TOWARDS POLYNOMIAL CONVERGENCE FOR VARIATIONAL QUANTUM ALGORITHMS USING LANGEVIN DYNAMICS

Pablo Páez Velasco, ICMAT
C. Nicolás Cabrera, 13-15, 13-15, Fuencarral-El Pardo, 28049 Madrid
T: +34912999752, pablpaez@ucm.es / pablo.paez@icmat.es
Ángela Capel, Fachbereich Mathematik, Universität Tübingen, Germany / Department of Applied Mathematics and Theoretical Physics, University of Cambridge, United Kingdom.
Marco Castrillón López, Departamento de Álgebra, Geometría y Topología, Universidad Complutense de Madrid.
Sofyan Iblisdir, Departamento de Análisis Matemático y Matemática Aplicada, Universidad Complutense de Madrid.
Angelo Lucia, Departamento de Análisis Matemático y Matemática Aplicada, Universidad Complutense de Madrid / ICMAT.
David Pérez García, Departamento de Análisis Matemático y Matemática Aplicada, Universidad Complutense de Madrid / ICMAT.

One of the most promising types of algorithms to run on noisy intermediate-scale quantum computers are variational optimization algorithms [4]. In those algorithms one deals with a parametrized quantum circuit whose outputs are then a parametrized family F of n-particle quantum states.

Given an n-body observable H, that can be efficiently implemented (e.g. a locally interacting Hamiltonian), the goal is to obtain an approximation of the ground state and its associated energy.

The aim of our work is to study the continuous Langevin Dynamics (see [1, 2]) in a rather general setting, shown in Figure 2. Proving convergence results in such a setting may potentially lead to poly-time algorithms to solve the problem introduced in the previous paragraph, when considering depth-2 quantum circuits with gates acting on a logarithmic number of sites. Moreover, our results should be applicable to other circuits, under certain assumptions on their structure and the Hamiltonian considered.

We generalize some of the results from [3] to the Lie Group SU(n); proving that the Gibbs distribution associated to the process shown in Figure 2 does indeed "find" the global minimum of F. Furthermore, we prove that our setting satisfies a logarithmic Sobolev Inequality, which guarantees exponential convergence of the process to its associated Gibbs distribution (see [1]).

$$\min_{x \in \mathrm{SU}(n)^{\times r}} F(x), \quad F : \mathrm{SU}(n)^{\times r} \to \mathbb{R} \text{ non-convex},$$

$$dZ_t = -\mathit{grad}F(Z_t)\, dt + \sqrt{\frac{2}{\beta}}\, dW_t.$$

Figure 2 – Langevin Dynamics proposed to find the minimum of F

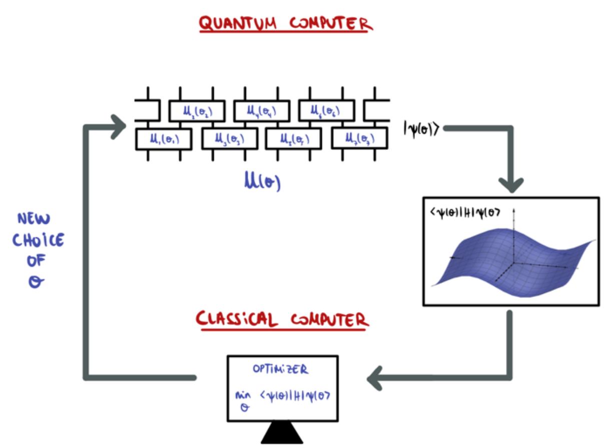

Figure 1 – General Variational Quantum Algorithm Scheme

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