TENSORIZING HIGH-DIMENSIONAL DENSITIES

José Ramón Pareja Monturiol, UCM - ICMAT (CSIC)
Nicolás Cabrera, 13-15, 28049, Madrid
T:+34 912999739, joserapa@ucm.es
Alejandro Pozas-Kerstjens, UNIGE
David Pérez-García, UCM - ICMAT (CSIC)

Tensor networks, originally stemming from condensed matter physics, are factorizations of high-dimensional tensors into networks of smaller tensors that have found successful applications in physics, mathematics, and more recently, in machine learning.

A significant challenge in using a tensor network as a compressed and efficient representation of a tensor is identifying the smaller tensors that make up the network. One method to build a tensor network is through iterative optimization, as seen in the Density Matrix Renormalization Group (DMRG) algorithm, used to compute ground states of quantum many-body systems as Matrix Product States (MPS), also known as Tensor Trains (TT) [1]. This method returns a tensor already in tensor network form, rather than decomposing a given tensor. Alternatively, from a provided tensor, one can construct tensor networks directly using sequential matrix factorization techniques, such as singular value decomposition (SVD) [2, 3]. However, this kind of decomposition is only feasible for low-dimensional regimes, where the original tensor is fully accessible.

For high-dimensional tensors, algorithms like TT-Cross [4] and TT-Recursive Sketching (TT-RS) [5] have been developed to construct tensor networks using only partial information from the tensor. While both algorithms can decompose functions expressed as high-dimensional tensors, the latter is more appropriate for sparse functions like densities. However, the accuracy of TT-RS depends on effectively extracting relevant information from the tensor. For example, for Markovian densities, each MPS core can be efficiently recovered using information from neighboring sites only.

In our work, we propose an adaptation of the TT-RS algorithm [5] that utilizes the information from the given density function (the high-dimensional tensor to decompose), along with a set of samples extracted from the corresponding distribution. This scenario is common in machine learning, where both the trained model (the density) and a training or test dataset are available. This technique can also be applied in condensed matter physics, as it facilitates finding tensor network representations of states given with black-box access. This approach may aid in the interpretability and analysis of Quantum Neural States [6].

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