Many-body quantum heat engines based on free-Fermion systems

Davide Rossini

Atomtronics, 28 May 2024
Benasque Science Center, Spain

UNIVERSITÀ DI PISA

INFN
Istituto Nazionale
Fisica Nucleare
Sezione di Pisa

In collaboration with

Roberto Arezzo

SISSA, Trieste

Giulia Piccitto

Univ. Catania

Michele Campisi

CNR - NEST, Pisa

G. Piccitto, M. Campisi, D. Rossini, New J. Phys. **24**, 103023 (2022).
V. R. Arezzo, G. Piccitto, D. Rossini, arXiv:2403.11645 (2024).

Outline of the talk

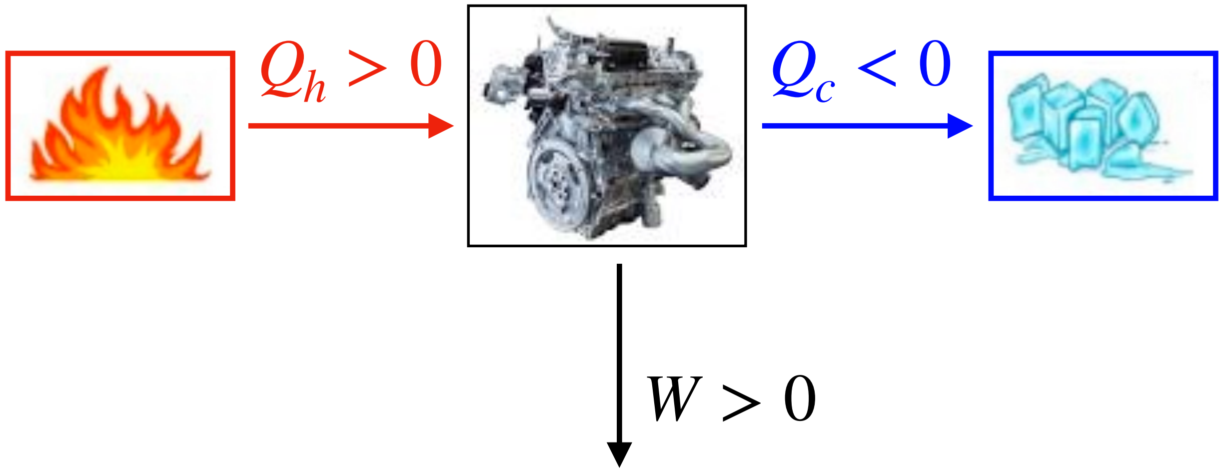
- Thermodynamics of microscopic systems
- Free-fermion systems
- Quantum Otto cycle
  - Ideal engine
  - Finite-duration engine

Thermodynamics of a thermal engine

I principle $dE = \delta Q - \delta W$

+

II principle $\sum_{i,\,\text{cycle}} \frac{Q_i}{T_i} \leq 0$



Thermodynamic cycles

Heater: $Q_h < 0,\;\; Q_c < 0,\;\; W < 0.$

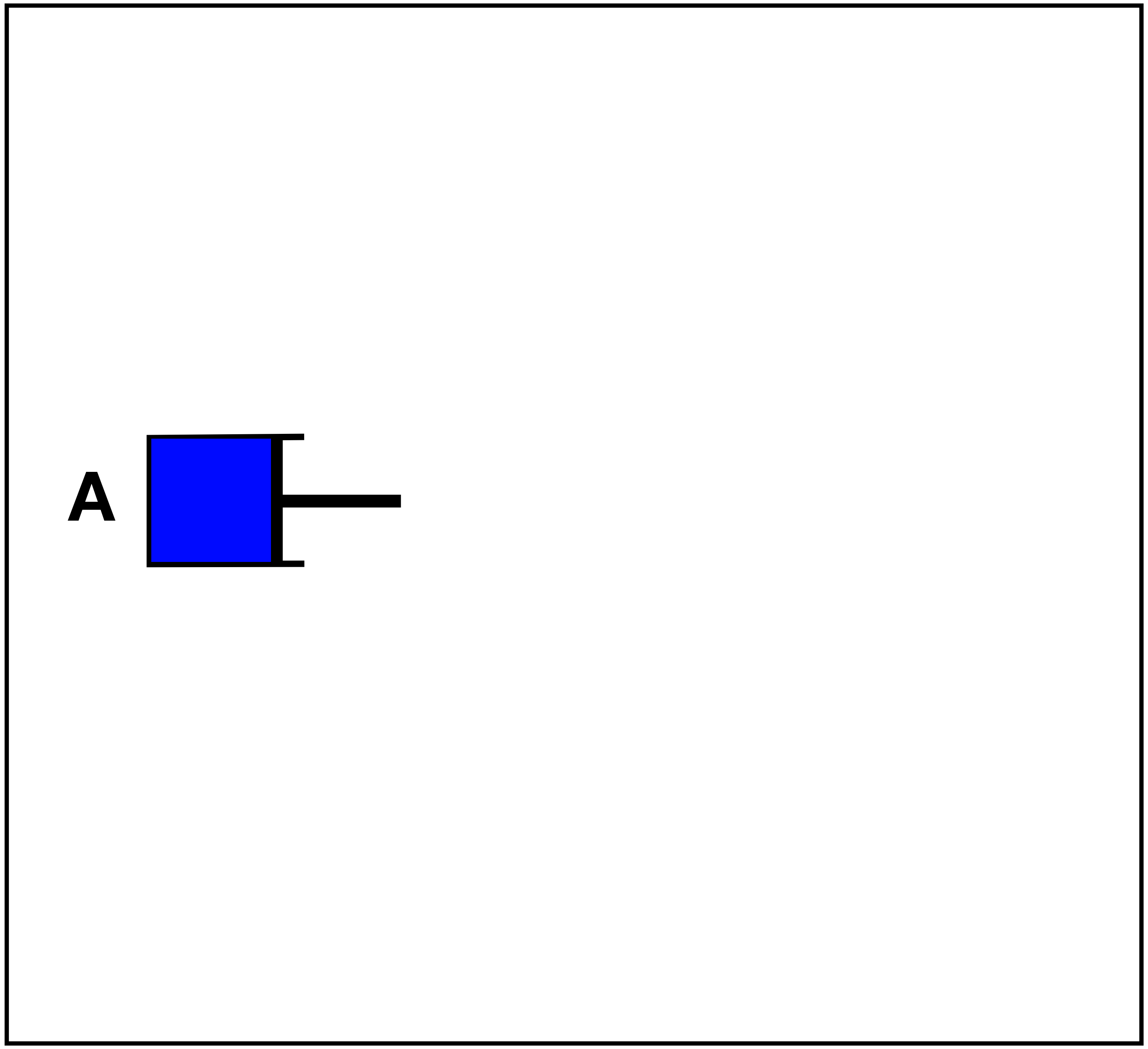
Accelerator: $Q_h > 0,\;\; Q_c < 0,\;\; W < 0.$

Heat engine: $Q_h > 0,\;\; Q_c < 0,\;\; W > 0.$

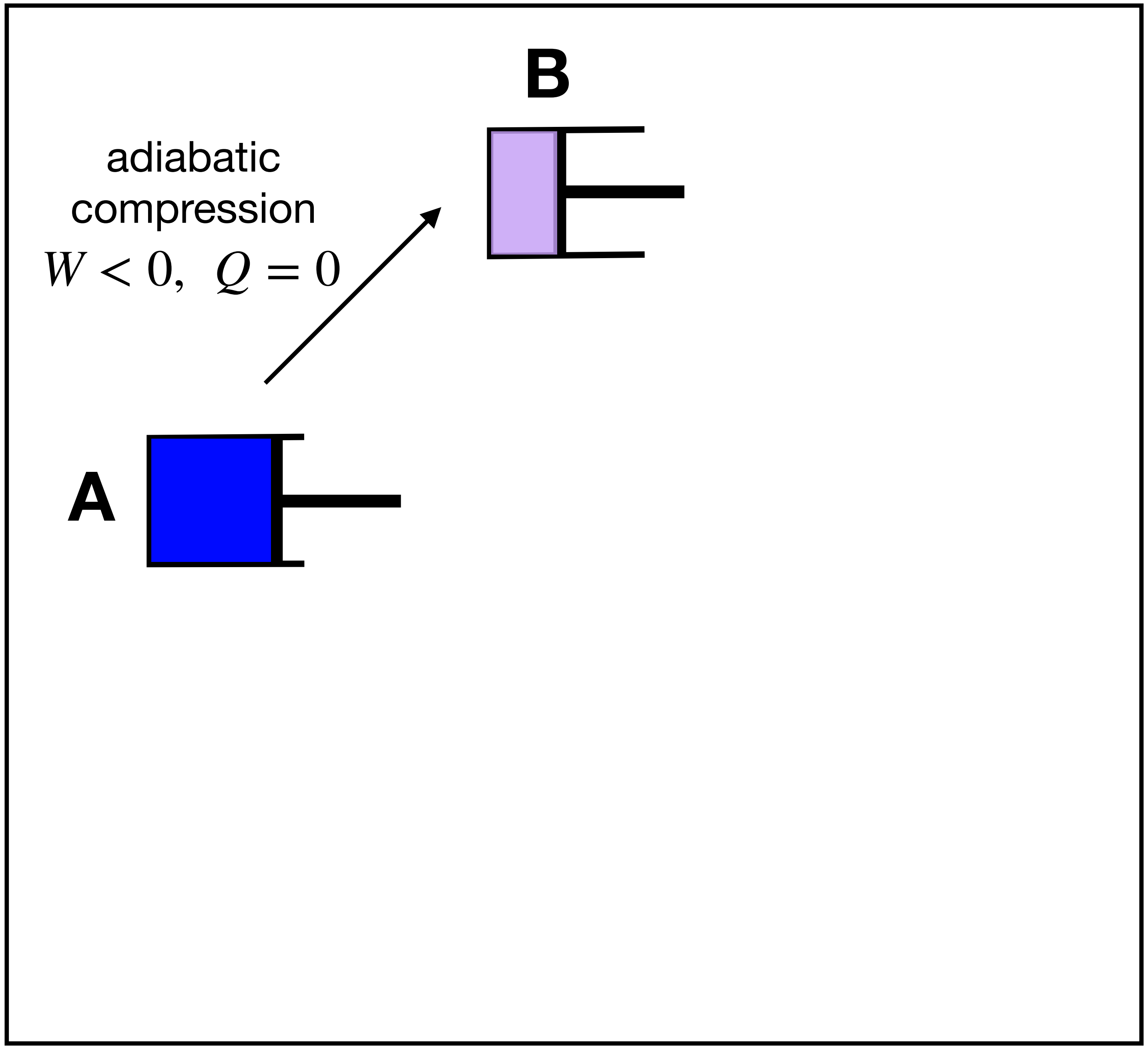
Refrigerator: $Q_h < 0,\;\; Q_c > 0,\;\; W < 0.$

