Resonance Cascades and Number Theory or three ways to search for the violators of the Goldbach conjecture using cold atoms

Oleksandr Marchukov, Andrea Trombettoni, Giuseppe Mussardo, Donatella Cassettari, Maxim Olshanii

Atomtronics

may 2024

UMASS BOSTON

NSF

I would not have been able to give this talk today

I would not have been able to give this talk today if not for a helping hand of this man

Francesco Lorenzi

who at a risk to his own health, arrested my slide,

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Dominik Pfeiffer

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over there, on Saturday.

I w is talk today if not
for

wh ted my slide,
wh
he
we
ov
as a part of a bigger expedition to Portillón de Benás

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over there, on Saturday.
I slid, roughly, over here:

Add to the list of what you will need in Benasque in 2026...

```
Things I will need in Benasque

- print out flight papers
- print out the lodging papers
- умывалки (necesser)
- passport
- pills
- powerbars
- glasses
- shores
- sunblock/bug_repellent
- euros
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good friends, ...

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Francesco Lorenzi

Dominik Pfeiffer

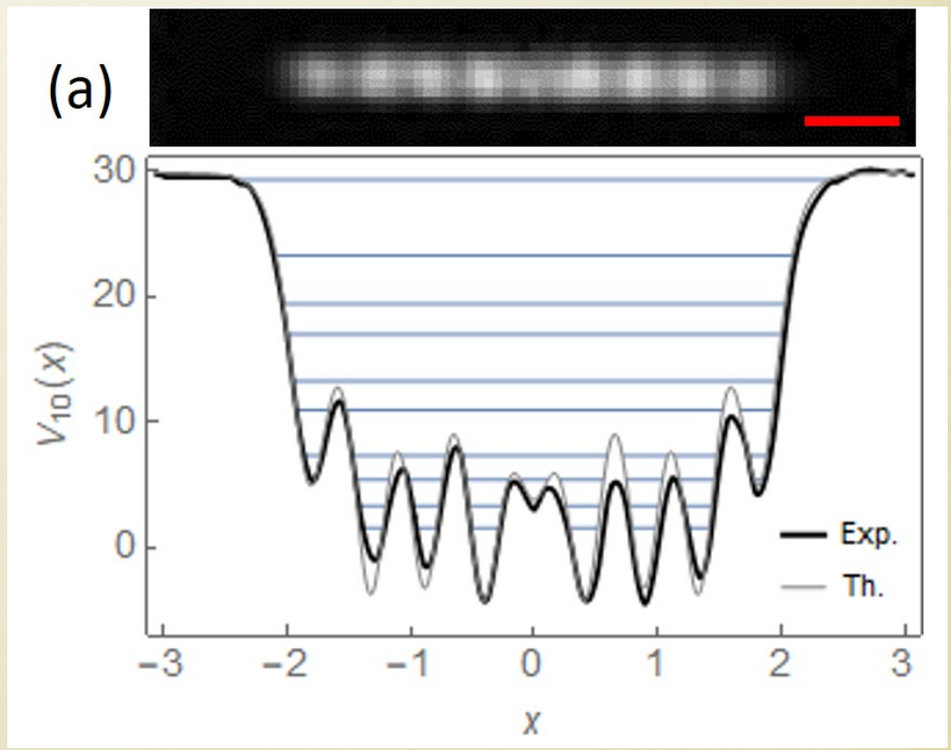
good friends, ...

and an ice ax

Why *Atomtronics* is the best conference in the Universe

Everything I am going
to present is uniquely inspired
by Donatella Cassettari's talk
at Atomtronics 2022. It resulted in
1 grant, 2 papers submitted, and 3 to be submitted within
a month

prime numbers



This talk happens to be nonlinear, the way the project is:

resonant cascades

ℕ\{9} toy model

Goldbach conjecture via resonant cascades (traditional **AMO**)

Goldbach conjecture via Grover protocol (**Quantum Information**)

Goldbach conjecture via an **atomtronic band-stop filter**

Goldbach conjecture

Intro 1

Resonance Cascades

This article contains spoilers. Read at your own risk!

This article is about **the quantum event**. For **the ensuing battle**, see **Black Mesa Incident**.

"I never thought I'd see a Resonance Cascade, let alone create one..."
—Black Mesa scientist[src]

