Instability of matter waves in optical lattices under Floquet driving

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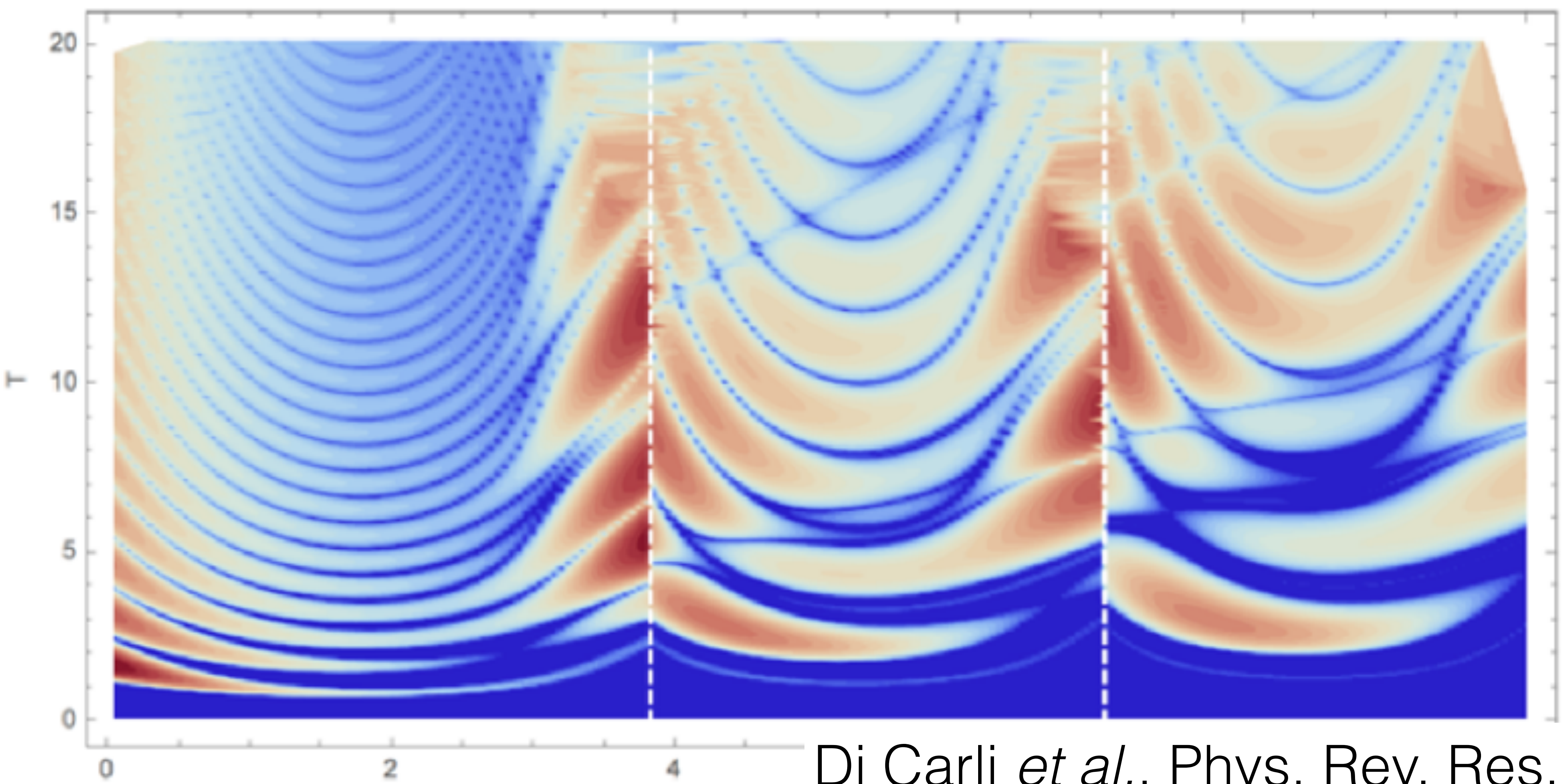


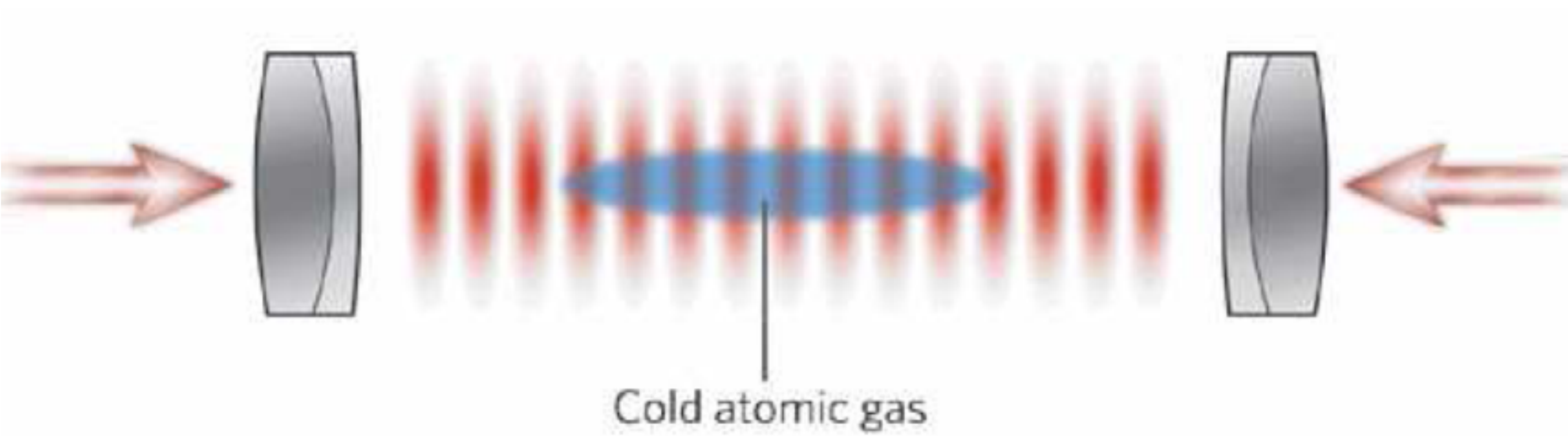
Di Carli *et al.*, Phys. Rev. Res. 5, 033024 (2023)
Cruickshank *et al.*, arXiv:2401.05265

Outline

- Introduction to “Floquet engineering”
- the good: precise control of Hamiltonians
- the bad: heating effects
- the ugly: dynamical and modulational instabilities
- Summary and conclusions

- BECs are highly controllable, and have excellent coherence properties

- can apply optical lattice potentials (“crystals of light”) to control their properties



- described well by the Bose-Hubbard Hamiltonian

$$H_{\rm latt} = -J \sum_j \left[a_j^\dagger a_{j+1} + {\rm H.c.} \right] + \frac{U}{2} \sum_j n_j \left(n_j - 1 \right)$$

> "Atomtronics is an emerging field seeking to realize atomic circuits exploiting ultra-cold atoms"
>
> \- L. Amico

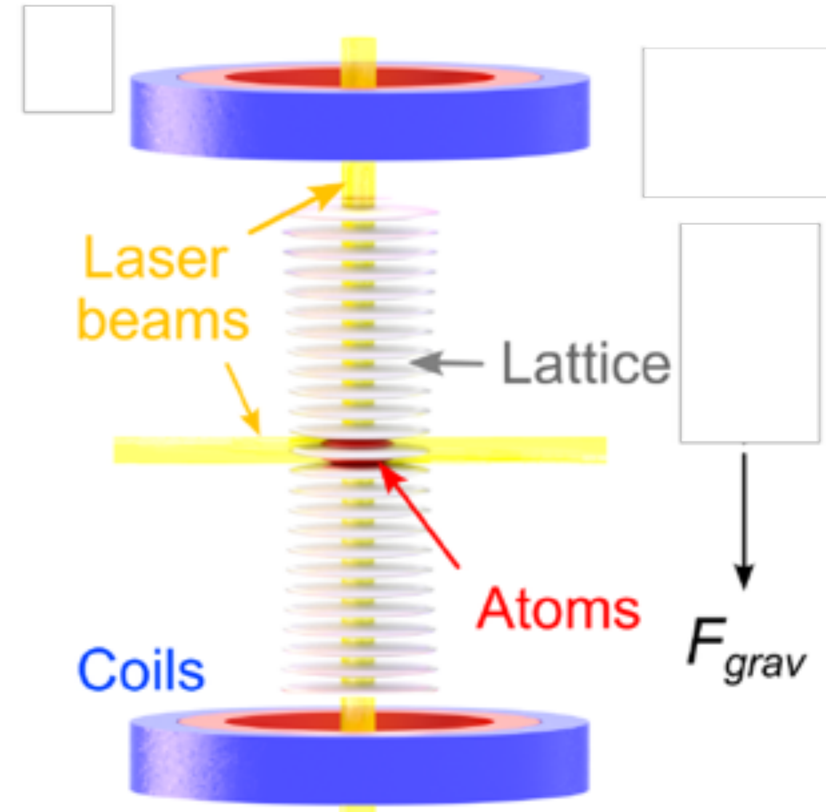
Electrons are charged, and respond to electric and magnetic fields

…but the condensed atoms are neutral

They *don't* have charge, but they *do* have mass

⇒ they can respond to *inertial* forces

- first choice: gravity
- second choice: accelerate the lattice



In the rest frame of the lattice (which is non-inertial)
⇒ the atoms see a force, $F_{in} = m\, a$

$$H = H_0 + K \sum_j x_j n_j$$

⇒ the lattice potential tilts, equivalent to a uniform *E*-field,

and varying *K* with time gives a time-dependent potential

“Floquet engineering”

Consider a general time-dependent Hamiltonian:

$$H(t) = H_0 + H_I(t)$$

- if $H_I(t)$ is periodic, $H_I(t) = H_I(t + T)$
- find solutions of $\left[i\hbar\, \partial_t - H(t)\right] |\psi_n\rangle = \epsilon_n |\psi_n\rangle$

for high frequencies, long timescale dynamics are described by an effective static Hamiltonian, $H(t) \rightarrow H_{\mathrm{eff}}$

H_{eff} obtained by series expansion in orders of $1/\omega$

Floquet-Magnus expansion:

$$H_F^{(0)} = \frac{1}{T}\int_{t_0}^{T+t_0} \mathrm{d}t\, H(t) = H_0,$$

$$H_F^{(1)}[t_0] = \frac{1}{2!Ti\hbar}\int_{t_0}^{T+t_0} \mathrm{d}t_1 \int_{t_0}^{t_1} \mathrm{d}t_2\, [H(t_1), H(t_2)],$$

and so on for higher orders

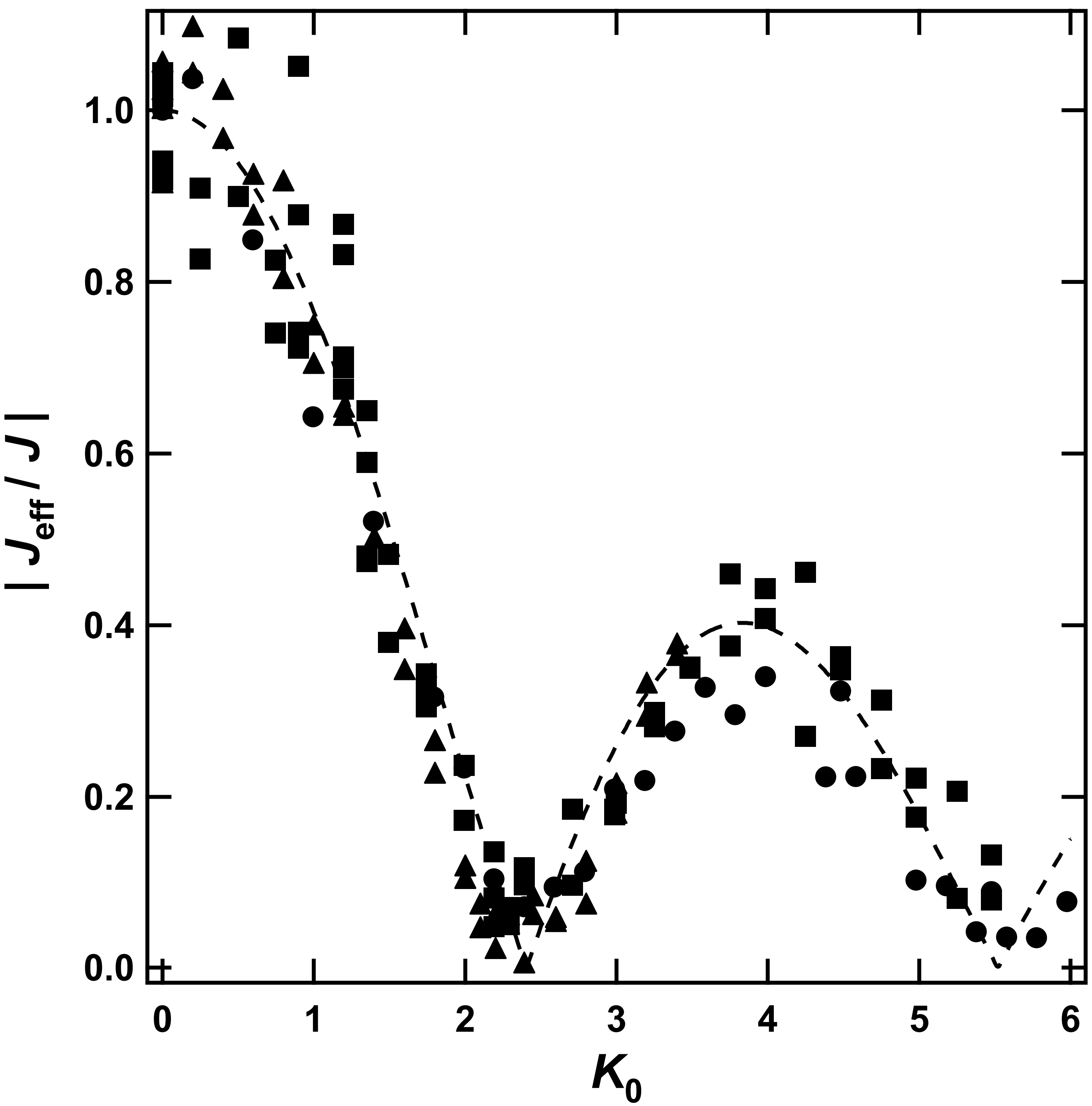
Note that the terms arise from commutators

