UNDECIDABILITY OF THE SPECTRAL GAP IN SYMMETRIC HAMILTONIANS

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The problem of determining the existence of a spectral gap was shown undecidable for one ([1]) or more dimensions ([3]). The 2-dimensional result constructs a family of translationally-invariant, nearest-neighbour Hamiltonians on a square lattice for which determining whether the system is gapped or gapless is an undecidable problem.

However, due to the artificiality of the result and the fact that symmetric behaviours can be found in most physical systems, we study if this undecidability property still holds for a 2-dimensional system whose Hamiltonian presents rotational symmetry.

In order to show that symmetric systems also present this undecidable property, we still use the Halting problem ([5]) as our starting point, as well as the rigid properties of Robinson's Tiling ([4]). Nonetheless, we use a different approach to the Tiling Problem, as done in [2], that allows us to incorporate the flexibility needed for describing a rotational invariant Hamiltonian.

References:

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