The **Resonance Cascade** was a cataclysmic quantum event that occurred after the insertion of Xen crystal sample "GG-3883" into the Anti-Mass Spectrometer at the Black Mesa Research Facility, by Dr. Gordon Freeman, caused the machinery to undergo a catastrophic malfunction, culminating in the Black Mesa Incident.

Contents [hide]

Resonance Cascade

General information

Type
Unstable resonance reaction

Location
Black Mesa Research Facility

Others like you also viewed
Combine

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e create one..."

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83" into the

ity, by Dr.

strophic

Resonance Cascade

General information

Type
Unstable resonance reaction

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Black Mesa Research Facility

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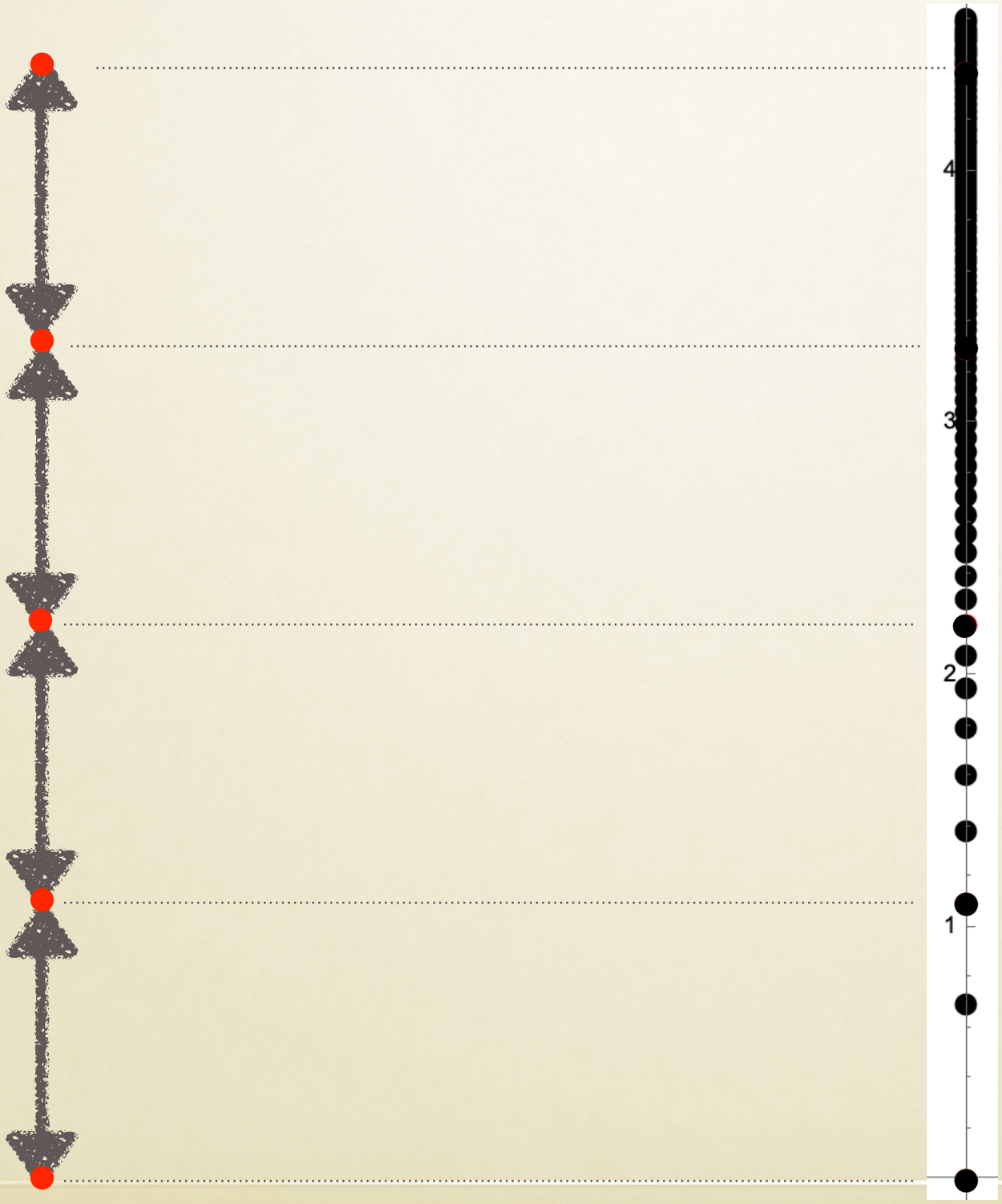
An arithmetic progression

?

$\subset$

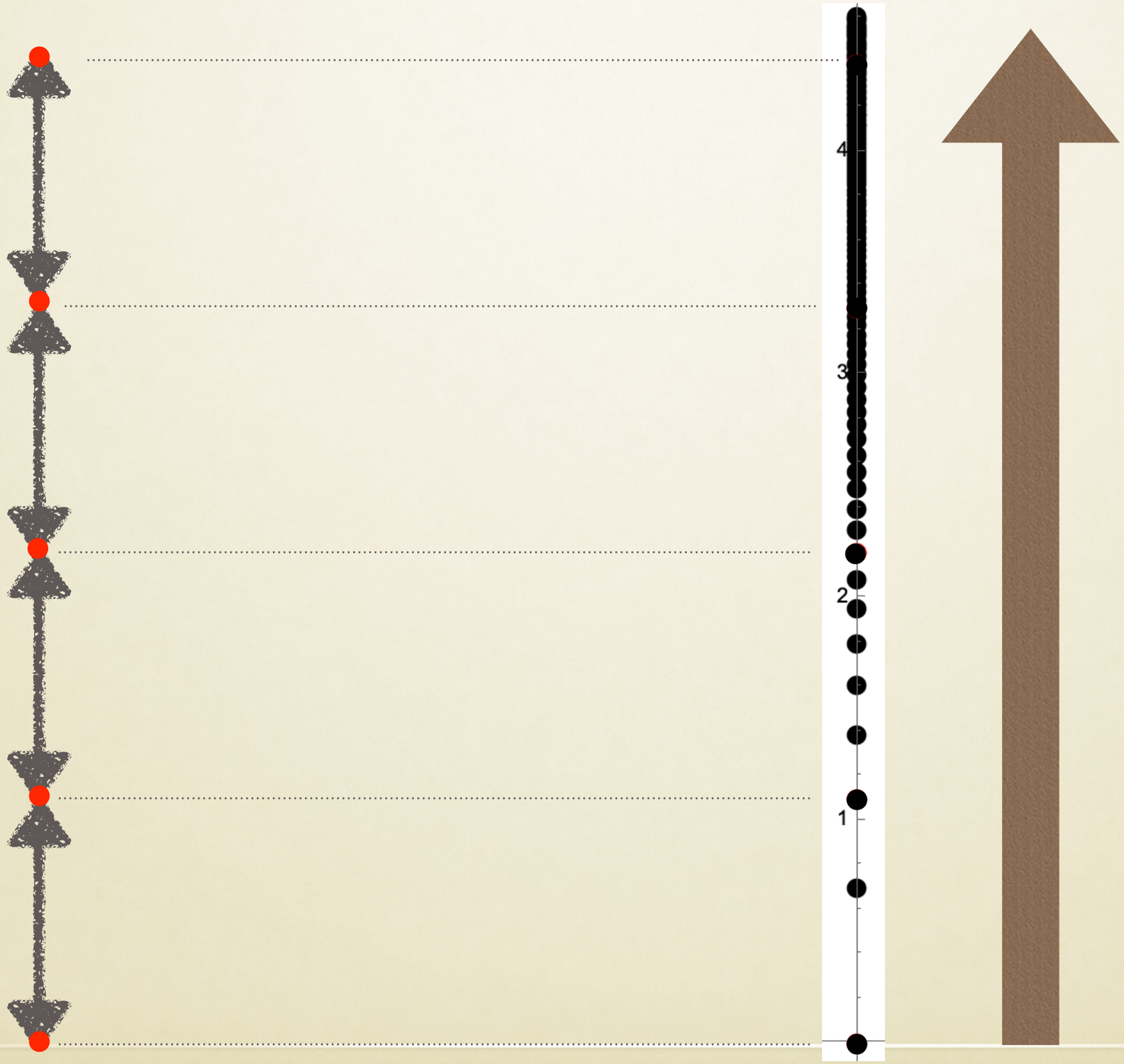
Another set of numbers



A monochromatic drive

Is there an unhindered driven resonant upward mobility?

A quantum spectrum



A monochromatic drive

Is there an unhindered driven resonant upward mobility?

Or not?

A quantum spectrum

A variation on the theme

Is this arithmetics progression contiguous?

Monochromatic drive

Is there an unhindered driven upward mobility?

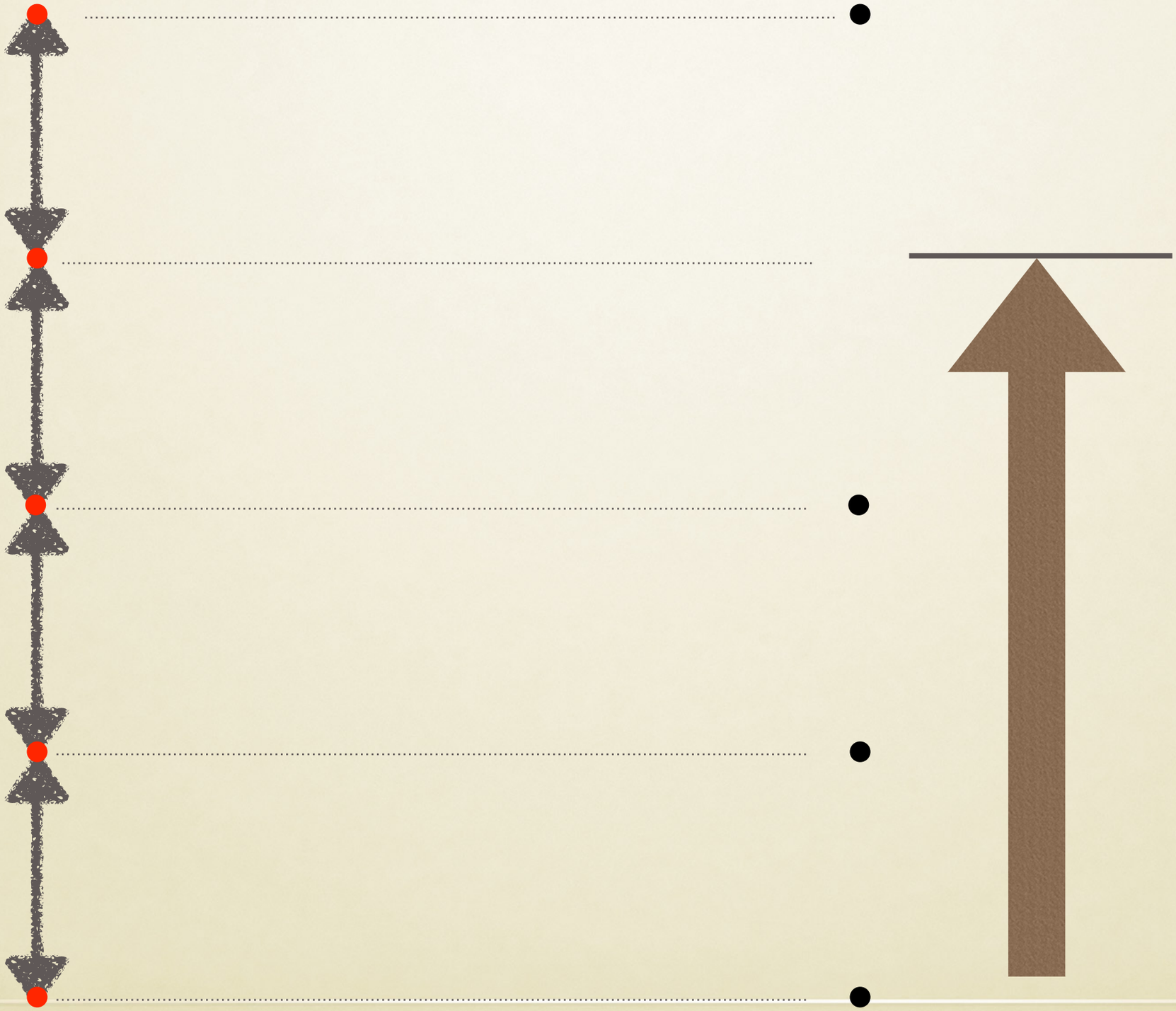
A quantum spectrum

Monochromatic drive

Is there an unhindered driven upward mobility?

Or not?

A quantum spectrum



Identity	Is this contiguous arithmetic progression...		... a subset of this set?
$n_1 n_2 = n_3$	$\ln(n^m)$	$\overset{?}{\subset}$	$\ln(\mathbb{N})$
$(n_1^2 + n_2^2)(n_3^2 + n_4^2) = n_5^2 + n_6^2$ Diophantus-Brahmagupta-Fibonacci identity	$\ln((n_1^2 + n_2^2)^m)$	$\overset{?}{\subset}$	$\ln(\mathbb{N}^2 + \mathbb{N}^2)$
$(n_1^2 + kn_2^2)(n_3^2 + kn_4^2) = n_5^2 + kn_6^2$ Brahmagupta identity	$\ln((n_1^2 + kn_2^2)^m)$	$\overset{?}{\subset}$	$\ln(\mathbb{N}^2 + k\mathbb{N}^2)$

$(n_1^2 + n_2^2 + n_3^2)(n_4^2 + n_5^2 + n_6^2) \neq n_7^2 + n_8^2 + n_9^2$

$11 \times 373 = (1^2 + 1^2 + 3^2) \times (2^2 + 12^2 + 15^2) = 4103 \neq n_7^2 + n_8^2 + n_9^2$

