Ultracold dipolar bosons trapped in atomtronic circuits

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Outline

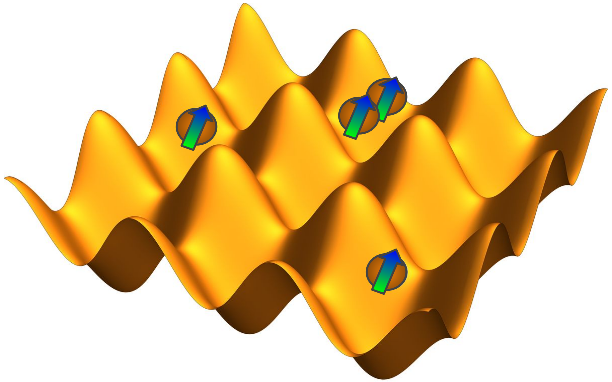
- Bose-Hubbard model with dipolar interactions
- Mott-like and Dipole-favoured regimes
- Average site occupation
- Ground state properties
- Correlations and dominant Fock states
- Energy gap and condensed fraction
- Summary and conclusions

M. Rovirola et al., Arxiv:2403.11620

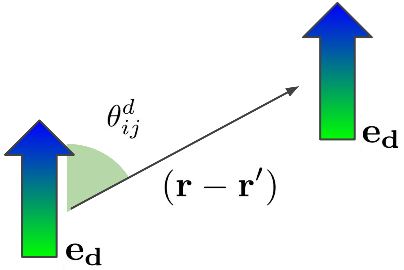
Bose-Hubbard model with dipolar interaction

$$\hat{\mathcal{H}} = -J \sum_{<i,j>}^{N_s} [\hat{a}_i^\dagger \hat{a}_j + h.c.] + \frac{U}{2} \sum_{i}^{N_s} \hat{n}_i(\hat{n}_i - 1) + \sum_{\substack{i,j \\ i \neq j}}^{N_s} \frac{V_{ij}^d}{2} \hat{n}_i \hat{n}_j$$

Nearest neighbour hopping — Contact interaction — Dipolar interaction



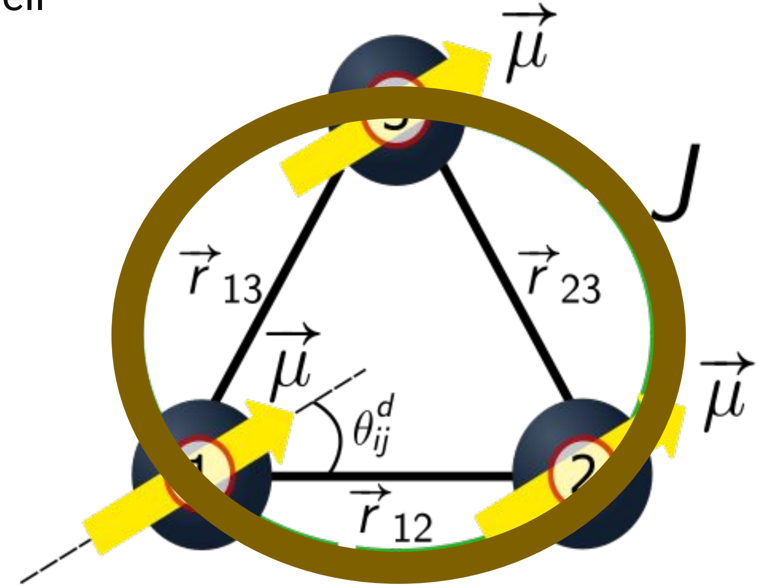
Dipolar interaction



$$V_{ij}^d = U_d \frac{1 - 3\cos^2 \theta_{ij}^d}{|\mathbf{r} - \mathbf{r}'|^3}$$

