Mimicking the Bosonic Su-Schrieffer-Heeger model with a lattice of rings

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Quantum Transport

universidade de aveiro

Ring potentials

+

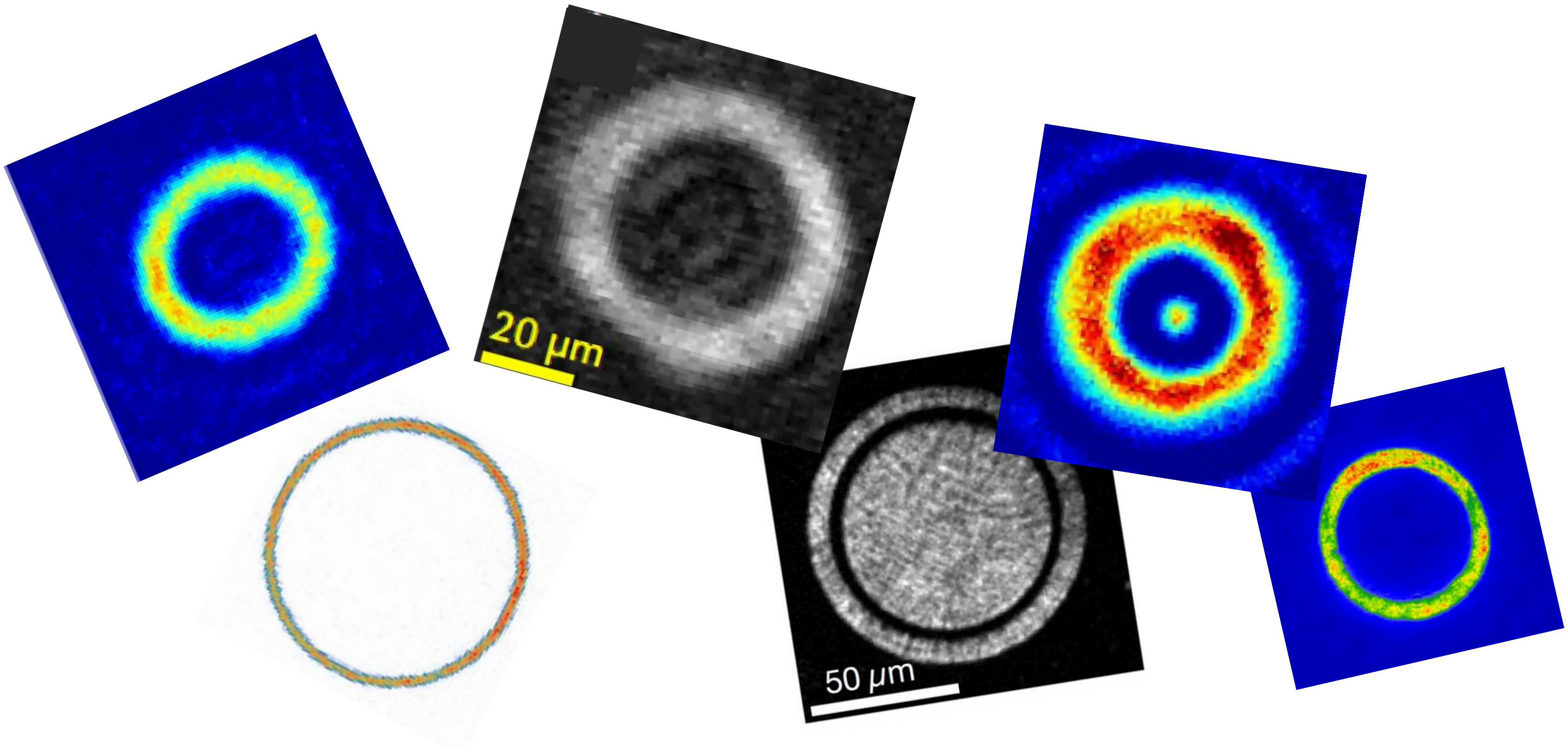
Su-Schrieffer-Heeger (SSH) model

⬇

Bosonic orbital SSH in a lattice of rings

Ring potentials

Ring potentials



L. Amico *et al*., AVS Quantum Sci. **3**, 039201 (2021).
L. Amico *et al*., Rev. Mod. Phys. **94**, 041001 (2022).

Ring potentials

OAM transfer

Light beams

M. Andersen *et al*., Physical Review Letters **97**, 170406 (2006).
S. Franke-Arnold, Philosophical Transactions Mathematical Physical & Engineering Sciences **375**, 20150435 (2017).

Weak link rotation

A. Ramanathan *et al*., Physical Review Letters **106**, 130401 (2011).
K. C. Wright *et al*., Physical Review Letters **110**, 025302 (2013).

Temperature quench

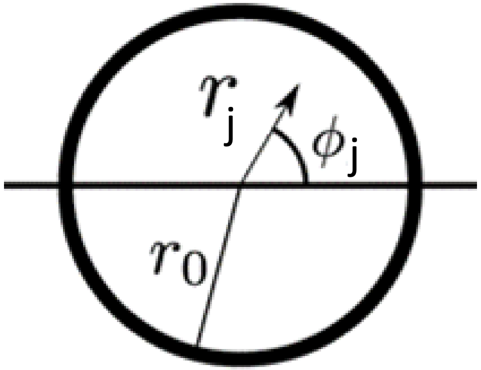
L. Corman *et al*., Physical Review Letters **113**, 135302 (2014).

Alternatively

p band of a conventional optical lattice

G. Wirth M. Ölschläger, and A. Hemmerich, Nature Physics **7**, 147 (2011).
X. Li and W. V. Liu, Reports on Progress in Physics **79**, 116401 (2016).
A. Kiely *et al*., J. of Phys. B: Atomic, Molecular and Optical Physics **49**, 215003 (2016).
T. Kock *et al*., J. of Phys. B: Atomic, Molecular and Optical Physics **49**, 042001 (2016).

Ring potentials



winding number, $n=\pm l$, l being the orbital angular momentum quantum number

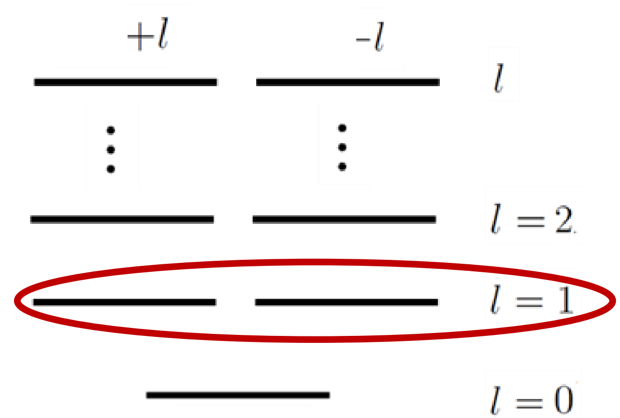
$$\Psi_j^n(r_j,\phi_j) = \langle \vec{r} \,|\, j,n \rangle = \psi(r_j)e^{in(\phi_j-\phi_0)}$$

index of the ring

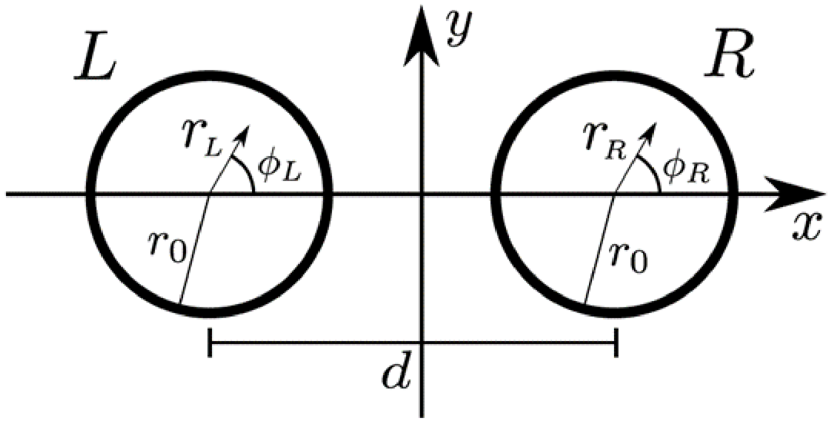
radial part

phase origin

$$E=E_0+E_c\,l^2$$



Ring potentials



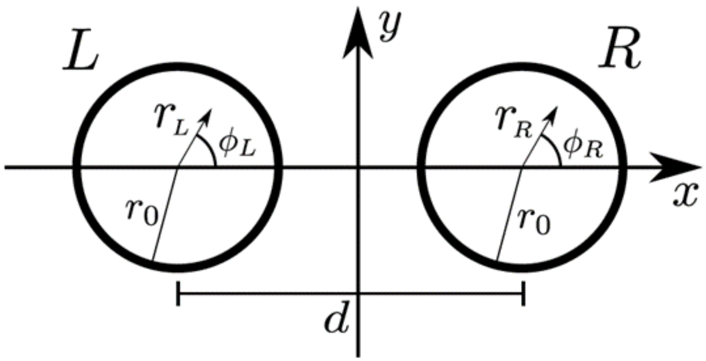
$$J_{j,n}^{k,n\prime} = e^{i(n-n')\phi_0} \int d^2r \; \Psi_{j,n}\,(\phi_0 = 0)\, H \; \Psi_{k,n'}(\phi_0 = 0)$$

j,k=L,R

$$\Psi_j^n(r_j,\phi_j) = \langle \vec{r} \,|\, j,n \rangle = \psi(r_j) e^{in(\phi_j - \phi_0)}$$

J.Polo *et al.*, Phys. Rev. A **93**, 033613 (2016).

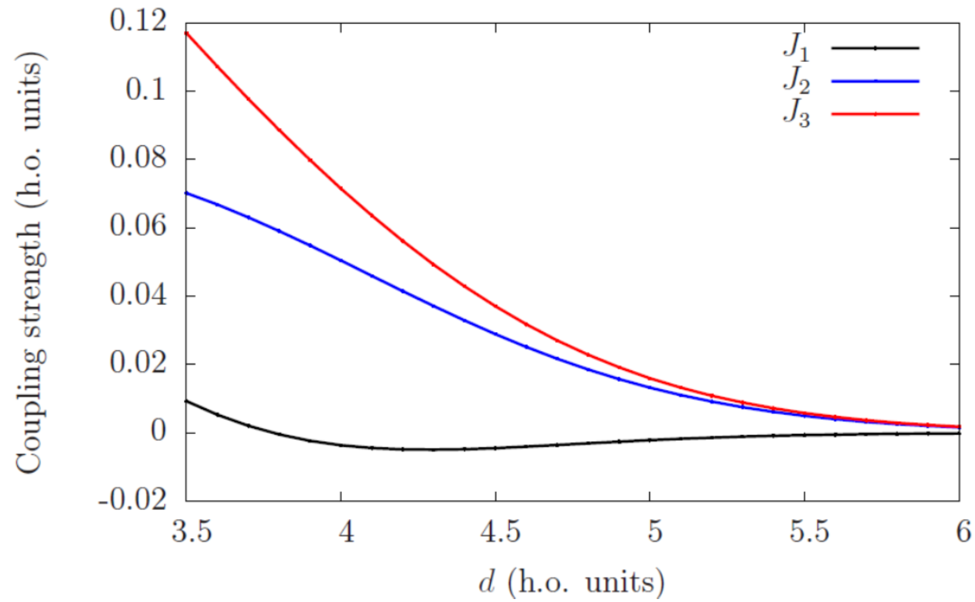
Ring potentials



$J_{L,n}^{L,-n} = |J_{L,n}^{L,-n}| e^{2in\phi_0}$ → J_1

$J_{L,n}^{R,n}$ → J_2

$J_{L,n}^{R,-n} = |J_{L,n}^{R,-n}| e^{2in\phi_0}$ → J_3



J_1: self coupling

J_2: cross coupling, no OAM exchange

J_3: cross coupling, OAM exchange

J.Polo *et al.*, Phys. Rev. A **93**, 033613 (2016).

