Clustering in transformer models in ML and its role in sentiment analysis

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MODCONFLEX
Modelling and control of flexible structures interacting with fluids

Funded by the European Union

Motivation

New machine learning tool in everyday life: **ChatGPT**.

G for **Generative** **P** for **Pre-trained** **T** for **Transformer**

Motivation

New machine learning tool in everyday life: **ChatGPT**.

Generative: learn to predict the next word/pixel.

"A dog doing math research".[1]

[1] Source: https://pixlr.com/image-generator/

Motivation

New machine learning tool in everyday life: **ChatGPT**.

Pre-trained: trained on a massive corpus of text/images.

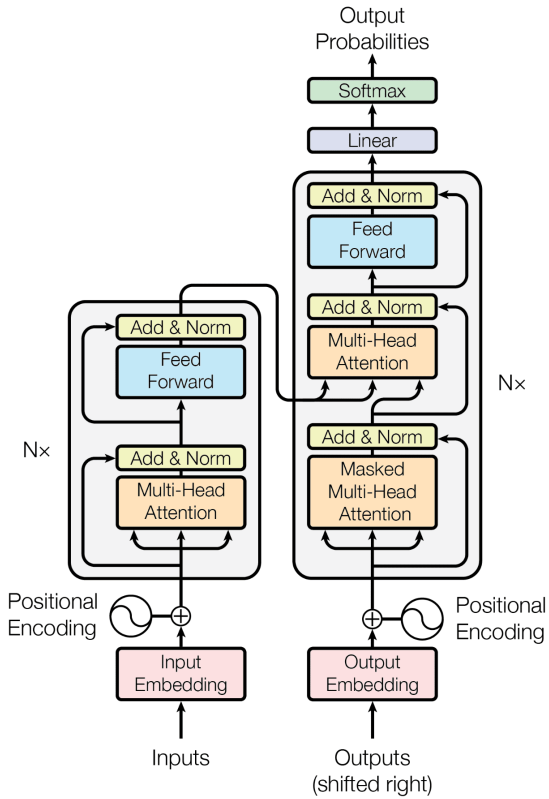
Evolution of number of words in training datasets.[2]

[2] Source: https://www.lesswrong.com/posts/asqDCb9XzXnLjSfgL/trends-in-training-dataset-sizes

Motivation

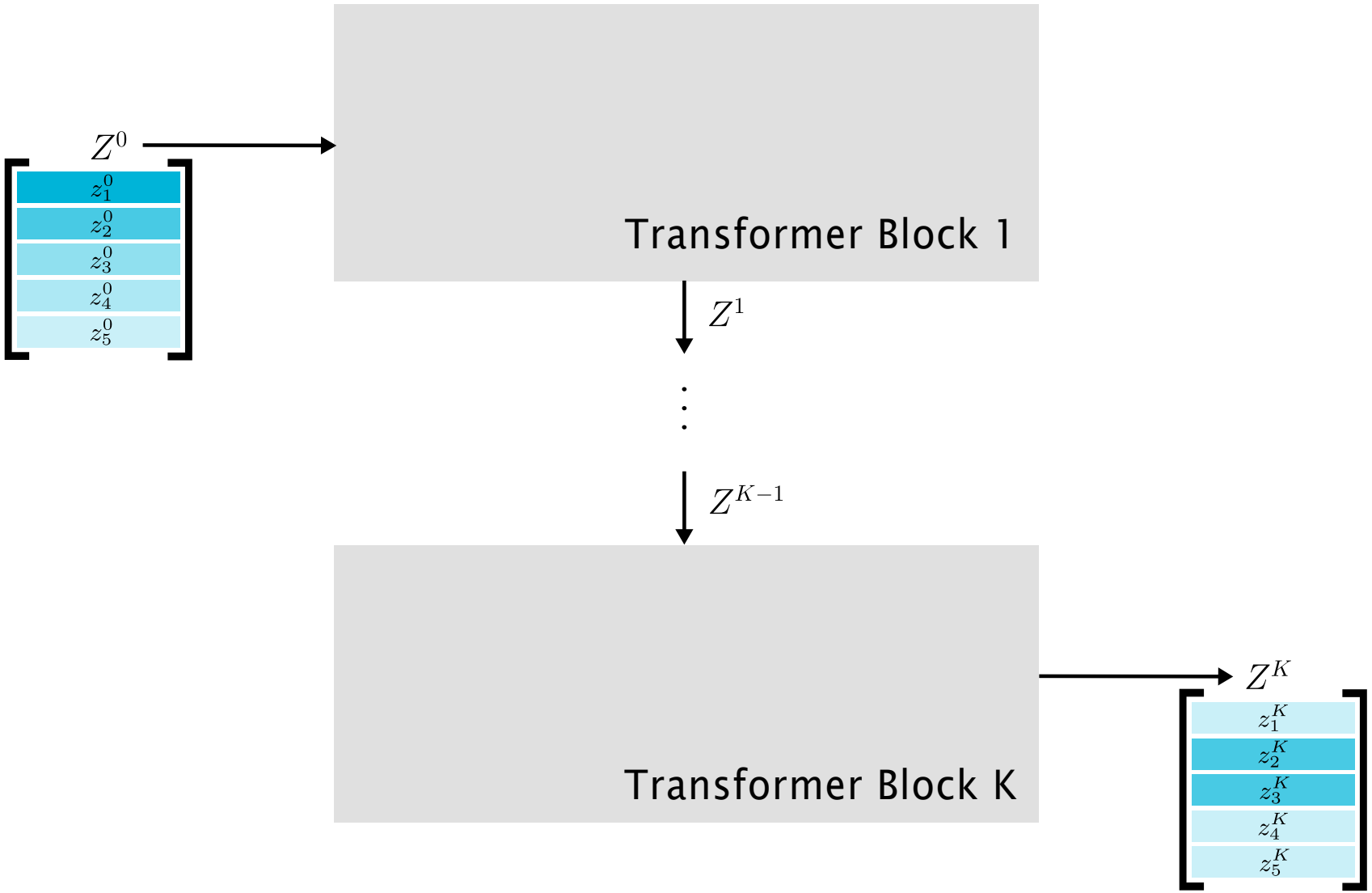
New machine learning tool in everyday life: **ChatGPT**.

Transformer: deep neural network architecture.