A. Solfanelli, M. Falsetti, M. Campisi, PRB **101**, 054513 (2020)

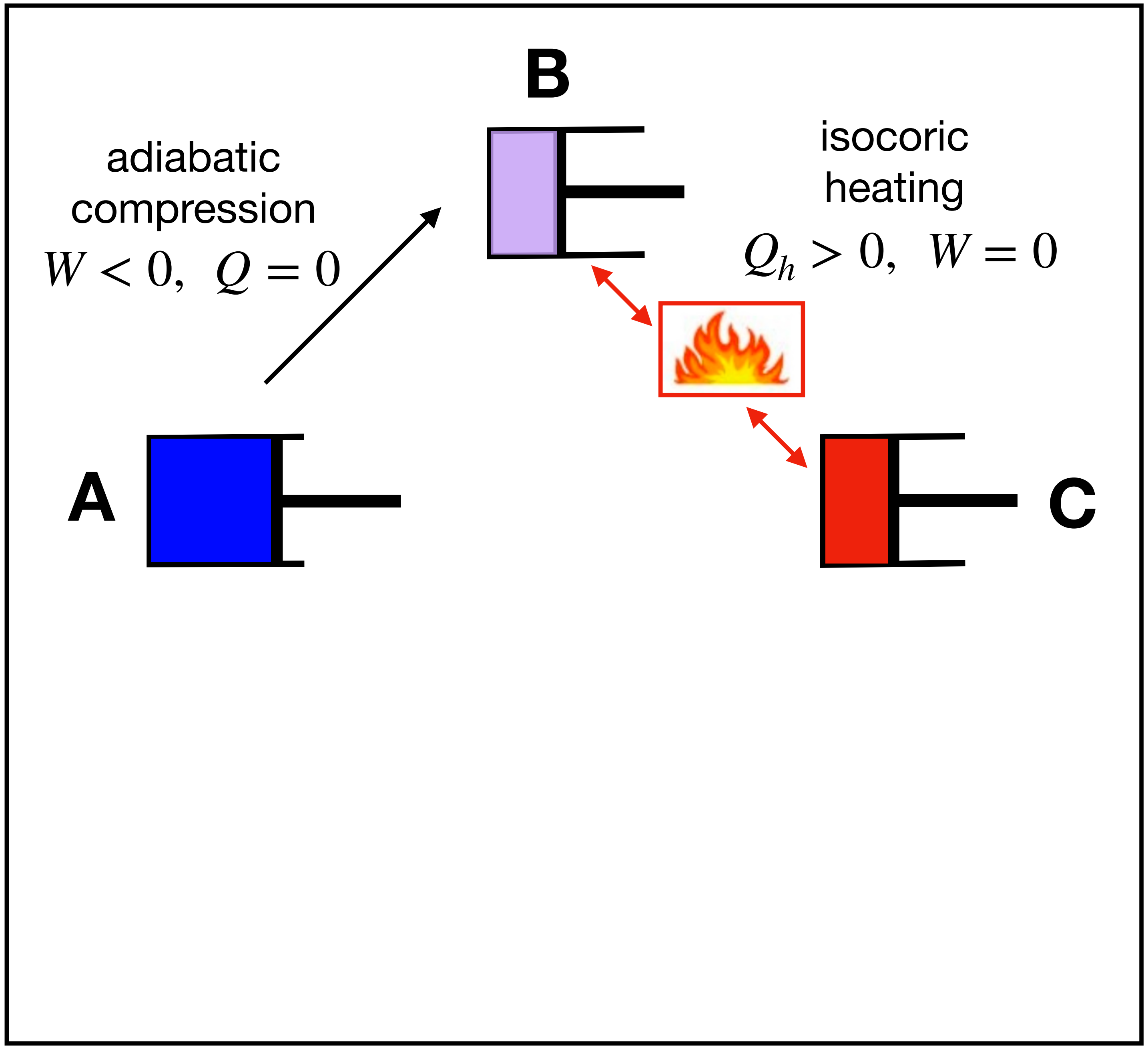
The Otto cycle



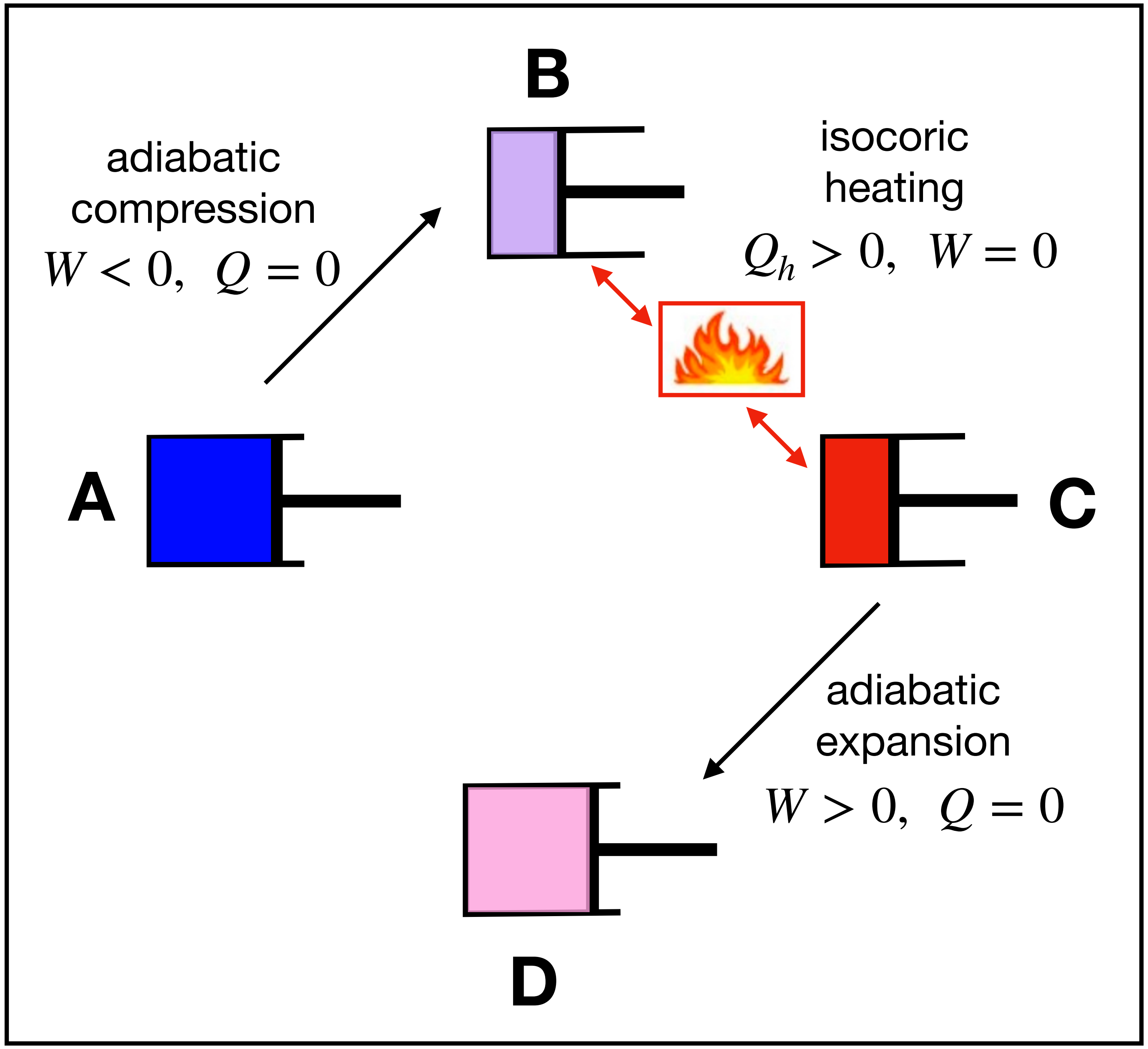
The Otto cycle



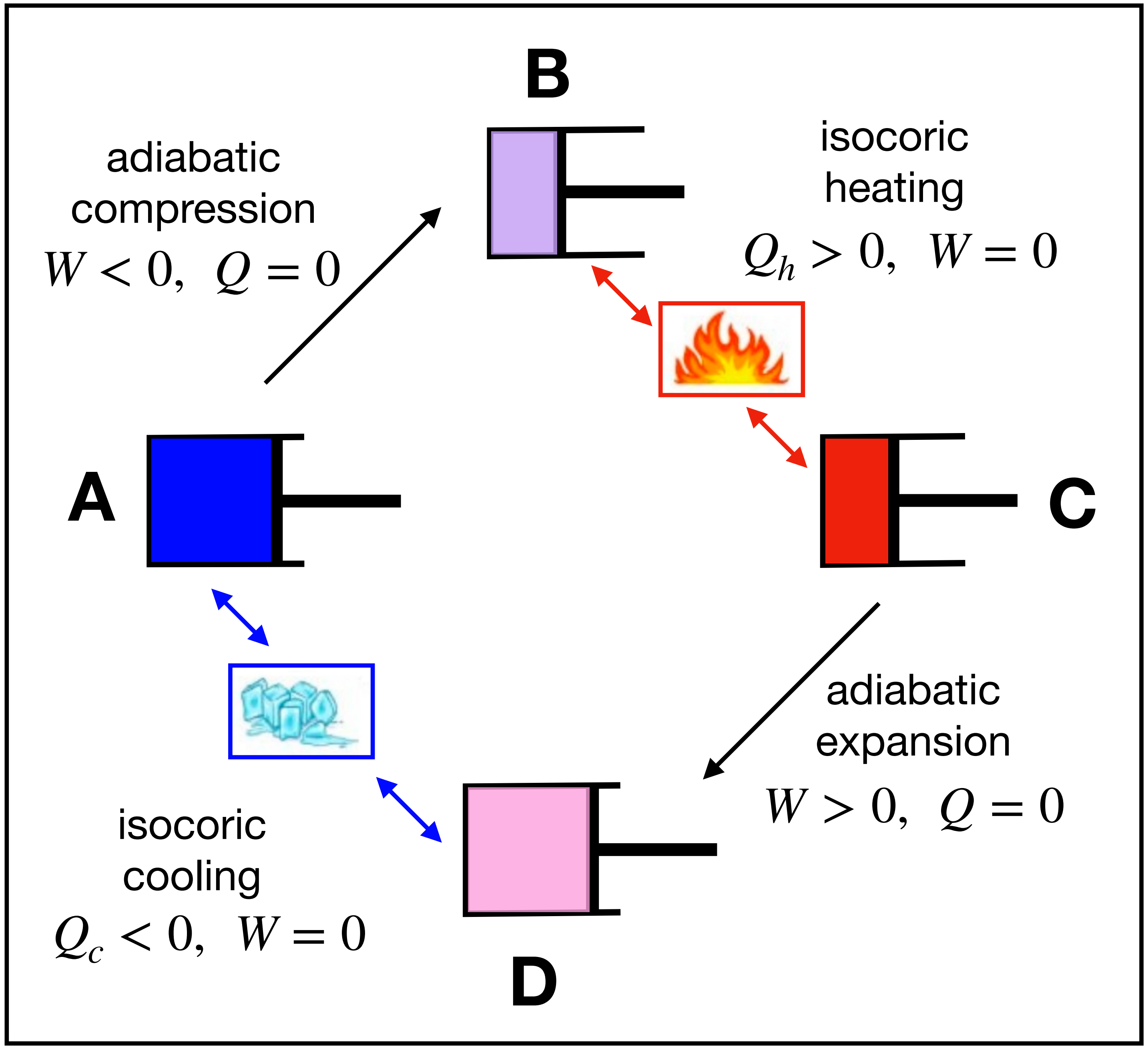
The Otto cycle



The Otto cycle



The Otto cycle



B

adiabatic
compression
$W < 0, \ \ Q = 0$

isocoric
heating
$Q_h > 0, \ \ W = 0$

A

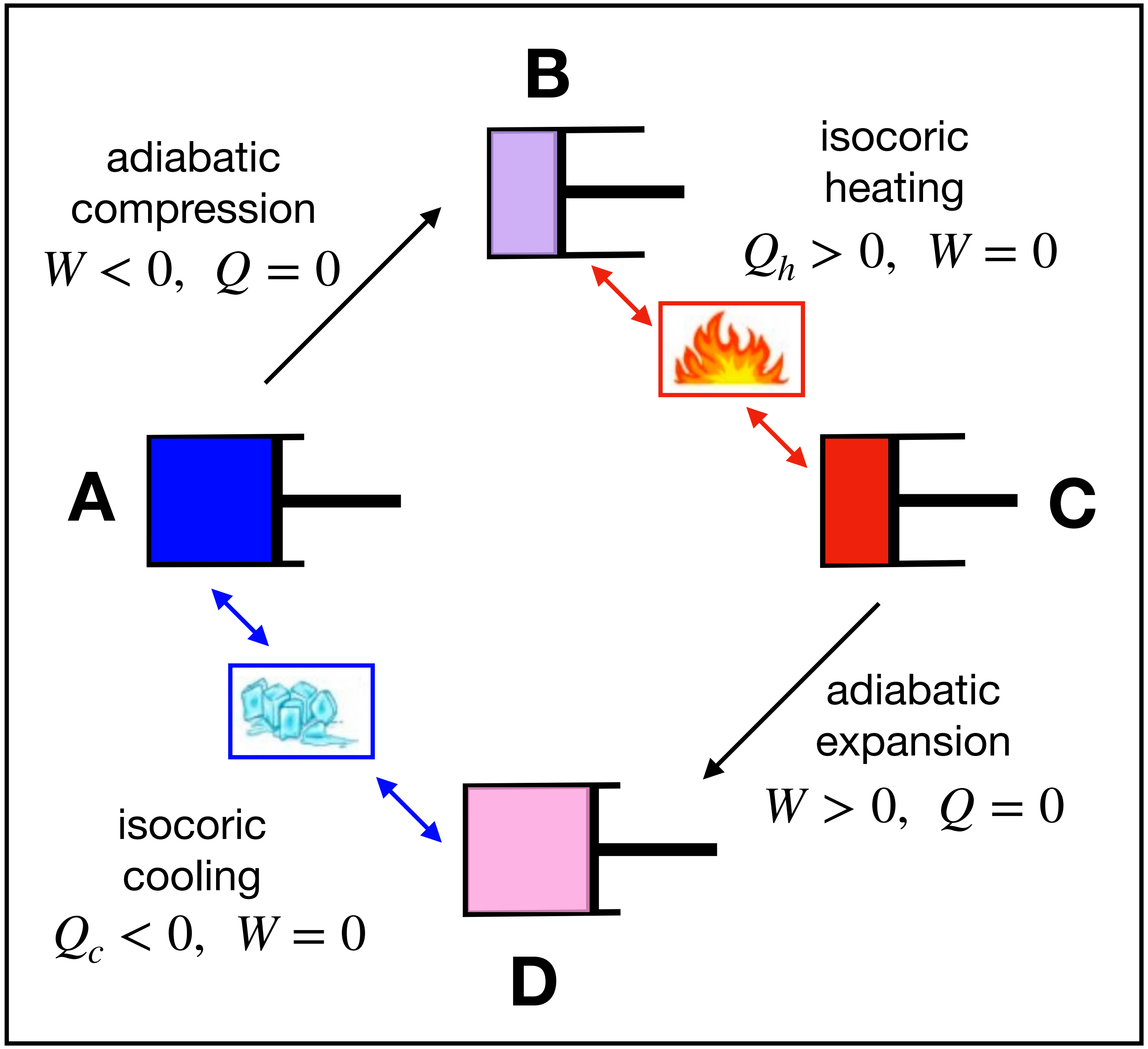
C

adiabatic
expansion
$W > 0, \ \ Q = 0$

isocoric
cooling
$Q_c < 0, \ \ W = 0$

D

The Otto cycle



Going quantum??

Thermodynamics of microscopic systems

Classical systems: heat & work can be easily defined.

Quantum systems: additional care must be taken.

R. Alicki, J. Phys. A **12**, L103 (1979)
H.H. Quan, Y. Liu, C. Sụn, F. Nori, PRE **76**, 031105 (2007)

Thermodynamics of microscopic systems

Classical systems: heat & work can be easily defined.

Quantum systems: additional care must be taken.

$$dE(\lambda) = \mathrm{Tr}\big[dH(\lambda)\,\rho(\lambda)\big] + \mathrm{Tr}\big[H(\lambda)\,d\rho(\lambda)\big]$$

R. Alicki, J. Phys. A **12**, L103 (1979)
H.H. Quan, Y. Liu, C. Sụn, F. Nori, PRE **76**, 031105 (2007)

Thermodynamics of microscopic systems

Classical systems: heat & work can be easily defined.

Quantum systems: additional care must be taken.

$$dE(\lambda) = \mathrm{Tr}\big[dH(\lambda)\,\rho(\lambda)\big] + \mathrm{Tr}\big[H(\lambda)\,d\rho(\lambda)\big]$$

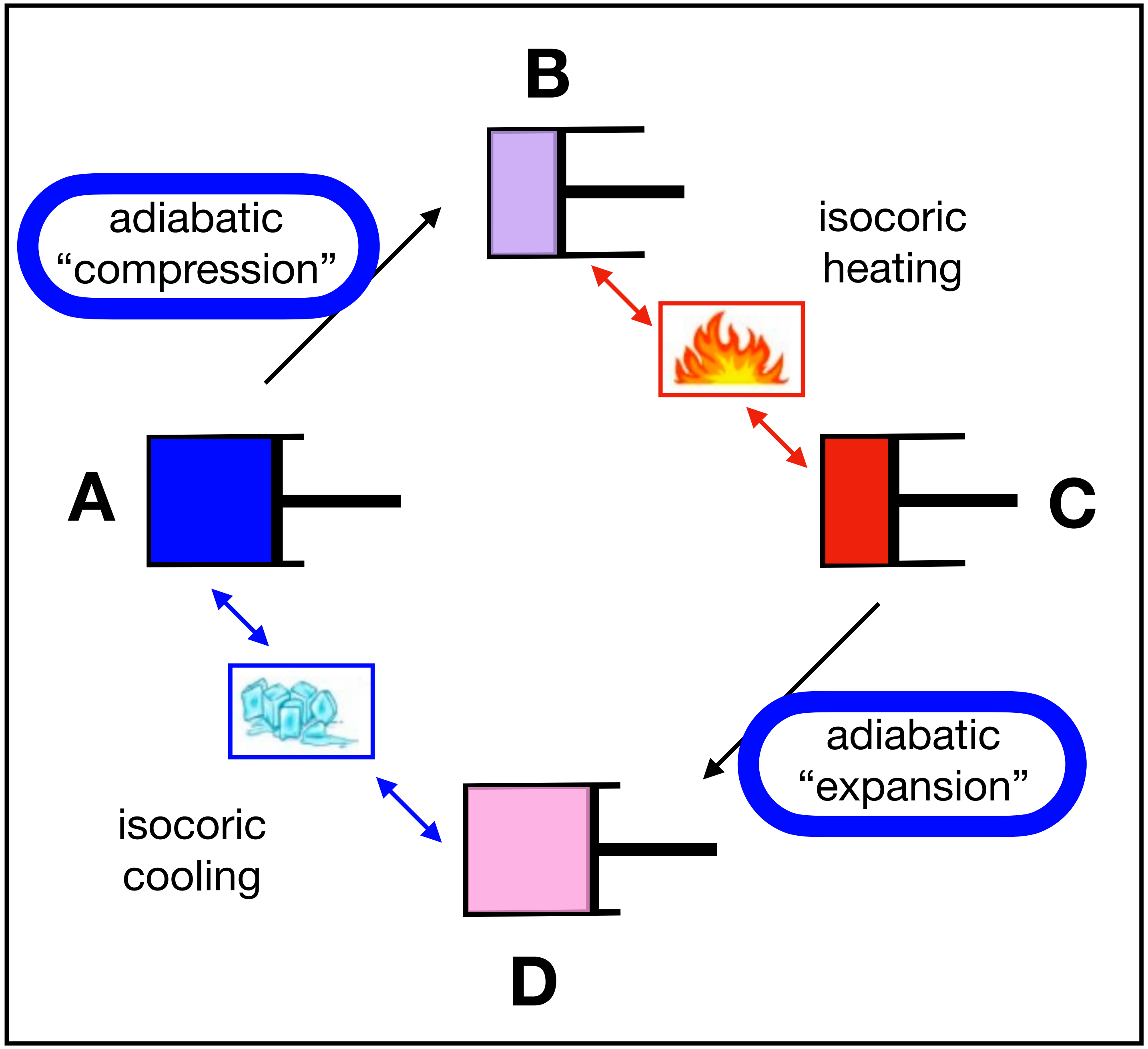
modification of the spectral
structure of the system

variation of the state
of the system

➡ identifying W and Q with the above two terms is not obvious

R. Alicki, J. Phys. A **12**, L103 (1979)
H.H. Quan, Y. Liu, C. Sụn, F. Nori, PRE **76**, 031105 (2007)

The Otto cycle



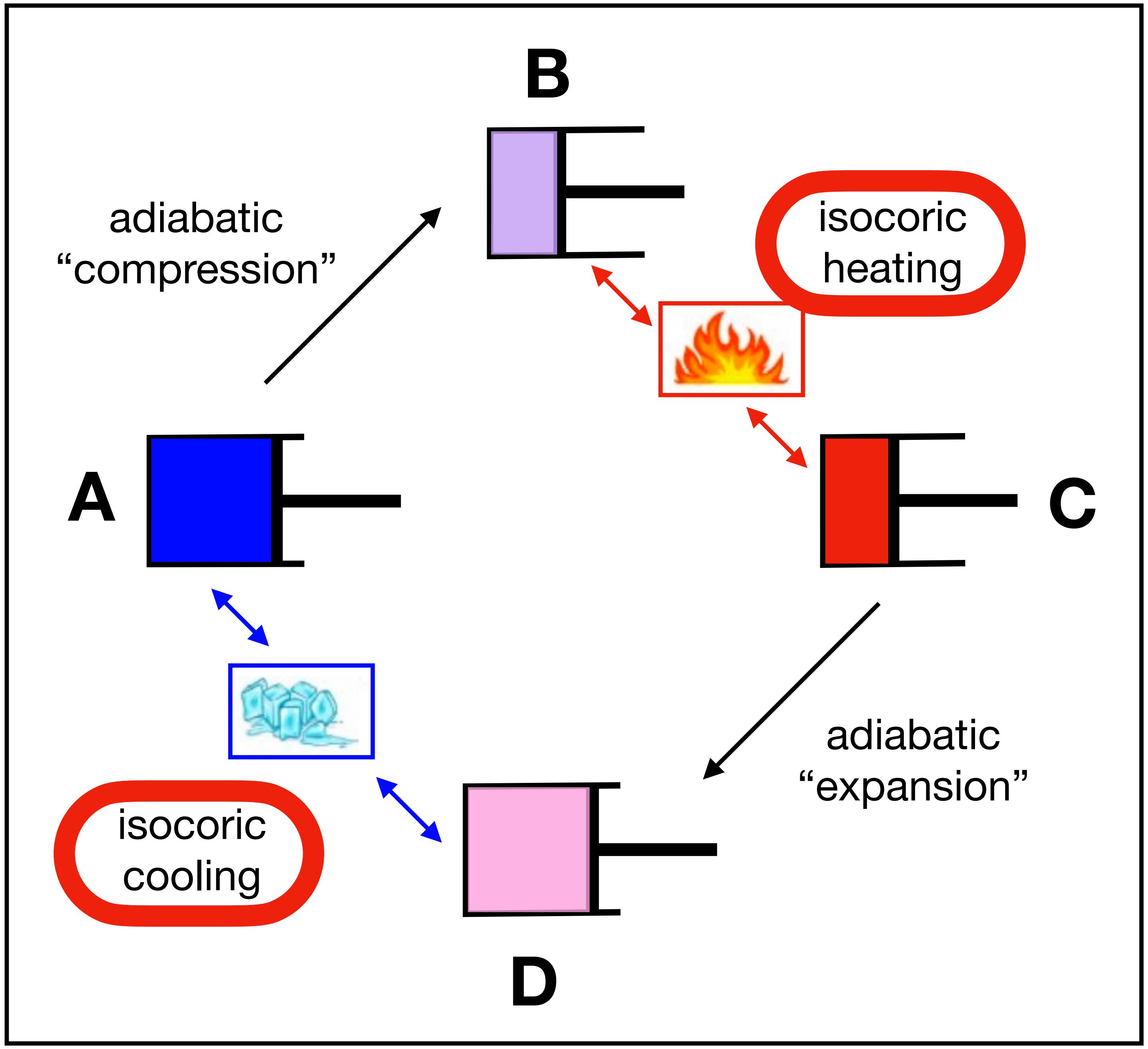
Going quantum??

