Dissipation Driven Phase Transitions in the non-Hermitian Kondo model (Atomtronic implementation)




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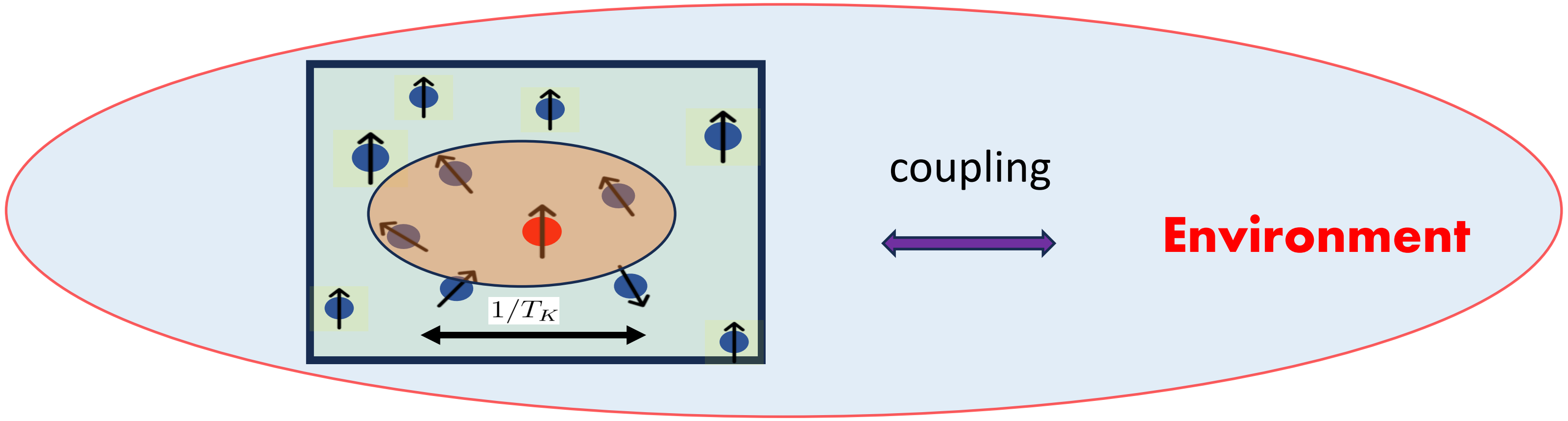


Atomtronics@Benasque, May 2024

Kondo effect in an open environment

Motivations :

i. Study a well known phenomenon under new circumstances

ii. Many conceptual questions arise

iii. Benchmark for numerical methods e.g. extend DMRG to complex variables

- Modeling **open** systems
  - energy, particles not conserved, dissipation into environment
  - non unitary evolution
  - New phase transitions

- Non Hermitian Hamiltonians, PT symmetry or not e.g $H = \hat{p}^2 + \hat{x}^2 + (i\hat{x})^N$ *C. Bender*

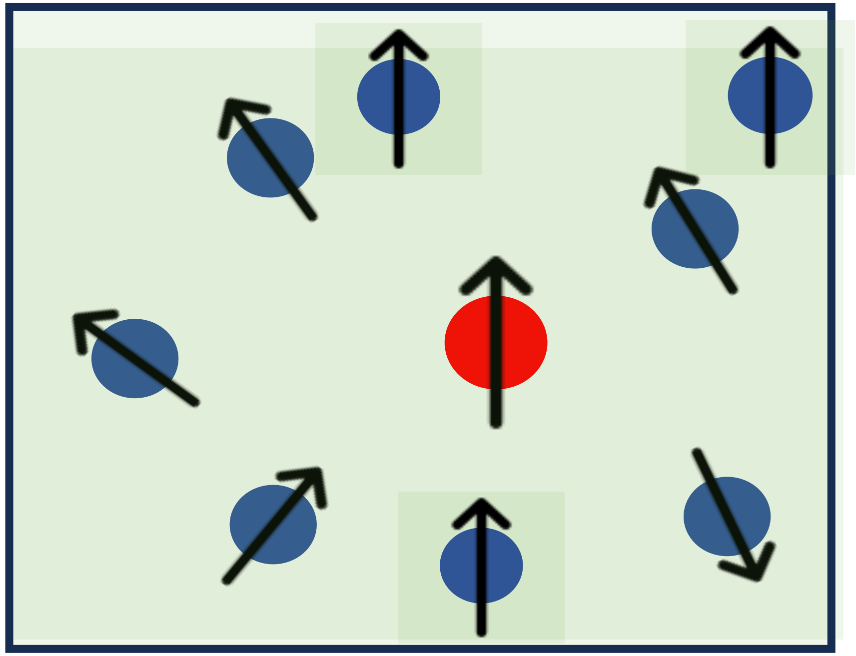
- The *Hermitian* Kondo model, old problem, well studied

  originated in metals, electrons scattering off a quantum impurity

- Study a *non-Hermitian,* Lindbladian generalization
  - Experiments using atomic gases (underway)
  - New phases emerge
  - competition energetics and dissipation

Kondo effect in a metal

- Magnetic impurity in a sea of gapless fermions: $H_{\text{metal}} + H_{\text{imp}}$



$$H_{\text{metal}} = \sum_k \epsilon_k \psi^\dagger_{ka} \psi_{ka}$$

Metal: Fermi sea, gas of electrons

$$H_{\text{imp}} = J \vec{S}_{\text{imp}} \cdot \vec{s}(0)$$

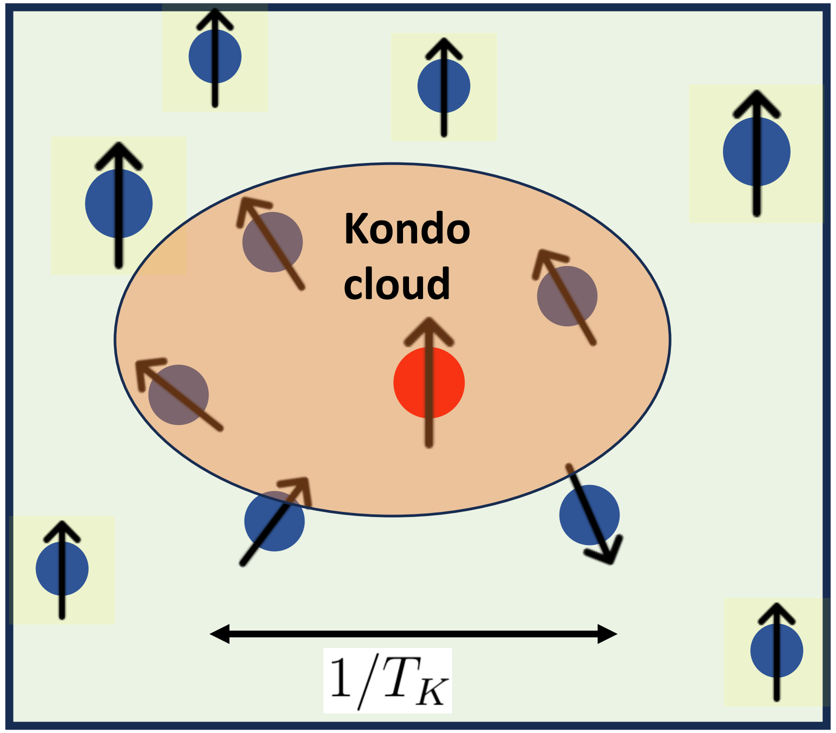
Kondo impurity coupled to the Fermi sea

$$\vec{s}(0) = \psi^\dagger_a(0) \vec{\sigma}_{ab} \psi_b(0)$$

Electrons spin density at impurity site

Kondo effect in a metal

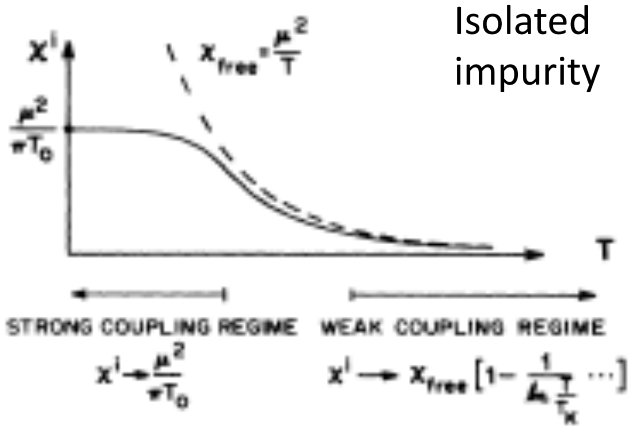
Magnetic impurity in a sea of gapless fermions



Closed Kondo system

- For $J > 0$ spin ½ impurity is screened
  - *- Singlet Ground State*
  - *- Many-body screening (Kondo cloud)*

