Novel approaches in quantum computing

from near term quantum devices to quantum machine learning

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EM TCCM Master Course on
Advanced Computational Techniques
Barcelona, October 23rd – 27th, 2023

Foto: Erik Lucero/Google

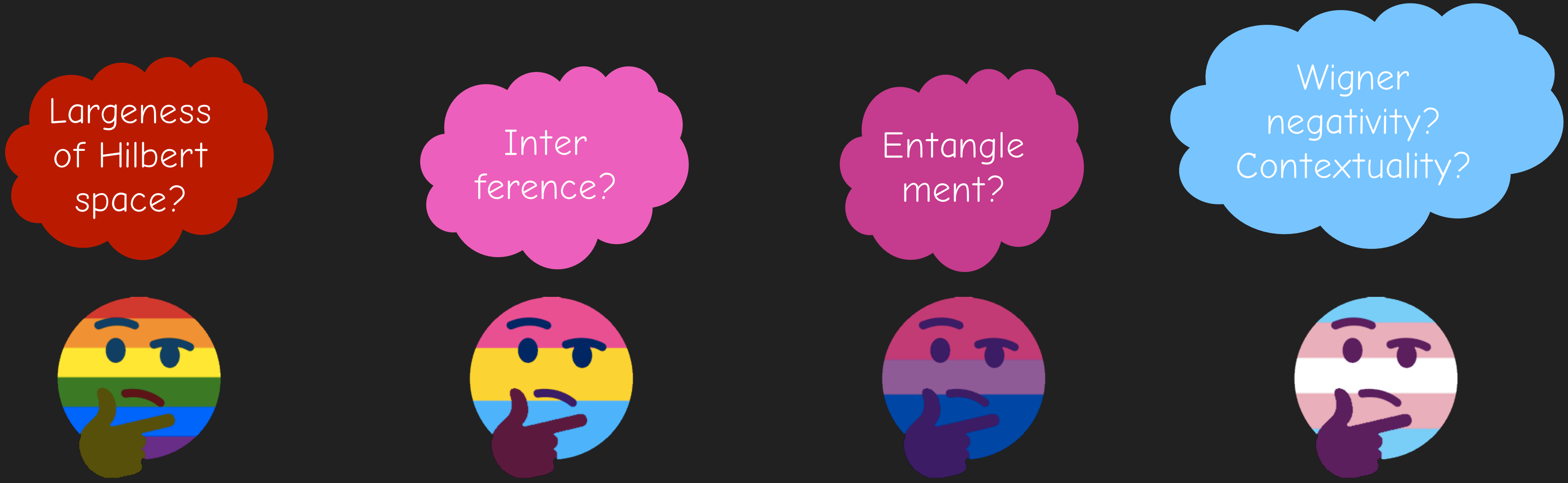
14:30 – 16:30: Novel approaches in quantum computing: from near term quantum devices to quantum machine learning

1. What Powers quantum computers: classical simulation of quantum computers, stabilizer formalism and magic states

2. Quantum advantage proofs

3. Quantum machine learning

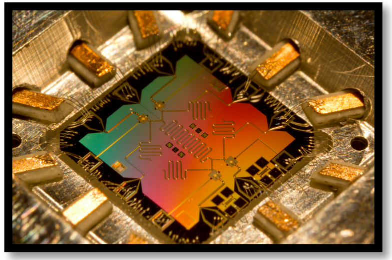
What makes quantum computing work?



How can we develop quantum applications?
When are quantum computers classically simulable?