Ring potentials

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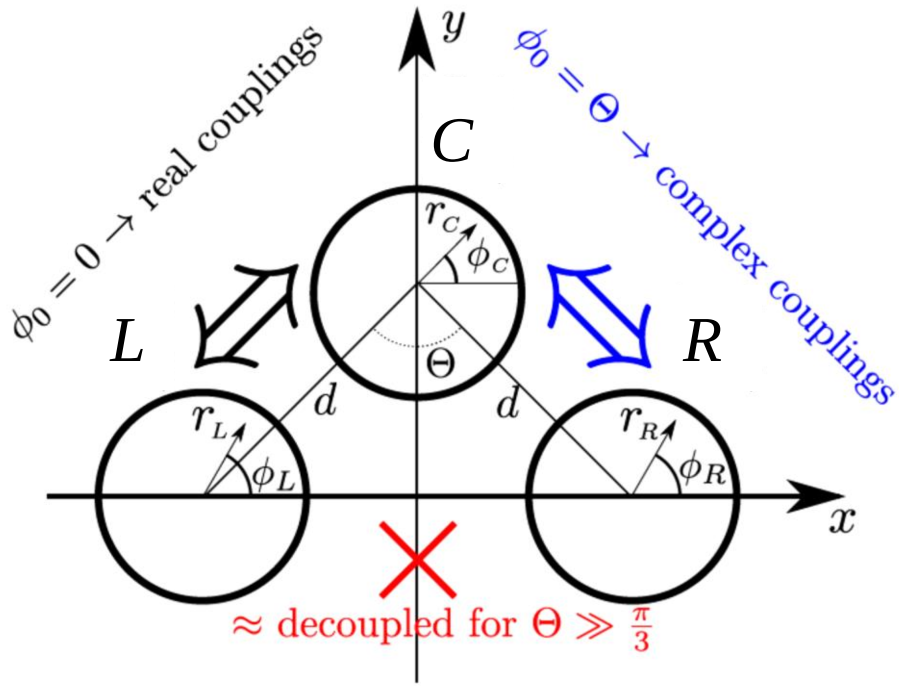


Along one direction, $\phi_0 \neq 0 \rightarrow J_1$ and J_3 couplings acquire phases

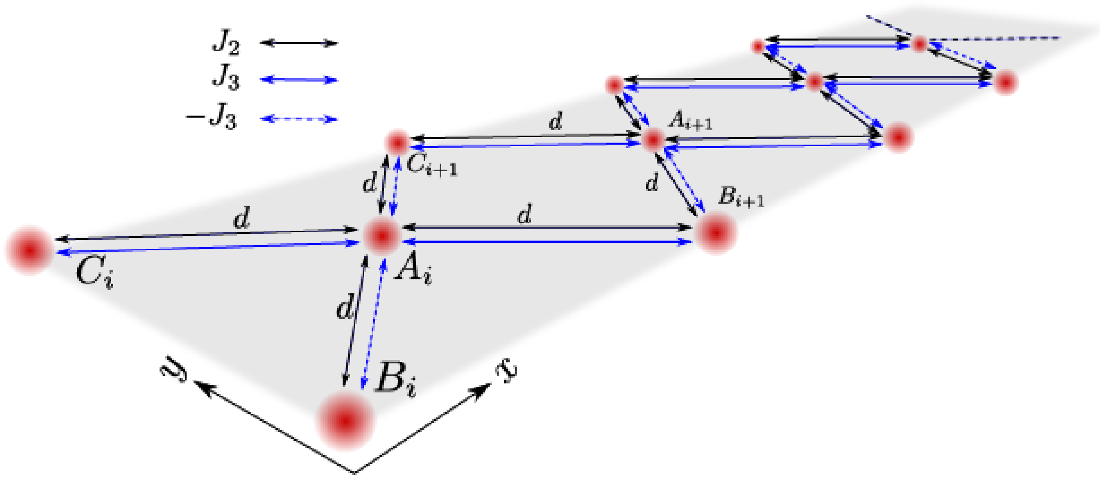
Fast decay of the couplings with d → L and R sites decoupled for $\theta >> \pi/3$

Self-coupling at site C has contributions from L and R sites →vanishes for $\theta = \pi/2$

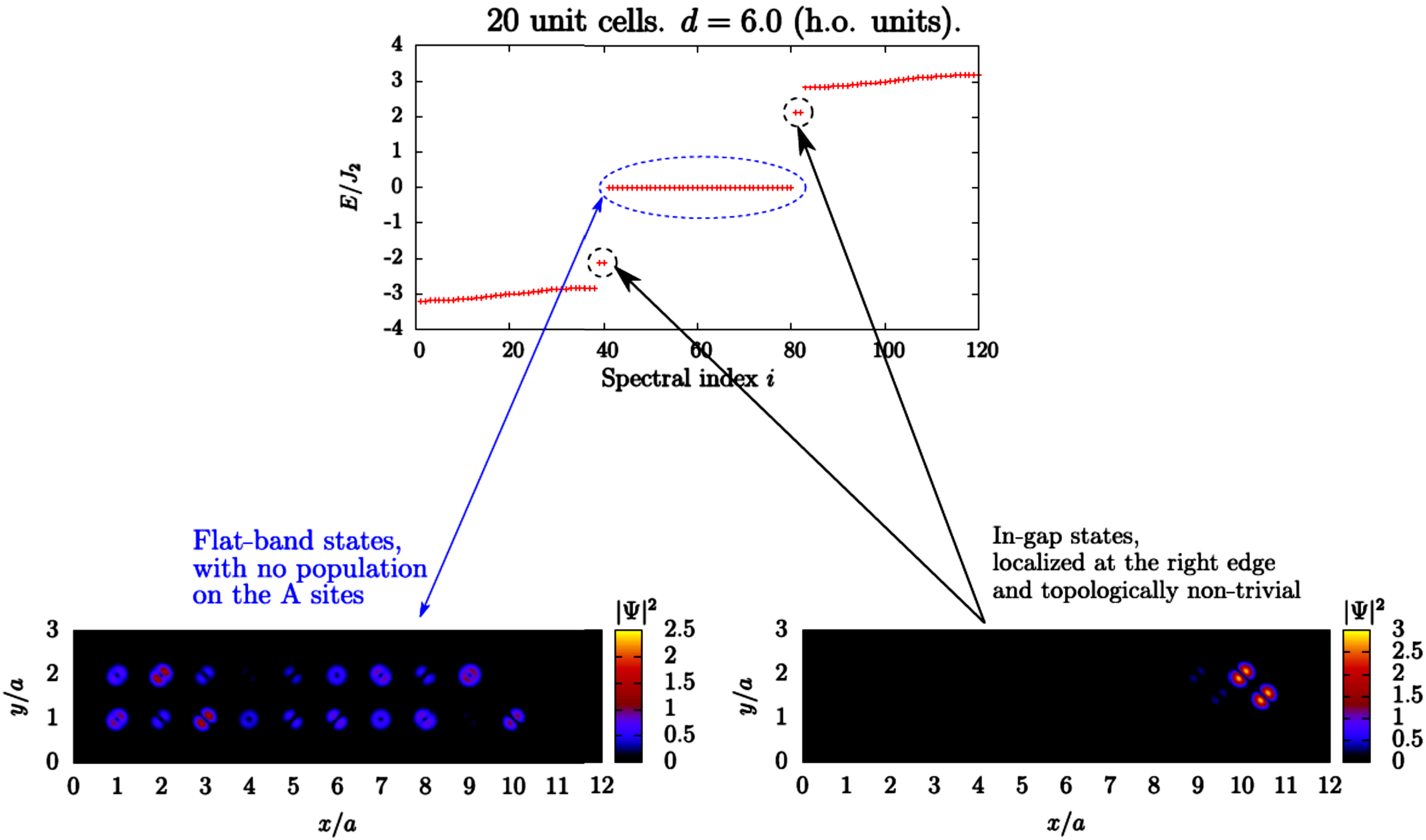
J_1: self coupling

J_2: cross coupling, no OAM exchange

J_3: cross coupling, OAM exchange



G. Pelegrí *et al.*, Phys. Rev. A **95**, 013614 (2017).
G. Pelegrí *et al.*, Phys. Rev. A **99**, 023612 (2019).
G. Pelegrí *et al.*, Phys. Rev. A **99**, 023613 (2019).



