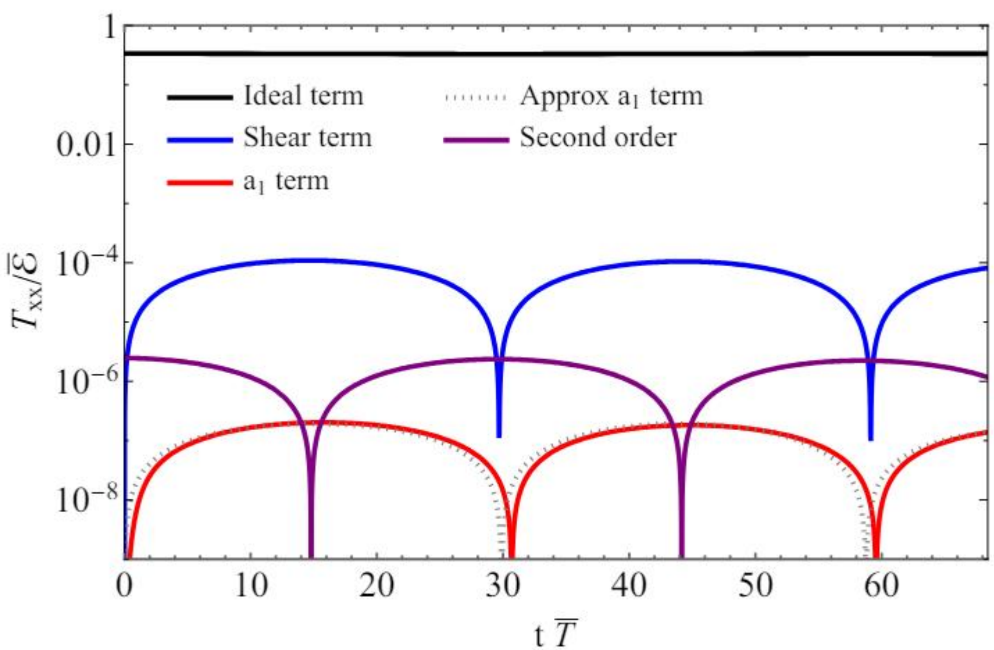

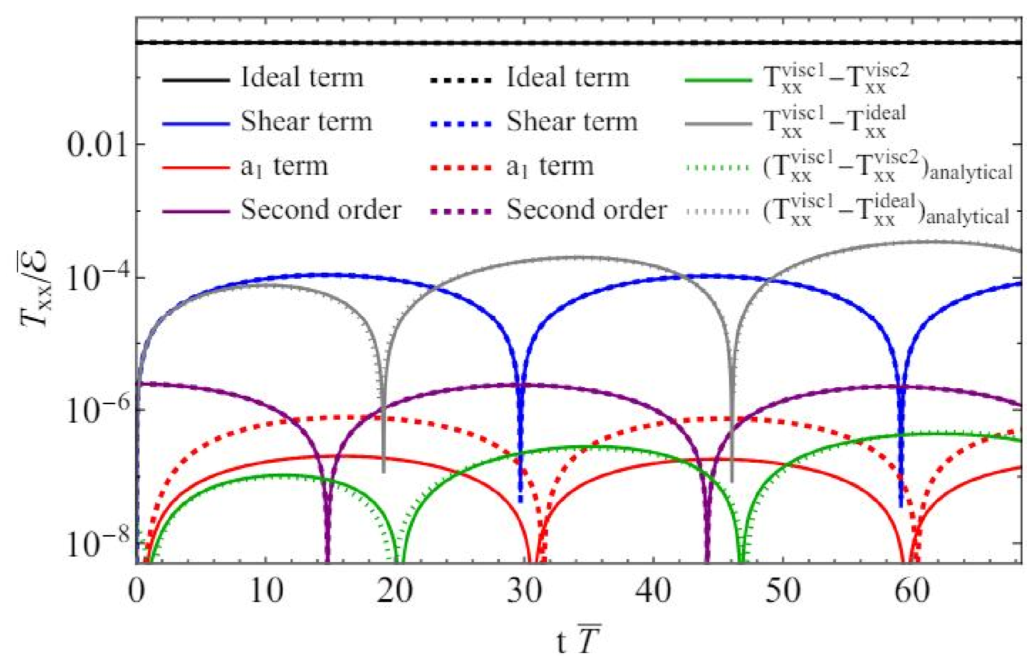

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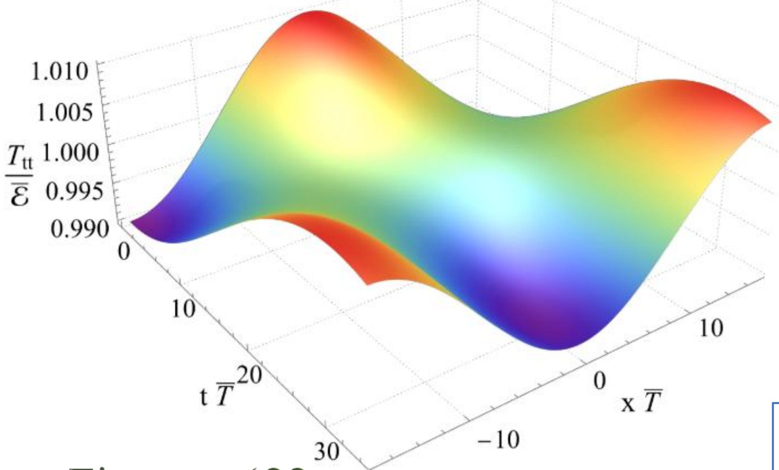

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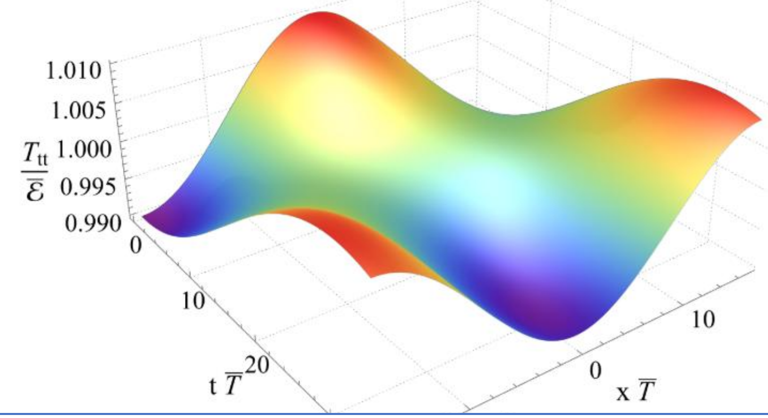

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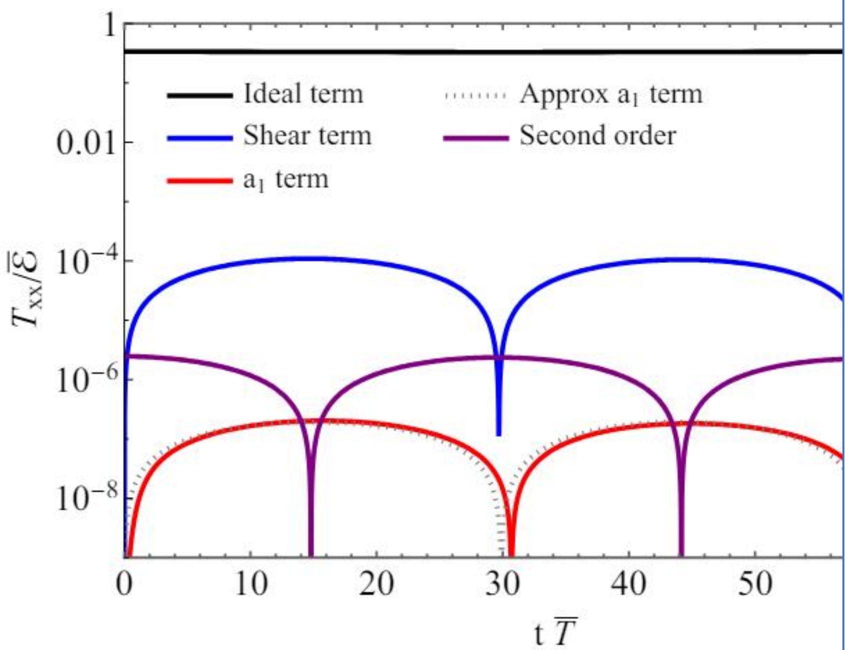
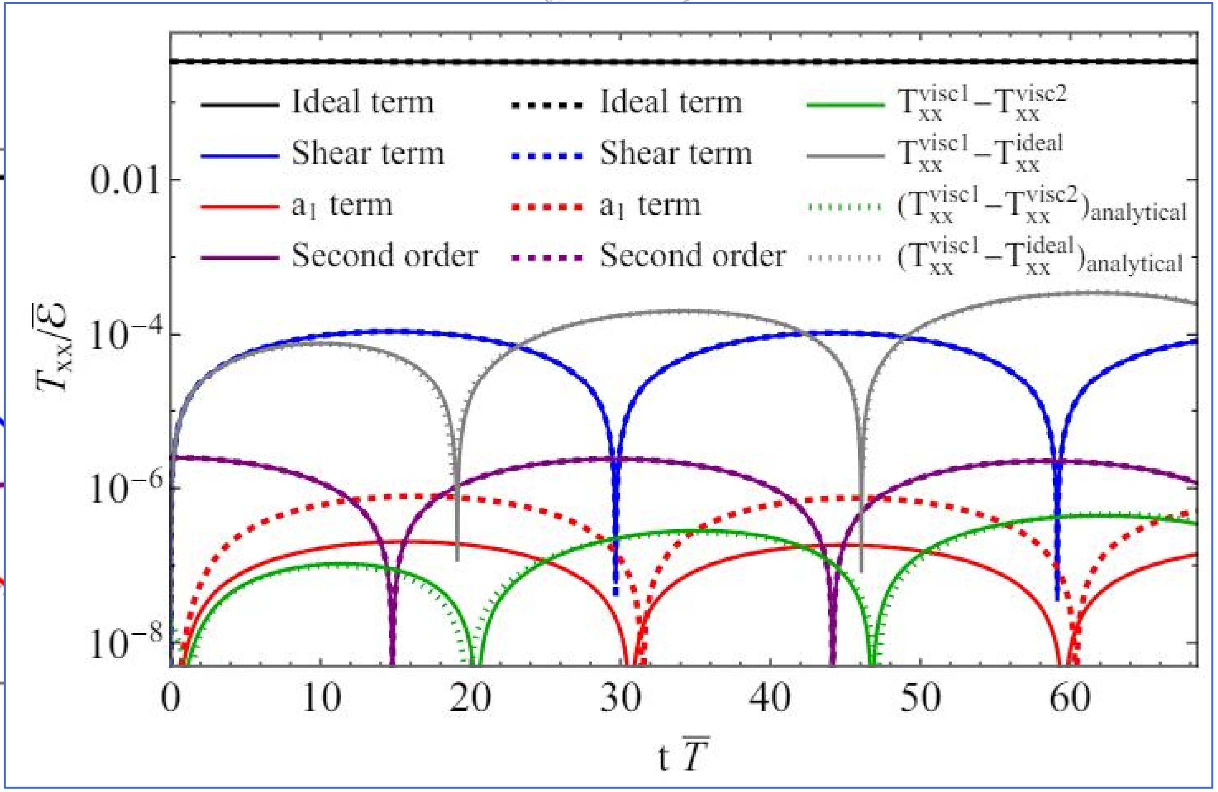
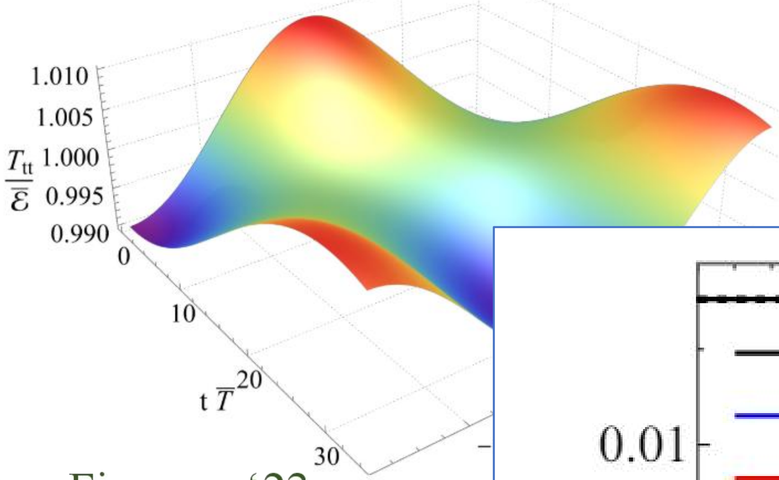


