LIE ALGEBRAIC INVARIANTS IN LINEAR QUANTUM OPTICS

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Linear quantum optics is a promising candidate for obtaining a quantum computational advantage. However, linear optics without post-selection is not powerful enough to produce any quantum state from a given input state, which limits its computational capabilities. Thus, we need a deeper understanding of linear optical evolution. In this work, we use the Lie algebra of passive linear optical Hamiltonians to find conserved quantities in the evolution of a mixed quantum state. We obtain the invariants by projecting the density matrix onto this Lie algebra and taking the norm and the spectrum of the projected density matrix. This gives us invariant quantities along the passive linear optical evolution (for example, two coherent states). Thus, if two input and output states have different invariants, it will be impossible to design a linear optical experiment that evolves one into the other. These invariants allow us to narrow the search when trying to prepare entangled resources useful for quantum computation, like Bell or noon states, from easy-to-prepare states, like Fock states. Using the invariants, we prove the impossibility of the following state preparations: 1) a Fock state from another Fock state (except by permutation of modes); 2) a perfect Bell state from a Fock state; 3) a noon state from a Fock state; 4) a W state from a GHZ state; 5) a photon-added coherent state from a coherent state and a Fock state. We conclude that future state preparation algorithms will need to take into account the necessary conditions given by our invariants to weed out impossible linear optical evolutions.

[1] Parellada, P. V., Garcia, V. G. I., Moyano-Fernández, J. J., & Garcia-Escartin, J. C. (2023). No-go theorems for photon state transformations in quantum linear optics. Results in Physics, 54, 107108. https://doi.org/10.1016/j.rinp.2023.107108