$\ln((n_1^2 + \ldots + n_4^2)^m) \quad \overset{?}{\subset} \quad \ln(\mathbb{N}^2 + \mathbb{N}^2 + \mathbb{N}^2 + \mathbb{N}^2)$

$(n_1^2 + n_2^2 + n_3^2 + n_4^2) \times$
$\times (n_5^2 + n_6^2 + n_7^2 + n_8^2)$
$= n_9^2 + n_{10}^2 + n_{11}^2 + n_{12}^2$

$\forall n,\ n = n_1^2 + n_2^2 + n_3^2 + n_4^2$

Lagrange's four-square theorem

$n, m \in \mathbb{N} \qquad e \in 2\mathbb{N} \qquad o \in 2\mathbb{N} + 1 \qquad p \in \mathbb{P}$

A variation on the theme

Identity	Is this arithmetic progression contiguous?		
$\forall n,\ n = n_1^2 + n_2^2 + n_3^2 + n_4^2$	$n_1^2 + n_2^2 + n_3^2 + n_4^2$	$\overset{?}{=}$	$\mathbb{N}$
Lagrange's four-square theorem			
$\forall e,\ e = p_1 + p_2$	$p_1 + p_2$	$\overset{?}{=}$	$2\mathbb{N}$
Goldbach conjecture			
$\forall e,\ e = p_1 + (p_2 p_3 \text{ or } p_2)$	$p_1 + (p_2 p_3 \text{ or } p_2)$	$\overset{?}{=}$	$2\mathbb{N}$
Jingrun theorem			
$\forall o,\ o = p_1 + 2p_2$	$p_1 + 2p_2$	$\overset{?}{=}$	$2\mathbb{N} + 1$
Lemoin conjecture			
$\forall o,\ o = p_1 + p_2 + p_3$	$p_1 + p_2 + p_3$	$\overset{?}{=}$	$2\mathbb{N} + 1$
Helfgott theorem (almost) (2013)			

$n, m \in \mathbb{N}$ $e \in 2\mathbb{N}$ $o \in 2\mathbb{N} + 1$ $p \in \mathbb{P}$

$p_1 + p_2$ prime spectra?

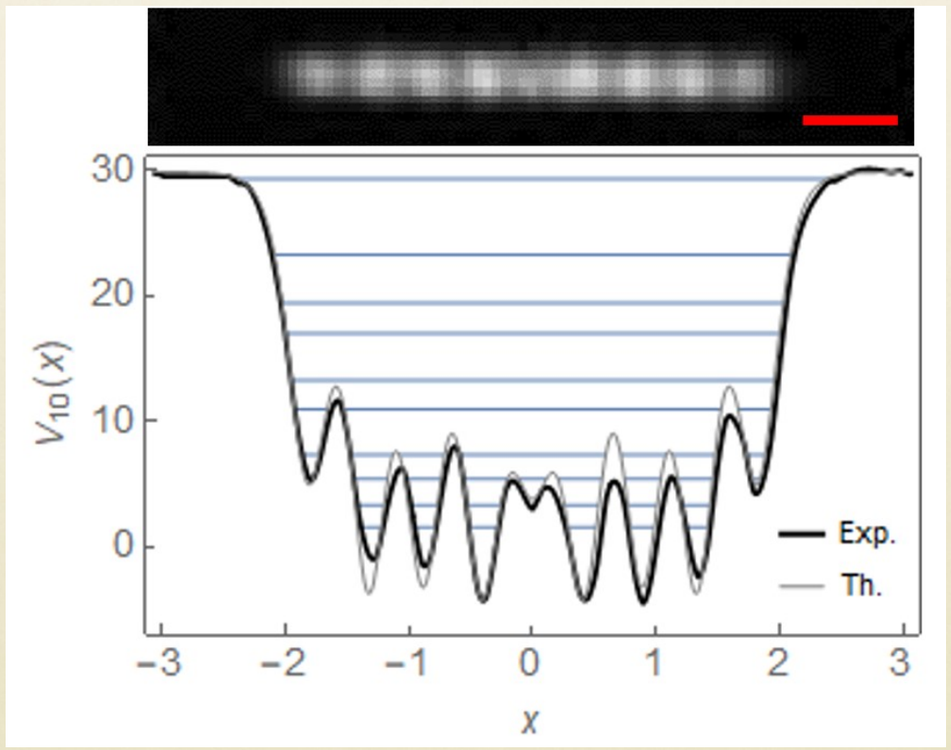
$p_1 + (p_2 p_3$ or $p_2)$

$p_1 + 2p_2$

$p_1 + p_2 + p_3$

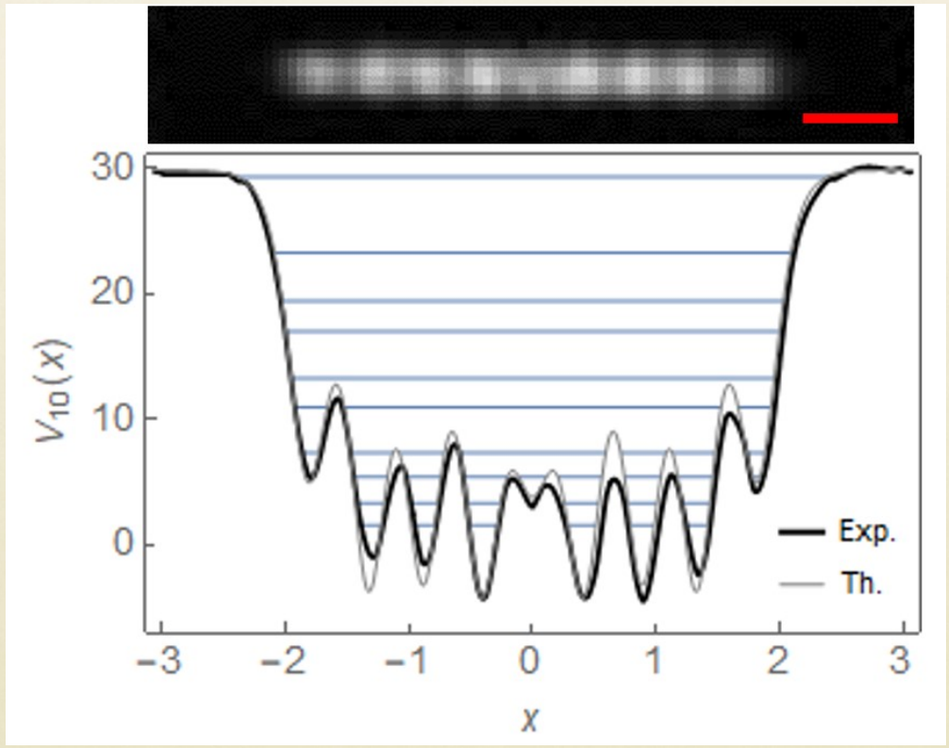
See talk by Donatella Cassettari, U St Andrews

prime numbers



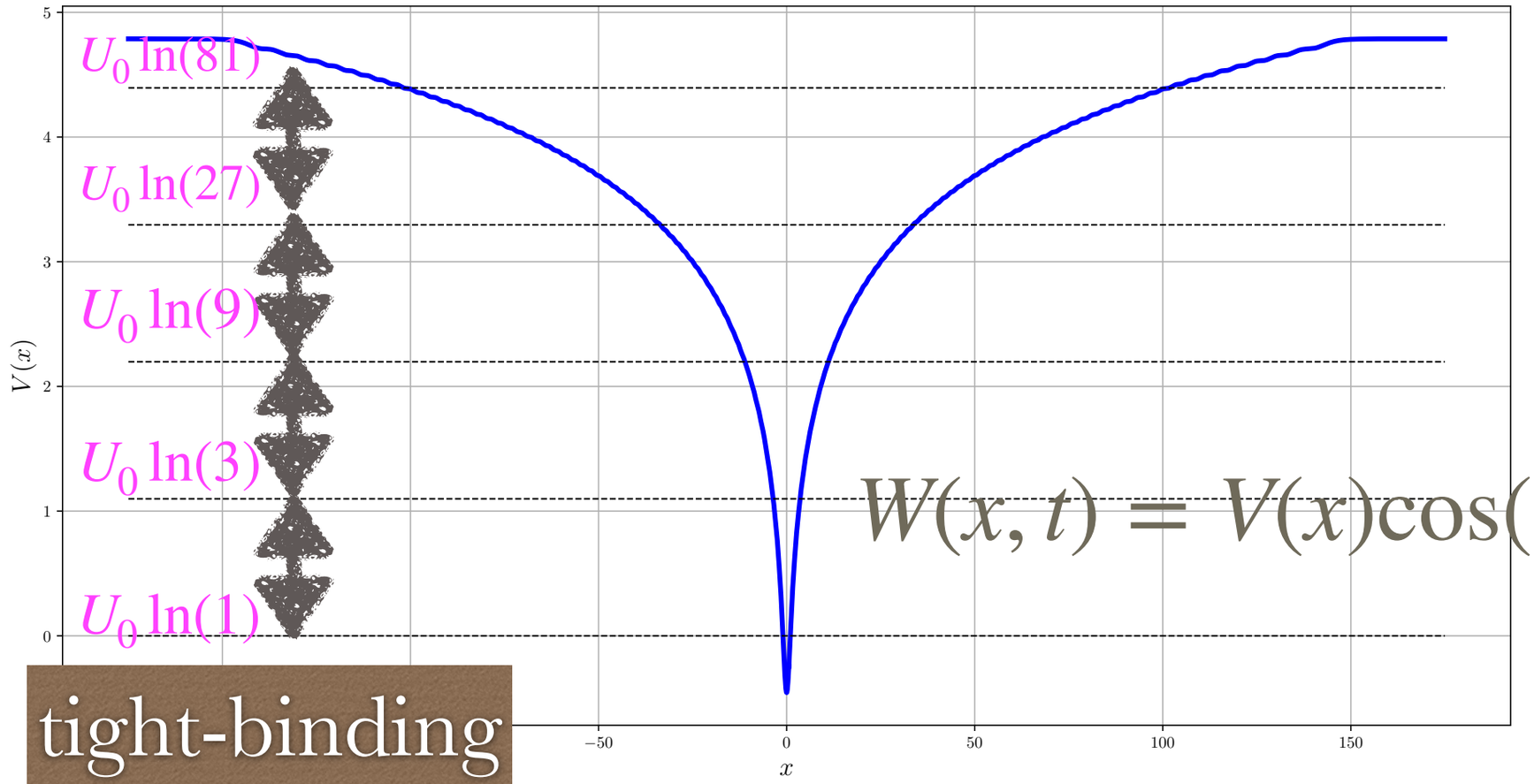
SUSY techniques: see talk by Andrea Trombettoni and Giuseppe Mussardo

prime numbers



Toy problem: "is 9 integer?"

The ln(n)- and ln(n)\ln(9) -potentials



$$W(x,t) = V(x)\cos(U_0 \ln(3) t/\hbar)$$

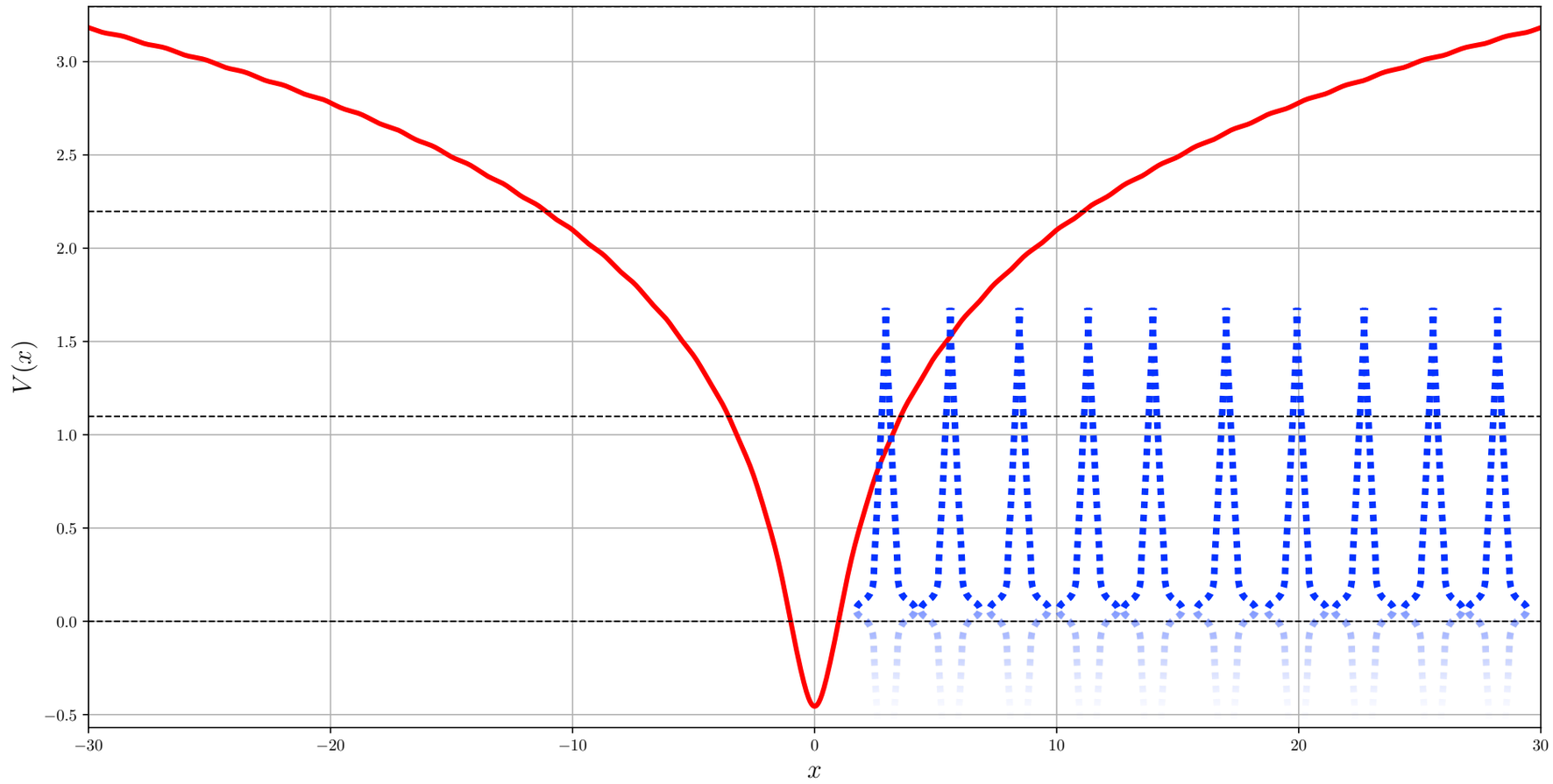
tight-binding model

$$E_n = U_0 \ln(n)$$

$$U(x) \approx U_0 \ln\left(\sqrt{\frac{2}{\pi}}\frac{x}{a}\right)$$

$$a \equiv \frac{\hbar}{\sqrt{mU_0}}$$

An example of a perturbation that gives tight binding with an approximately homogeneous hopping



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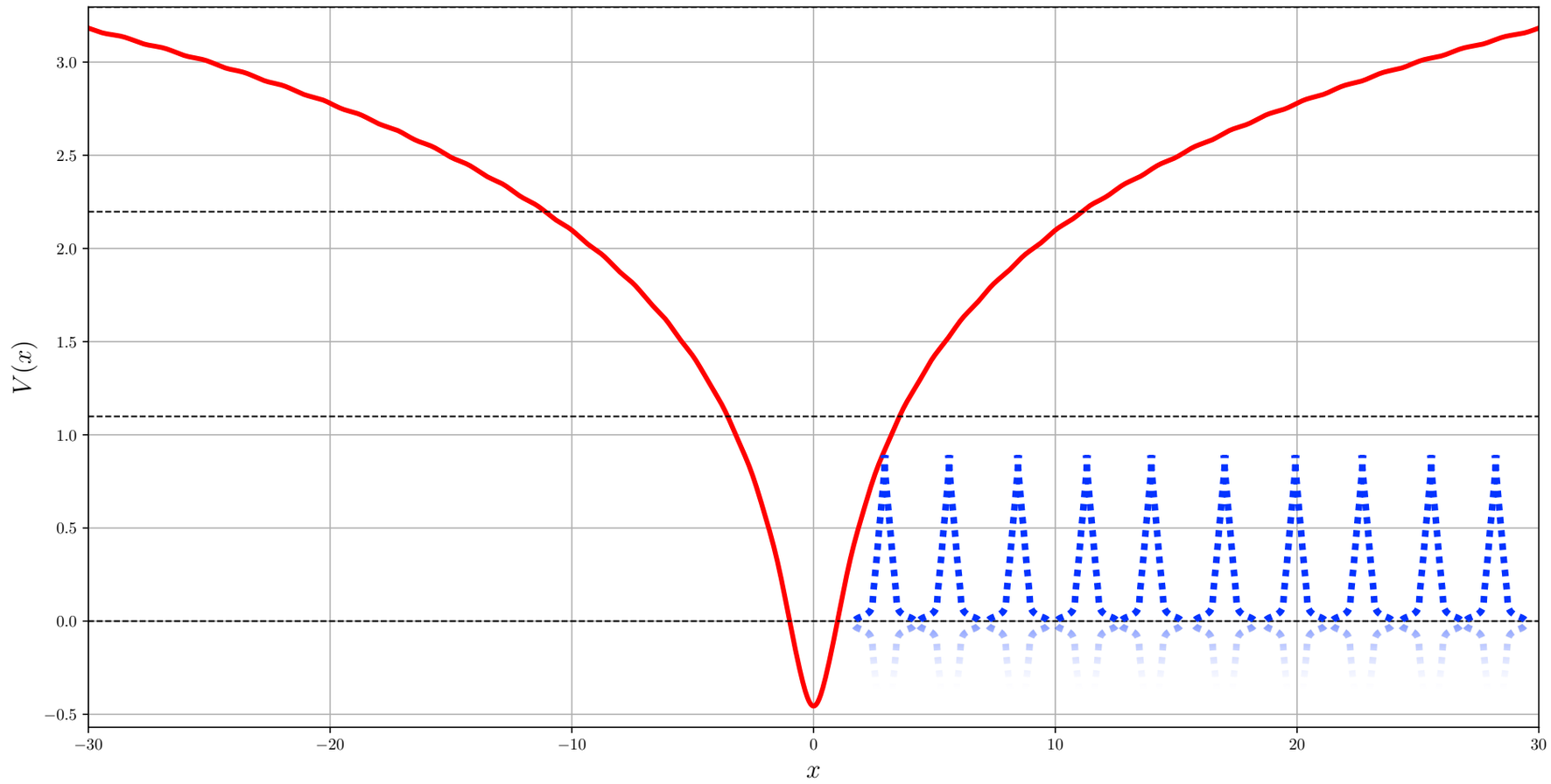

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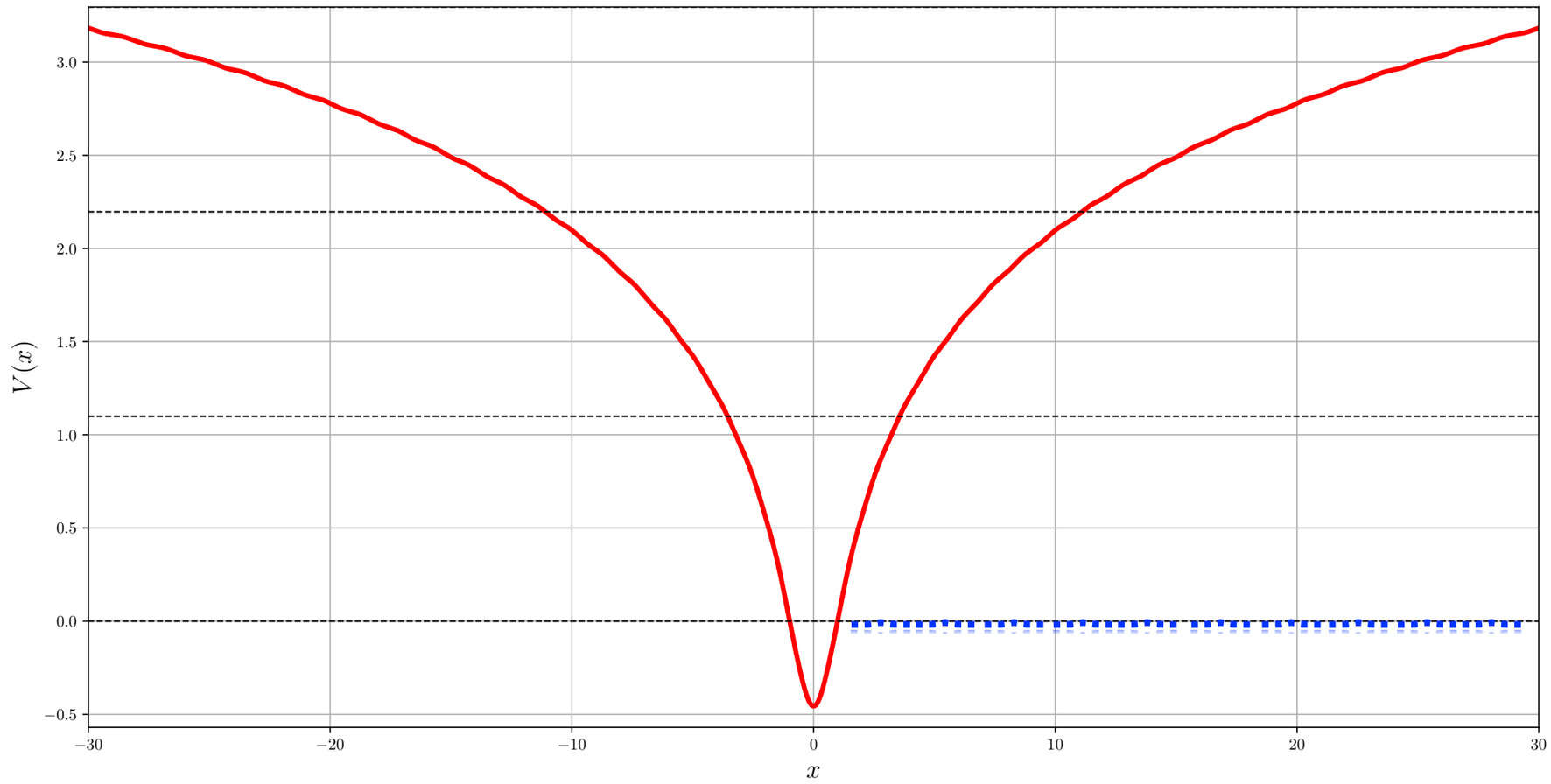

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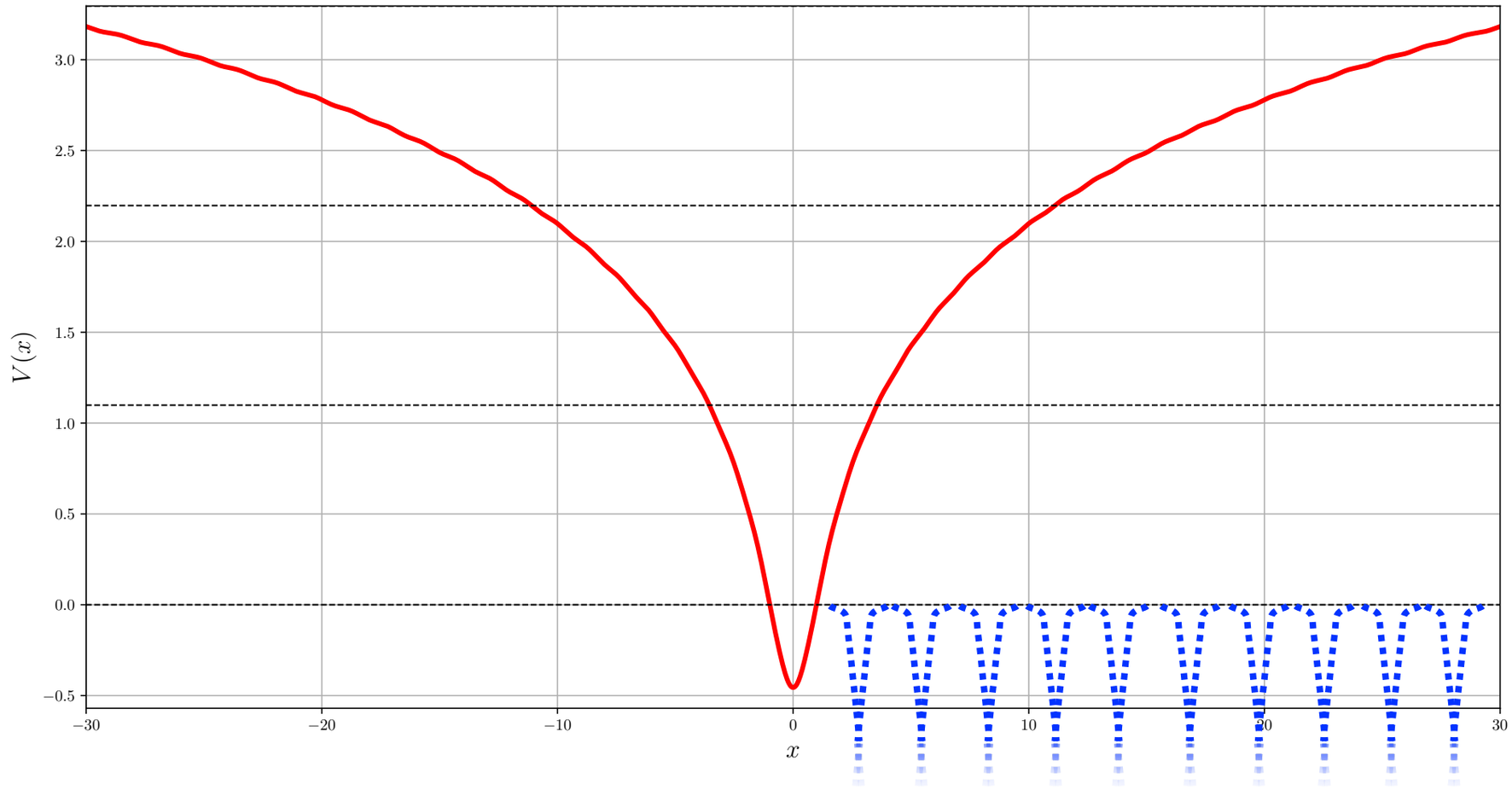