- Magnetic susceptibility and resistivity increase as $T \to 0$



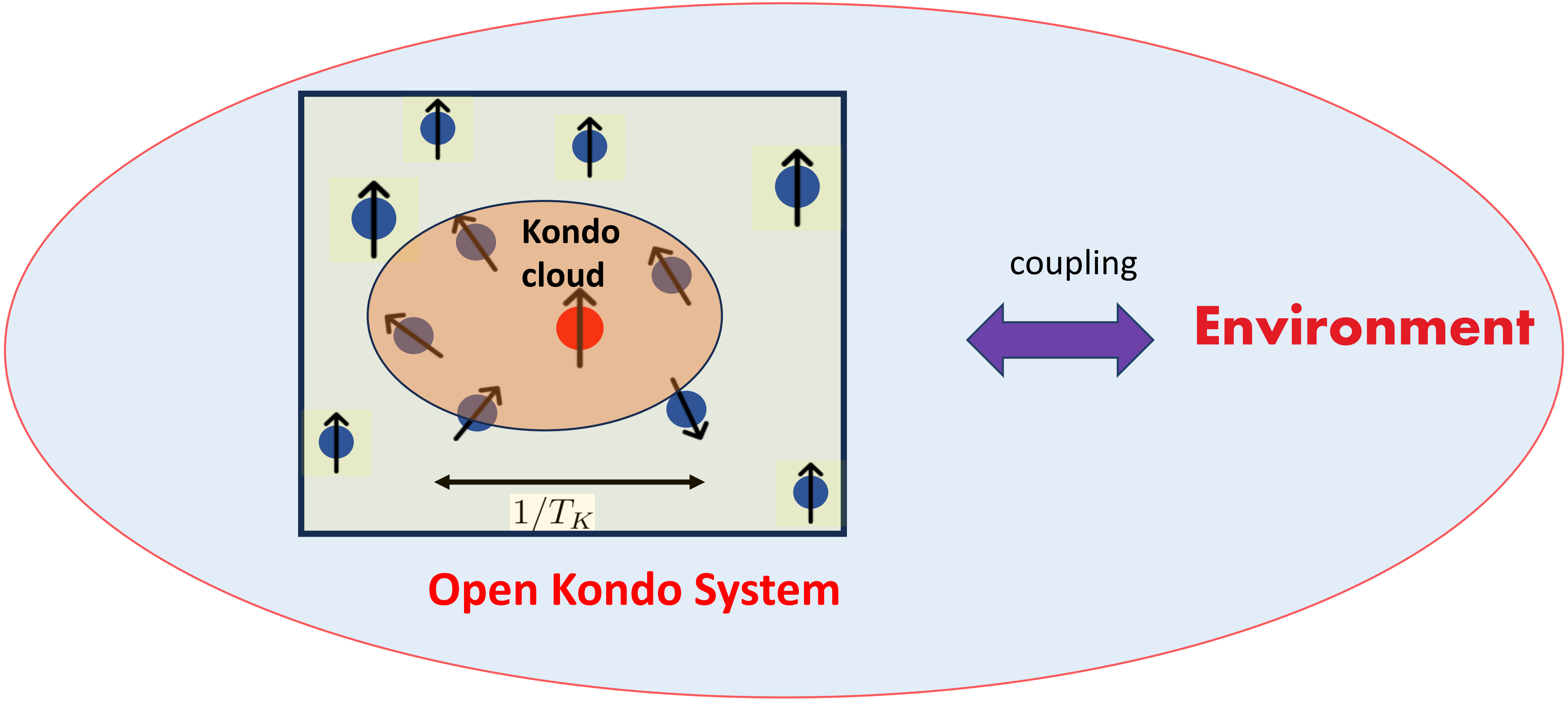
- RG interpretation: $\beta(J) = D\dfrac{dJ}{dD} = \rho_0 J^2 + \dfrac{1}{2}\rho_0^2 J^3 + \cdots$
  - ***- only one phase*** *System flows to strong coupling*

- Non-perturbative scale $T_K \approx De^{-\pi/2J}$
- Impurity D.O.S $\rho_{\text{imp}}(E) = \dfrac{T_K}{\pi(E^2 + T_K^2)}$

Open Kondo system

Kondo system coupled to an environment:

Example: AMO system of a two orbital fermionic quantum gas
- Atoms (e.g. ^{173}Yb) in a metastable excited state play the role of a spin S = 1/2 impurities
- Atoms in their ground state are mobile and act as itinerant spin S = 1/2 fermions.
- Two-body losses due to inelastic scattering between the impurity and itinerant atoms are enhanced in the singlet channel. *(I. Bloch group 2018)*



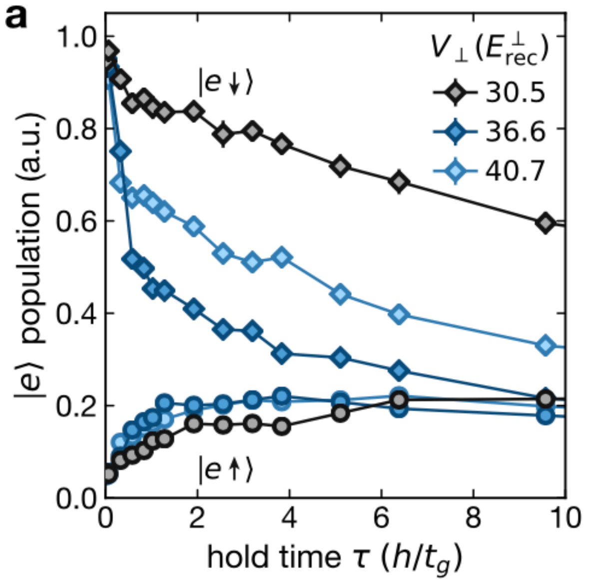
Experiment

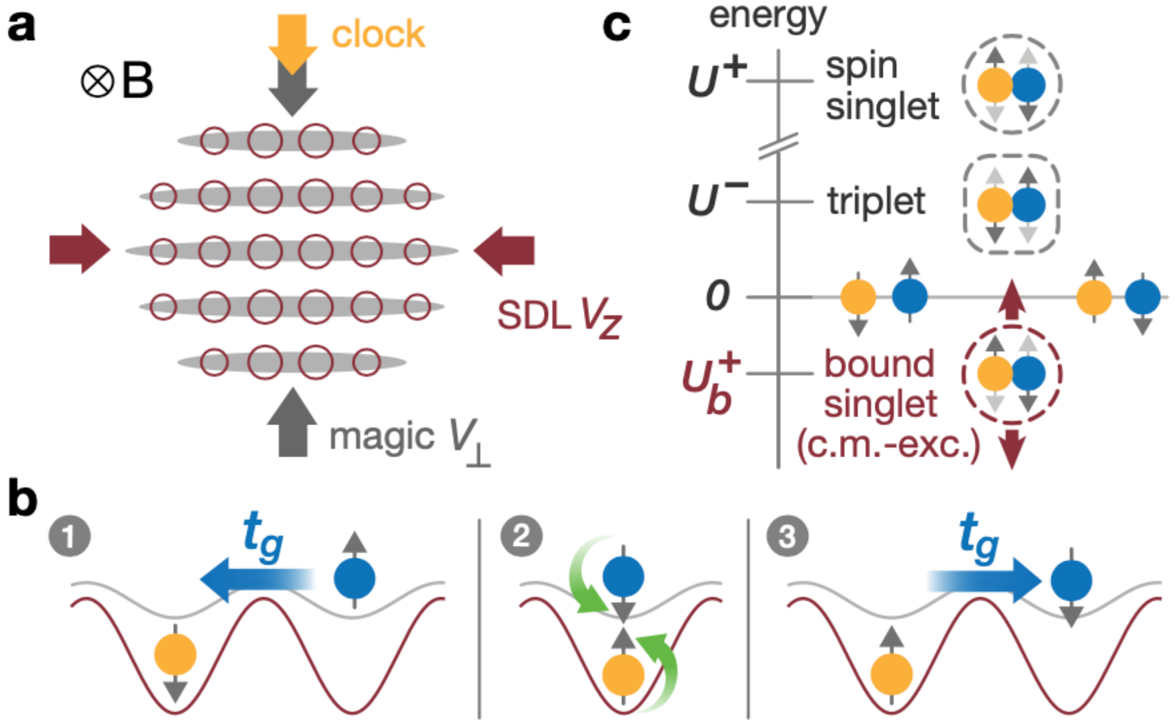
Riegger et al '18 (I. Bloch group)

- Ultra cold gases with orbital and spin degrees of freedom allow localized and mobile magnetic moments
  e.g. Ytterbium ^{173}Y

- Two orbitals can be represented in the two lowest-lying states of their electronic spin singlet and triplet manifold, $^{1}S_0$ (denoted |g⟩) and $^{3}P_0$ (|e⟩).