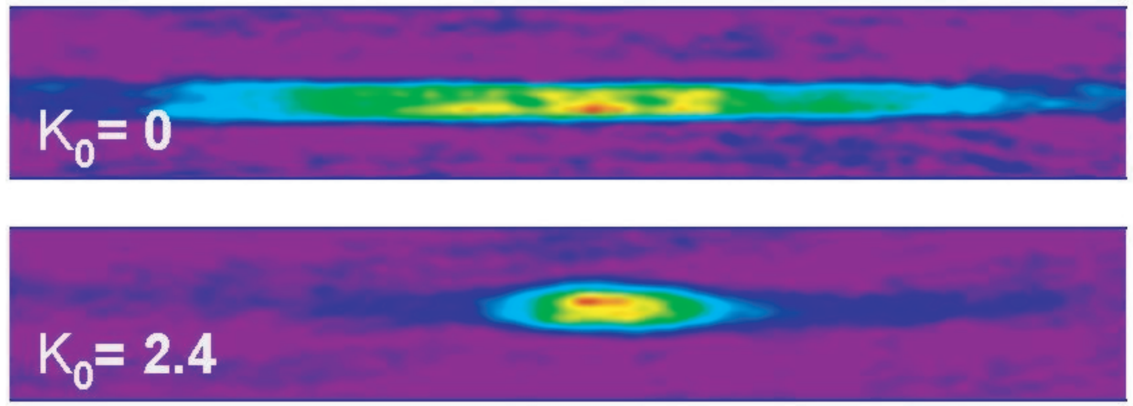
If we have a general Hamiltonian, $H = T + U + V$

any term can be driven, and will produce non-trivial dynamics if it does not commute with the others

Specific case: sinusoidal driving, $K(t) = K \cos \omega t$

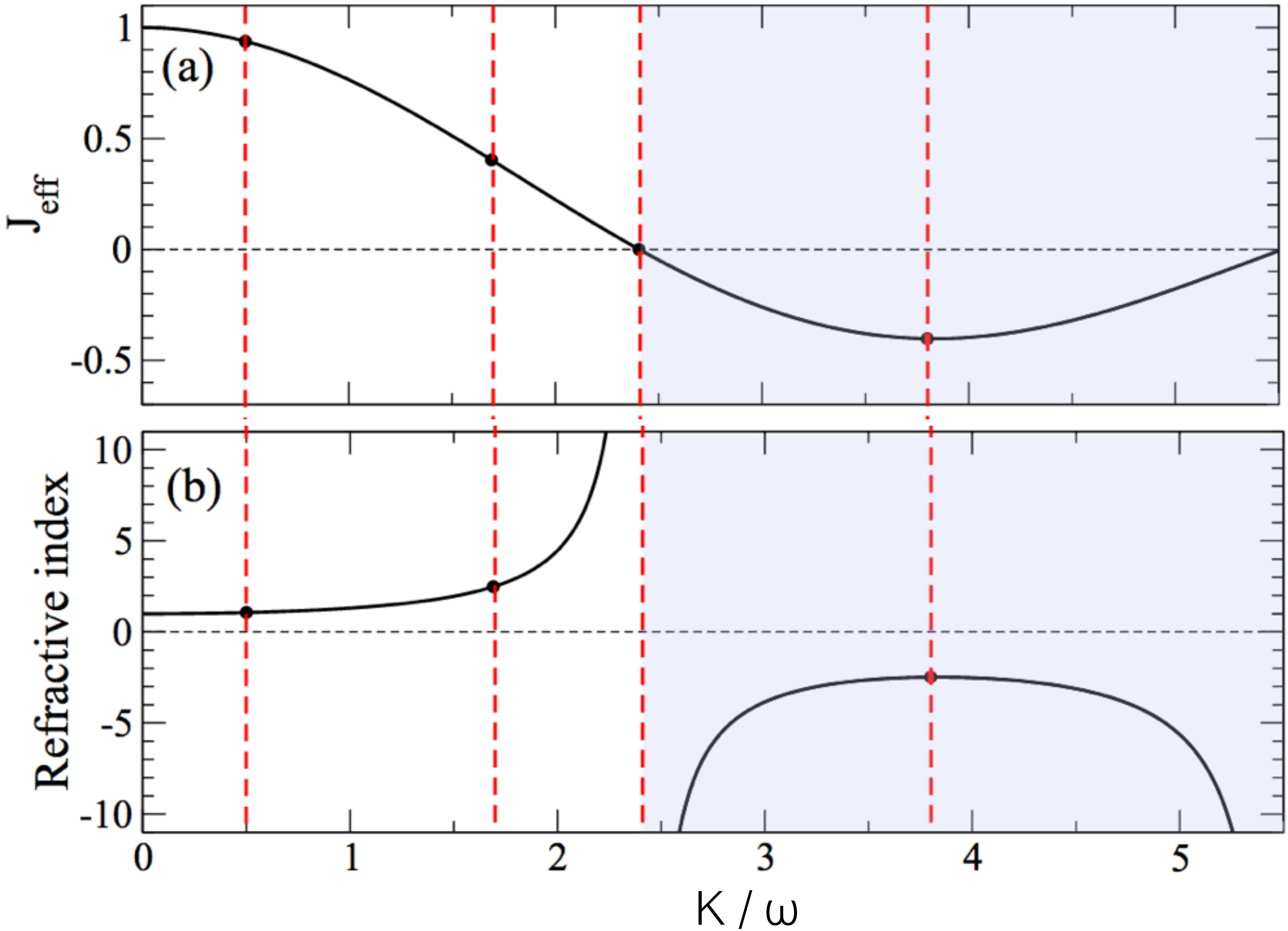
$\Rightarrow$ $\boldsymbol{J_{eff} = J\, J_0\, (K/\omega)}$, coherent control of tunneling

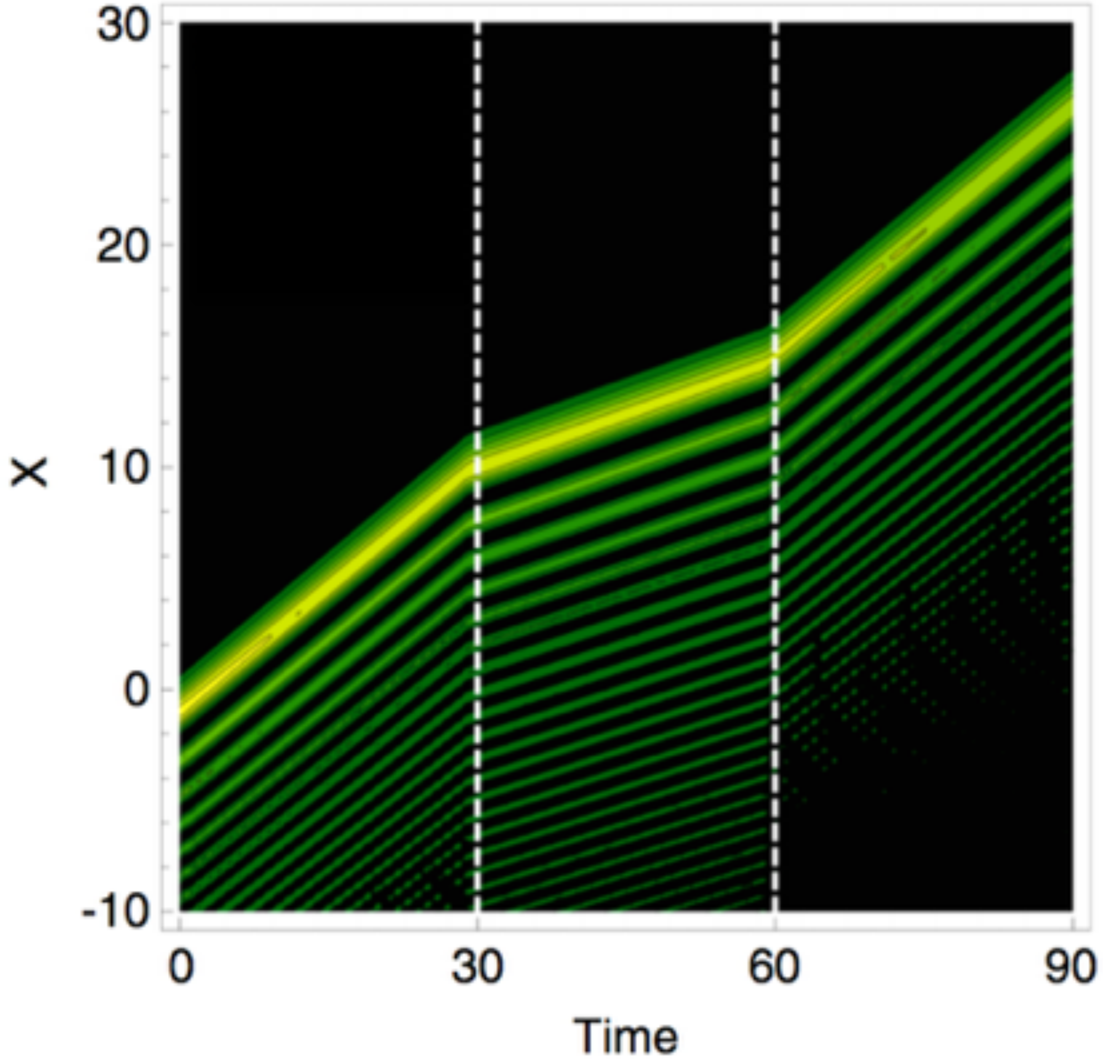




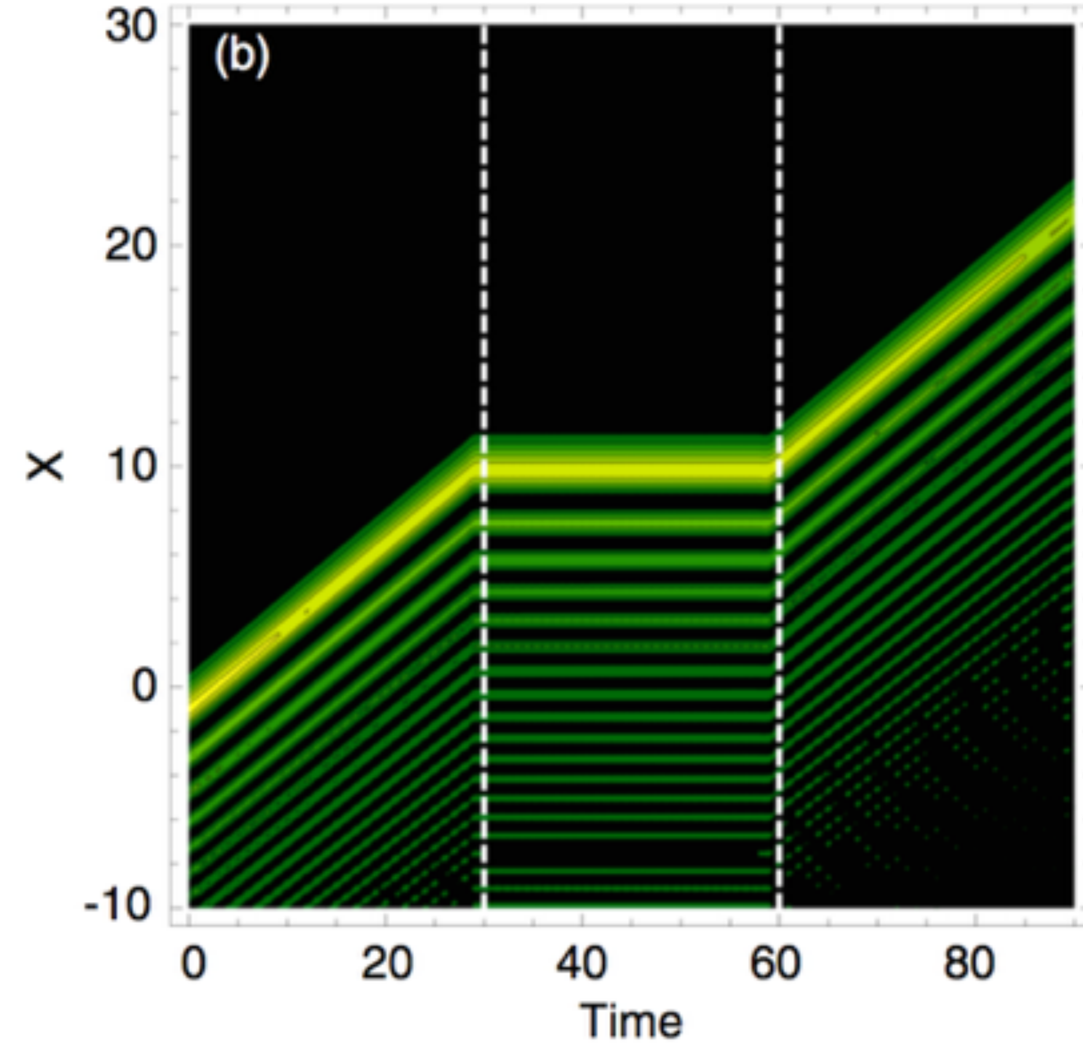
Lignier *et al*, PRL 99, 220403 (2007)