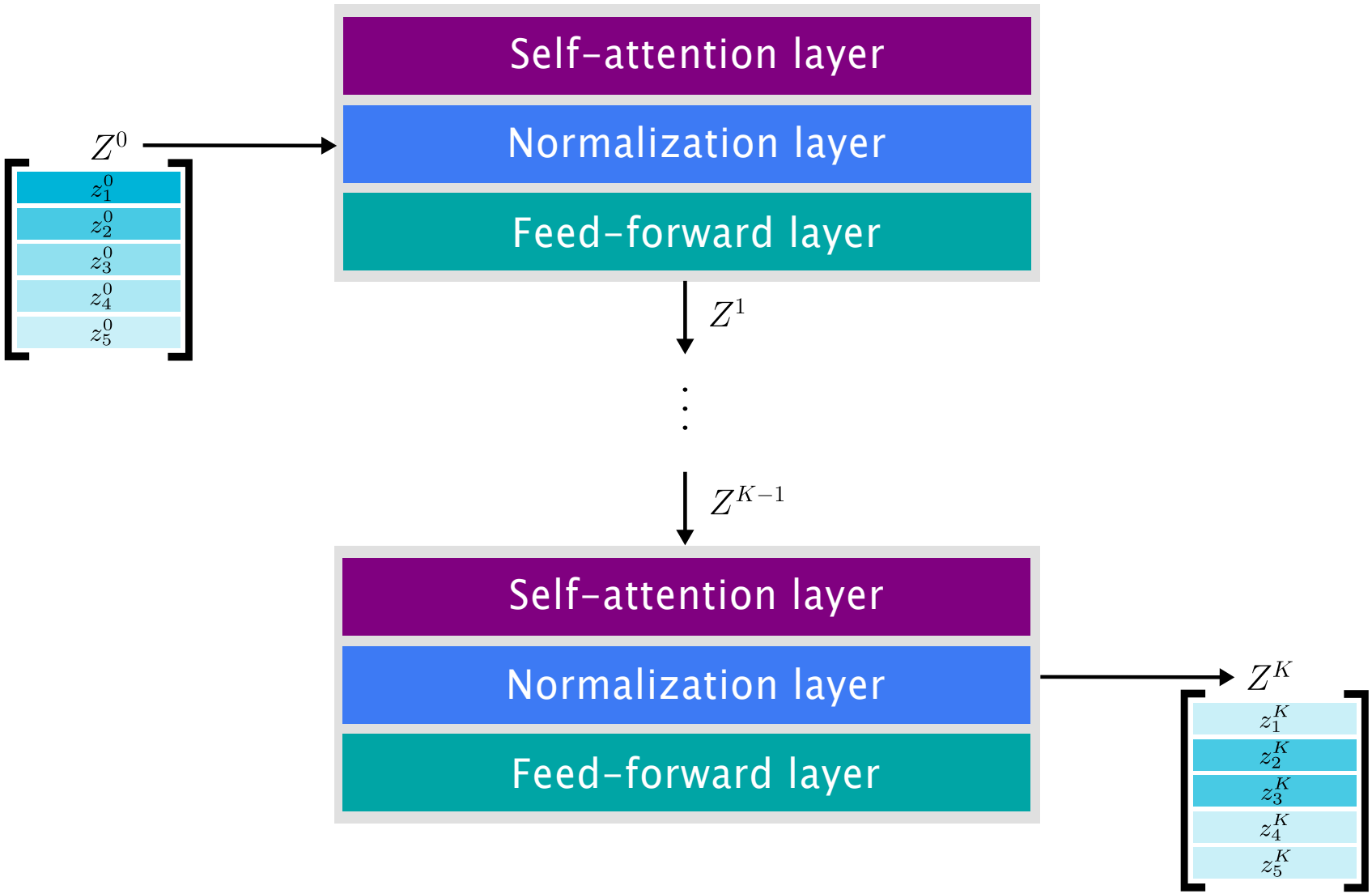


Original transformer architecture in [VSP+17].

Modeling of transformers



Modeling of transformers



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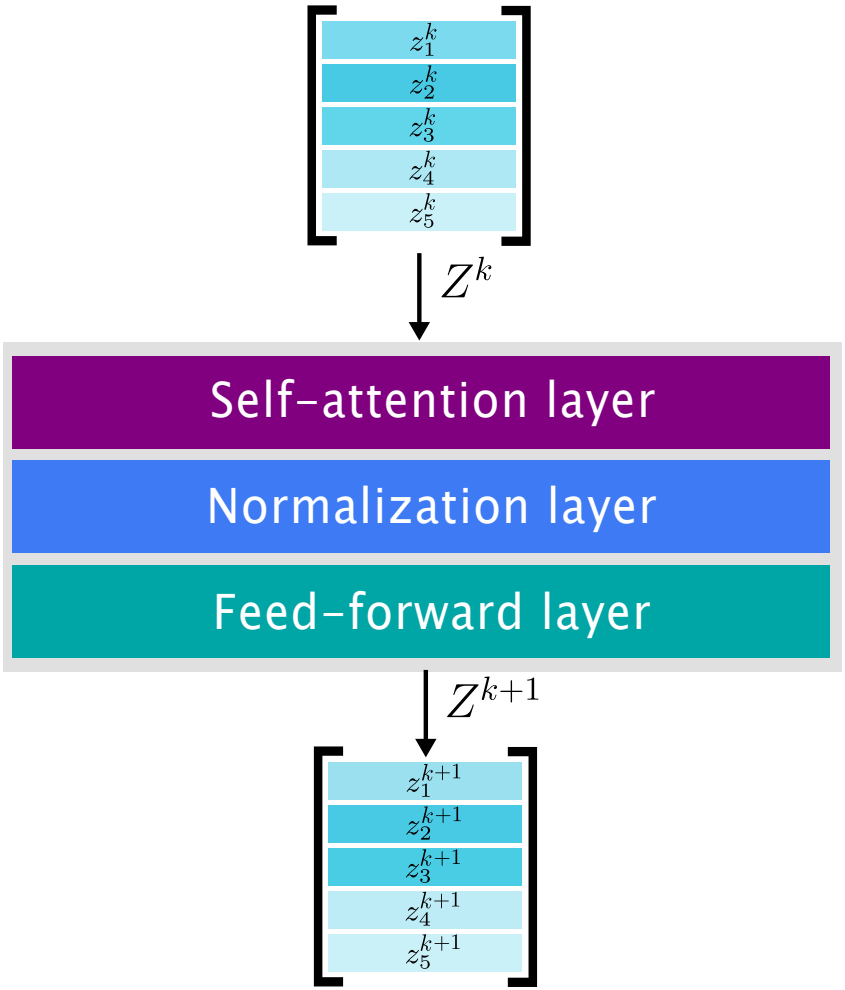

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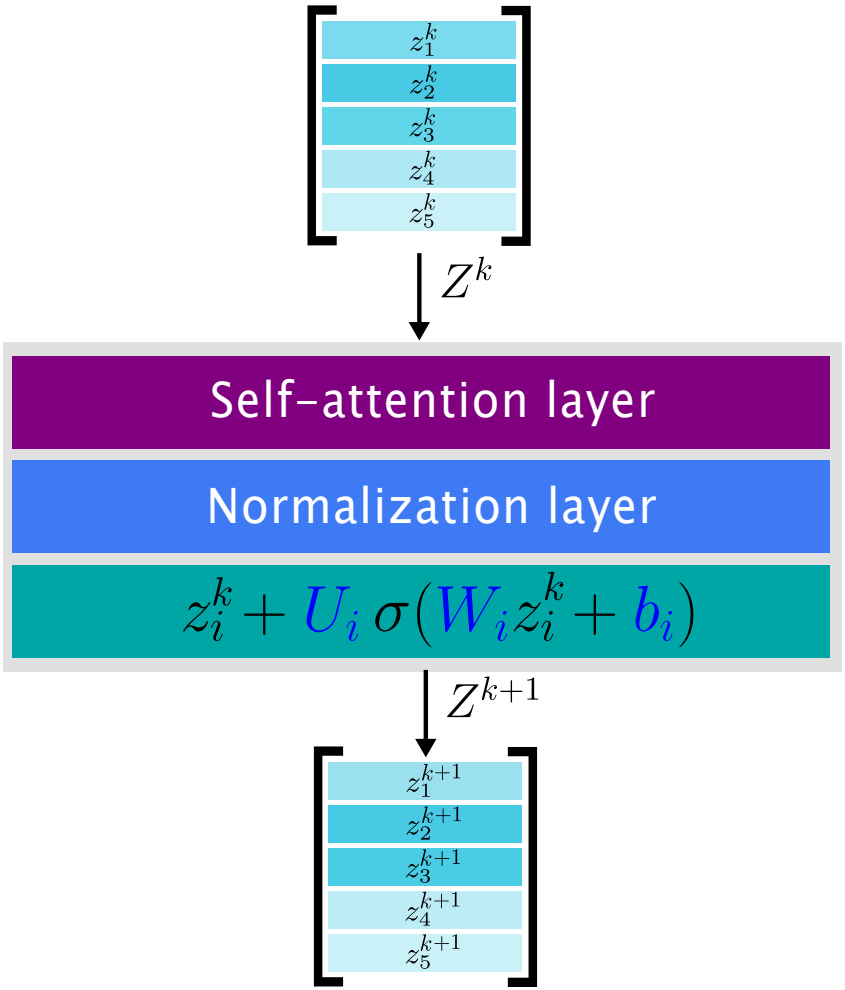