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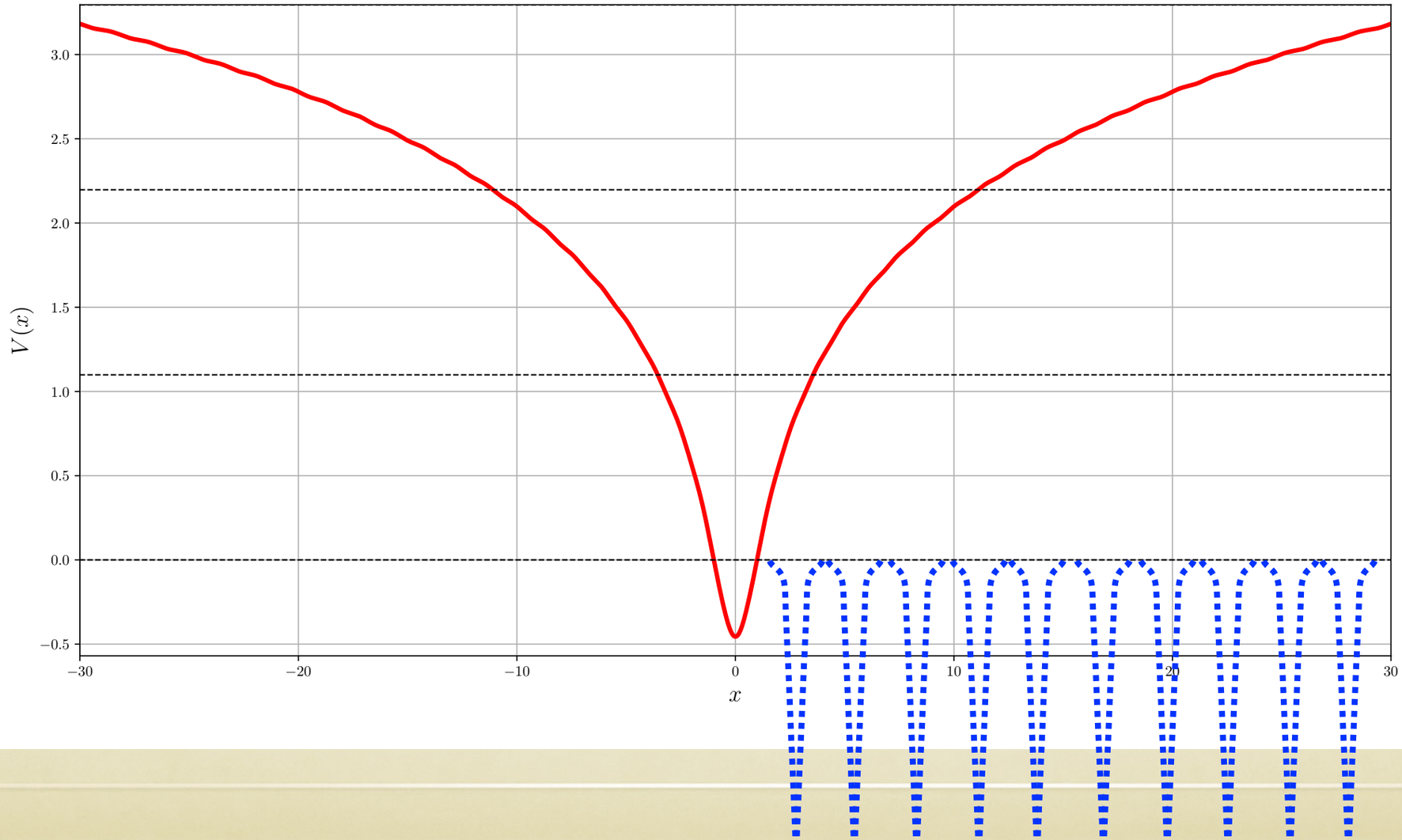

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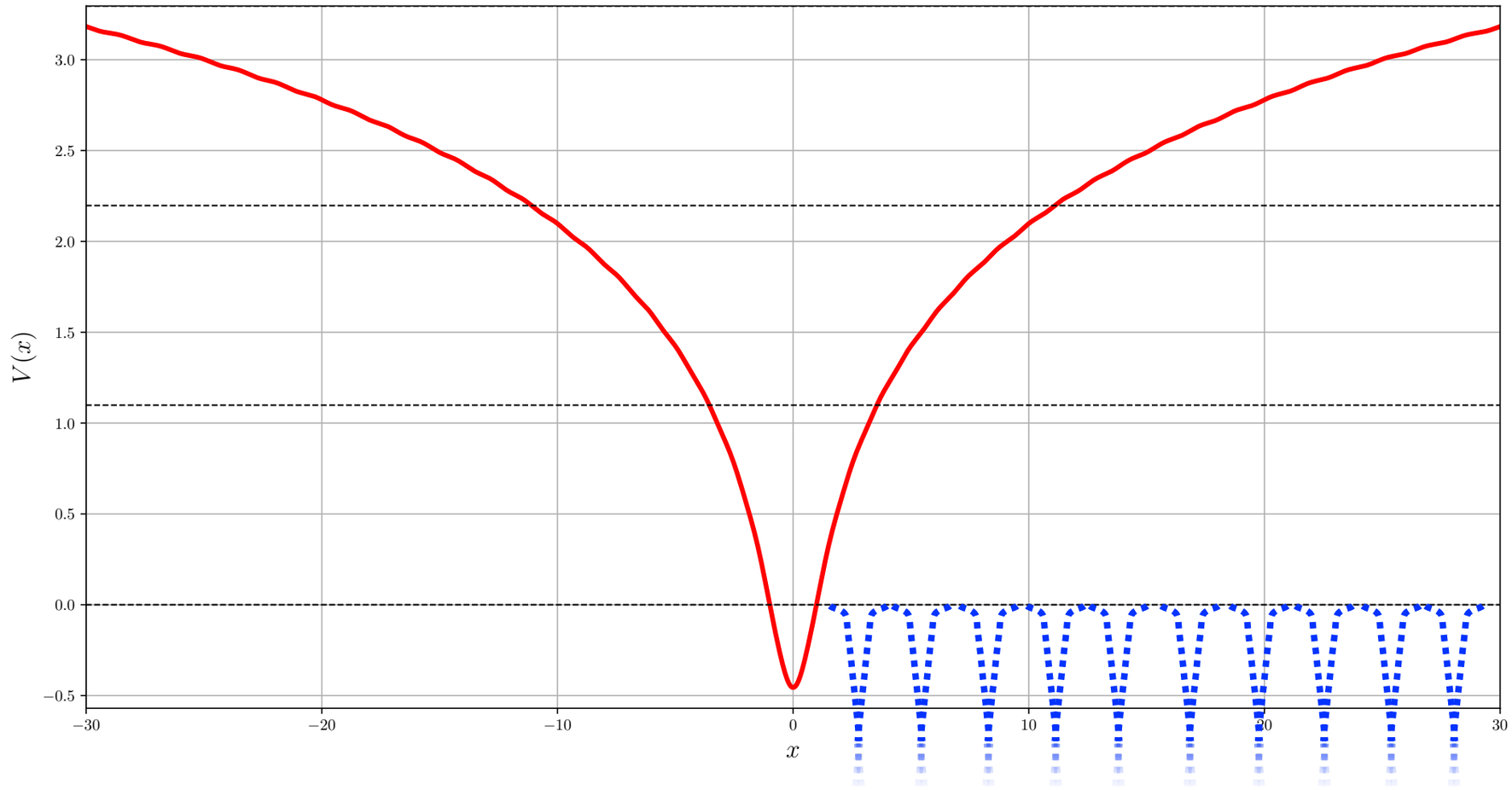

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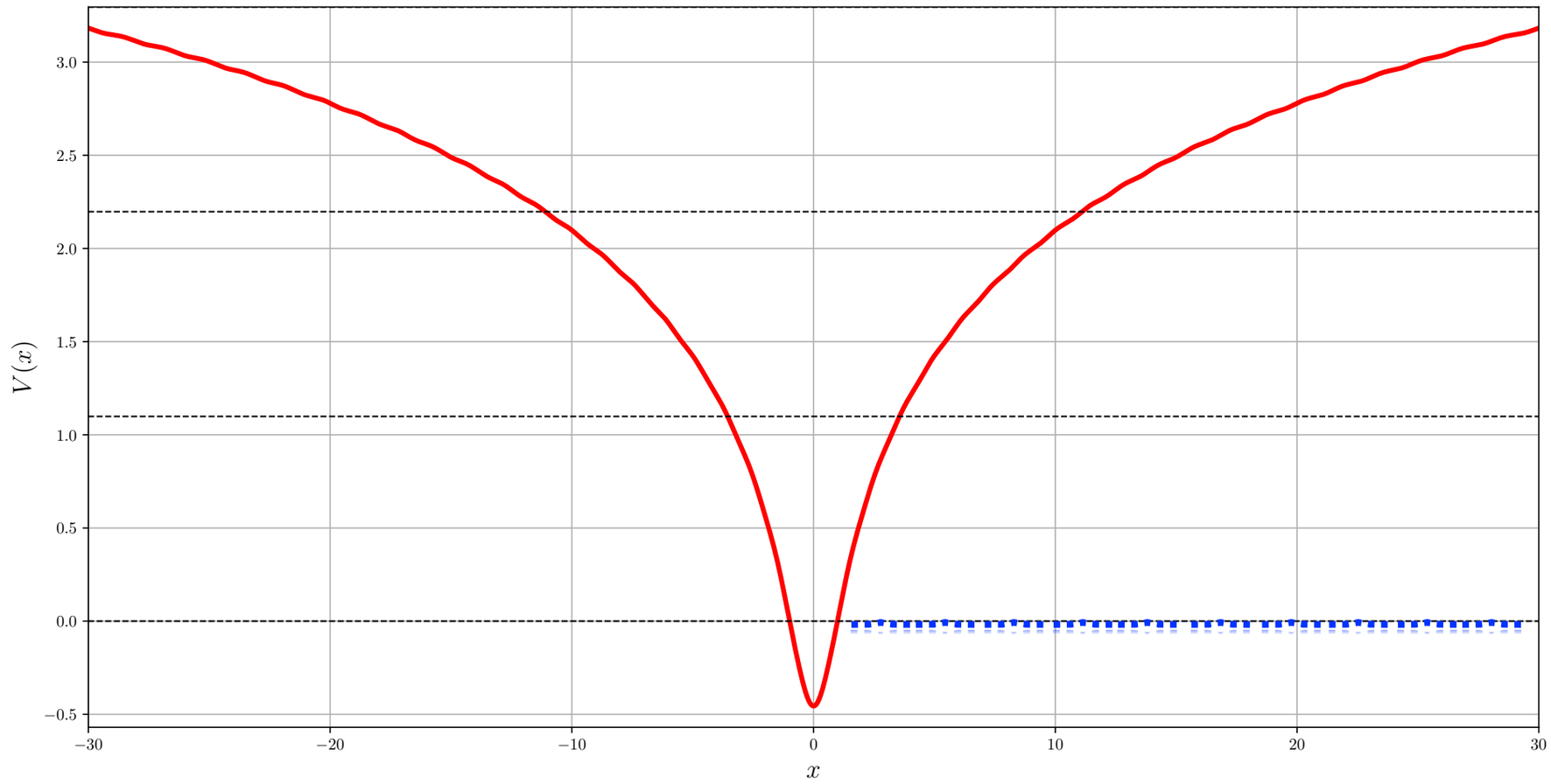

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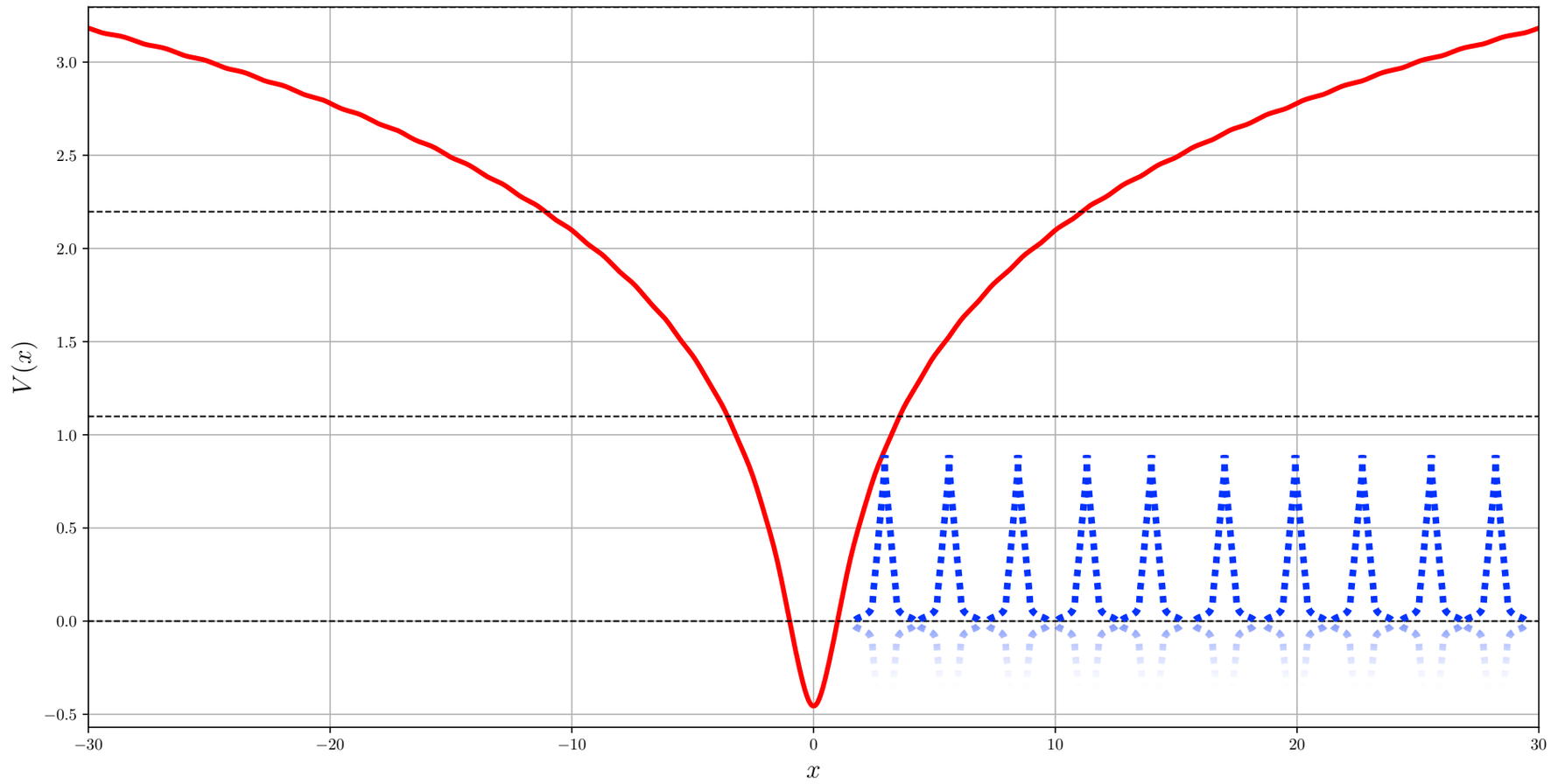

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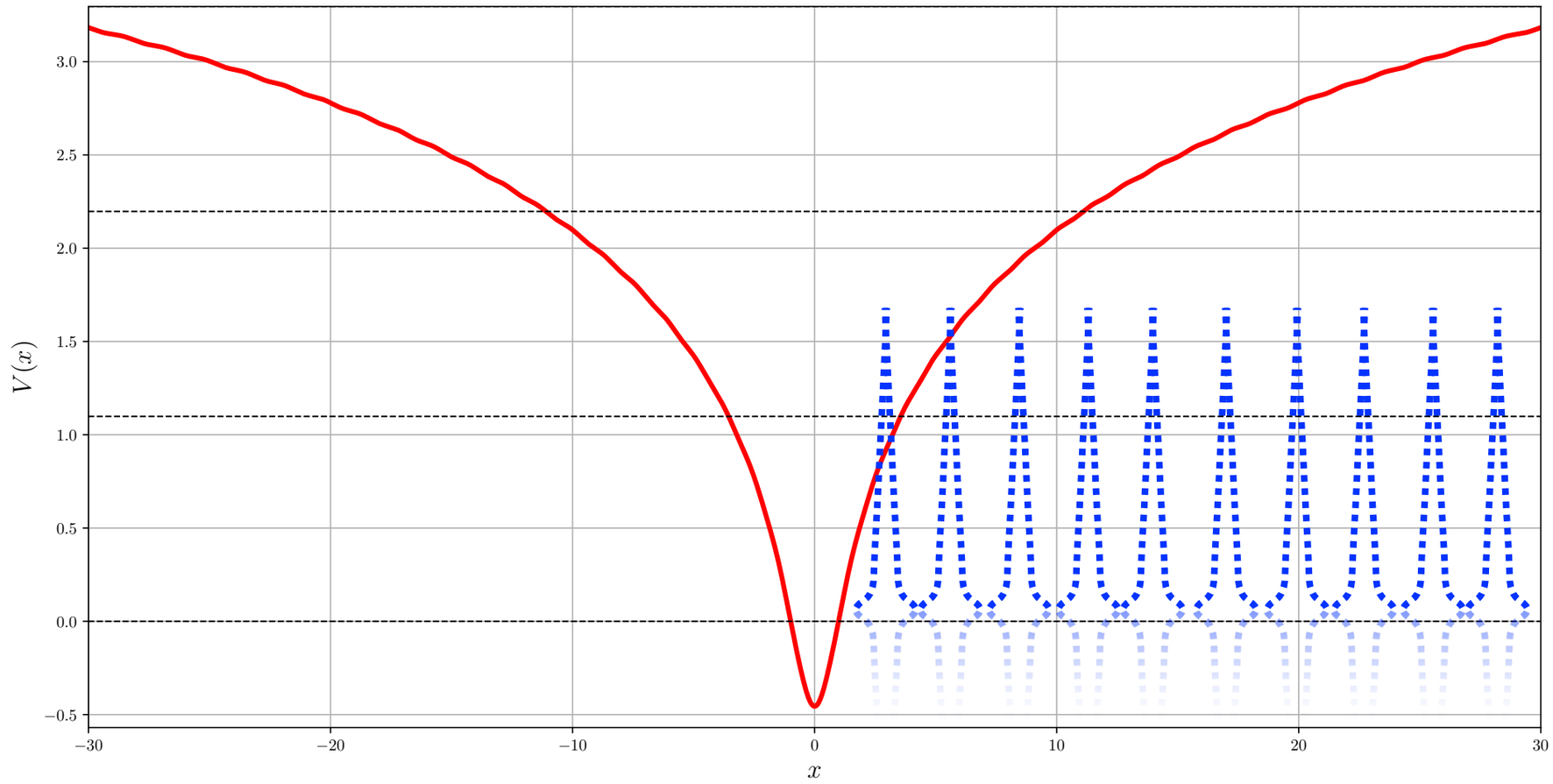
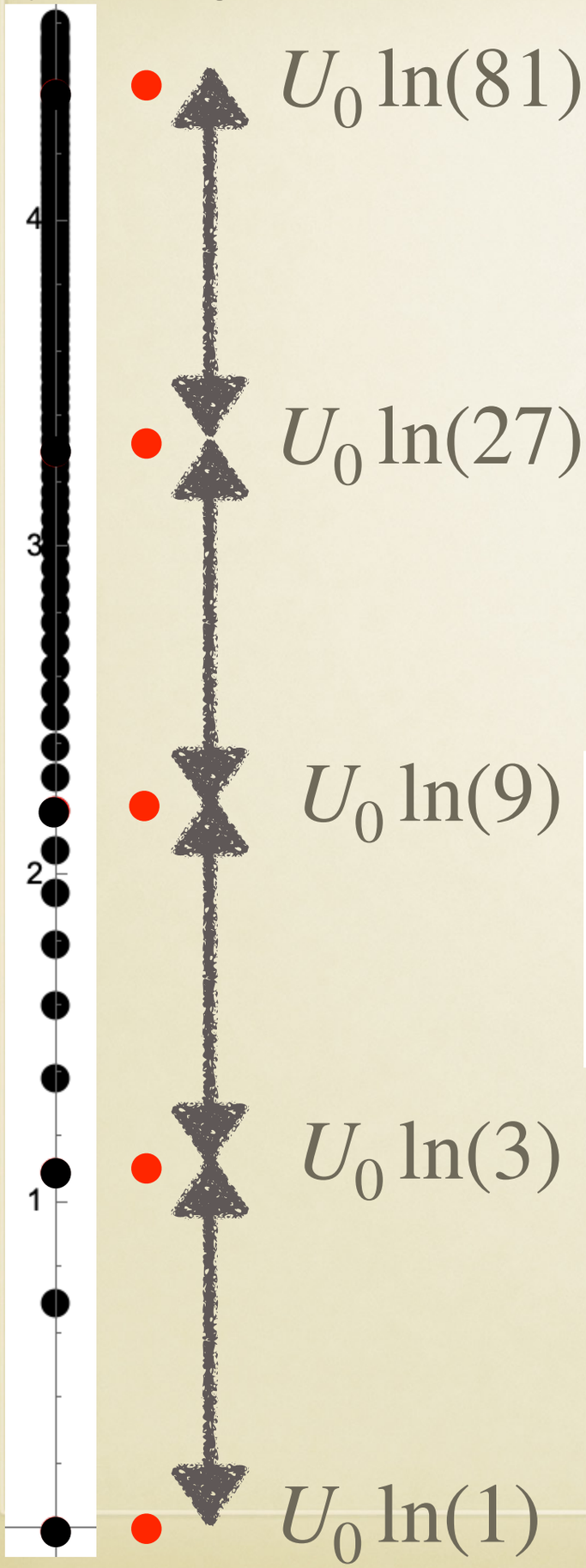
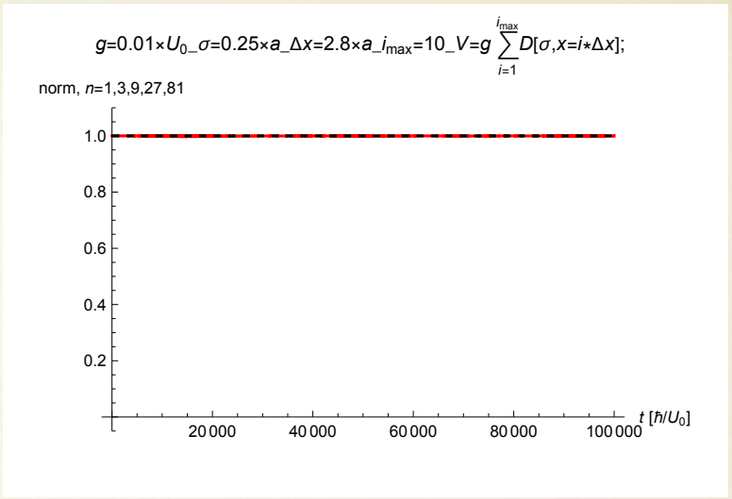


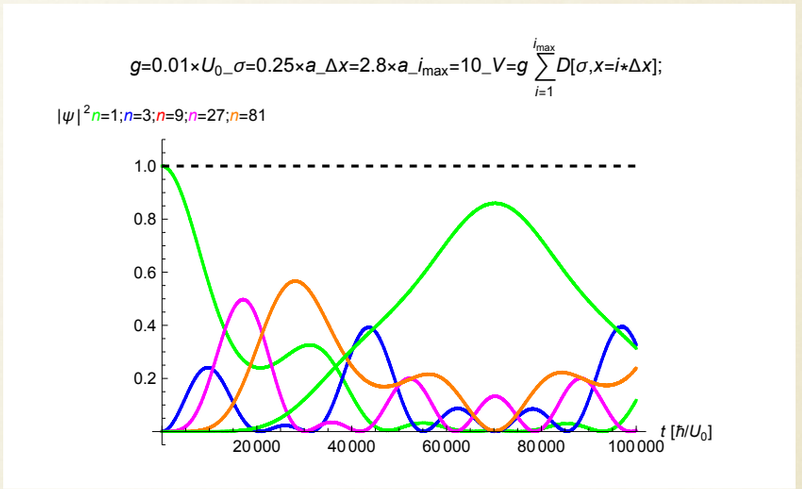
$E_n = U_0 \ln n$

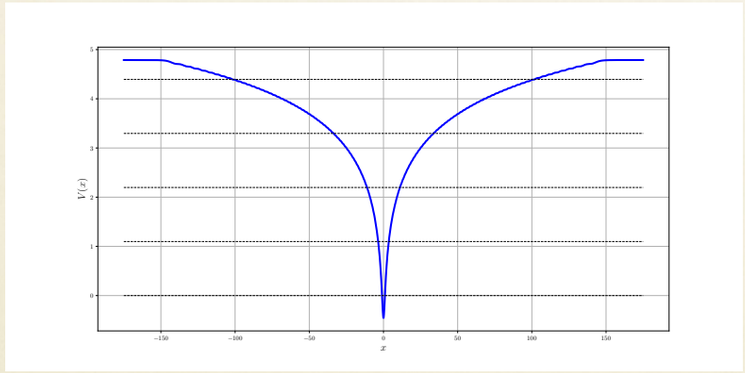


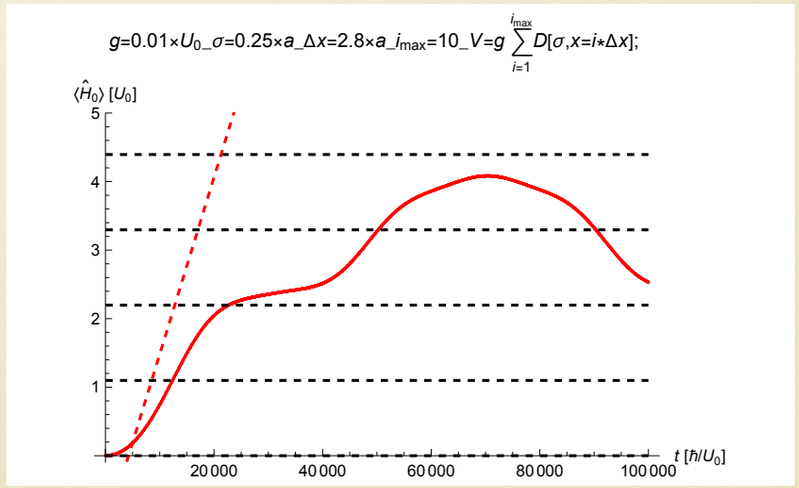
- $U_0 \ln(81)$
- $U_0 \ln(27)$
- $U_0 \ln(9)$
- $U_0 \ln(3)$
- $U_0 \ln(1)$

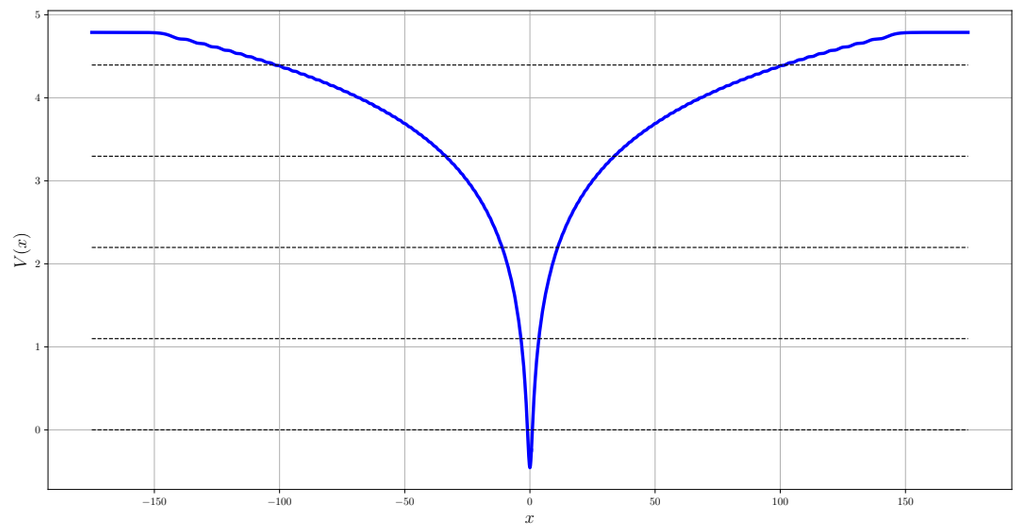
Our Universe



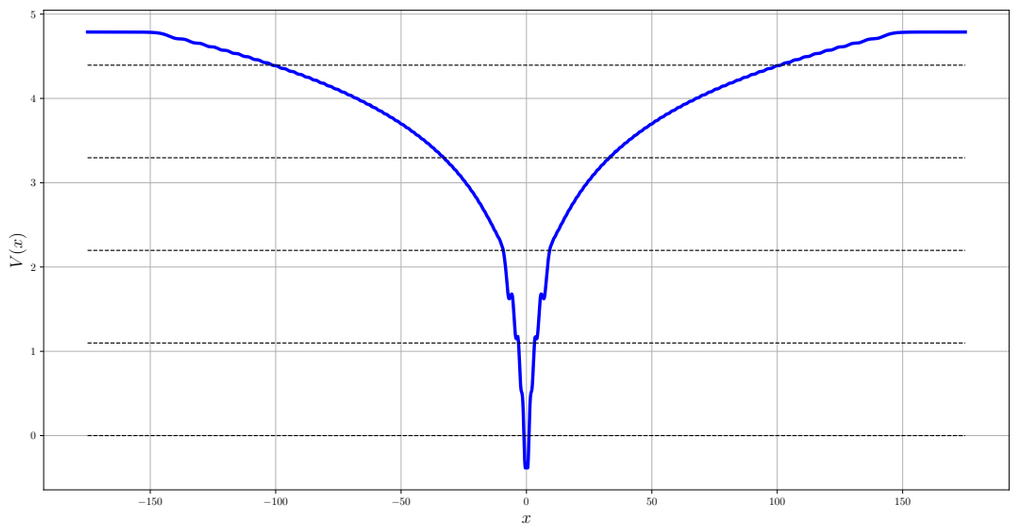








ln(n)-spectrum potential

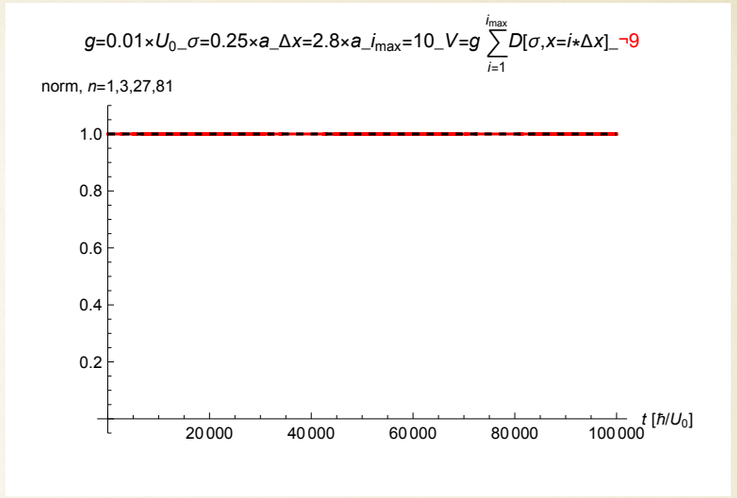


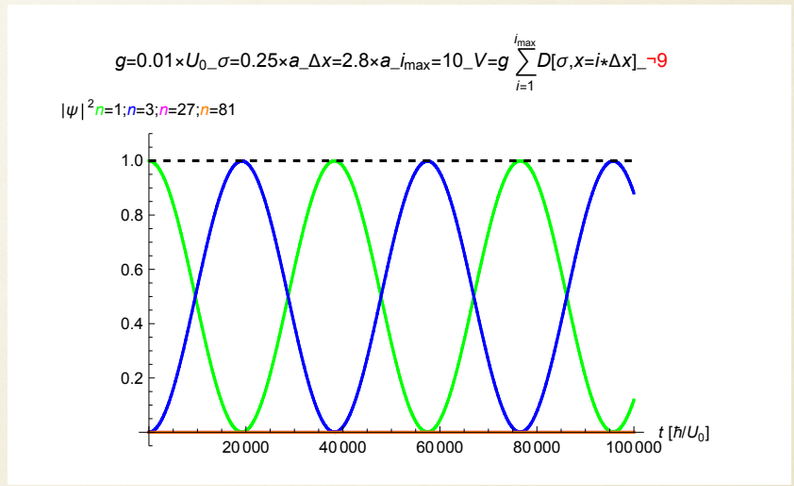
ln(n)\{9}-spectrum potential

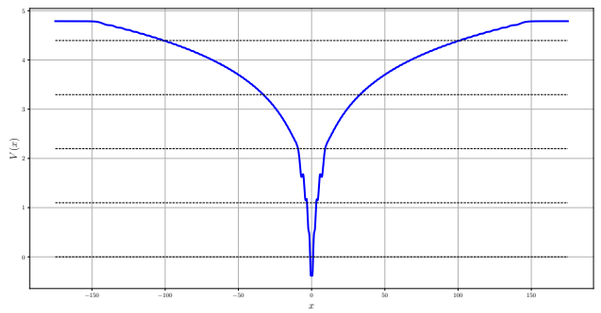
$E_n = U_0 \ln n$

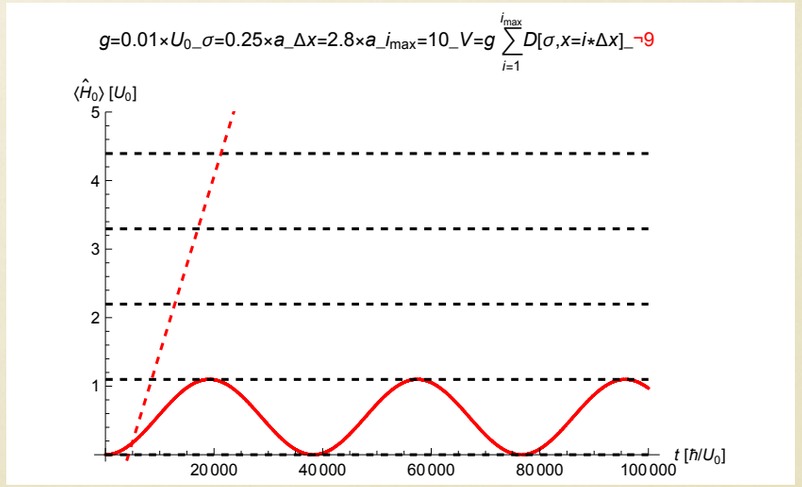
- $U_0 \ln(81)$
- $U_0 \ln(27)$
- $U_0 \ln(9)$
- $U_0 \ln(3)$
- $U_0 \ln(1)$

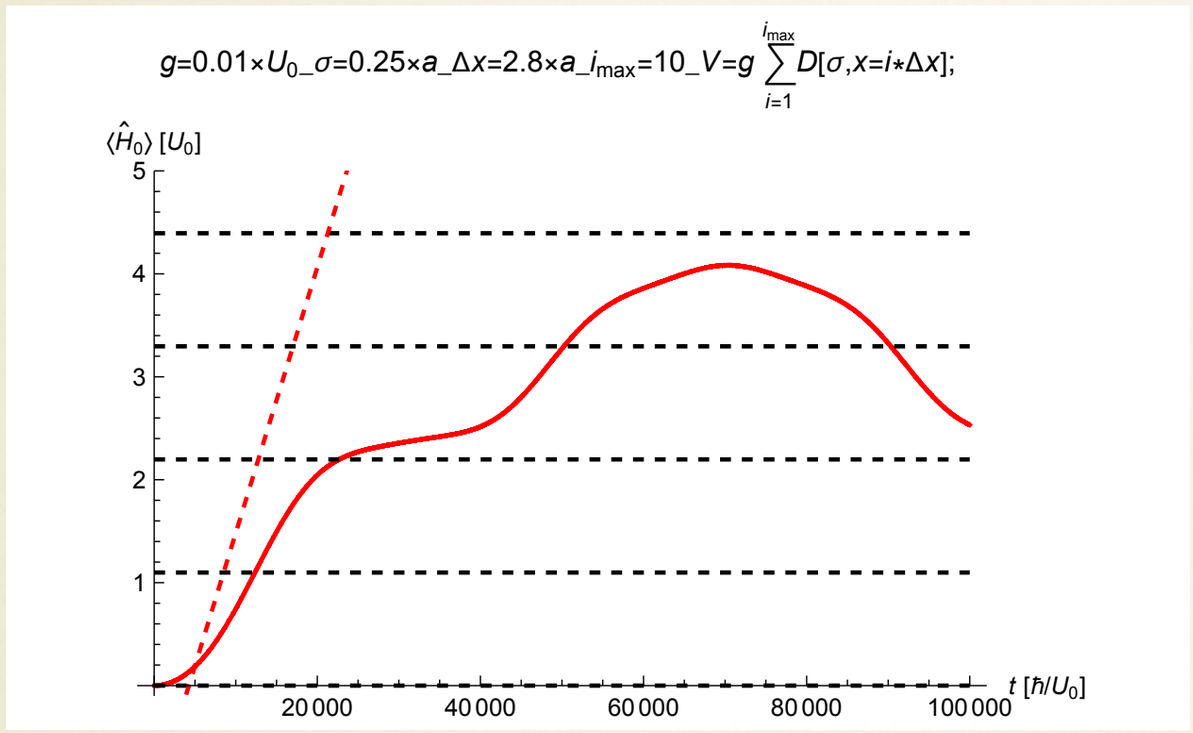
A Universe where 9 is not a natural

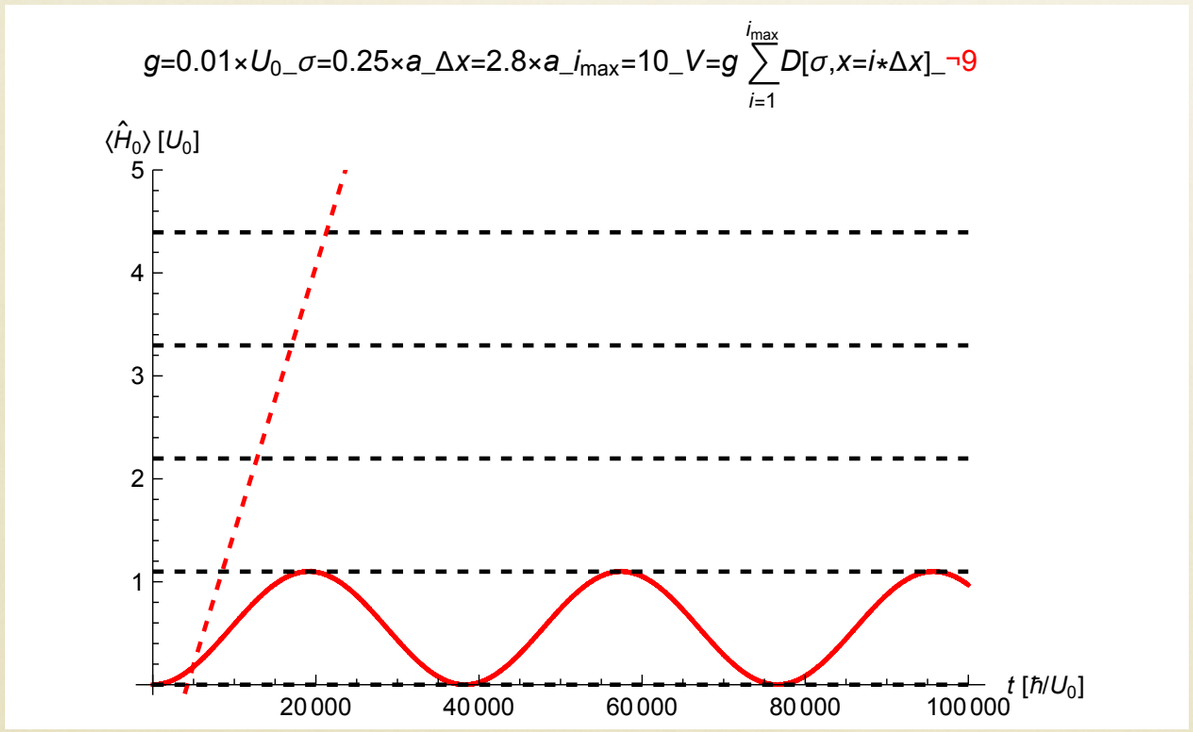












In the tightest collaboration with Oleksandr Marchukov

OK, now we know that 9 is a natural number... so what?

Goldbach conjecture is next

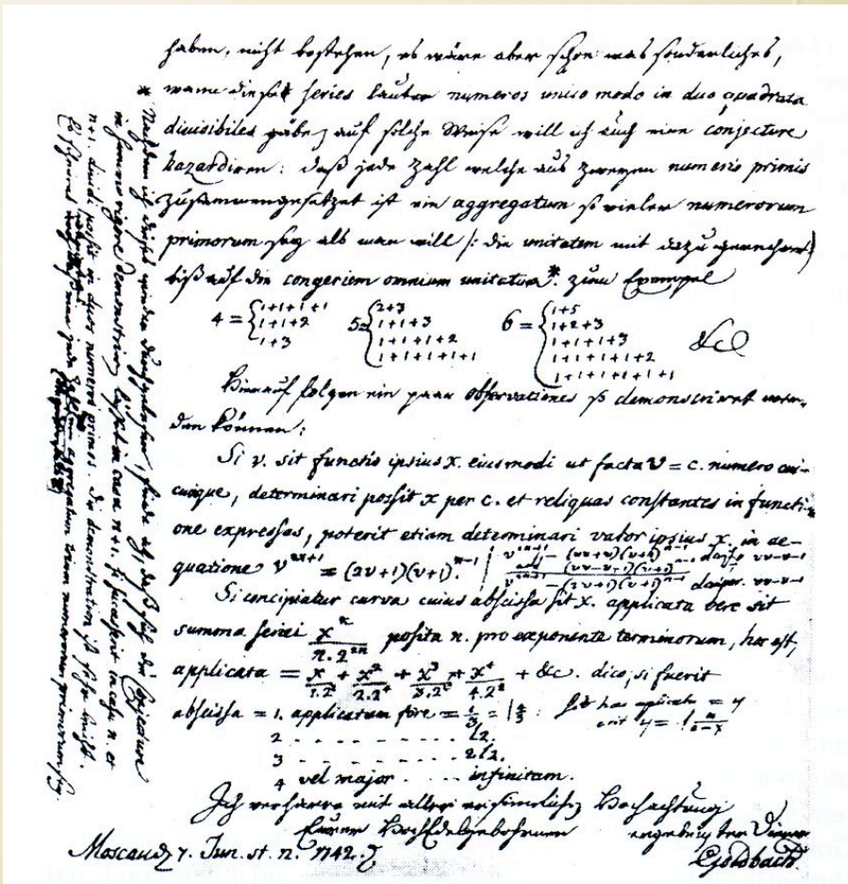
Intro 2 Goldbach Conjecture

Goldbach conjecture:

$$\forall n : n = \text{even} > 2, \exists p_1, p_2 : n = p_1 + p_2$$

Tested numerically up to $n = 4 \times 10^{18}$ [Tomás Oliveira e Silva, Siegfried Herzog, and Silvio Pardi, *Mathematics of Computation* **83**, 2033 (2013).]

Christian Goldbach (1690 - 1764)



Letter from Goldbach to Euler dated 7 June 1742

Three ways to search for the violators of the Goldbach conjecture:

1. Resonant cascades (traditional AMO)
2. A Goldbach-Grover scheme (quantum info)
3. An atomtronic low-pass filter (atomtronics)

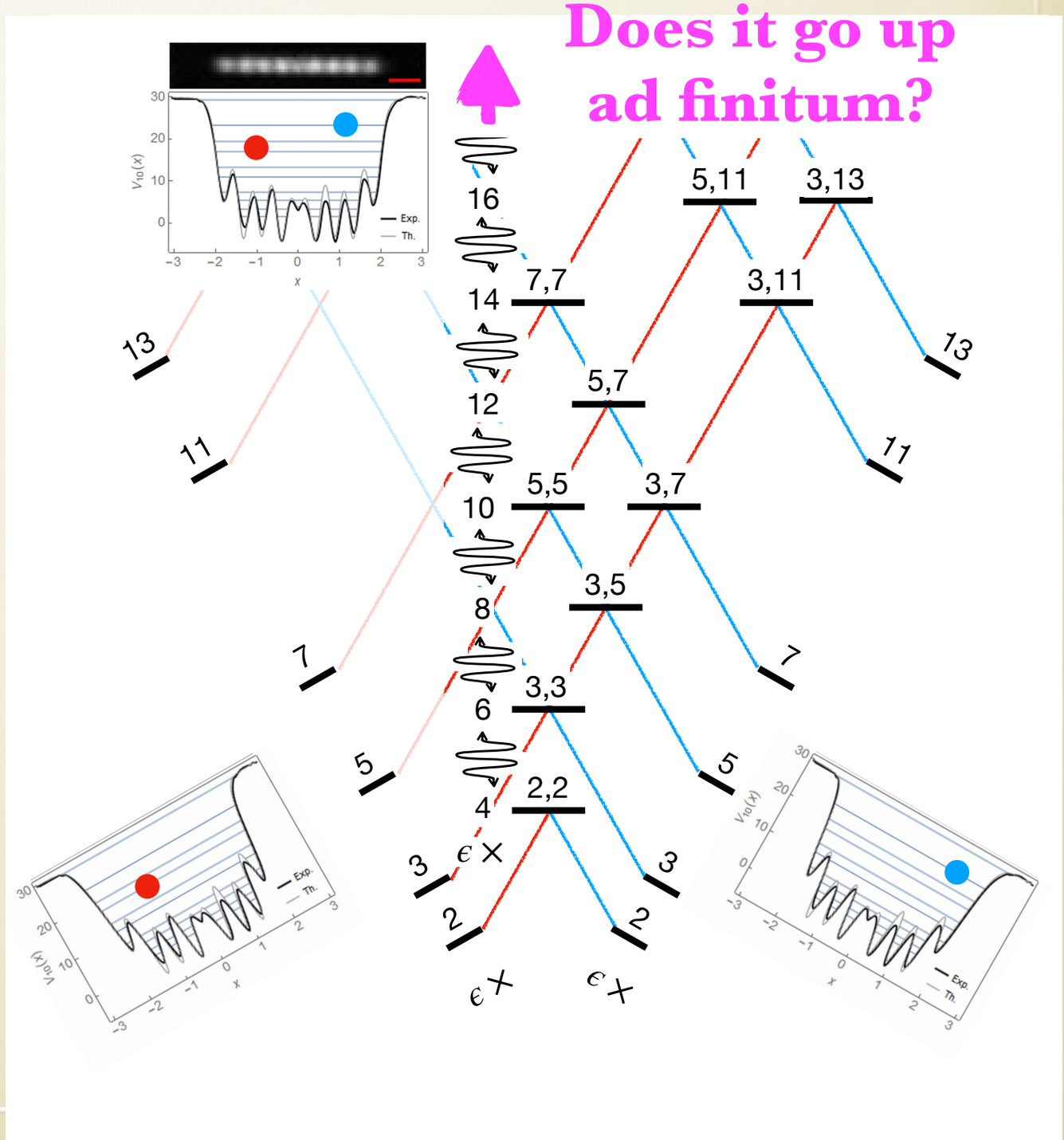
A resonant cascade whose contiguity is predicated on validity of the Goldbach conjecture