Adiabatic transformations:

Unitary evolution $\lambda(t)$ changes the energy levels of $H[\lambda(t)]$.

$$Q = 0; \quad W \equiv -\Delta E = -\int_{t_i}^{t_f} \partial_{t'} \left\langle H[\lambda(t')] \right\rangle_{\rho(t')} dt'$$
$$= \left\langle H[\lambda(t_i)] \right\rangle_{\rho(t_i)} - \left\langle H[\lambda(t_f)] \right\rangle_{\rho(t_f)}$$

The Otto cycle



Going quantum??

Adiabatic transformations:

Unitary evolution $\lambda(t)$ changes the energy levels of $H[\lambda(t)]$.

$$Q = 0; \quad W \equiv -\Delta E = -\int_{t_i}^{t_f} \partial_{t'} \big\langle H[\lambda(t')] \big\rangle_{\rho(t')} \, dt'$$

$$= \big\langle H[\lambda(t_i)] \big\rangle_{\rho(t_i)} - \big\langle H[\lambda(t_f)] \big\rangle_{\rho(t_f)}$$

Isochoric transformations:

λ is fixed. The system in contact with a bath does not evolve unitarily.

$$W = 0; \quad Q = \langle H(\lambda) \rangle_{\rho(t_f)} - \langle H(\lambda) \rangle_{\rho(t_i)}$$

Heat engines with quantum many-body systems

The working medium can be a gas of interacting atoms

Jaramillo et al., Quantum supremacy of many-particle thermal machines, NJP **18**, 075019 (2016)
J. Bengtsson et al., Quantum Szilard engine with attractively interacting bosons, PRL **120**, 100601 (2018)
Y. Chen et al., Interaction-driven many-particle quantum heat engine and its universal behavior, npj Quant. Inf. **5**, 88 (2019)
N. Yunger Halpern et al., Quantum engine based on many-body localization, PRB **99**, 024203 (2019).
F. Carollo et al., Nonequilibrium quantum many-body Rydberg atom engine, PRL **124**, 170602 (2020).
T. Fogarty and T. Busch, A many-body heat engine at criticality, Quantum Sci. Technol. **6**, 015003 (2021)
M. Boubakour, T. Fogarty, and T. Busch, Interaction-enhanced quantum heat engine, PRR **5**, 013088 (2023)
R. S. Watson et al., Interaction-driven quantum many-body engine enabled by atom-atom correlations arXiv:2308.05266
…

or even a system of several interacting quantum spins

Q. Wang, Performance of quantum heat engines under the influence of long-range interactions PRE **102** 012138 (2020)
B.S. Revathy et al, Universal finite-time thermodynamics of many-body q. machines from KZ scaling, PRR **2**, 043247(2020)
A. Solfanelli et al., Quantum heat engine with long-range advantages, NJP **25**, 033030 (2023)
L. A. Williamson and M. J. Davis, Many-body enhancement in a spin-chain quantum heat engine, PRB **109**, 024310 (2024)
…

Free-fermion systems

$$H = \sum_{i,j} D_{i,j} c_i^\dagger c_j + \tfrac{1}{2}\left(O_{i,j} c_i^\dagger c_j^\dagger + \mathrm{hc}\right),$$

$$D = D^\dagger, \quad O = -O^T$$

can be cast into a free-quasiparticle model through a Bogoliubov transformation:

$$H = \sum_k \omega_k \left(b_k^\dagger b_k - \frac{1}{2}\right)$$

$$\omega_k \equiv \omega_k(\lambda)$$

spectrum of the
(fermonic) quasiparticles

Thermal state: $\rho_\beta(\lambda) \propto e^{-\beta H(\lambda)}$

$$\langle b_k^\dagger b_k \rangle_{\rho_\beta(\lambda)} = \frac{1}{1 + e^{-\beta \omega_k(\lambda)}} \equiv f[\beta, \omega_k(\lambda)]$$

Fermi-Dirac distribution

Ideal transformations for free fermions

Isotherm: λ: $\lambda_i \to \lambda_f$ varies slowly in time, the system stays in thermal equilibrium with a bath at temperature β^{-1}

$$W \equiv -\Delta F = -\int_{\lambda_i}^{\lambda_f} \sum_k \left[\partial_\lambda \omega_k(\lambda)\right] \left[\frac{1}{1+e^{\beta\omega_k(\lambda)}} - \frac{1}{2}\right] d\lambda; \quad Q = \Delta E + W$$

Ideal transformations for free fermions

Isotherm: λ: $\lambda_i \to \lambda_f$ varies slowly in time, the system stays in thermal equilibrium with a bath at temperature β^{-1}

$$W \equiv -\Delta F = -\int_{\lambda_i}^{\lambda_f} \sum_k \left[\partial_\lambda \omega_k(\lambda)\right] \left[\frac{1}{1+e^{\beta\omega_k(\lambda)}} - \frac{1}{2}\right] d\lambda; \qquad Q = \Delta E + W$$

Adiabatic: no heat exchange; λ: $\lambda_i \to \lambda_f$ varies slowly in time, the quantum adiabatic theorem can be invoked

$$Q = 0; \qquad W \equiv -\Delta E = -\sum_k \left[\frac{1}{1+e^{\beta\omega_k(\lambda_i)}} - \frac{1}{2}\right] \left[\omega_k(\lambda_f) - \omega_k(\lambda_i)\right]$$

Ideal transformations for free fermions

Isotherm: λ: $\lambda_i \to \lambda_f$ varies slowly in time, the system stays in thermal equilibrium with a bath at temperature β^{-1}

$$W \equiv -\Delta F = -\int_{\lambda_i}^{\lambda_f} \sum_k \left[\partial_\lambda \omega_k(\lambda)\right] \left[\frac{1}{1+e^{\beta\omega_k(\lambda)}} - \frac{1}{2}\right] d\lambda; \quad Q = \Delta E + W$$

Adiabatic: no heat exchange; λ: $\lambda_i \to \lambda_f$ varies slowly in time, the quantum adiabatic theorem can be invoked

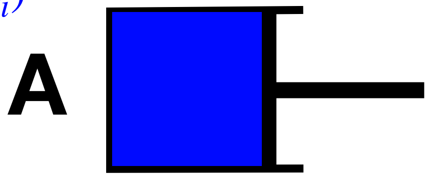
$$Q = 0; \quad W \equiv -\Delta E = -\sum_k \left[\frac{1}{1+e^{\beta\omega_k(\lambda_i)}} - \frac{1}{2}\right] \left[\omega_k(\lambda_f) - \omega_k(\lambda_i)\right]$$

Isochoric: no variations of λ; initially equilibrium with a bath β_i, then thermalization with another bath β_f

$$W = 0; \quad Q \equiv \langle H \rangle_{\rho_f} - \langle H \rangle_{\rho_i} = \sum_k \omega_k \left[f(\beta_2, \omega_k) - f(\beta_1, \omega_k)\right]$$

Ideal quantum Otto cycle

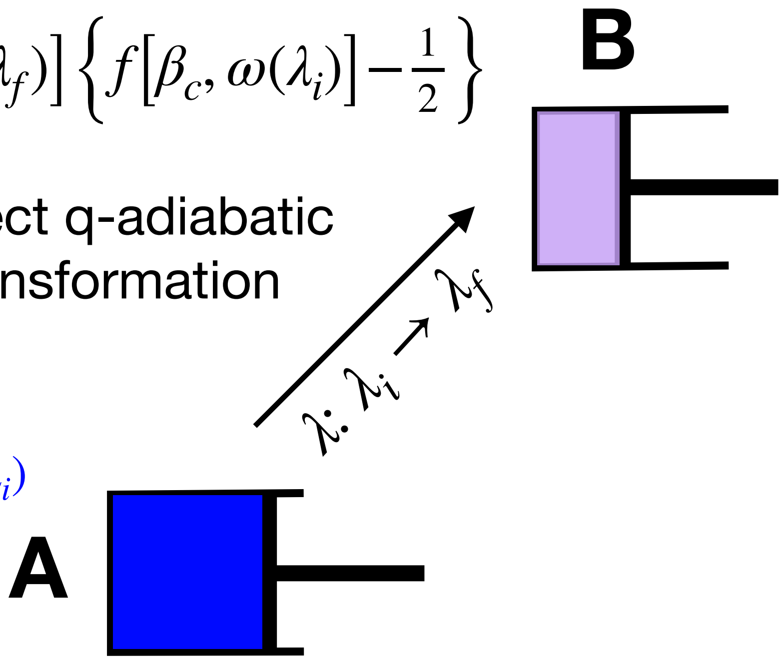
$\rho_A \propto e^{-\beta_c H(\lambda_i)}$



Ideal quantum Otto cycle

$$W_{AB} = \sum_k \left[\omega_k(\lambda_i) - \omega_k(\lambda_f)\right]\left\{f\left[\beta_c, \omega(\lambda_i)\right] - \frac{1}{2}\right\}$$

$\rho_A \propto e^{-\beta_c H(\lambda_i)}$



perfect q-adiabatic transformation

λ: $\lambda_i \to \lambda_f$

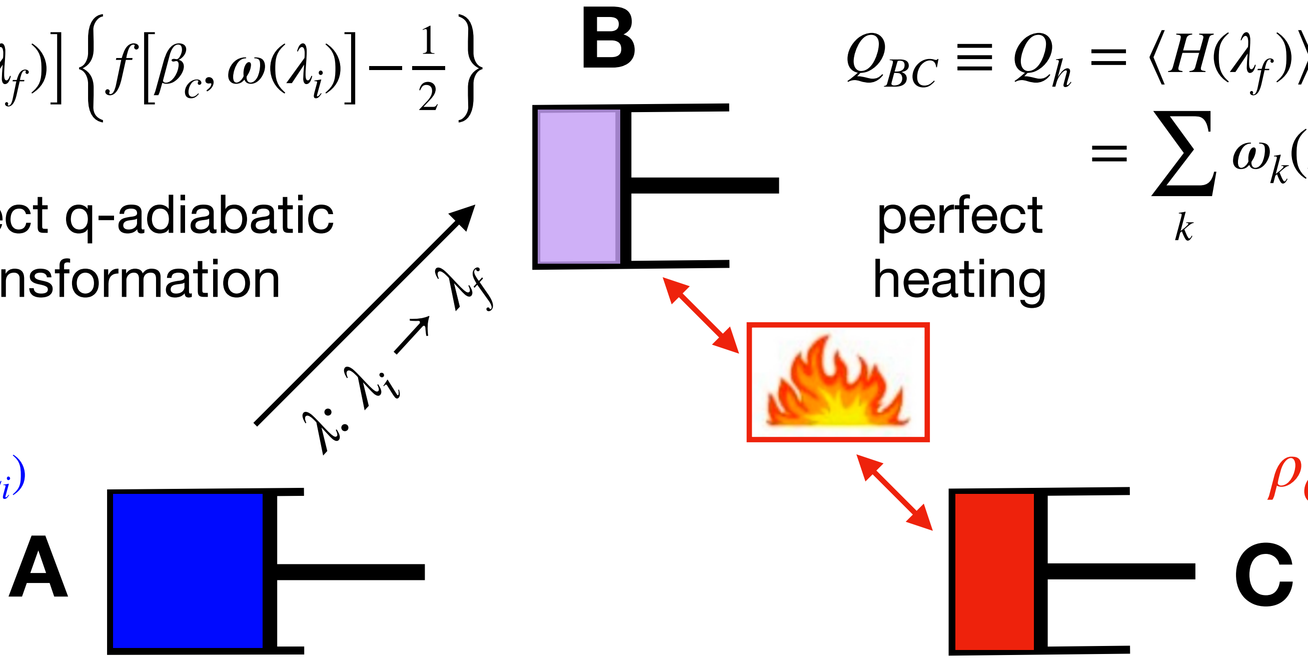
Ideal quantum Otto cycle

$$W_{AB} = \sum_k \left[\omega_k(\lambda_i) - \omega_k(\lambda_f)\right] \left\{ f\left[\beta_c, \omega(\lambda_i)\right] - \frac{1}{2} \right\}$$

$$Q_{BC} \equiv Q_h = \langle H(\lambda_f) \rangle_{\rho_f} - \langle H(\lambda_f) \rangle_{\rho_i}$$
$$= \sum_k \omega_k(\lambda_f) \left(\Delta f_k\right)_{hc}$$

perfect q-adiabatic transformation

perfect heating

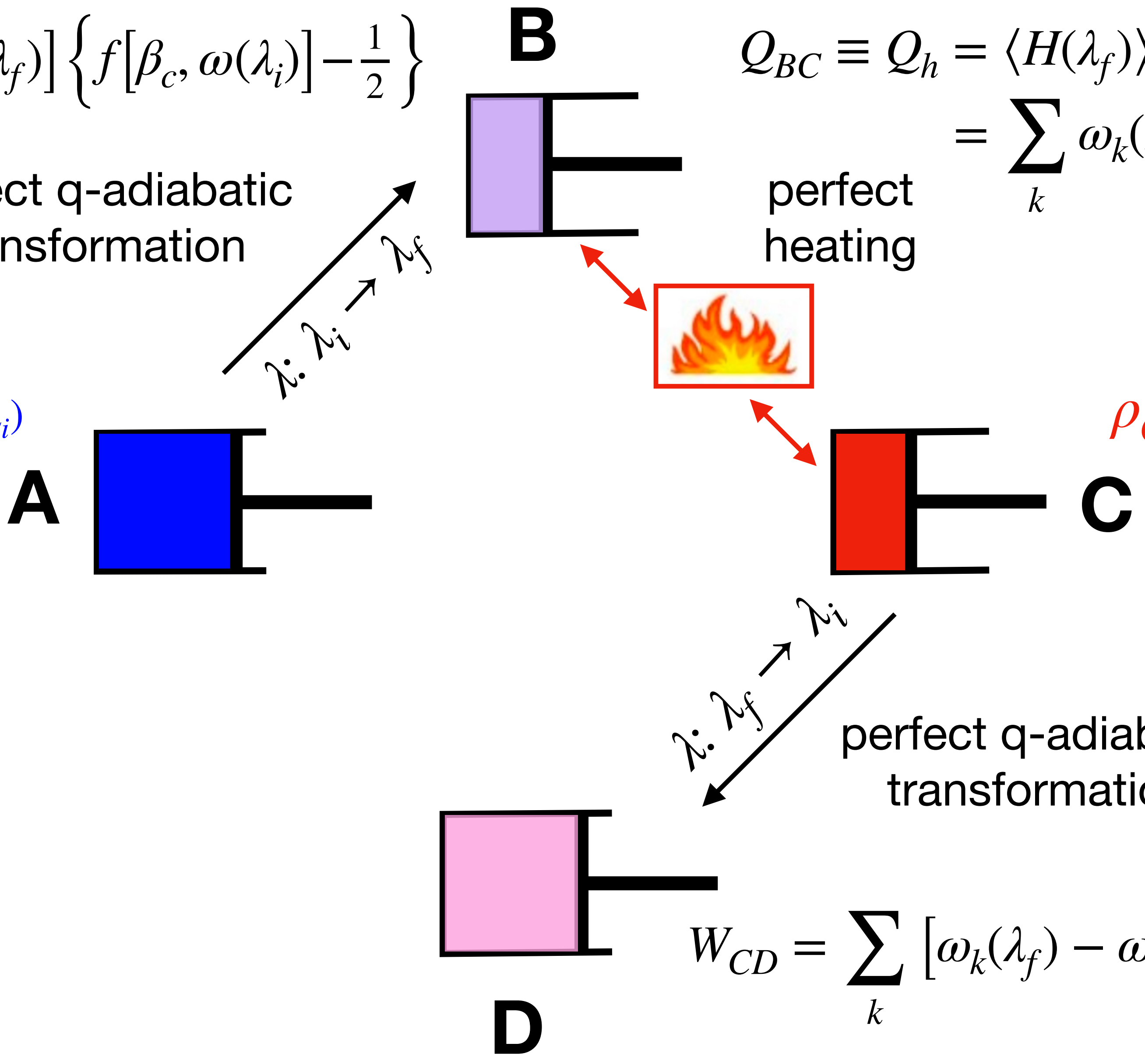


$$\left(\Delta f_k\right)_{hc} \equiv f\left[\beta_h, \omega_k(\lambda_f)\right] - f\left[\beta_c, \omega_k(\lambda_i)\right]$$