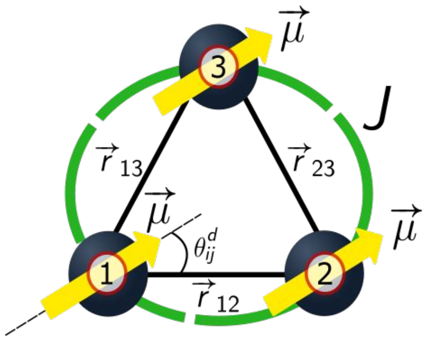
The system

Fully connected triple well

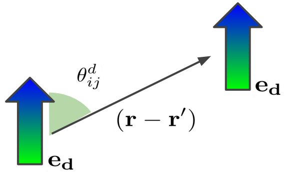


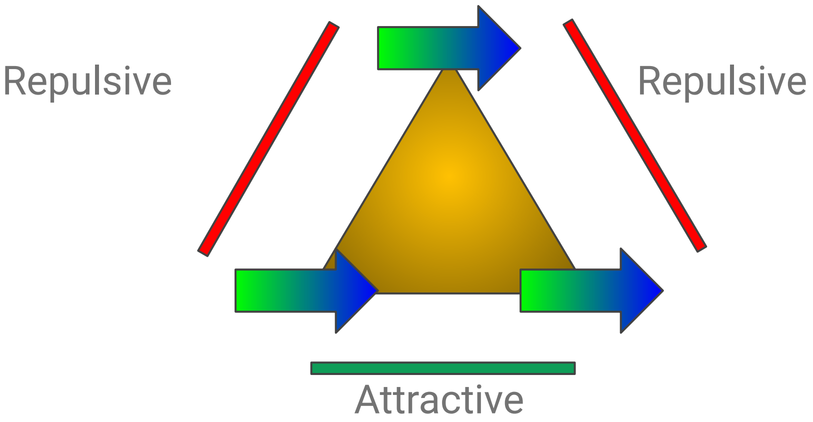
K. Wittmann W. et al., Phys. Rev. E 105, 034204 (2022)
A. Richaud et al., Sci Rep 9, 6908 (2019)
A. Gallemí et al., Phys. Rev. A 88, 063645 (2013)

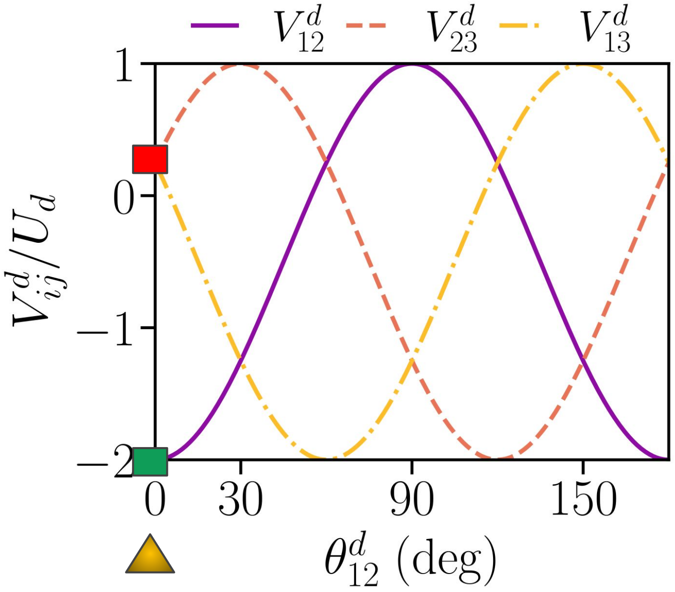
The system: equilateral triple well



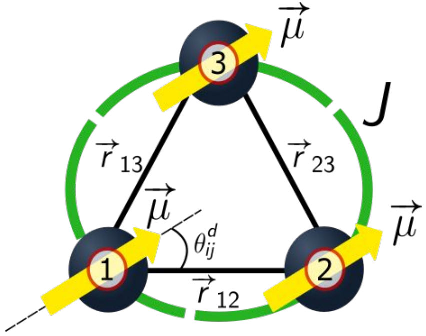
$$V^d_{ij} = U_d\,(1 - 3\cos^2\theta^d_{ij})$$