Identity

Is this arithmetic progression contiguous?

$$\forall e,\ e = p_1 + p_2 \qquad p_1 + p_2 \overset{?}{=} 2\mathbb{N}$$

Goldbach conjecture



Dictionary: one-body transition = twin prime connection between partitions

Need two-body interactions: **not every** even n is representable as

$$n = p_1 + p_2$$

where p_1 is a lower twin prime.

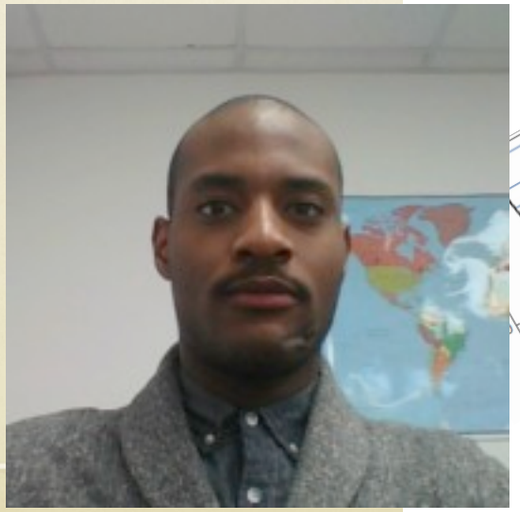
Smallest example:

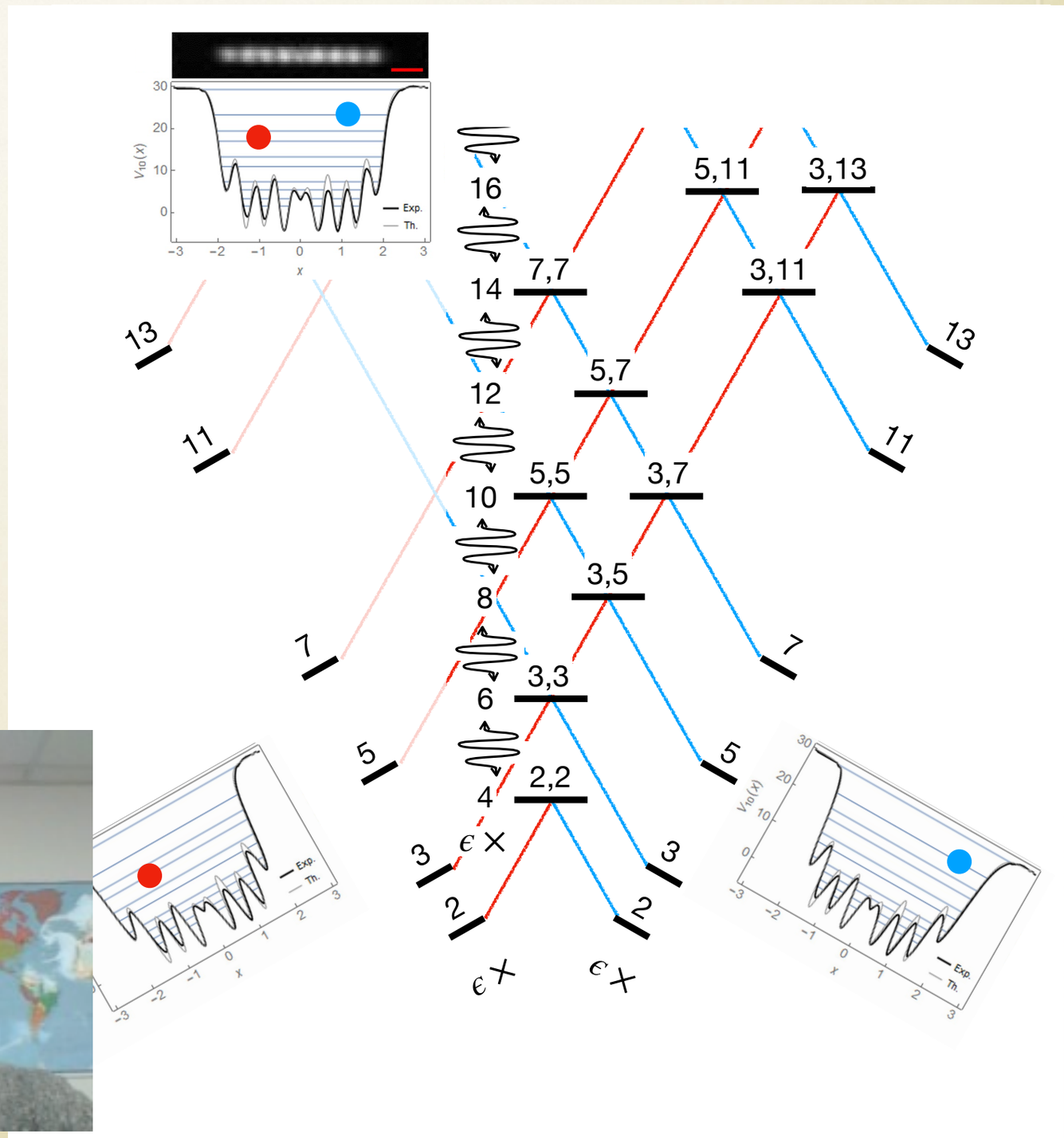
$$n = 38 = 7 + 31 = 19 + 19$$

Indeed:

$$n + 2 = 40 = 3 + 37 = 11 + 29 = 17 + 23$$

How many such incidents: UMB Mathematics Honors Thesis, Benz De Mitchell Pierre





Quantum dynamics of atoms in number-theory-inspired potentials

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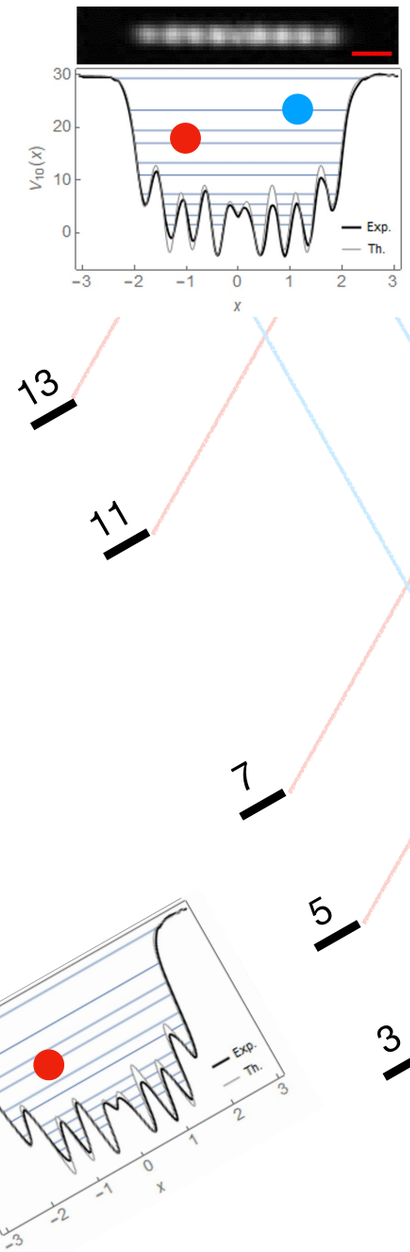
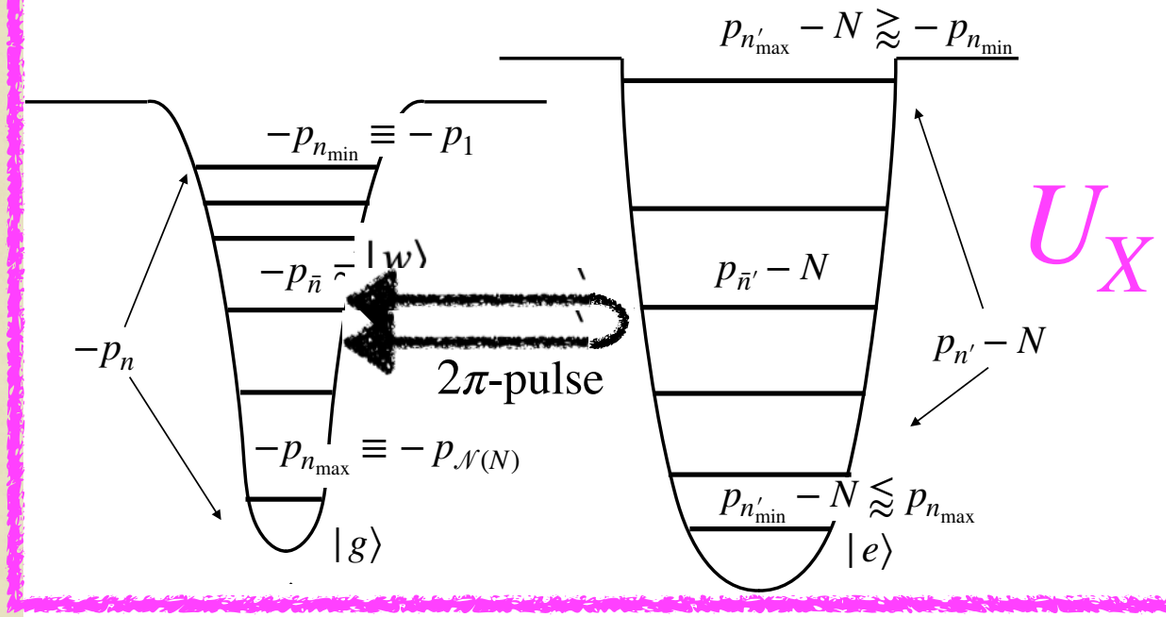
(Dated: May 20, 2024)

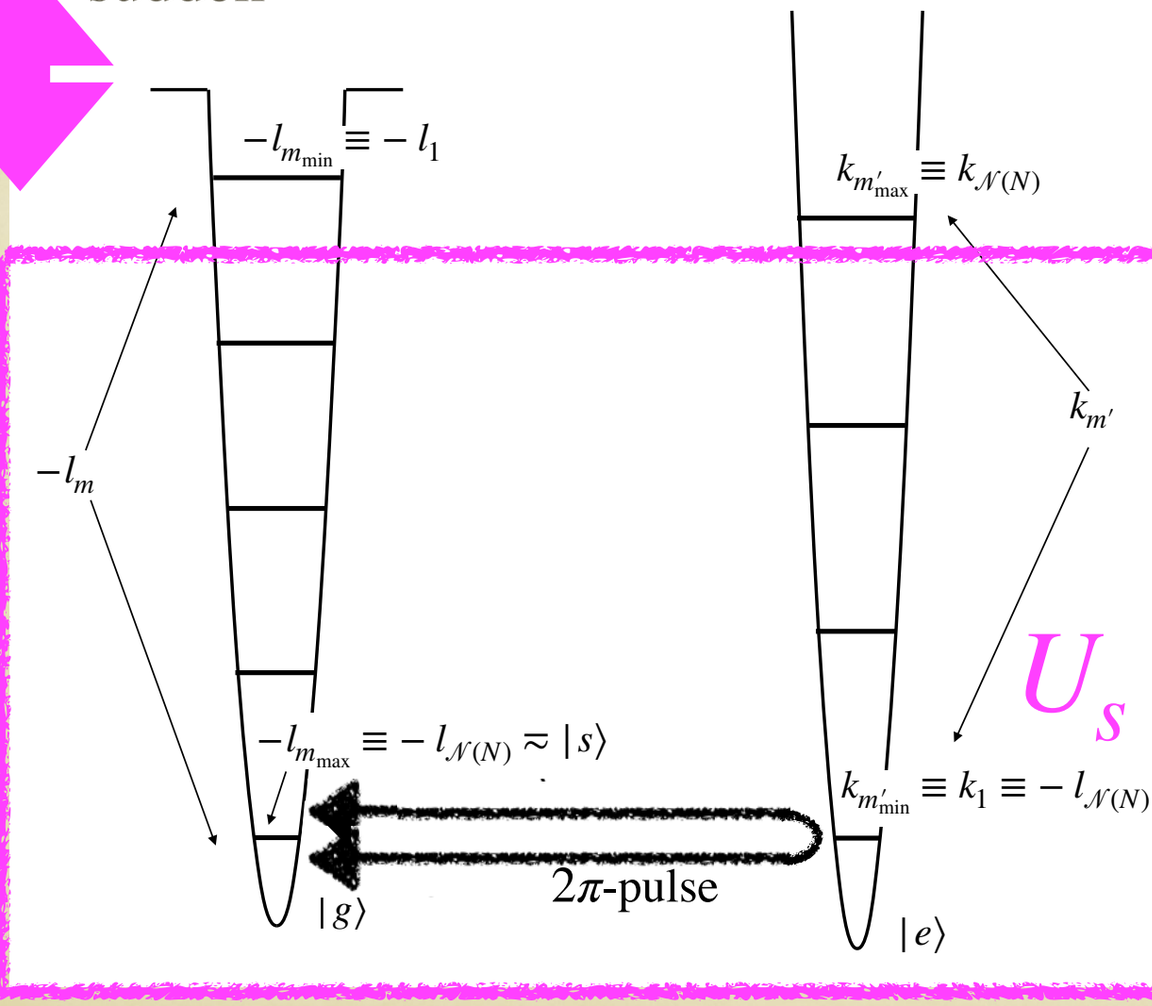
paper in preparation



Goldbach-Grover



sudden



Grover oracle (almost):

$$|w\rangle = ?$$

standard: $\hat{U}_w = \hat{I} - 2|w\rangle\langle w|$

Is there a $|w\rangle$?

our: $\hat{U}_X = \hat{I} + \begin{pmatrix} 1 \\ \text{or} \\ 0 \end{pmatrix} \{-2|w\rangle\langle w|\}$

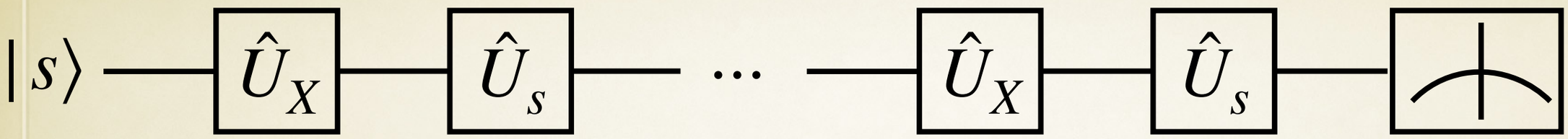
Violation of Goldbach Conjecture

Grover diffusion:

$$\hat{U}_s = \hat{I} - 2|s\rangle\langle s|$$

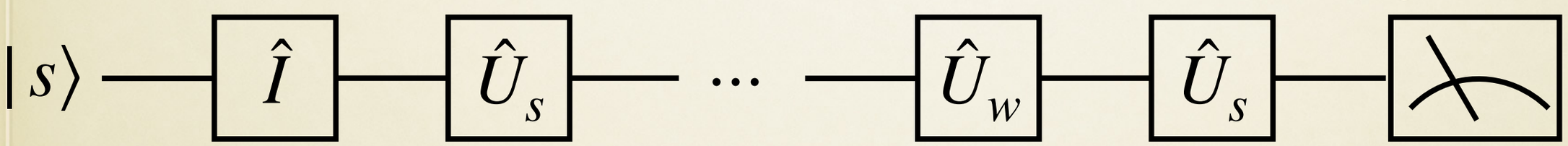
$$|s\rangle = \frac{1}{\sqrt{\#p\text{'s}}} \sum_p |p\rangle\langle p|$$

$\#p\text{'s} \sim \ln^2(N)$



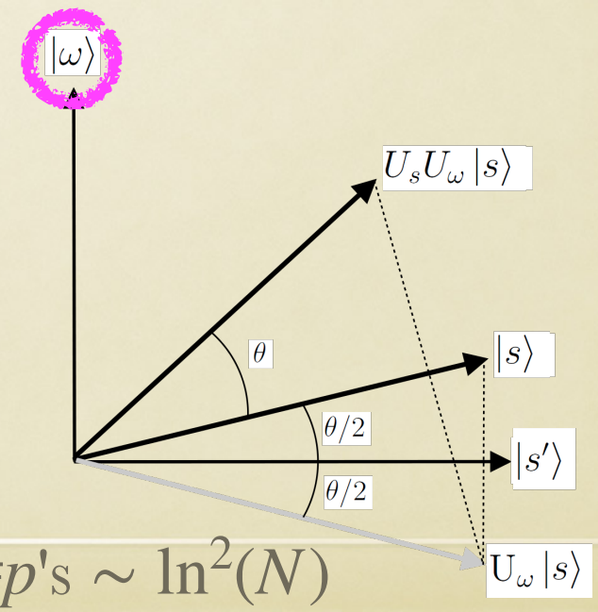
Repeat $\frac{\pi}{4}\sqrt{\#p\text{'s}}$ times

If N does not violate Goldbach conjecture

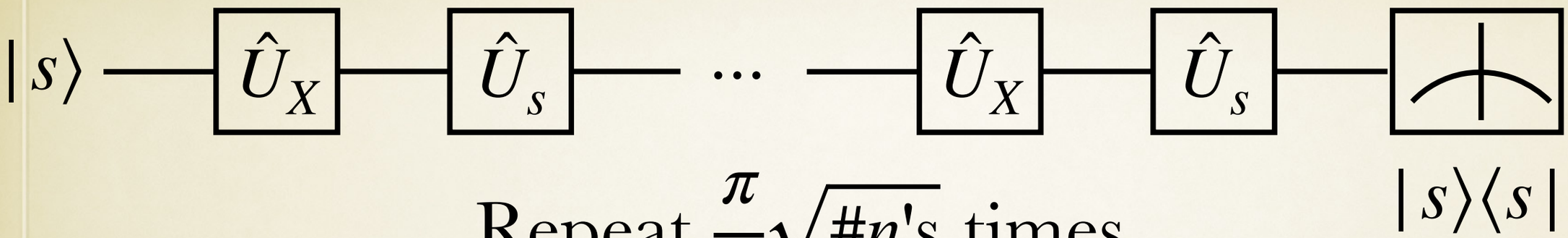


Hilbert space = all candidates for the lowest prime involved in Goldbach partitions if N

$|w\rangle$ = true lowest prime involved in Goldbach partitions of N

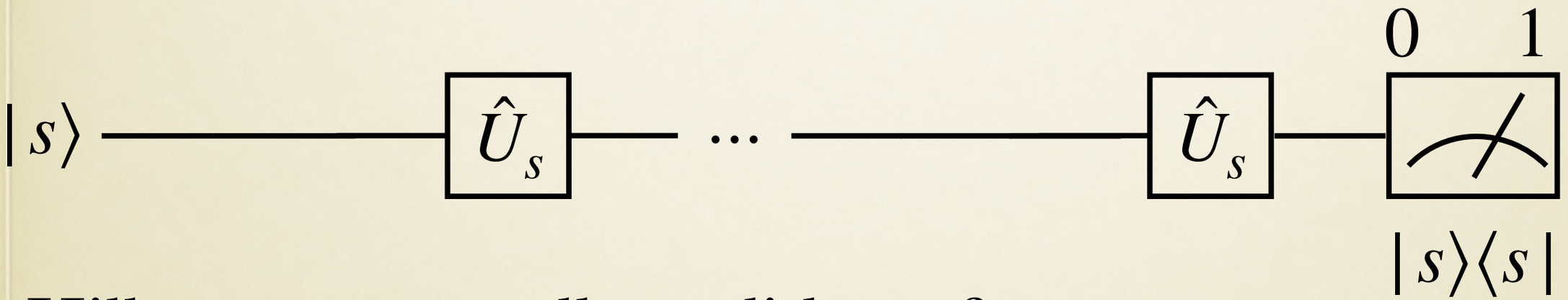


$\#p\text{'s} \sim \ln^2(N)$



Repeat $\frac{\pi}{4}\sqrt{\#n\text{'s}}$ times

If N violates Goldbach conjecture



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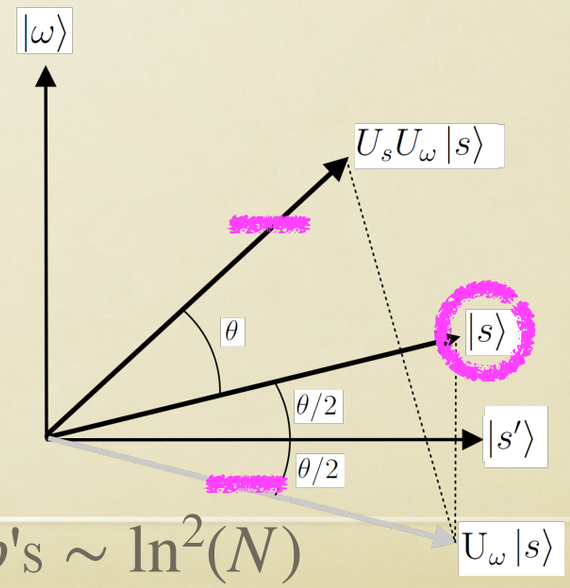
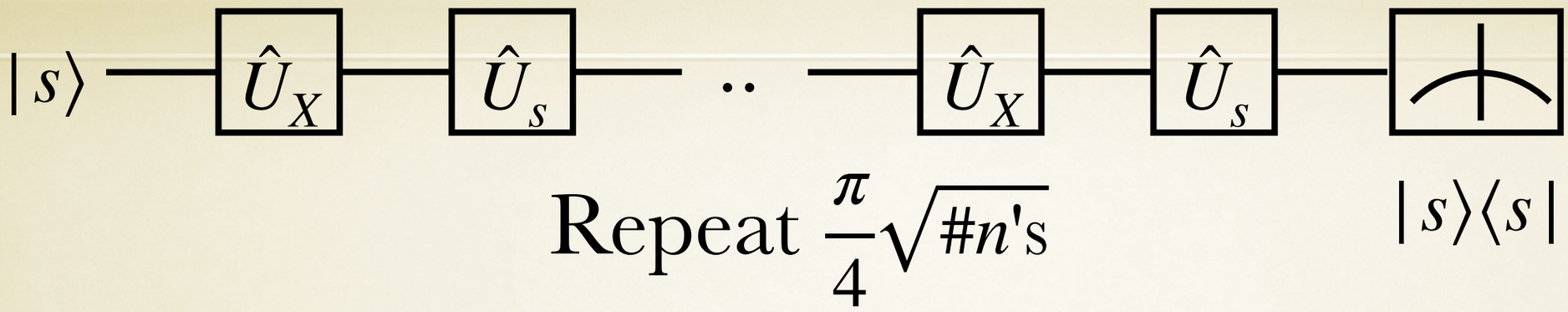


$\#p\text{'s} \sim \ln^2(N)$



Quantum advantage:

Per even number N:

$\mathcal{O}(\sqrt{\mathcal{N}(N)})$ 😎,