☺ Quantum computers promise huge advantages for computation

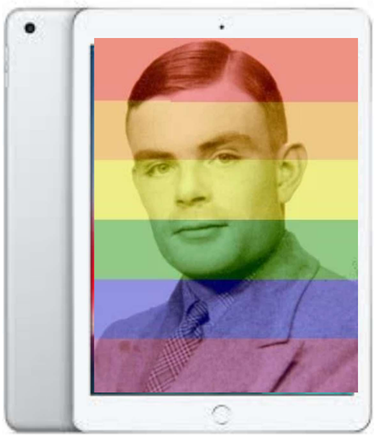
☺ 50-1000 qubit devices are being developed



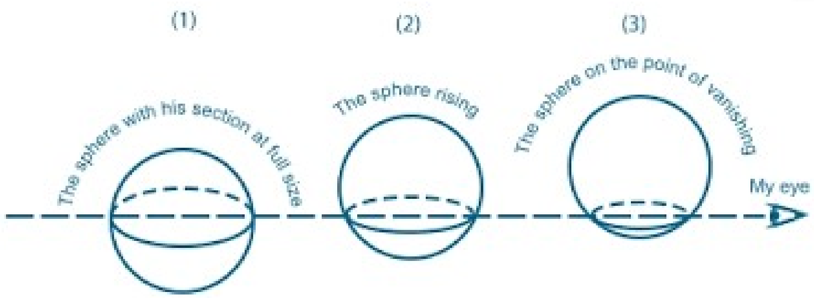
☹ Quantum applications are hard to find and extremely difficult to build