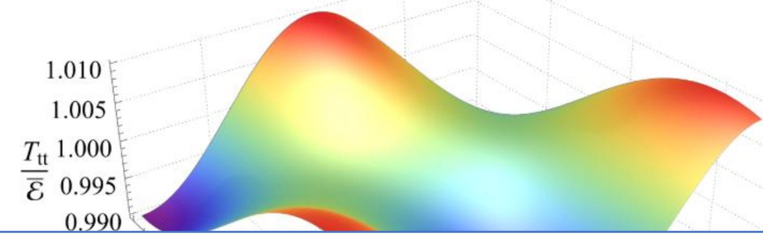


Evolutions in different frames (criterion C)

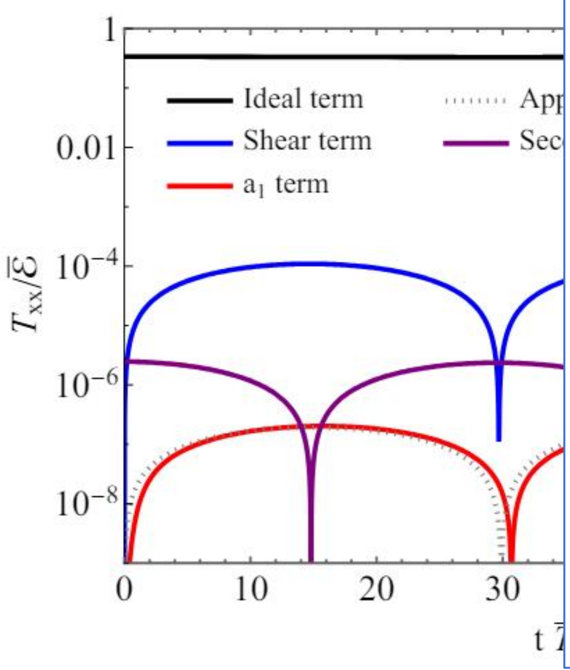
Frame $\{a_1, a_2\} = \{5,5\}$

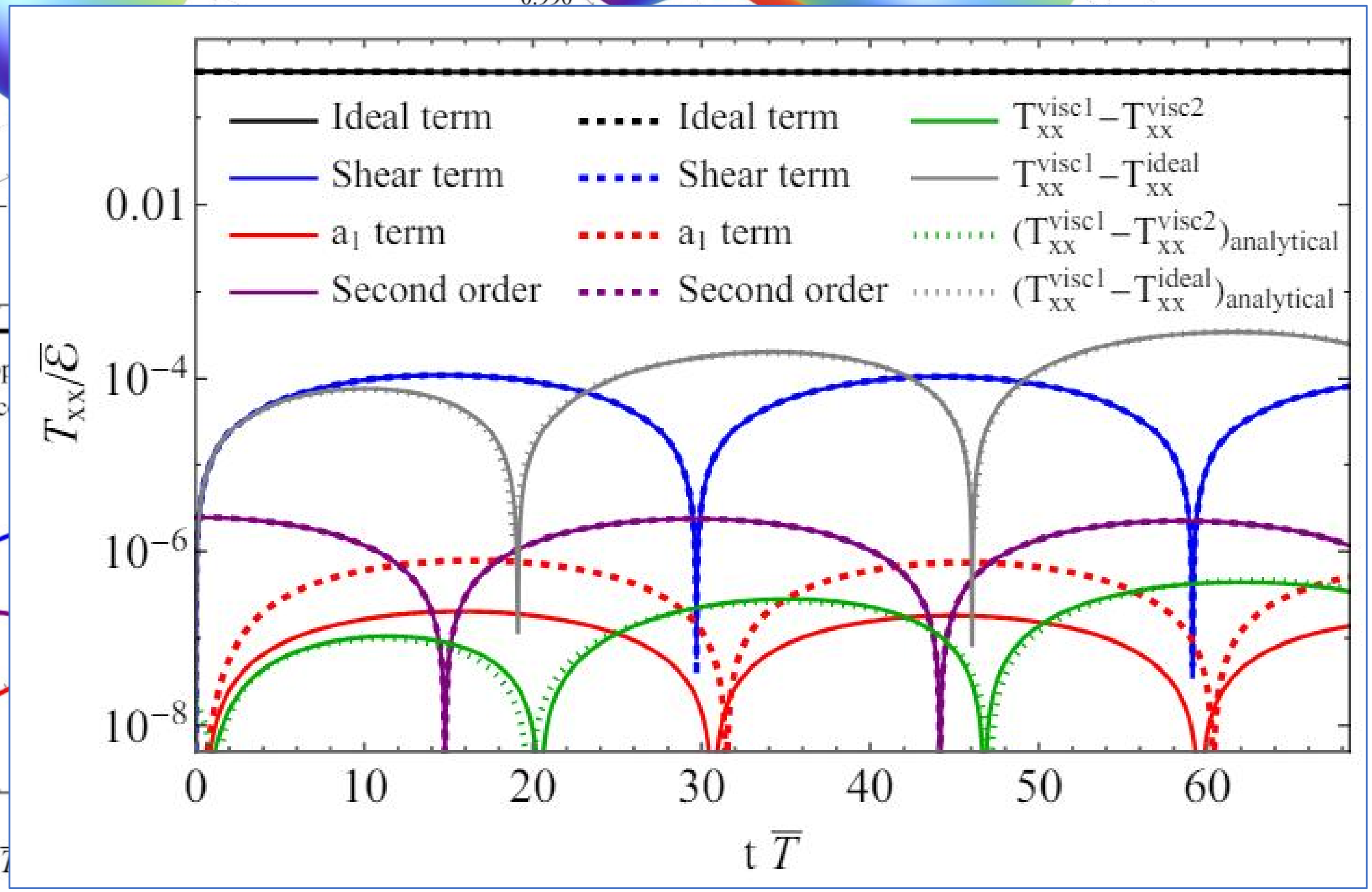


Frame $\{a_1, a_2\} = \{10,10\}$



Bea, Figueras '23





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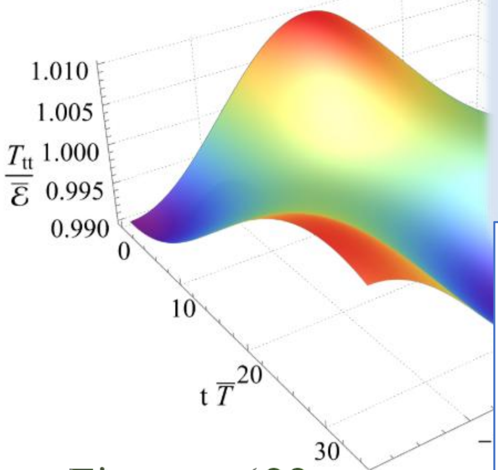
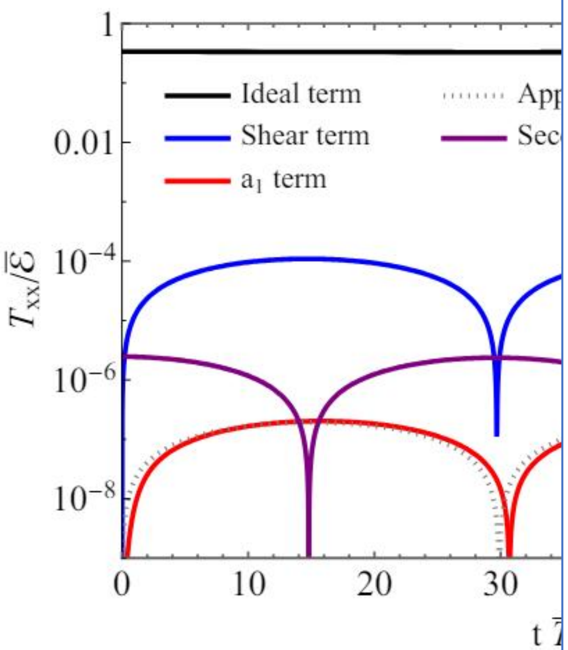