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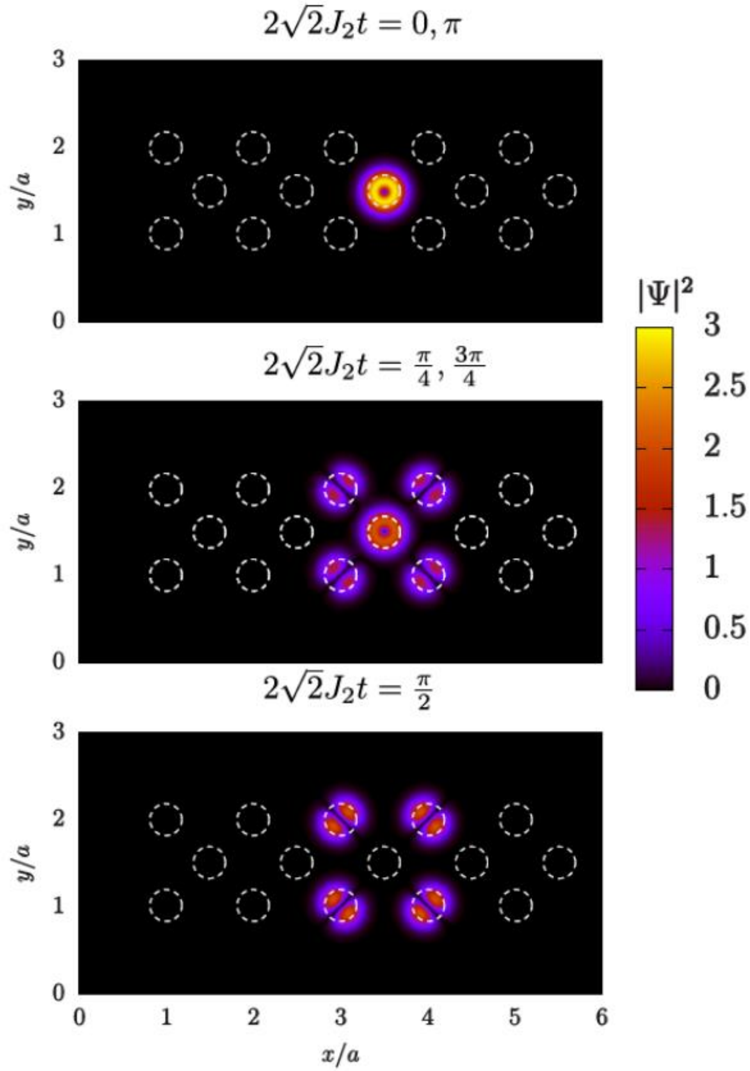
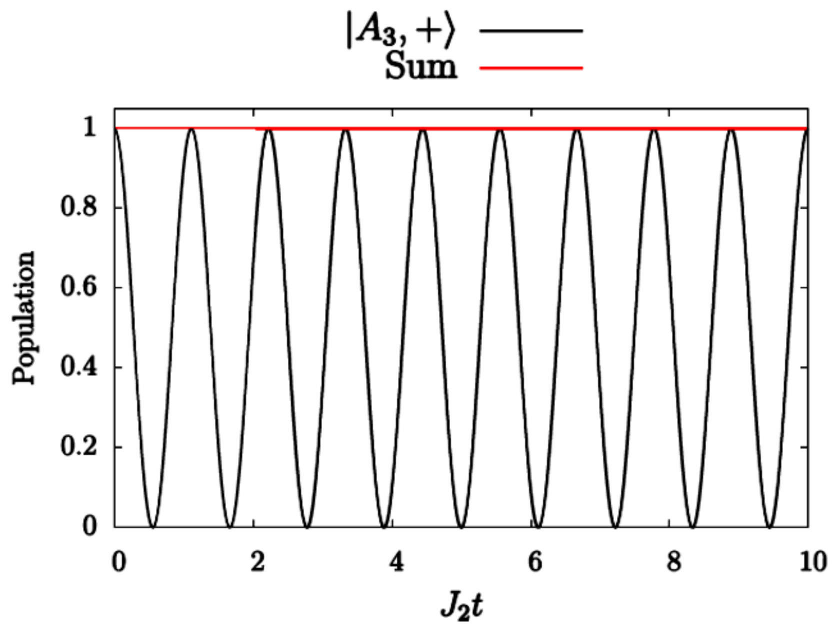
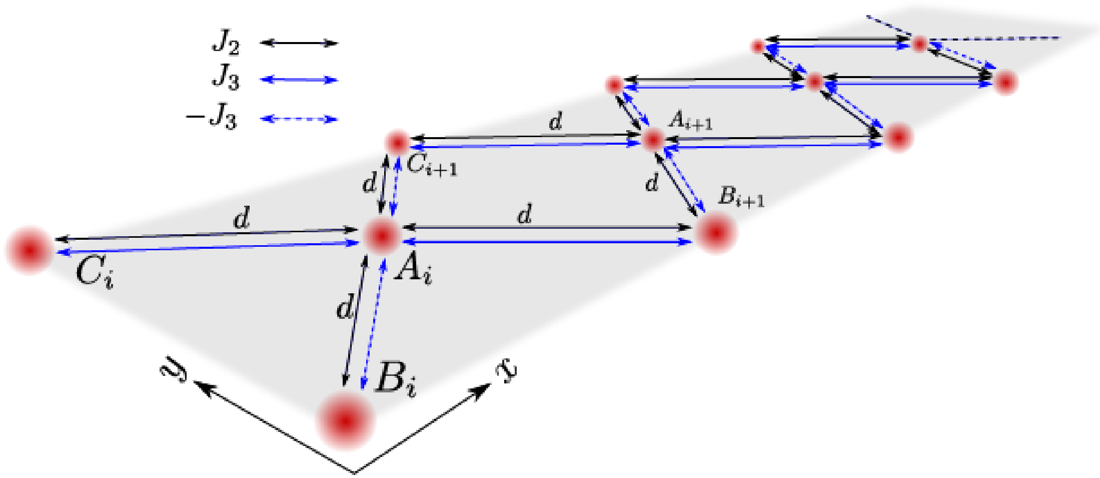
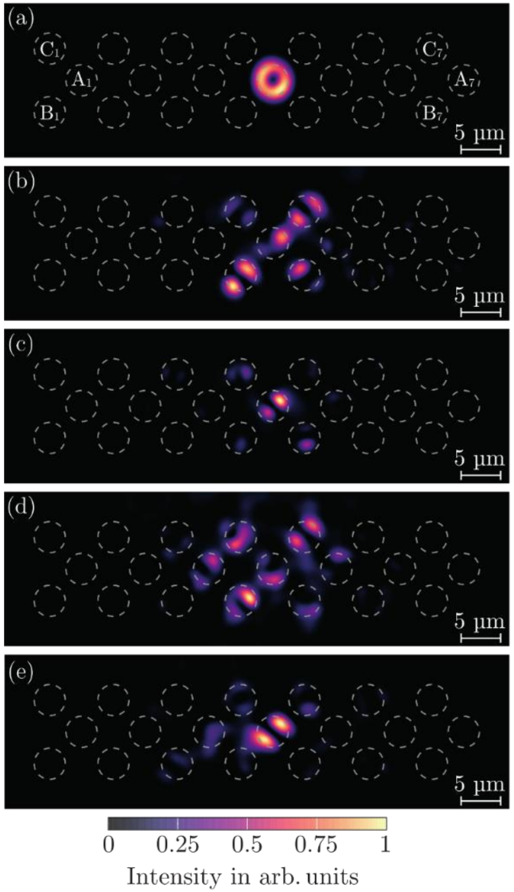
Aharonov-Bohm caging





C. Jörg *et al.*, Light Sci Appl **9**, 150 (2020).





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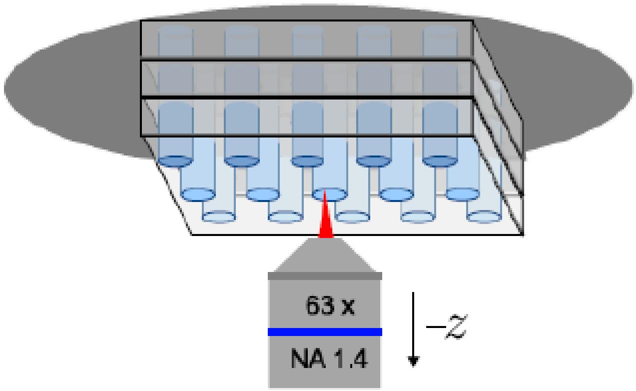
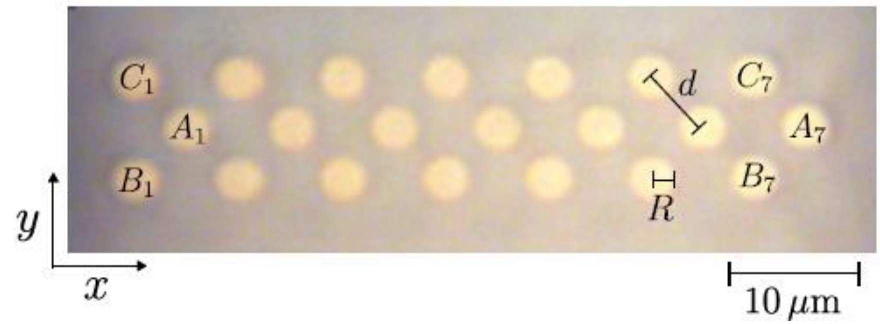


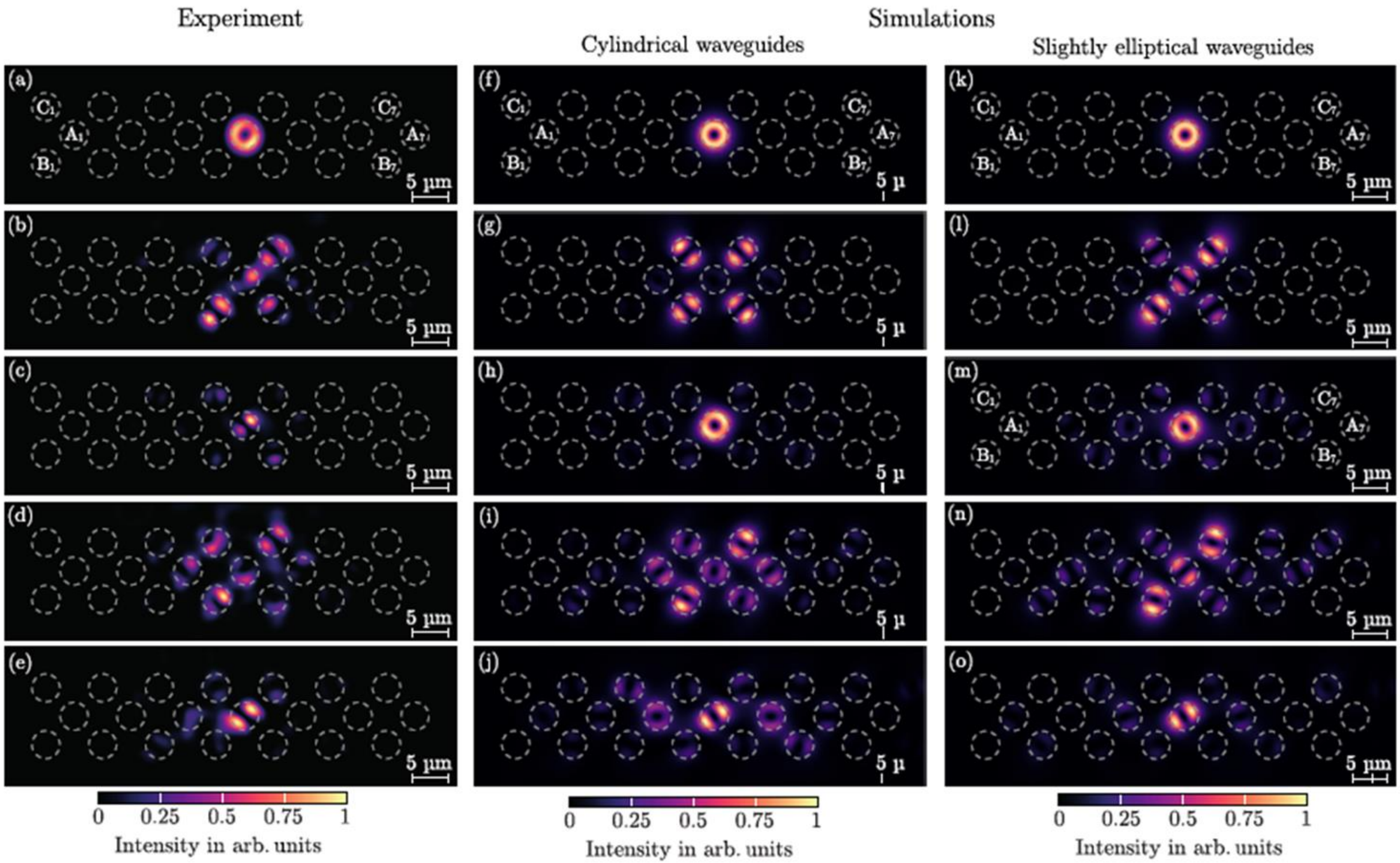


- Nanoscribe system
- Photo resist IP-Dip
- Samples: 0.25-1 mm

- 7 unit cells
- R= 1.9 μm
- Δn ~ 0.008
- d = 5.5 μm

C. Jörg, G. Queraltó, M. Kremer, G. Pelegrí, J. Schulz, A. Szameit, G. von Freymann, J. Mompart, V. Ahufinger, Light Sci Appl **9**, 150 (2020).