Notions of Classicality

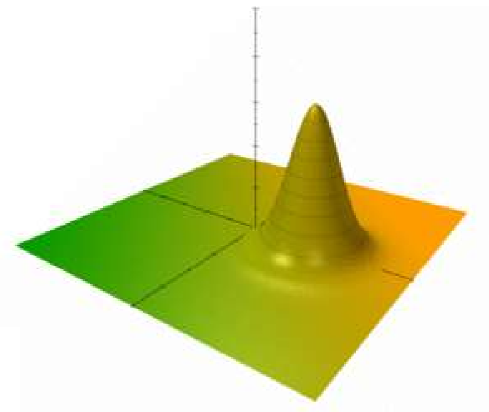
Classical Simulability



Non-contextuality

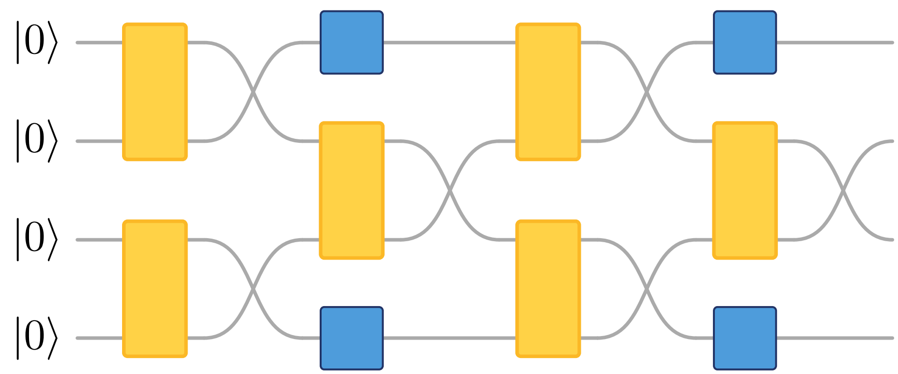


Wigner Positivity

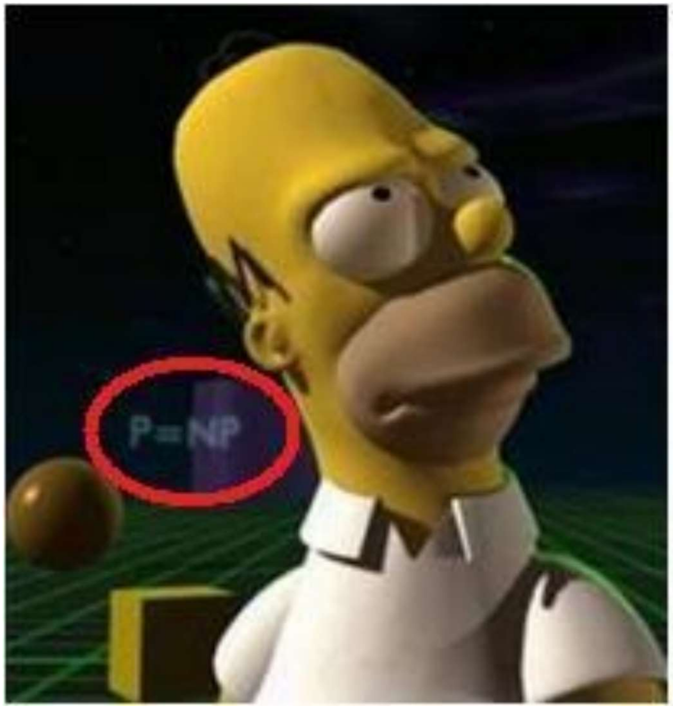


Vocabulary: circuit complexity

- (Qu)bit number: number of (quantum) bits in a circuit.
- Circuit size: number of elementary gates in a circuit
- Depth: number of layers in a circuit



Complexity theory



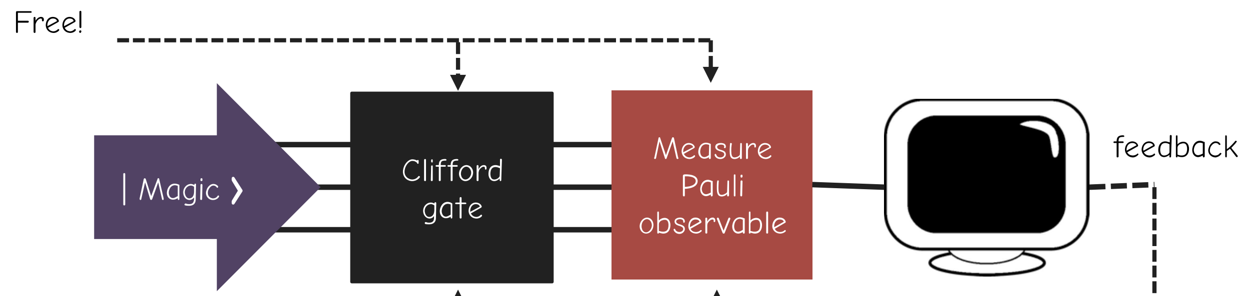
Vocabulary: complexity classes

- Efficient algorithms solve n-bit problems using circuits of O(poly(n)) gates
- P is the class of problems that can be solved classically efficiently
  - Example: nxn matrix multiplication, $O(n^3)$ classical algorithm
- NP are problems that can be verified classically efficiently
  - Example: checking integer solution of an integer polynomial equation $p(x)=0$
- Analogue quantum classes BQP and QMA

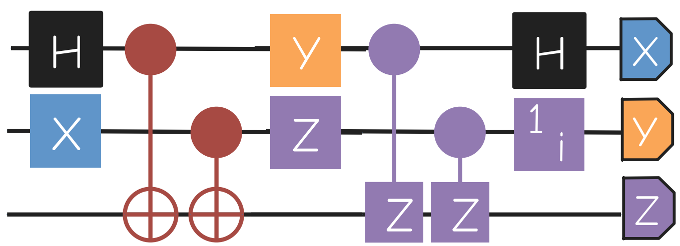
Notions of classical simulability

- Quantum tasks: quantum circuits can generate probability distributions and compute expectation values
- Weak simulation: efficiently sampling from a output distribution of an n-qubit poly(n)-size circuit (exactly or with 1/poly(n) additive error)
- Estimation of observables. efficiently computing an expectation value with in poly(n) time with 1/poly(n) additive error
- Strong simulation: efficiently computing output probabilities exactly or with huge precision (exactly, or constant relative error)