$\mathcal{N}(N)$ = #primes to browse

Per max N in consideration:

$\mathcal{O}(\ln(N_{\max}))$ 😟;

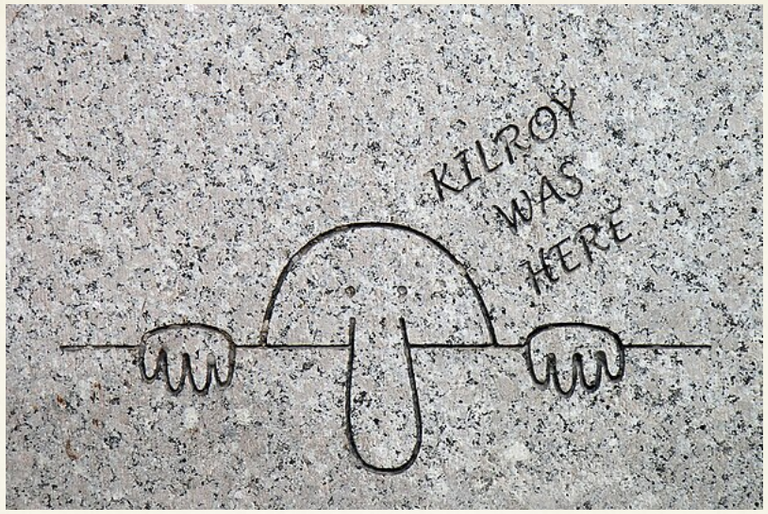
current classical: $N_{\max} = 4 \times 10^{18}$

$\mathcal{O}(\ln(N_{\max})) \sim 45$.

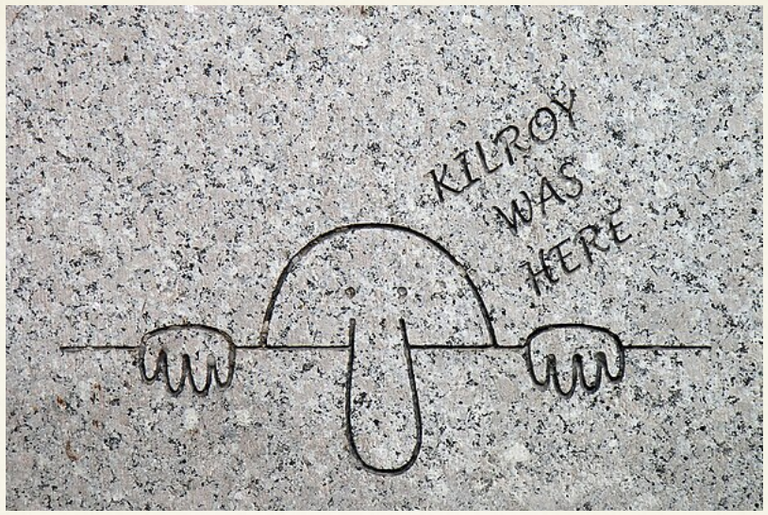
Oleksandr V. Marchukov, Andrea Trombettoni, Giuseppe Mussardo, Maxim Olshanii [arXiv:2404.00517] submitted to *Quantum*

$\#p\text{'s} \sim \ln^2(N)$

Goldbach atomtronic parallel band-stop filter



Engraving of Kilroy on the National World War II Memorial in Washington, D.C.



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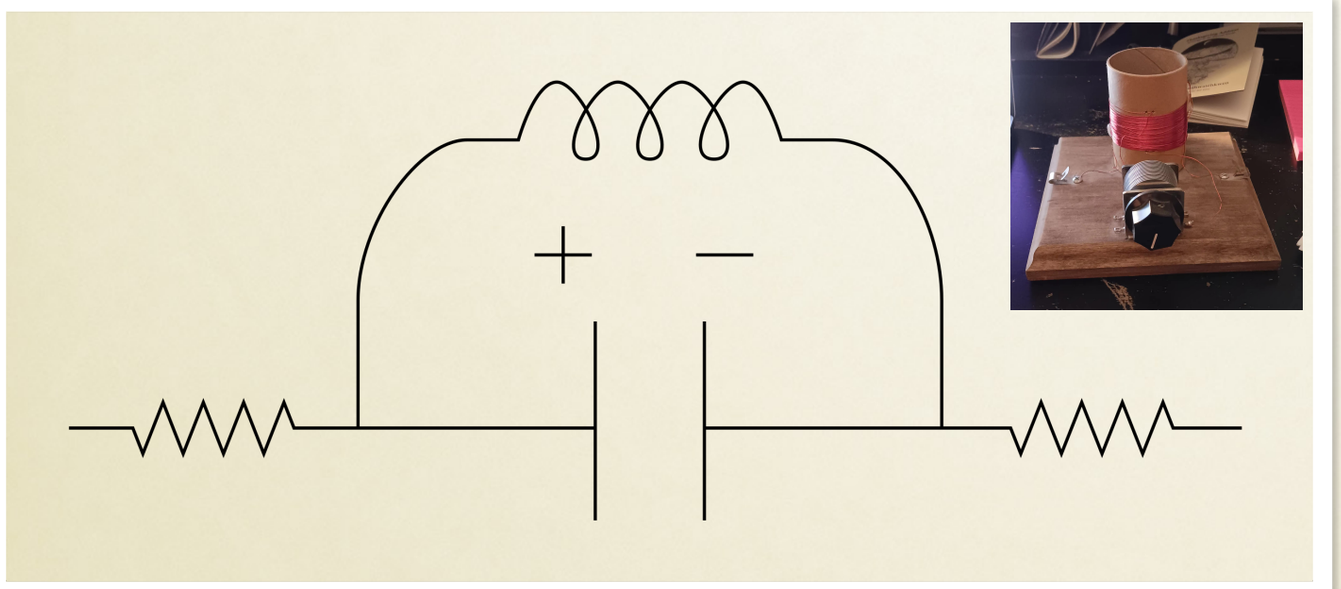
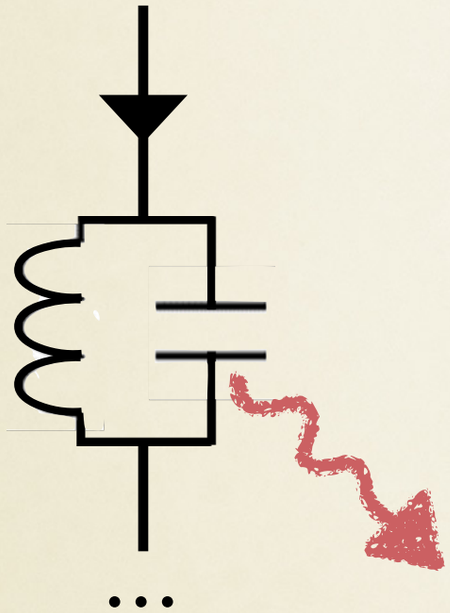


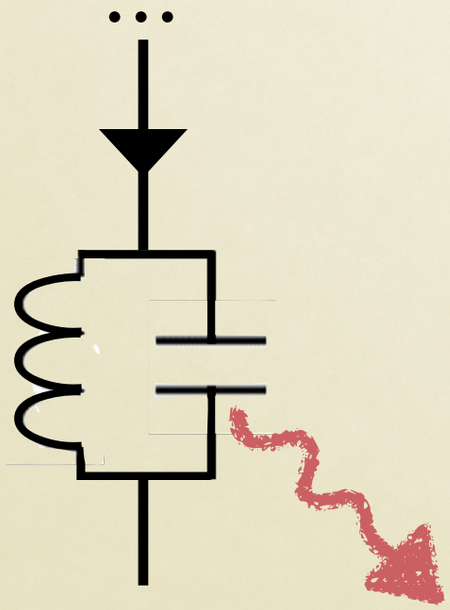
Kilroy/Chad as an RLC circuit arranged to create a band-stop filter (a.k.a. wave trap, a.k.a. wave discriminator) filter, originally drawn in Thomas Pynchon's 1963 novel *V.*

Our Universe: Goldbach conjecture is (likely) valid

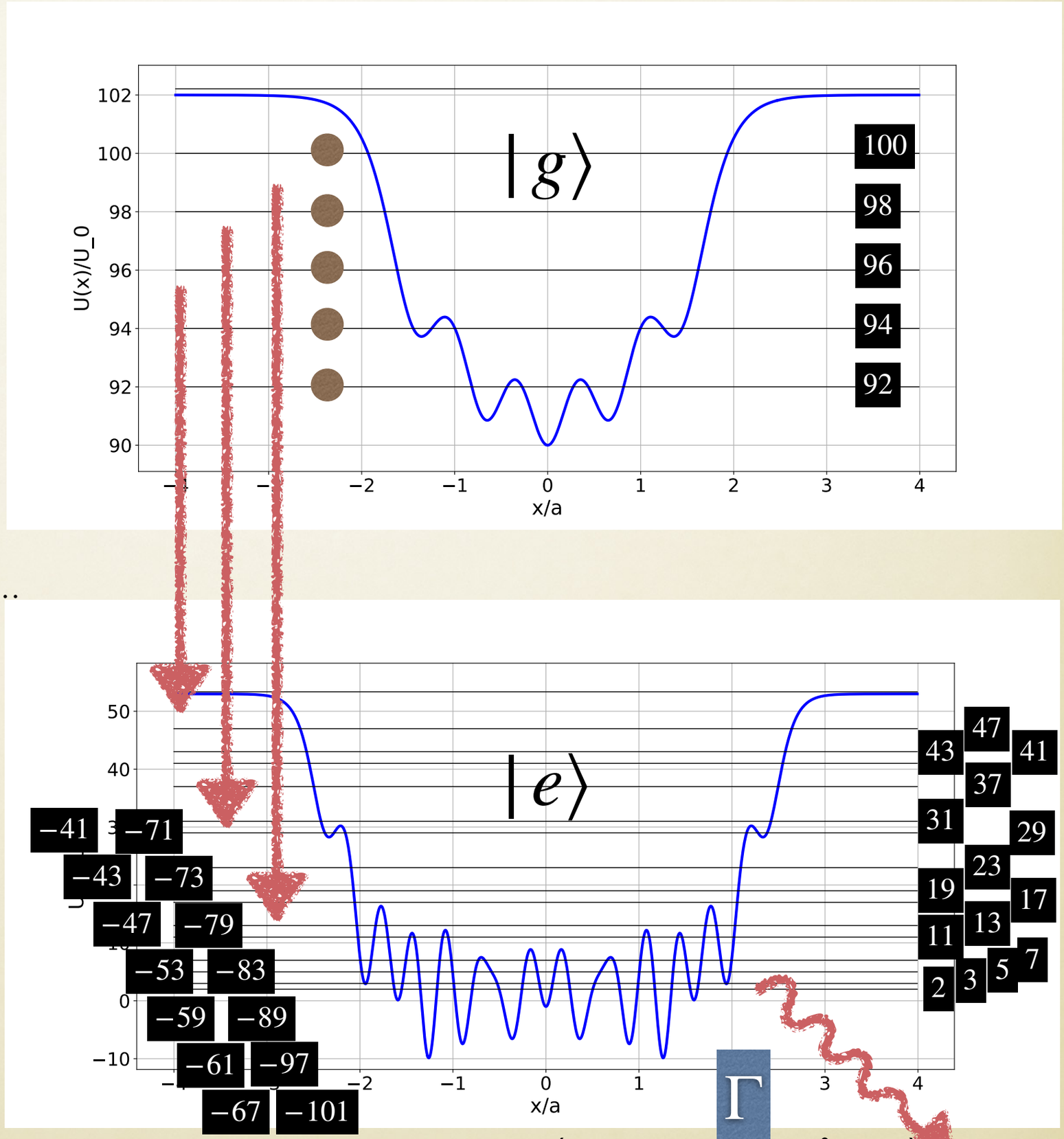
$\omega_{\text{in}} = \ldots, 92, 94, 96, 98, 100, \ldots$



$\omega_{\text{filter}} = \ldots, 3+41, \ldots, 3+89, \ldots, 3+101, \ldots$
$\ldots, 19+79, \ldots, 19+83, \ldots, 19+89, \ldots$

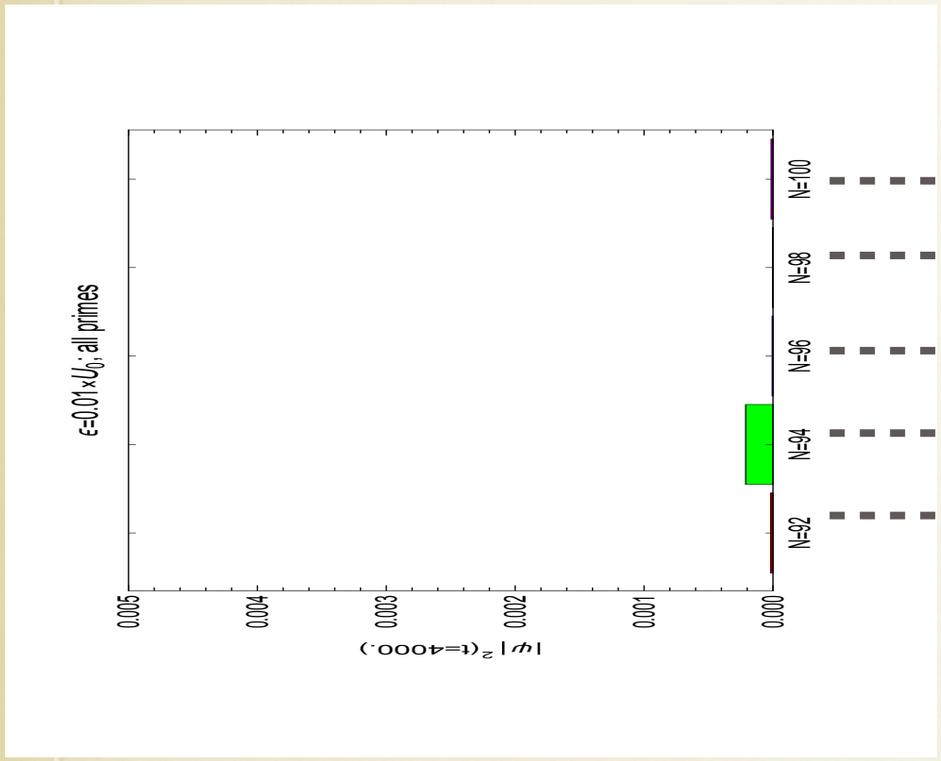


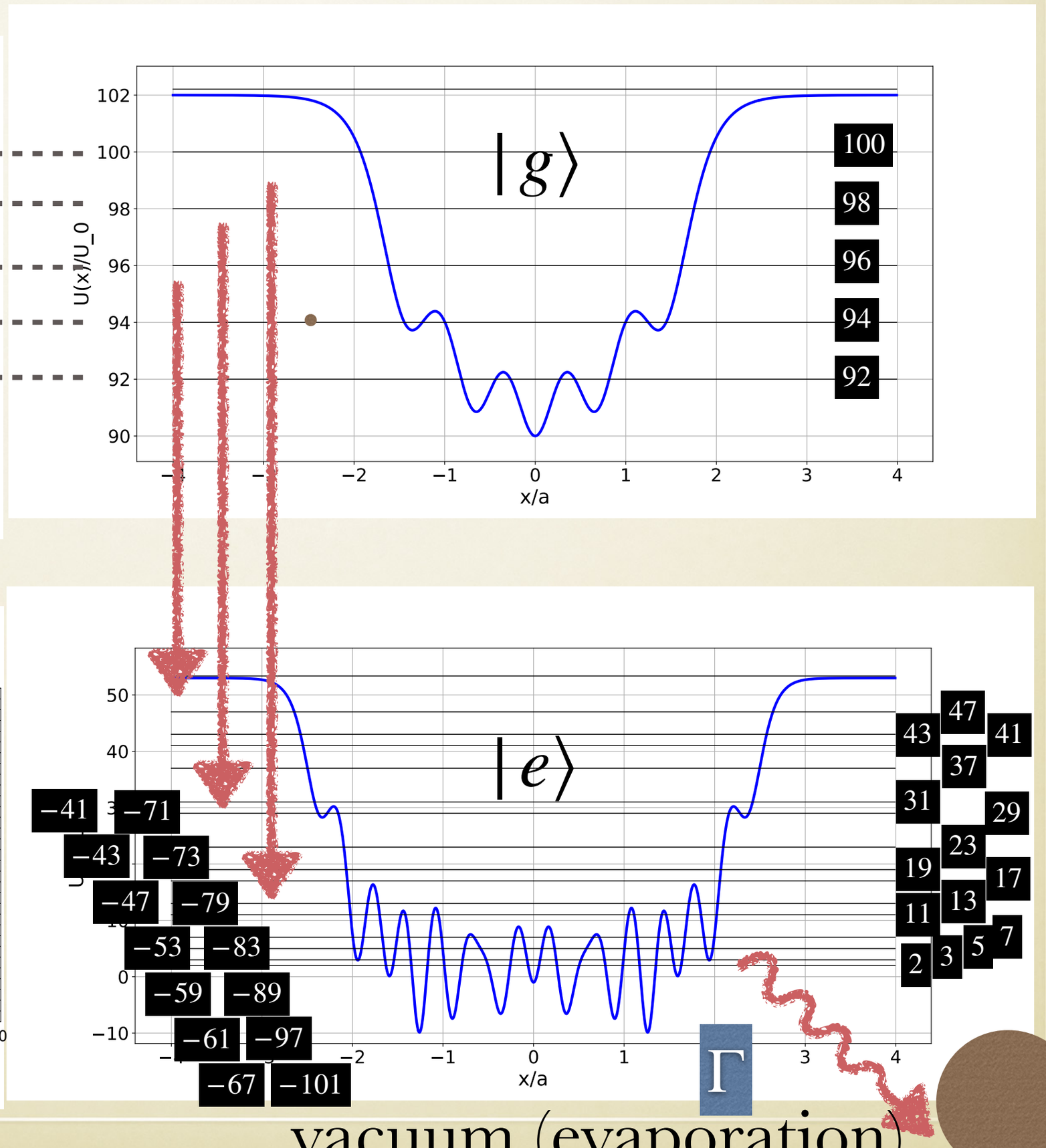
$\omega_{\text{out}} = \varnothing$

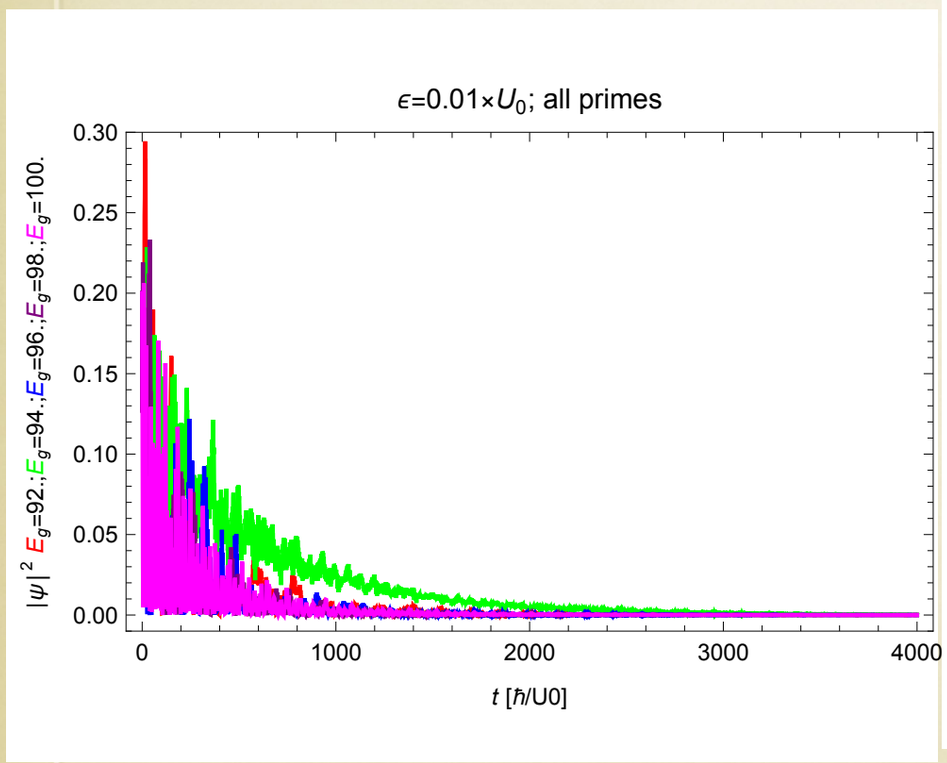


vacuum (evaporation)

Our Universe: Goldbach conjecture is (likely) valid



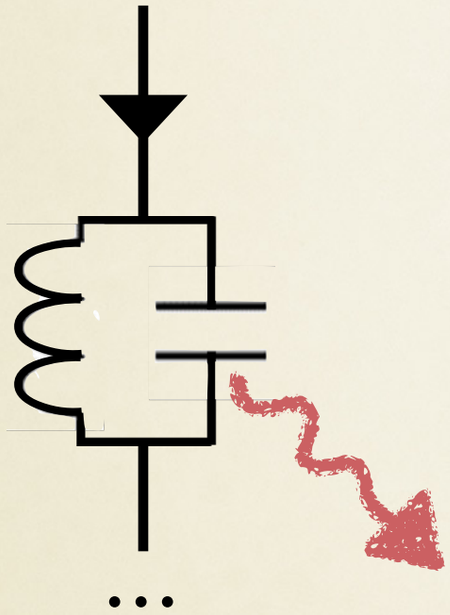




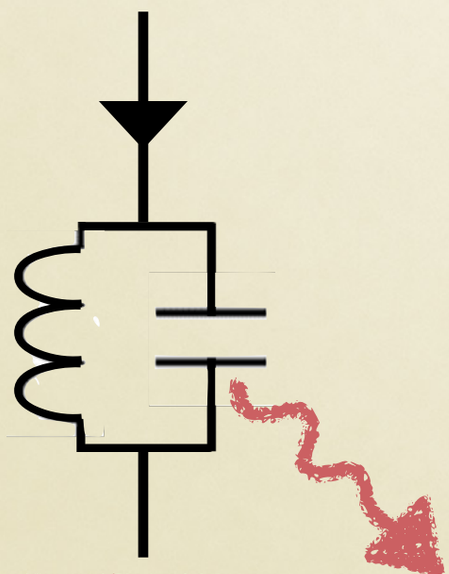
vacuum (evaporation)

An alternate Universe: 61, 67, and 79 are not prime ⇒ ⇒ 98 violates Goldbach conjecture

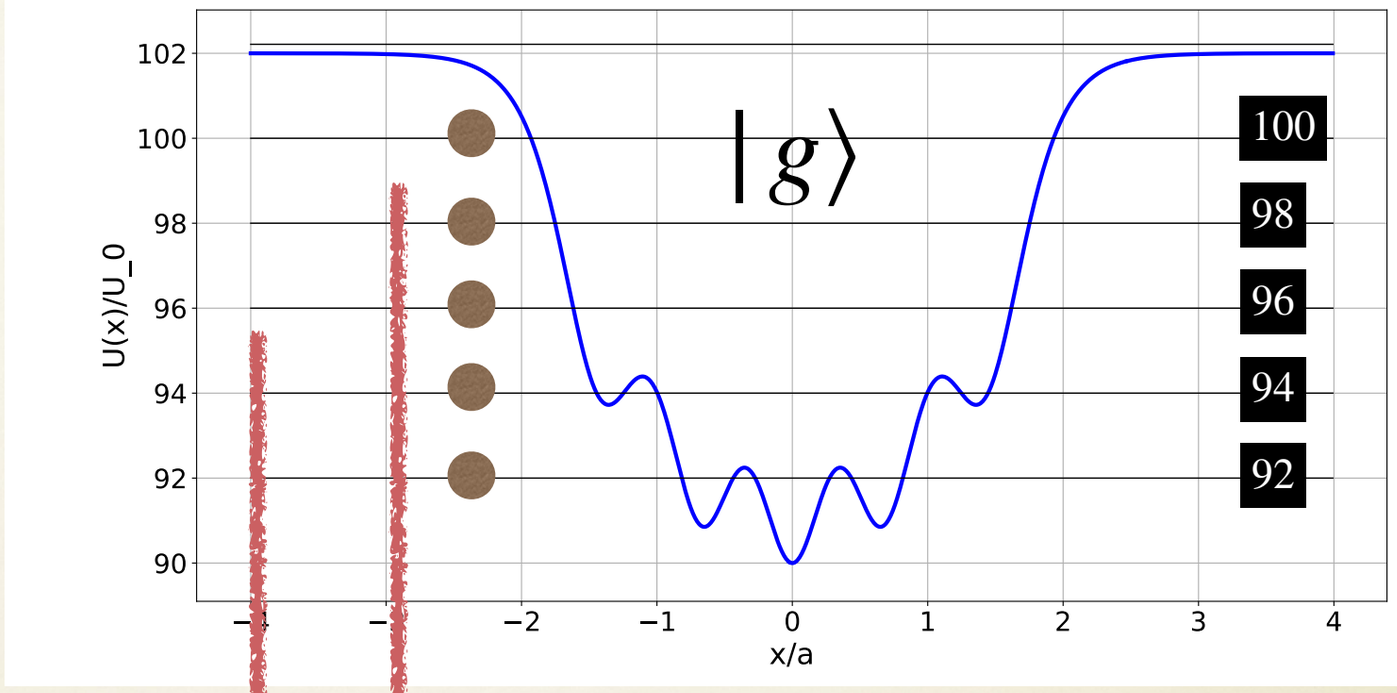
$\omega_{\text{in}} = \ldots, 92, 94, 96, 98, 100, \ldots$

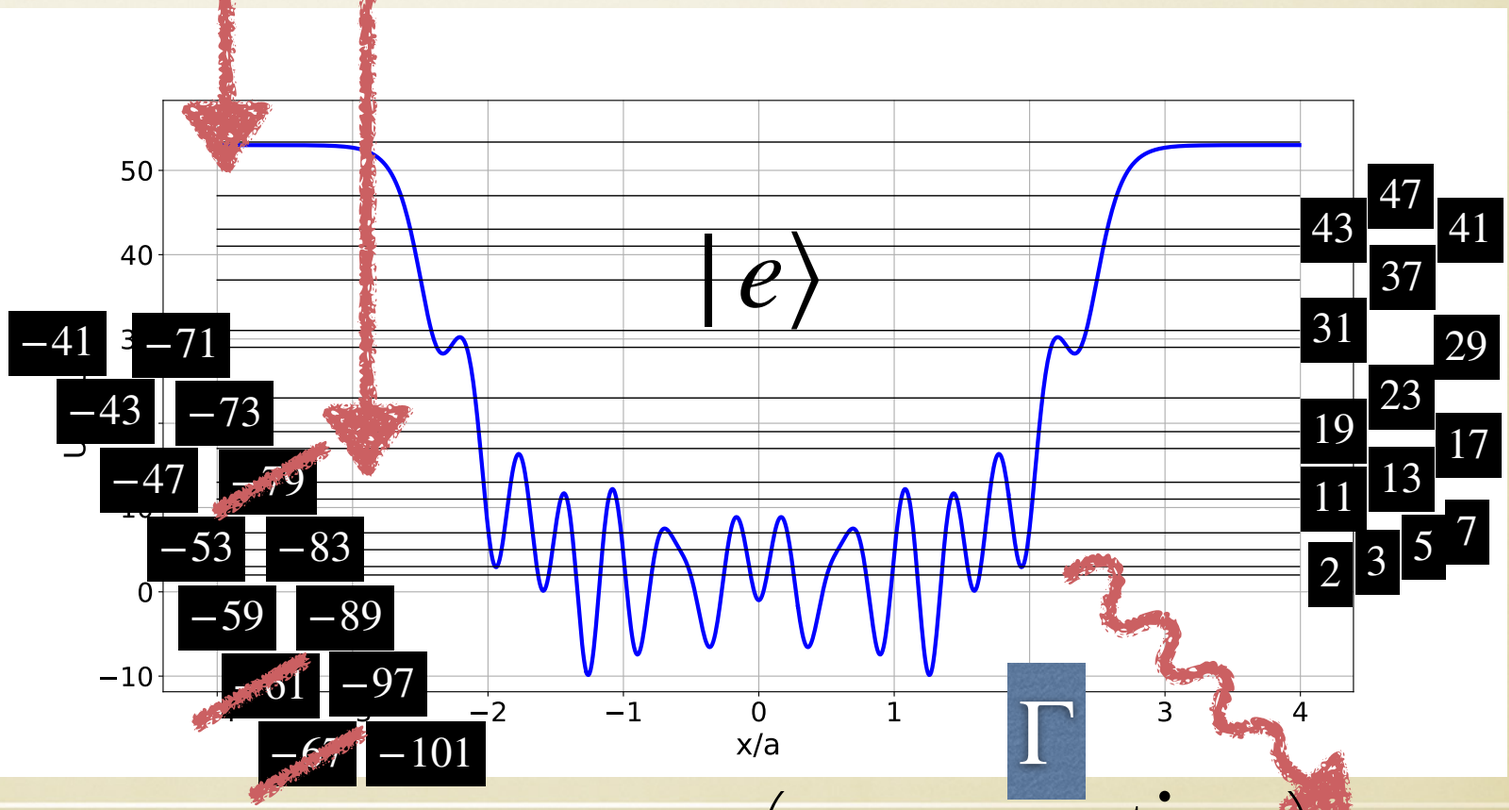


$\omega_{\text{filter}} = \ldots, 3+41, \ldots, 3+89, \ldots, 3+101, \ldots$
$\ldots, 19+79, \ldots, 19+83, \ldots, 19+89, \ldots$



$\omega_{\text{out}} = 98$





vacuum (evaporation)

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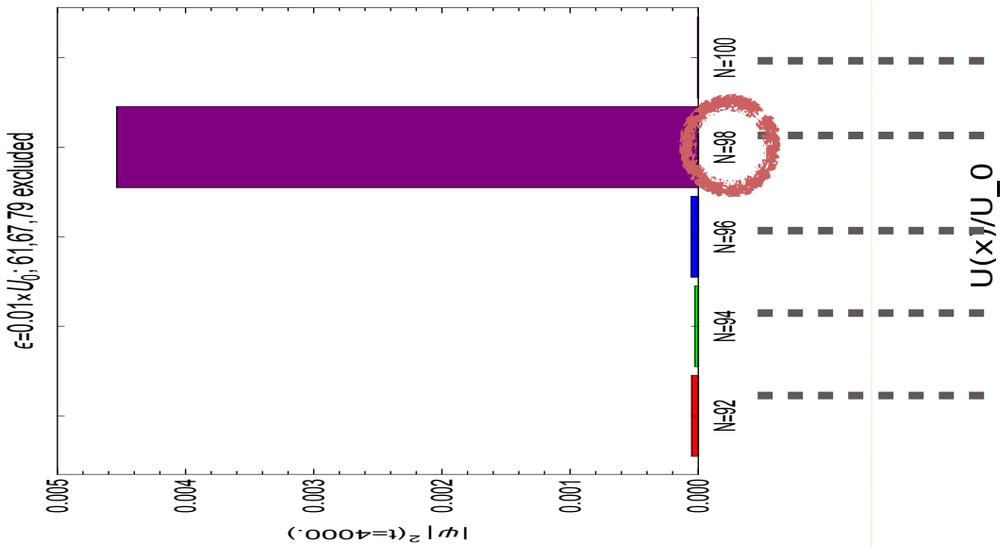
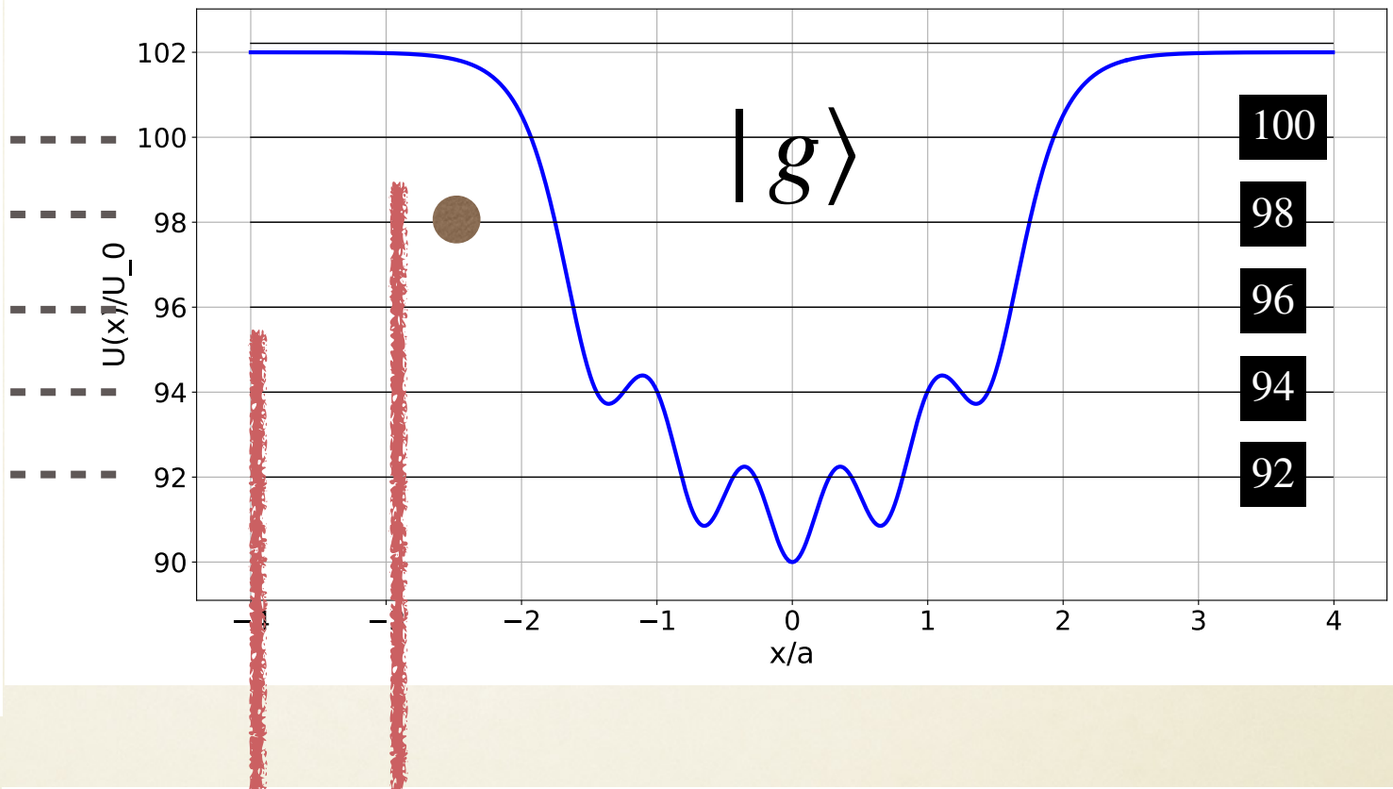
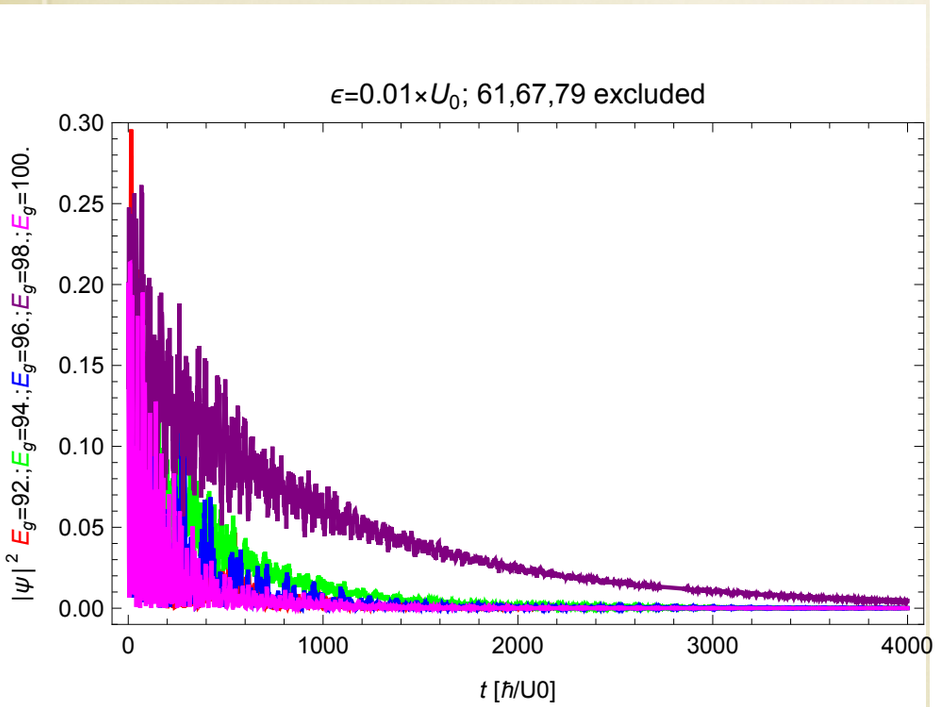
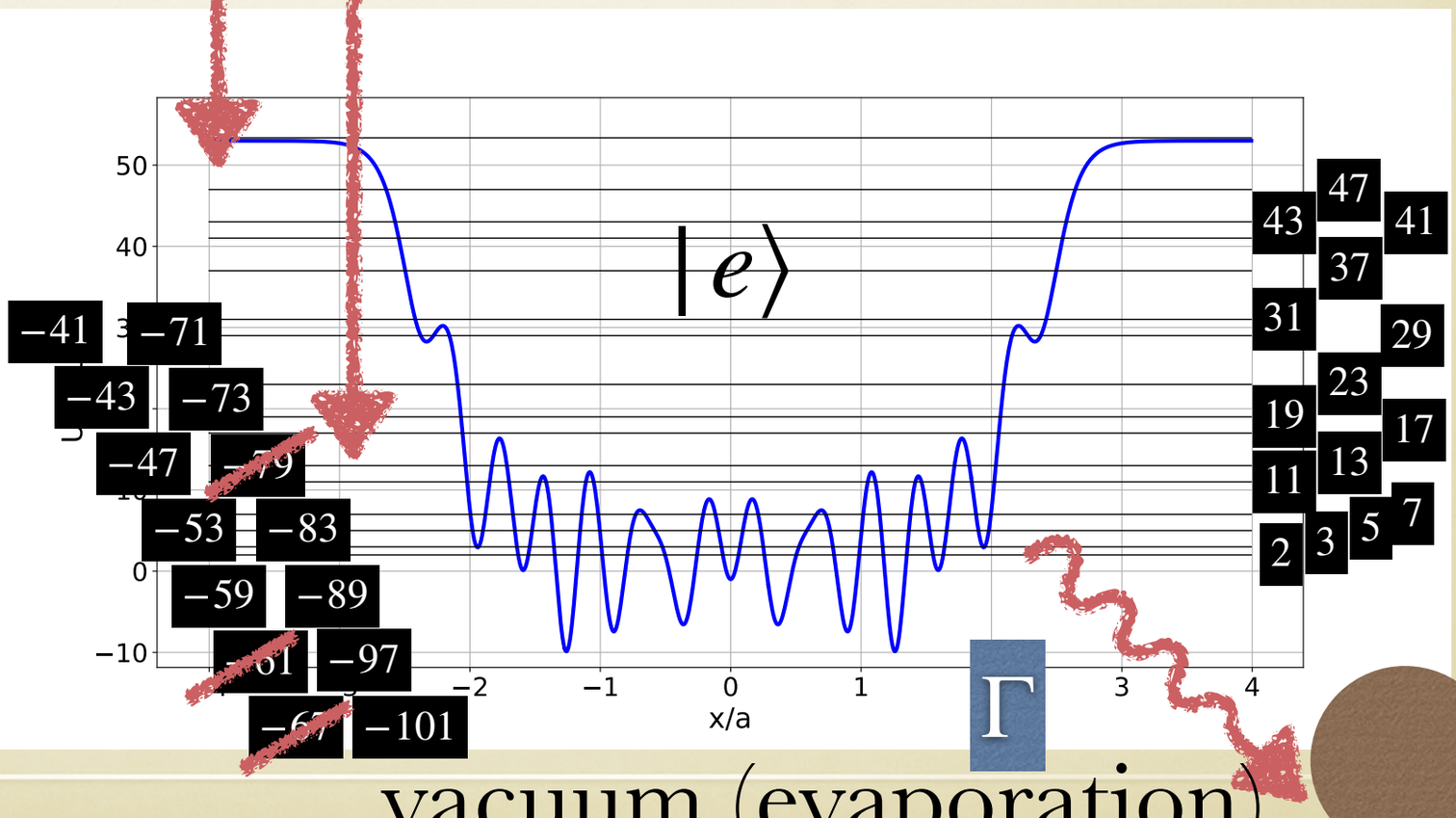








vacuum (evaporation)