See also: S. Mukherjee et al., Physical Review Letters **121**, 075502 (2018).
M. Kremer et al., Nature Communications **11**, 907 (2020).



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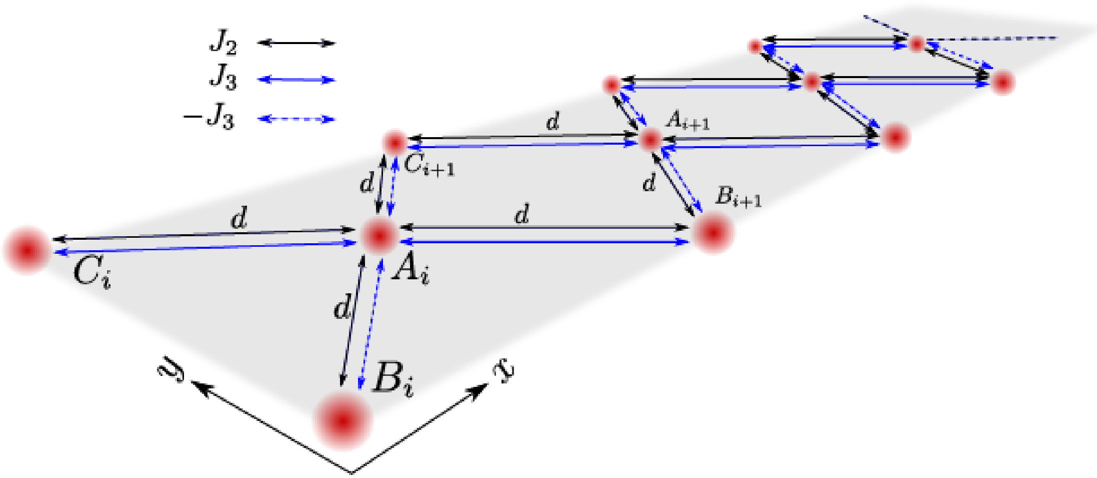

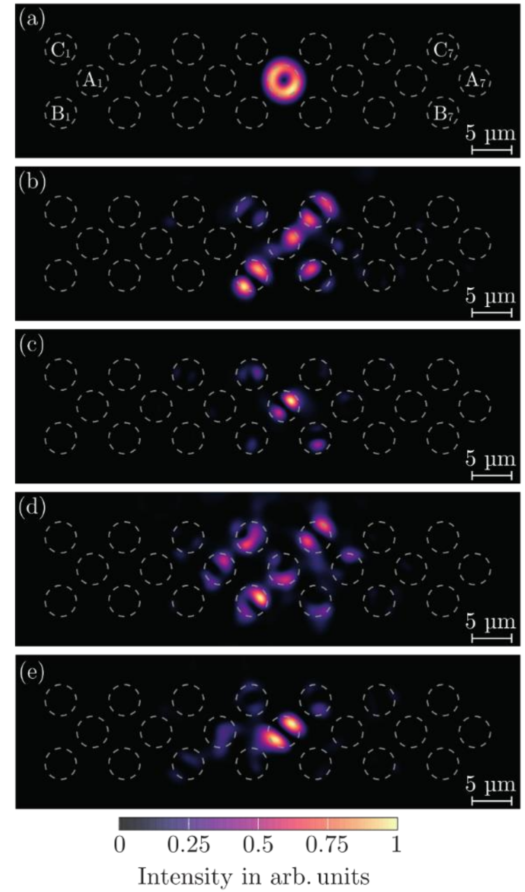

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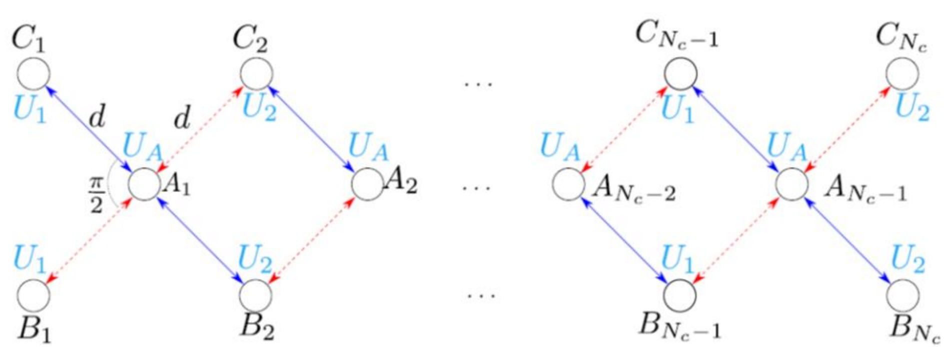
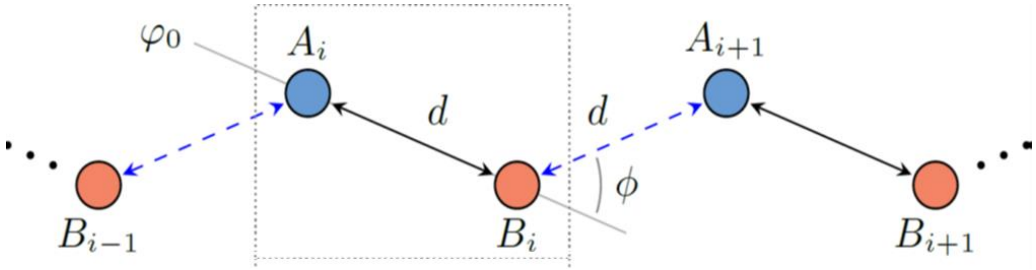


G. Pelegrí *et al.*, Physical Review Research **2**, 033267 (2020).



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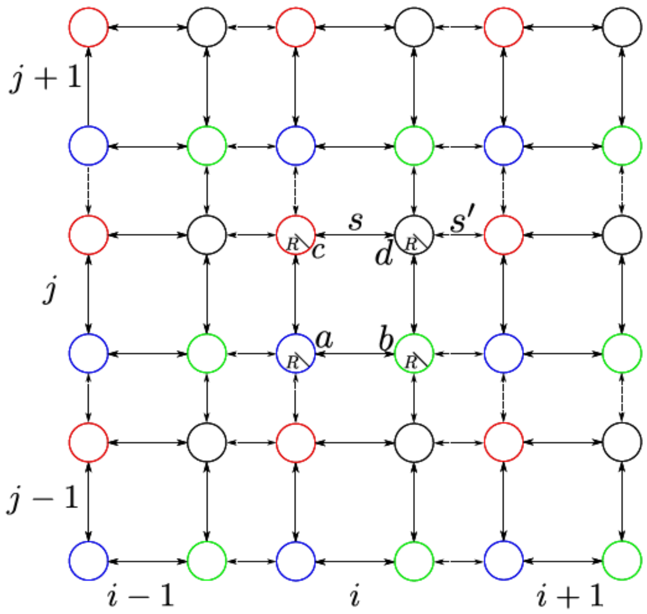
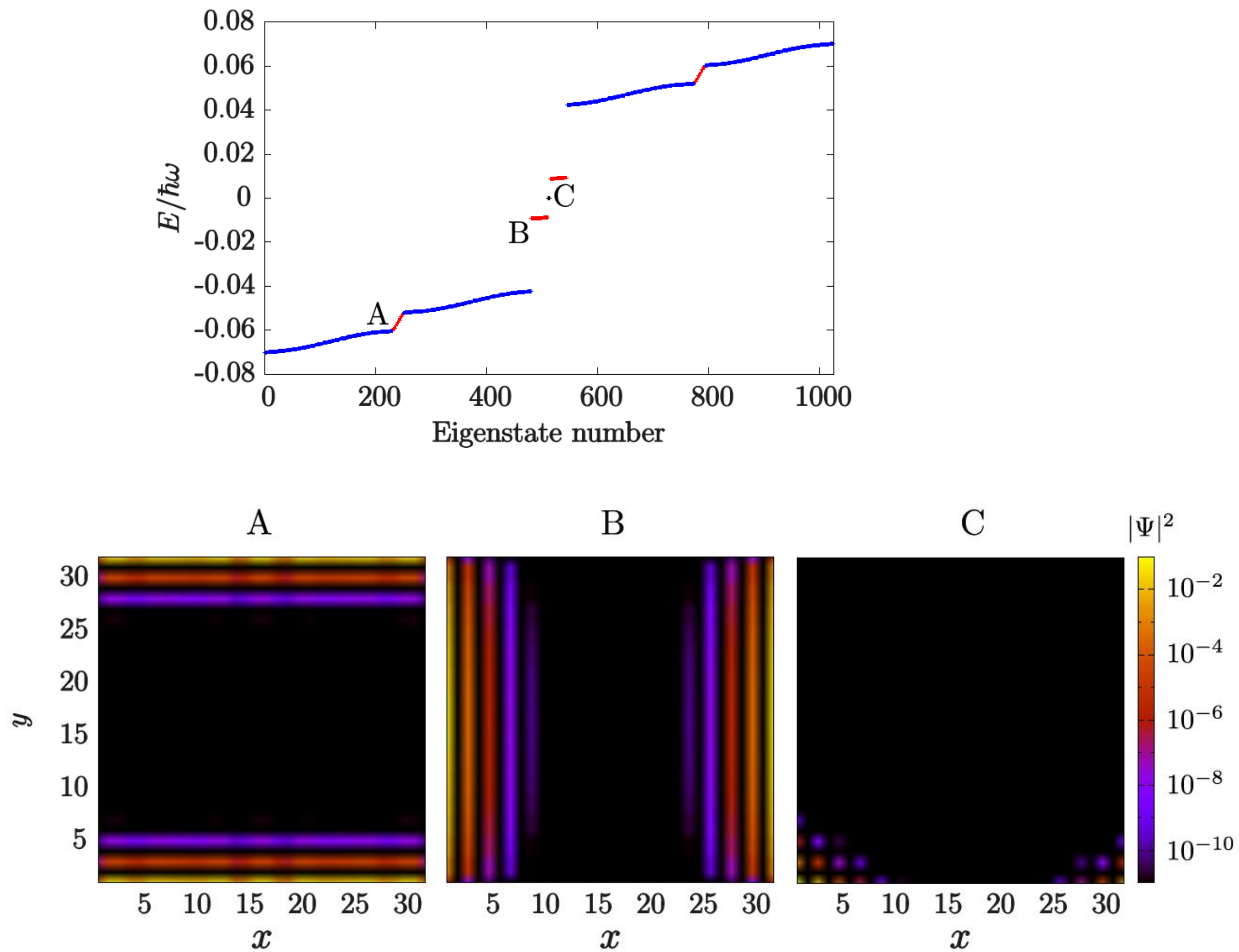
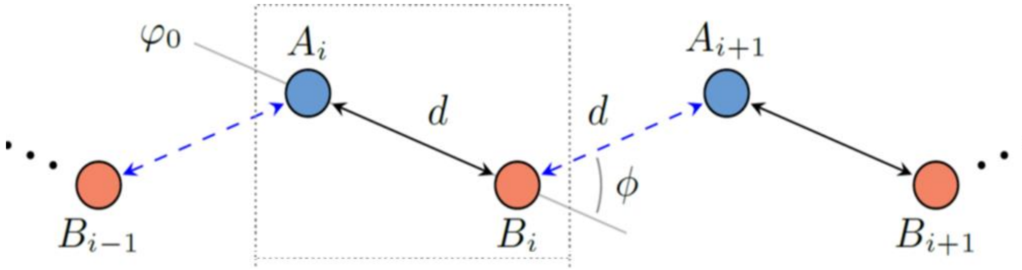