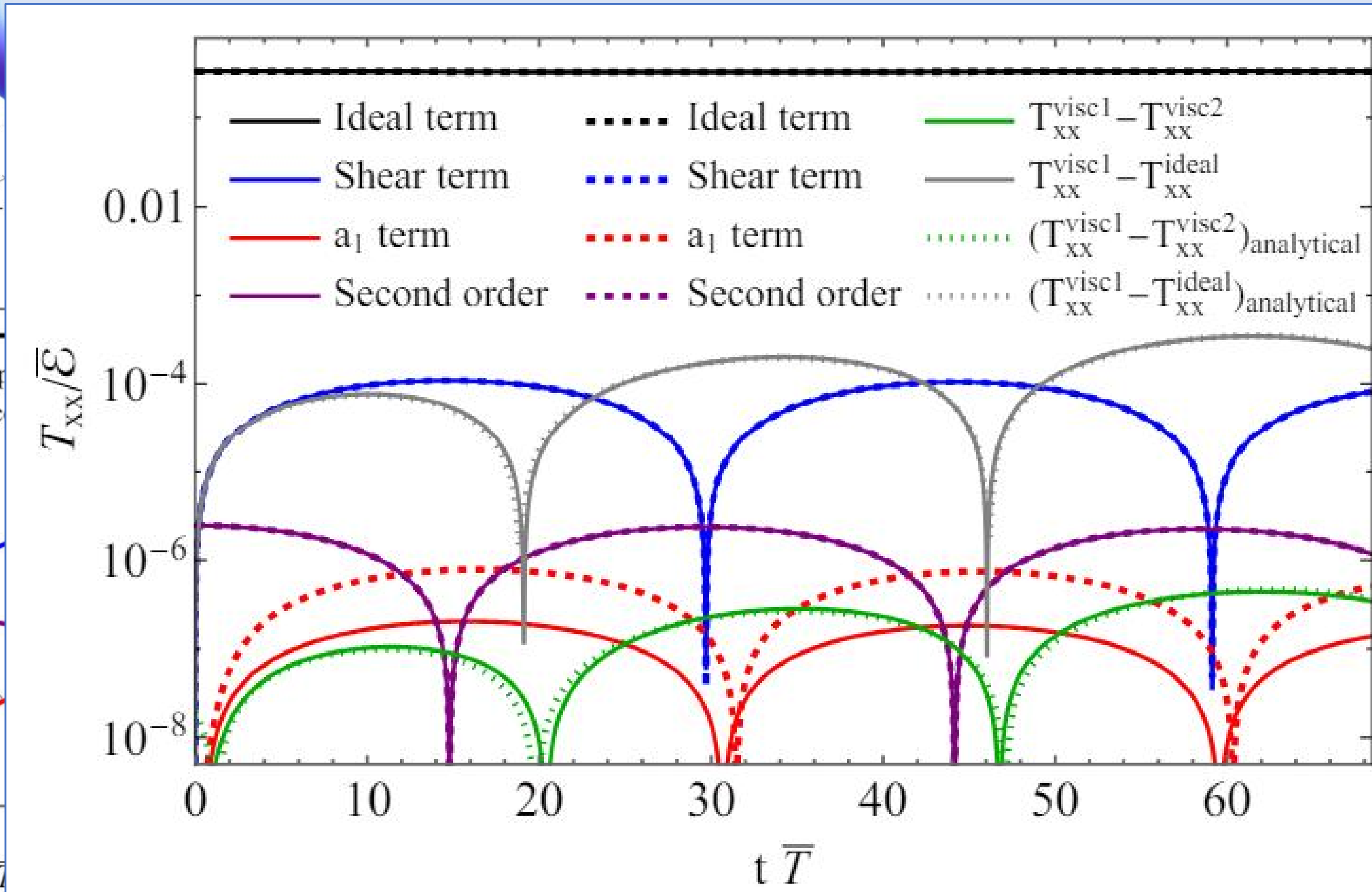
Bea, Figueras '23



Criterion C Perform a third evolution using ideal hydro

$$T_{\mu\nu}^{ideal} - T_{\mu\nu}^{visc} >> T_{\mu\nu}^{visc1} - T_{\mu\nu}^{visc2}$$

The physics to first order is independent of the arbitrarily chosen frame



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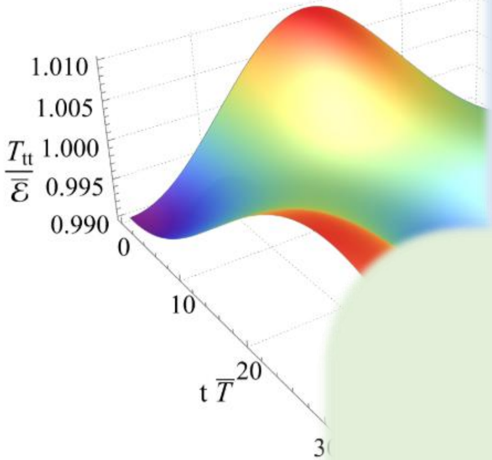

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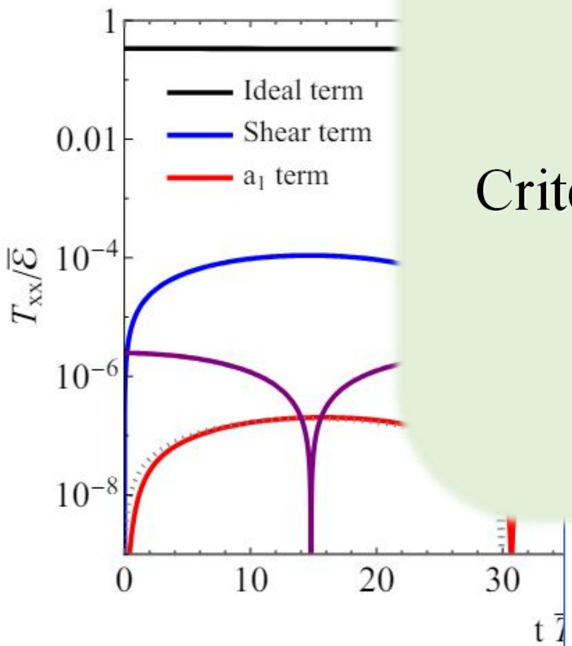

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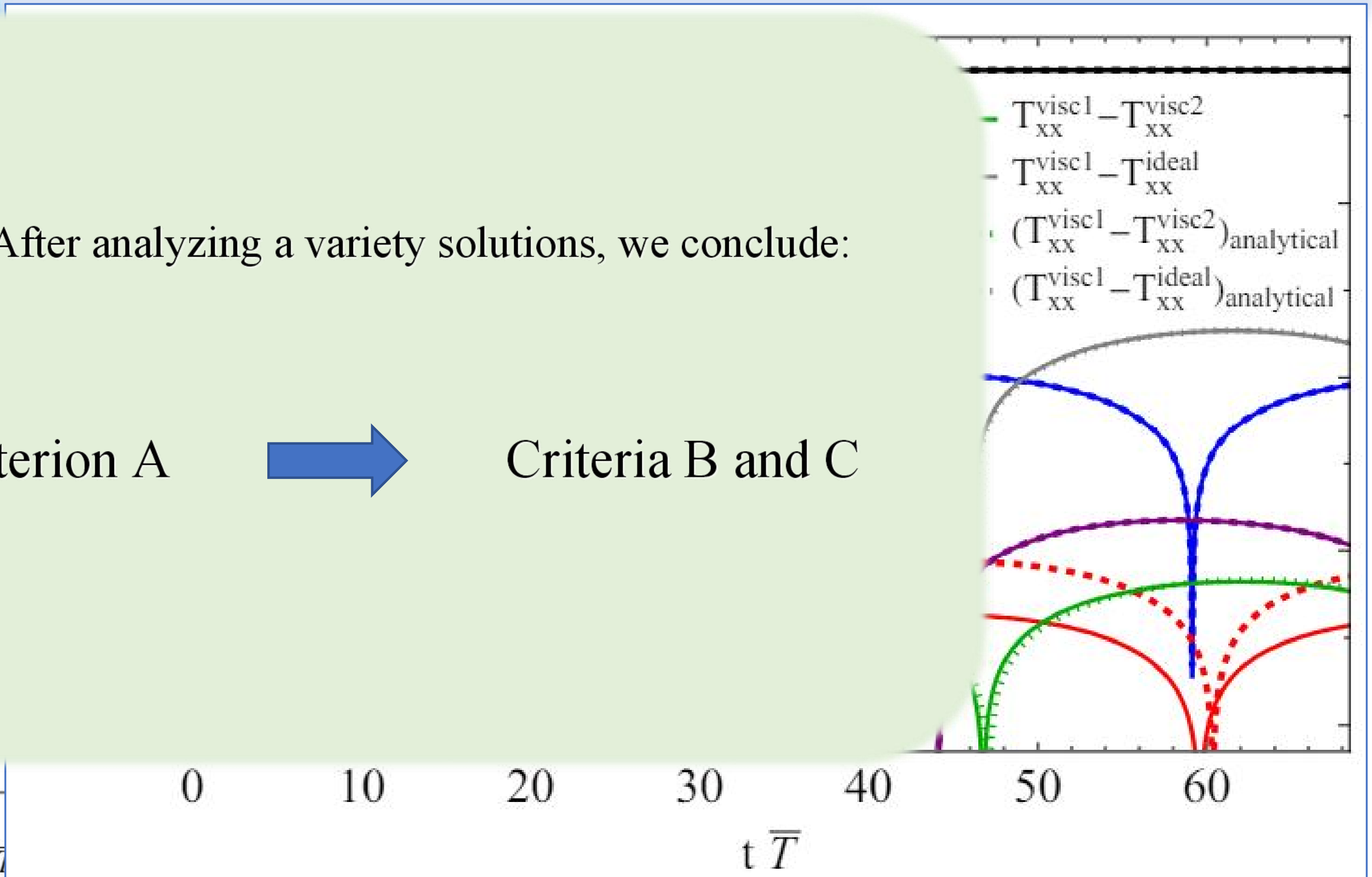