Ideal quantum Otto cycle

$$W_{AB} = \sum_k \left[\omega_k(\lambda_i) - \omega_k(\lambda_f)\right]\left\{f\left[\beta_c, \omega(\lambda_i)\right] - \frac{1}{2}\right\}$$

$$Q_{BC} \equiv Q_h = \langle H(\lambda_f)\rangle_{\rho_f} - \langle H(\lambda_f)\rangle_{\rho_i}$$
$$= \sum_k \omega_k(\lambda_f)\,\left(\Delta f_k\right)_{hc}$$



perfect q-adiabatic transformation

perfect heating

$\rho_A \propto e^{-\beta_c H(\lambda_i)}$

$\rho_C \propto e^{-\beta_h H(\lambda_f)}$

perfect q-adiabatic transformation

$$W_{CD} = \sum_k \left[\omega_k(\lambda_f) - \omega_k(\lambda_i)\right]\left\{f\left[\beta_h, \omega(\lambda_f)\right] - \frac{1}{2}\right\}$$

$$\left(\Delta f_k\right)_{hc} \equiv f\left[\beta_h, \omega_k(\lambda_f)\right] - f\left[\beta_c, \omega_k(\lambda_i)\right]$$

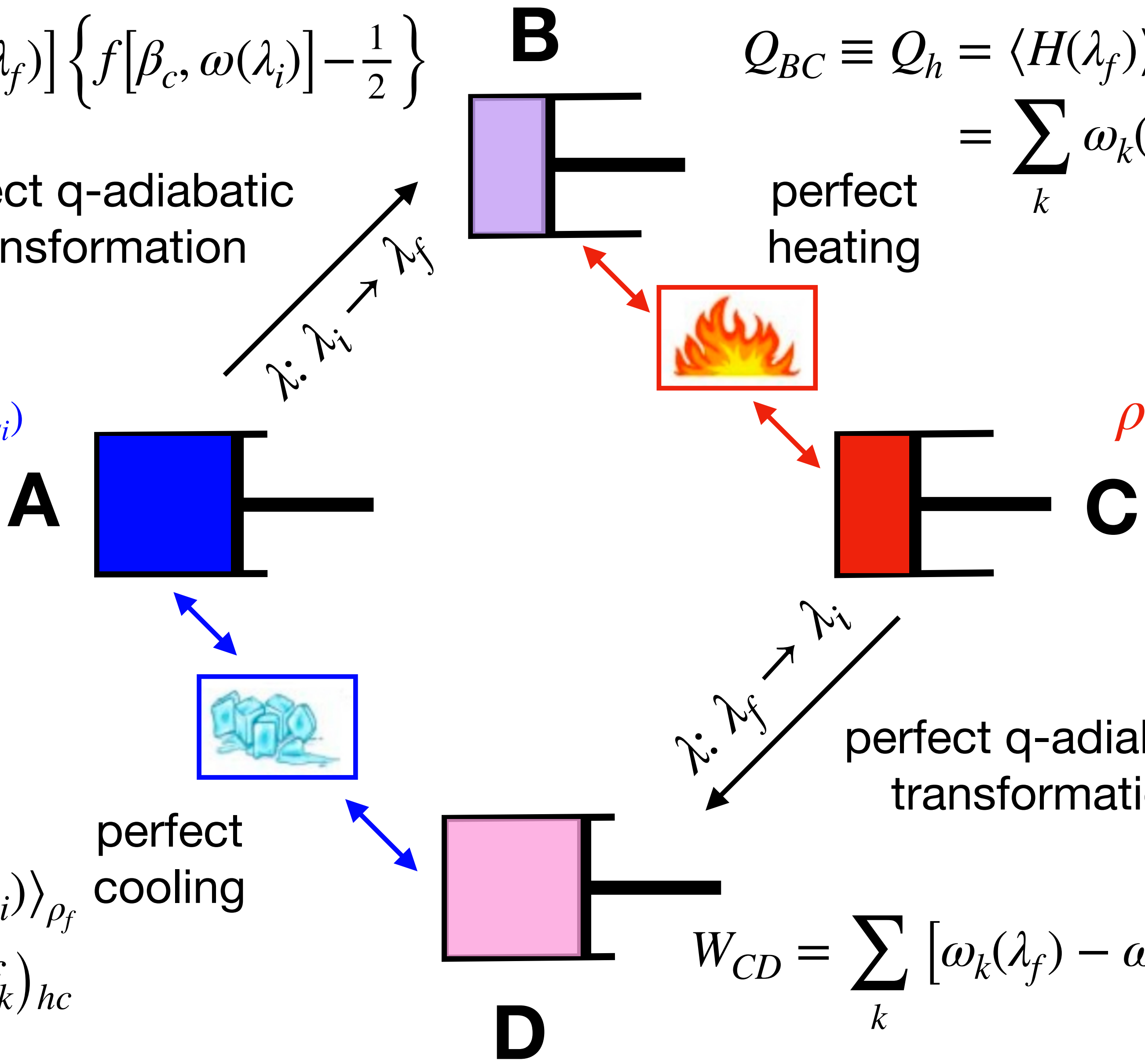
Ideal quantum Otto cycle

$$W_{AB} = \sum_k \left[\omega_k(\lambda_i) - \omega_k(\lambda_f)\right] \left\{ f[\beta_c, \omega(\lambda_i)] - \tfrac{1}{2} \right\}$$

perfect q-adiabatic transformation

$$Q_{BC} \equiv Q_h = \langle H(\lambda_f) \rangle_{\rho_f} - \langle H(\lambda_f) \rangle_{\rho_i}$$
$$= \sum_k \omega_k(\lambda_f) \left(\Delta f_k\right)_{hc}$$

perfect heating



$\rho_A \propto e^{-\beta_c H(\lambda_i)}$

$\rho_C \propto e^{-\beta_h H(\lambda_f)}$

perfect q-adiabatic transformation

perfect cooling

$$Q_{DA} \equiv Q_c = \langle H(\lambda_i) \rangle_{\rho_i} - \langle H(\lambda_i) \rangle_{\rho_f}$$
$$= -\sum_k \omega_k(\lambda_i) \left(\Delta f_k\right)_{hc}$$

$$W_{CD} = \sum_k \left[\omega_k(\lambda_f) - \omega_k(\lambda_i)\right] \left\{ f[\beta_h, \omega(\lambda_f)] - \tfrac{1}{2} \right\}$$

$$W \equiv W_{AB} + W_{BC} = Q_h - Q_c$$

$$\left(\Delta f_k\right)_{hc} \equiv f\left[\beta_h, \omega_k(\lambda_f)\right] - f\left[\beta_c, \omega_k(\lambda_i)\right]$$

Realistic quantum Otto cycle

A) Non-perfect quantum adiabaticity (adiabatic strokes):

Excitations generated during the unitary dynamics of a finite duration T

-> full description of the dynamics $U(t) = \mathrm{Texp}\left[e^{-i\int_0^t ds\, H(\lambda(s))}\right]$

Realistic quantum Otto cycle

A) Non-perfect quantum adiabaticity (adiabatic strokes):

Excitations generated during the unitary dynamics of a finite duration T

-> full description of the dynamics $U(t) = \mathrm{Texp}\left[e^{-i\int_0^t ds\, H(\lambda(s))}\right]$

B) Non-perfect thermalization (isochoric strokes):

System in contact with the reservoirs for a finite time τ

-> microscopic modeling of the system-bath dynamics

Realistic quantum Otto cycle

A) Non-perfect quantum adiabaticity (adiabatic strokes):

Excitations generated during the unitary dynamics of a finite duration T
-> full description of the dynamics $U(t) = \mathrm{Texp}\left[e^{-i\int_0^t ds\, H(\lambda(s))}\right]$

B) Non-perfect thermalization (isochoric strokes):

System in contact with the reservoirs for a finite time τ
-> microscopic modeling of the system-bath dynamics

Semi-analytic treatment for free-fermion systems

A) Non-perfect quantum adiabaticity

$H(\lambda_i) \to H(\lambda_f)$ in a **finite time** T

$$\boxed{U(t) = \mathrm{Texp}\left\{e^{-i\int_0^t ds\, H[\lambda(s)]}\right\}}$$

$$W = -\int_0^T \frac{\partial}{\partial_t}\Big\langle H[\lambda(t)]\Big\rangle_{\rho(t)} dt = \langle H(\lambda_i)\rangle_{\rho(0)} - \langle H(\lambda_f)\rangle_{\rho(T)}$$

Initial state

time-evolved state:
$\rho(T) = U(T)\rho(0)U(T)$

A) Non-perfect quantum adiabaticity

$H(\lambda_i) \rightarrow H(\lambda_f)$ in a **finite time** T

$$\boxed{U(t) = \mathrm{Texp}\left\{e^{-i\int_0^t ds\, H[\lambda(s)]}\right\}}$$

$$W = -\int_0^T \frac{\partial}{\partial_t}\left\langle H[\lambda(t)]\right\rangle_{\rho(t)} dt = \langle H(\lambda_i)\rangle_{\rho(0)} - \langle H(\lambda_f)\rangle_{\rho(T)}$$

Initial state

time-evolved state:
$\rho(T) = U(T)\rho(0)U(T)$

Bogoliubov formalism:

Heisenberg representation & Nambu spinors:

$\Psi = \left(c_1, \cdots, c_N, c_1^\dagger, \cdots, c_N^\dagger\right)^T, \quad \Phi = \left(b_1, \cdots, b_N, b_1^\dagger, \cdots, b_N^\dagger\right)^T; \qquad c_j^H(T) = U^\dagger(T)\, c_j\, U(T)$

Equilibrium : $\Psi = \mathbb{U}\,\Phi$ where $H = \Psi^\dagger\, \mathbb{H}\, \Psi, \quad \mathbb{H}^{\mathbb{D}} = \mathbb{U}^\dagger\, \mathbb{H}\, \mathbb{U}$

Dynamics: $\Psi^H(T) = \mathbb{U}(T)\, \Phi^H(0)$ with $\boxed{\partial_t \mathbb{U}(t) = -2i\, \mathbb{H}[\lambda(t)]\, \mathbb{U}(t)}$

A) Non-perfect quantum adiabaticity

$$W_{AB} = \sum_k \{\omega_k(h_i) - \boxed{\tilde{\omega}_k(h_f)}\} \left\{f[\beta_c, \omega_k(h_i)] - \frac{1}{2}\right\}$$

$$Q_{BC} \equiv Q_h = \sum_k \omega_k(\lambda_f) \left\{f[\beta_h, \omega_k(\lambda_f)] - \frac{1}{2}\right\} - \boxed{\tilde{\omega}_k(\lambda_f)} \left\{f[\beta_c, \omega_k(\lambda_i)] - \frac{1}{2}\right\}$$

$$W_{CD} = \sum_k \{\omega_k(h_f) - \boxed{\tilde{\omega}_k(h_i)}\} \left\{f[\beta_h, \omega_k(h_f)] - \frac{1}{2}\right\}$$

$$Q_{DA} \equiv Q_c = -\sum_k \boxed{\tilde{\omega}_k(\lambda_i)} \left\{f[\beta_h, \omega_k(\lambda_f)] - \frac{1}{2}\right\} - \omega_k(\lambda_i) \left\{f[\beta_c, \omega_k(\lambda_i)] - \frac{1}{2}\right\}$$

$$\underline{\tilde{\omega}_k(\lambda) = \left[\mathbb{U}^\dagger(T)\, \mathbb{H}(\lambda)\, \mathbb{U}(T)\right]_{kk}}$$

B) Non-perfect thermalization

Microscopic treatment for free fermions in contact with thermal baths.
Quadratic coupling to n baths & Markov approximation:

$$\boxed{d_t\rho(t) = -\,i[H,\rho(t)] + \mathscr{D}[\rho(t)]}$$

nonlocal Lindblad master eq.

$$\mathscr{D}[\rho] = \sum_{n,k} \gamma_{n,k}\Big[\big(1 - f(\beta_n,\omega_k)\big)\big(2b_k\rho b_k^\dagger - \{b_k^\dagger b_k,\rho\}\big) + f(\beta_n,\omega_k)\big(2b_k^\dagger\rho b_k - \{b_k b_k^\dagger,\rho\}\big)\Big]$$

bath coupling constants

Fermi-Dirac distribution

A. D'Abbruzzo & DR, PRA **103**, 052209 (2021)

B) Non-perfect thermalization

Microscopic treatment for free fermions in contact with thermal baths.
Quadratic coupling to n baths & Markov approximation:

$$\boxed{d_t\rho(t) = -\,i[H,\rho(t)] + \mathscr{D}[\rho(t)]}$$

nonlocal Lindblad master eq.

$$\mathscr{D}[\rho] = \sum_{n,k} \gamma_{n,k}\Big[\big(1-f(\beta_n,\omega_k)\big)\big(2b_k\rho b_k^\dagger - \{b_k^\dagger b_k,\rho\}\big) + f(\beta_n,\omega_k)\big(2b_k^\dagger\rho b_k - \{b_k b_k^\dagger,\rho\}\big)\Big]$$

bath coupling constants

Fermi-Dirac distribution

each site coupled to a distinct bath at temperature β ensures exponential convergence to a unique state $\rho_\beta \propto e^{-\beta H}$

$$\langle b_k^\dagger b_k\rangle(t) = f(\beta,\omega_k)\big(1-e^{-2\mathcal{J}t}\big) + \langle b_k^\dagger b_k\rangle_{\rho_i}\, e^{-2\mathcal{J}t}$$

$$\mathcal{J} \sim \Sigma_n \gamma_{n,k}\,,\ \ \forall k$$

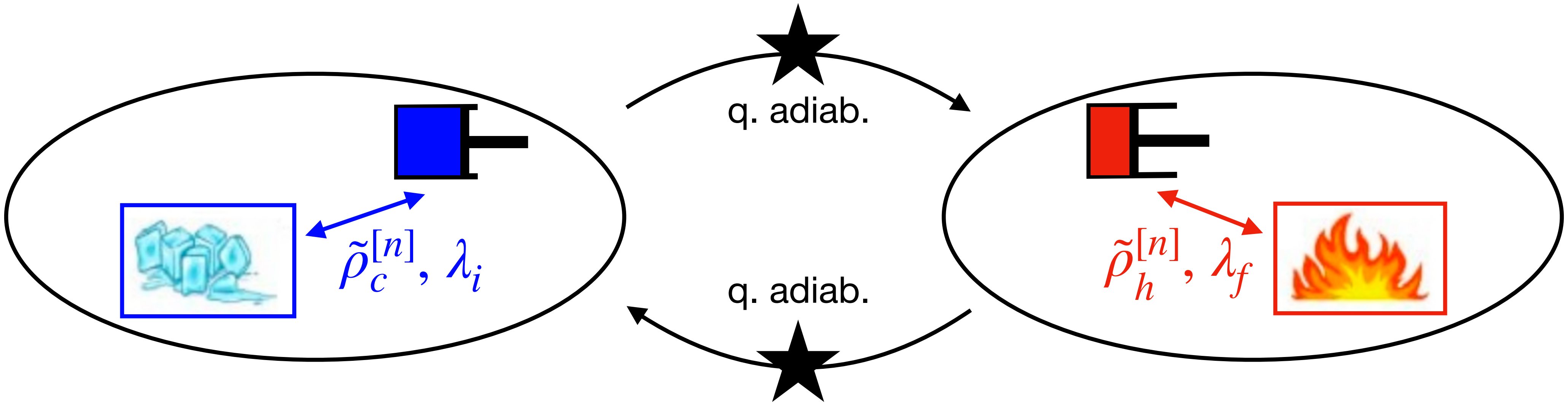
A. D'Abbruzzo & DR, PRA **103**, 052209 (2021)