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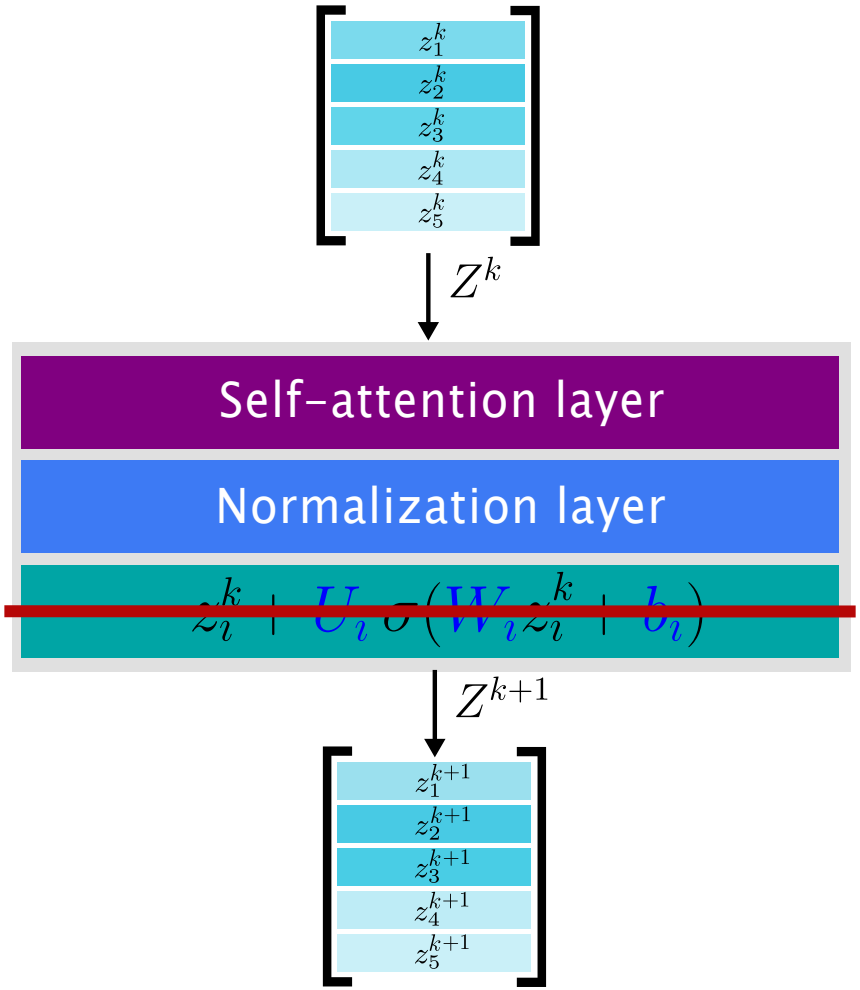

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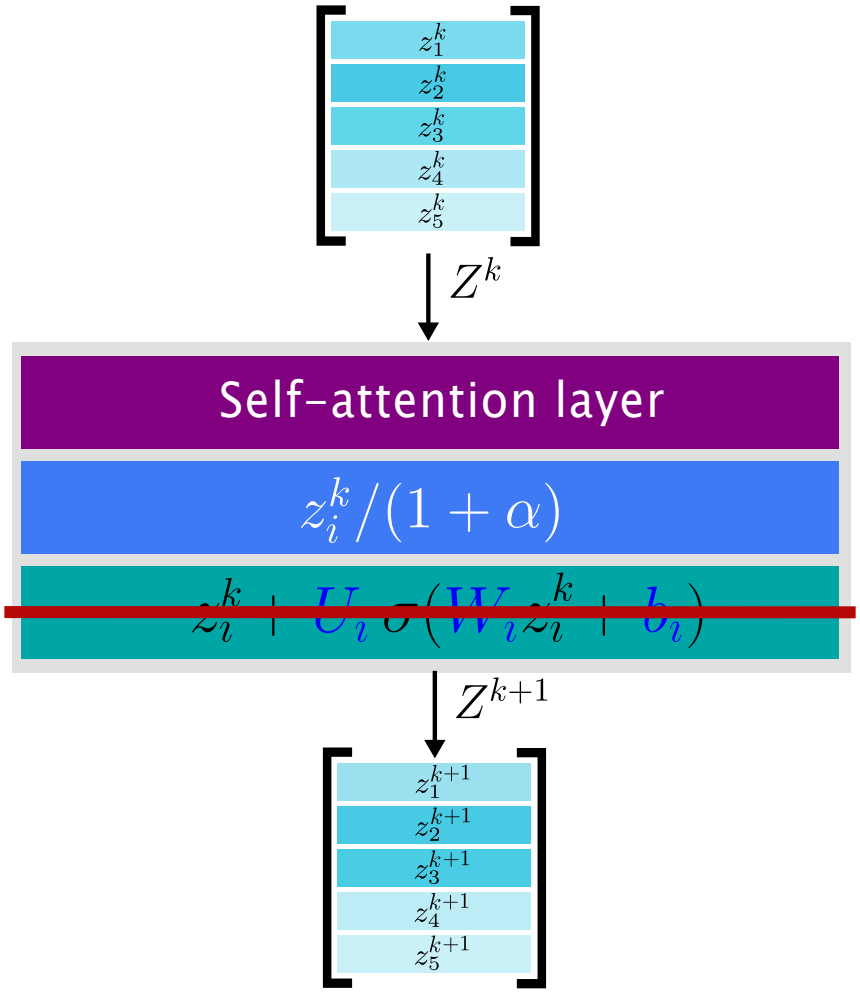
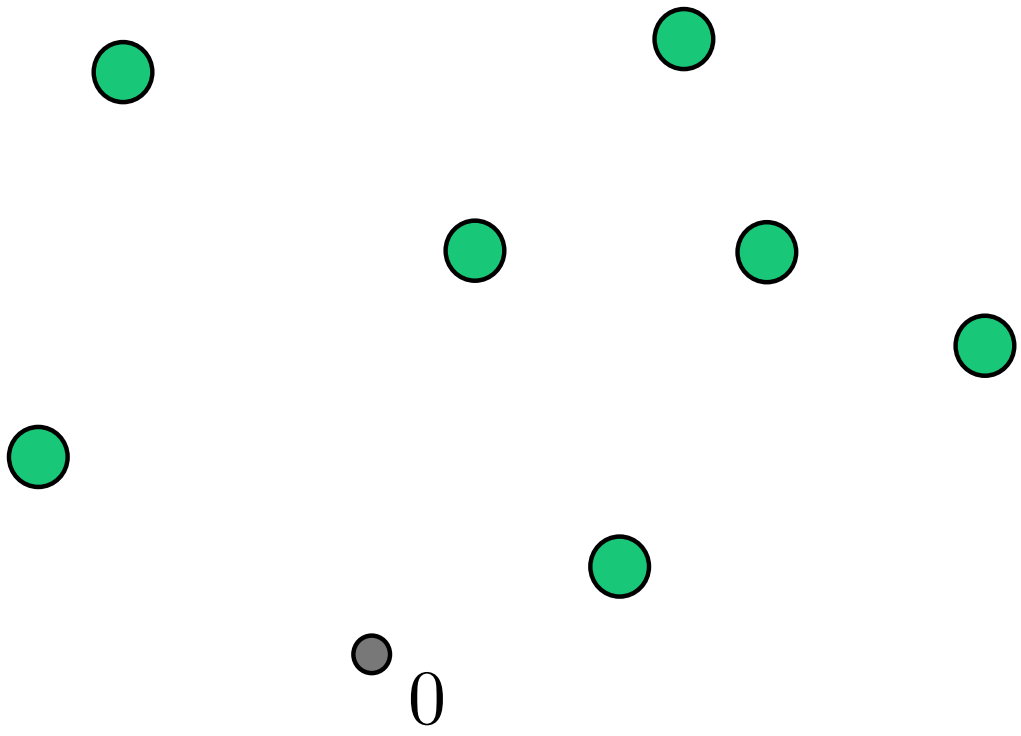