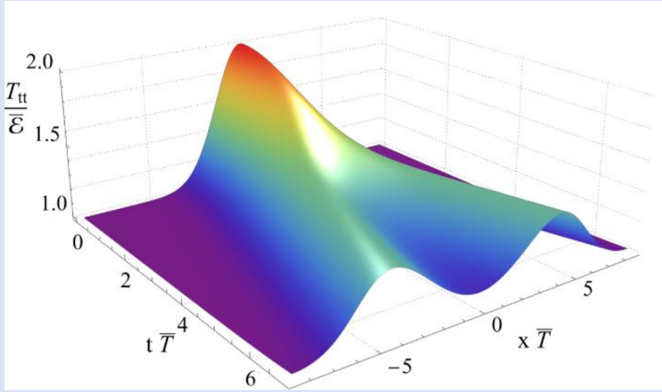
After analyzing a variety solutions, we conclude:

Criterion A → Criteria B and C

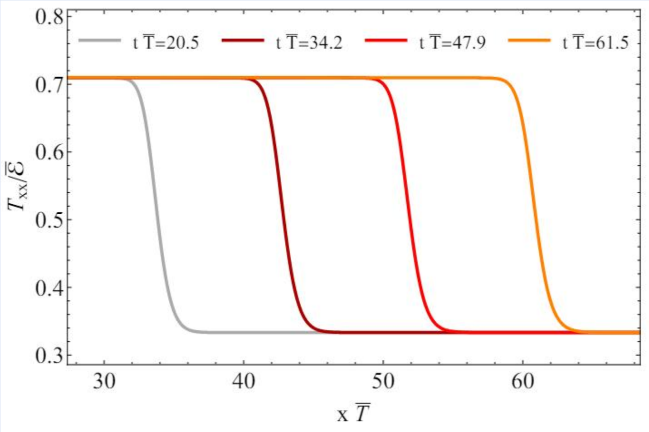
Evolving relativistic Navier-Stokes

We study solutions well in the non-linear regime:

Bea, Figueras '23



Large amplitude gaussian perturbation



Shockwaves

Evolving relativistic Navier-Stokes

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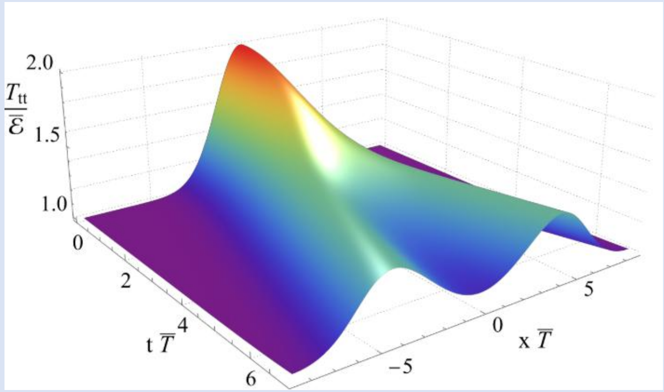
Bea, Figueras '23



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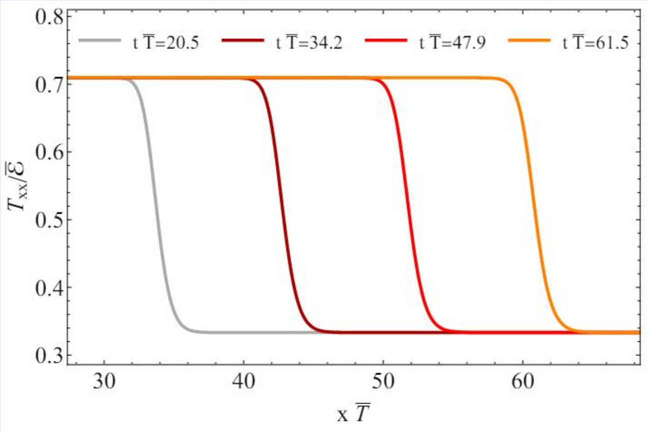
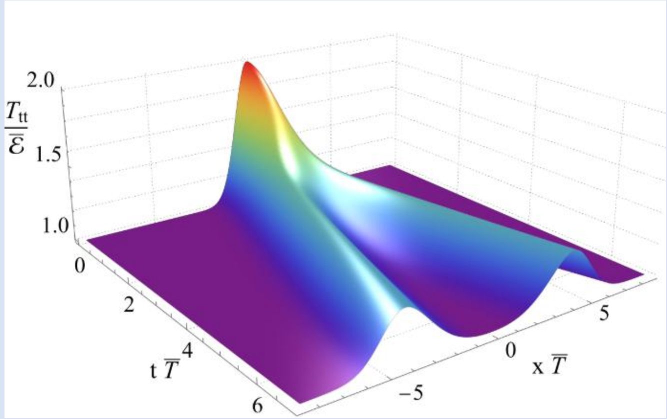
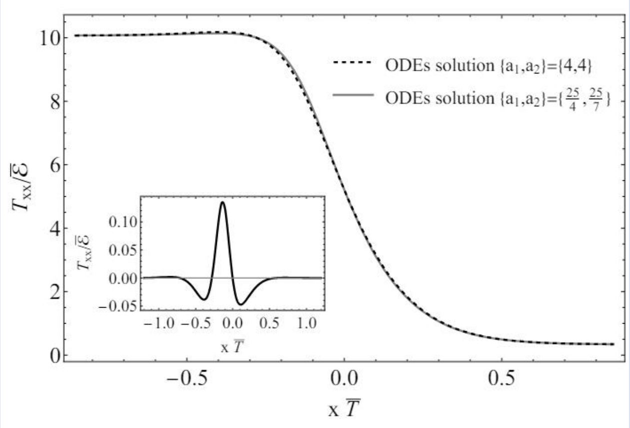


Shockwaves

Motivated by the physics of the quark-gluon plasma:

→ We also study solution marginally in the hydrodynamic regime





It is robust! ➡ Criterion A only marginally satisfied but still Criterion C satisfied

Navier-Stokes: Initial data

→ If we are given initial data: we change frame to our working causal frame

Navier-Stokes: Initial data

⟶ If we are given initial data: we change frame to our working causal frame

Use the prescription of effective field theory:

$$\epsilon \to \epsilon + \mathcal{A}\,, \qquad u^\mu \to u^\mu + \frac{\mathcal{Q}^\mu}{\epsilon + p}$$

$$\mathcal{A} := a_1\,\eta \left(\frac{3}{4}\frac{\dot{\epsilon}}{\epsilon} + \nabla \cdot u\right)$$

$$\mathcal{Q}^\mu := a_2 \eta \left(\dot{u}^\mu + \frac{1}{4}\frac{\nabla_\perp^\mu \epsilon}{\epsilon}\right)$$

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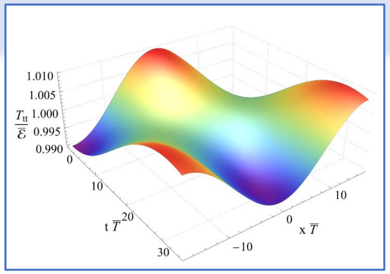

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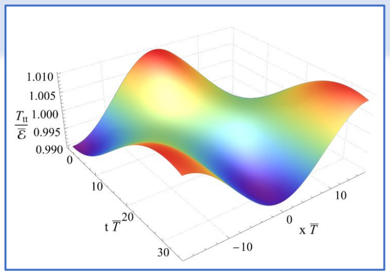
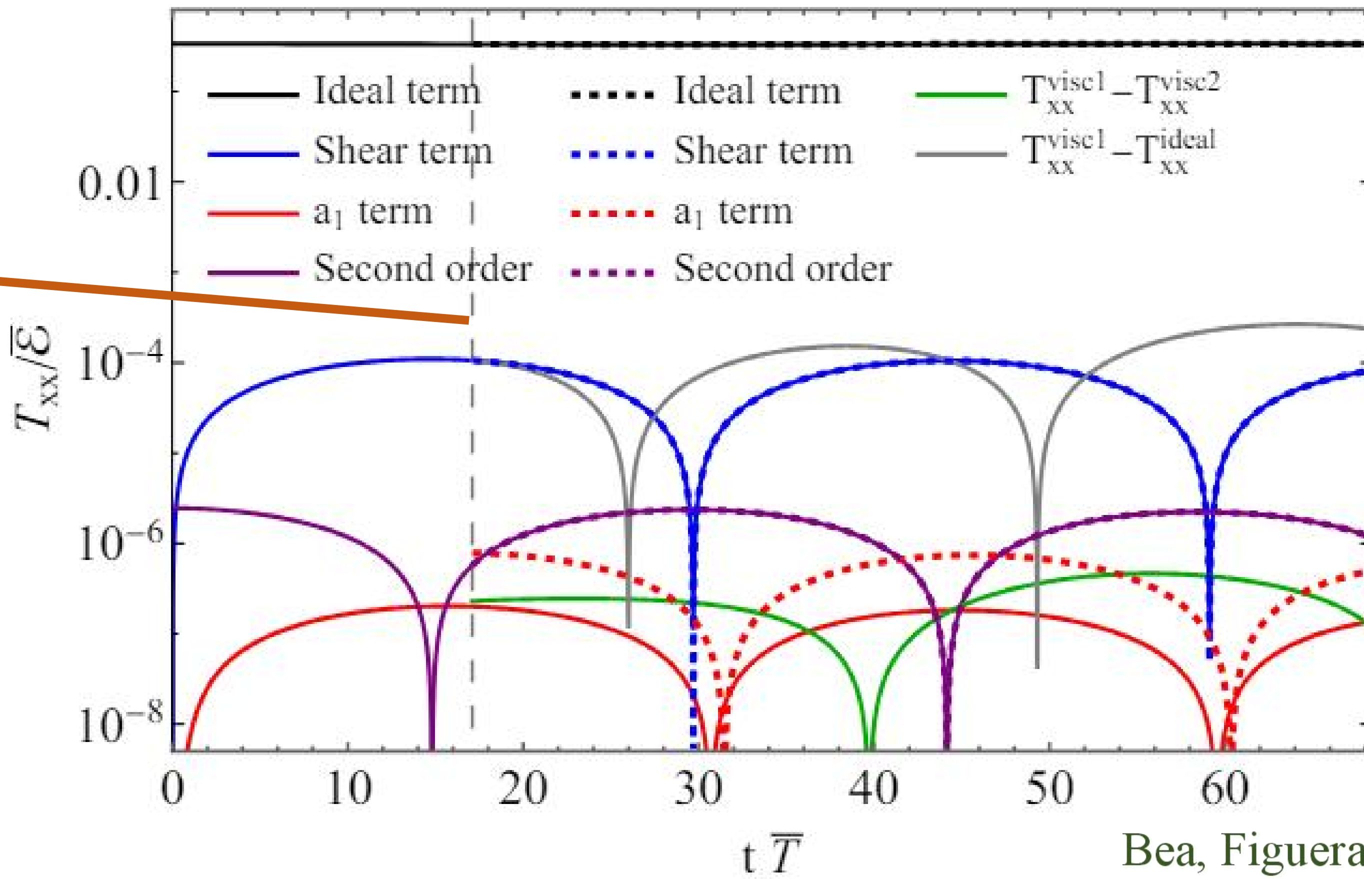