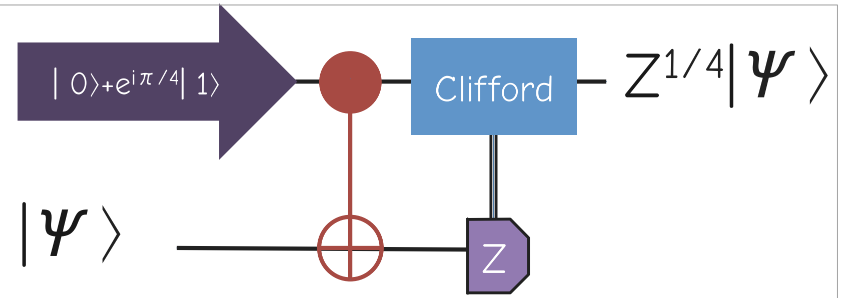
Clifords & Magic



Bravyi, Kitaev, PRA 2005



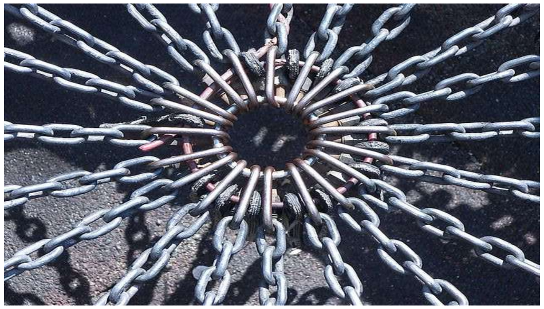
Clifford Gates



Magic States

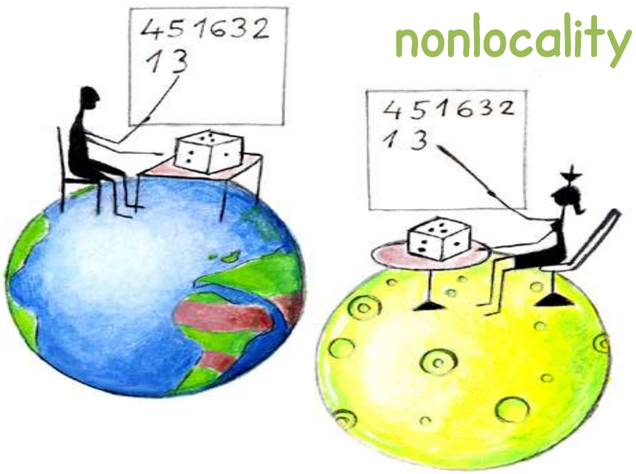
Clifford circuits exhibit quantum weirdness

entanglement

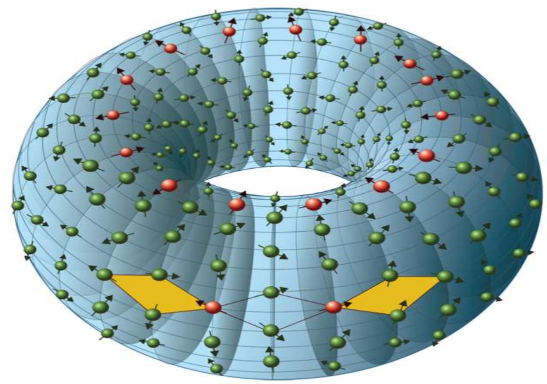


teleportation

nonlocality



ability to resist noise

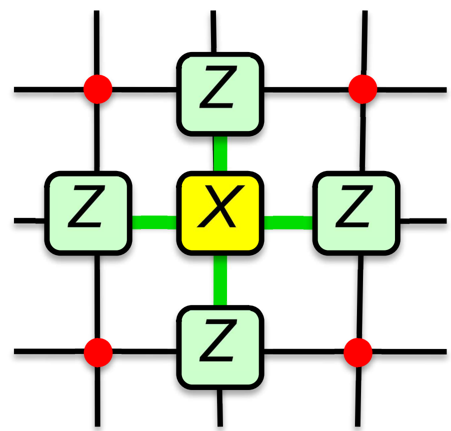


Stabilizer states

- **Finite class** of n-qubit quantum states
  - But many interesting **examples**:

    basis states, bell pairs, GHZ states, cluster states

- States defined in terms of **operators** (stabilizers)
  - Description is **efficient**
  - Spirit of MPS, PEPS Gaussian states.