G. Pelegrí *et al.*, Phys. Rev. B **100**, 205109 (2019).

G. Pelegrí *et al.*, Phys. Rev. B **100**, 205109 (2019).





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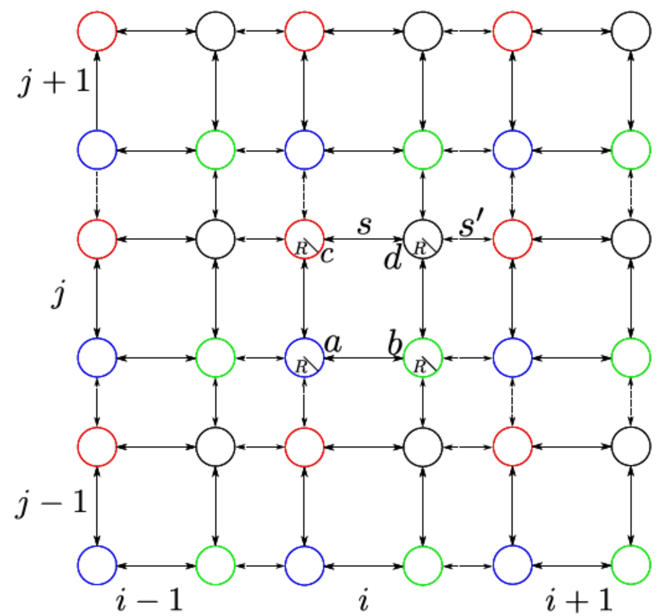
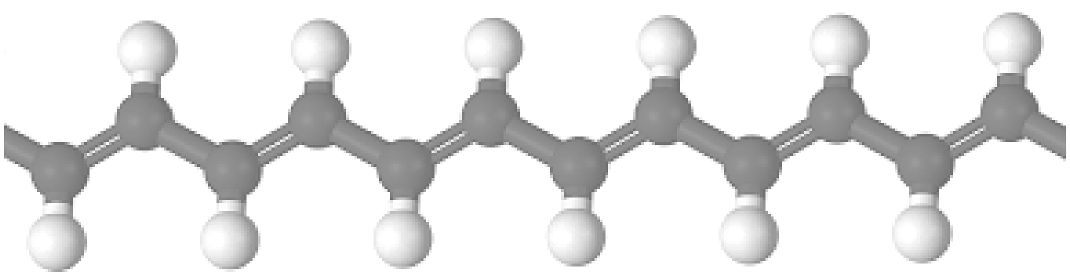


G. Pelegrí *et al.*, Phys. Rev. B **100**, 205109 (2019).

E. Nicolau *et al.*, Europhysics Letters **145**, 35001 (2024).

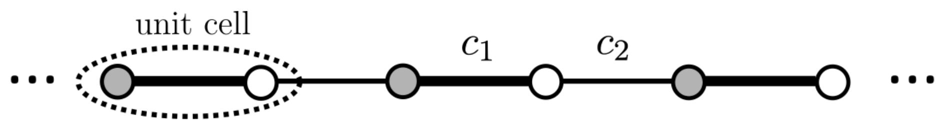
Su-Schieffer-Hegger model

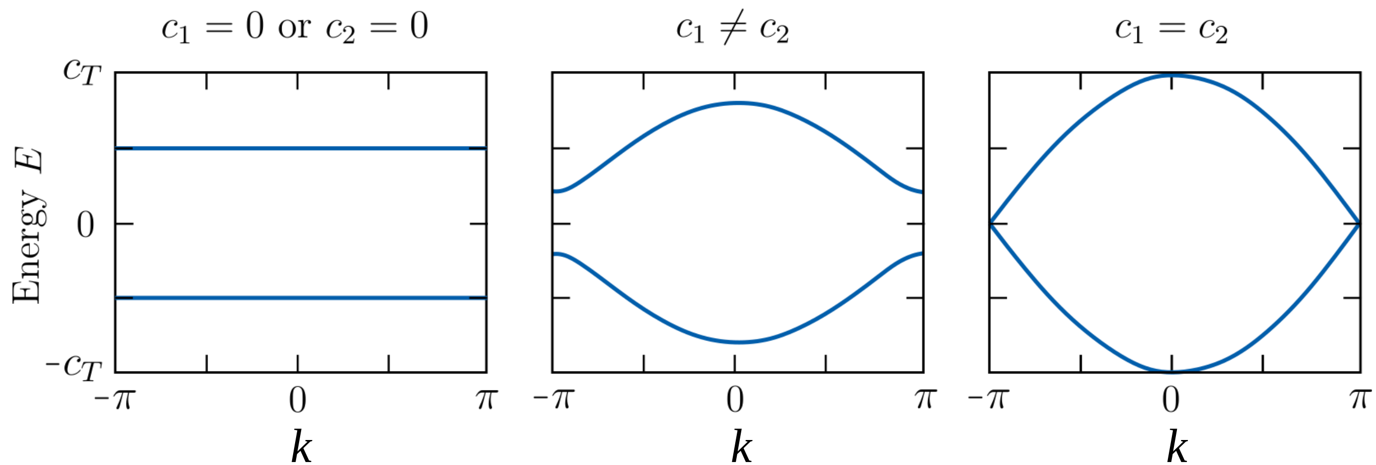


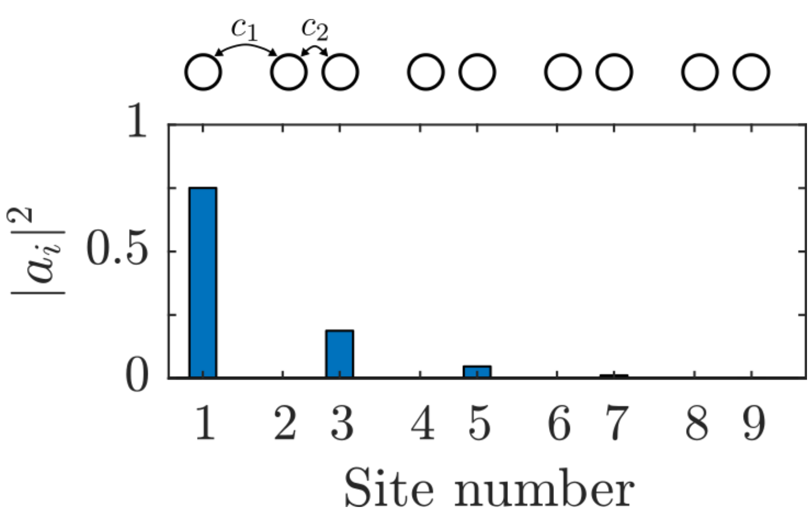
W. P. Su, J. R. Schrieffer, A. J. Heeger, Phys. Rev. Lett. **42**, 1698 (1979).

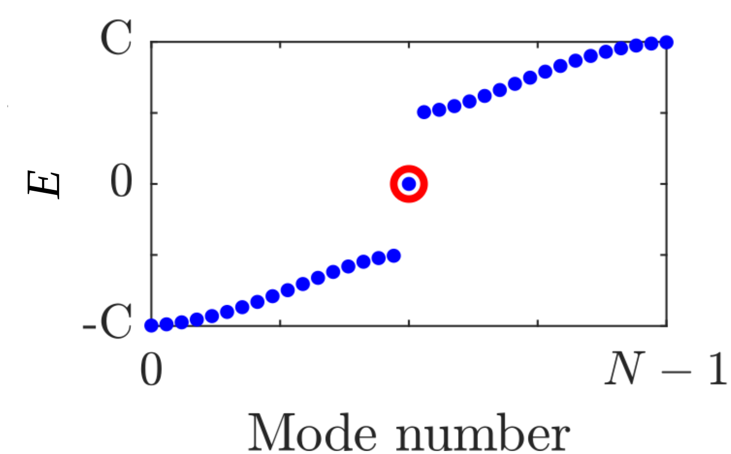
Su-Schieffer-Hegger model

W. P. Su, J. R. Schrieffer, A. J. Heeger, Phys. Rev. Lett. **42**, 1698 (1979).









SSH in a lattice of rings

SSH in a lattice of rings

1 atom

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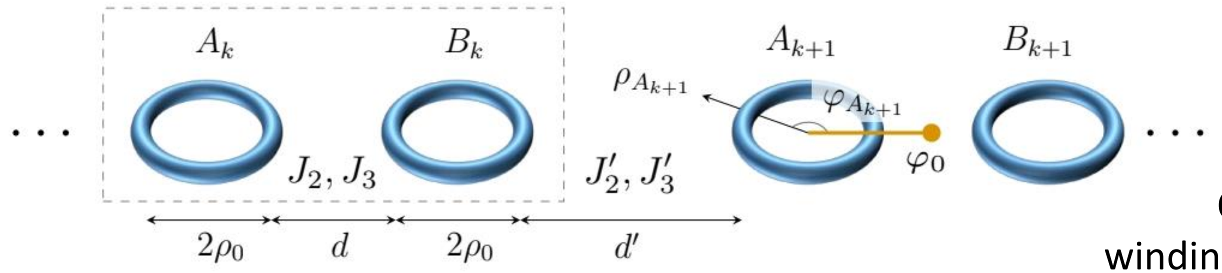
SSH structure



OAM: l=1
winding number: n=±1
circulation: α=±

Single-particle

$$\hat{\mathcal{H}}^0_{l=1} = J_2 \sum_{k=1}^{N_c} \sum_{\alpha=\pm} \hat{a}_k^{\alpha\dagger} \hat{b}_k^{\alpha} + J_2' \sum_{k=1}^{N_c-1} \sum_{\alpha=\pm} \hat{b}_k^{\alpha\dagger} \hat{a}_{k+1}^{\alpha} + J_3 \sum_{k=1}^{N_c} \sum_{\alpha=\pm} \hat{a}_k^{\alpha\dagger} \hat{b}_k^{-\alpha} + J_3' \sum_{k=1}^{N_c-1} \sum_{\alpha=\pm} \hat{b}_k^{\alpha\dagger} \hat{a}_{k+1}^{-\alpha} + \text{ H.c.}$$