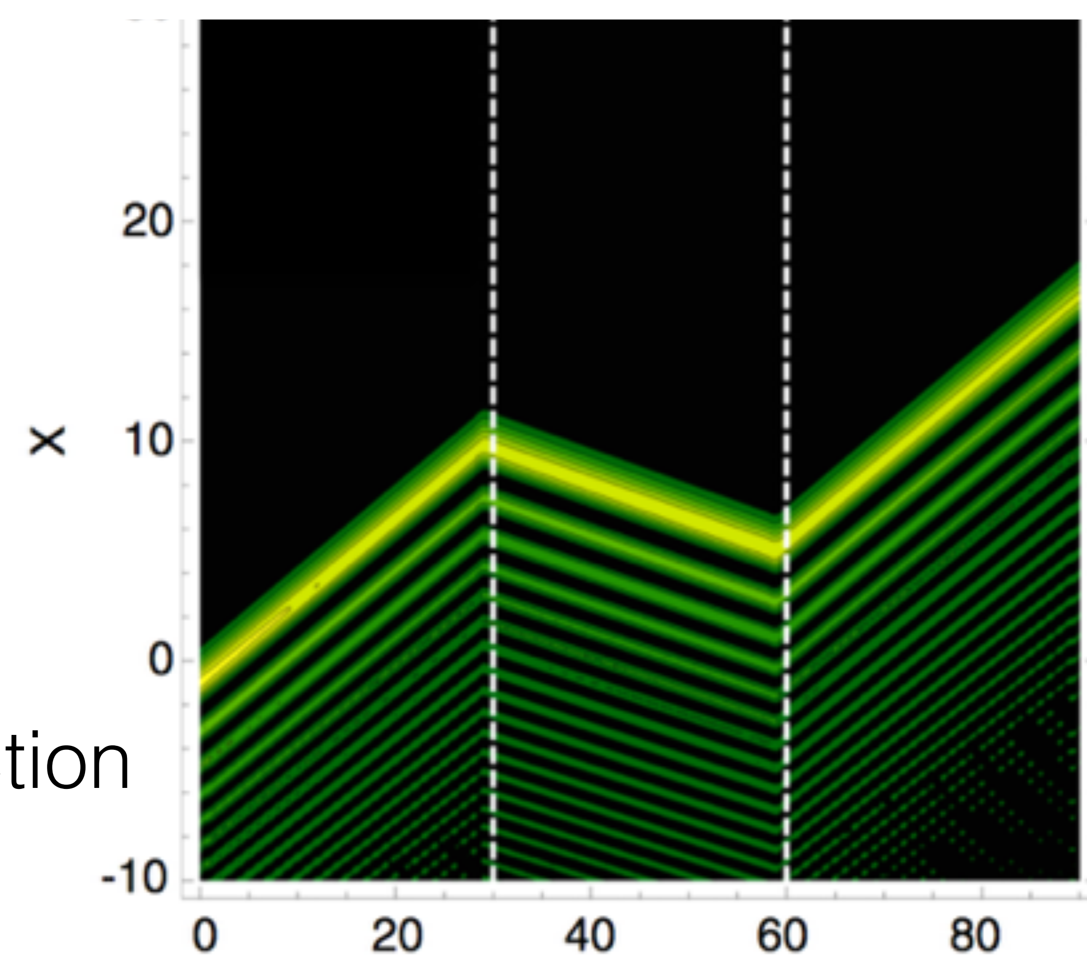
The Good


(a)
(b)
J_{eff}
Refractive index
1
0.5
0
-0.5
10
5
0
-5
-10
0
1
2
3
4
5
K / ω



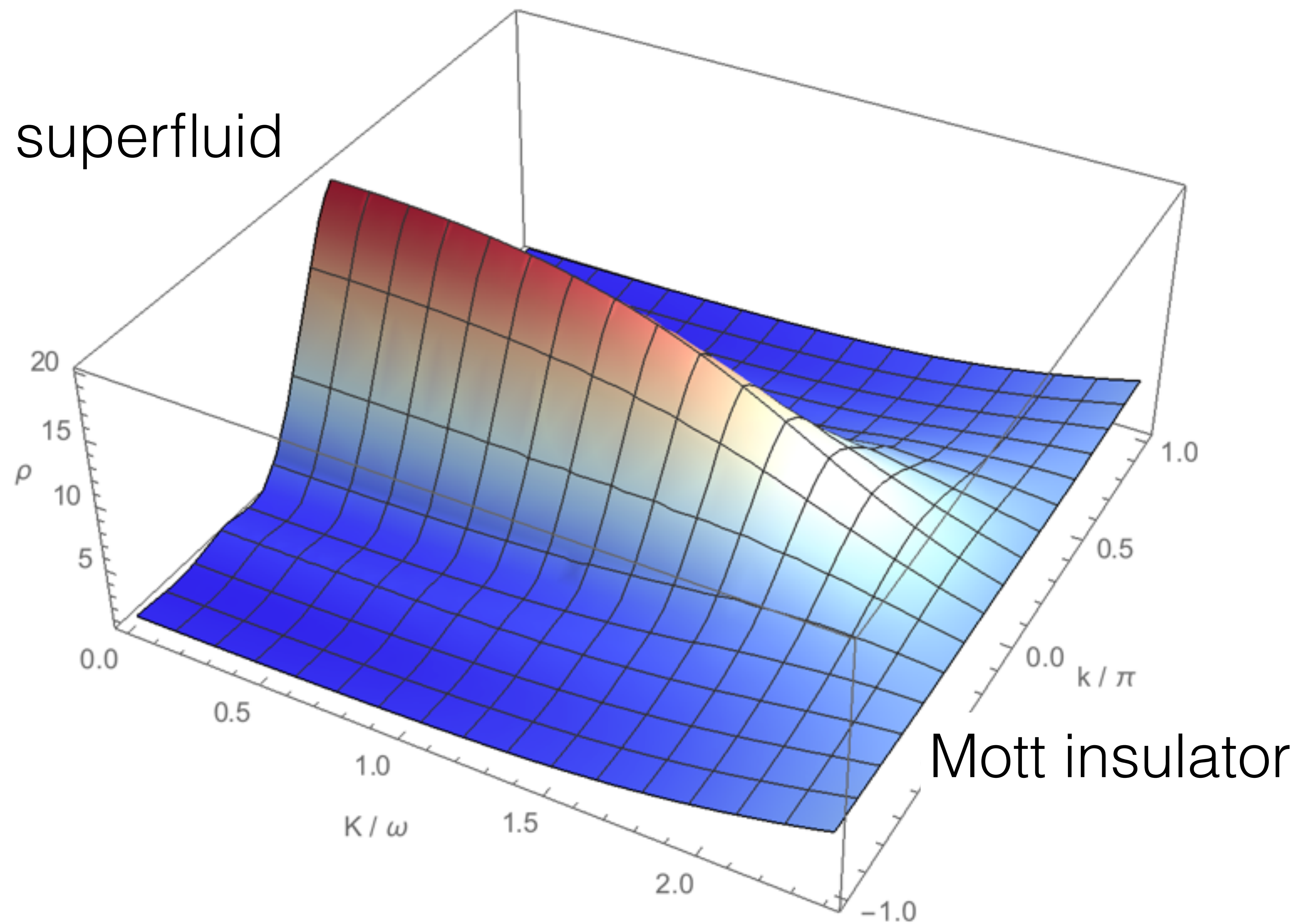
standard "refraction"



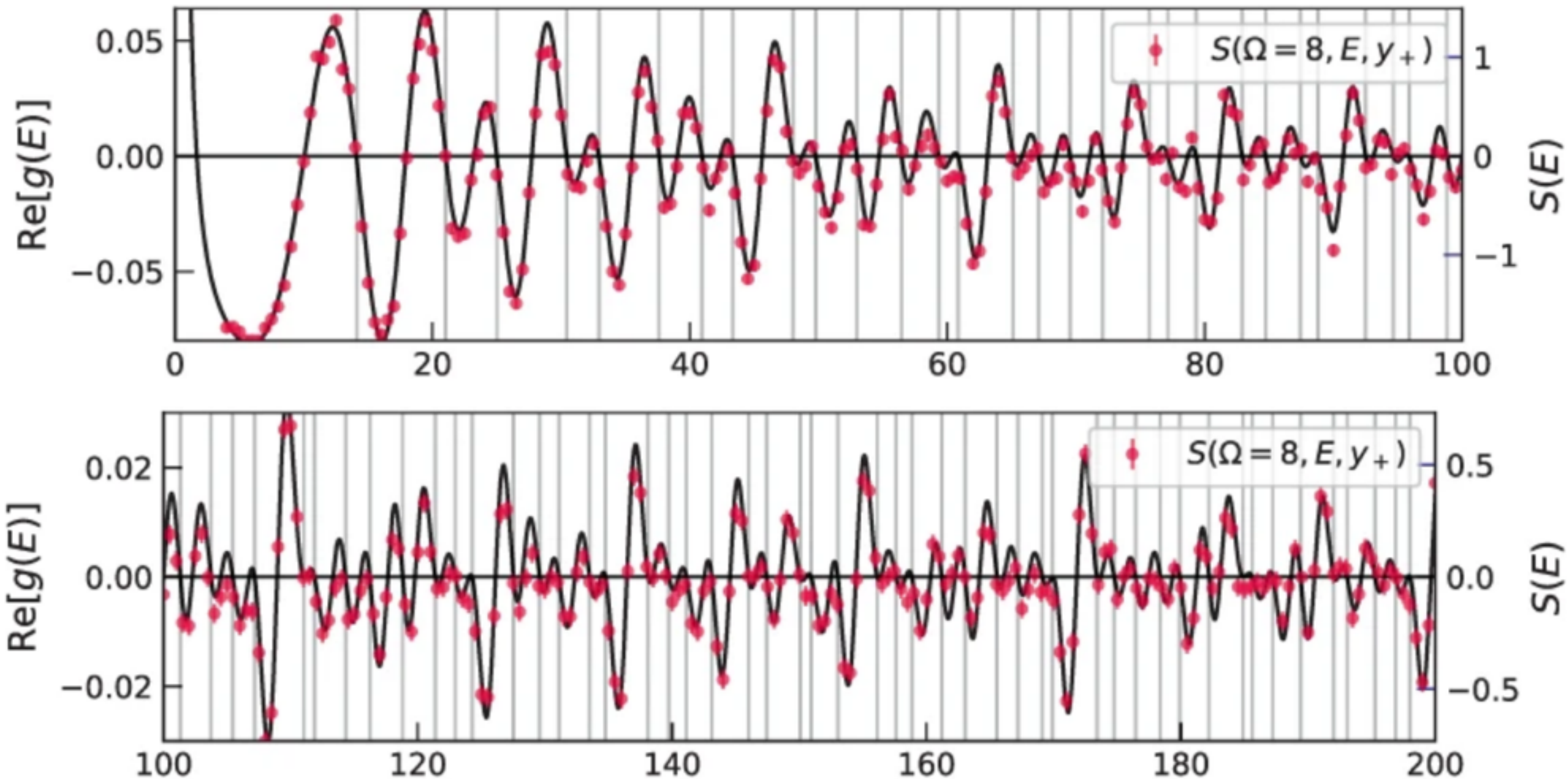


negative refraction

- manipulating the *amplitude* of J_{eff} can be used to control the Mott transition

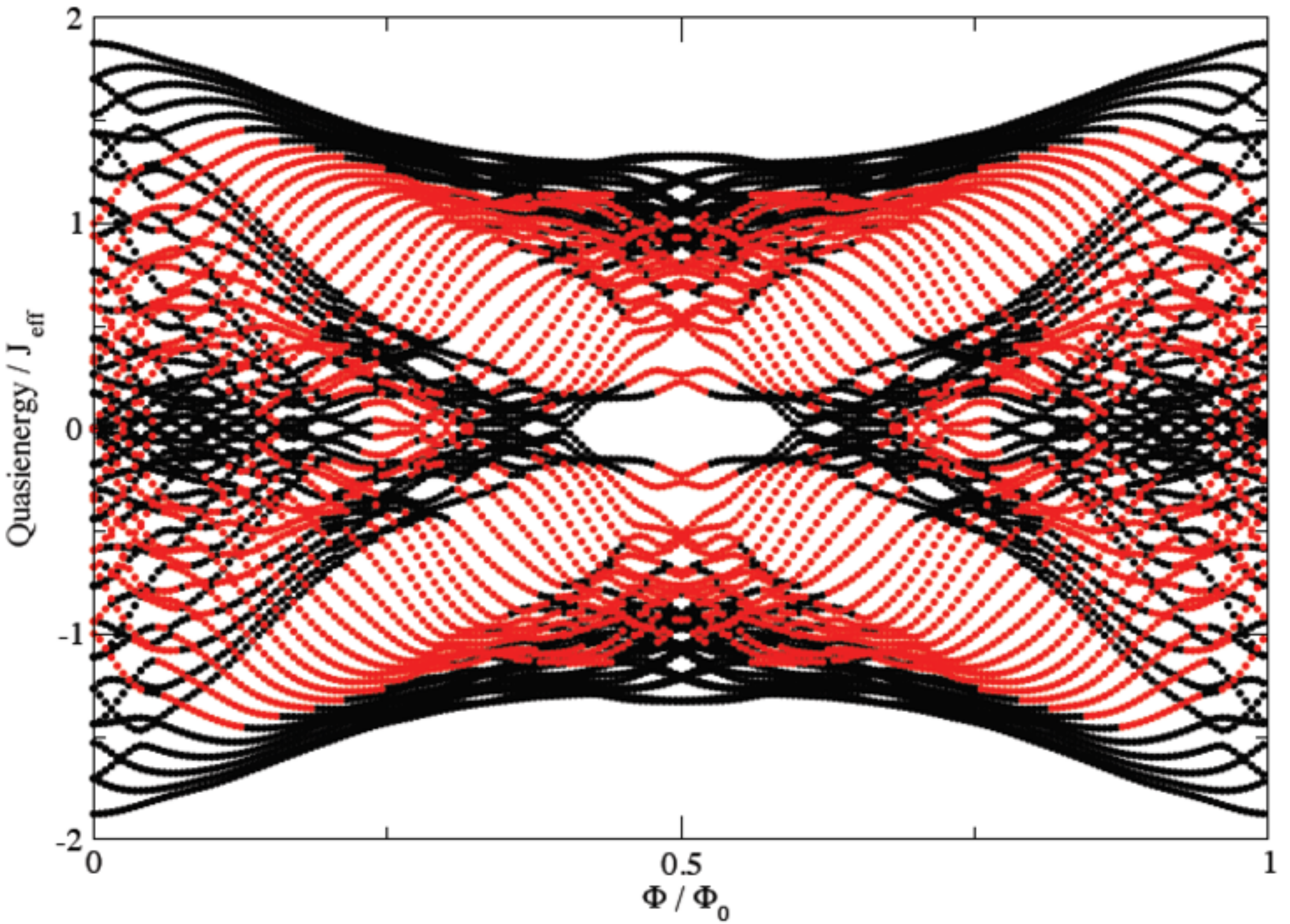


- or by setting $J_{\mathrm{eff}}(y) \propto \zeta(1/2 + iy)$ we can study the Riemann hypothesis



