Controllability of the Linearized Compressible Navier-Stokes System with Maxwell's Law[1]

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[1] Joint work with Subrata Majumdar (Instituto de Matemáticas, UNAM)

CNSE with Maxwell's law

▷ Let us consider the one-dimensional compressible Navier-Stokes system in the domain $(0, 2\pi)$:

$$\left.\begin{aligned} &\partial_t \hat{\rho} + \partial_x(\hat{\rho}\hat{u}) = 0 && \text{in } (0,T)\times(0,2\pi),\\ &\partial_t\left(\hat{\rho}\hat{u}\right) + \partial_x\left(\hat{\rho}\hat{u}^2\right) + \partial_x p = \partial_x \hat{S} && \text{in } (0,T)\times(0,2\pi). \end{aligned}\right\} \tag{1}$$

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- $\hat{\rho}$, $\hat{u}$, p, and $\hat{S}$ represent the density, velocity, pressure, and stress tensor of the fluid, respectively.

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- Assume that the pressure p satisfies the following constitutive law:

$$p(\hat{\rho}) = a\hat{\rho}^{\gamma}, \quad a > 0,\ \gamma \geq 1.$$

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- Here μ represents the fluid viscosity and κ denotes the relaxation time that characterizes the time delay in the response of the stress tensor to the velocity gradient.

Linearized system

▷ We consider the linearized system around the constant steady state $(\rho_s, u_s, 0)\,, \rho_s > 0, u_s > 0$ of (1):

$$
\left.\begin{aligned}
&\partial_t \rho + u_s \partial_x \rho + \rho_s \partial_x u = \mathbb{1}_{\mathcal{O}_1} f_1, && \text{in } (0,T)\times(0,2\pi),\\
&\partial_t u + u_s \partial_x u + a\gamma {\rho_s}^{\gamma-2} \partial_x \rho - \frac{1}{\rho_s}\partial_x S = \mathbb{1}_{\mathcal{O}_2} f_2, && \text{in } (0,T)\times(0,2\pi),\\
&\partial_t S + \frac{1}{\kappa} S - \frac{\mu}{\kappa}\partial_x u = \mathbb{1}_{\mathcal{O}_3} f_3, && \text{in } (0,T)\times(0,2\pi),\\
&\rho(t,0) = \rho(t,2\pi),\ u(t,0) = u(t,2\pi),\ S(t,0) = S(t,2\pi), && t \in (0,T),\\
&\rho(0,x) = \rho_0(x), \quad u(0,x) = u_0(x), \quad S(0,x) = S_0(x), && x \in (0,2\pi).
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Linearized system

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\partial_t u + u_s \partial_x u + a\gamma {\rho_s}^{\gamma-2} \partial_x \rho - \dfrac{1}{\rho_s} \partial_x S = \mathbb{1}_{\mathcal{O}_2} f_2, & \text{in } (0,T)\times(0,2\pi), \\
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\end{array}\right\} \quad (2)$$

- $\mathbb{1}_{\mathcal{O}_j}$ is the characteristic function of an open set $\mathcal{O}_j \subseteq (0,2\pi), j = 1,2,3.$
- f_1, f_2, f_3 are the controls.

Semigroup framework

- Let $(L^2(0,2\pi))^3$ be endowed with the inner product

$$\left\langle \begin{pmatrix} \rho \\ u \\ S \end{pmatrix}, \begin{pmatrix} \sigma \\ v \\ \tilde{S} \end{pmatrix} \right\rangle_{(L^2(0,2\pi))^3} = b\int_0^{2\pi} \rho\bar{\sigma}\,\mathrm{d}x + \rho_s \int_0^{2\pi} u\bar{v}\,\mathrm{d}x + \frac{\kappa}{\mu}\int_0^{2\pi} S\bar{\tilde{S}}\,\mathrm{d}x.$$

- We now define the unbounded operator $\left(\mathcal{A}, \mathcal{D}(\mathcal{A};(L^2(0,2\pi))^3)\right)$ in $(L^2(0,2\pi))^3$ by

$$\mathcal{D}(\mathcal{A};(L^2(0,2\pi))^3) = \left\{ \begin{pmatrix} \rho \\ u \\ S \end{pmatrix} \in (L^2(0,2\pi))^3 \;:\; (\rho,u,S)^\top \in H_p^1 \times H_p^1 \times H_p^1 \right\}$$

and

$$\mathcal{A} = \begin{bmatrix} -u_s\frac{d}{dx} & -\rho_s\frac{d}{dx} & 0 \\ -b\frac{d}{dx} & -u_s\frac{d}{dx} & \frac{1}{\rho_s}\frac{d}{dx} \\ 0 & \frac{\mu}{\kappa}\frac{d}{dx} & -\frac{1}{\kappa} \end{bmatrix}.$$