Topological characterization

Basis rotation to resolve unitary symmetry

$$\left|A_k^{s(a)}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|A_k^+\right\rangle \underset{(-)}{+} \left|A_k^-\right\rangle\right) \qquad \left|B_k^{s(a)}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|B_k^+\right\rangle \underset{(-)}{+} \left|B_k^-\right\rangle\right)$$

$$\begin{cases} \hat{\mathcal{H}}_s = t_s \sum_{k=1}^{N_c} \hat{a}_k^{s\dagger} \hat{b}_k^{s} + t_s' \sum_{k=1}^{N_c-1} \hat{a}_{k+1}^{s\dagger} \hat{b}_k^{s} + \text{H.c.} \\ \hat{\mathcal{H}}_a = t_a \sum_{k=1}^{N_c} \hat{a}_k^{a\dagger} \hat{b}_k^{a} + t_a' \sum_{k=1}^{N_c-1} \hat{a}_{k+1}^{a\dagger} \hat{b}_k^{a} + \text{H.c.} \end{cases}$$

Two decoupled SSH chains

$$t_a' = J_2' - J_3', \qquad t_a = J_2 - J_3,$$
$$t_s' = J_2' + J_3', \qquad t_s = J_2 + J_3.$$

SSH in a lattice of rings

1 atom

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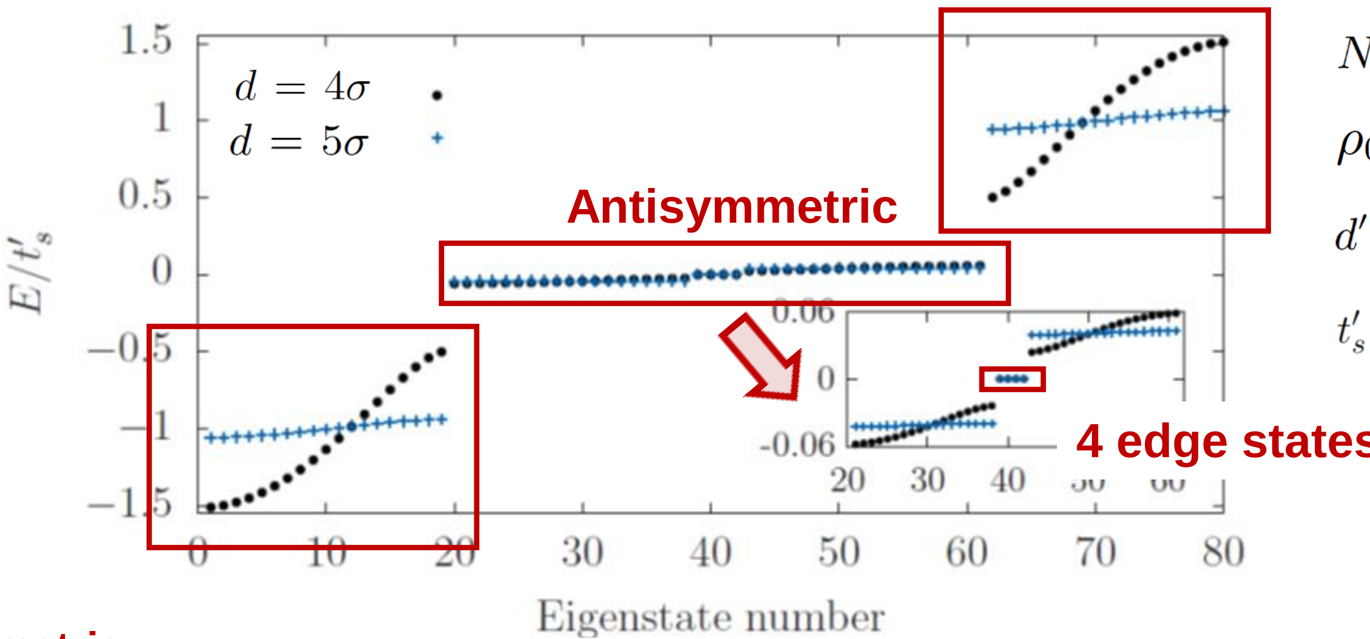
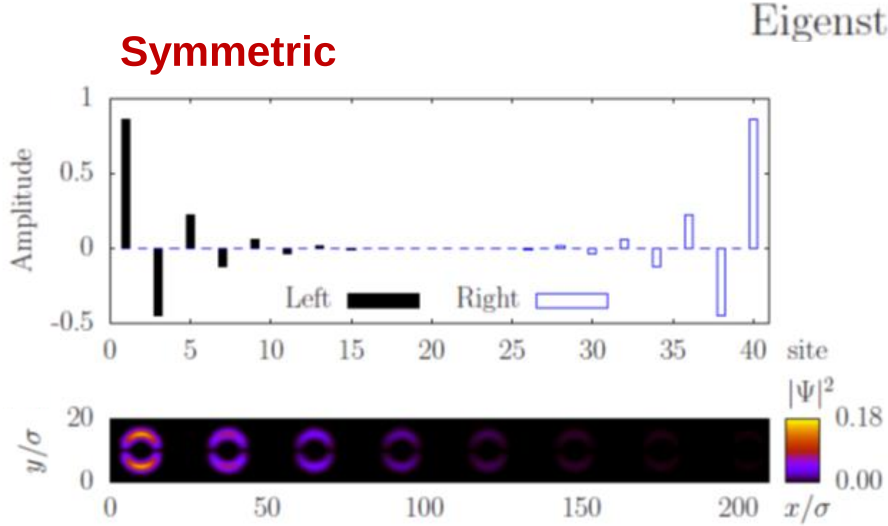


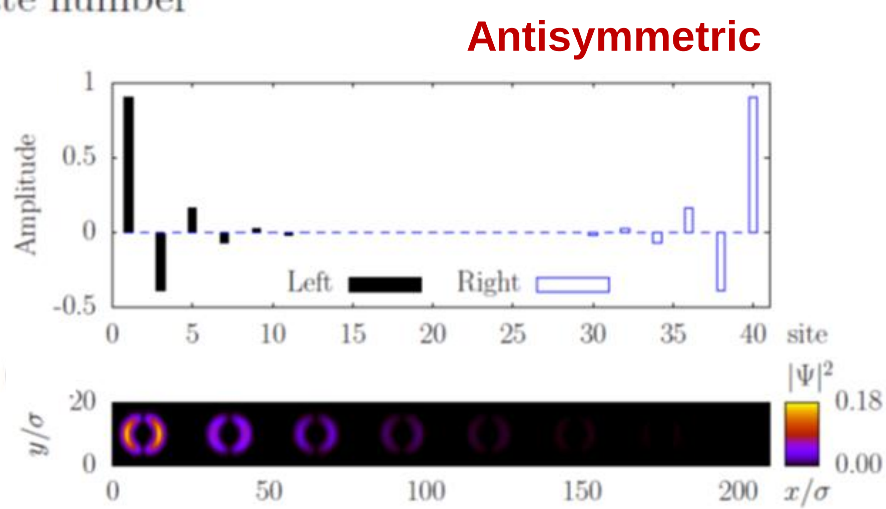
$N_c = 20$

$\rho_0 = 5\sigma$

$d' = 3.6\sigma$

$t'_s = J'_2 + J'_3$





SSH in a lattice of rings

2 atoms

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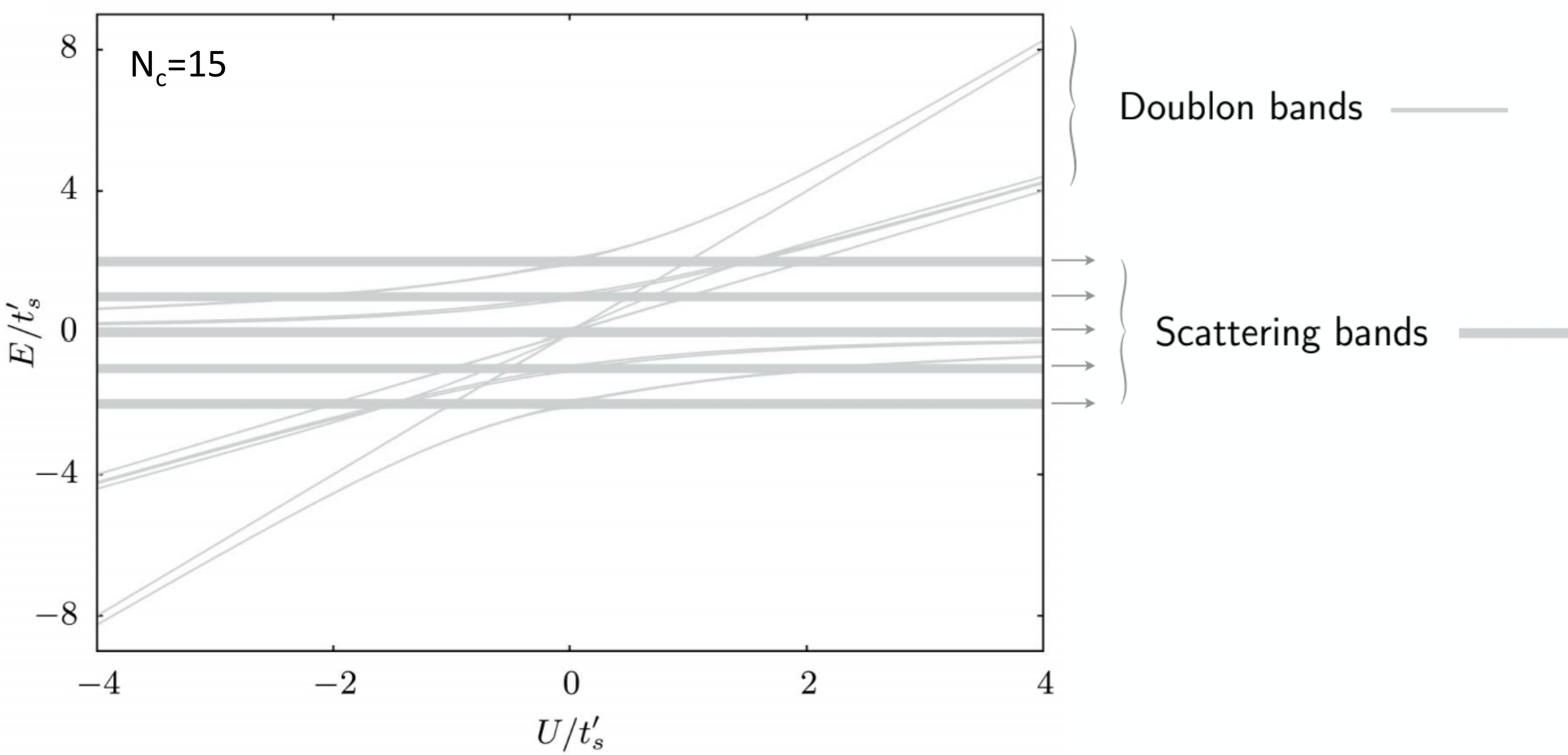
On-site boson interactions $\hat{\mathcal{H}}_{l=1}^{int} = \frac{U}{2}\sum_{j=A,B}\sum_{k=1}^{N_c}\left[\hat{n}_{j_k}^{+}(\hat{n}_{j_k}^{+}-1)+\hat{n}_{j_k}^{-}(\hat{n}_{j_k}^{-}-1)+4\hat{n}_{j_k}^{+}\hat{n}_{j_k}^{-}\right]$