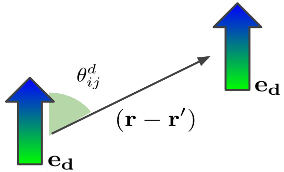


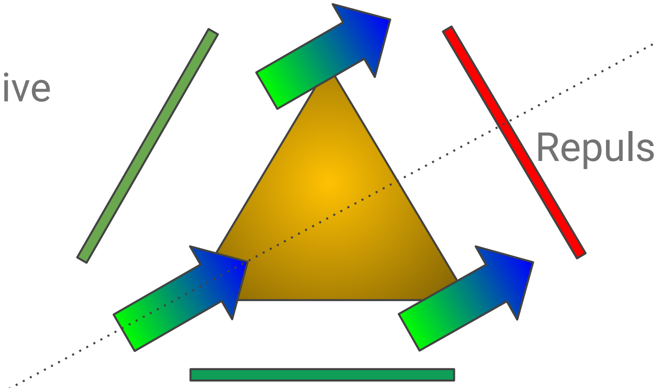


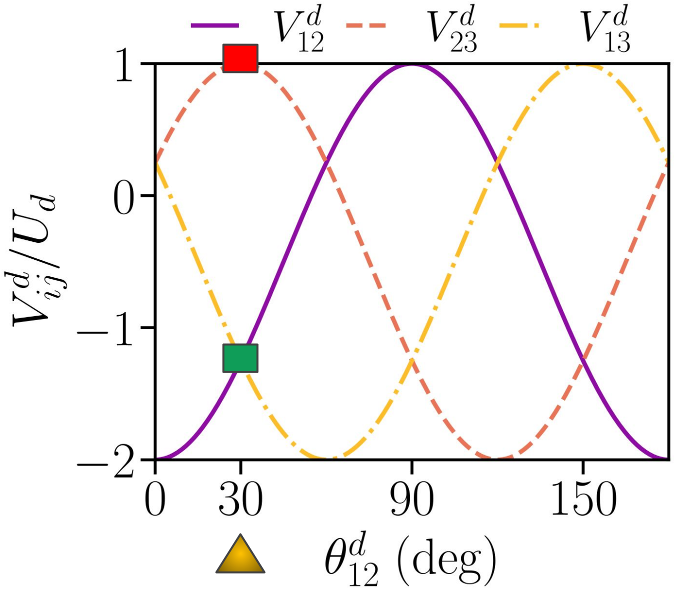
The system: equilateral triple well



$$V_{ij}^d = U_d \left(1 - 3\cos^2\theta_{ij}^d\right)$$







Bose-Hubbard regimes at atomic limit J=0

Mott insulator regime:

$$\hat{\mathcal{H}} = \frac{U}{2}\sum_{i}^{N_s} \hat{n}_i(\hat{n}_i - 1)$$

For repulsive contact interactions, ground state is a Mott insulator state

$$|N/3, N/3, N/3\rangle$$

Dipole-favoured regime:

$$\hat{\mathcal{H}} = \sum_{\substack{i,j \\ i \neq j}}^{N_s} \frac{V_{ij}^d}{2} \hat{n}_i \hat{n}_j$$

If there is any attractive dipolar interaction, ground state populates only those 2 wells

$$|N - n_i, n_i, 0\rangle$$

Mott-like and dipole-favoured regimes: transition between regimes

Mott state energy

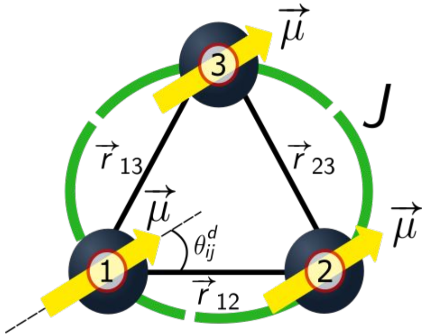
$$E_{|N/3,N/3,N/3\rangle} = \frac{N}{2}\left[U(N/3-1) - U_d N/3\right]$$