Definitions

1 qubit: Pauli gates

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

n qubits: Pauli operators

$$\alpha\, P_1 \otimes P_2 \otimes \cdots \otimes P_n \quad \begin{cases} \bullet\ P_i \in \{I, X, Y, Z\} \\ \bullet\ \alpha \in \{\pm 1, \pm i\} \end{cases}$$

- $|\psi\rangle$ **is a stabilizer state** if there exists a set of Pauli operators $\{\sigma_1, \dots, \sigma_r\}$ such that
  1. $|\psi\rangle$ is an **+1-eigenstate** with of every σ_i $\rightarrow$ $\boldsymbol{\sigma_i |\psi\rangle = |\psi\rangle}$
  2. $|\psi\rangle$ is uniquely defined,
  3. *all operators* σ_i, σ_j **commute**

Product states

- $|+\rangle = \frac{|0\rangle+|1\rangle}{\sqrt{2}}$ $\rightarrow$ uniquely stabilized by $\{X\}$
- $|0\rangle^n$, $|1\rangle^n$ $\rightarrow$ $\{Z_1, Z_2, \dots, Z_n\}$, $\{-Z_1, -Z_2, \dots, -Z_n\}$

Entangled states

Bell states

- $|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ $\rightarrow$ $\{X_1X_2, Z_1Z_2\}$
- $|\Phi^{-}\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$ $\rightarrow$ $\{-X_1X_2, Z_1Z_2\}$
- $|\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$ $\rightarrow$ $\{\pm X_1X_2, -Z_1Z_2\}$

GHZ states

- $\frac{|0\rangle^{\otimes n} + |1\rangle^{\otimes n}}{\sqrt{2}}$ $\rightarrow$ all Z_iZ_{i+1}, and $X_1X_2\cdots X_n$

Product states

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- $|\Phi^\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |1$
- $|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |$
- $|\Psi^\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$

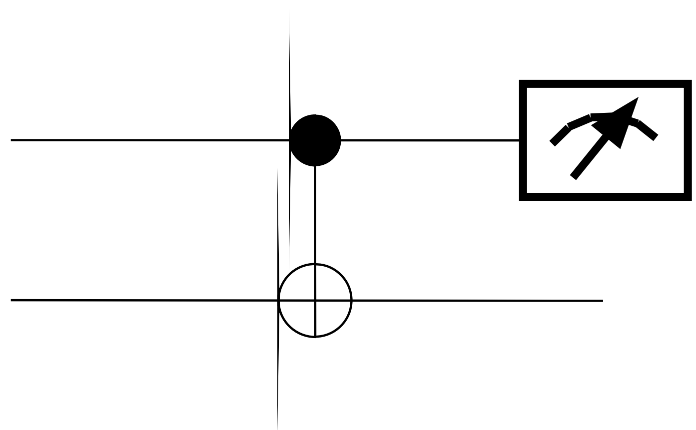
Mind that **n-qubit** states can be defined in terms of **very few** matrices! O(n)

GHZ states

- $\frac{|0\rangle^{\otimes n} + |1\rangle^{\otimes n}}{\sqrt{2}}$ → all $Z_i Z_{i+1}$, and $X_1 X_2 \cdots X_n$

Stabilizer operations

- Send **stabilizer states** to **stabilizer states**
- **Unitaries:** known as **Clifford** circuits
- **Measurements** of **Pauli** observables