- The control operator $\mathcal{B} \in \mathcal{L}((L^2(0,2\pi))^3;(L^2(0,2\pi))^3)$ is defined by

$$\mathcal{B}f = (1_{\mathcal{O}_1}f_1, 1_{\mathcal{O}_2}f_2, 1_{\mathcal{O}_3}f_3)^\top, \qquad f = (f_1,f_2,f_3)^\top \in (L^2(0,2\pi))^3.$$

Well-posedness

▷ With the above introduced notations, the system (2) can be rewritten as

$$\dot{z}(t) = \mathcal{A}z(t) + \mathcal{B}f(t), \quad t \in (0,T), \qquad z(0) = z_0, \tag{3}$$

• where $z(t) = (\rho(t,\cdot), u(t,\cdot), S(t,\cdot))^\top, z_0 = (\rho_0, u_0, S_0)^\top$, and $f(t) = (f_1(t,\cdot), f_2(t,\cdot), f_3(t,\cdot))^\top$.

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- where $z(t) = (\rho(t,\cdot), u(t,\cdot), S(t,\cdot))^\top$, $z_0 = (\rho_0, u_0, S_0)^\top$, and $f(t) = (f_1(t,\cdot), f_2(t,\cdot), f_3(t,\cdot))^\top$.

Theorem 1

The operator $(\mathcal{A}, \mathcal{D}(\mathcal{A}; (L^2(0,2\pi))^3))$ is the ***infinitesimal generator of a strongly continuous semigroup*** *$\{\mathbb{T}_t\}_{t\geq 0}$ on $(L^2(0,2\pi))^3$. Further, for any $f \in L^2(0,T;(L^2(0,2\pi))^3)$ and for any $z_0 \in (L^2(0,2\pi))^3$, (3) admits a unique solution $(\rho,u,S) \in C([0,T];(L^2(0,2\pi))^3)$ with*

$$\|(\rho,u,S)\|_{C([0,T];(L^2(0,2\pi))^3)} \leqslant C\Big(\|z_0\|_{(L^2(0,2\pi))^3} + \|f\|_{L^2(0,T;(L^2(0,2\pi))^3)}\Big).$$

Problem statement

▷ Let $T > 0$. Then for any $(\rho_0, u_0, S_0)^\top, (\rho_1, u_1, S_1)^\top \in (L^2(0, 2\pi))^3$, can we find controls $f_i \in L^2\left(0, T; L^2(\mathcal{O}_i)\right)$, $i = 1, 2, 3$, such that the corresponding solution $(\rho, u, S)^\top$ of (2) with initial condition $(\rho_0, u_0, S_0)^\top$, satisfy

$$(\rho, \mathbf{u}, \mathbf{S})^\top(\mathbf{T}, \mathbf{x}) = (\rho_\mathbf{1}, \mathbf{u_1}, \mathbf{S_1})^\top(\mathbf{x}), \text{ for all } x \in (0, 2\pi)?$$

Controllability results (Control acts locally)

Theorem 2

Let $f_2 = 0 = f_3$ *in* (2) *and* $\mathcal{O}_1 \subset (0, 2\pi)$. *Then there exists a* $T_0 > 0$ *such that the system* (2) *is* ***exactly controllable*** *in* $L^2(0, 2\pi) \times \dot{L}^2(0, 2\pi) \times \dot{L}^2(0, 2\pi)$ *at time* $T > T_0$, *by an interior control* $f_1 \in L^2\left(0, T; L^2(\mathcal{O}_1)\right)$ *for the density.*

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Remark 1

- The system is also ***exactly controllable*** at time $T > T_0$ by velocity or stress control.

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Remark 1

- The system is also ***exactly controllable*** at time $T > T_0$ by velocity or stress control.

- In the above theorem, the waiting time T_0 is of the form

$$T_0 = 2\pi\left(\frac{1}{|\beta_1|} + \frac{1}{|\beta_2|} + \frac{1}{|\beta_3|}\right),$$

  where $\beta_i, i = 1, 2, 3$ are the velocity of the characteristics equations associated to the linear system.