B) Non-perfect thermalization

System in contact with a thermal reservoir for a **finite time τ**

$$\Theta_{c/h} \equiv \mathrm{diag}\left\{ f\left[\beta_{c/h}, \omega_k(\lambda_{i/f})\right] \right\}$$

$$\Gamma^{[n]}_{c/h} \equiv \mathrm{diag}\left\{ \mathrm{Tr}\left[b_k^\dagger b_k \, \tilde{\rho}^{[n]}_{c/h} \right] \right\} \; ; \qquad \Gamma^{[0]}_c = \Theta_c$$



B) Non-perfect thermalization

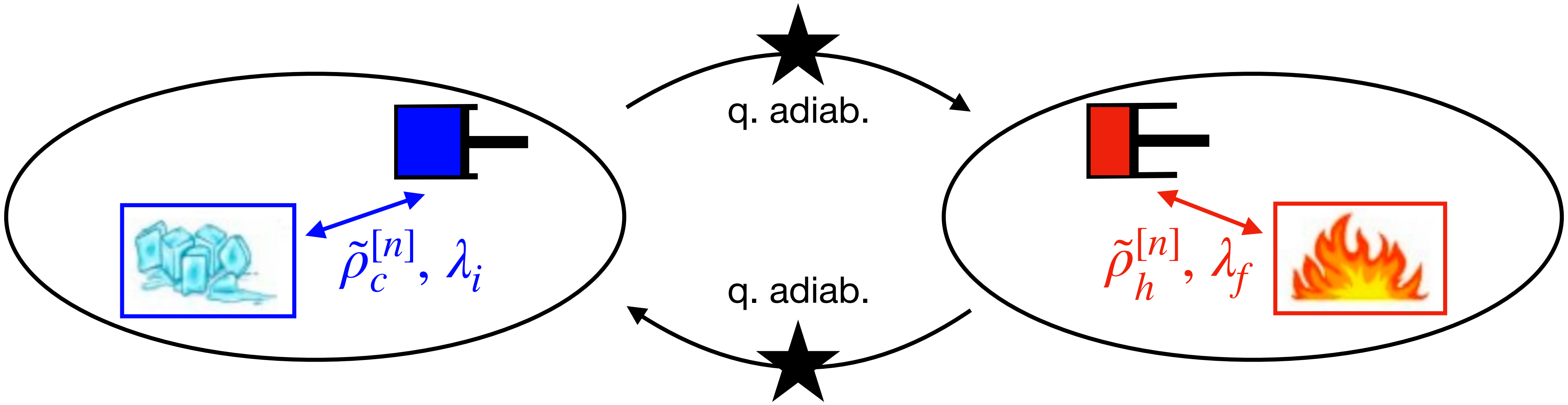
System in contact with a thermal reservoir for a **finite time τ**

$$\Theta_{c/h} \equiv \mathrm{diag}\left\{ f\left[\beta_{c/h}, \omega_k(\lambda_{i/f})\right] \right\}$$

$$[\mathcal{J} = 1]$$

$$\Gamma^{[n]}_{c/h} \equiv \mathrm{diag}\left\{ \mathrm{Tr}\left[b^\dagger_k b_k \, \tilde{\rho}^{[n]}_{c/h}\right] \right\} \; ; \qquad \Gamma^{[0]}_c = \Theta_c$$

$$h(x) \equiv (1 - e^{-x})^{-1}$$



$$\Gamma^{[n]}_h = \Theta_h(1 - e^{-2\tau}) + \Gamma^{[n-1]}_c e^{-2\tau}$$

$$\Gamma^{[n]}_c = \Theta_c(1 - e^{-2\tau}) + \Gamma^{[n]}_h e^{-2\tau}$$

B) Non-perfect thermalization

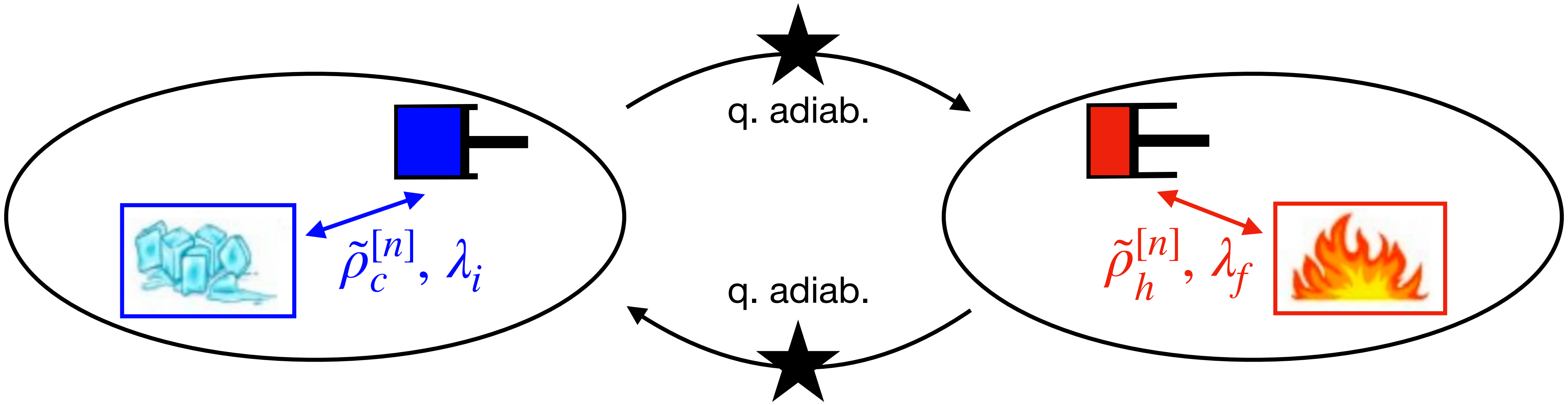
System in contact with a thermal reservoir for a **finite time τ**

$$\Theta_{c/h} \equiv \mathrm{diag}\left\{ f\left[\beta_{c/h}, \omega_k(\lambda_{i/f})\right] \right\}$$

$$\Gamma^{[n]}_{c/h} \equiv \mathrm{diag}\left\{ \mathrm{Tr}\left[b_k^\dagger b_k\, \tilde{\rho}^{[n]}_{c/h}\right] \right\} \; ; \qquad \Gamma^{[0]}_c = \Theta_c$$

$$[\mathcal{J} = 1]$$

$$h(x) \equiv (1 - e^{-x})^{-1}$$



$$\Gamma^{[n]}_h = \Theta_h(1 - e^{-2\tau}) + \Gamma^{[n-1]}_c e^{-2\tau}$$

$$\Gamma^{[n]}_c = \Theta_c(1 - e^{-2\tau}) + \Gamma^{[n]}_h e^{-2\tau}$$

$$\xrightarrow[\text{steady limit}]{}$$

$$\Gamma^{[+\infty]}_h = h(2\tau)\left(\Theta_h + e^{-2\tau}\,\Theta_c\right)$$

$$\Gamma^{[+\infty]}_c = h(2\tau)\left(\Theta_c + e^{-2\tau}\,\Theta_h\right)$$

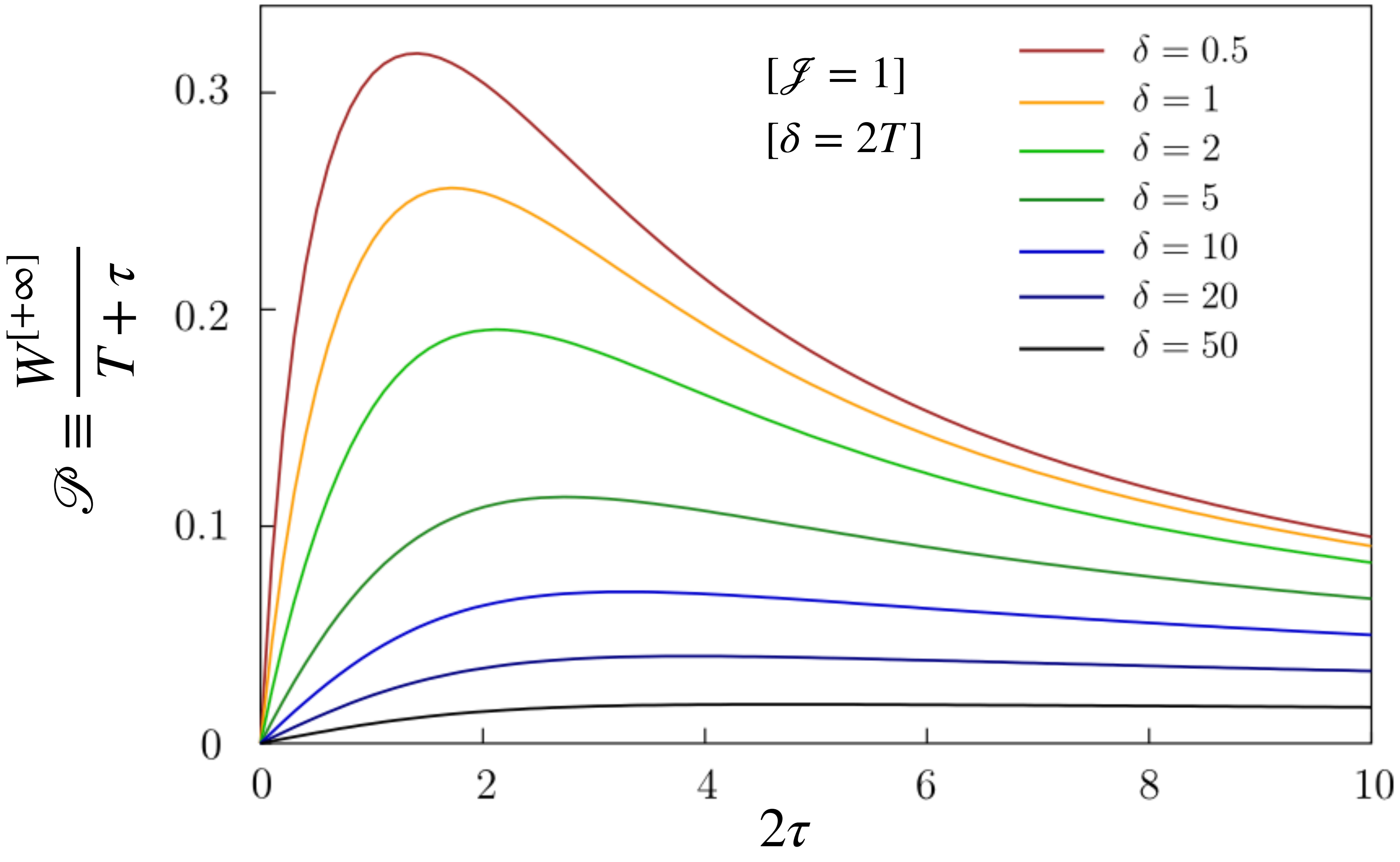
Converges exponentially to:

$$\boxed{\begin{aligned} W^{[+\infty]} &= g(\tau)\, W_{\mathrm{ideal}} \\ Q^{[+\infty]}_{c/h} &= g(\tau)\, Q_{c/h,\,\mathrm{ideal}} \end{aligned}}$$

$$\underline{g(\tau) \equiv \tanh(\tau)}$$

B) Non-perfect thermalization

System in contact with a thermal reservoir for a **finite time τ**



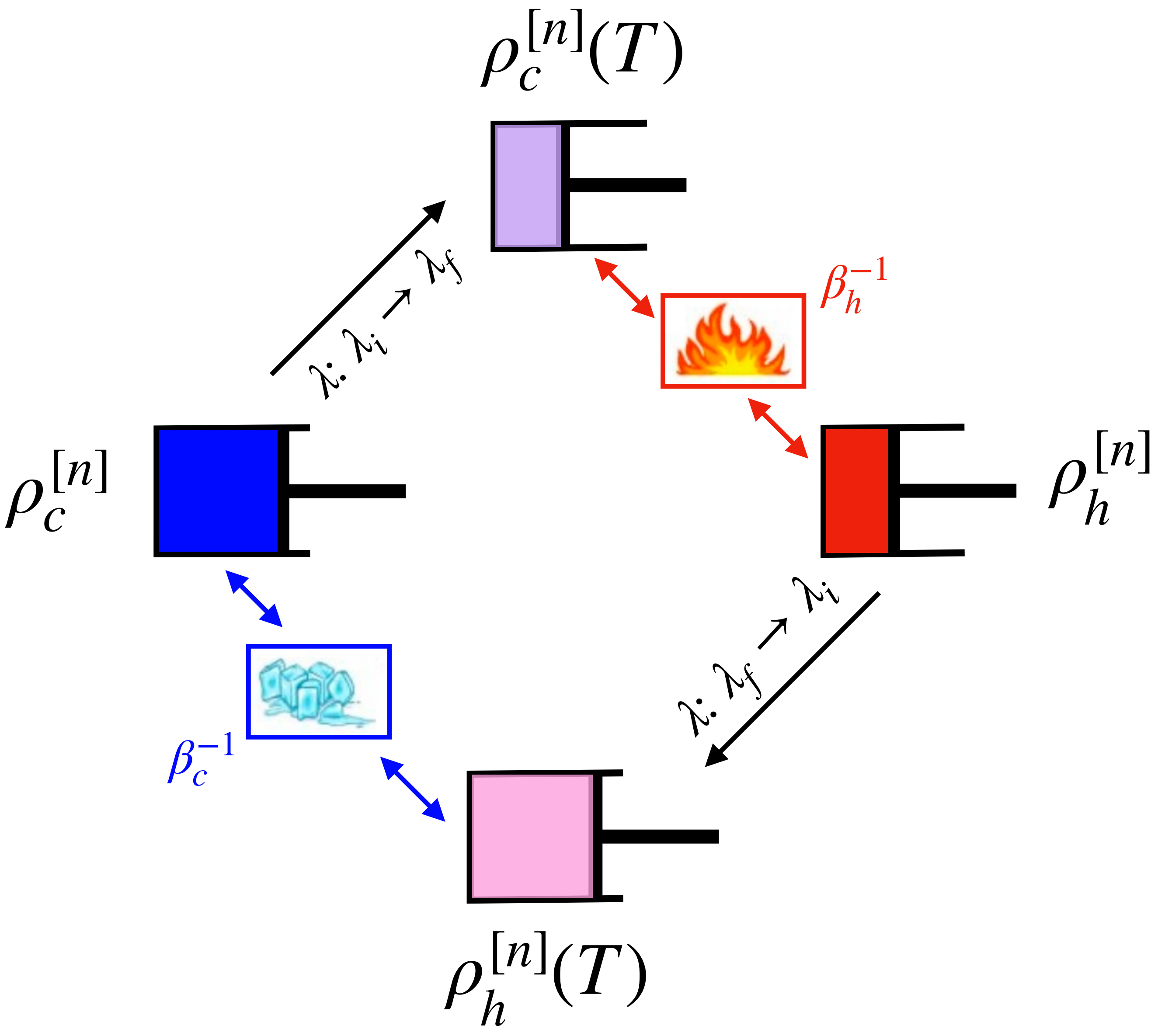
Converges exponentially to:

$$\boxed{\begin{aligned} W^{[+\infty]} &= g(\tau)\, W_{\text{ideal}} \\ Q_{c/h}^{[+\infty]} &= g(\tau)\, Q_{c/h,\,\text{ideal}} \end{aligned}}$$

$$\underline{g(\tau) \equiv \tanh(\tau)}$$

AB) Non-perfect thermalization & quantum adiabaticity

Non-diagonal components of the Bogoliubov quasiparticles correlator become non vanishing:



$$\left(\Lambda_c^{[n]}\right)_{jl} = \left\langle \Phi_j \Phi_l^\dagger \right\rangle_{\rho_c^{[n]}}$$

$$\left(\Lambda_{c,T}^{[n]}\right)_{jl} = \left\langle \Phi_j \Phi_l^\dagger \right\rangle_{\rho_c^{[n]}(T)}$$

$$\left(\Lambda_h^{[n]}\right)_{jl} = \left\langle \Phi_j \Phi_l^\dagger \right\rangle_{\rho_h^{[n]}}$$

$$\left(\Lambda_{h,T}^{[n]}\right)_{jl} = \left\langle \Phi_j \Phi_l^\dagger \right\rangle_{\rho_h^{[n]}(T)}$$

Expressions in terms of series expansions, easy to be computed numerically in the limit-cycle $n \to \infty$.