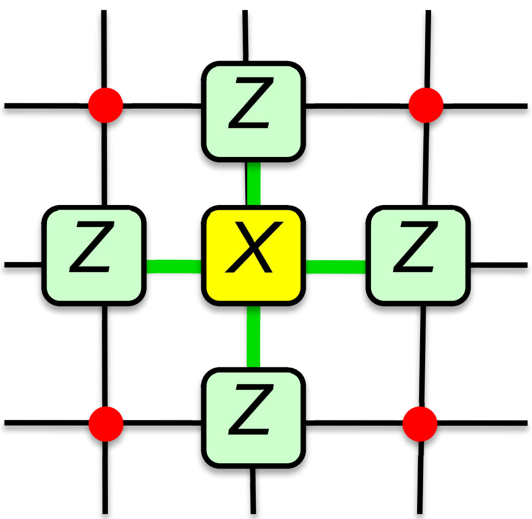
Clifford gates

- Send **Pauli operator** sto **Pauli operators**
- There are **simple rules** to compute the action.
- □ Act nicely on **stabilizer states!**

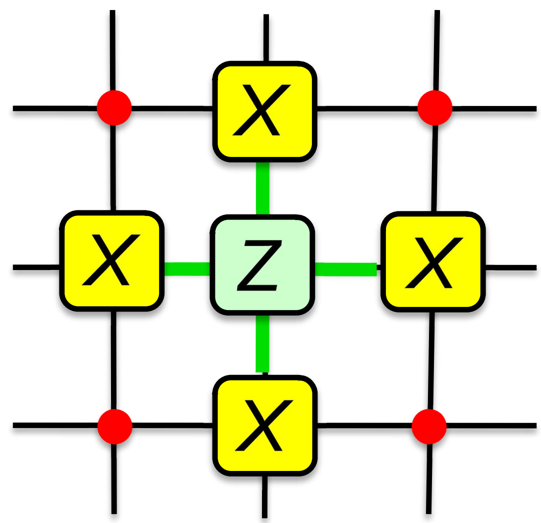
- $|\psi\rangle$ is stab state with stabilizer $\boldsymbol{\sigma}$
- $\boldsymbol{U}$ is ***Clifford***

$$\boldsymbol{U}$$

- $\boldsymbol{U}|\boldsymbol{\psi}\rangle$ is stab state with stab group $\boldsymbol{S}' = \boldsymbol{U}\boldsymbol{\sigma}\boldsymbol{U}^{\dagger}$



$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$



Clifford gates

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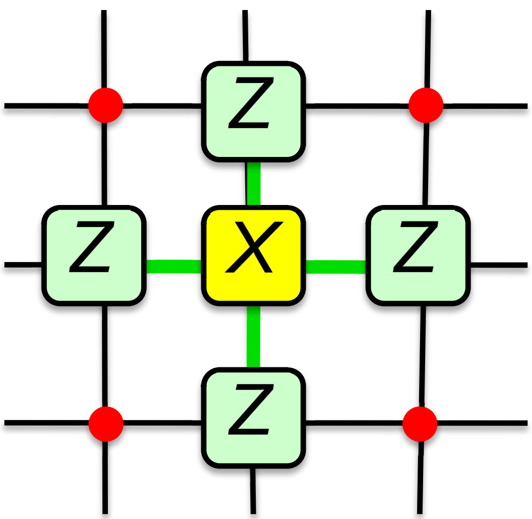

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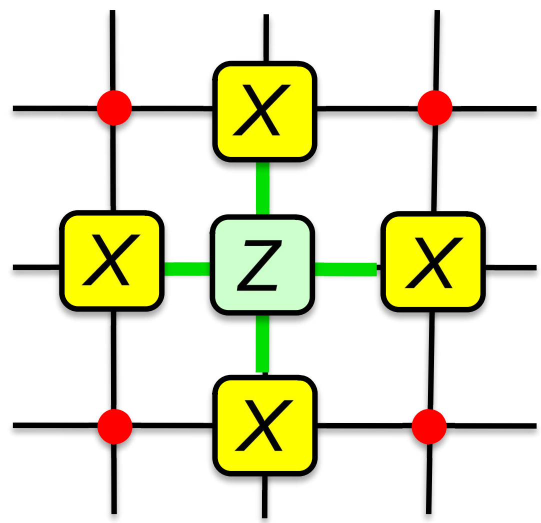