$\rho_0 = 5\sigma \quad d = 5\sigma \quad d' = 3.6\sigma$



SSH in a lattice of rings

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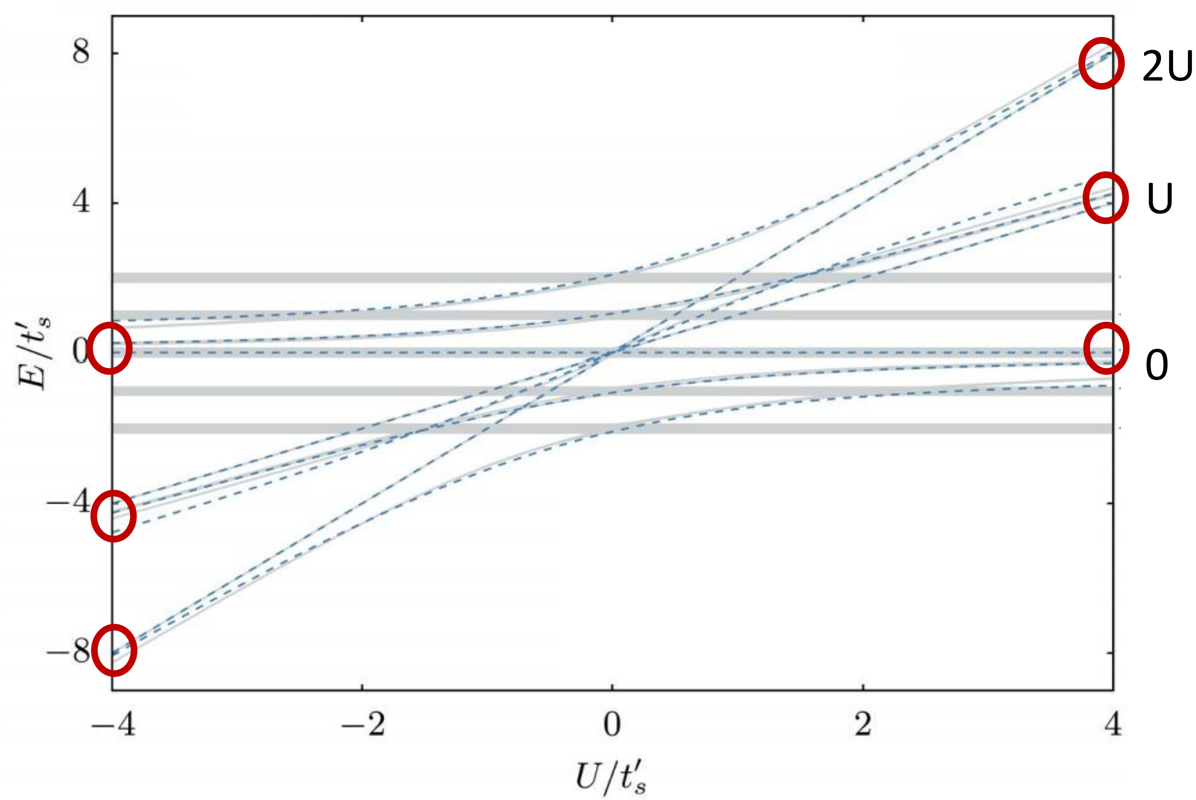
2 atoms

strong-link Hamiltonian



strong-link Hamiltonian

(10x10 matrix)

diagonalization

Doublon bands - - - - -

SSH in a lattice of rings

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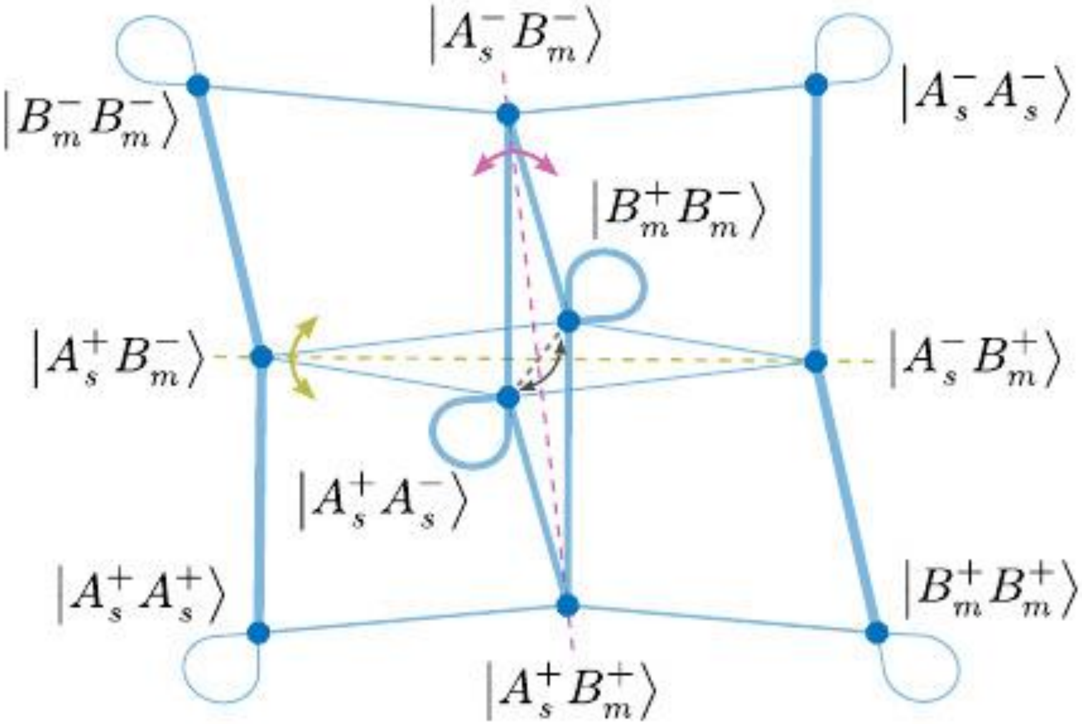
2 atoms

strong-link Hamiltonian



SSH in a lattice of rings

2 atoms

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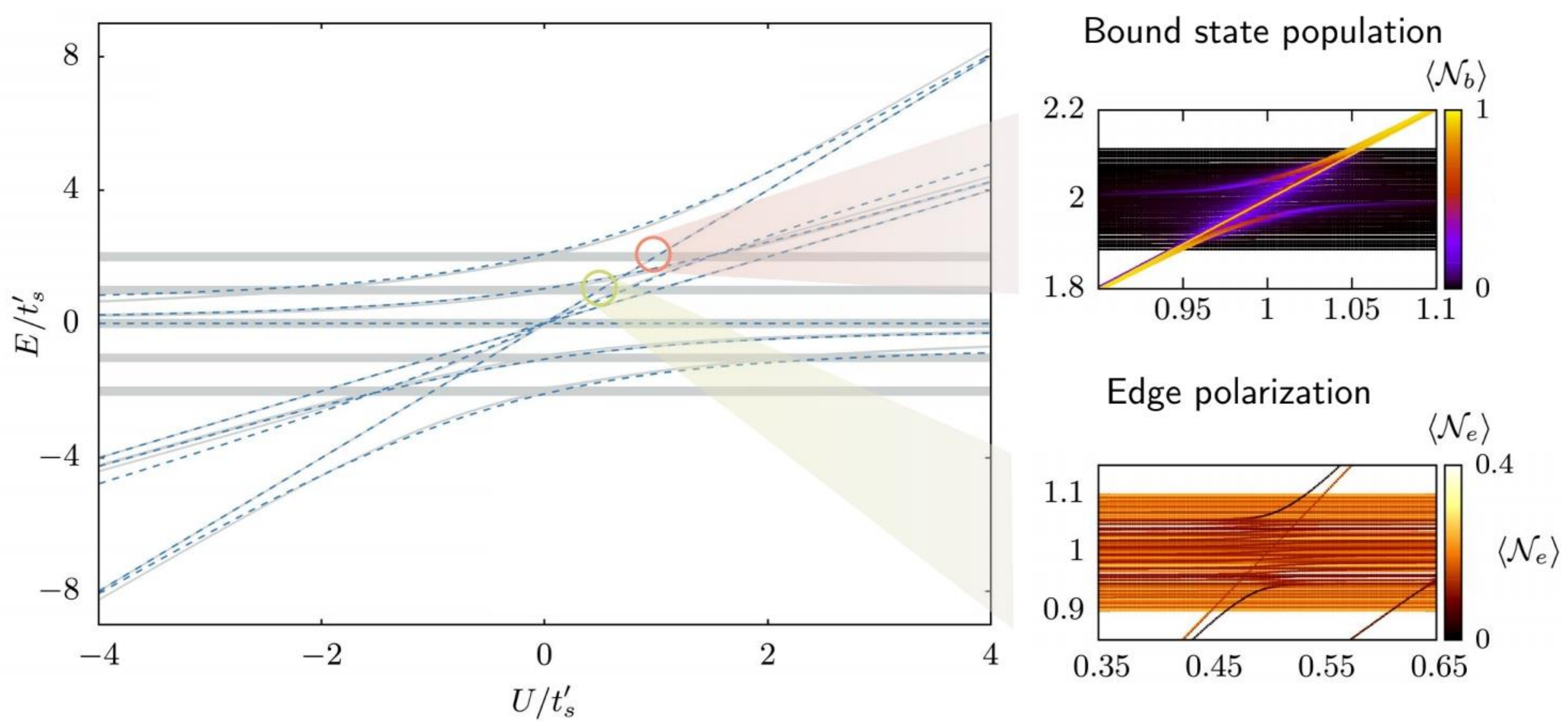
strong-link Hamiltonian

Crossings and avoided crossings



Exchange circulation symmetry → Well-defined parity → Avoided crossings for states of the same parity sector