The self-attention layer



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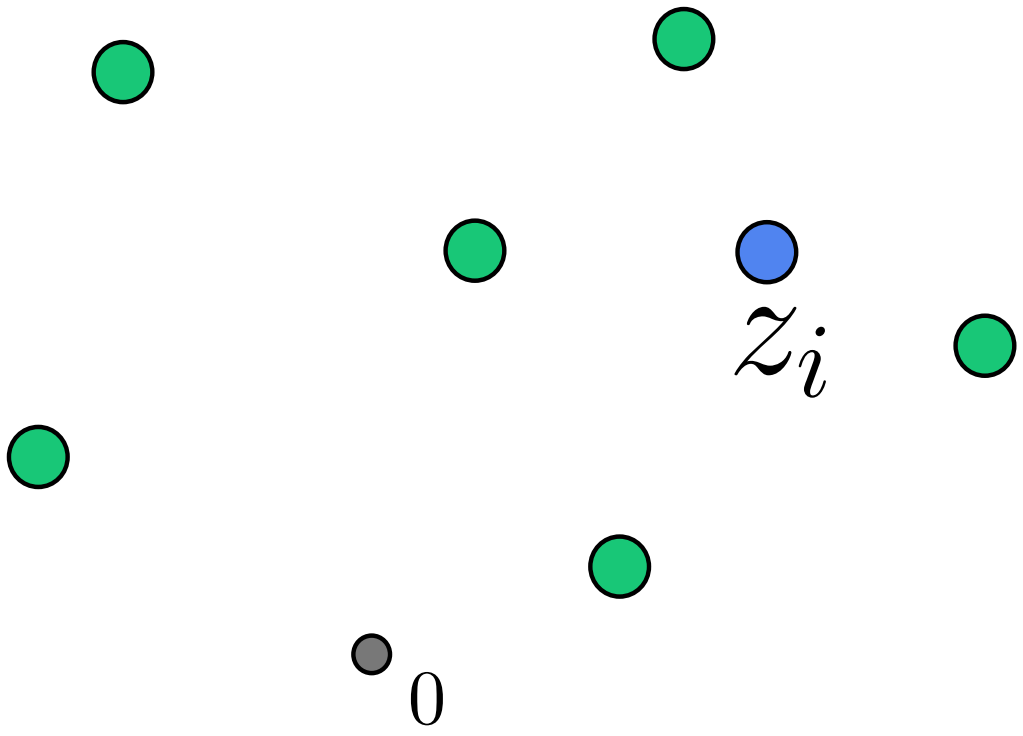

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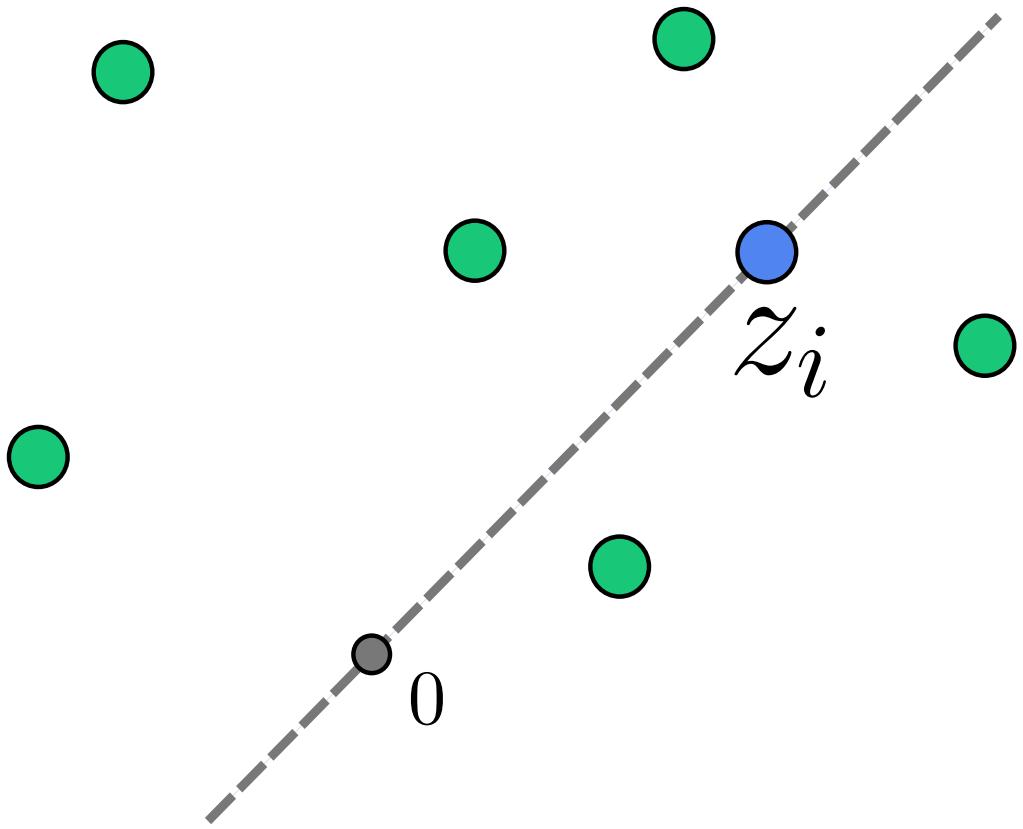

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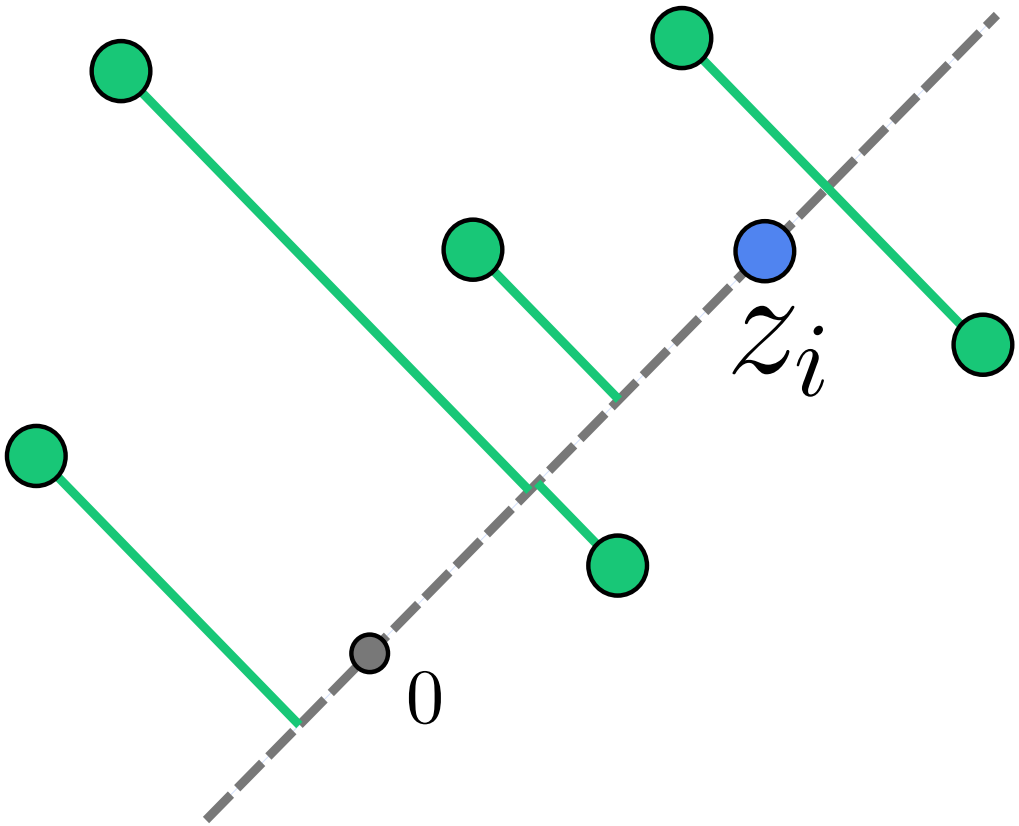