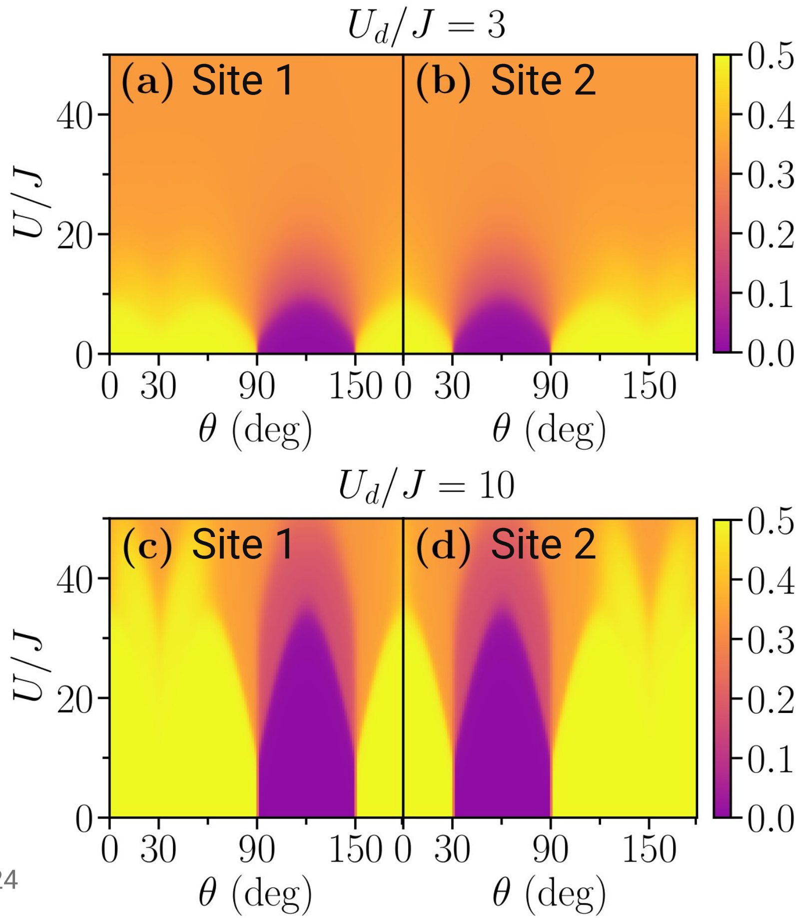
Dipole-favoured state energy

$$E_{|N-n_i,n_i,0\rangle} = \frac{U}{2}[(N-n_i)^2 + n_i^2 - N] + U_d[1-3\cos^2(\theta_{12})](Nn_i - n_i^2)$$

$$U_c(\theta) = \frac{U_d\left[N^2 + 6(N-n_i)n_i(1-3\cos^2\theta)\right]}{2\left[3(N-n_i)n_i - N^2\right]}$$

Average site occupation N=6

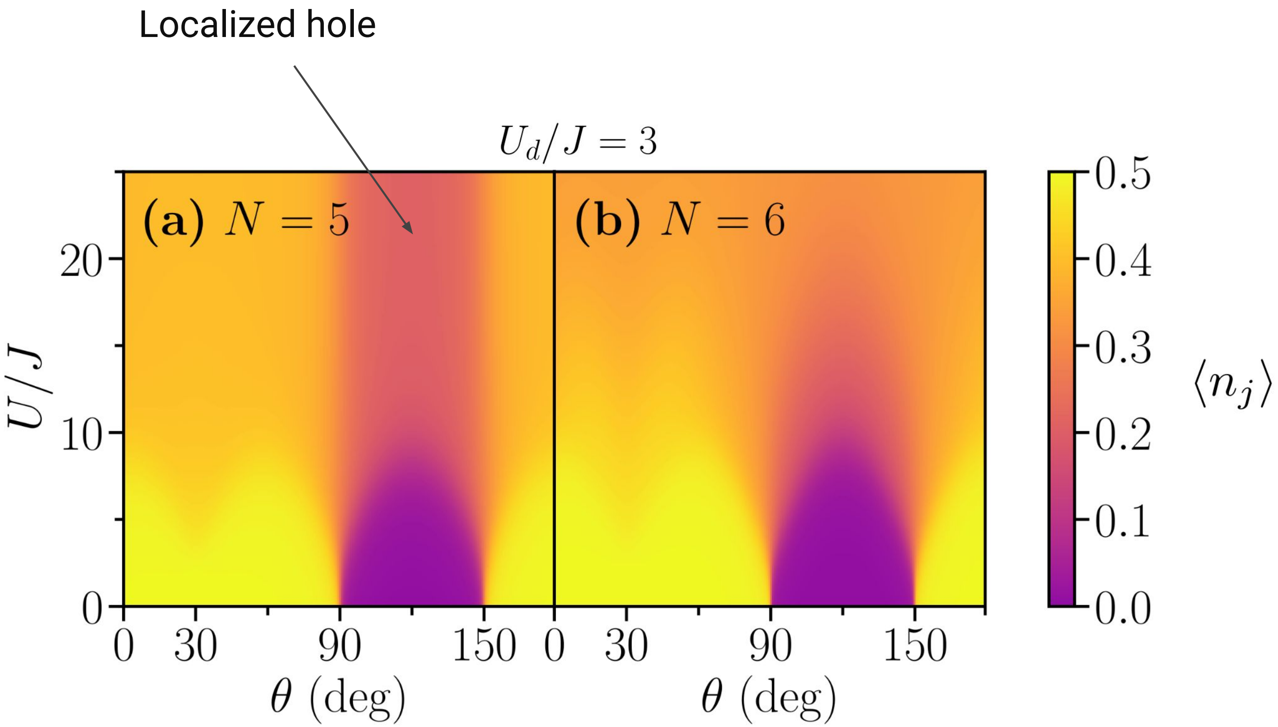




$$\langle n_j \rangle = \langle \Psi_{\mathrm{GS}} | \hat{n}_i | \Psi_{\mathrm{GS}} \rangle / N$$