- Use state-dependent optical lattices





Open quantum systems:
Lindbladian (Born-Markov approx) Master equation

Simple derivation
arXiv: 1110.2122

- Lindblad equation for density matrix of the system ρ

$$\dot{\rho}(t) = [H, \rho(t)] + \sum_{\kappa} \left(L_\kappa \rho(t) L_\kappa^\dagger - \frac{1}{2} L_\kappa^\dagger L_\kappa \rho(t) - \frac{1}{2} \hat{\rho}(t) L_\kappa^\dagger L_\kappa \right)$$

Unitary dynamics — Jump terms — No-jump terms

- Rewriting the Lindbladian

$$\dot{\rho}(t) = -i \left(H_{\text{eff}}\, \rho - \rho H_{\text{eff}}^\dagger \right) + \sum_{\kappa} L_\kappa \rho L_\kappa^\dagger$$

with

$$H_{\text{eff}} = H - \frac{i}{2} \sum_{\kappa} L_\kappa^\dagger L_\kappa$$

Markovian environment:

i. Born approx.:
Weak coupling to environment

ii. Markov approx:
Env correlation time $\ll$ total relaxations time

$$\tau_{env} \ll \tau_{rel} \sim \Delta/\mu^2$$

Δ Typical system level spacing

μ Coupling to env

iii. Jump operators L_κ time ind.

$$1/\Delta \ll \tau_{rel}$$

The Kondo Model in dissipative media

- The Kondo Hamiltonian, ($c_{\mathbf{k}\sigma}$ describe mobile atoms in gs, f_σ metastable atoms in excited state)

$$H = \sum_{\mathbf{k},\sigma} \epsilon_k c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \sum_{\mathbf{k},\mathbf{k}',\sigma\sigma'} J c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}'\sigma'} \sigma_{\sigma\sigma'} \cdot \mathbf{S}_{imp} + \sum_{\mathbf{k},\mathbf{k}',\sigma} J' c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}'\sigma}$$

With $\mathbf{S}_{imp} = \sum_{\sigma\sigma'} f^\dagger_\sigma \vec{\sigma}_{\sigma\sigma'} f'_\sigma$ the local impurity spin

f_σ local fermion

- Add 2-body loss operators

$$L_\pm = \sqrt{2\gamma^\mp_{eg}} \frac{1}{\sqrt{2}} \left(f_\downarrow c_\uparrow(0) \pm f_\uparrow c_\downarrow(0) \right)$$

$$L_{\uparrow\uparrow} = \sqrt{2\gamma^+_{eg}} f_\uparrow c_\uparrow(0)$$

$$L_{\downarrow\downarrow} = \sqrt{2\gamma^-_{eg}} f_\downarrow c_\downarrow(0)$$

Nakagawa, Kawakami, Ueda
PRL 121, 203001 (2018)

- Obtain the Effective Non-Hermitian Hamiltonian (dropping jump operators)

$$H_{eff} = \sum_{\mathbf{k},\sigma} \epsilon_k c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \sum_{\mathbf{k},\mathbf{k}',\sigma\sigma'} (J_\Re + iJ_\Im) c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}'\sigma'} \sigma_{\sigma\sigma'} \cdot \mathbf{S}_{imp} + \sum_{\mathbf{k},\mathbf{k}',\sigma} (J'_\Re + iJ'_\Im) c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}'\sigma}$$

imaginary part $J_\Im$ is related to the rate $\Gamma_0 = DJ_\Im$ of two-body losses due to inelastic scattering.

Perturbative analysis (complex coupling $J = J_{\Re} + iJ_{\Im}$)

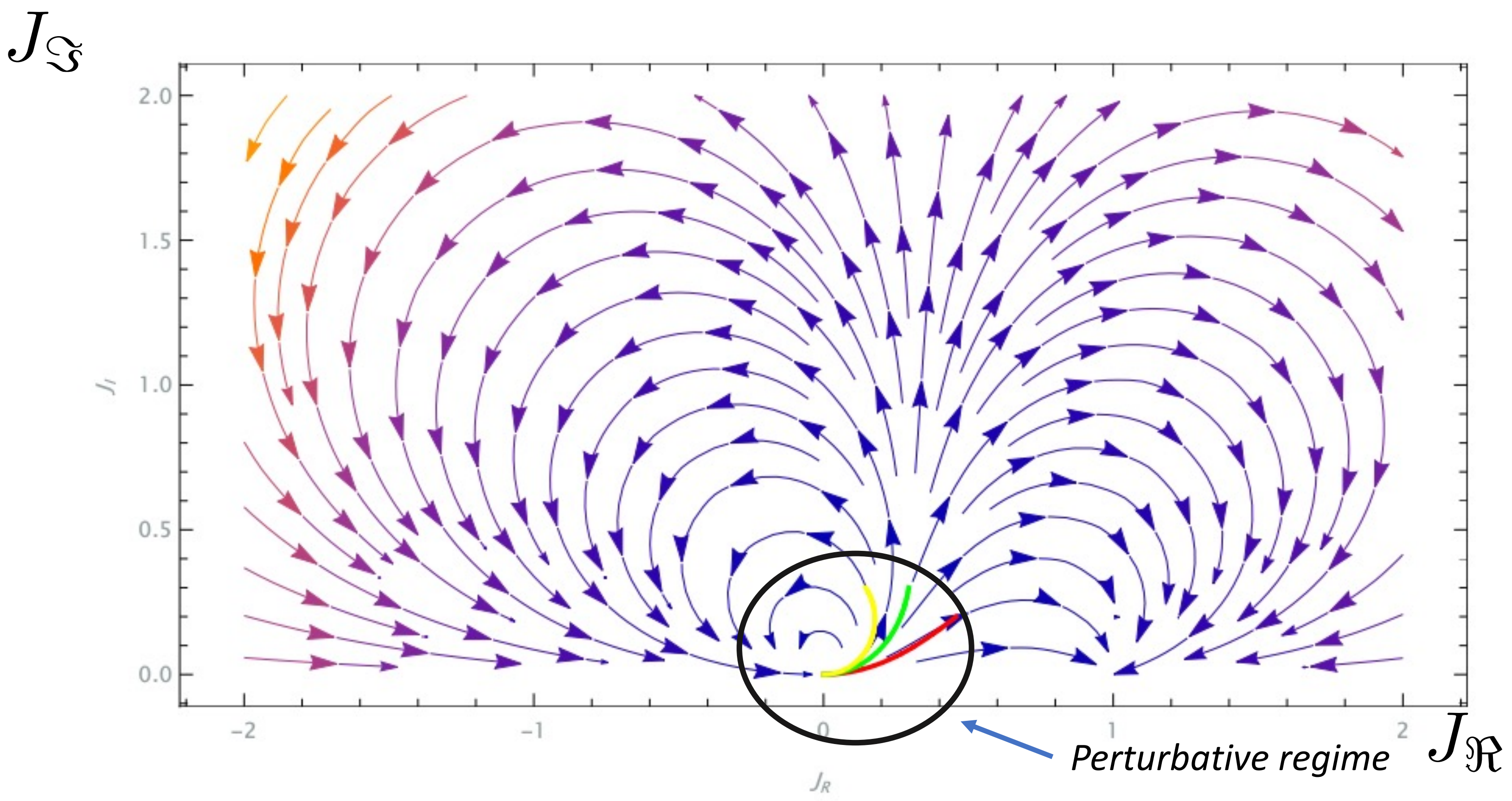


Figure: 2-loop RG for Non-Hermitian Kondo

- RG analysis Indicates two phases:

  screened and **unscreened**

  Nakagawa, Kawakami, Ueda
  PRL 121, 203001 (2018)

- We shall find other phases in between

$$\beta(J) = D\frac{dJ}{dD} = \rho_0 J^2 + \frac{1}{2}\rho_0^2 J^3 + \cdots$$

The Kondo Hamiltonian $(3d \to 1d)$

$$H_{3d} = \sum_{\mathbf{k},\sigma} \epsilon_k c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \sum_{\mathbf{k},\mathbf{k}',\sigma\sigma'} (J_\Re + iJ_\Im) c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}'\sigma'} \sigma_{\sigma\sigma'} \cdot \mathbf{S}_{imp} + \sum_{\mathbf{k},\mathbf{k}',\sigma} (J'_\Re + iJ'_\Im) c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}'\sigma}$$

- Linearize to obtain low energy effective Hamiltonian

Route 1: sum modes, linearize

$$\psi_{\epsilon a} \equiv \int d^3k\, \delta(\epsilon_{\vec{k}} - \epsilon)\, c_{\vec{k}a}$$

$$\{\psi_{\epsilon a}, \psi^\dagger{}_{\epsilon' b}\} = \delta_{ab}\, \delta(\epsilon - \epsilon')\nu(\epsilon)$$

$$\psi_a(x) = \int_{-D}^{D} \frac{d\epsilon}{\sqrt{\nu}}\, e^{i\epsilon x}\, \psi_{\epsilon a}$$

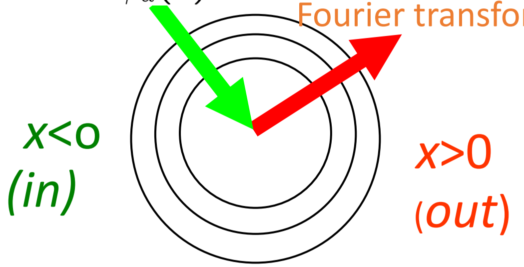
Route 2: impurity geometry, keep s-waves

$c_{\vec{k},a} \to c_{k,l,m,a}$ spherical modes

$\to c_{k,o,o,a}$ s-waves

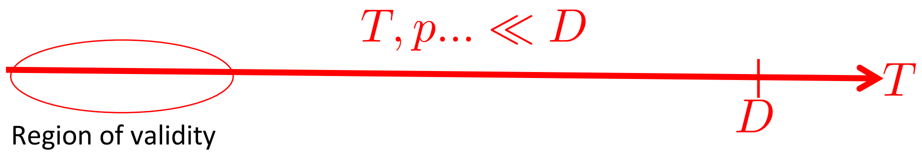
$\to c_{k_F+q,a}$ Linearize around k_F

$\to \psi_a(x)$ Fourier transform



$$H_{1d} = -i \int \psi^\dagger_\sigma(x)\, \partial_x\, \psi_\sigma(x) + (J_\Re + iJ_\Im)\psi^\dagger_\sigma(0)\vec{\sigma}_{\sigma\sigma'}\psi_{\sigma'}(0) \cdot \mathbf{S}_{imp} + (J'_\Re + iJ'_\Im)\psi^\dagger_\sigma(0)\psi_\sigma(0)$$

- Linearization valid for:



Equivalently, take the scaling limit: $D \to \infty$

Construction of eigenstates, eigenvalues

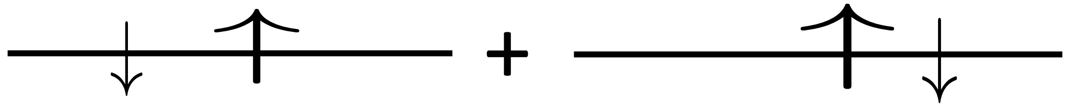
- We wish to solve $$H|F\rangle = E|F\rangle$$

with
$$|F\rangle = \int F_{\sigma_1 \ldots \sigma_{N^e}, \sigma_0}(x_1 \ldots x_{N^e}) \prod_{j=1}^{N^e} \psi^\dagger_{\sigma_j}(x_j)|0, \sigma_0\rangle$$

- Equivalently
$$hF_{\sigma_1 \ldots \sigma_{N^e}, \sigma_0}(x_1 \ldots x_{N^e}) = EF_{\sigma_1 \ldots \sigma_{N^e}, \sigma_0}(x_1 \ldots x_{N^e})$$

with
$$h = -i \sum_{j=1}^{N^e} \partial_j + (J_{\Re} + iJ_{\Im}) \sum_j \vec{\sigma}_j \cdot \mathbf{S}_{imp}\, \delta(x_j) + (J'_{\Re} + iJ'_{\Im}) \sum_j \delta(x_j)$$

- Eigenfunctions are composed of plain waves, e.g. Consider a single electron:



$A^{10}_{a_1,a_0} e^{ikx}$ + $A^{01}_{a_1,a_0} e^{ikx}$ → $E = k$

Impurity – electron S-matrix (derived from Hamiltonian)

$$F_{a_1 a_0}(x) = e^{ikx}[A^{10}_{a_1 a_0}\theta(-x) + A^{01}_{a_1 a_0}\theta(x)]$$ where $$A^{01}_{a_1 a_0} = \sum_{b_1 b_0} [S^{10}]^{b_1 b_0}_{a_1 a_0} A^{10}_{b_1 b_0}$$

Construction of eigenstates, eigenvalues

- The eigen functions take the form $F^{k_1 \dots k_N}_{\sigma_1 \dots \sigma^e_N, \sigma_0}(x_1 \dots x_N) = \sum_Q \sum_{\vec{\sigma}} \theta(x_Q) A^Q_{\vec{\sigma}} e^{i \sum k_j x_j}$

  Where we divided configuration space into $N!$ regions, $N = N^e + 1$, according to the ordering $x_{Q1} \leq x_{Q2} \leq \cdots \leq 0 \leq \cdots \leq x_{QN^e}$

- The amplitudes $A^Q_{\vec{\sigma}}$ are connected by S-matrices S^{jk}, S^{j0}

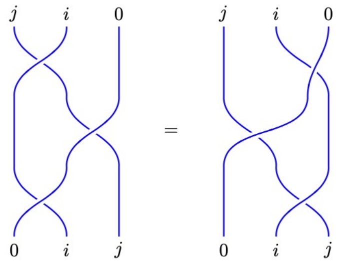


Figure: Graphical representation of the yang Baxter equation

- The impurity-electron S-matrix $S^{j0} = e^{-i\chi} \frac{I^{j0} - ic' P^{j0}}{1 - ic'}, c' \in \mathbb{C}$

  with $c' = \frac{2J}{1 - \frac{3J^2}{4}} = c\, e^{i\phi}, \; c \in \mathbb{R}$

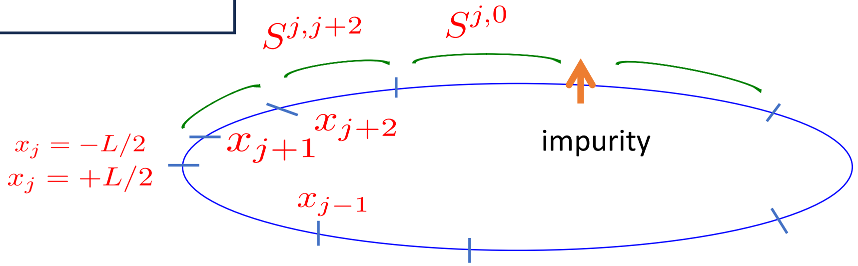
  The electron-electron S-matrix $S^{ij} = P^{ij}$

  Satisfy the Yang-Baxter (Braiding) equation: $S^{ij} S^{i0} S^{j0} = S^{j0} S^{i0} S^{ij}$ guaranteeing **consistency**

- Impose periodic boundary conditions

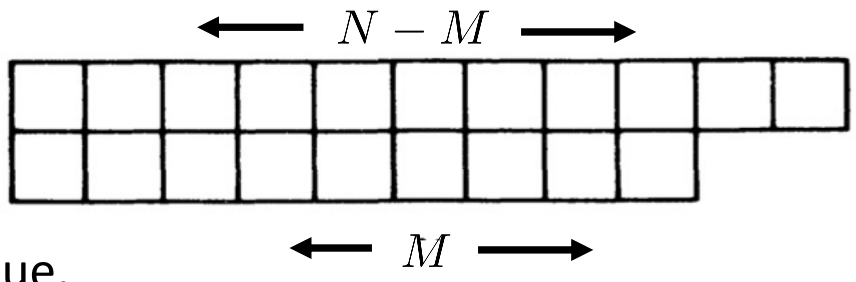
$$e^{-ik_j L} A_{\sigma_1 \dots \sigma_N} = (Z_j)^{\sigma'_1 \dots \sigma'_N}_{\sigma_1 \dots \sigma_N} A_{\sigma'_1 \dots \sigma'_N}$$

  with the **transfer matrix** $Z_j = S^{1j} .. S^{jN^e} S^{j0}$



- Solve for the momenta k_j, and the energy : $E = \sum_j k_j$

The Bethe Ansatz Equations



M Is the number of *down* spins

$N-M$ The number of *up* spins

$N=N^e+1$ Number of spins

$S=(N-2M)/2$ The total spin

- Diagonalizing the transfer matrix by means of the Scattering technique, we obtain: (for a state with M *spins down* and $N-M$ *spins up*)

Solution Expressed in terms of *M* spin - momenta: $\Lambda_1 \cdots \Lambda_M$

$$e^{ik_j L} = \prod_{\gamma=1}^{M} \frac{\Lambda_\gamma - 1 + ic/2}{\Lambda_\gamma - 1 - ic/2}$$

$$\prod_{\delta=1,\delta\neq\gamma}^{M} \frac{\Lambda_\delta - \Lambda_\gamma + ic}{\Lambda_\delta - \Lambda_\gamma - ic} = \left(\frac{\Lambda_\gamma - 1 - i\frac{c}{2}}{\Lambda_\gamma - 1 + i\frac{c}{2}}\right)^{N^e} \left(\frac{\Lambda_\gamma - 1 + e^{-i\phi} - i\frac{c}{2}}{\Lambda_\gamma - 1 + e^{-i\phi} + i\frac{c}{2}}\right)$$

Impurity term: it drives the Λ_γ roots to **complex** values

- Where:

$$\frac{2J}{1-\frac{3J^2}{4}} = c\, e^{i\phi}, \quad c \in \mathbb{R}$$

- The phase ϕ plays a central role:
it determines the coupling to environment
- For $\phi = 0$ one obtains the Hermitian BA eqns

- The allowed momenta

$$k_j = \frac{2\pi}{L} n_j + \frac{i}{L} \sum_{\gamma=1}^{M} \log \frac{\Lambda_\gamma - 1 + ic/2}{\Lambda_\gamma - 1 - ic/2}$$

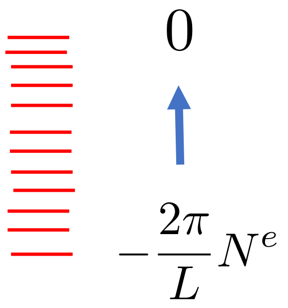
Note: The $\Lambda_1 \cdots \Lambda_M$ are complex for $\phi \neq 0$ and so also k_j are complex

The Energy

Charge part — Spin part

- The energy: $$E=\sum_j k_j = \frac{2\pi}{L}\sum n_j + i\frac{N}{L}\sum_{\gamma=1}^{M}\log\frac{\Lambda_\gamma - 1 + ic/2}{\Lambda_\gamma - 1 - ic/2}$$

Energies are complex , since $\Lambda_1\cdots\Lambda_M$ are complex for $\phi\neq 0$



- Charge-spin decoupling

- The charge quantum numbers n_j satisfy $$\frac{2\pi n_j}{L}\geq -2\pi D = -2\pi\frac{N^e}{L}$$

- The spin momenta Λ_γ solutions of the BAE determine the spin dynamics :

Free electrons term — The impurity term

$$\prod_{\delta=1,\delta\neq\gamma}^{M}\frac{\Lambda_\delta-\Lambda_\gamma+ic}{\Lambda_\delta-\Lambda_\gamma-ic}=\left(\frac{\Lambda_\gamma-1-i\frac{c}{2}}{\Lambda_\gamma-1+i\frac{c}{2}}\right)^{N^e}\left(\frac{\Lambda_\gamma-1+\cos(\phi)+i\sin(\phi)-i\frac{c}{2}}{\Lambda_\gamma-1+\cos(\phi)+i\sin(\phi)+i\frac{c}{2}}\right)$$