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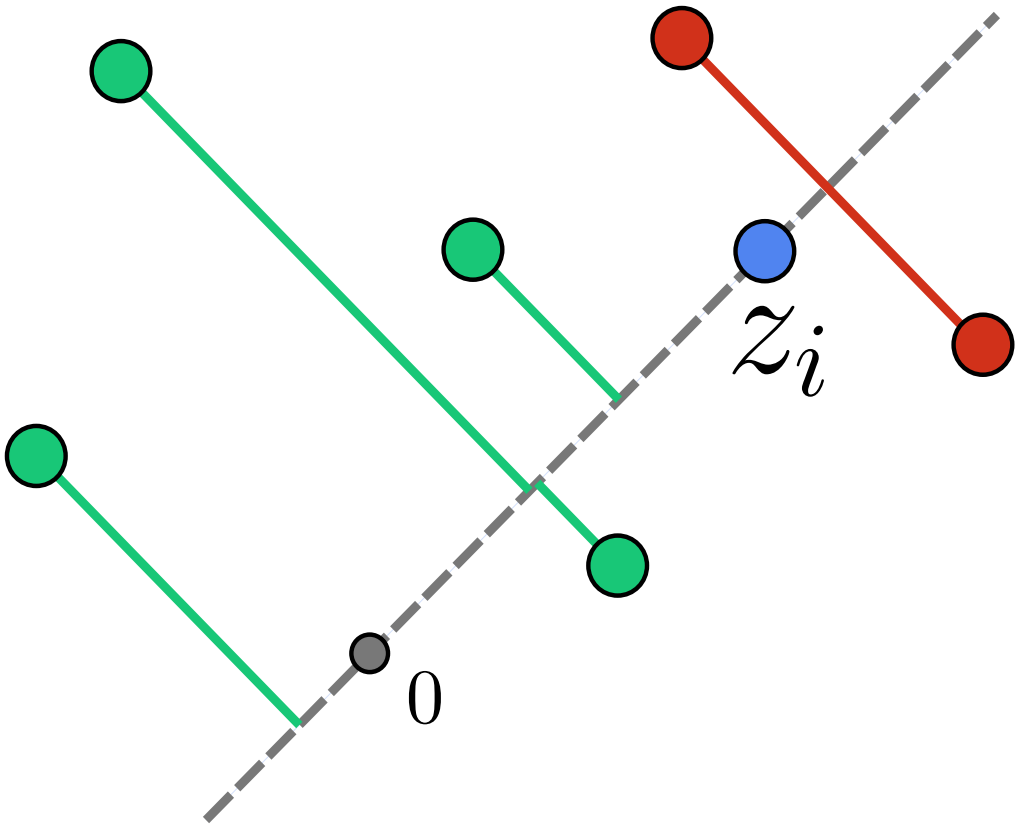

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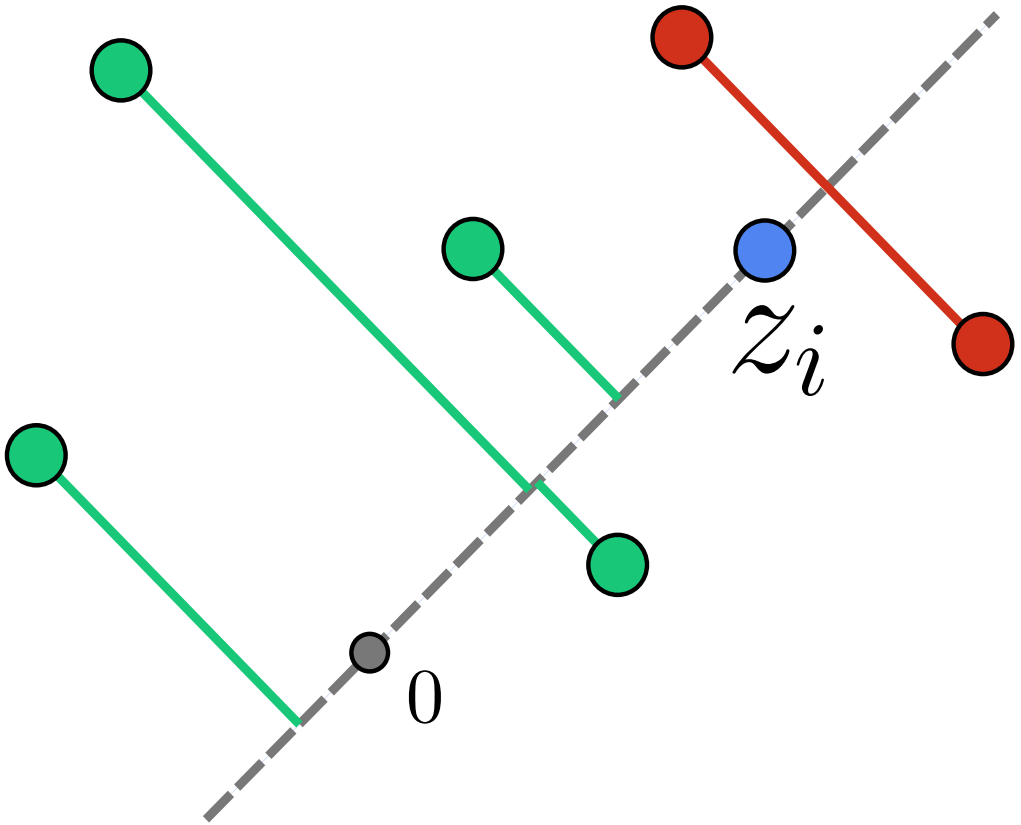
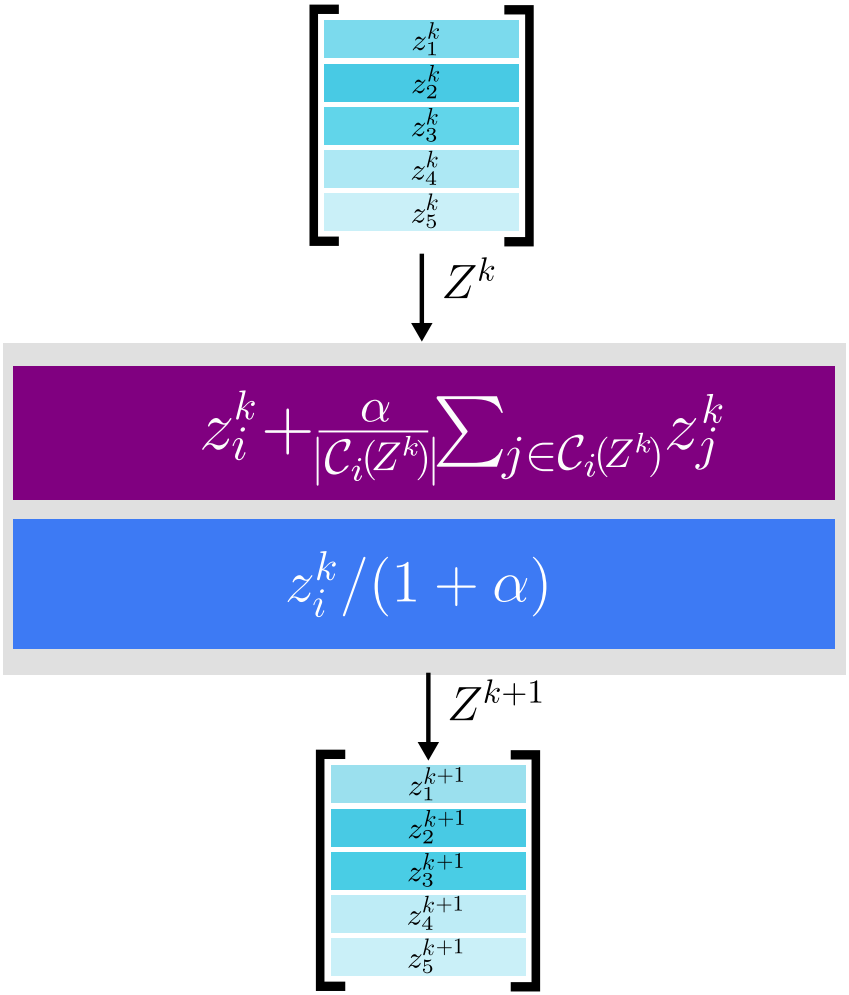