can obtain over 80 zeros, with accuracy of <1 %

- we can also manipulate the *phase* of J_{eff} to simulate magnetic fields



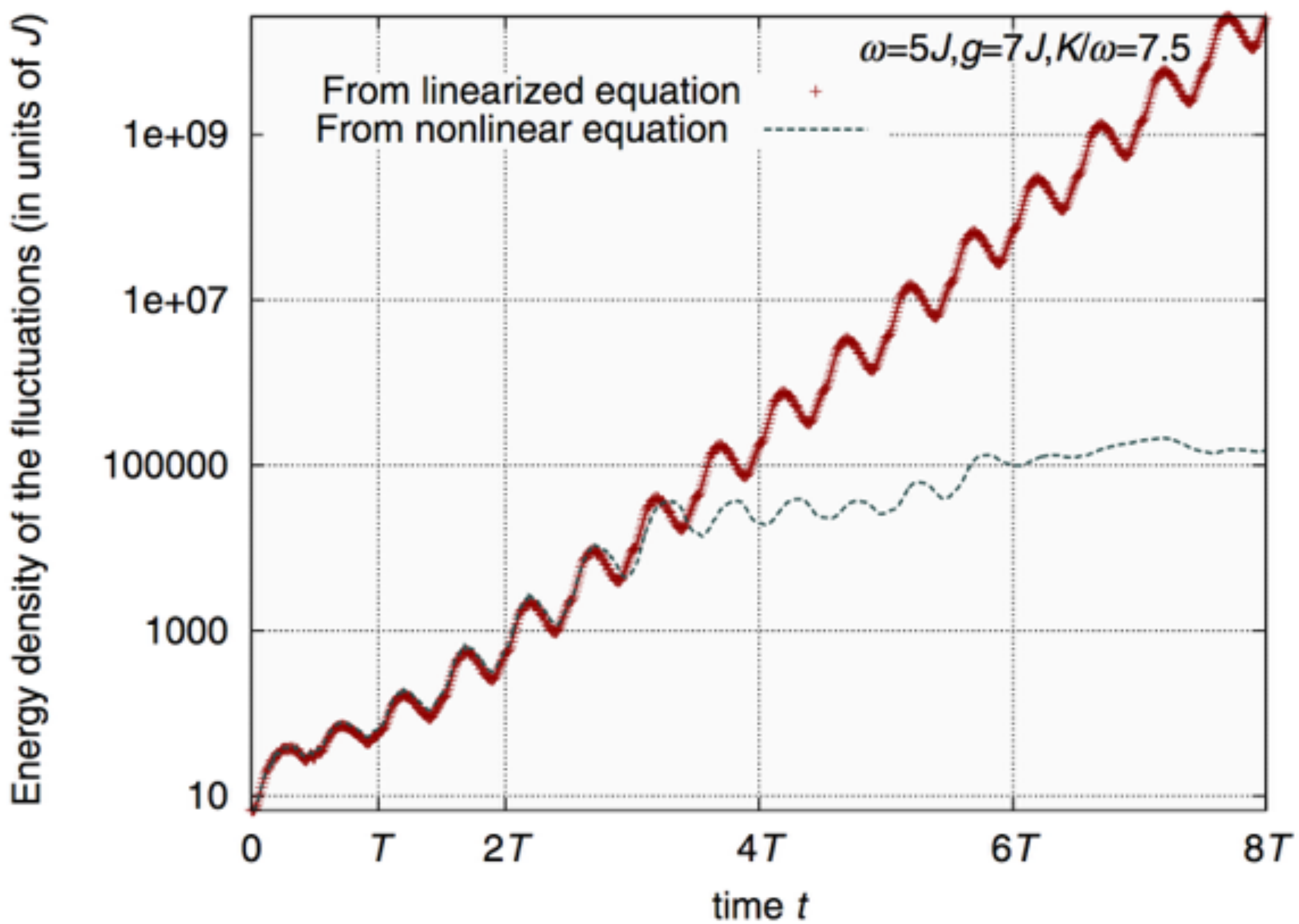
Hofstadter butterfly structure

The Bad

The problem:

Typically driving will heat the system up

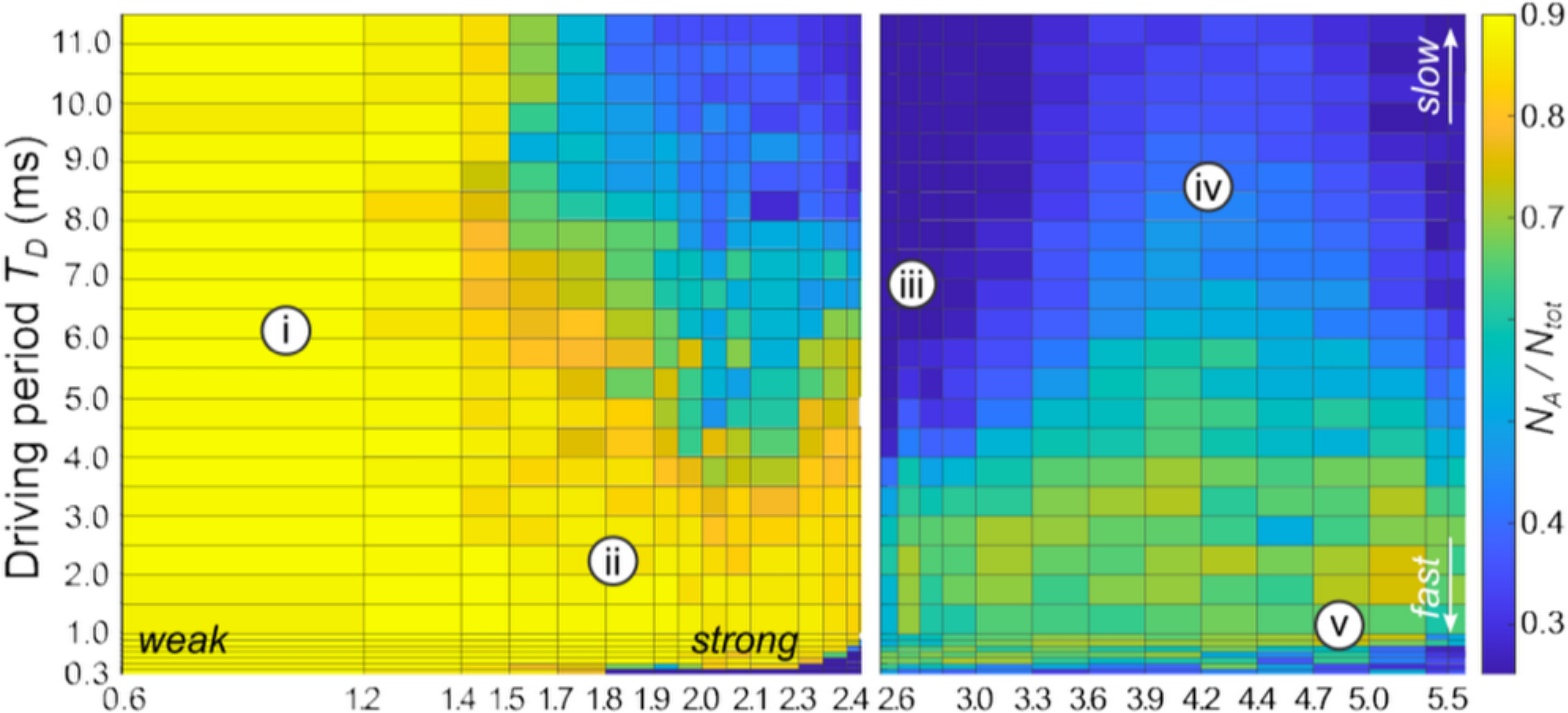
no local symmetries ⇒ thermalise to infinite temperature



Lellouch et al., PRX 7, 021015 (2016)

However, under some circumstances, the system only absorbs energy weakly

⇒ a long-lived "pre-thermal state"



Driving strength, K/ω

- what is the cause of the instabilities?
- can we find stable regions of parameter-space?
- is the high-frequency regime different from the low-frequency regime?

The “Ugly”

Procedure

- classical field approximation:
  boson operators a_n → c-numbers α_n

- produces a discretised Gross-Pitaevskii equation,
  with $g = U N$

- perturb around the Floquet states:

$$\alpha'_n(t) = \alpha_n(t) \left(1 + u(t)e^{iqn} + v^*(t)e^{-iqn}\right)$$

where q is the excitation's momentum wrt the condensate

Substituting in the equation of motion, and expanding to first order in *u* and *v* gives:

$$i\frac{d}{dt}\begin{pmatrix} u(t) \\ v(t) \end{pmatrix} = \mathcal{L}(q,t)\begin{pmatrix} u(t) \\ v(t) \end{pmatrix}$$

time-dependent Bogoliubov-de Gennes equations

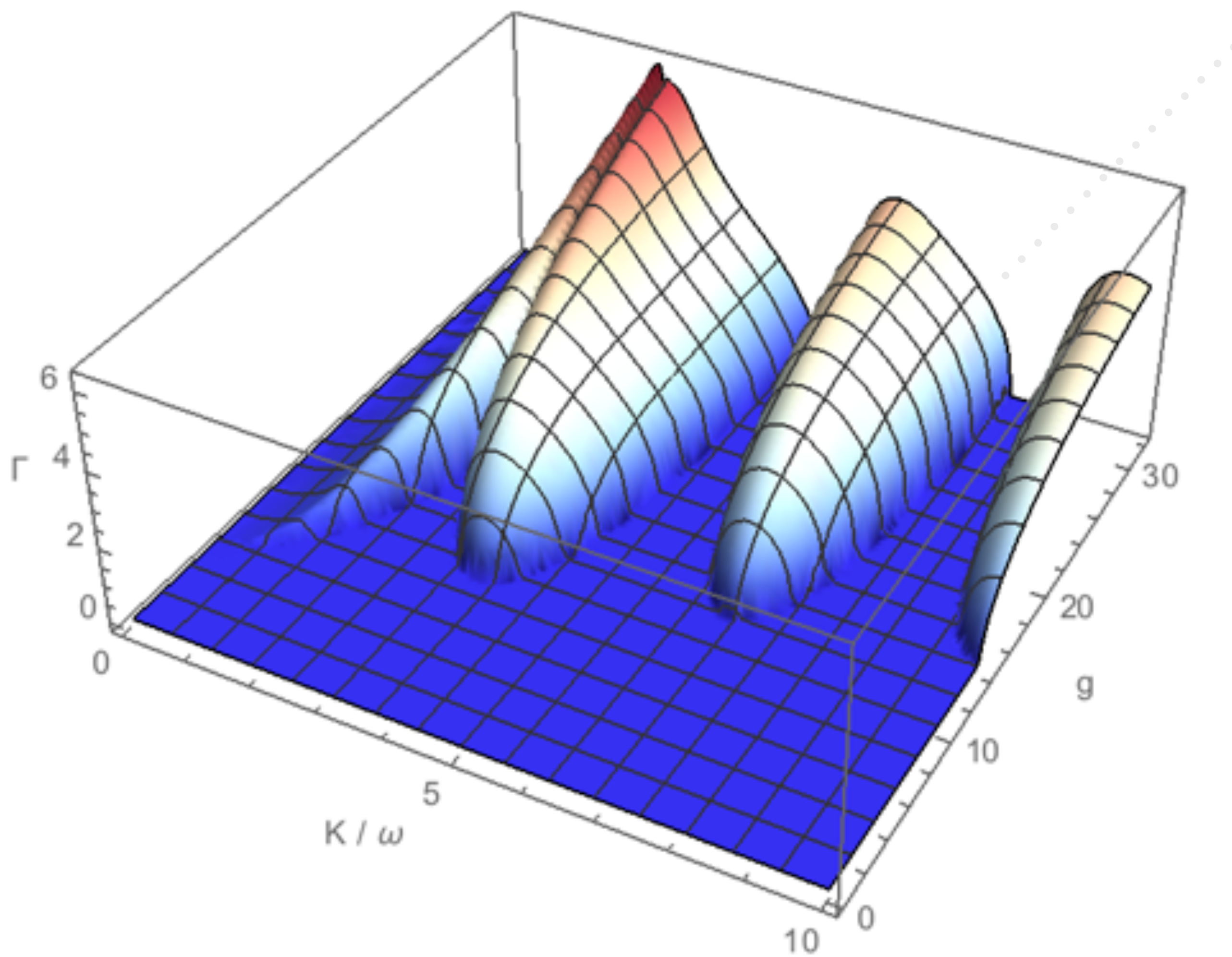
$$\begin{aligned}
\mathcal{L}_{11}(q,t) &= 4J\sin(q/2)\sin(q/2 - K/\omega\sin\omega t) + g, \\
\mathcal{L}_{12}(q,t) &= g = -\mathcal{L}_{21}(q,t), \\
\mathcal{L}_{22}(q,t) &= -4J\sin(q/2)\sin(q/2 + K/\omega\sin\omega t) - g
\end{aligned}$$