Observability inequality

▷ Consider the following adjoint system of (2):

$$\left.\begin{aligned}
&\partial_t \sigma + u_s \partial_x \sigma + \rho_s \partial_x v = 0, && \text{in } (0,T)\times(0,2\pi),\\
&\partial_t v + b\partial_x \sigma + u_s \partial_x v - \frac{1}{\rho_s}\partial_x \tilde{S} = 0, && \text{in } (0,T)\times(0,2\pi),\\
&\partial_t \tilde{S} - \frac{1}{\kappa}\tilde{S} - \frac{\mu}{\kappa}\partial_x v = 0, && \text{in } (0,T)\times(0,2\pi),\\
&\sigma(t,0) = \sigma(t,2\pi),\ v(t,0) = v(t,2\pi),\ \tilde{S}(t,0) = \tilde{S}(t,2\pi), && t \in (0,T),\\
&\sigma(T,x) = \sigma_T(x), \quad v(T,x) = v_T(x), \quad \tilde{S}(T,x) = \tilde{S}_T(x), && x \in (0,2\pi).
\end{aligned}\right\} \tag{4}$$

Proposition 3

Let $T > 0$. Then the system (2) is ***exactly controllable*** *in $(L^2(0,2\pi))^3$ at time $T > 0$ using a control f_1 in $L^2(0,T;L^2(0,2\pi))$ with support in $\mathcal{O}_1$ acting in the density equation, if and only if, there exists a positive constant $C_T > 0$ such that for any $(\sigma_T, v_T, \tilde{S}_T)^\top \in (L^2(0,2\pi))^3$, $(\sigma, v, \tilde{S})^\top$, the solution of (4), satisfies the following observability inequality:*

$$\int_0^{2\pi} |\sigma_T(x)|^2 \, \mathrm{d}x + \int_0^{2\pi} |v_T(x)|^2 \, \mathrm{d}x + \int_0^{2\pi} |\tilde{S}_T(x)|^2 \, \mathrm{d}x \leqslant C_T \int_0^T \int_{\mathcal{O}} |\sigma(t,x)|^2 \, \mathrm{d}x \, \mathrm{d}t.$$

Spectral analysis of the linearized operator

Proposition 4

The spectrum of the linearized operator consists of 0 *and three sequences* λ_n^1, λ_n^2 *and* λ_n^3 *of eigenvalues. Furthermore:*

(a) *All the eigenvalues have negative real part.*

(b) *The eigenvalues behave asymptotically as*

$$\lambda_n^1 = -\,\omega_1 + i\beta_1 n + O\left(\frac{1}{|n|}\right),$$

$$\lambda_n^2 = -\,\omega_2 + i\beta_2 n + O\left(\frac{1}{|n|}\right),$$

$$\lambda_n^3 = -\,\omega_3 + i\beta_3 n + O\left(\frac{1}{|n|}\right).$$

- $\beta_j, j = 1, 2, 3$ *are the distinct real roots of the equation*

$$r^3 + 2u_s r^2 + \left(u_s^2 - b\rho_s - \frac{\mu}{\kappa\rho_s}\right) r - \frac{\mu u_s}{\kappa\rho_s} = 0,$$

and $\omega_j = \frac{\beta_j^2 + 2u_s\beta_j + u_s^2 - b\rho_s}{\kappa\left(3\beta_j^2 + 4u_s\beta_j + u_s^2 - b\rho_s - \mu/\kappa\rho_s\right)} \neq \omega_i$ *for* $i \neq j$.

(c) *Multiple eigenvalues can occur only for finitely many* n.

Spectrum of the linearized operator

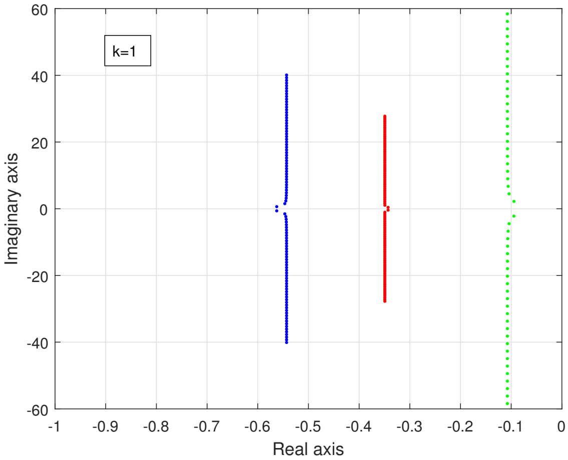

Figure: Eigenvalues of $\mathcal{A}$ in the complex plane for $|n|$ varies from 1 to 30 when $\mu = \rho_s = u_s = b = 1$ and k=1.