$$\mathcal{C}_i(Z) = \{j \in [n] \, : \, \langle z_i, z_j \rangle = \max_{\ell \in [n]} \langle z_i, z_\ell \rangle\}.$$

$$z_i^+ = z_i + \frac{\alpha}{|\mathcal{C}_i(Z)|} \sum_{j \in \mathcal{C}_i(Z)} z_j$$

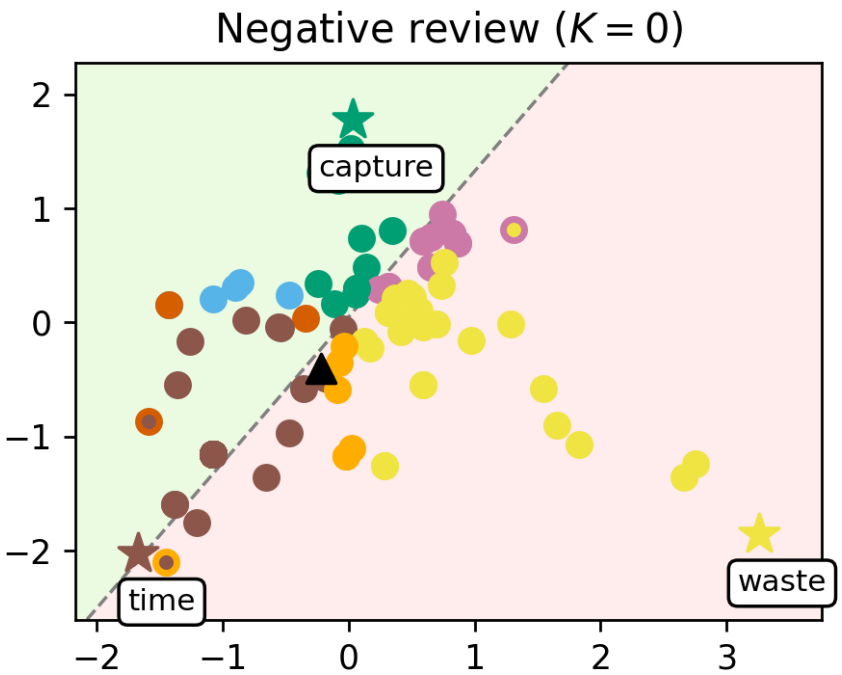
Pure-attention hardmax transformers

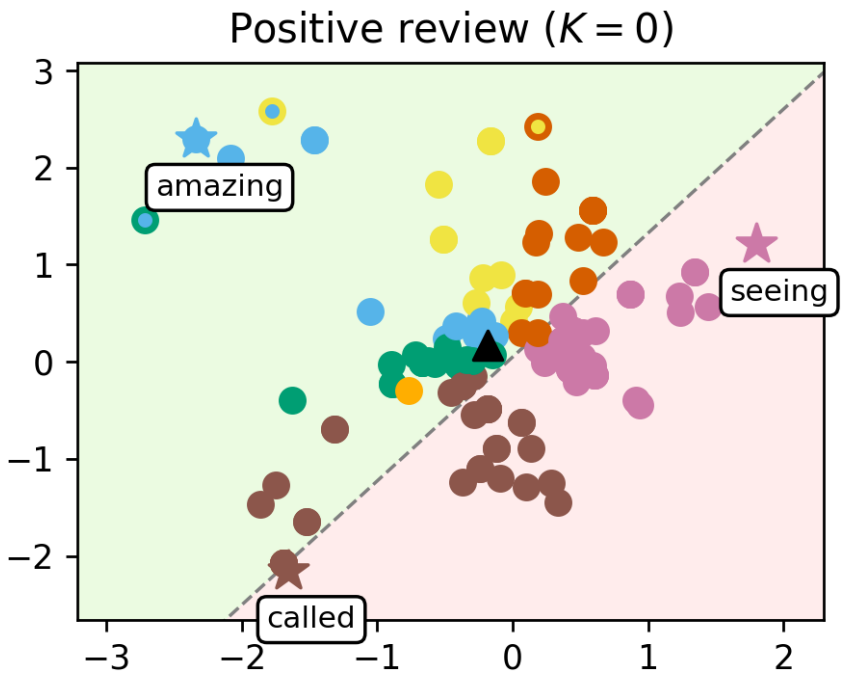


$$z_i^{k+1} = z_i^k + \frac{\alpha}{1+\alpha} \frac{1}{|\mathcal{C}_i(Z^k)|} \sum_{j \in \mathcal{C}_i(Z^k)} \left(z_j^k - z_i^k \right), \quad k \geq 0.$$

Interpretability through dynamics and control

- $k \to \infty$ asymptotics: clustering first proved for similar continuous transformer models in [GLPR23a], [GLPR23b].