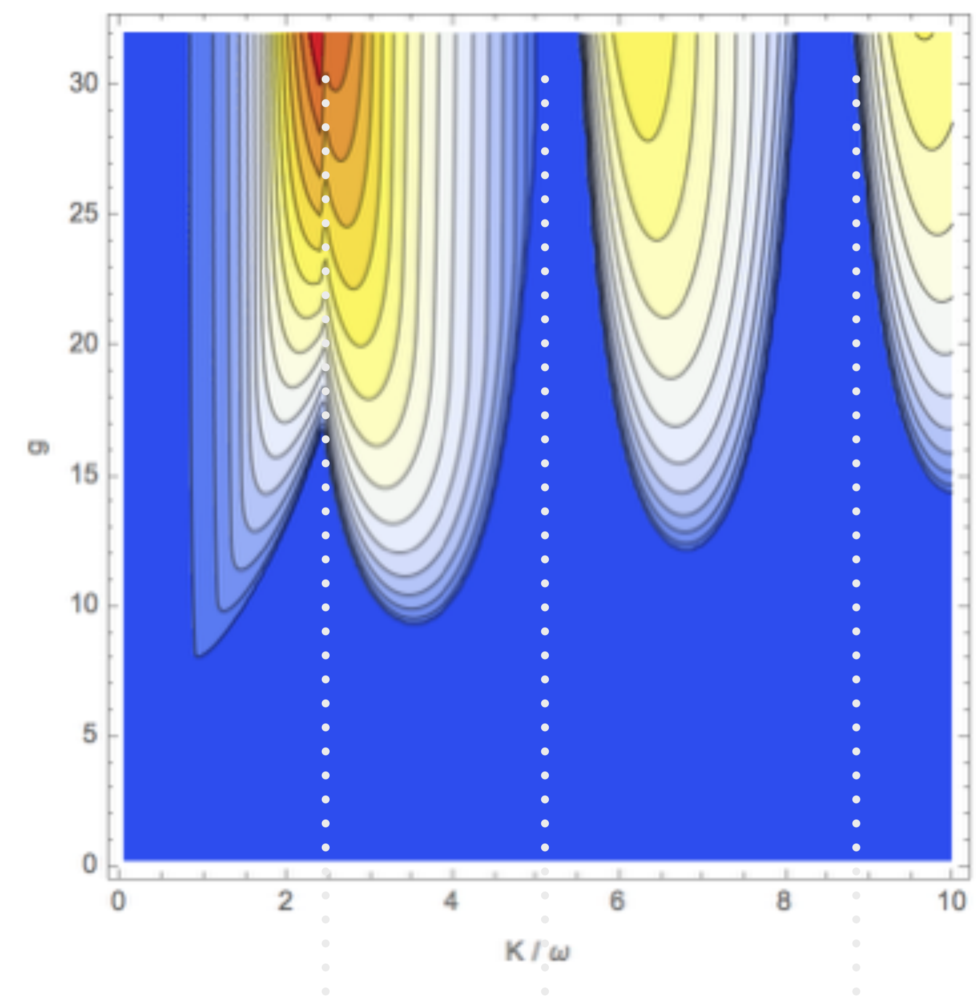
CEC, Phys. Rev. A 79, 063612 (2009)

- Note that *L(q,t)* is also *T*-periodic ⇒ Floquet theorem applies

- eigenvalues govern the growth of excitations

- imaginary component implies exponential growth

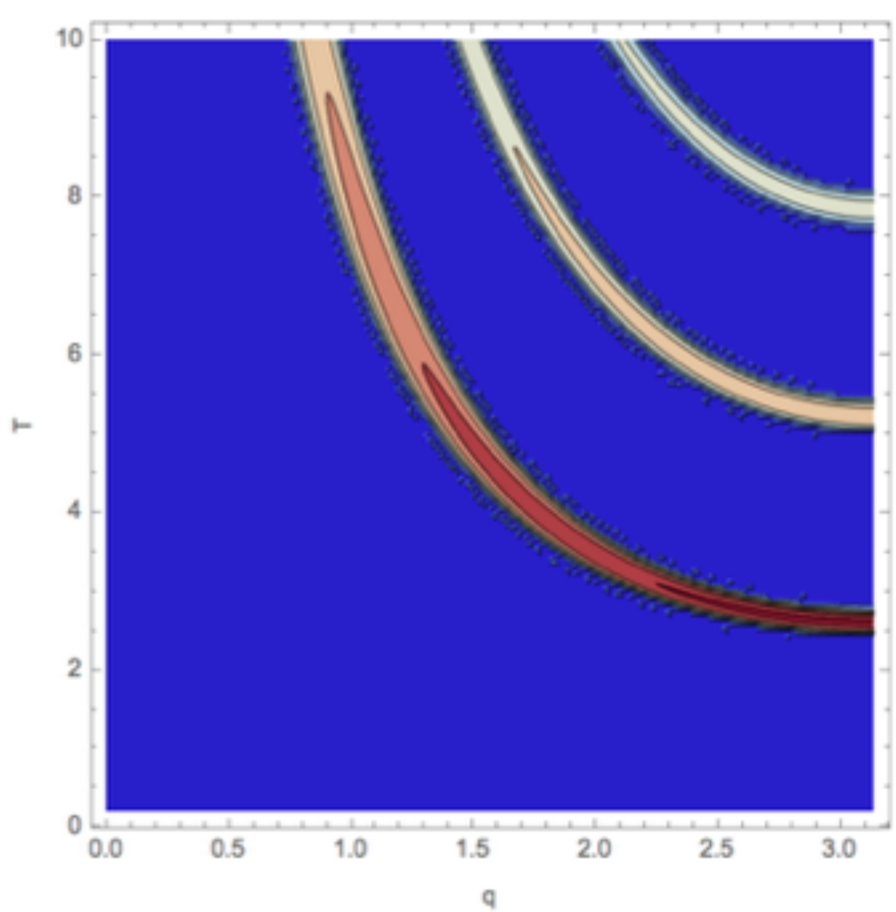


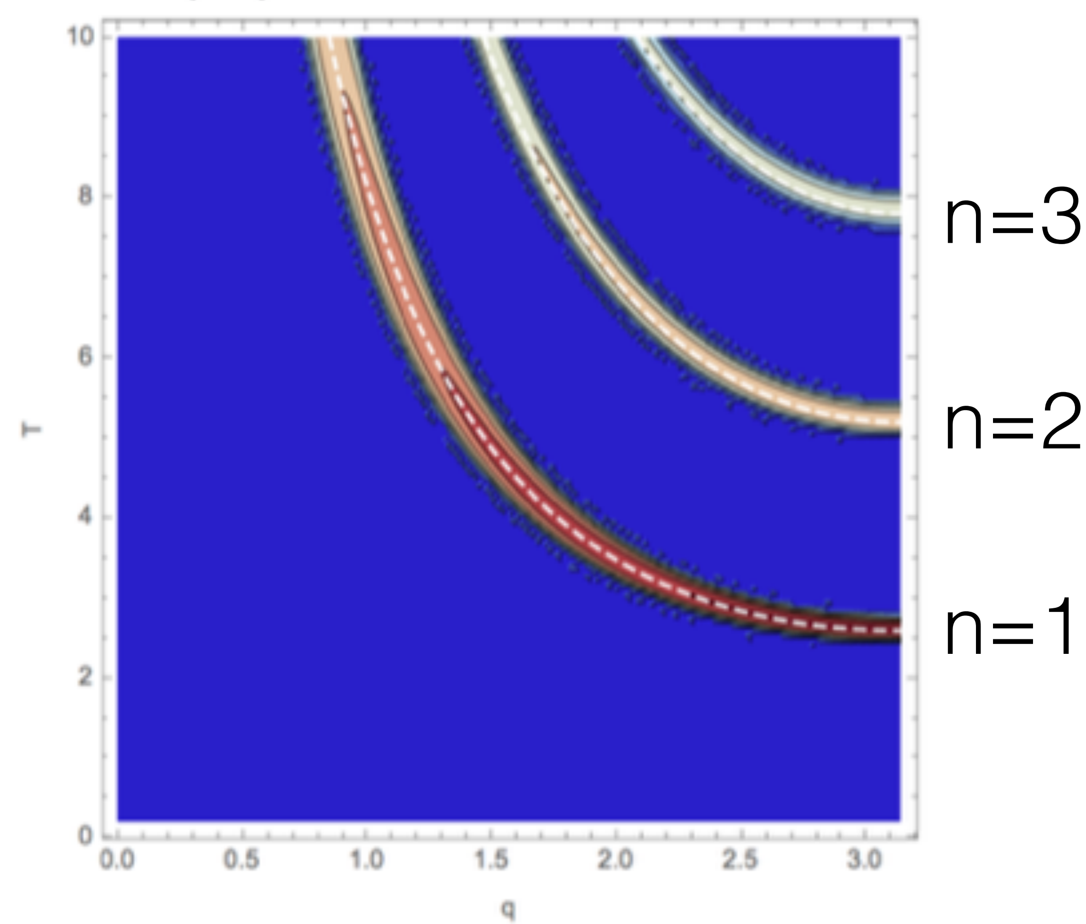


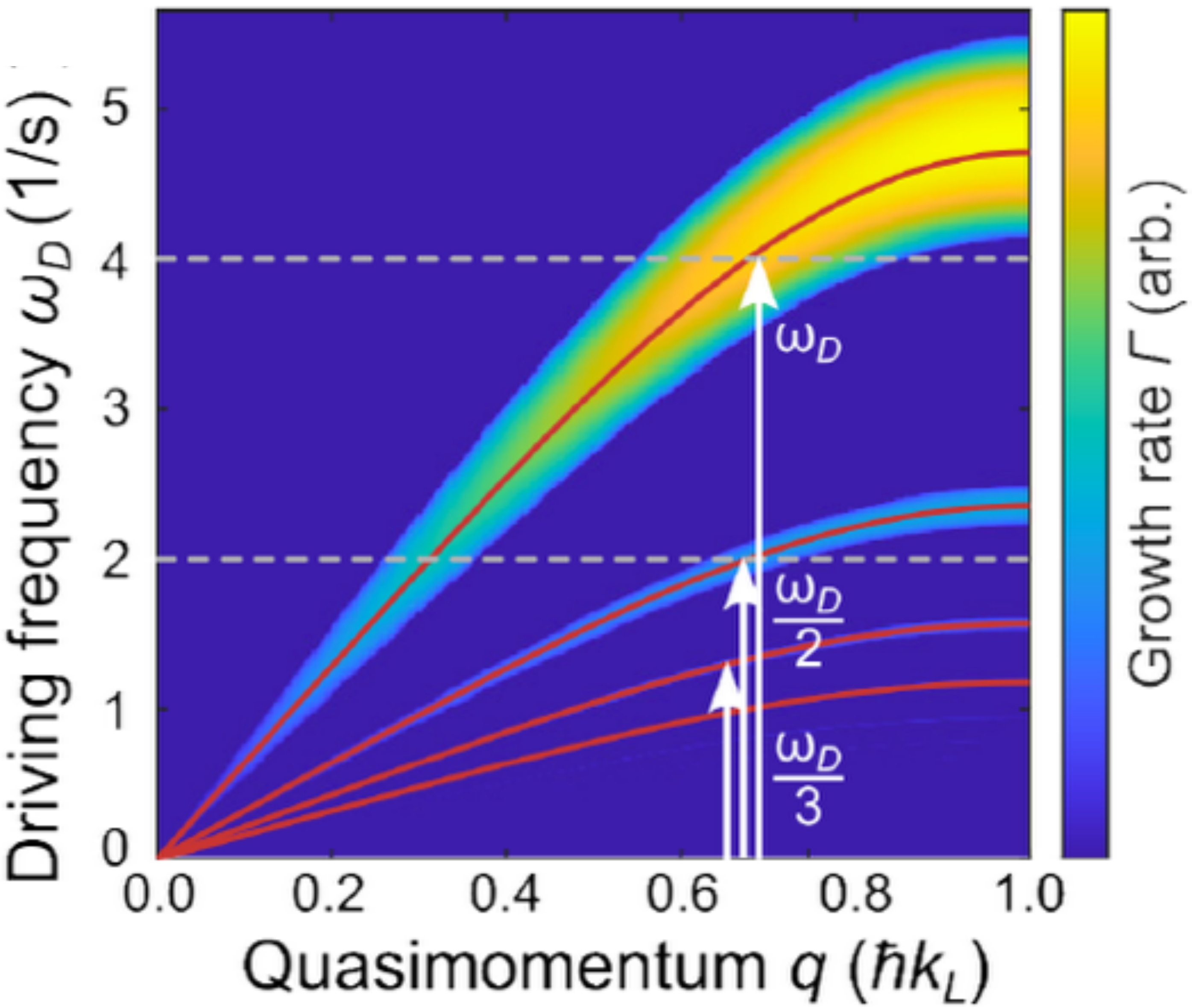
At high frequencies, we can obtain an expression for the eigenvalues of *L:*

$$E_{\text{ave}}(q) = \sqrt{4J_{\text{eff}}\sin^2(q/2)\left(4J_{\text{eff}}\sin^2(q/2) + 2g\right)}$$