Example:

Otto engine based on a quantum Ising-chain medium

The quantum Ising chain

A free-fermion model, after Jordan-Wigner transforming fermions into qubits:

$$H_{\text{Ising}}(h) = -\sum_j \left(\sigma_j^x \sigma_{j+1}^x + h\,\sigma_j^z\right)$$

$$\boxed{\sigma_j^- = e^{-i\pi \sum_{\ell=1}^{j-1} c_\ell^+ c_\ell}\, c_j}$$

$$\omega_k(h) = 2\sqrt{1 + h^2 - 2h\cos k}\,, \quad k \in (0,\pi)$$

(thermodynamic limit)

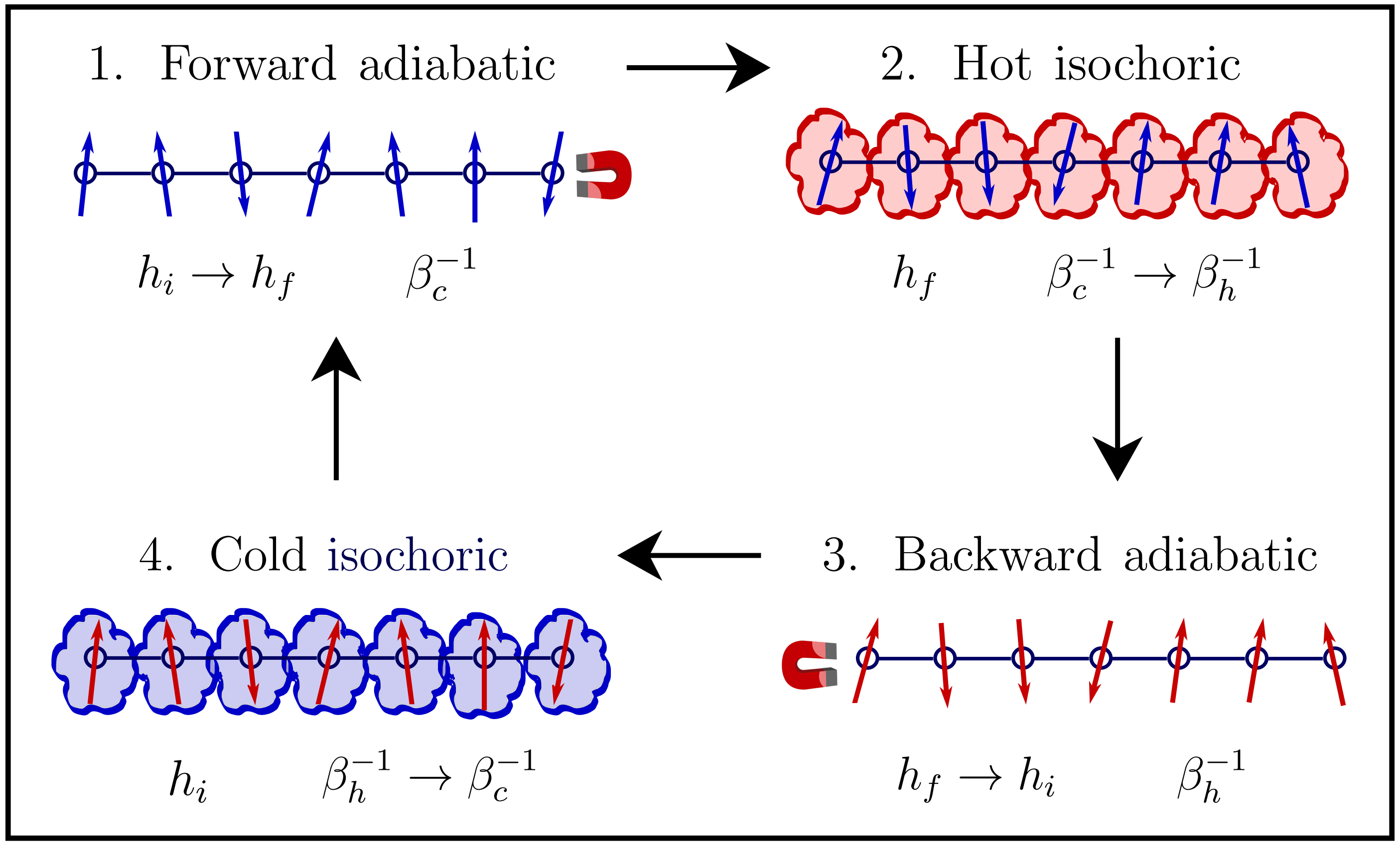
The quantum Ising chain

A free-fermion model, after Jordan-Wigner transforming fermions into qubits:

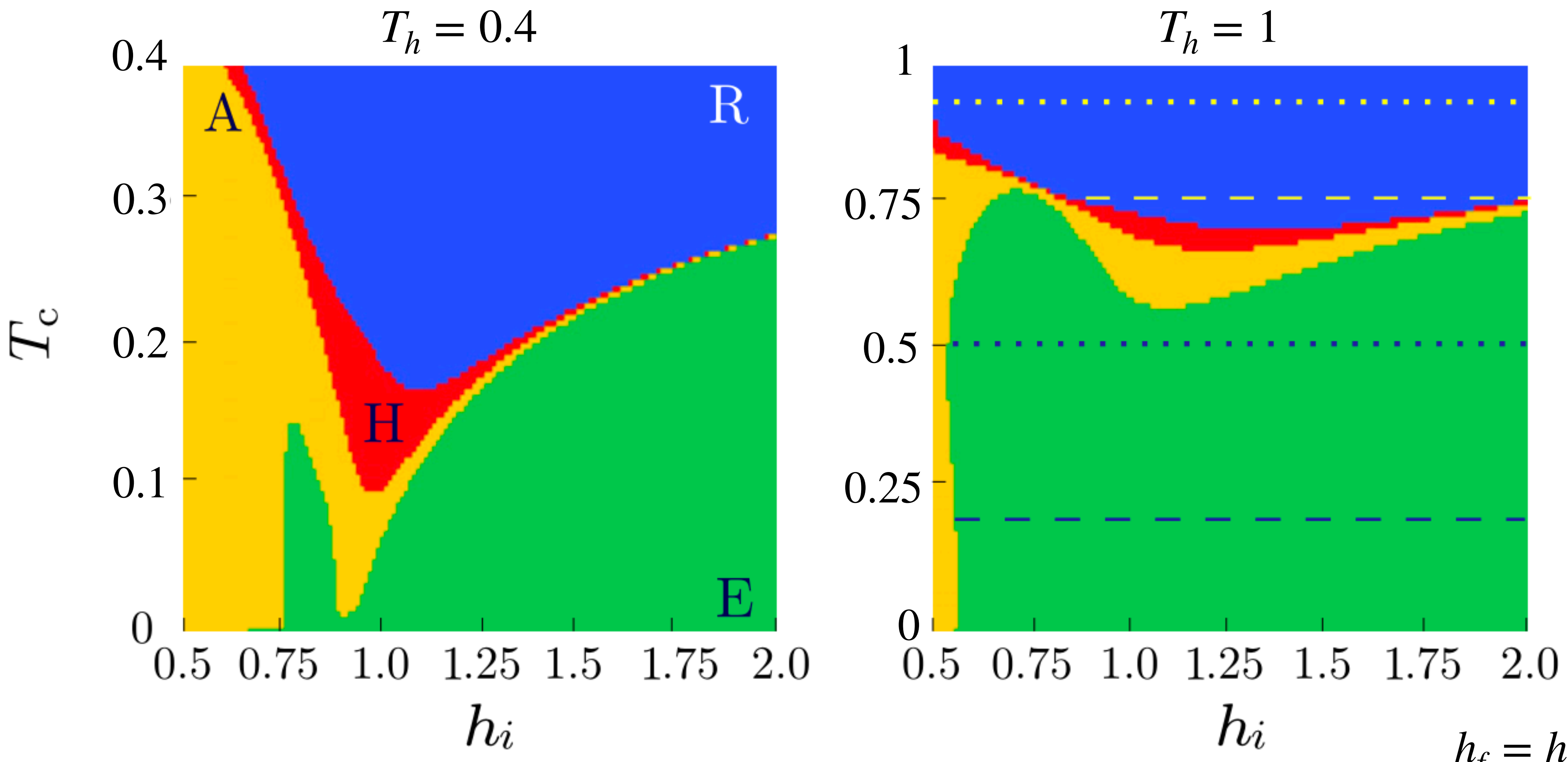
$$H_{\text{Ising}}(h) = -\sum_{j} \left(\sigma_j^x \sigma_{j+1}^x + h\, \sigma_j^z \right)$$

$$\boxed{\sigma_j^- = e^{-i\pi \sum_{\ell=1}^{j-1} c_\ell^+ c_\ell}\, c_j}$$

$$\omega_k(h) = 2\sqrt{1 + h^2 - 2h\cos k}\,, \quad k \in (0,\pi)$$ (thermodynamic limit)



Ideal Ising Otto cycle



$h_f = h_i + 0.5$

H: heater $(Q_h, Q_c, W < 0)$

A: accelerator $(Q_h > 0, \ \ Q_c, W < 0)$

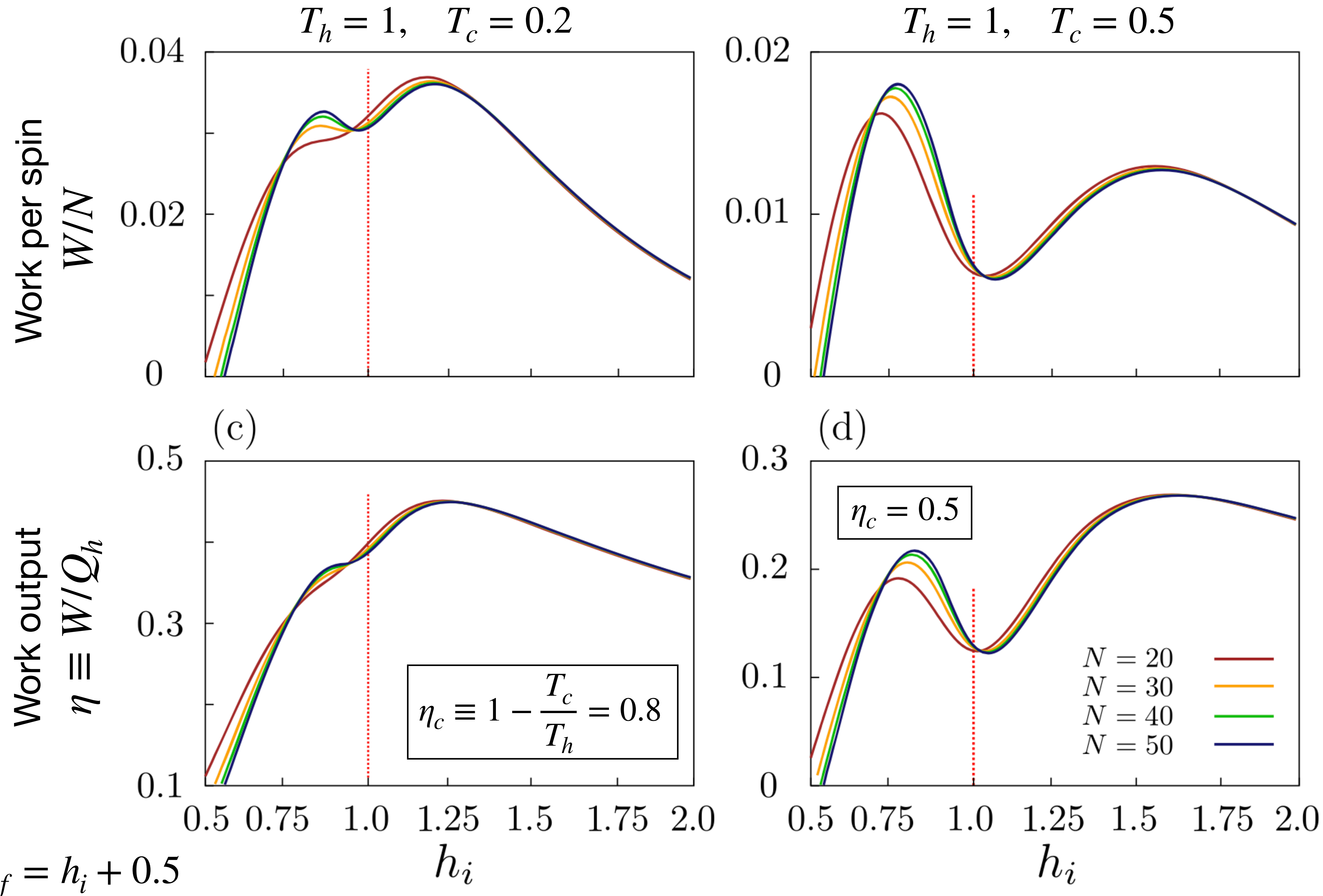
E: heat engine $(Q_c < 0 \ \ Q_h, W > 0)$

R: refrigerator $(Q_c > 0, \ \ Q_h, W < 0)$

Quantum criticality $(h = 1)$:

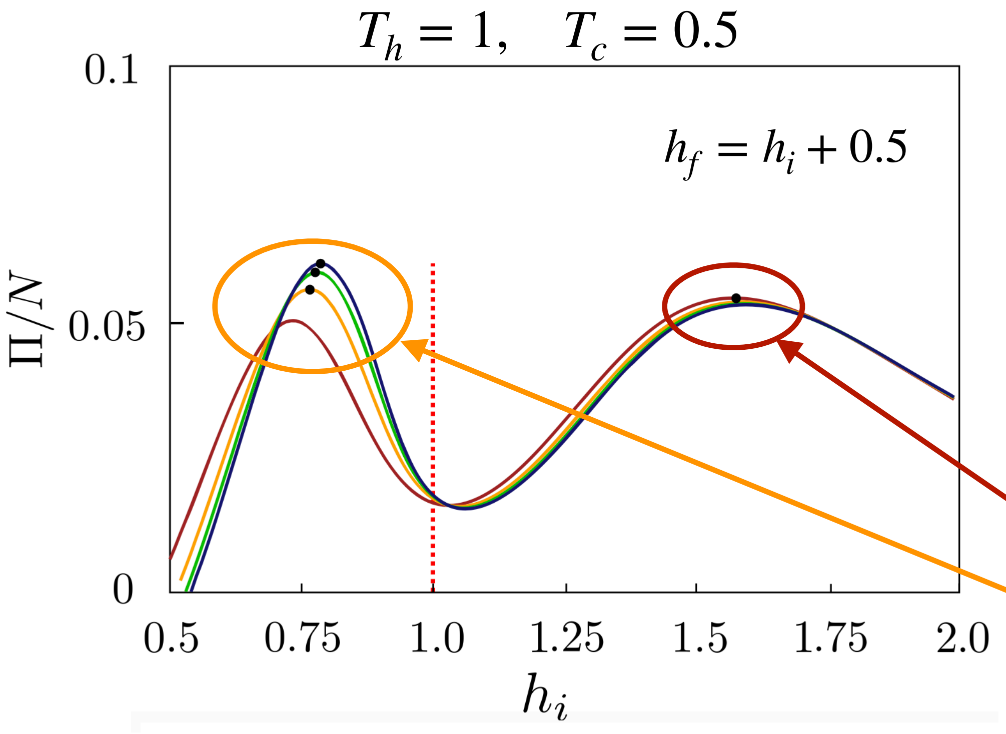
- closure of the energy gap
- divergence of the magnetic susceptibility
- divergence of the specific heat

Heat engine



$h_f = h_i + 0.5$

Heat engine



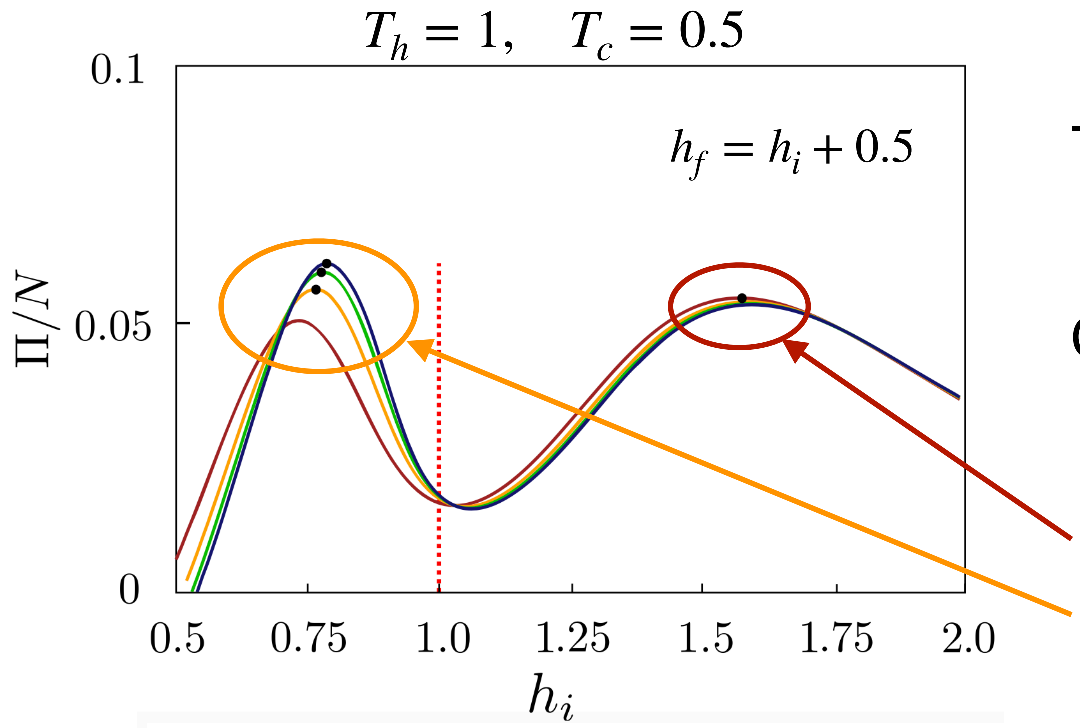
Thermodynamic performance: $\Pi = \dfrac{W}{\eta_C - \eta}$