Ingham inequality

Proposition 5
Let $T > 2\pi\left(\frac{1}{|\beta_1|}+\frac{1}{|\beta_2|}+\frac{1}{|\beta_3|}\right)$. *Then there exist positive constants* C *and* C_1 *depending on* T *such that for* $g(t) = \sum_{n\in\mathbb{Z}^*}\sum_{l=1}^{3} a_n^l e^{\overline{\lambda_n^l}(T-t)}$ *with* $\sum_{n\in\mathbb{Z}^*}\sum_{l=1}^{3} |a_n^l|^2 < \infty$, *the following inequality holds:*

$$C\sum_{n\in\mathbb{Z}^*}\sum_{l=1}^{3}|a_n^l|^2 \le \int_0^T |g(t)|^2\,\mathrm{d}t \le C_1\sum_{n\in\mathbb{Z}^*}\sum_{l=1}^{3}|a_n^l|^2.$$

▷ The proof of this inequality relies on the construction of a family biorthogonal to the family of exponentials $\{e^{-\lambda_n^l t}, n\in\mathbb{Z}^*, l=1,2,3\}$.

Methodology of the proof

▷ ***Exact controllability*** of the linear system is ***equivalent*** to a certain ***observability inequality*** satisfied by the solution of the corresponding adjoint problem.

▷ We proved the observability inequality using the spectral analysis of the linearized operator.

- The spectrum of the linear operator consists of three sequences of complex eigenvalues whose ***real parts converge*** to three distinct finite numbers, and the ***imaginary parts behave as*** n for $|n| \to \infty$.
- The eigenfunctions of the linearized operator and its adjoint form ***Riesz bases***.
- Using the series representation of the solution of the adjoint problem and a ***hyperbolic type Ingham inequality***, we proved the ***observability inequality***.

Controllability results (Control acts everywhere)

Theorem 6

Let $f_2 = 0 = f_3$ *in* (2) *and* $\mathcal{O}_1 = (0, 2\pi)$*. Then for any* $T > 0$ *the system* (2) *is* ***exactly controllable*** *in* $L^2(0, 2\pi) \times \dot{L}^2(0, 2\pi) \times \dot{L}^2(0, 2\pi)$ *at time* $T > 0$*, by a control* $f_1 \in L^2\left(0, T; L^2(0, 2\pi)\right)$ *acting everywhere in the density.*

Controllability results (Control acts everywhere)

Theorem 6

Let $f_2 = 0 = f_3$ in (2) and $\mathcal{O}_1 = (0, 2\pi)$. Then for any $T > 0$ the system (2) is ***exactly controllable*** *in $L^2(0,2\pi) \times \dot{L}^2(0,2\pi) \times \dot{L}^2(0,2\pi)$ at time $T > 0$, by a control $f_1 \in L^2\left(0, T; L^2(0,2\pi)\right)$ acting everywhere in the density.*

Remark 2

Additionally, we achieve ***exact controllability*** of the system (2) at time $T > 0$ by means of interior control acting either velocity or stress equation applied everywhere in the domain.

Methodology of the proof

▷ We used ***direct method*** by constructing the control explicitly to prove the controllability.

- The eigenfunctions of $\mathcal{A}$, the linear operator associated to the system (2) forms a Riesz Basis.
- System (2) can be ***projected onto each finite dimensional eigenspaces*** for each $n \in \mathbb{Z}$.
- Any given time $T > 0$, each ***finite dimensional system is controllable*** using ***Hautus Test*** and construct the control using the finite-dimensional controllability operator.
- ***Summing up these finite dimensional controls***, we can construct a control for the whole system.

Conclusion

• We thoroughly study the controllability aspects of the compressible Navier-Stokes system with Maxwell's law linearized around a non-zero velocity in $(L^2(0, 2\pi))^3$ with ***periodic boundary*** condition using distributed L^2-controls.

• We give the proof of a suitable ***Ingham-type inequality*** which helps to derive the required ***observability inequality***.

• We can obtain the above results for the system with ***boundary controls***.

• Also, we have ***lack of controllability*** of the system in ***small time*** when the ***control acts locally*** in the domain or in the boundary.

Open problems

- ***Does T_0 represent the minimal time for the exact controllability of the system?***

  Determine the minimal time $T_{min} > 0$, such that the system is exactly controllable at $T \geq T_{min}$ and the system is not exactly controllable at $T < T_{min}$ is a challenging open problem.

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- ***Does T_0 represent the minimal time for the exact controllability of the system?***

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- ***What controllability results can we obtain for the system with Dirichlet boundary conditions?***

  The proof is based on explicit computation of the eigenvalues and eigenfunctions of the linear operator; hence, it is confined to specific boundary conditions (periodic in this case). Thus, it is interesting to see what controllability result we can get for the Dirichlet boundary conditions.

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"I can't change the direction of the wind, but I can adjust my sails to always reach my destination." - Jimmy Dean