Data {e,ux} at time t

Frame {a1,a2}={5,5}



Bea, Figueras '23

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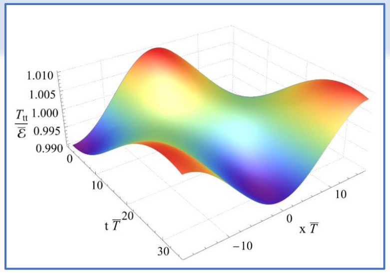


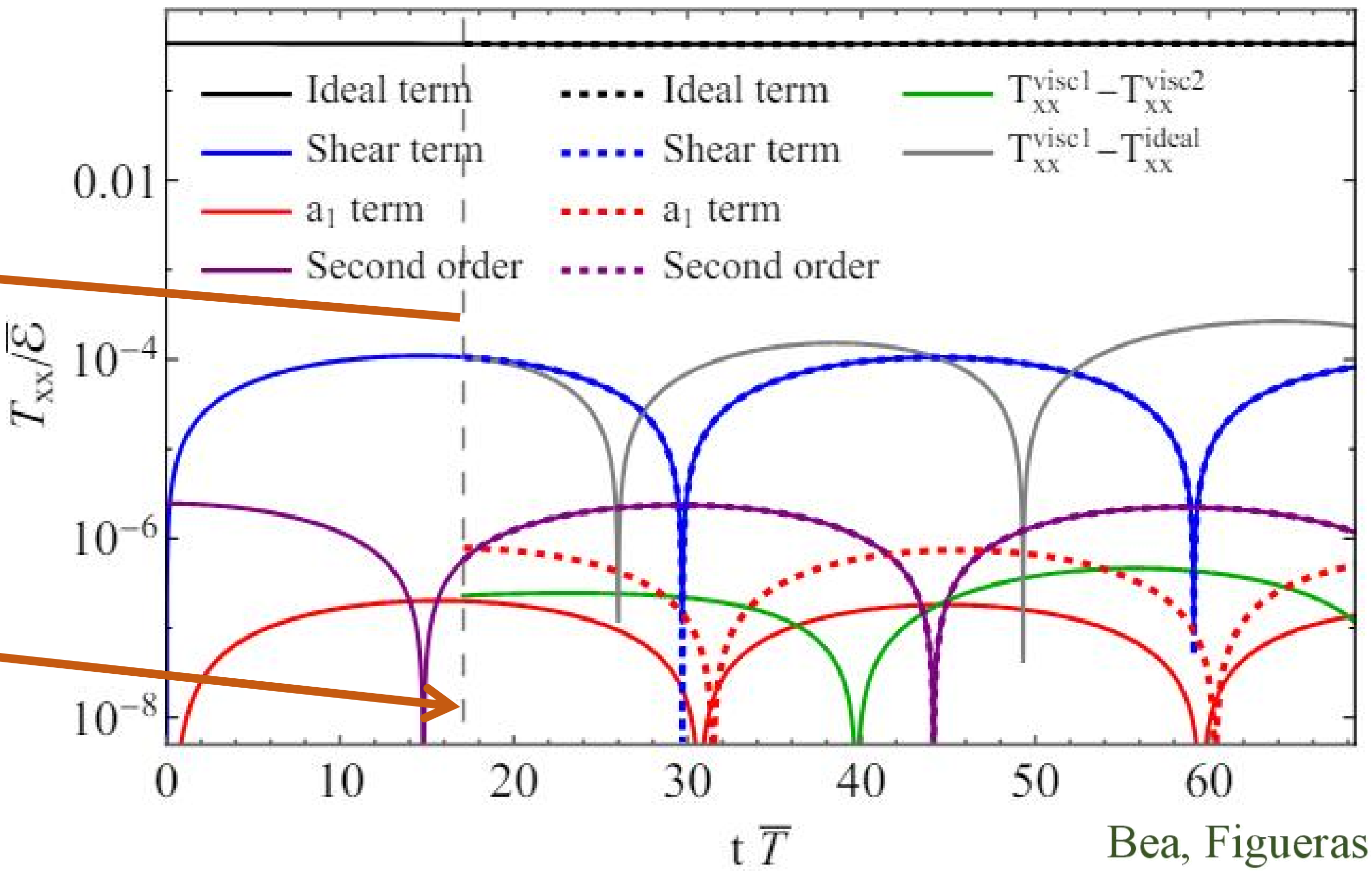
Data {e,ux} at time t

Frame {a1,a2}={5,5}

↓

Frame {a1,a2}={10,10}

Use it as initial data for another evolution



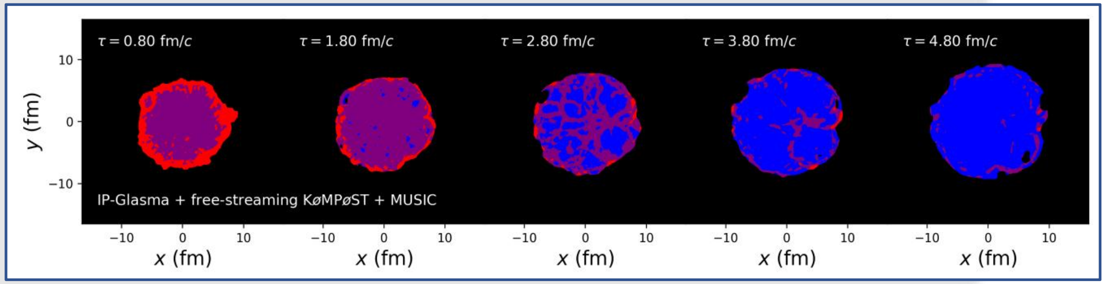
Bea, Figueras '23

→ As the change is of second order, we could use same initial data as in Landau frame

Towards Navier-Stokes description of the QGP

MIS description of the quark-gluon plasma

But these evolutions find limitations
We have seen explicit examples



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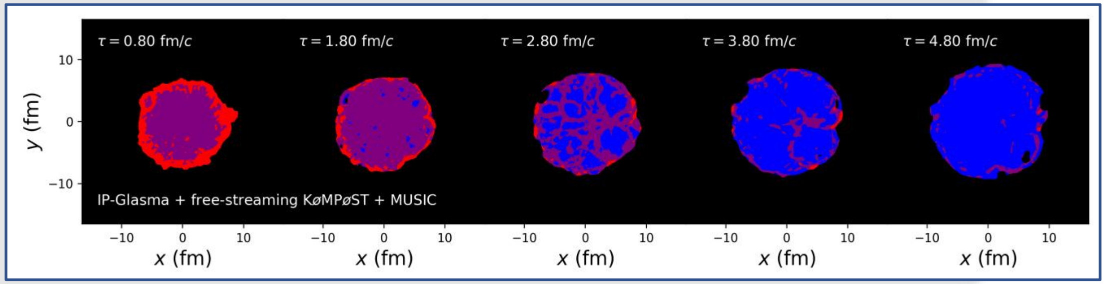
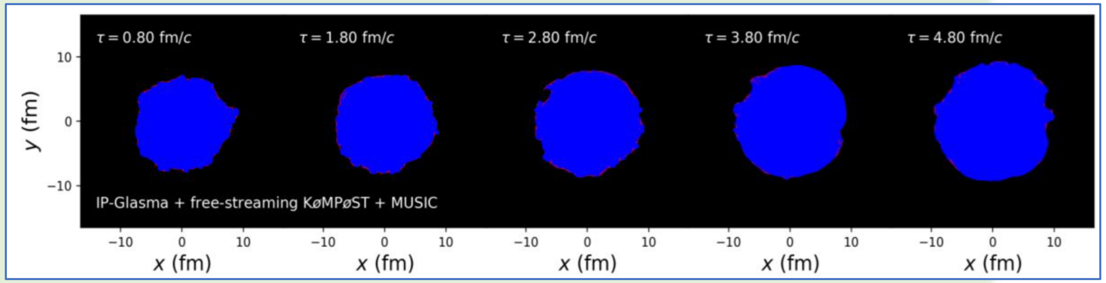


Navier-Stokes description of the quark-gluon plasma

Well behaved once a causal frame is chosen



Now that we have good control on the frame dependence and initial data, we can proceed with implementation

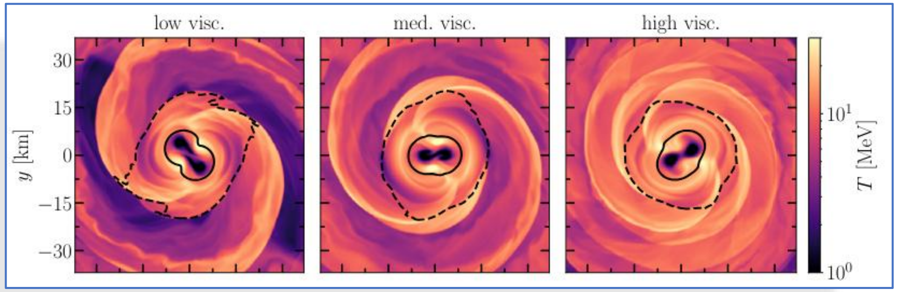
➡ Description of the experimental data for radial flows in central collisions

[In progress...]