In general: Clifford circuits

products of poly(n) gates from this set

Pauli gates

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

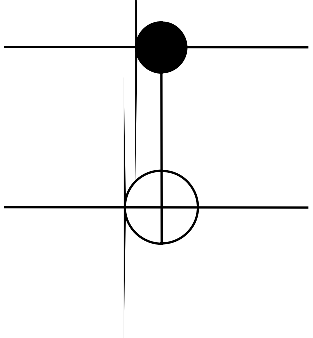
$P := \frac{\pi}{4}$ phase gate

$$P = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

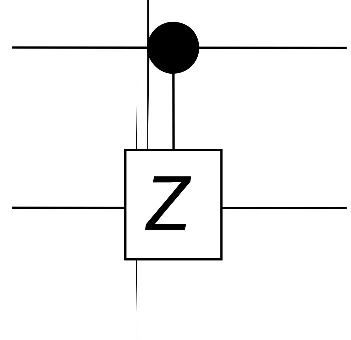
Hadamard

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

controlled-NOT



controlled-Z



Stabilizer measurements

any tensor product of **Pauli** operators is a stabilizer observable

Typical measurements

- Standard basis
- Communication (dense coding)

Observable

- $Z_1, Z_2, \ldots, Z_n$ sequentially
- Bell measurement: both XX and ZZ

- $|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ → $\{X_1X_2, Z_1Z_2\}$
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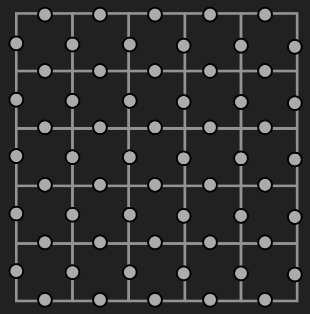
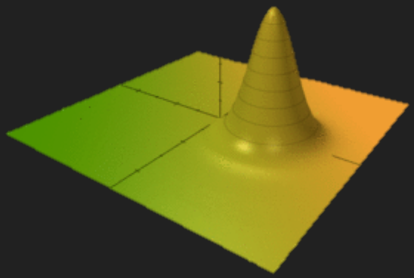
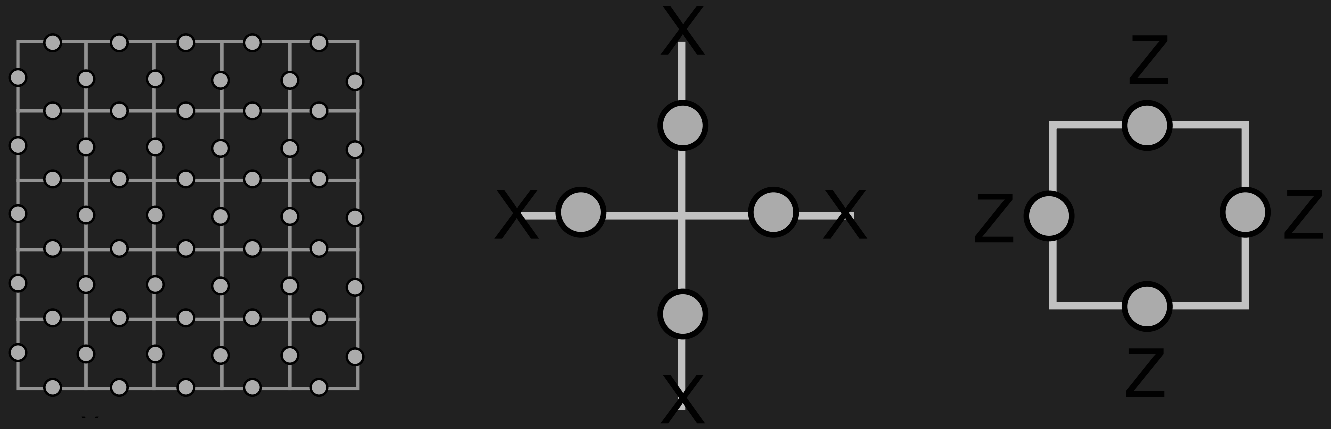
Classical simulation methods for Clifford circuits