- Take the thermodynamic limit: $N\to\infty, L\to\infty$ holding D fixed

- The the scaling limit (Universality) $D\to\infty$

Solutions of the BAE: the dense set $\mathcal{C}$ and the bound mode

$$\prod_{\delta=1,\delta\neq\gamma}^{M} \frac{\Lambda_\delta - \Lambda_\gamma + ic}{\Lambda_\delta - \Lambda_\gamma - ic} = \left(\frac{\Lambda_\gamma - 1 - i\frac{c}{2}}{\Lambda_\gamma - 1 + i\frac{c}{2}}\right)^{N^e} \left(\frac{\Lambda_\gamma - 1 + \cos(\phi) + i\sin(\phi) - i\frac{c}{2}}{\Lambda_\gamma - 1 + \cos(\phi) + i\sin(\phi) + i\frac{c}{2}}\right)$$

Impurity term (not a phase)

- The impurity term determines the nature of the solutions according to value of $\alpha = \pi \sin\phi / c$
- For large N the roots Λ_γ fall close to the real line to order $1/N^2$ forming a dense set $\mathcal{C}$

Numerical solution: the Bethe roots are driven to the real line as $N \to \infty$

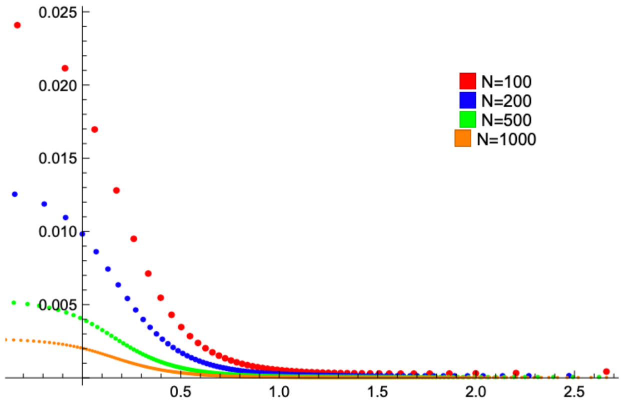


Figure: ground state Bethe roots for $c = \sqrt{1.5}$ and $\phi = \frac{\pi}{5}$

- Also have **isolated solution: Impurity string** $\Lambda_{IS} = 1 - \frac{ic}{2} - e^{-i\phi}$ **-** denotes a **local Bound mode**
- When impurity Bound mode is occupied $|B\rangle$, when unoccupied $|U\rangle$

The RG invariant scales and the Phase Diagram

- The model is characterized by two RG invariants:

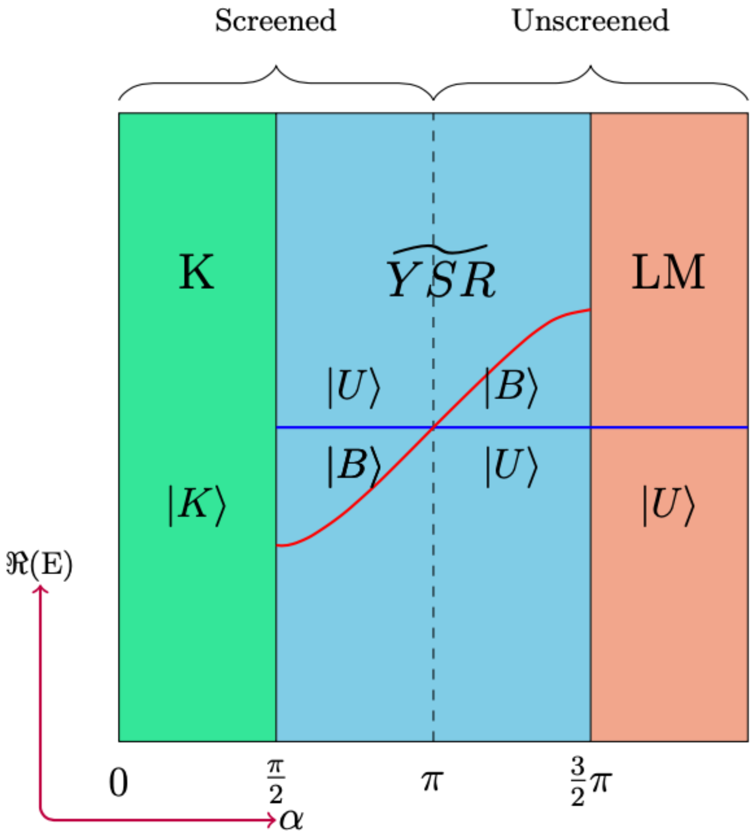
$$T_K = 2D\, e^{-\pi \cos\phi/c}$$ RG scale

$$\alpha = \pi \sin\phi/c$$ Coupling to environment

- Equivalently the complex Kondo scale

$$T_0 = 2De^{-\pi/c'} = T_K e^{i\alpha},$$

- **The parameter α determines the phase diagram**



- In terms of couplings in Hamiltonian $2\alpha/\pi \simeq J_i/J_r^2, \quad J_i \ll 1$

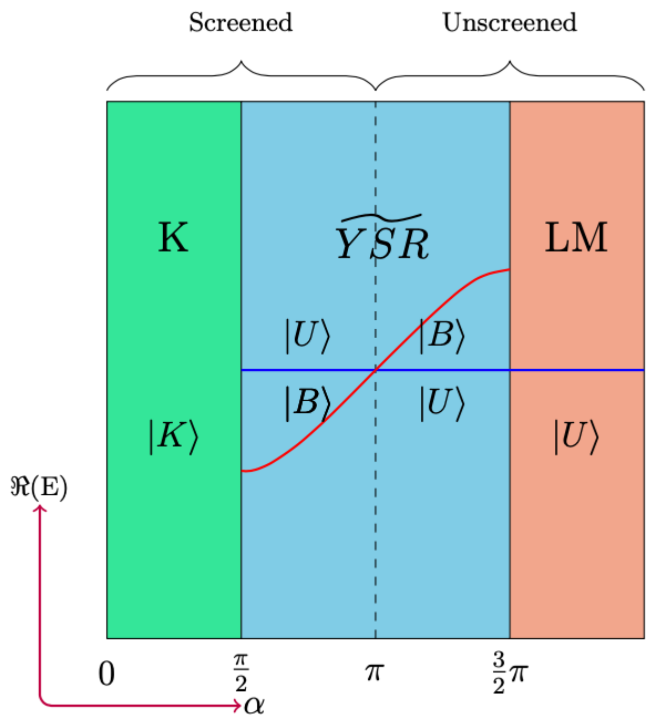
The Phase Diagram

- The impurity term determines the phase diagram of the model $\left(\frac{\Lambda_\gamma - 1 + \cos(\phi) - i\sin(\phi) - i\frac{c}{2}}{\Lambda_\gamma - 1 + \cos(\phi) - i\sin(\phi) + i\frac{c}{2}}\right)$

  - Becomes the impurity term of the Hermitian model when $\phi \to 0$
  - The imaginary part of the denominator changes sign when $\alpha = \frac{\sin\phi}{c}$ crosses π
  - **Local bound state exits in the region** $\pi/2 < \alpha < 3\pi/2$

The RG invariant α determines the phase structure of the model

- K denotes the Kondo phase where the impurity is screened by the Kondo cloud in the ground state |K⟩
- LM denotes the local moment phase where the impurity is unscreened in the ground state |U ⟩
- $\widetilde{YSR}$ phase – the impurity is screened by single particle bound mode in |B⟩ state, but not screened in |U ⟩ state.



$0 < \alpha < \pi/2$ The Kondo phase

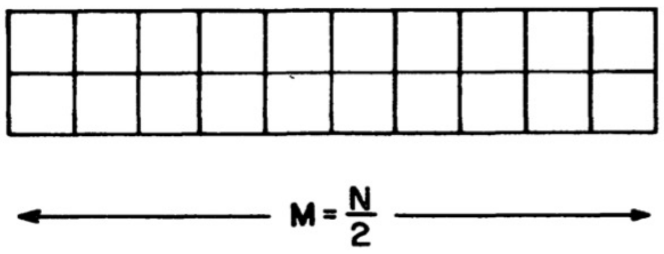
The ground state is a singlet – Impurity is screened by a multiparticle cloud

- Define density $\sigma(\Lambda_\gamma) = 1/(\Lambda_{\gamma+1} - \Lambda_\gamma)$, then $\sum_\gamma f(\Lambda_\gamma) = \int d\Lambda \sigma(\Lambda) f(\Lambda)$

- From BAE, the ground state solution density : $\sigma_K(\Lambda) = \frac{1}{2c}\left[N^e \operatorname{sech}\left(\frac{\pi(\Lambda - 1)}{c}\right) + \operatorname{sech}\left(\frac{\pi(\Lambda - 1 + e^{-i\phi}}{c}\right)\right]$

- The number of down spins $M = \int d\Lambda \sigma_0(\Lambda) = \frac{N^e + 1}{2} = \frac{N}{2}$

- The ground state spin



$$S = \frac{N^e + 1}{2} - \int \sigma_0(\Lambda) \mathrm{d}\Lambda = \frac{N^e + 1}{2} - \left[\frac{N^e}{2} + \frac{1}{2}\right] = 0$$