Carnot efficiency: $\eta_C = 1 - \dfrac{T_c}{T_h}$

"*paramagnetic*" peak

"*critical*" peak

Heat engine

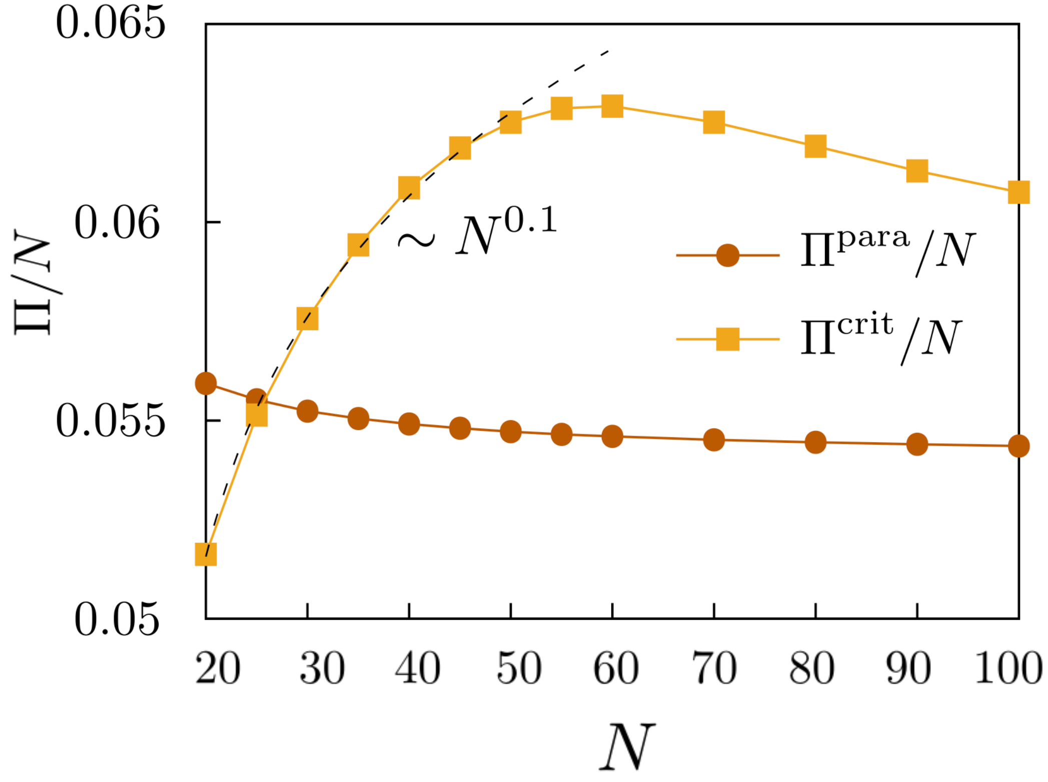


Thermodynamic performance: $\Pi = \dfrac{W}{\eta_C - \eta}$

Carnot efficiency: $\eta_C = 1 - \dfrac{T_c}{T_h}$

"*paramagnetic*" peak

"*critical*" peak



Paramagnetic peak: scales linearly with N

Critical peak: scales more than linearly with N

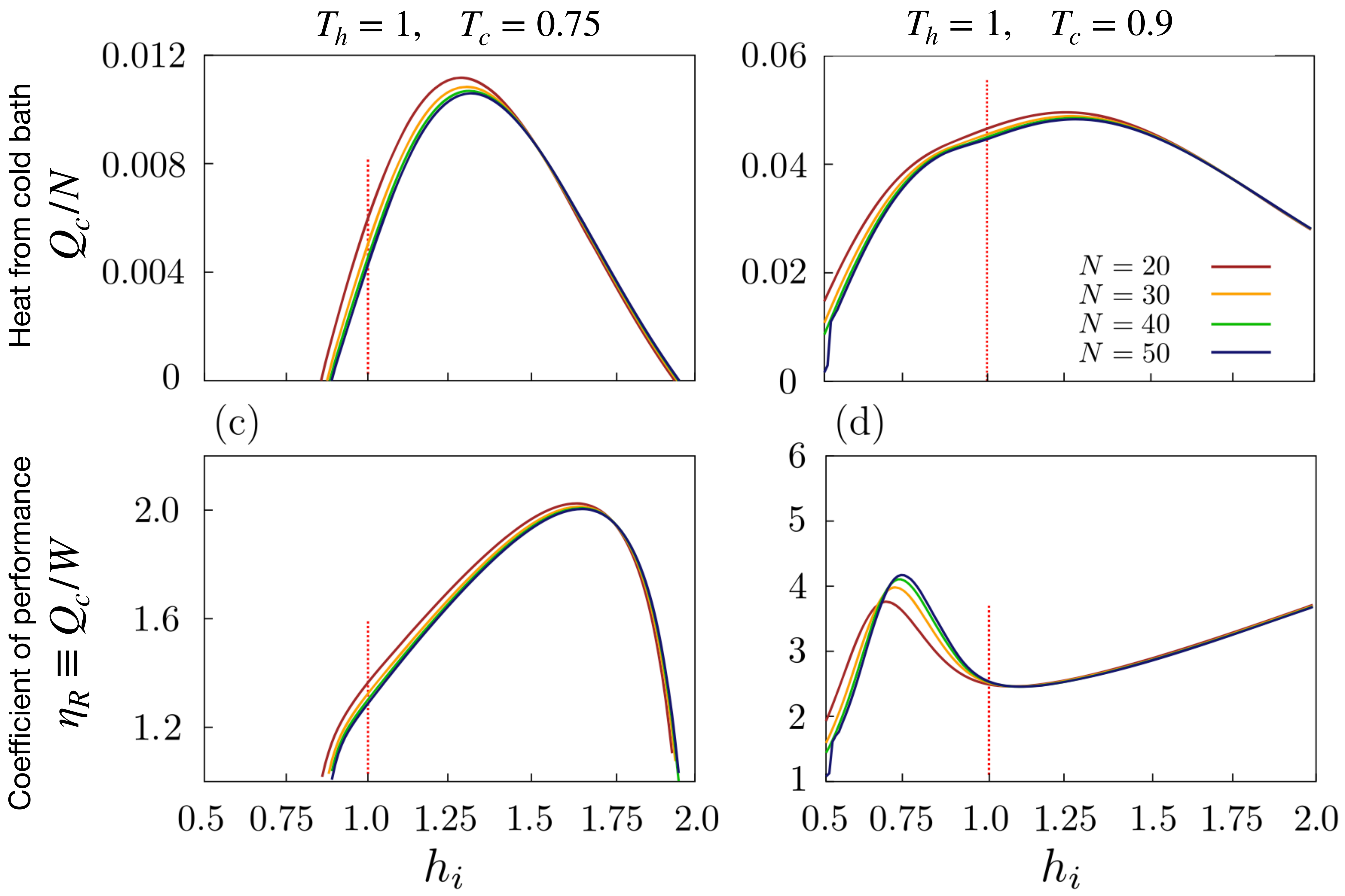
Performance enhancement by criticality?

$$\Pi^{\mathrm{crit}}/N \sim N^{\alpha}, \quad \alpha > 0$$

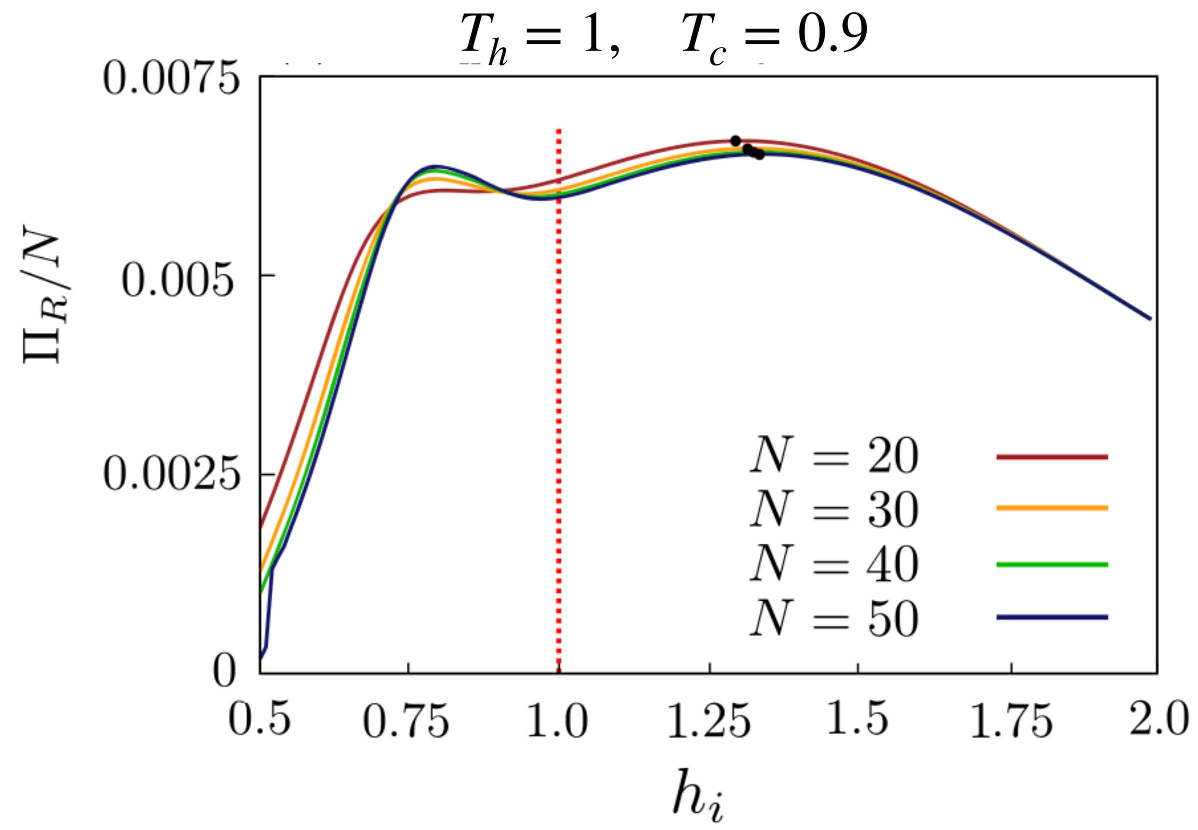
- α increases when cooling down the system
- crossover effect

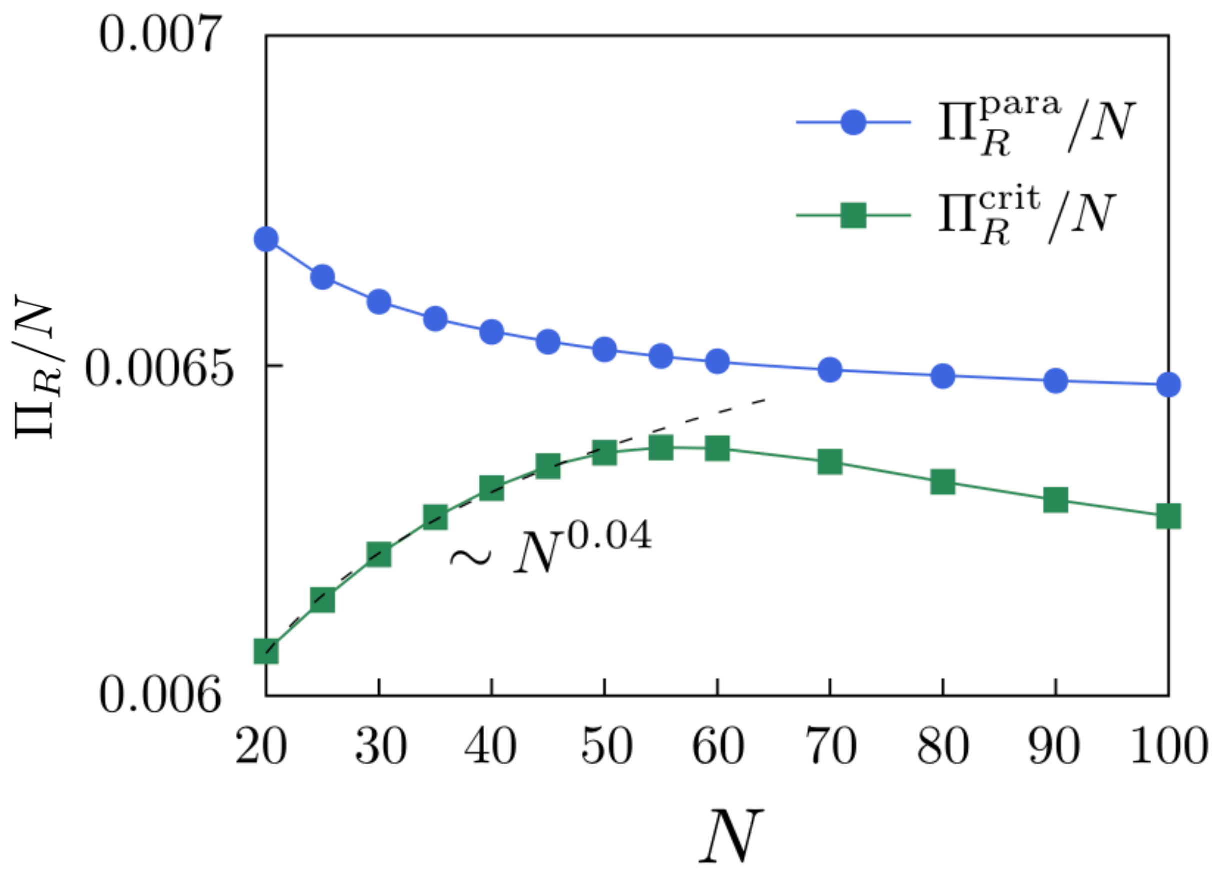
M. Campisi, R. Fazio, Nat. Comm. **7**, 11895 (2016)

Refrigerator



Refrigerator





As for the heat engine:

- double-peak structure
- paramagnetic peak weakly affected by the system size
- super-extensive scaling of the critical peak for moderate N values