```
ϵ = 0.01;
V0 = 1.;
σ = 0.1;
Δx = 0.1;
Γ = .01;
```

$U_k(x) \approx U_0\ a\ \delta(x\text{-}x_k)$

$x_k=\Delta x\ (k\text{-}1)$

$k=1, 2, \ldots, N_{\text{peaks}}$

$U(x) = \sum_{k=1}^{N_{\text{peaks}}} \text{weights}[[k]]\, V_k$

$V(x,t) = \epsilon\, U(x) \sum_{n_f=1}^{N_{\text{large\_primes}}} \text{Cos}\left[\omega_{n_f}\, t\right]$

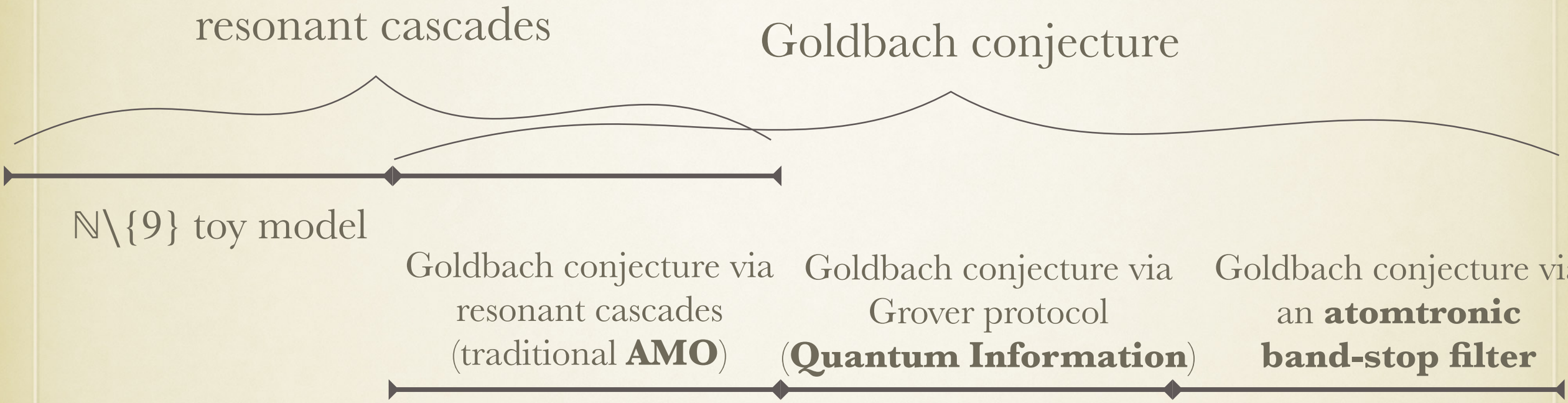
```
Npeaks = 31;
```