$\langle n_j \rangle$

Average site occupation

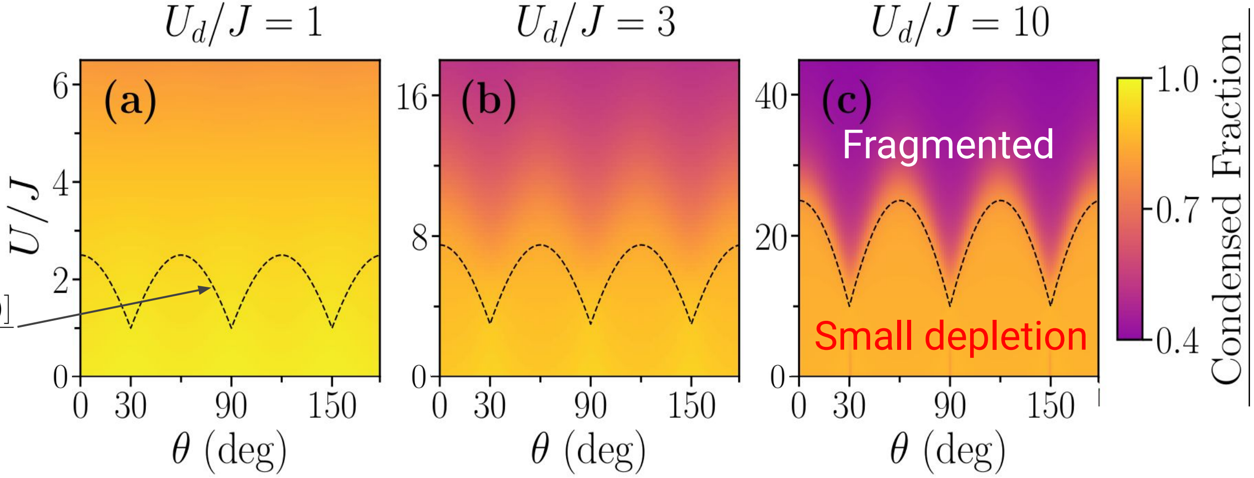


Quantum many body properties N=3: Condensed fraction

$$\hat{\rho}_{OBDM} = \sum_{j}^{N_s} \sum_{k}^{N_s} \frac{\langle \Psi_{GS} | \hat{a}_j^\dagger \hat{a}_k | \Psi_{GS} \rangle}{N} \; |j\rangle \langle k|$$

Highest eigenvalue
=
population of single particle orbit

$$U_c(\theta) = \frac{U_d \left[N^2 + 6(N - n_i)n_i(1 - 3\cos^2\theta)\right]}{2\left[3(N - n_i)n_i - N^2\right]}$$