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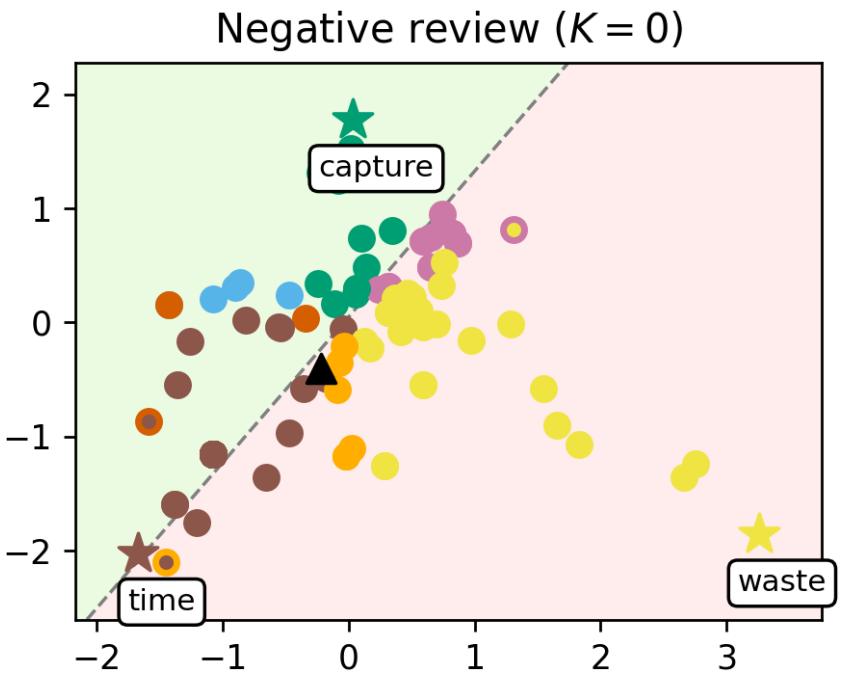
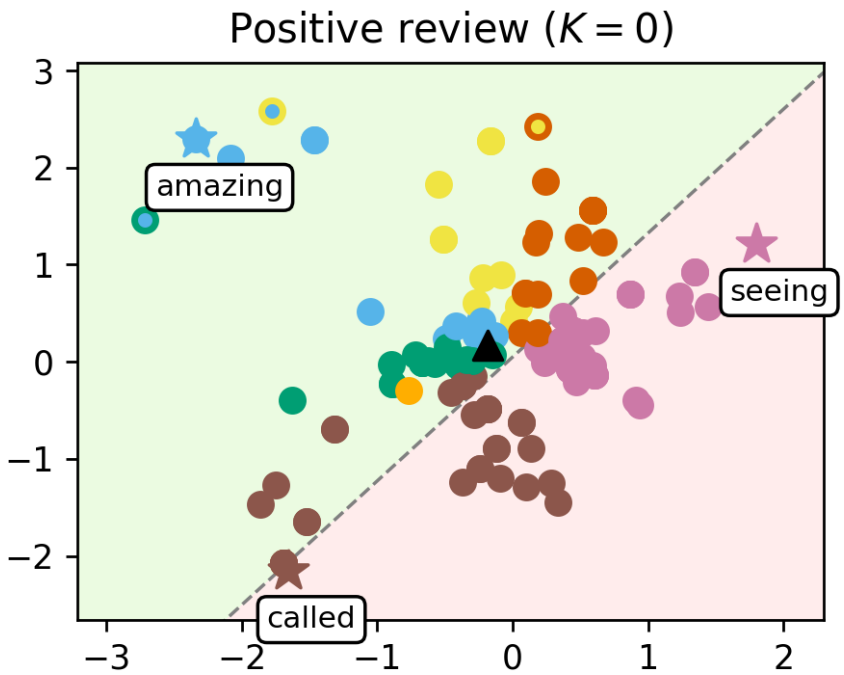
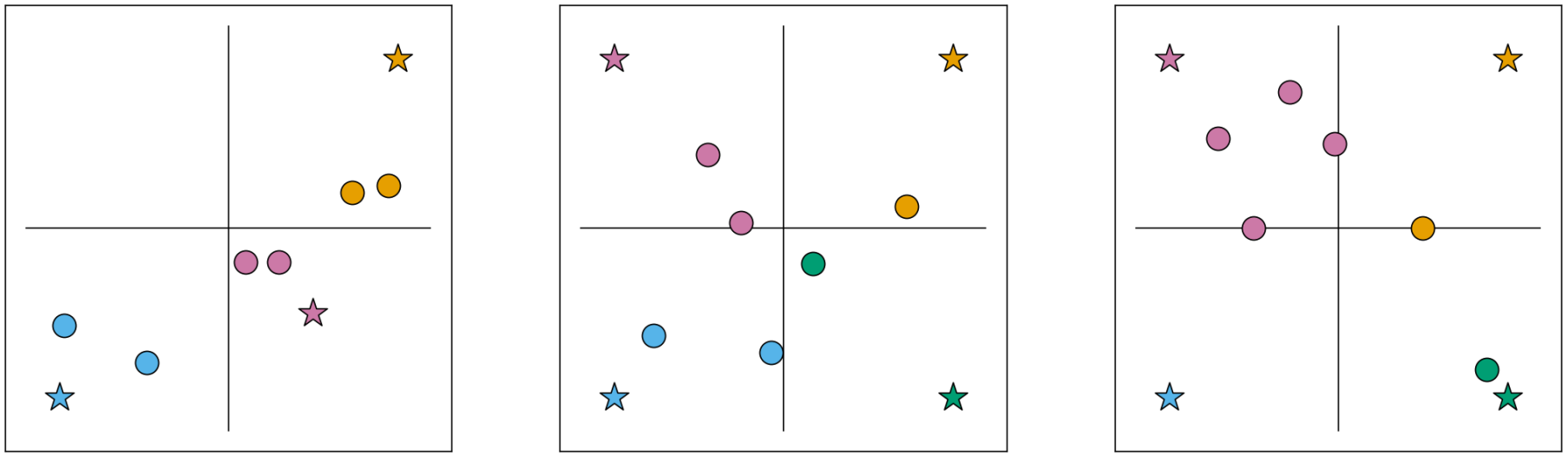




- **Q1:** When and how do discrete dynamics exhibit clustering?
- **Q2:** Is clustering capturing the 'context' in a text?

Asymptotic dynamics



Point z_i is a **leader** if there exists a layer $k \in \mathbb{N}$ s.t. $\mathcal{C}_i(Z^k) = \{i\}$.

Emergence and characterization of cluster points

Q1: When and how do discrete dynamics exhibit clustering?

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Theorem (A. A, G. Fantuzzi, E. Zuazua 2024)

Assume $z_1^0, \dots, z_n^0 \in \mathbb{R}^d$ are nonzero and distinct. There exist

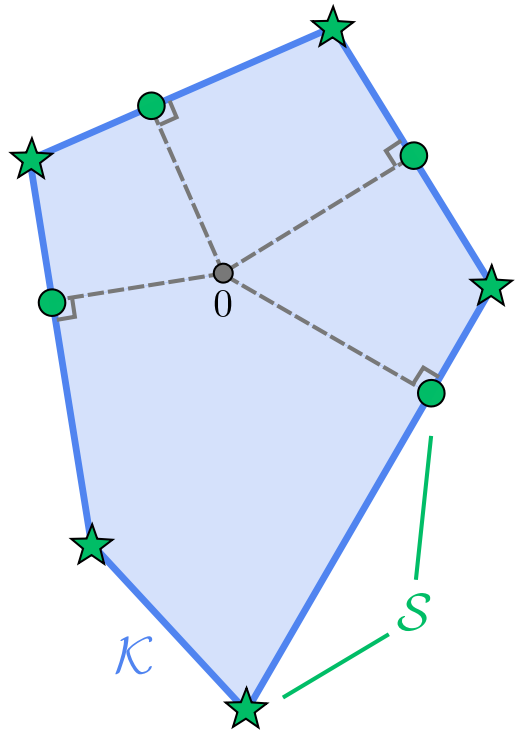
(a) a convex polytope $\mathcal{K}$, and

(b) a finite set $\mathcal{S} \subset \partial\mathcal{K}$

such that:

(i) $z_i^k \to s \in \mathcal{S}$ as $k \to \infty$ for all i.

(ii) Every $s \in \mathcal{S}$ is a limit value of a leader or a convex combination of them.

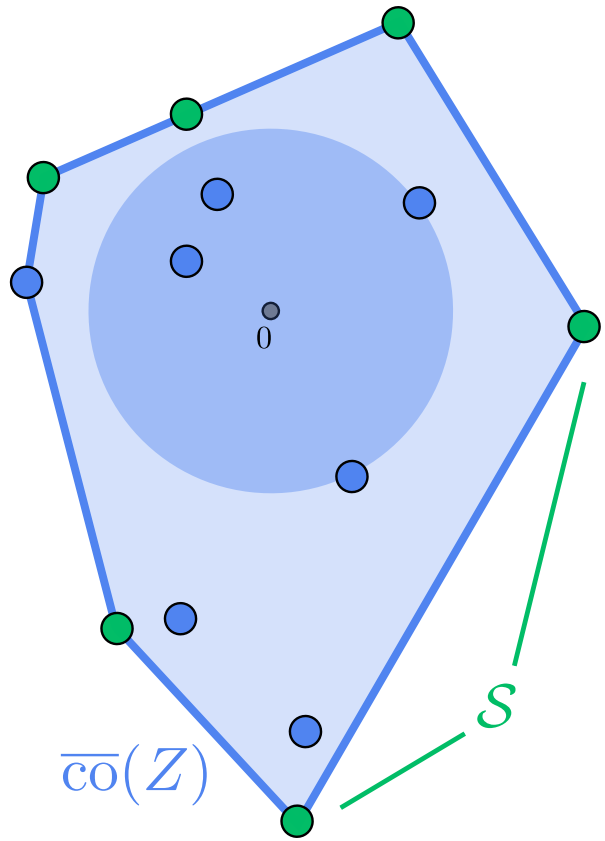


Idea of the proof

Step 1: convergence to an equilibrium at the boundary of a convex polytope $\mathcal{K}$. Two competing forces:

- Convex hull of tokens shrinks,
- Norm of tokens strictly grows when not close to finite set $\mathcal{S} \subset \partial\mathcal{K}$.