$0 < \alpha < \pi/2$ The Kondo phase: the magnetization

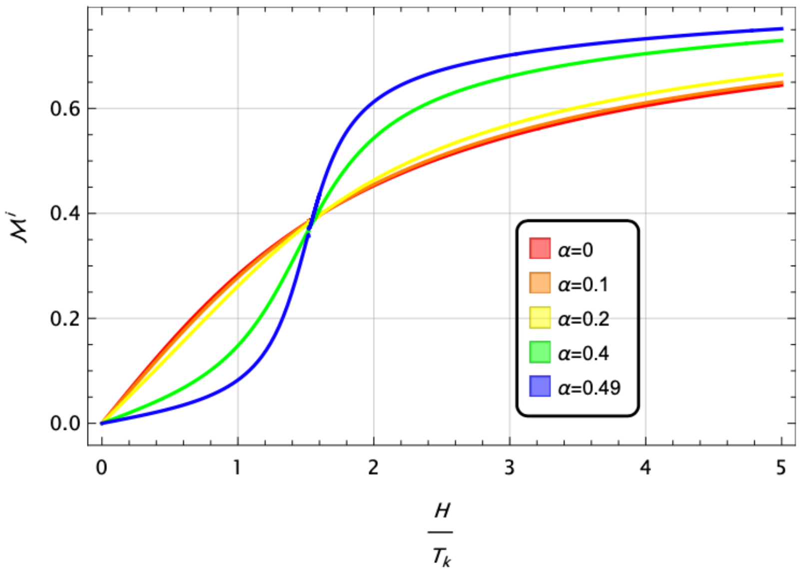
- Applying a magnetic field excites a macroscopic number of spinons from $-\infty$ to B

$$\sigma_B(\Lambda) + \int_B^{\infty} K\left(\Lambda - \Lambda'\right)\sigma_B\left(\Lambda'\right) d\Lambda' = f(\Lambda)$$

with $$f(\Lambda) = \frac{N^e}{\pi}\frac{c/2}{(c/2)^2 + (\Lambda-1)^2} + \frac{1}{\pi}\frac{c/2}{(c/2)^2 + (\Lambda - 1 + e^{-i\phi})^2}$$

- The real part of the impurity magnetization (Solve by Wiener – Hopf)

$$\mathcal{M}^i = \begin{cases} \frac{\mu}{\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \frac{\left(k+\frac{1}{2}\right)^{k-\frac{1}{2}}}{e^{k+\frac{1}{2}}} \left(\frac{2\pi}{e}\right)^{-k-\frac{1}{2}} \left(\frac{H}{T_K}\right)^{2k+1} \cos(\alpha(2k+1)), & H \le \sqrt{\frac{2\pi}{e}}T_K \\ \mu\left[1 - \int_0^{\infty} \frac{\mathrm{d}t}{t}\pi^{-\frac{3}{2}}\sin(\pi t)\Gamma\left(t+\frac{1}{2}\right)\left(\frac{2\pi}{e}\right)^{t} e^{t(1-\log(t))}\left(\frac{T_K}{H}\right)^{2t}\cos\left(2t\alpha\right)\right], & \sqrt{\frac{2\pi}{e}}T_K \le H \end{cases}$$



- The model still flows to strong coupling at low field, screening the impurity and to weak coupling at high field

$0 < \alpha < \pi/2$ The Kondo phase : The energy

- The energy eigenvalues $E = \sum_j k_j = \frac{2\pi}{L}\sum n_j + i\frac{N}{L}\sum_{\gamma=1}^{M}\log\frac{\Lambda_\gamma - 1 + ic/2}{\Lambda_\gamma - 1 - ic/2}$

- The **ground state energy** $E_{0K} = -\frac{\pi N^2}{2L} - iD\log\left(\frac{\Gamma(\frac{1}{2}-\frac{i}{2c'})\Gamma(1+\frac{i}{2c'})}{\Gamma(\frac{1}{2}+\frac{i}{2c'})\Gamma(1-\frac{i}{2c'})}\right)$ has real and imaginary parts

  - Real part $\Re(E_{0K})$ energy of the state

  - Imaginary part $\Im(E_{0K}) = -\Gamma_0 + \mathcal{O}(D/\log(2D/T_K))$ with $\Gamma_0 = DJ_{\Im}$

- The **Excitations**

  spin excitations: holes in the dense set of Λ - solutions, quartets, strings, bound states ..

  - Fundamental spin excitations: *spinons* $S^z = 1/2, \mathcal{E}(\Lambda) = 2De^{\frac{\pi}{c}(\Lambda-1)}$, Λ is the location of the hole (complex)

  Complex excitation energy $\mathcal{E}(\Lambda) = E + i\Gamma(E)$

  $\Gamma(E)$ is the decay rate ($\Gamma(E)$ < 0) or enhancement rate ($\Gamma(E)$ > 0) of the spinon states relative to the Kondo ground state.

  - Two spinons can be combined symmetrically or anti-symmetrically to form triplet and singlet spin excitations.

  Charge excitations decoupled spinless holons, holes

$0 < \alpha < \pi/2$ The Kondo phase: The DOS

- **Excitations: spinons carry spin ½, no charge, complex energy** $\mathcal{E} = 2De^{\pi\Lambda} = E + i\Gamma(E)$

- The spinon DOS $\rho(\mathcal{E})d\mathcal{E} = \sigma(\Lambda)\frac{d\Lambda}{d\mathcal{E}}d\mathcal{E}$ is given by $\rho_K(\mathcal{E}) = \frac{L}{2\pi} + \frac{1}{\pi}\frac{T_0}{\mathcal{E}^2 + T_0^2}$

bulk Impurity

- Define DOS in terms of real energy $\tilde{\rho}_K(E)dE = \rho_K(\mathcal{E})d\mathcal{E}$ it can be found in terms of $\Gamma(E)$, $\tilde{\rho}_K(E) = \Re(\rho_K(E)) + \mathcal{O}(1/L)$,

$$\tilde{\rho}_K(E) = \rho_K(E + i\Gamma(E))(1 + i\partial\Gamma(E)/\partial E) \in \mathbb{R}$$

- Spinon decay rate $\Gamma(E) \simeq \frac{1}{L}\tanh^{-1}\left(\frac{2ET_K}{E^2 + T_K^2}\sin\alpha\right) \ll 1,$ Valid for most couplings and energies, but log singularity for $\alpha \to \pi/2, E \to T_K$

- For most couplings and energies $\Gamma(E) \geq 0$ hence the states with one spinon have a <u>longer</u> lifetime than the Kondo ground state itself. This indicates that it is dynamically advantageous to remove a state from the Kondo cloud.

- This lowers the amplitude for a singlet state to be formed at the impurity site

- we expect the Kondo state to be dynamically unstable against depopulation of the screening cloud through single spinon excitations (experimental prediction), but it is a 1/L effect.

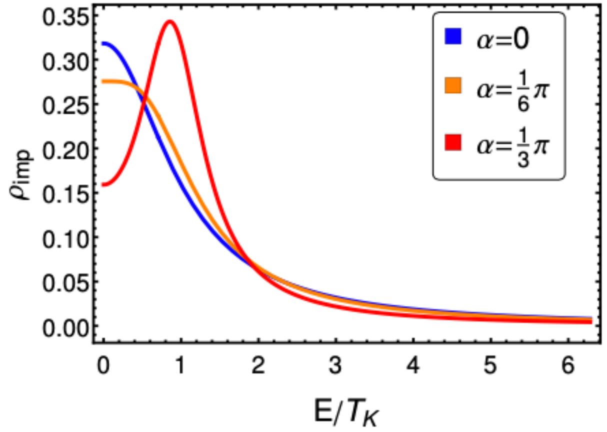
$0 < \alpha < \pi/2$

The Kondo phase: the DOS

- We note:
  - States with one spinon have a longer lifetime than the Kondo ground state itself, $\Gamma(E) \geq 0$
  - it is dynamically advantageous to remove a state from the Kondo cloud
  - this lowers the amplitude for a singlet state to be formed at the impurity site, avoiding losses.
  - But the time scale for such a process being ∝ L, so Kondo state is dynamically stable

- The impurity DOS

$$\rho_{\rm imp}\left[\frac{E}{T_K}\right] = \frac{\cos\alpha}{\pi T_K}\frac{1+(E/T_K)^2}{1+2(E/T_K)^2\cos 2\alpha+(E/T_K)^4},$$



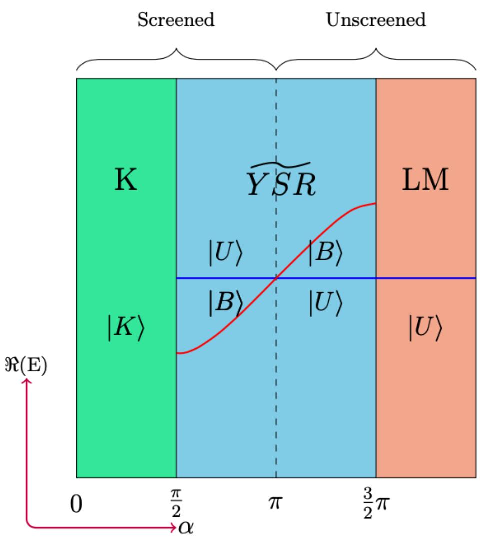
Shift observed by STM

- As α varies from 0 to π/2, the impurity DOS changes from a pure Kondo behavior at α = 0 with a peak at $E=0$, to a situation where the peak is shifted to $E_\alpha = \sqrt{2\sin\alpha - 1}\, T_K$ when $\alpha \geq \pi/6$. Eventually, when $\alpha \to (\pi/2)^-$ the DOS develops a delta peak at $E = T_K:\ \tilde{\rho}(E)) = L/2\pi + 1/2\ \delta(E - T_K)$

- Therefore, as α increases (so does the bare loss rate $J_\Im$) the number of modes that contribute to the Kondo screening of the impurity decreases until there remains one single mode at $E = T_K$ when α → π/2.