2 atoms

SSH in a lattice of rings

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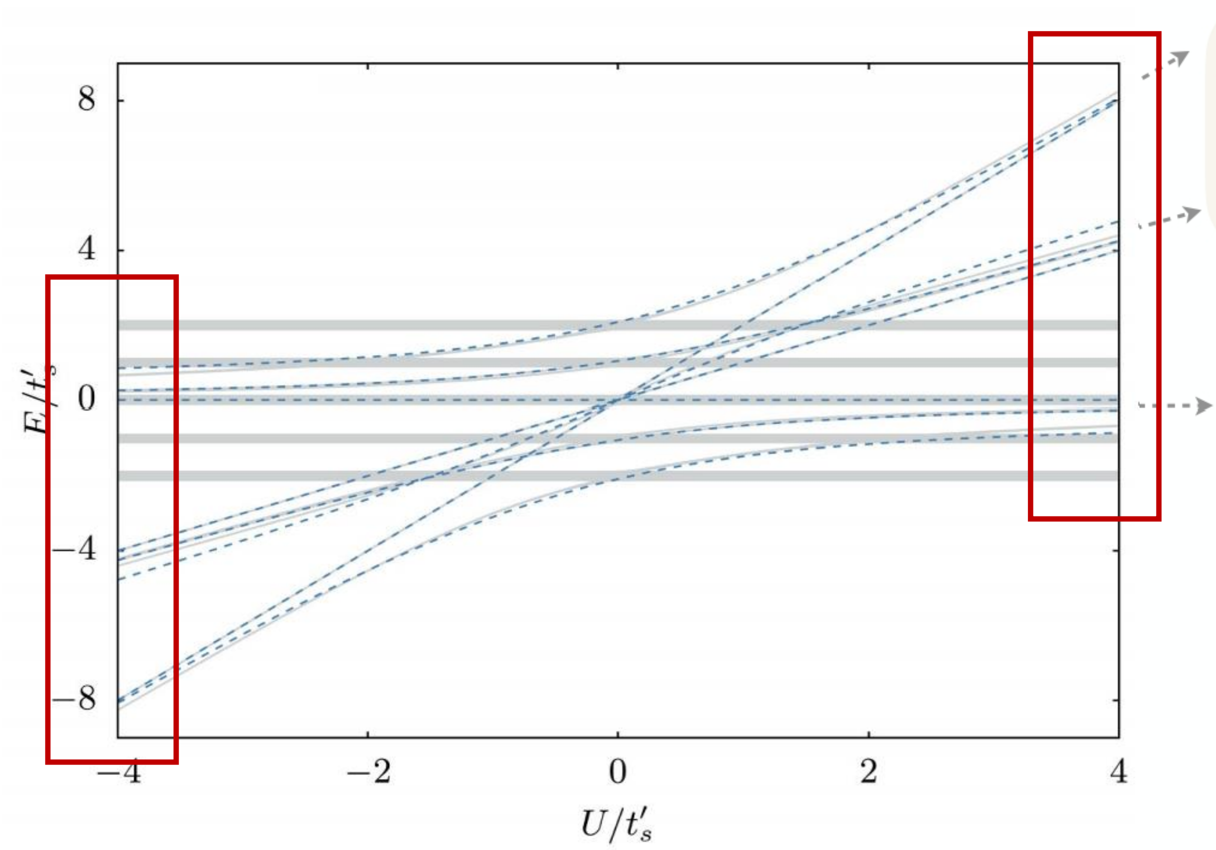
strongly interacting limit

Strongly-interacting subspaces $|U| \gg J$



Doublons

$\mathcal{B}$ Opposite circulation

$\mathcal{A}$ Same circulation

Particles in different sites

Second order perturbation theory: introduce couplings J as a perturbation

SSH in a lattice of rings

2 atoms

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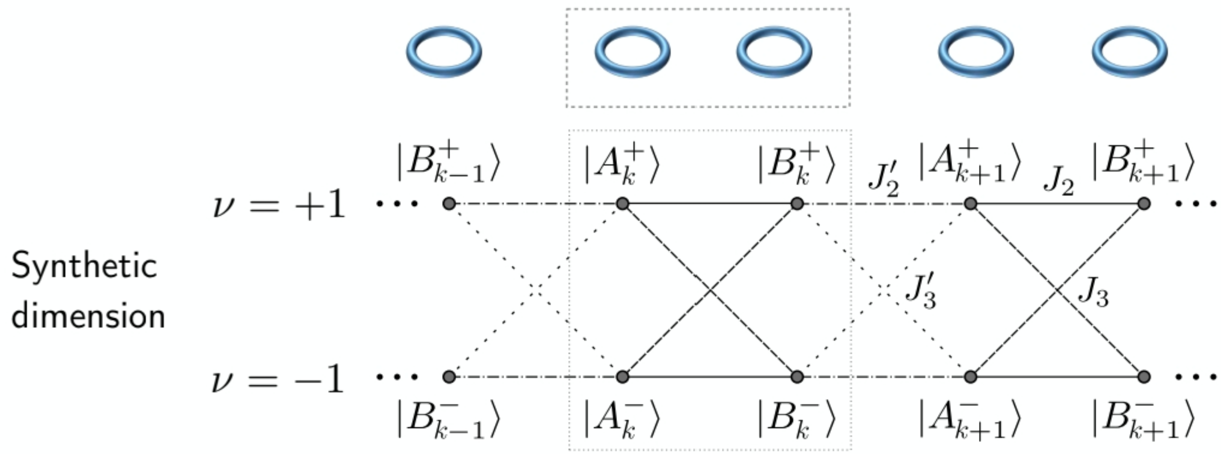

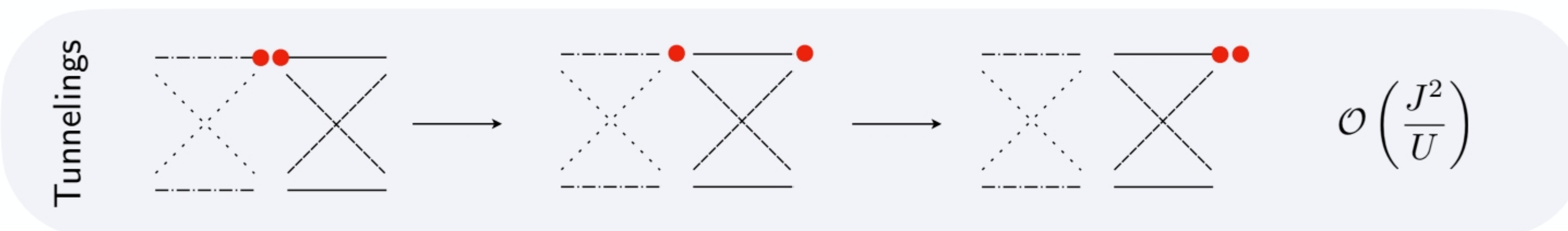

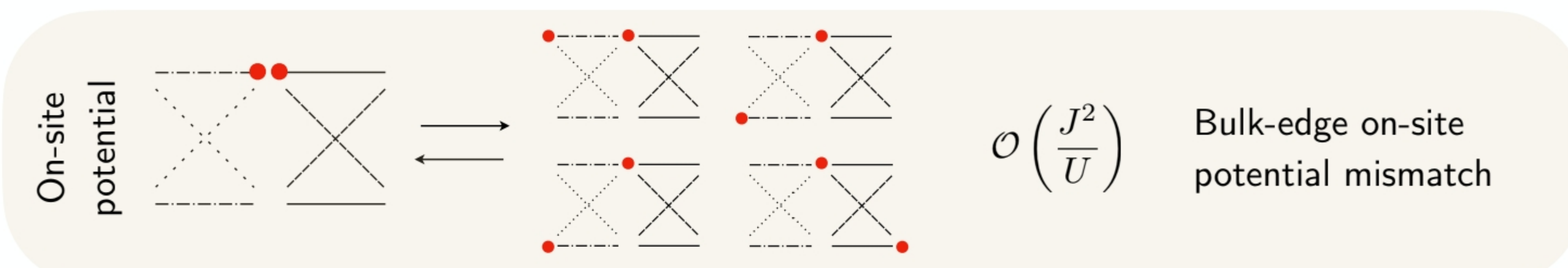

2 atoms

SSH in a lattice of rings

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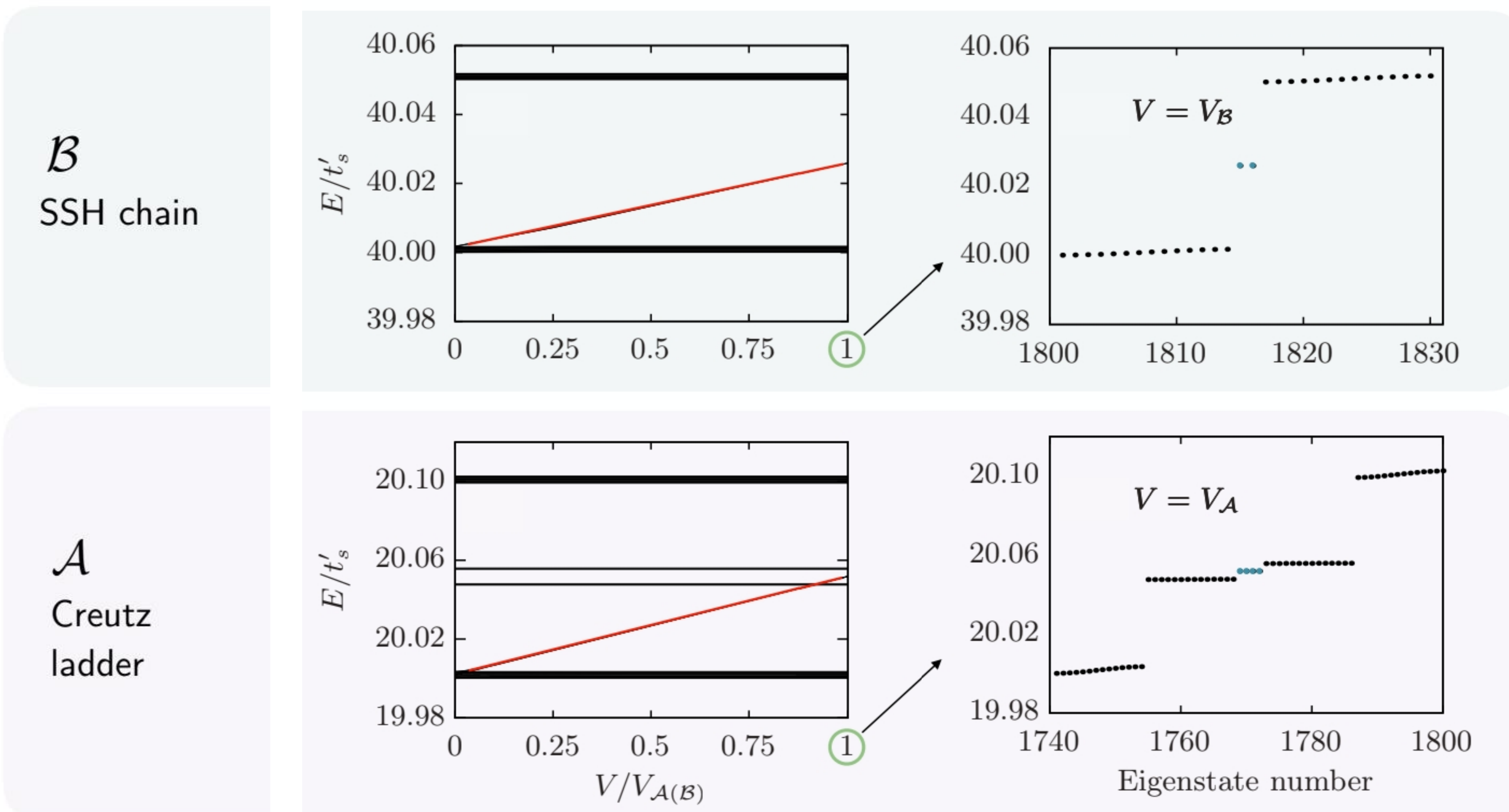
strongly interacting limit

Effective single-particle models



edge potential correction

(chiral symmetry, quantized Zak phase)

Tamm-Shockley states

Topologically protected edge states

Conclusions

Ultracold atoms carrying orbital angular momentum in lattices constitute a novel platform to explore topology for single atoms and interacting few-atoms systems.

As an example…

Single particle **we have characterized the model topologically through an exact mapping.**

Two particles **we have analyzed the interplay between interactions and topology in the dimerized limit, in terms of a strong-link Hamiltonian, and in the strongly interacting limit, by means of perturbation theory.**

Thank you for your attention