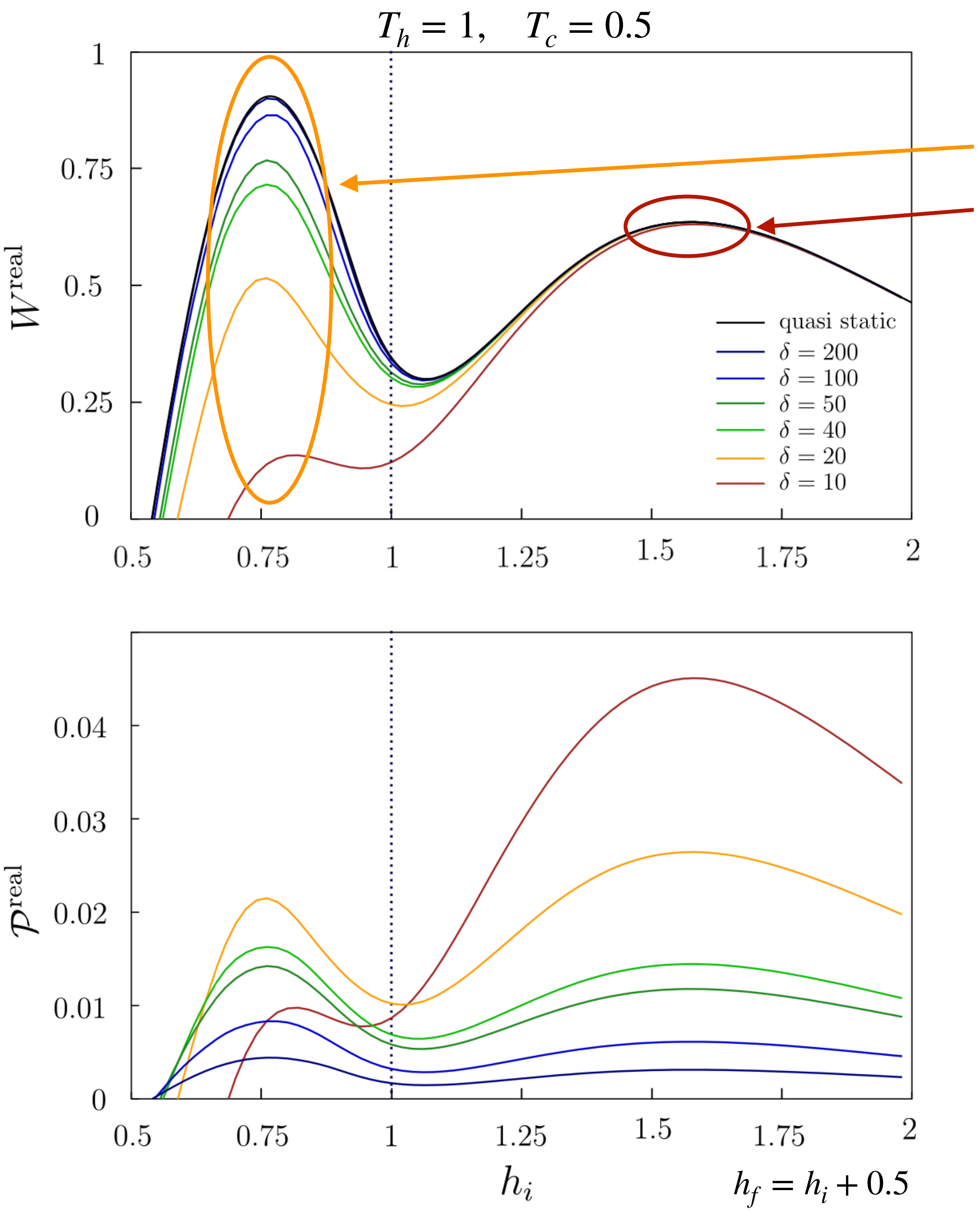
Less perceptible role of quantum criticality, than for the heat engine

Non-ideal Ising Otto cycle

Finite adiabaticity time: $T \equiv \delta/2$

Finite thermalization time: $\tau \equiv \gamma/2, \quad [\mathcal{J} = 1]$

Imperfect quantum adiabaticity

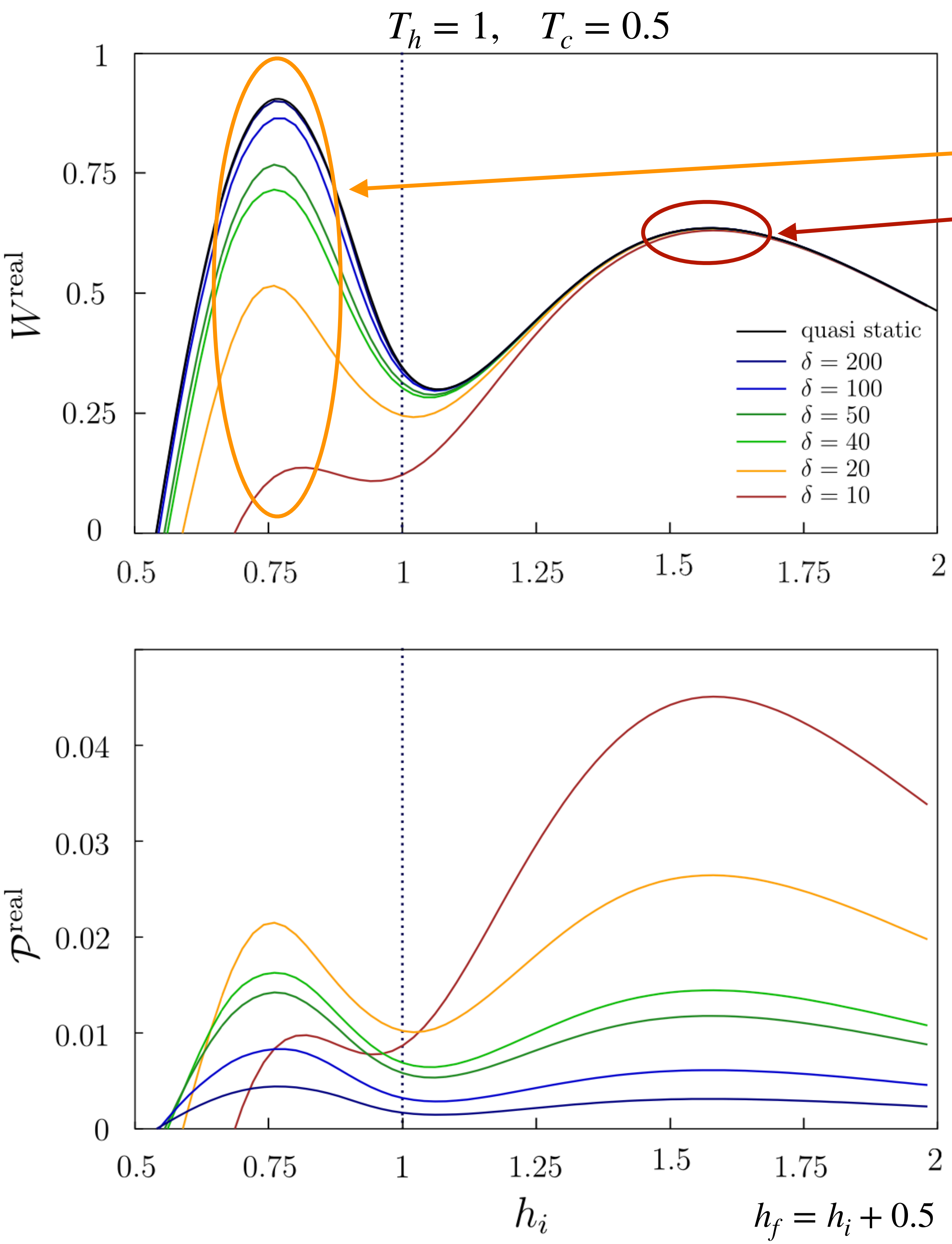


“*critical*” peak

“*paramagnetic*” peak

nearly perfect
thermalization: $\gamma = 5$

Imperfect quantum adiabaticity

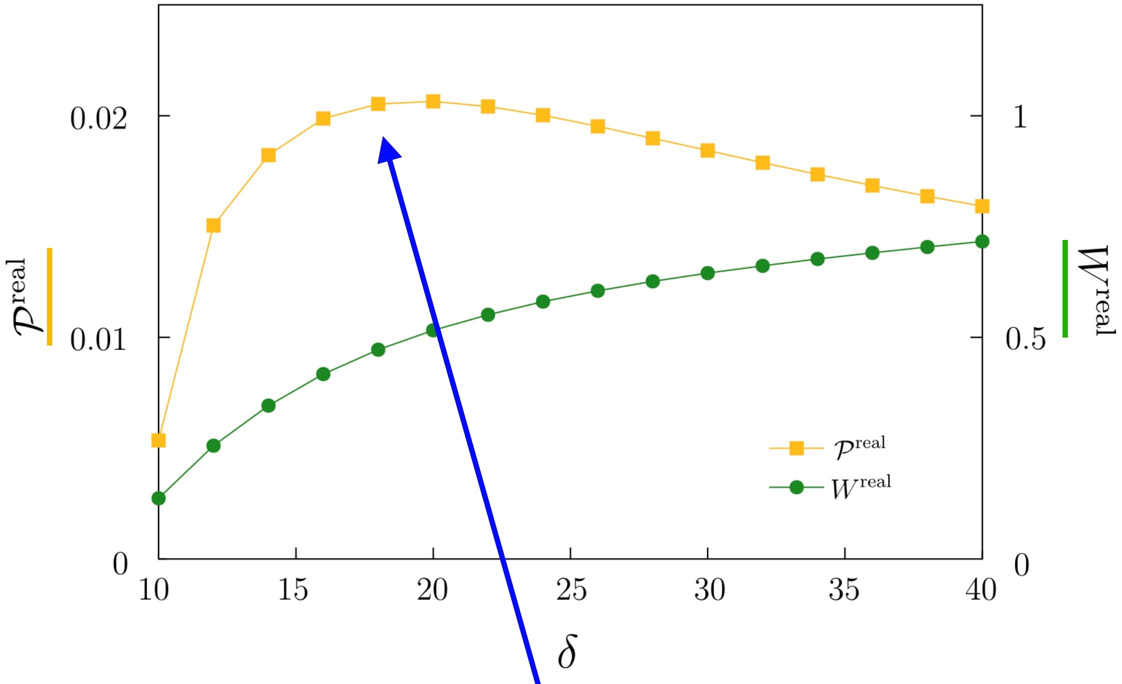


nearly perfect
thermalization: $\eta = 5$

"*critical*" peak

"*paramagnetic*" peak

work and power @ critical peak

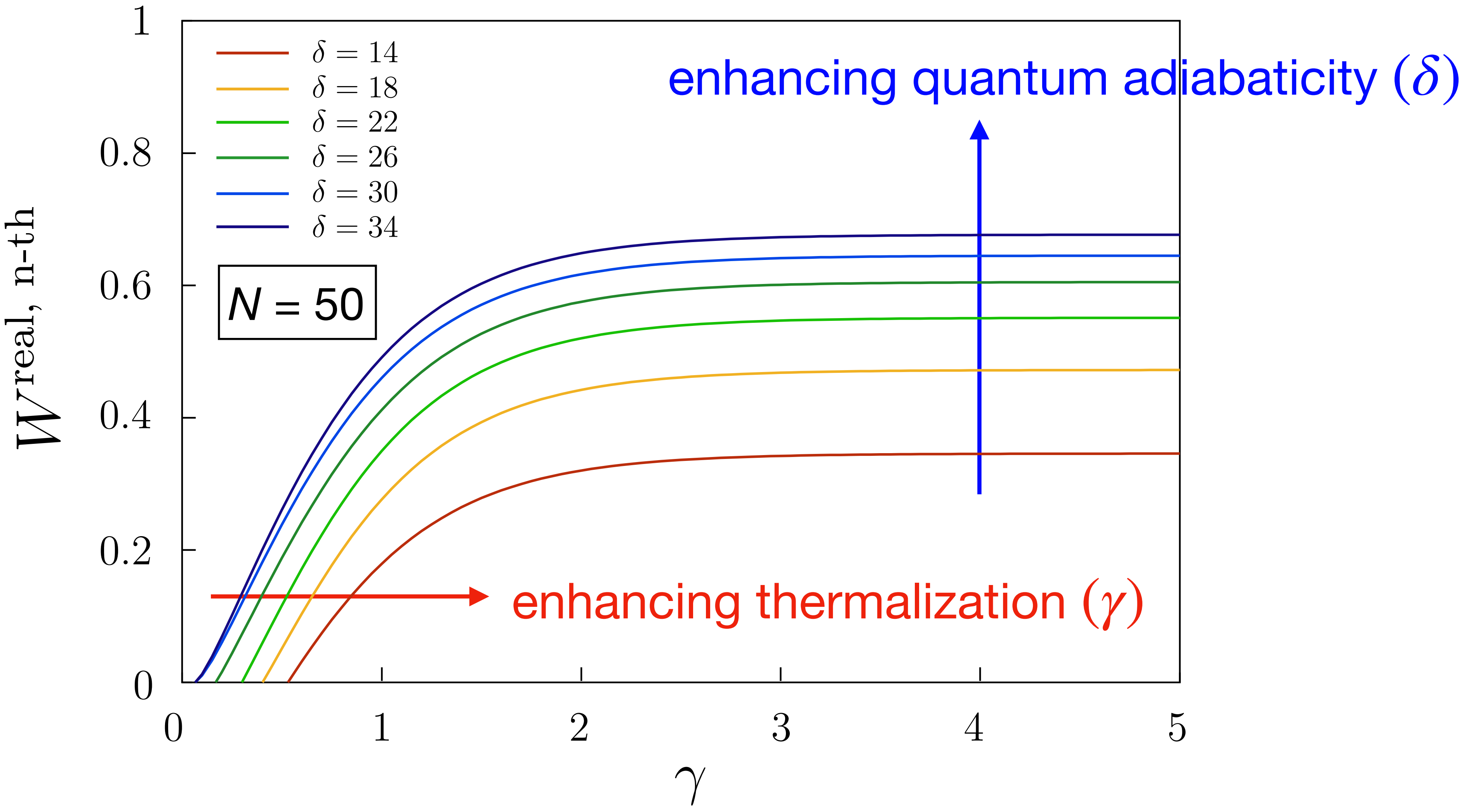


optimal working point:

power non monotonic in δ

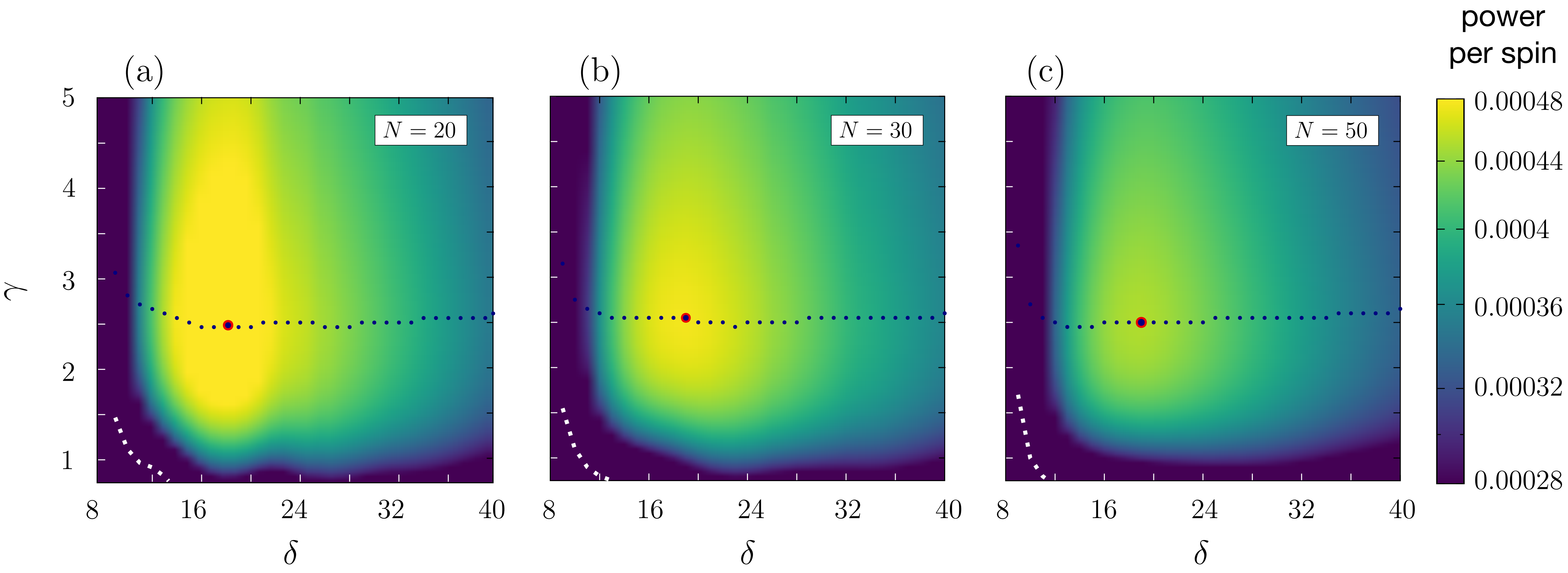
Non-ideal Ising Otto cycle

Work done by the engine @ the critical peak $[\, h_i = 0.76, \;\; h_f = h_i + 0.5 \,]$



- Curves for the ***work*** are all ***monotonic***, both in $T = \delta/2$ and in $\tau = \gamma/2$.
- Imperfect thermalization & quantum non-adiabaticity can modify the engine operation mode.

Non-ideal Ising Otto cycle



- The ***power*** is ***nonmonotonic***, both in $T = \delta/2$ and in $\tau = \gamma/2$.
  (time scales obviously depend on the Hamiltonian spin-spin coupling strength *J*, as well as on the system-bath coupling strength $\mathscr{J}$).
- Functional form & position of power output are weakly affected by the system size *N*.
- <u>Power output is optimized @ small *N*.</u>
  (the gap closes with *N*, thus work extraction at large *N* is affected by fast quenches).

Summary

- Free-fermion Otto engines can operate in different modes
- Optimal working point
- Not obvious how criticality might enhance performances
- Variational optimization of (quasi)-adiabatic strokes?
- Role of fluctuations @ criticality?
- Different universality classes / role of interactions?