- We interpret this transfer of spectral weight towards T_K as announcing the apparition of a bound state when $\alpha > \pi/2$

$\pi/2 < \alpha < 3\pi/2$ The Bound-Mode phase $\widetilde{YSR}$



A new isolated solution appears in addition to the dense set of solutions of BAE $\mathcal{C}$ –
Impurity String **(IS)**

$$\Lambda_{IS} = 1 - \frac{ic}{2} - e^{-i\phi}$$

- The state may be occupied (in addition to $\mathcal{C}$) denote: $|B\rangle$, or unoccupied, denote $|U\rangle$

  IS energy of the occupied state w.r.t. unoccupied state

$$E_{IS} = E_{0B}(N^e) - \frac{1}{2}(E_{0U}(N^e+1) + E_{0U}(N^e-1)) = -\frac{\pi}{2L} + (E_b + i\Gamma_b)$$

where $\boxed{E_b = -T_K \sin\alpha \quad \text{and} \quad \Gamma_b = T_K \cos\alpha \leq 0}$

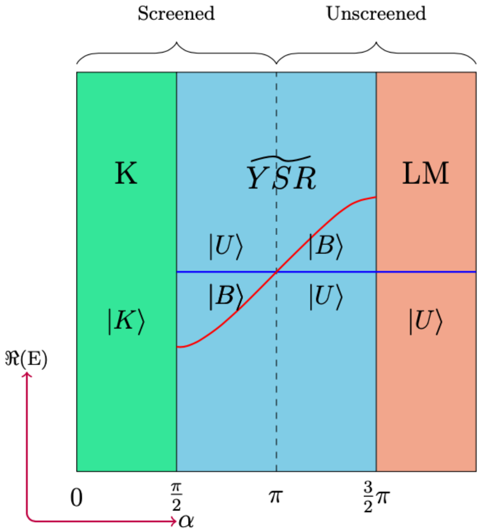
- Occupying the IS state in the range $\pi/2 \leq \alpha \leq \pi$ lowers the energy leading to a ground state $|B\rangle$, while in the range $\pi \leq \alpha \leq 3\pi/2$, not occupying the IS lowers the energy leading to a state $|U\rangle$

- In the state $|B\rangle$ the impurity is screened by a single particle bound mode localized near it, the impurity and the bound mode forms a singlet. (similar to YSR state in a SC)

- The bound state energy imaginary part $\Gamma_b = T_K \cos\alpha < 0$ gives the bound state a finite lifetime. Hence, in the $\widetilde{YSR}$ phase, even in the regime $\pi/2 \leq \alpha \leq \pi$, the impurity is eventually found to be unscreened at long enough times $\Gamma_0^{-1} \gg t \gg |\Gamma_b|^{-1}$.

- This reflects the fact that the quantum phase transition between the Kondo and unscreened phases is dynamically induced by losses.

$\pi/2 < \alpha < 3\pi/2$ The Bound-Mode phase $\widetilde{YSR}$



- **The state** $|U\rangle$ **-** the bound mode is not occupied

  - The root density $\sigma_U(\Lambda) = \frac{1}{2c}\frac{N^e}{\cosh\frac{\pi}{c}(\Lambda-1)} + \frac{1}{2\pi}\left(\frac{1}{i(\Lambda-\Lambda_{IS})} + \Psi\left(\frac{i(\Lambda-\Lambda_{IS})}{2c}\right) - \Psi\left(\frac{i(\Lambda-\Lambda_{IS})+c}{2c}\right)\right)$

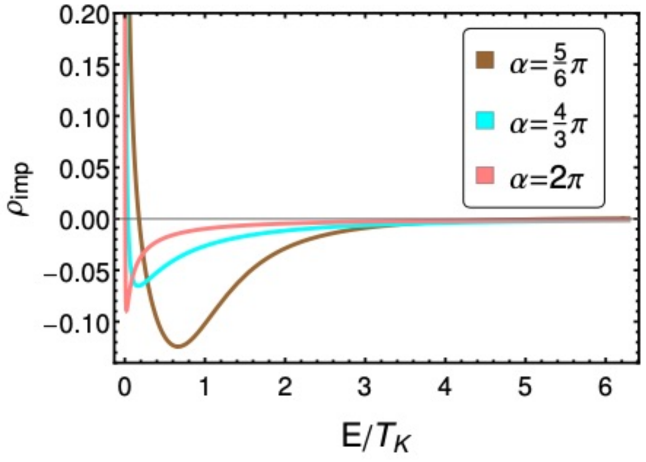
  - The number of solutions $M = \int_{\mathbb{R}} \mathrm{d}\Lambda\, \sigma_U(\Lambda) = \frac{N^e}{2}$ (Here choose N^e even. If N^e odd need add hole)

  Spin of state $S^z = \pm 1/2$: impurity unscreened

  - The ground state energy $E_{0U} = -\frac{\pi N^2}{2L} - iD\log\left(\frac{\Gamma(\frac{i}{2c'})\Gamma(1+\frac{i}{2c'})}{\Gamma^2(\frac{1}{2}+\frac{i}{2c'})}\right)$

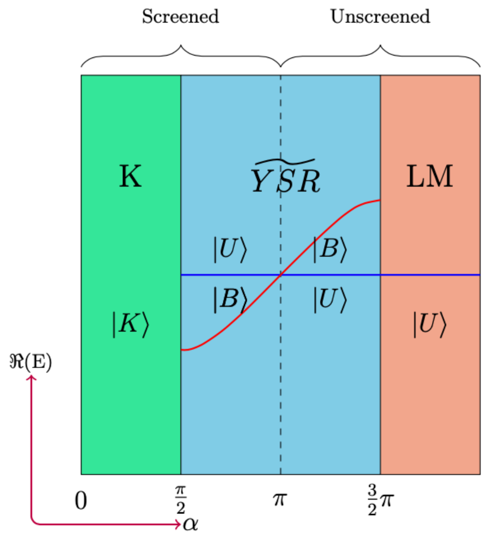
  - The spinon DOS as a function of real E we find $\tilde{\rho}_U(E) = \Re\rho_U(\mathcal{E})$

    where $\rho_U(\mathcal{E}) = \sigma_U(\Lambda)\frac{d\Lambda}{d\mathcal{E}}$ and $\mathcal{E} = E + i\Gamma(E)$



- We observe that the contribution of the spinons to the DOS can be positive as well as negative depending on the energy. We conjecture that the positive contribution corresponds to a partial screening of the impurity and negative contribution to the DOS as the signature that spinons do not participate to the screening of the impurity

$\pi/2 < \alpha < 3\pi/2$ The Bound-Mode phase $\widetilde{YSR}$



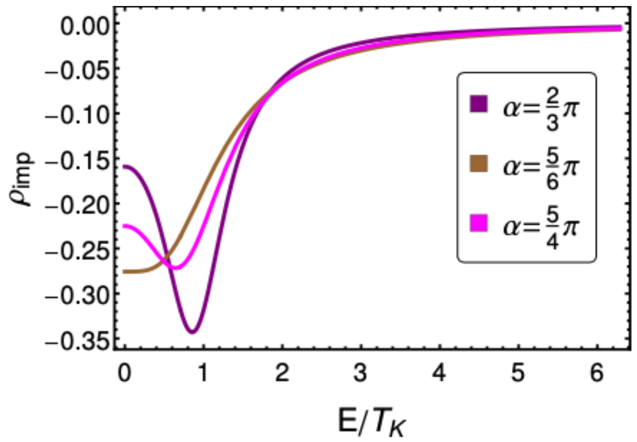
- **The state** $|B\rangle$ - the bound state is occupied

- The continuous root density and ground state energy are analytic continuations of the Kondo phase to region $\pi/2 < \alpha < 3\pi/2$

$$\sigma_B(\Lambda) \equiv \sigma_K(\Lambda), \quad E_{0B} \equiv E_{0K}, \quad \alpha \in (\pi/2, 3\pi/2).$$

- The spinon DOS

$$\tilde{\rho}_B(E) = \Re\tilde{\rho}_K(E) + \delta(E - E_b)$$



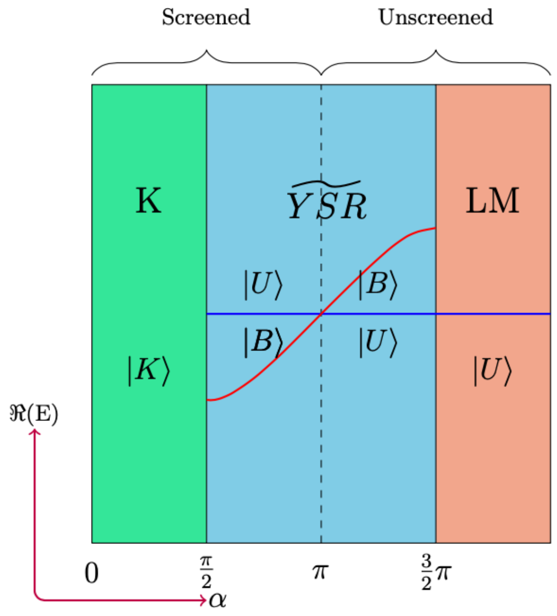
- $\Re\tilde{\rho}_K(E)$ is the analytic continuation to $\alpha \in (\pi/2, \pi)$

- The delta function contribution comes from the bound state.