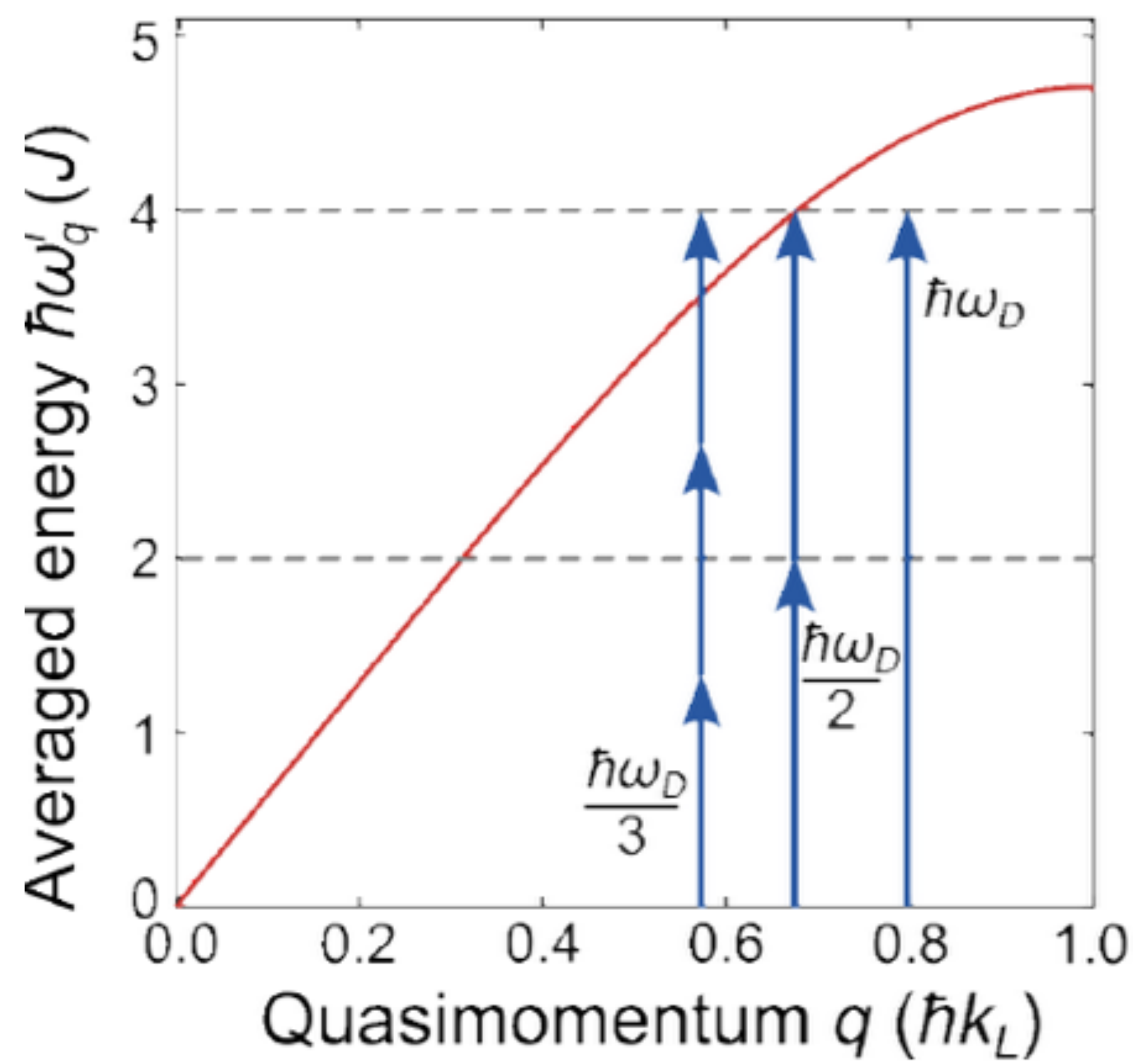
Resonances occur when $E_{\text{ave}}(q) = n\omega$







Driving frequency ω_D (1/s)
Quasimomentum q ($\hbar k_L$)
Growth rate Γ (arb.)
ω_D
$\frac{\omega_D}{2}$
$\frac{\omega_D}{3}$
0.0
0.2
0.4
0.6
0.8
1.0
0
1
2
3
4
5



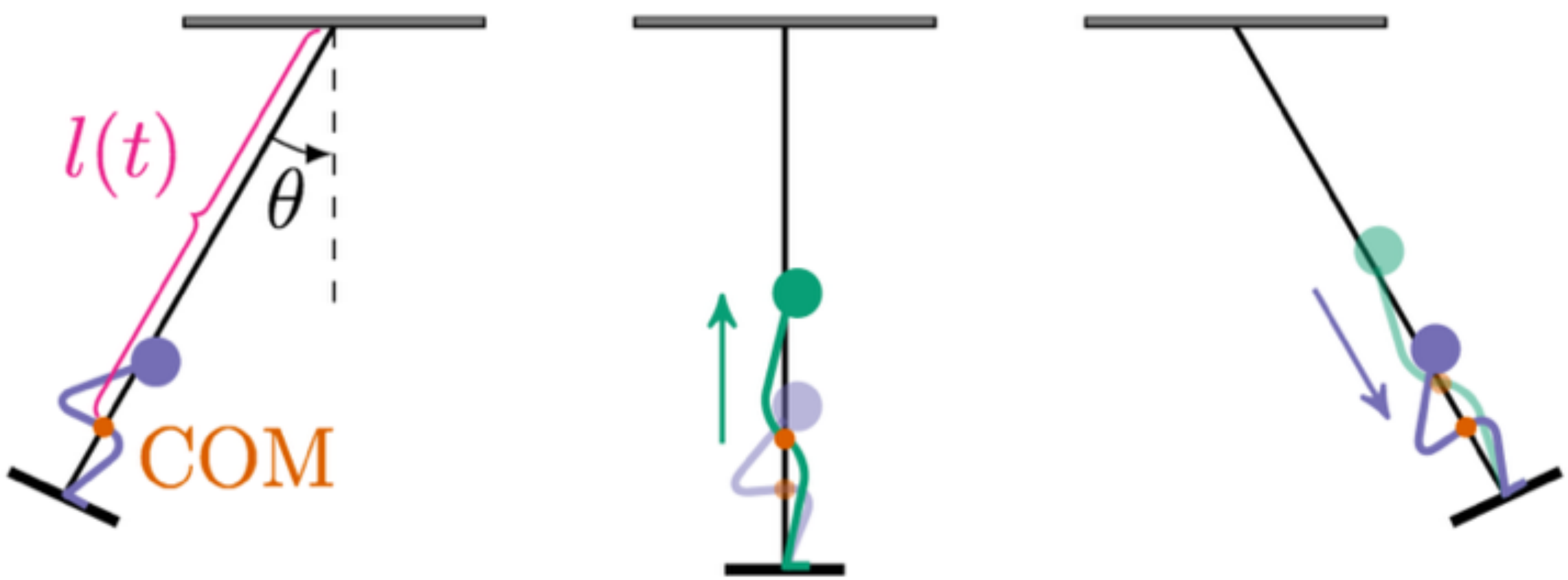
Averaged energy $\hbar\omega'_q$ (J)
Quasimomentum q ($\hbar k_L$)
$\hbar\omega_D$
$\frac{\hbar\omega_D}{2}$
$\frac{\hbar\omega_D}{3}$
0.0
0.2
0.4
0.6
0.8
1.0
0
1
2
3
4
5

This is an example of **parametric resonance**

In detail:

$$i\partial_t \begin{pmatrix} \tilde{u}'_{\mathbf{q}} \\ \tilde{v}'_{\mathbf{q}} \end{pmatrix} = \left[E_{\text{ave}}(\mathbf{q})\hat{1} + \hat{W}_{\mathbf{q}}(t) + \sinh(2\theta_{\mathbf{q}}) \begin{pmatrix} 0 & h_{\mathbf{q}}(t)e^{-2iE_{\text{ave}}(\mathbf{q})t} \\ -h_{\mathbf{q}}(t)e^{2iE_{\text{ave}}(\mathbf{q})t} & 0 \end{pmatrix} \right] \begin{pmatrix} \tilde{u}'_{\mathbf{q}} \\ \tilde{v}'_{\mathbf{q}} \end{pmatrix} .$$

note the factor of 2!



- just like pumping a swing

“kiiking” https://www.youtube.com/watch?v=TWbcsEDrmFE

At low frequencies, we can track the momentum of the system over one period

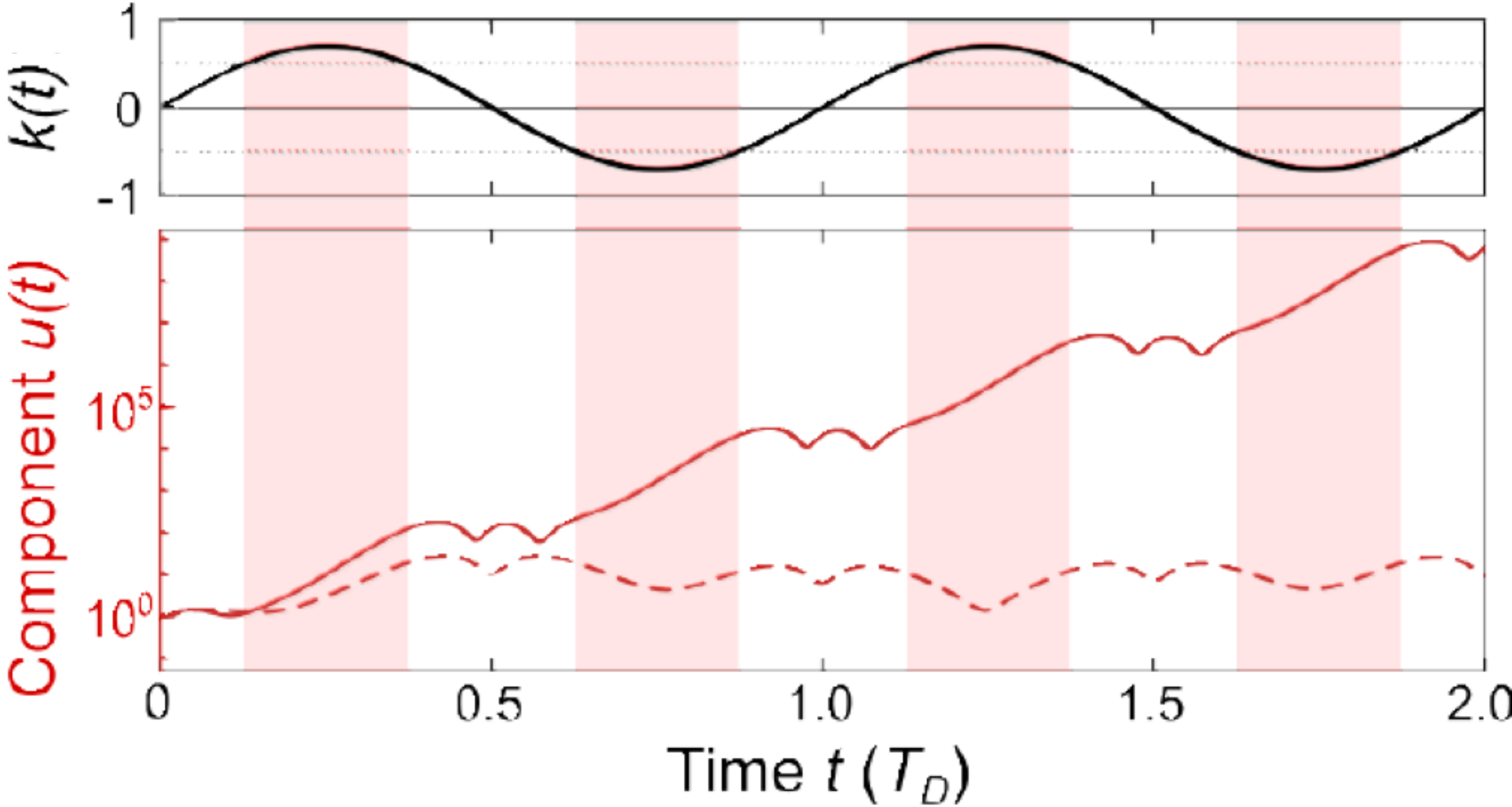
- this depends on the *micromotion* of the Floquet state