Towards viscous neutron star mergers

MIS Neutron star mergers

Evolutions with MIS find limitations
We have seen explicit examples



Chabanov, Rezzolla '23 (a)
Chabanov, Rezzolla '23 (b)

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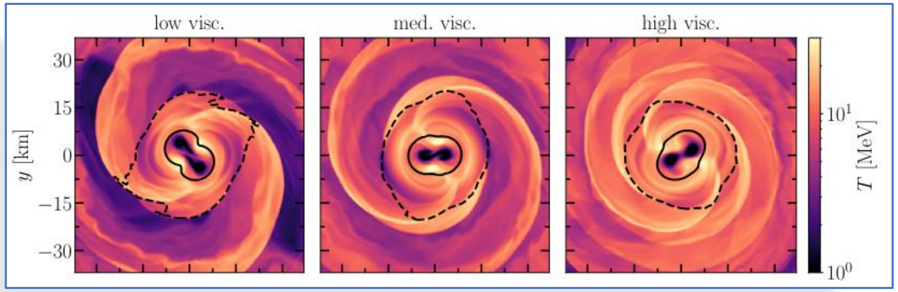

Chabanov, Rezzolla '23 (a)
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Navier-Stokes neutron star mergers

Now that we have control on the numerical evolutions: first steps of this implementation

Bea, Bezares, Figueras, Palenzuela, Shum [in progress]

→ 3+1 decomposition of the equations

→ First tests in spherical symmetry

Towards viscous neutron star mergers

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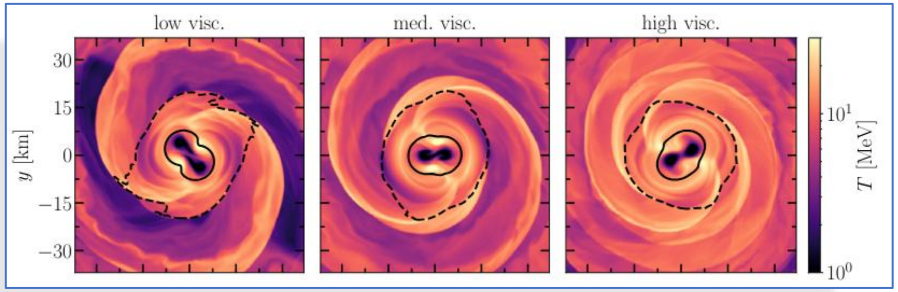

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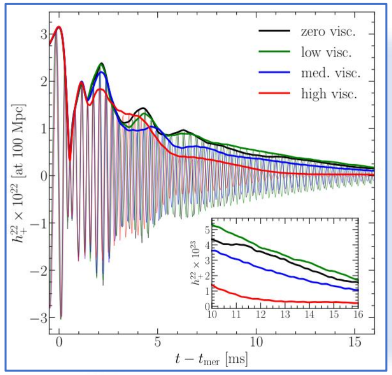
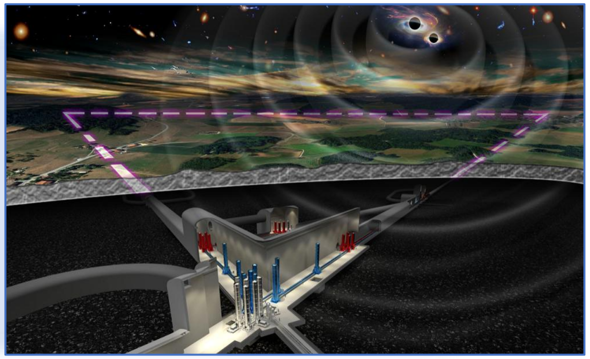


Relevant to obtain accurate templates for future experiments like Einstein telescope



Summary & main message

- MIS theories present limitations at a fundamental level (well-posedness)

  ⟶ Shown in explicit examples

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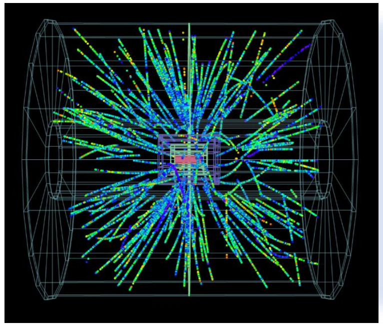
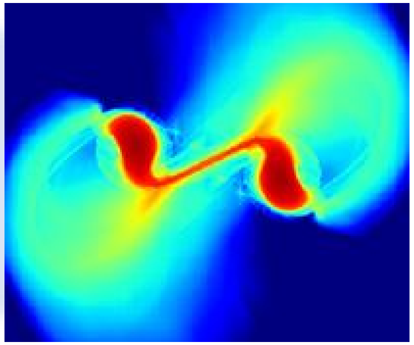


→ Ready for implementation in the QGP and NS mergers as a promising alternative to MIS



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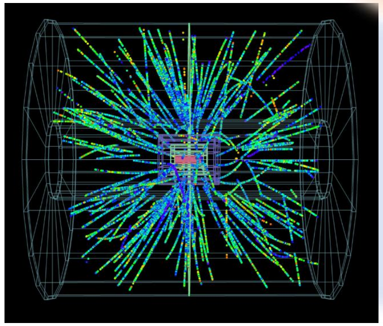
Thank you!!



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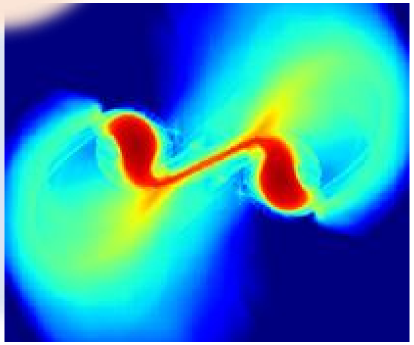


Backup slides

Backup slide 1: MIS equations

- Conformal theory

- Ideal hydrodynamics

$$T^{\mu\nu} = \epsilon\, u^\mu\, u^\nu + p\, \Delta^{\mu\nu} \quad \Rightarrow \quad \nabla_\mu T^{\mu\nu} = 0$$

Well posed!!

- First order hydro: **Landau frame**

$$T^{\mu\nu} = \epsilon\, u^\mu\, u^\nu + p\, \Delta^{\mu\nu} - \eta\sigma^{\mu\nu} \quad \Rightarrow \quad \nabla_\mu T^{\mu\nu} = 0$$

Ill-posed…

- Usual fix: **MIS-type**

$$T^{\mu\nu} = \epsilon\, u^\mu\, u^\nu + p\, \Delta^{\mu\nu} + \Pi^{\mu\nu}$$

$$\Pi^{\mu\nu} = -\eta\sigma^{\mu\nu} - \eta\tau_\pi \left(\dot{\sigma}^{<\mu\nu>} + \frac{3}{2}\sigma^{\mu\nu}\nabla\cdot u\right)$$

Problems alleviated!

⇓ New variable

$$T^{\mu\nu} = \epsilon\, u^\mu\, u^\nu + p\, \Delta^{\mu\nu} + \Pi^{\mu\nu}$$

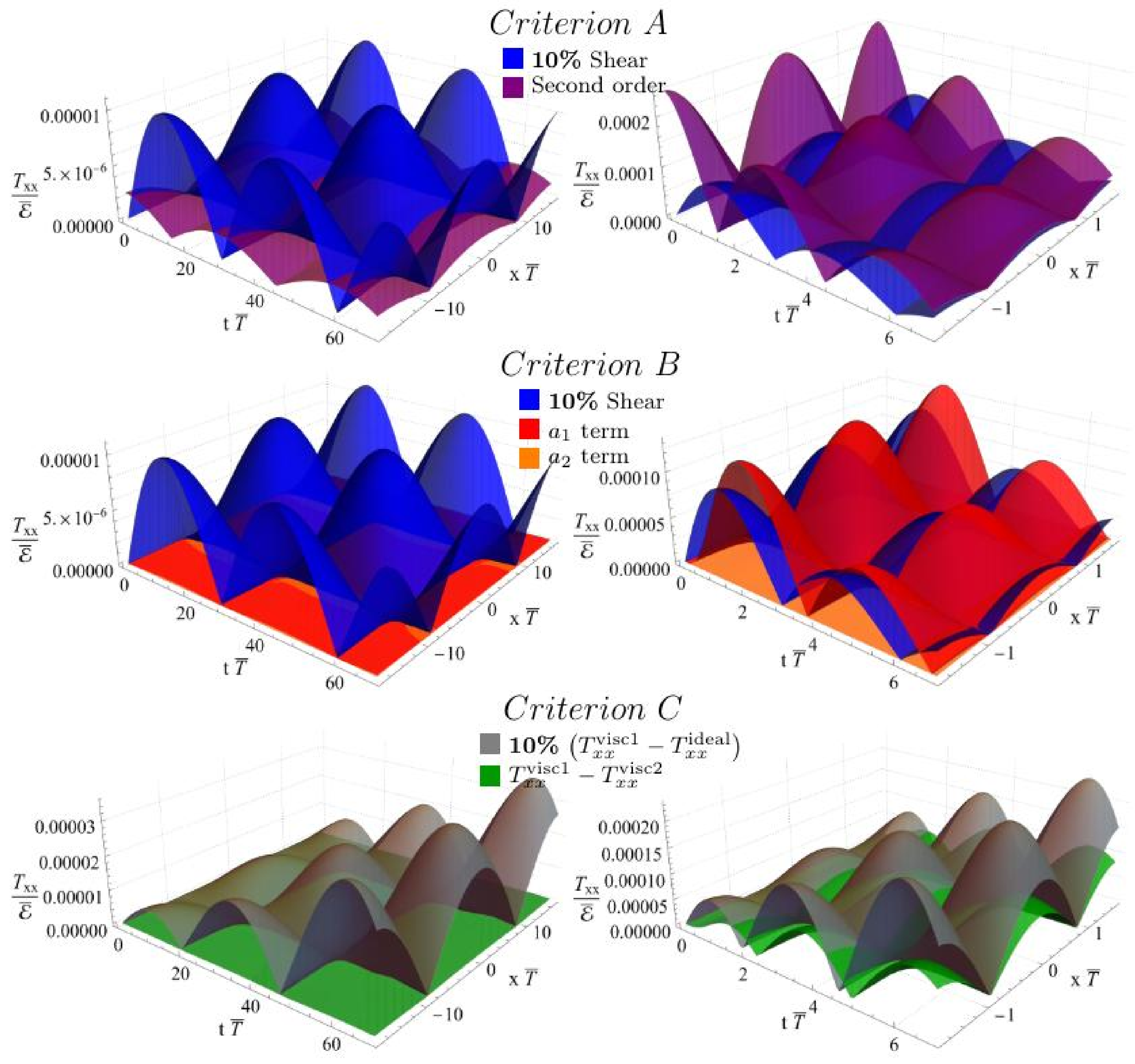
$$\Pi^{\mu\nu} = -\eta\sigma^{\mu\nu} - \tau_\pi \left(\dot{\Pi}^{<\mu\nu>} + \frac{3}{2}\Pi^{\mu\nu}\nabla u\right)$$