- The impurity contribution to the spinon DOS is always negative. However, due to the positive delta function contribution the integrated density of state is positive and equals to ½ as in the Kondo phase.

- We interpret the negative contribution of the spinons as the signature that they do not participate to the screening of the impurity in the |B⟩ state, only the bound state does.

$\alpha > 3\pi/2$ The Local Moment Regime



- For $\alpha > 3\pi/2$ there is no single particle bound state the impurity can not be completely screened.

- For N^e even, the total spin of the ground state is S=1/2 and it is described by the analytical continuation of $|U\rangle$ to values of $\alpha > 3\pi/2$

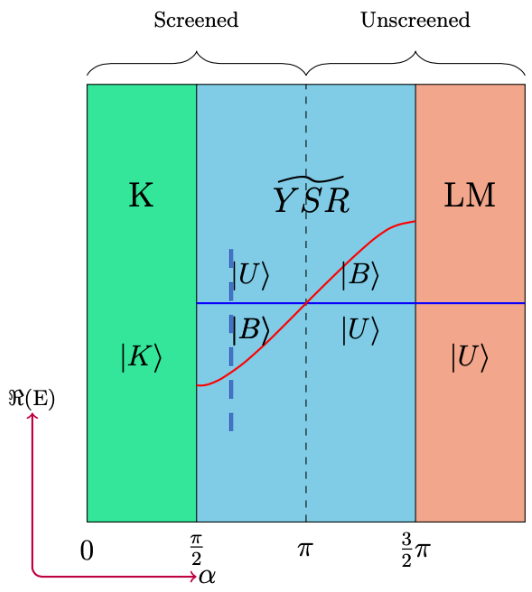
$$\sigma_U(\Lambda) = \frac{1}{2c}\frac{N^e}{\cosh\frac{\pi}{c}(\Lambda-1)} + \frac{1}{2\pi}\left(\frac{1}{i\Lambda} + \Psi\left(\frac{i\Lambda}{2c}\right) - \Psi\left(\frac{i\Lambda+c}{2c}\right)\right)$$

- The ground state root distribution and the density of states are given as above. The impurity is partially screened by the positive part of DOS.

Dissipation driven phase transition

We saw $E_b = -T_K \sin\alpha \quad \text{and} \quad \Gamma_b = T_K \cos\alpha \leq 0$

so bound state energy E_b is negative for α ≤ π and positive for α ≥ π.



On the basis of purely energetic considerations, the impurity is screened (resp. unscreened) in the region α ≤ π (resp. α ≥ π) with a first order phase transition at α = π where the two states cross.

However, since $\Gamma_b \leq 0$ even when α ≤ π, the impurity is eventually found unscreened at sufficiently large time $1/\Gamma_0 \gg t \gg \tau_b \equiv 1/|\Gamma_b|$

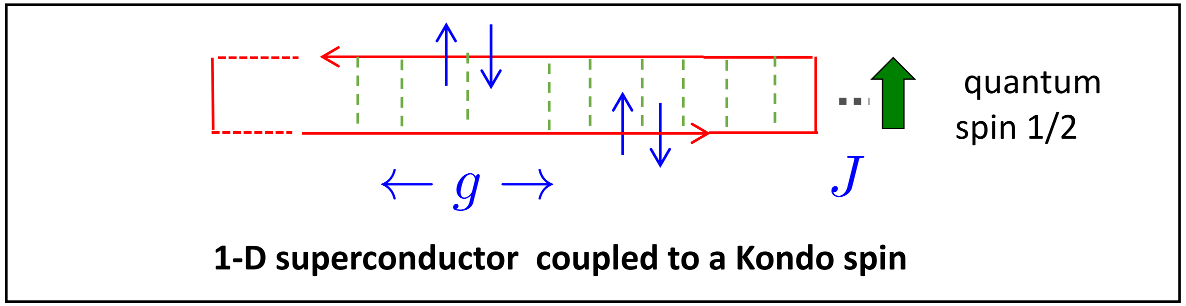
Indeed, consider the system with $\pi/2 < \alpha < \pi$:
prepare the state of the system at time *t = 0* in a linear combination of the unscreened and screened states $|\psi\rangle = u|U\rangle + b|B\rangle$.

After some time *t* (including losses)

$$|\psi(t)\rangle \sim (u|U\rangle + be^{-t/\tau_b}|B\rangle)e^{-\Gamma_0 t}$$

So that whenever $u \neq 0$ after a time $t \gg \tau_b$ the system will be in the unscreened state |U ⟩.

Kondo spin at the edge of a 1-d superconductor



Pasnoori, Rylands, Andrei
Phys. Rev. Research 2, 013006 (2020)

Pasnoori, Rylands, Andrei, Azaria
Phys. Rev. B 105, 174517 (2022)

Model is integrable for any *g* (bulk coupling) and *J* (impurity coupling)

$$H = H_0 + H_{\text{int}} + H_{\text{imp}}$$

$$\begin{cases} H_{\text{int}} = -g \int_{-\frac{L}{2}}^{0} dx\, \psi^\dagger_{Ra}\psi^\dagger_{Lc}\left(\vec{\sigma}_{ab}\cdot\vec{\sigma}_{cd}\right)\psi_{Rb}\psi_{Ld} \\ H_{\text{imp}} = -J\left(\vec{\sigma}_{ab}\cdot\vec{S}_{cd}\right)\psi^\dagger_{La}(0)\psi_{Rb}(0) \end{cases}$$

Charge conserving 4 fermion interaction term
- Open SC gap

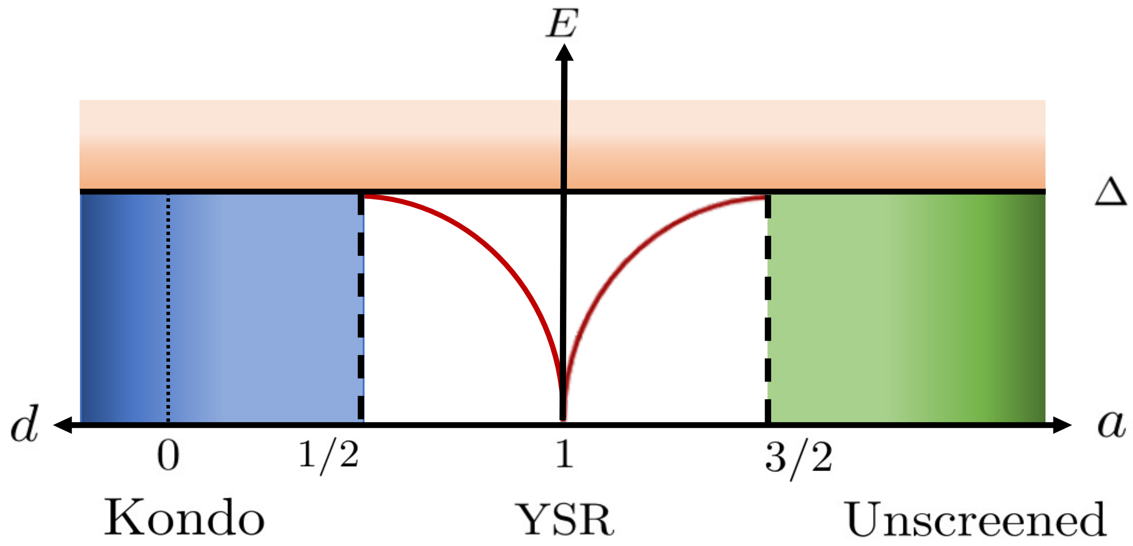
Open boundary conditions:

$$\psi^\dagger_{Ra}(0) = -\psi^\dagger_{La}(0) \qquad \psi^\dagger_{Ra}(-L/2) = -\psi^\dagger_{La}(-L/2)$$

Model is integrable for any *g* and *J*

Convenient parameters

Bulk parameter	$b = (1 - g^2/4)/2g$		
RG invariant	$d = \sqrt{b^2 - \frac{b(1-3J^2/4)}{J} - 1}$	$d \in \mathbb{R}$	$d = ia$



a **RG invariant**

$J \gg 2g$ $\quad J \sim 2g$ $\quad J \ll 2g$

- The bulk is gapped $\Delta = De^{-\pi b}$
- Three regimes - depending on relative values of *g* and *J*
  - Kondo $T_K = \Delta\sqrt{1+\cosh^2 \pi d} \underset{J \gg g}{\rightarrow} De^{-\pi/2J}$
  - YSR
  - Local moment

Conclusions and Outlook

- **Kondo correlations vs dissipation**: A dynamical phase transition occurs at $\alpha = \pi/2$ due to the inelastic collision between the impurity and itinerant atoms in a Kondo system in open quantum setting . Recall α is an RG invariant parameter which describes the coupling to the environment.

- For $\alpha < \pi/2$ the Kondo physics survives where the impurity is screened by multiparticle Kondo cloud.

- A dynamical phase appear when $\pi/2 < \alpha < 3\pi/2$ where two distinct kinds of states exist: one where bound mode is formed at the impurity site which screens the impurity spin and another where impurity is unscreened.

- For $3\pi/2 < \alpha$ the impurity cannot be screened.

- Note that even in the Kondo-phase only for narrow regime $\alpha < \pi/6$ the density of states has a Lorentzian peak centered at $E = 0$ but when $\alpha > \pi/6$ the peak in the spinon density starts to move away from $E = 0$ to $E = T_K$.

- **Dissipation plays role analogous to SC condensate**: removes electrons from the Kondo cloud