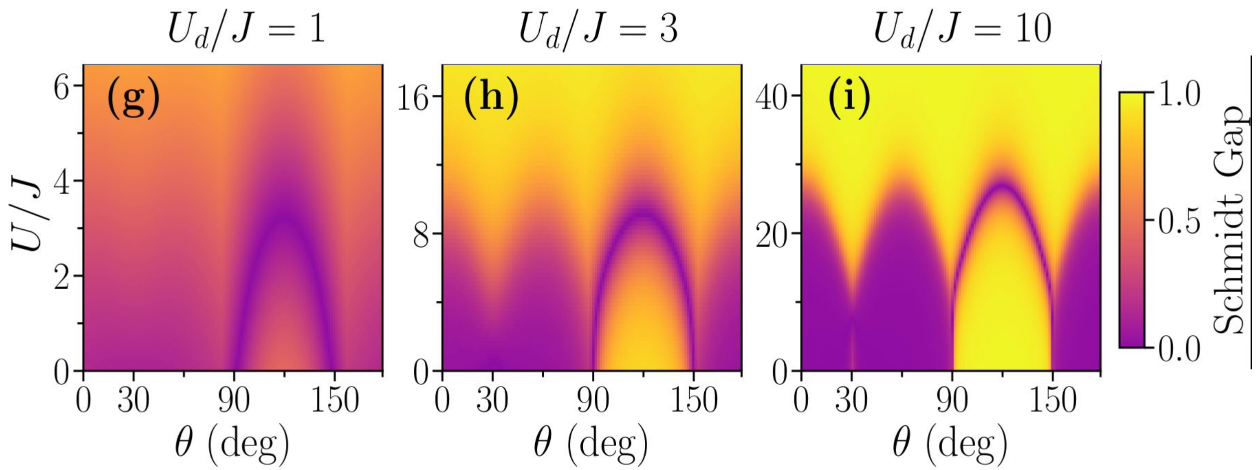


Quantum many body properties N=3: Schmidt Gap

Schmidt coefficients

Reduced 1-site density matrix

$$\hat{\rho}_j = \sum_{n_j=0}^{N} \lambda_{n_j} \, |n_j\rangle \langle n_j|$$

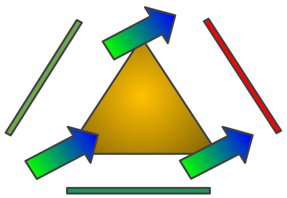


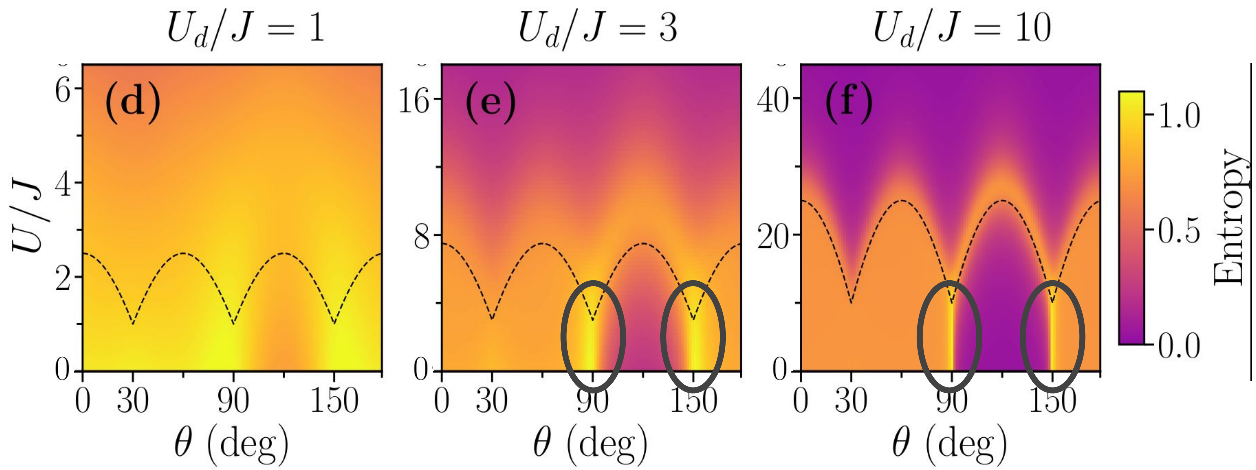
Quantum many body properties: Von Neumann entropy

Charles H. Bennett et al., Phys. Rev. A 53, 2046 (1996)
L. Amico et al., Phys. Rev. Lett. 78, 5022 (1997)
L. Dell'Anna et al., Phys. Rev. A 87, 053620 (2013)

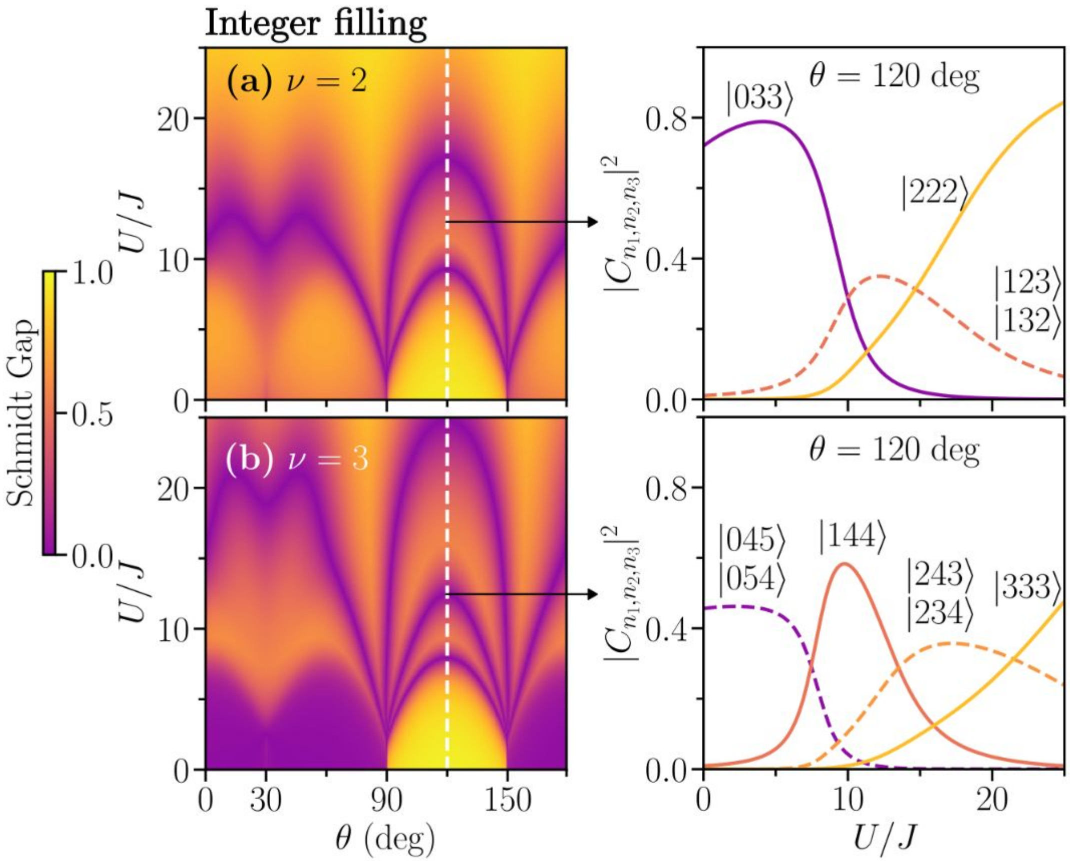
Reduced 1-site density matrix $\hat{\rho}_j = \sum_{n_j=0}^{N} \lambda_{n_j} \; |n_j\rangle \langle n_j|$