New equation

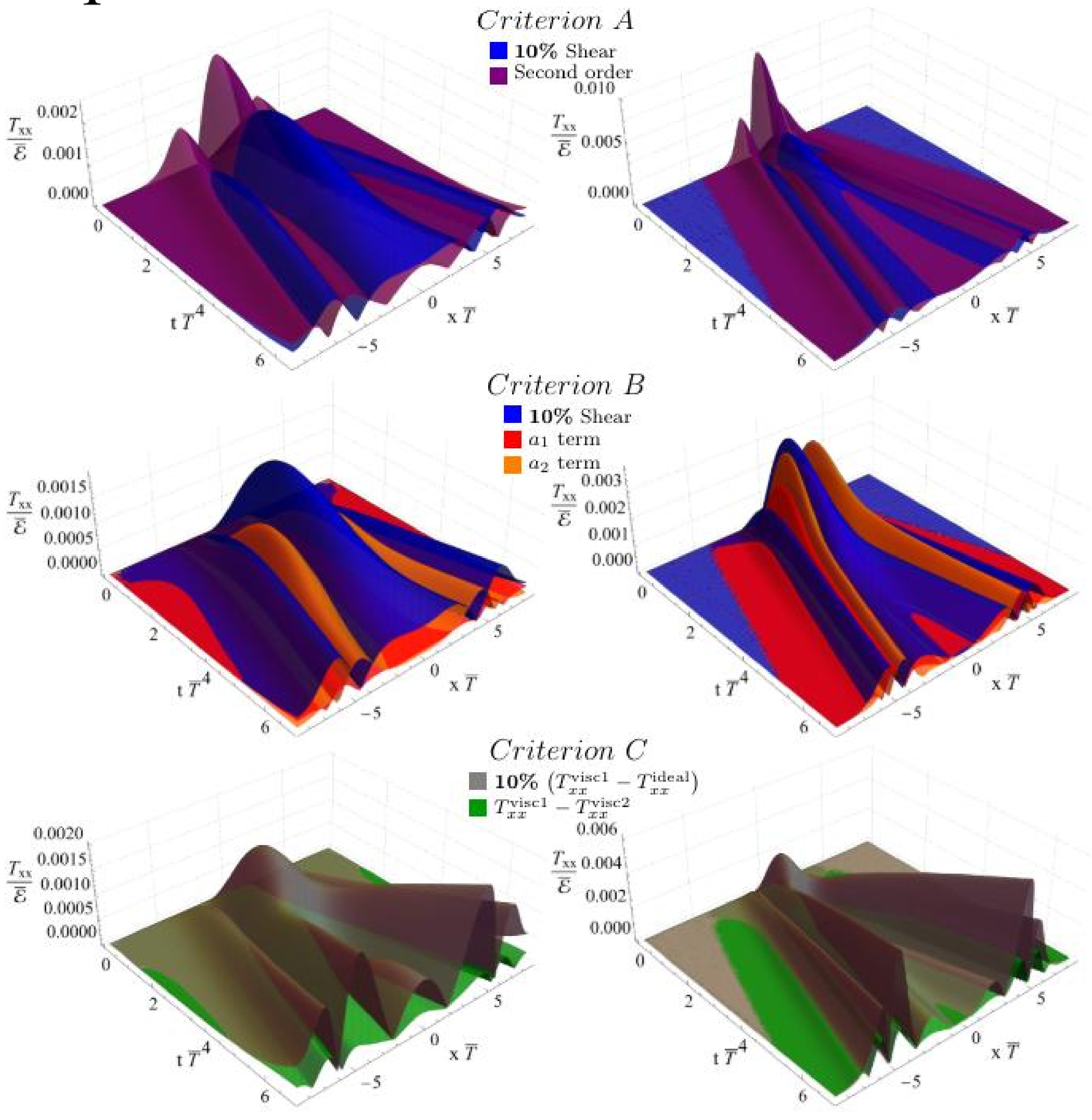
$$\Rightarrow \quad \nabla_\mu T^{\mu\nu} = 0$$

$$\Pi^{\mu\nu} = -\eta\sigma^{\mu\nu} - \tau_\pi \left(\dot{\Pi}^{<\mu\nu>} + \frac{3}{2}\Pi^{\mu\nu}\nabla u\right)$$