Step 2: vertices of $\mathcal{K}$ = leaders.



Role of clustering in sentiment analysis

Q2: What is the role of clustering in a real machine learning application? Is it capturing 'context'?

Role of clustering in sentiment analysis

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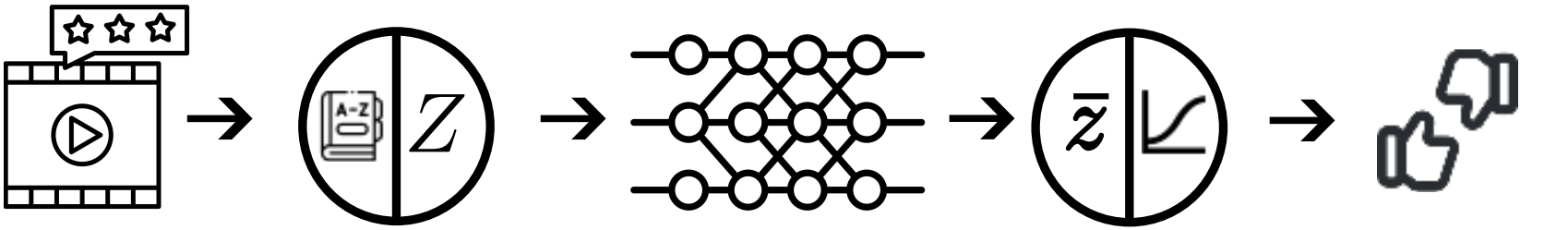
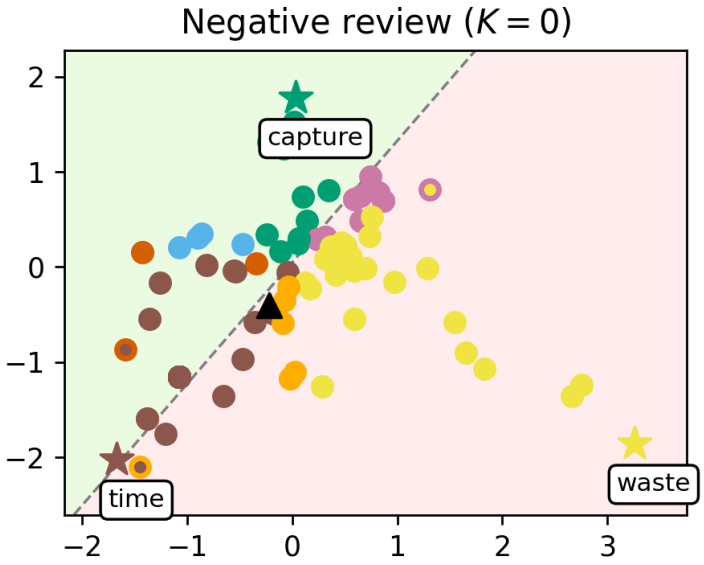
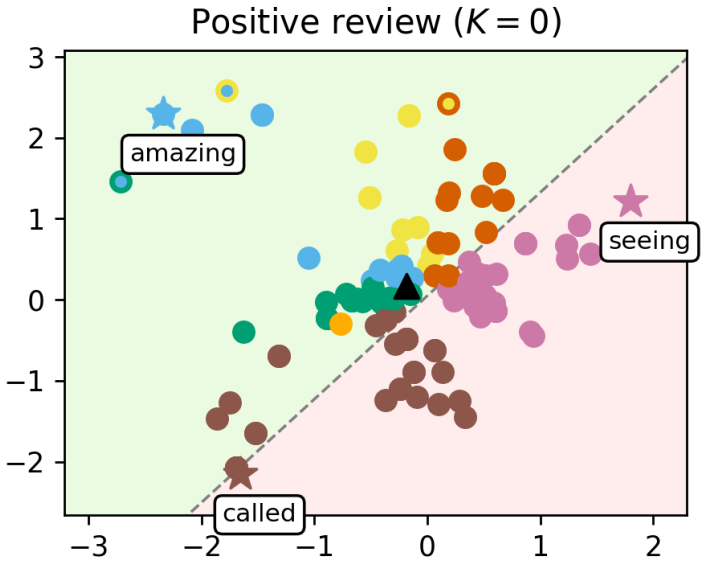
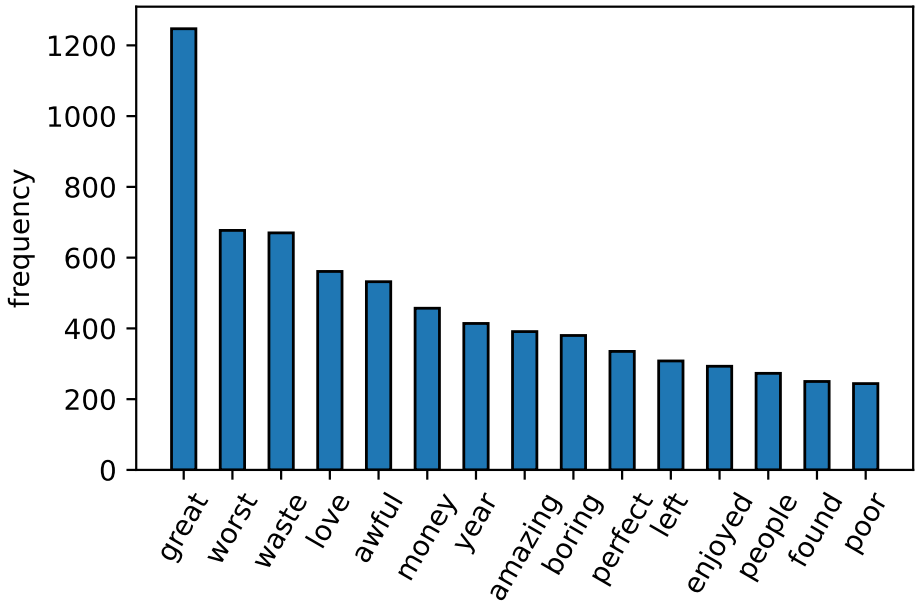
Task: Sentiment analysis of movie reviews.



Results


Negative review ($K = 0$)
capture
time
waste


Positive review ($K = 0$)
amazing
seeing
called


frequency
great
worst
waste
love
awful
money
year
amazing
boring
perfect
left
enjoyed
people
found
poor

Conclusions and perspectives

- We analyzed a transformer model, proving that it entails a clustering effect.
- Related clustering to emergence of context in the ML application of sentiment analysis.
- **Open questions:**
  - *Controlling* the leaders by appropriately choosing the matrix $A \in \mathbb{R}^{d\times d}$ in

$$\mathcal{C}_i(Z) = \{j \in [n] \,:\, \langle Az_i, z_j\rangle = \max_{\ell\in[n]} \langle Az_i, z_\ell\rangle\}.$$

  - What does clustering imply for physical systems, what are the *leaders* in, say, a flow past a cylinder?

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  - What does clustering imply for physical systems, what are the *leaders* in, say, a flow past a cylinder?

Thank you for your

$$\text{“}z_i + \frac{\alpha}{|\mathcal{C}_i(Z)|}\sum_{j\in\mathcal{C}_i(Z)} z_j\text{”}$$

arXiv:2407.01602