$$S_j^{VN} = -\,\mathrm{Tr}\left(\hat{\rho}_j \log \hat{\rho}_j\right) = -\sum_{n_j=0}^{N} \lambda_{n_j} \log \lambda_{n_j}$$

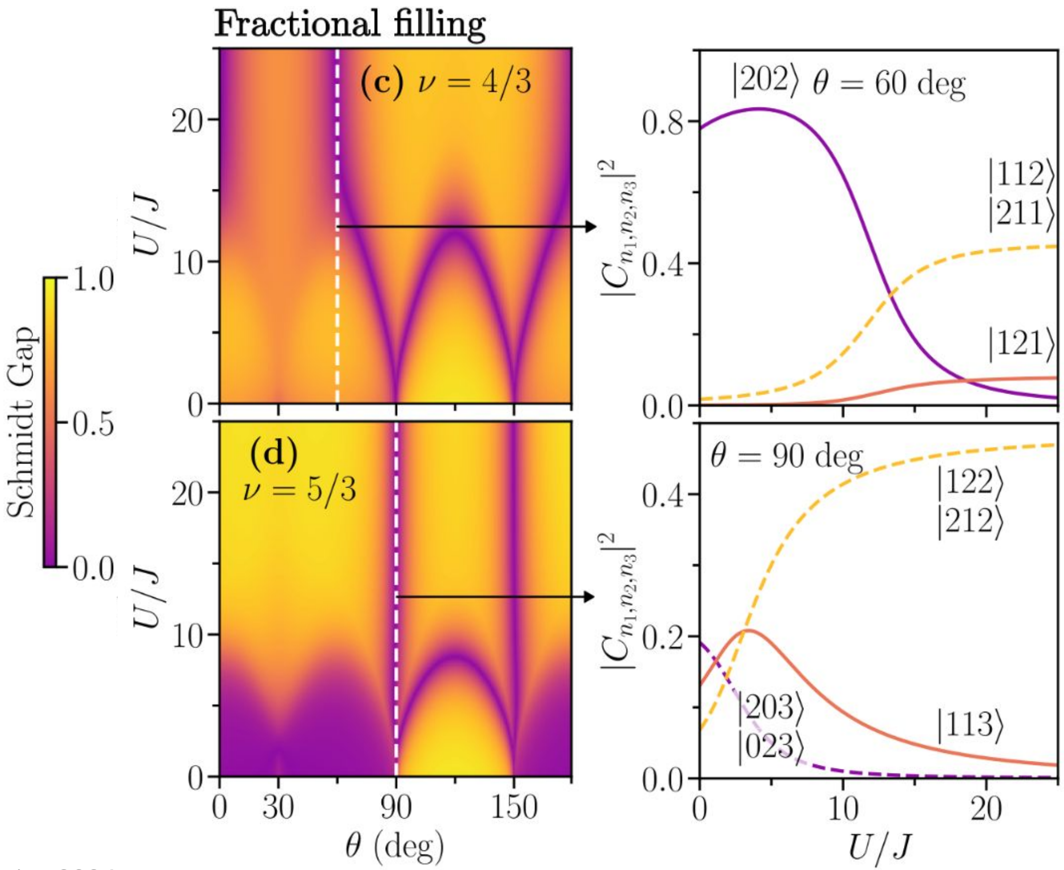




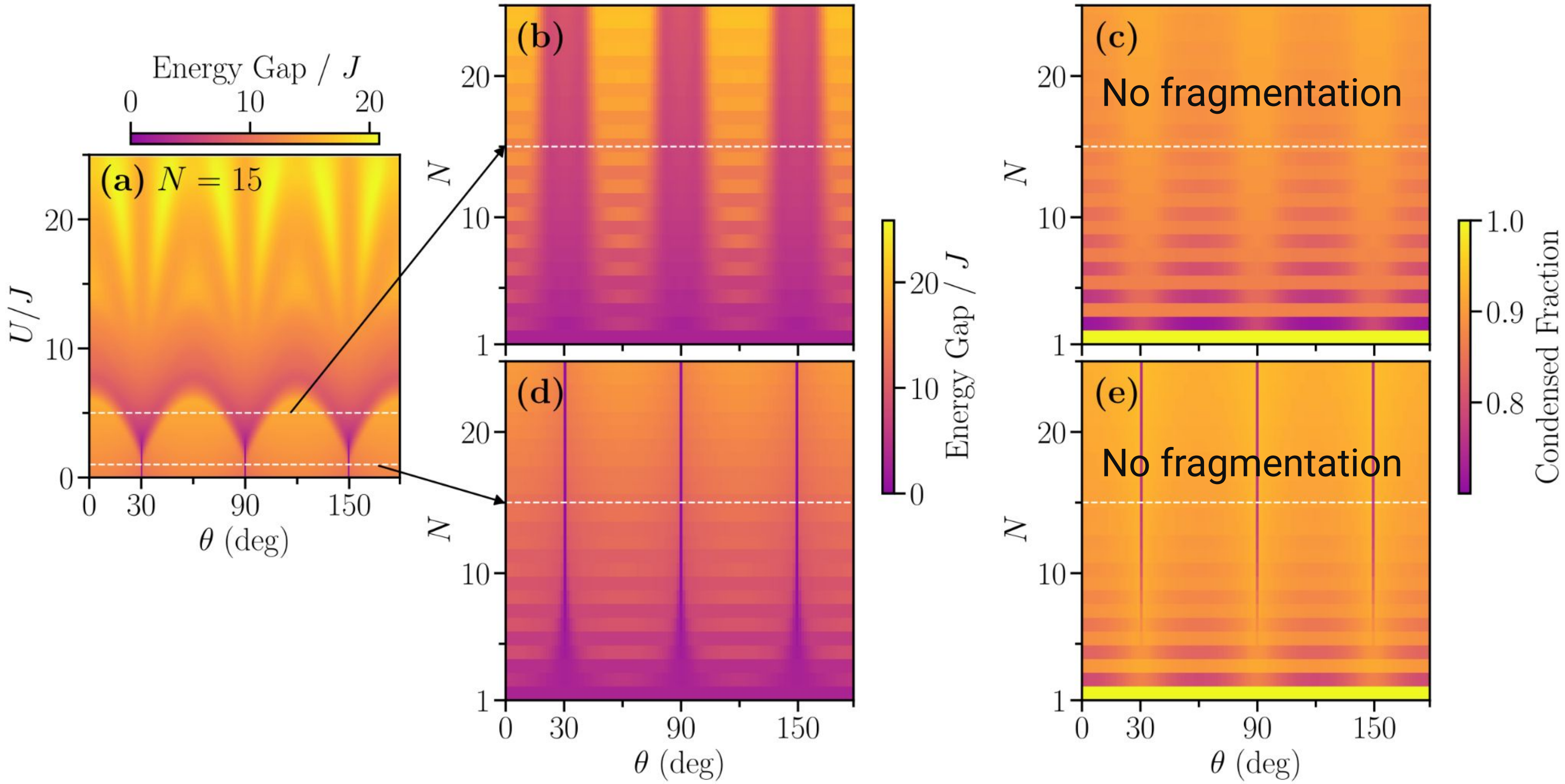
Correlations and dominant Fock states



Correlations and dominant Fock states



Energy gap and condensed fraction with number of particles



Summary and conclusions

- The triple well dipolar system presents a rich landscape of regimes as a function of the parameters of the system.
- We give a simple formula to calculate the critical interaction for which the system changes its regime.
- We demonstrate that correlations depends on the polarization and on the filling factor, finding entanglement for incommensurate filling regardless the on site interaction.

M. Rovirola, H. Briongos-Merino, B. Juliá-Díaz, M. Guilleumas Arxiv:2403.11620

Outlook

- Construct cat-like or correlated states manipulating the dipolar angle.

K. Wittmann W. et al., Phys. Rev. A 108, 033313 (2023)

- Prepare vortex states in the ring using magnetostirring.

L. Klaus et al., Nat. Phys. 18, 1453 (2022)

- Study coupled rings in different geometries

Thank you!