Backup slide 2



Backup slide 3

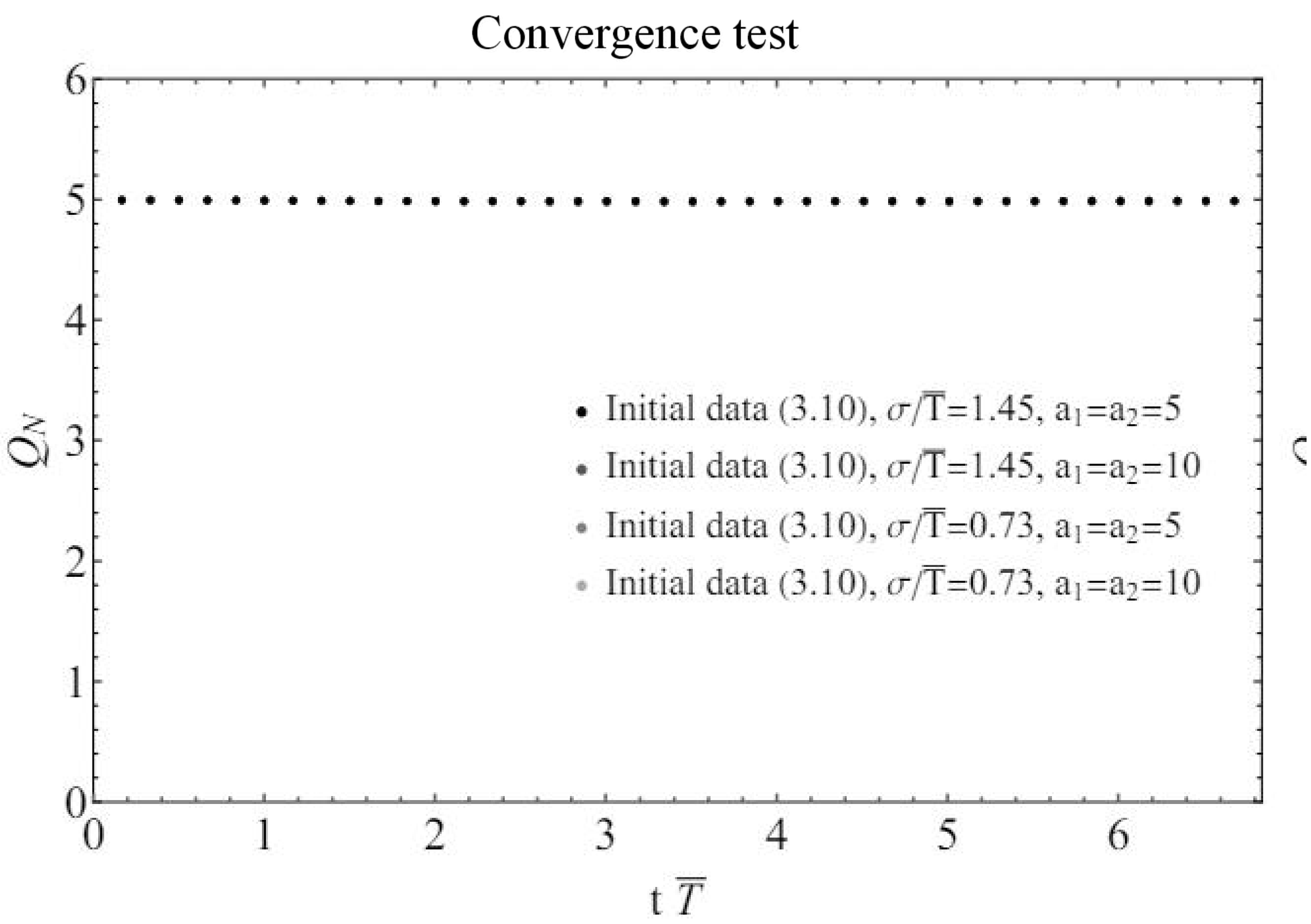


Backup slide 4

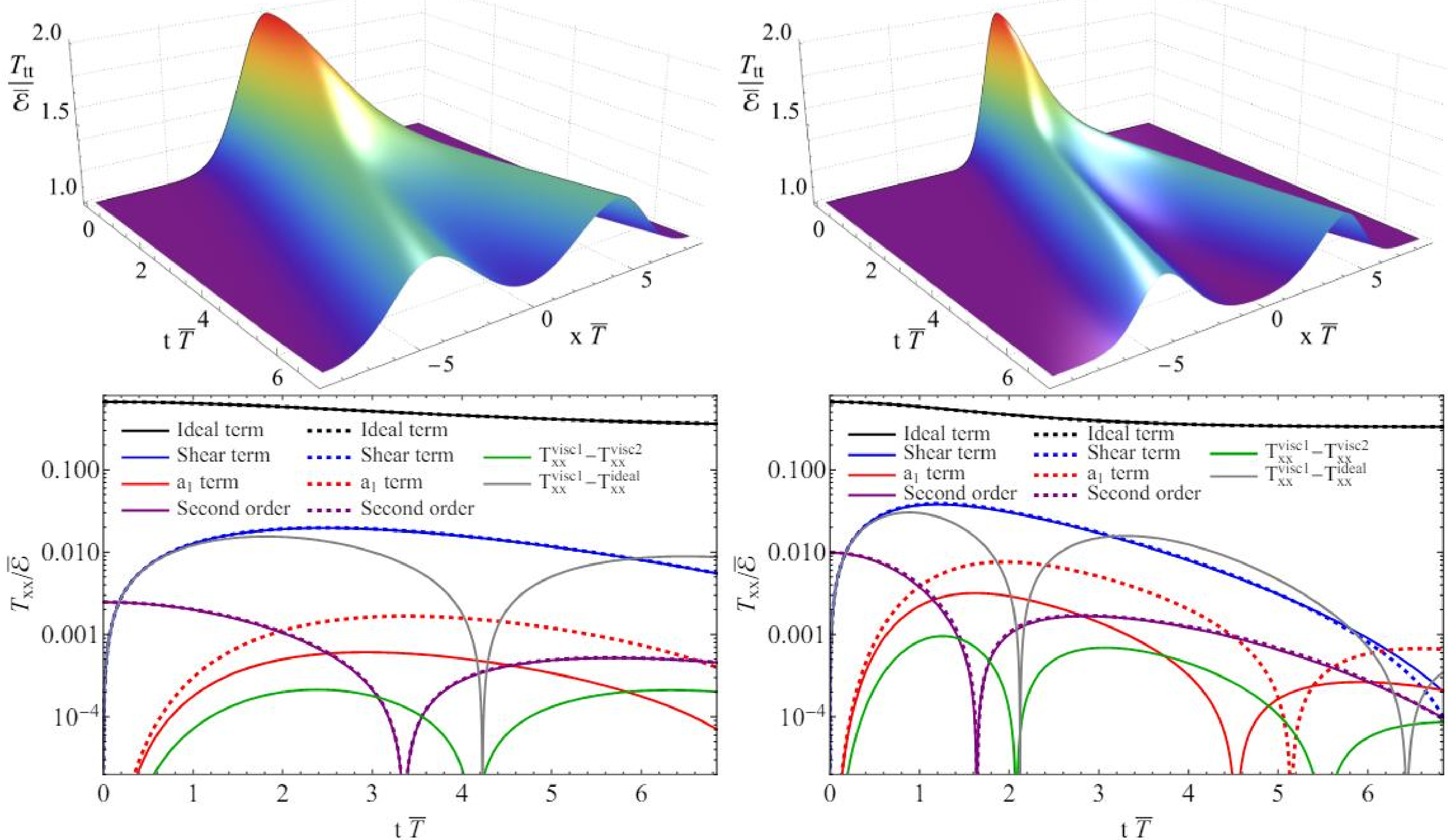
$$T^{\mu\nu} = \epsilon\, u^\mu\, u^\nu + p\,\Delta^{\mu\nu} - \eta\,\sigma^{\mu\nu}$$
$$+ \eta\,\tau_\pi \left(\dot{\sigma}^{\langle\mu\nu\rangle} + \frac{1}{3}\,\sigma^{\mu\nu}\,\nabla\cdot u \right) + \lambda_1\,\sigma^{\langle\mu}{}_{\rho}\sigma^{\nu\rangle\rho} + \lambda_2\,\sigma^{\langle\mu}{}_{\rho}\Omega^{\nu\rangle\rho} + \lambda_3\,\Omega^{\langle\mu}{}_{\rho}\Omega^{\nu\rangle\rho}$$

$$\eta\,\tau_\pi = \frac{s}{8\pi^2 T}\,(2 - \ln 2)\ ,$$
$$\lambda_1 = \frac{s}{8\pi^2 T}\ ,$$
$$\lambda_2 = \frac{s}{8\pi^2 T}\,(-2\ln 2)\ ,$$
$$\lambda_3 = 0\ .$$

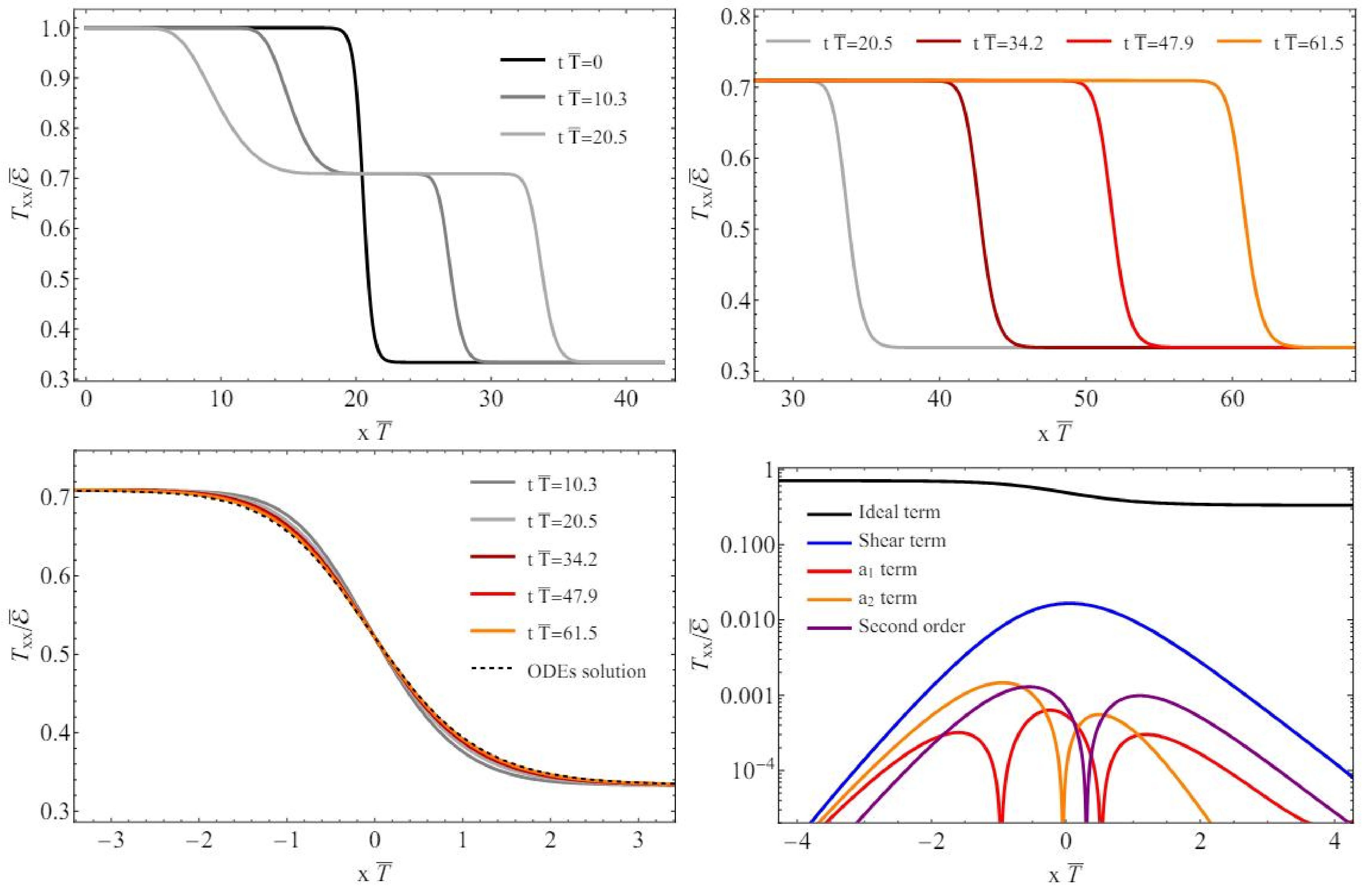
Backup slide 5



Backup slide 6



Backup slide 7



Holography

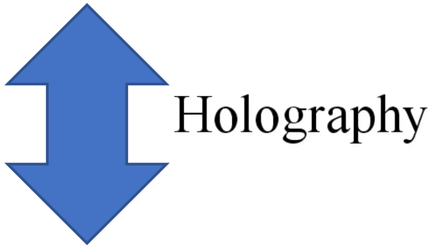
→ Excellent framework to study the applicability of hydrodynamics.

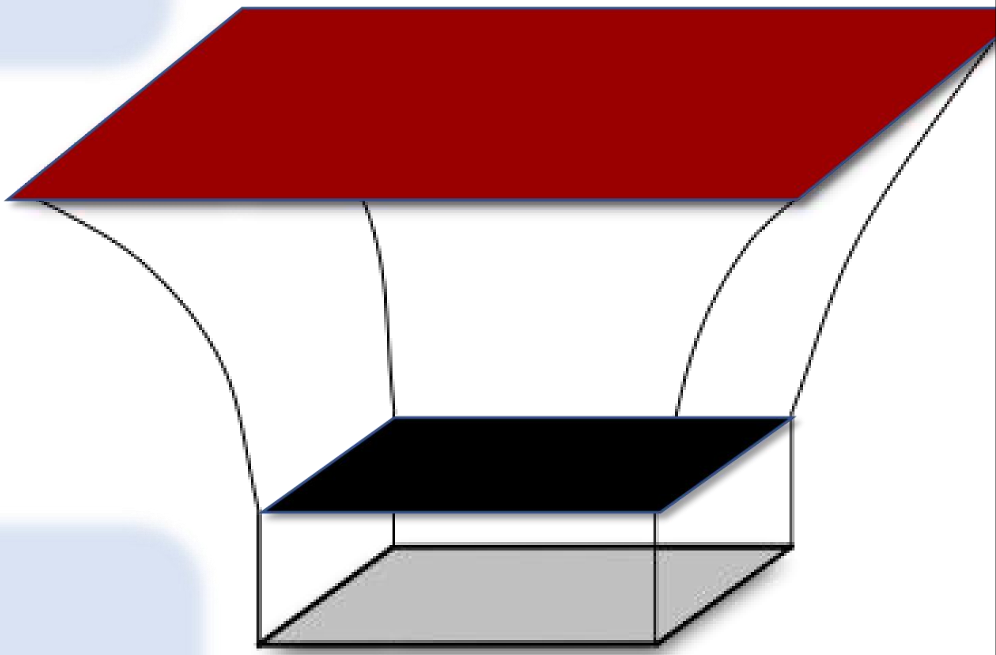
→ Far from equilibrium strongly coupled field theories from first principles.

Holography = AdS/CFT duality = gauge/gravity duality

Holography

- CFT on Minkowski in 3+1 dim
- Decoupled sector of the stress tensor $T^{\mu\nu}$.





- Gravity with Λ in 4+1 dim :

$$S \sim \int d^{3+1}x \sqrt{-g}\,(R - 2\Lambda)$$

Holography

- CFT on Minkowski in 3+1 dim
- Decoupled sector of the stress tensor $T^{\mu\nu}$.

Real-time quantum dynamics

Holography

Numerical Relativity

Dynamical classical gravity

- Gravity with Λ in 4+1 dim :

$$S \sim \int d^{3+1}x \sqrt{-g}\,(R - 2\Lambda)$$

Bantilan, Bea, Figueras '22
Bea, Figueras [in progress]