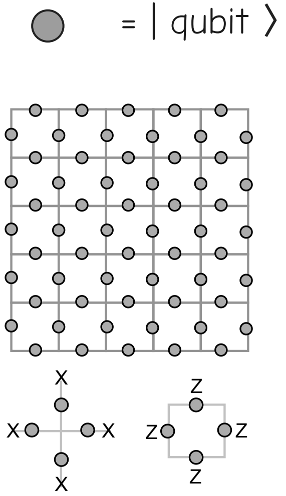


Stabilizer Formalism
(Gottesman-Knill theorem)

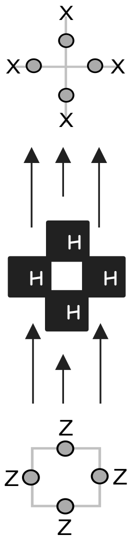


Stabilizer formalism for qubits

Stabilizer Code Formalism



Pauli Tracking

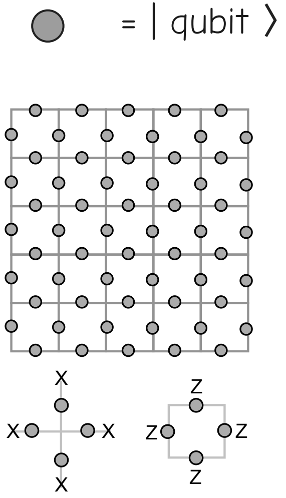


Gaussian Elimination

Gottesman, PhD Thesis 1998, Aaronson, Gottesman, PRA, 2004

Stabilizer formalism for qubits

Stabilizer Code Formalism



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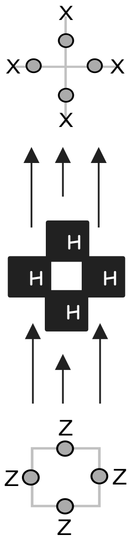
Pauli Tracking



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Gaussian Elimination

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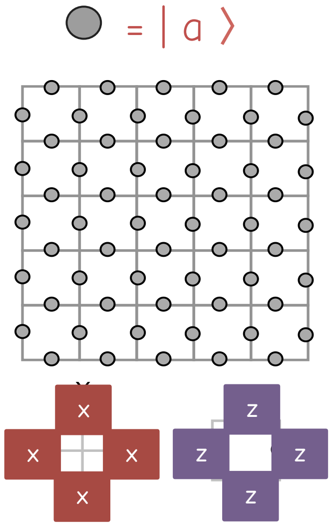
Stabilizer formalism on qudits

Qudit Paulis



- Integer arithmetic modulo d, over group $\mathbf{Z}_d$

- Qudit Cliffords preserve qudit Paulis.
Qudit Paulis are tensor products of
    - $\mathbf{X}(a)|b\rangle=|a+b\rangle$, $\mathbf{Z}(a)|b\rangle=\omega^{ab}|b\rangle$, $\omega=\exp(2\pi i/d)$

- Stabilizer formalism can efficiently classically simulate adaptive Clifford circuits with Pauli measurements (weak & strong simulation)

- Uses algorithms for abstract algebra: Gaussian elimination (odd prime d) Smith Normal forms (composite d)
    - Gottesman,*Chaos, Solitons & Fractals*, 96
    - DeBeaudrap, QIC, 2013,
    - Bermejo-Vega, Van Den Nest , QIC, 2014