$$|\Psi(t)\rangle = \sum_j c_j e^{-i\epsilon_j t}|\psi_j(t)\rangle$$

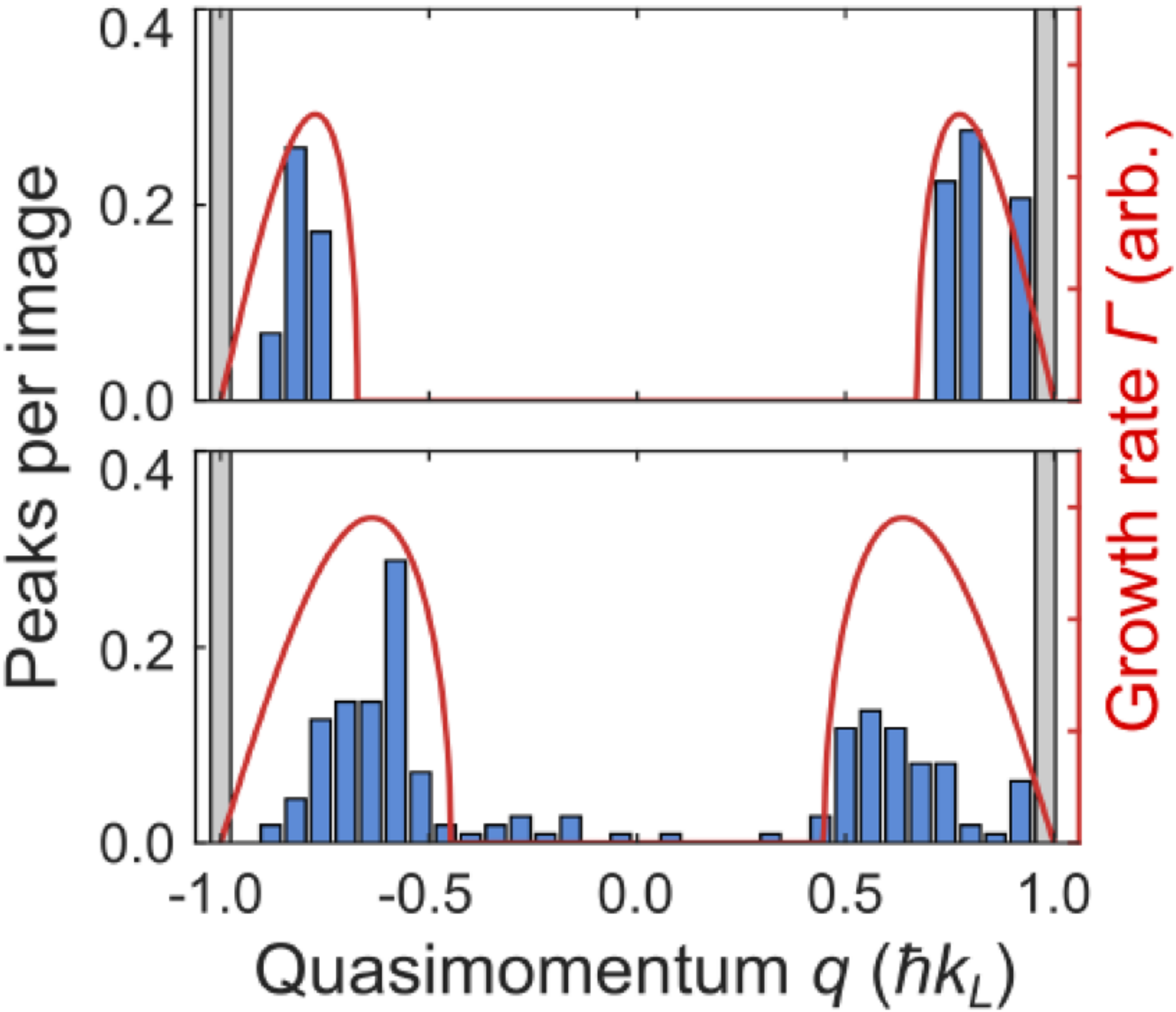
$$|\psi_j(t)\rangle = |\psi_j(t+T)\rangle$$

For certain momenta, the instantaneous eigenvalues of *L(q,t)* will become complex, and **modulational instability*** occurs

*Trombettoni *et al*., J. Phys. B 39, S231 (2006)

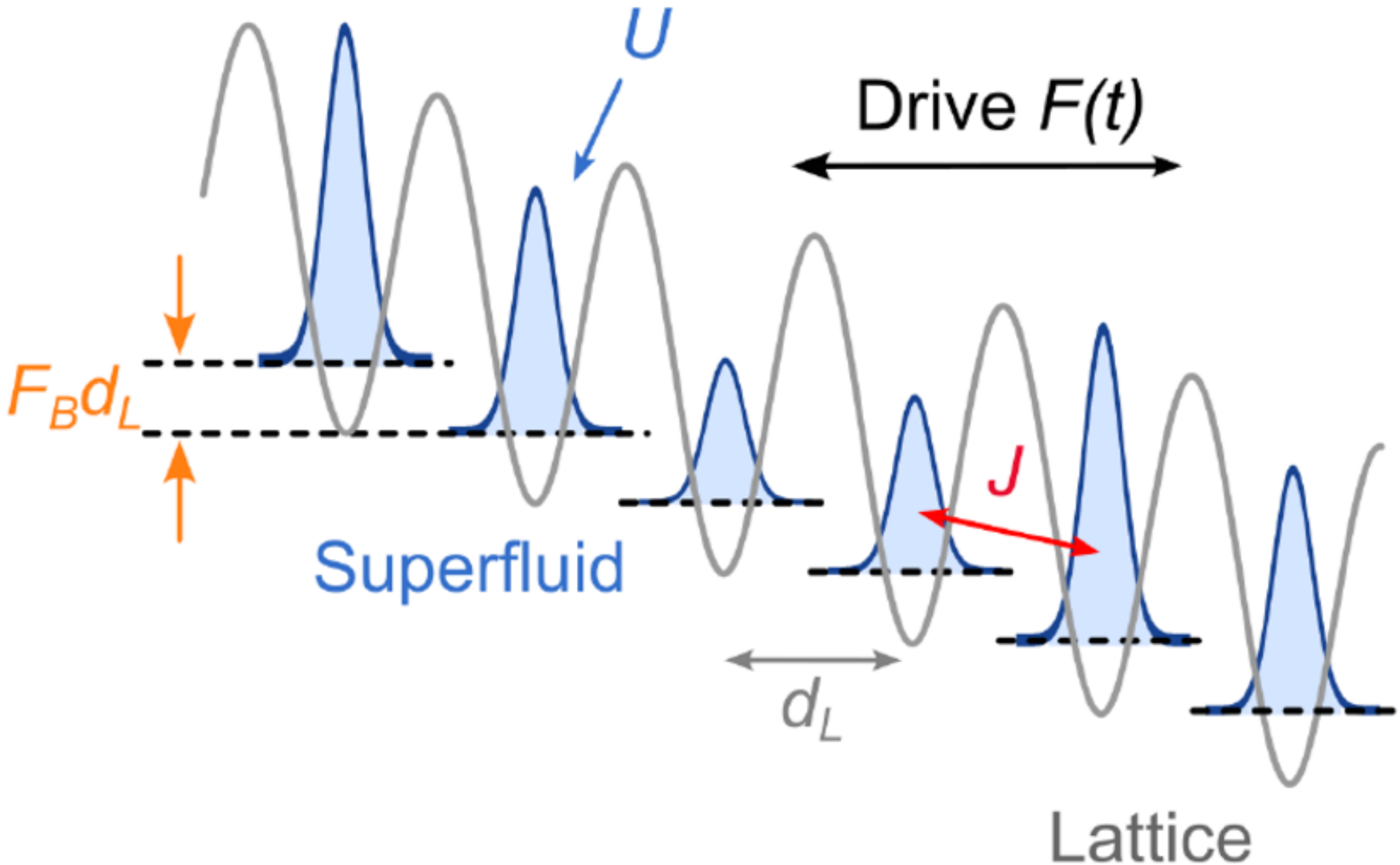


k(t)
1
0
-1
Component u(t)
10^5
10^0
0
0.5
1.0
1.5
2.0
Time t (T_D)



0.4
0.2
0.0
Peaks per image
Growth rate Γ (arb.)
-1.0
-0.5
0.0
0.5
1.0
Quasimomentum q (ħk_L)

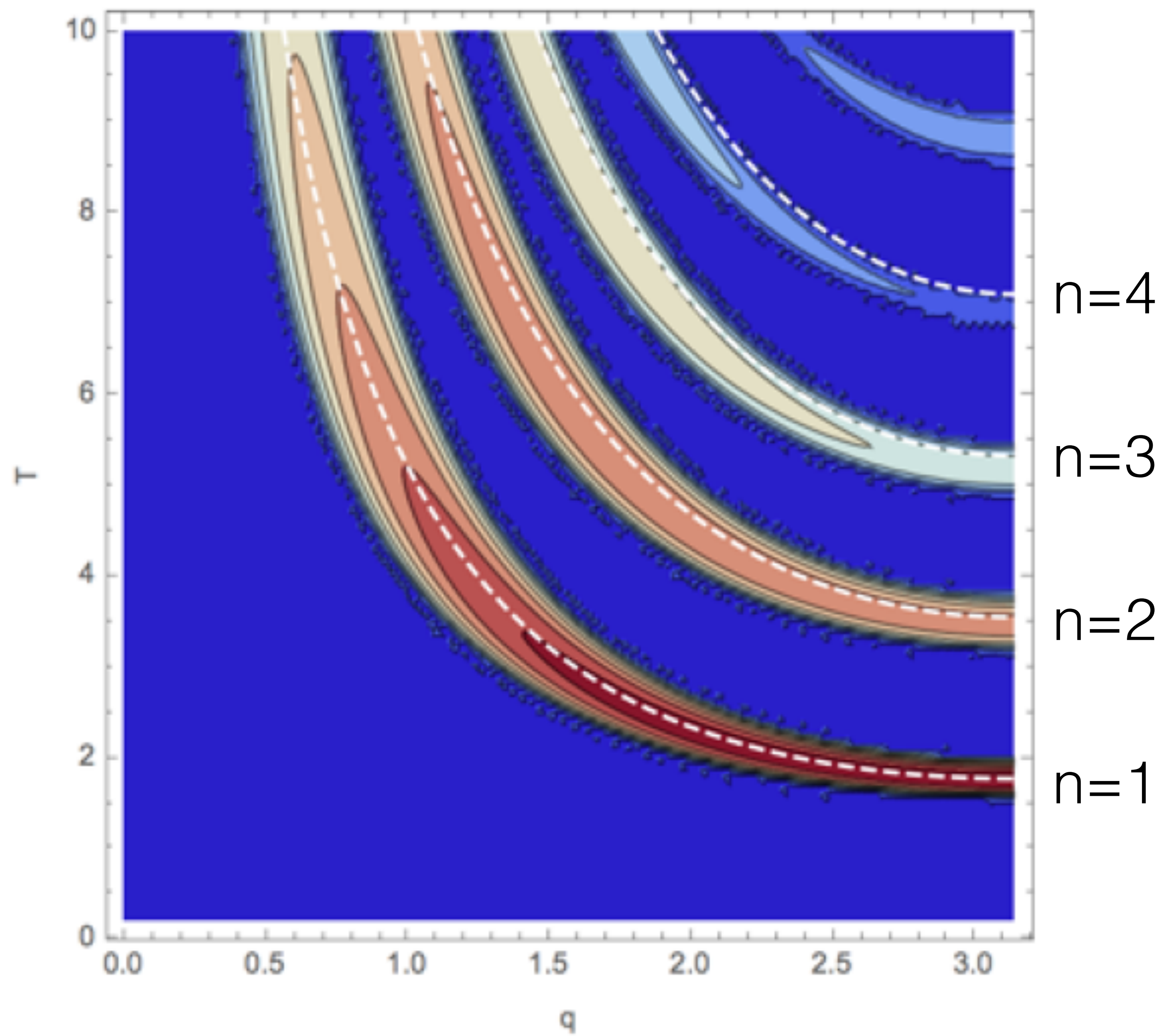
Let us add a static tilt to the driving, $\Delta E = m\,\omega$



At high frequencies, parametric instability again occurs,

but now $J_{\text{eff}} = J\ \mathcal{J}_m(K/\omega)$

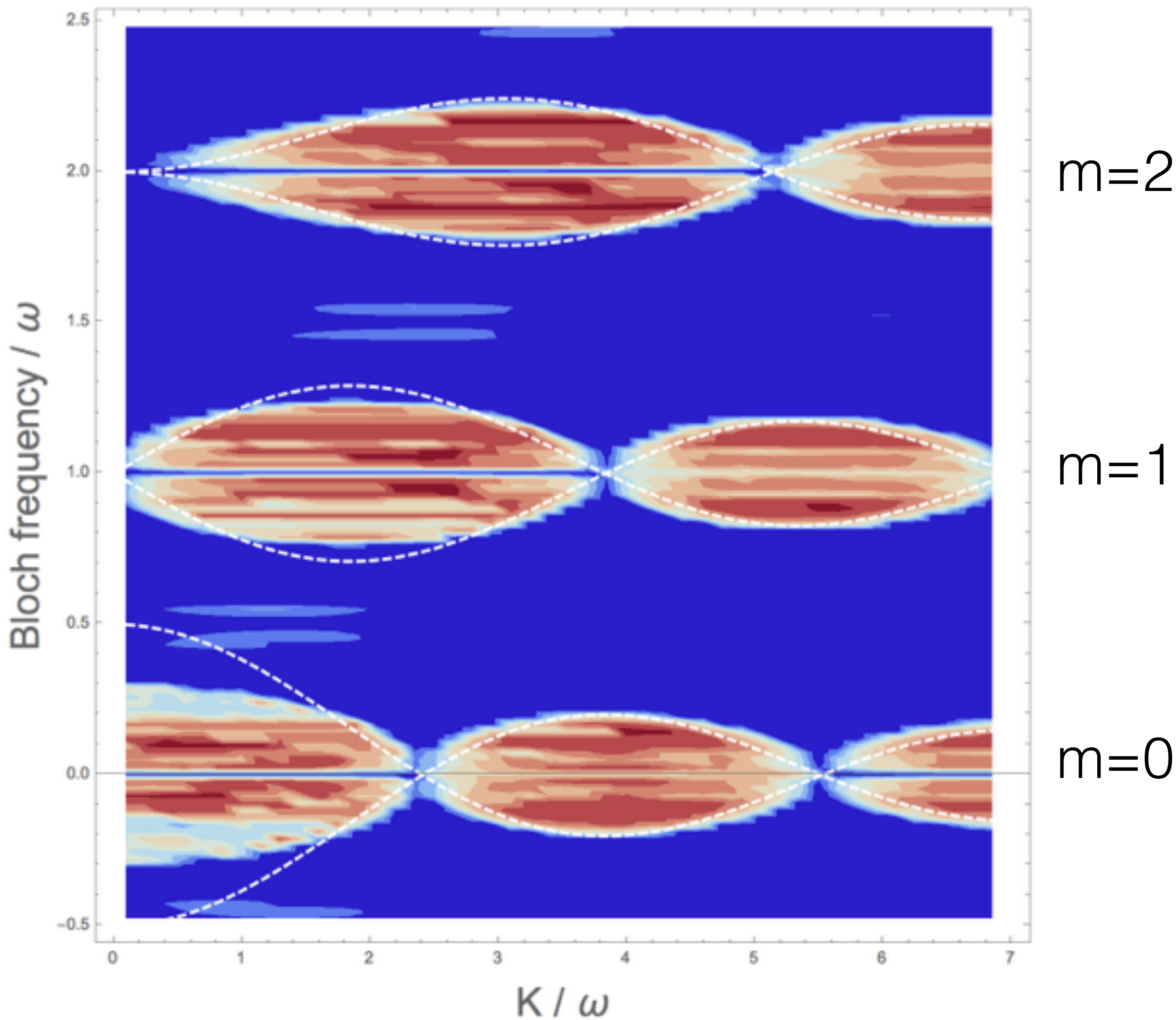
For *m=2:*



similar resonances, but governed by J_2

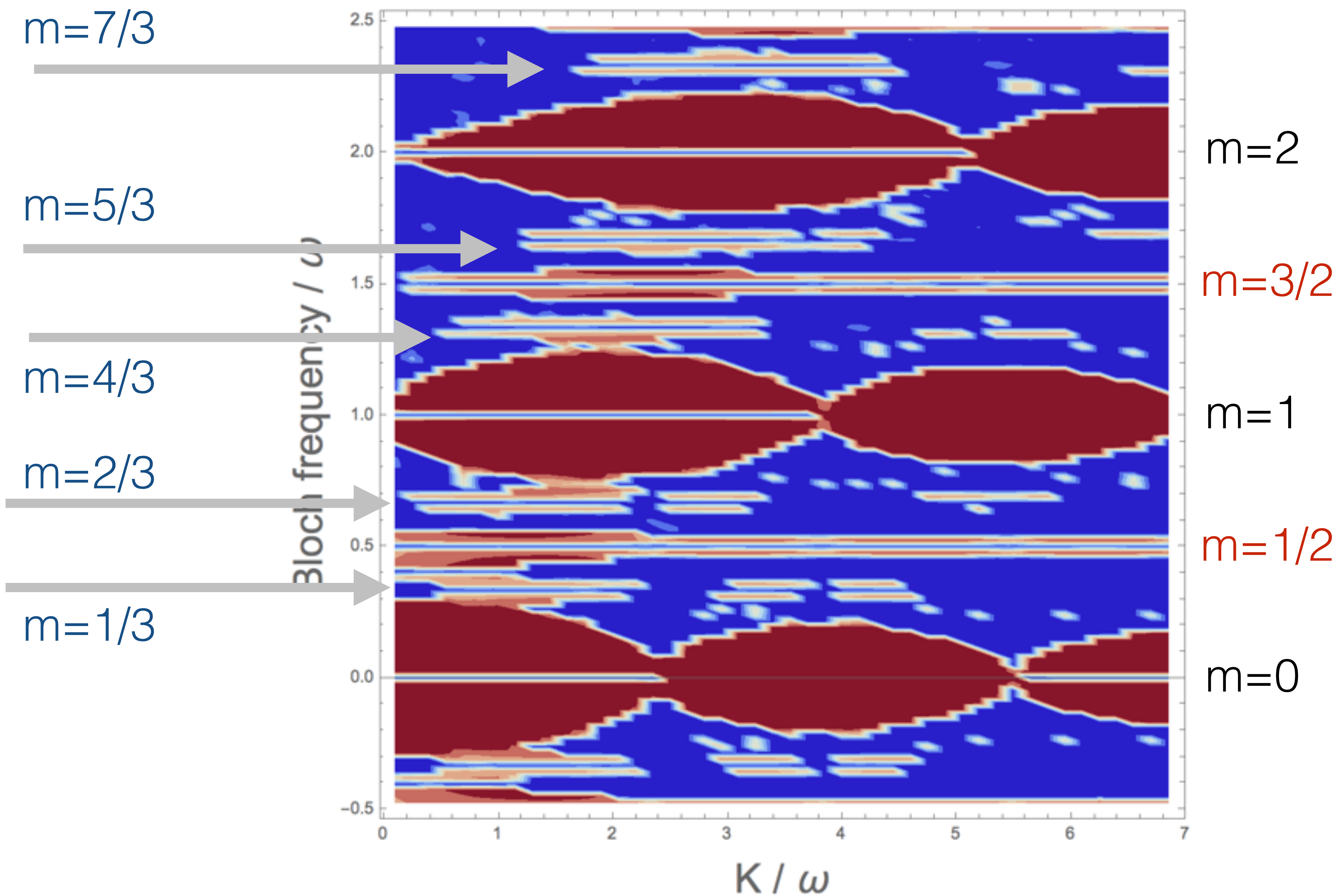
Finally, consider a non-resonant tilt, $\Delta E = (p/q)\,\omega$

Now we have to consider the Floquet problem over q periods of driving

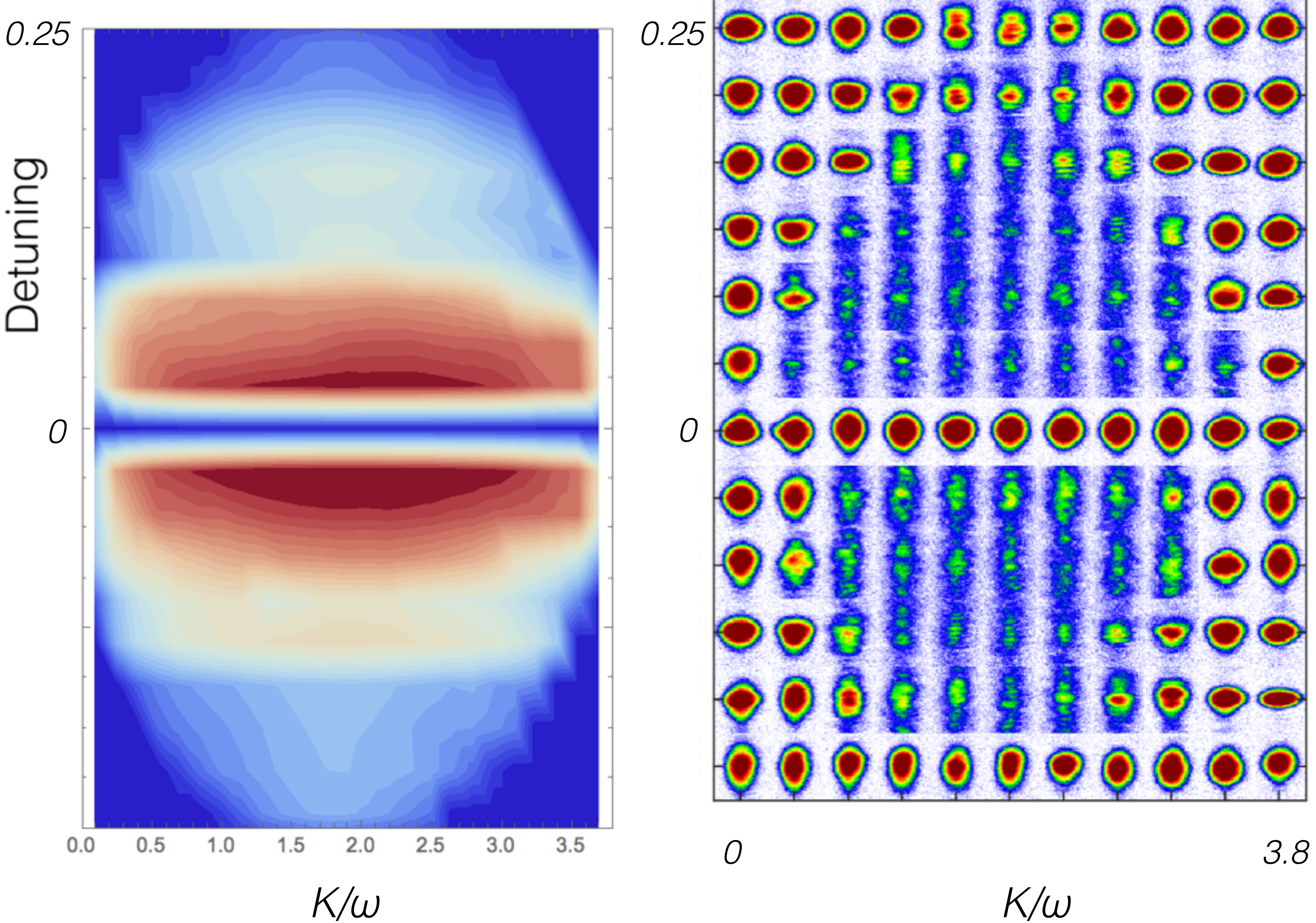


$q=42$

A logarithmic scale reveals more structure

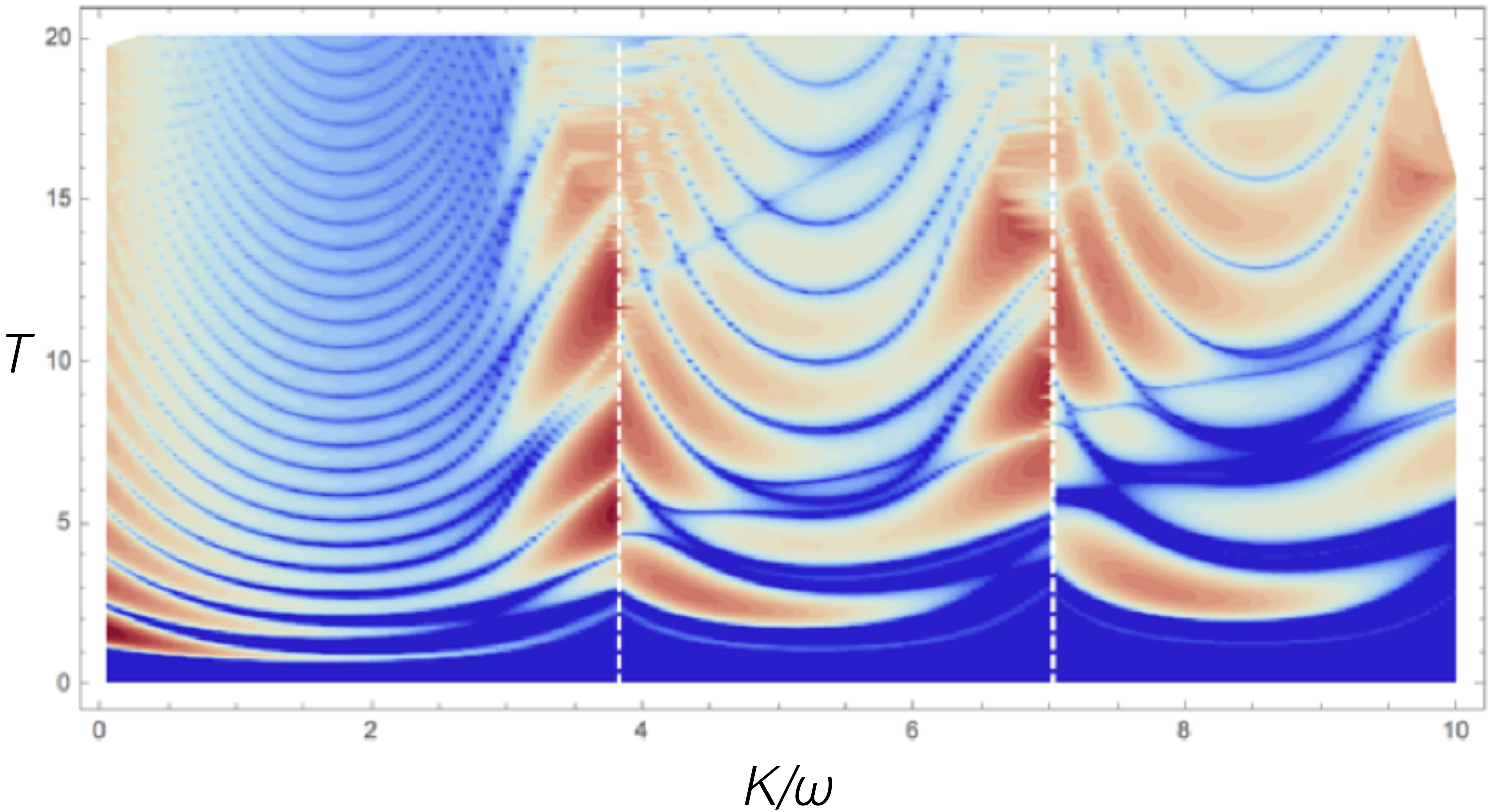


m=1, in detail



Summary and Conclusions

- Floquet engineering is an effective and powerful way to modify Hamiltonians
- two types of instability
  - dynamical instability, most evident at high frequencies
  - modulational instability, at low frequencies
- stability is best at high frequency
- at lower frequencies we can identify sweet spots where heating is minimised



20
15
10
5
0
T
0
2
4
6
8
10
K/ω

Thank you to my collaborators

- the cold atoms group at the University of Strathclyde

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Arthur La Rooij, Stefan Kuhr, and Elmar Haller

Thank you for your attention!