```
weights = Table[1. / Npeaks // N, {kk, 1, Npeaks}];
```

Speculative: Avogadro parallelism

Computation time $\sim 1/\Gamma$. It is independent of the number of the evens tested: **each atom is a very weak classical computer** that explores the paths to the evens through the primes. **But we can have a large number of atoms**.

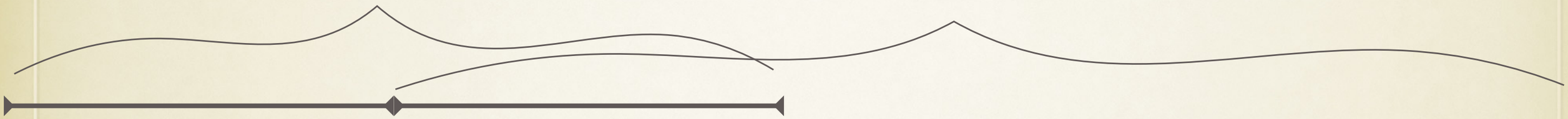
Summary


resonant cascades
Goldbach conjecture
ℕ\{9} toy model
Goldbach conjecture via resonant cascades (traditional **AMO**)
Goldbach conjecture via Grover protocol (**Quantum Information**)
Goldbach conjecture via an **atomtronic band-stop filter**

Summary

resonant cascades

Goldbach conjecture



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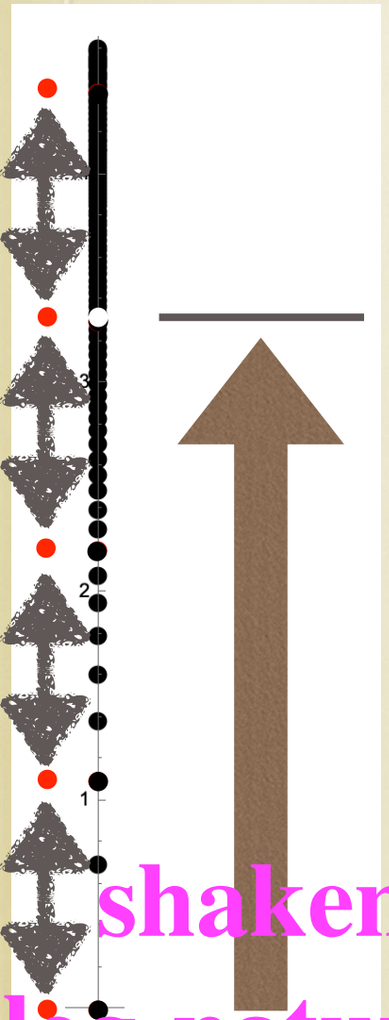

shaken log-natural potentials

Summary

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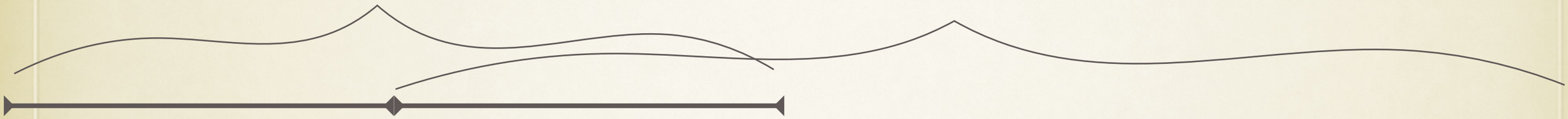

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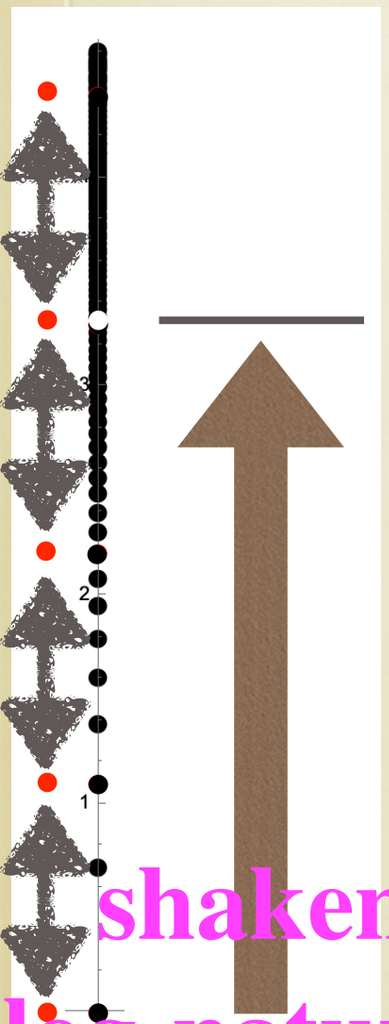

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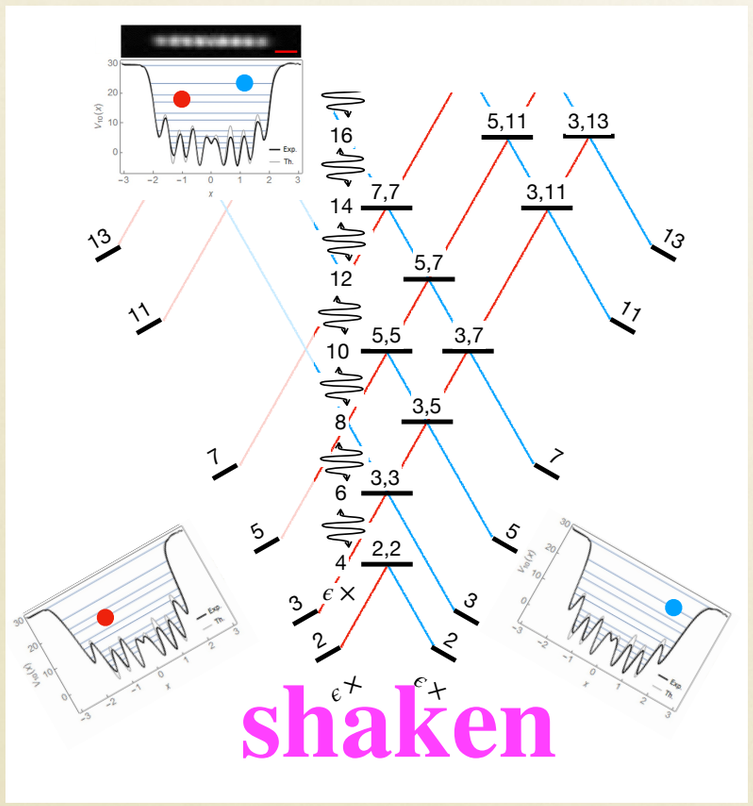

shaken prime potentials

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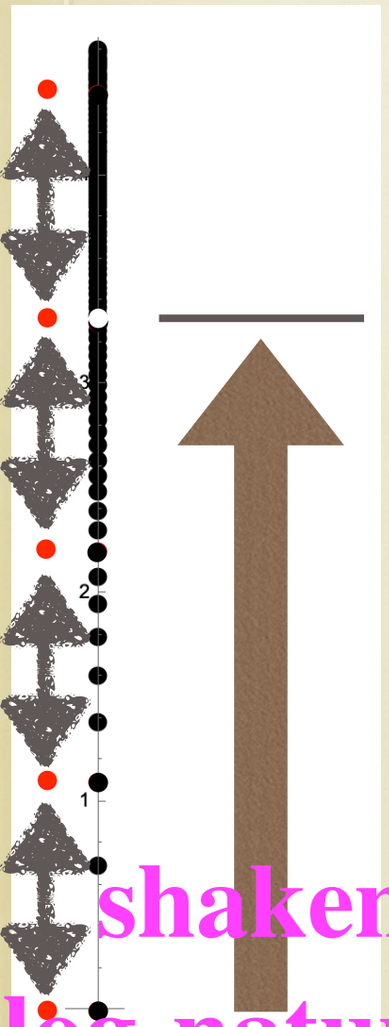

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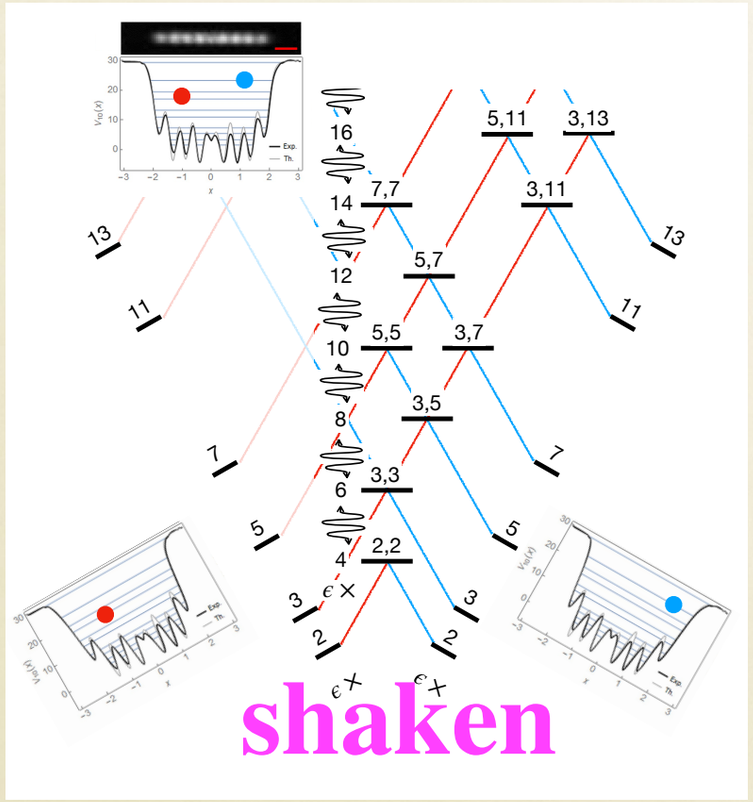
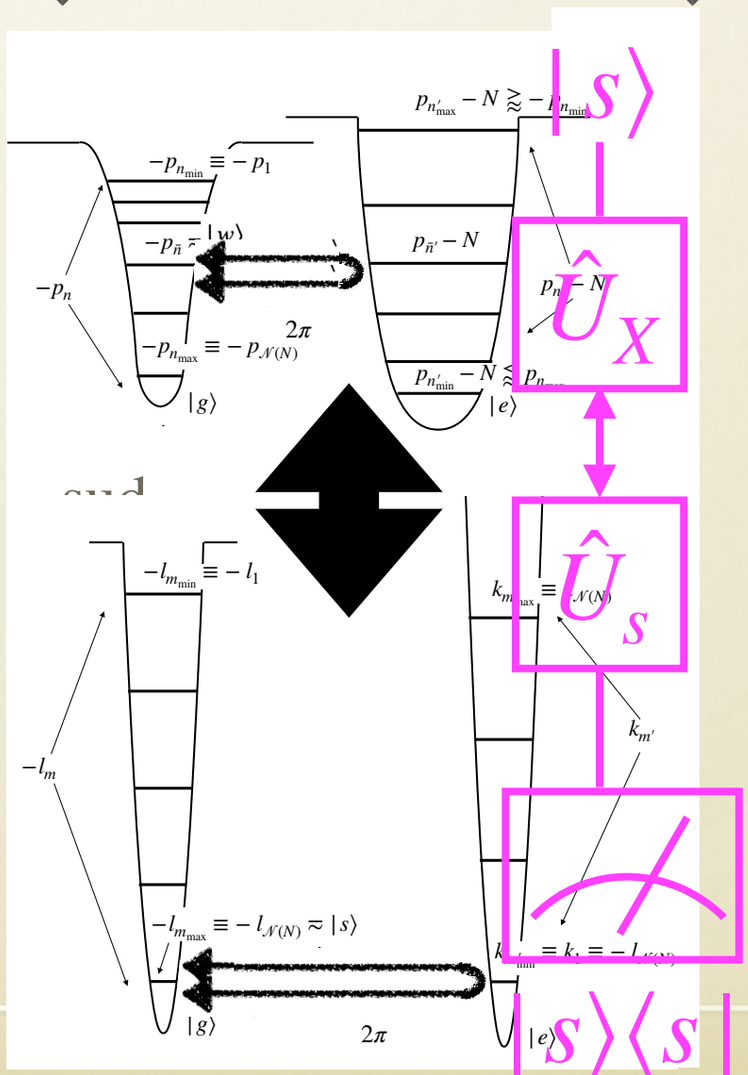


shaken prime potentials



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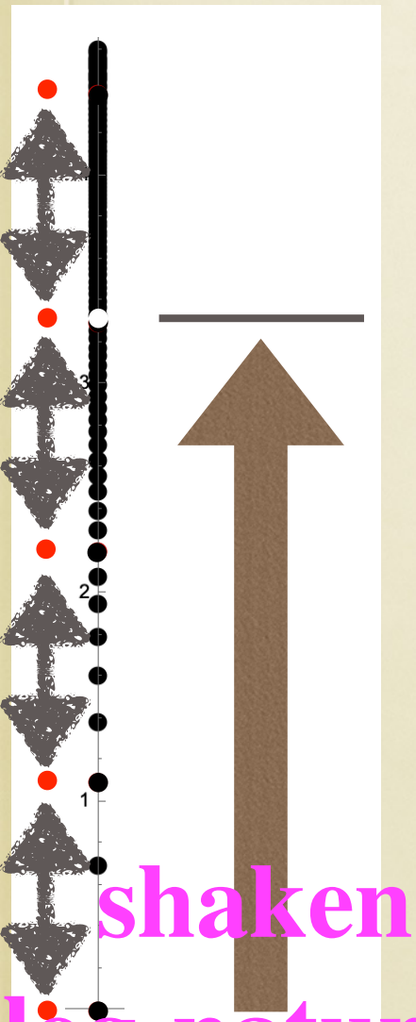
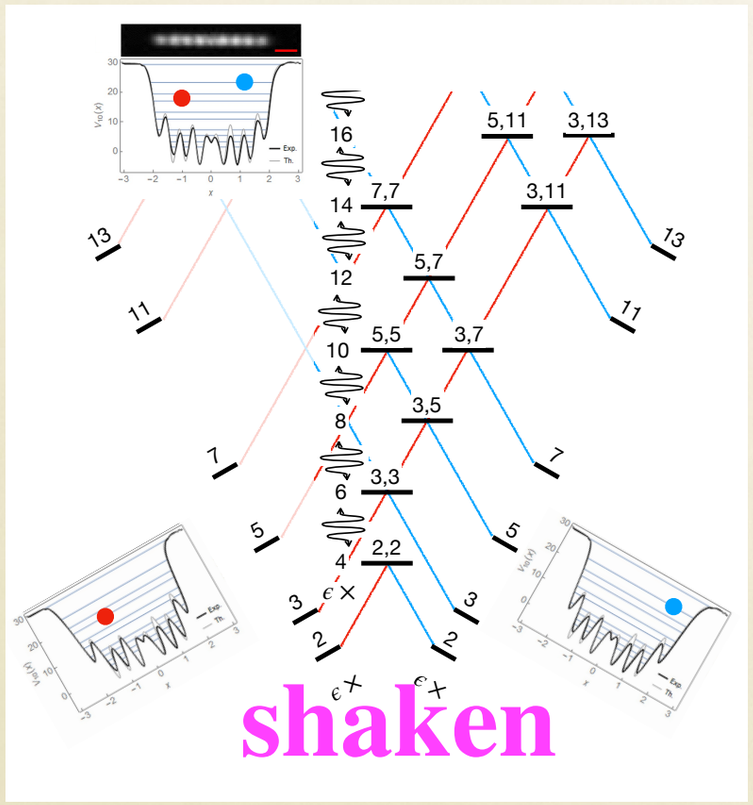
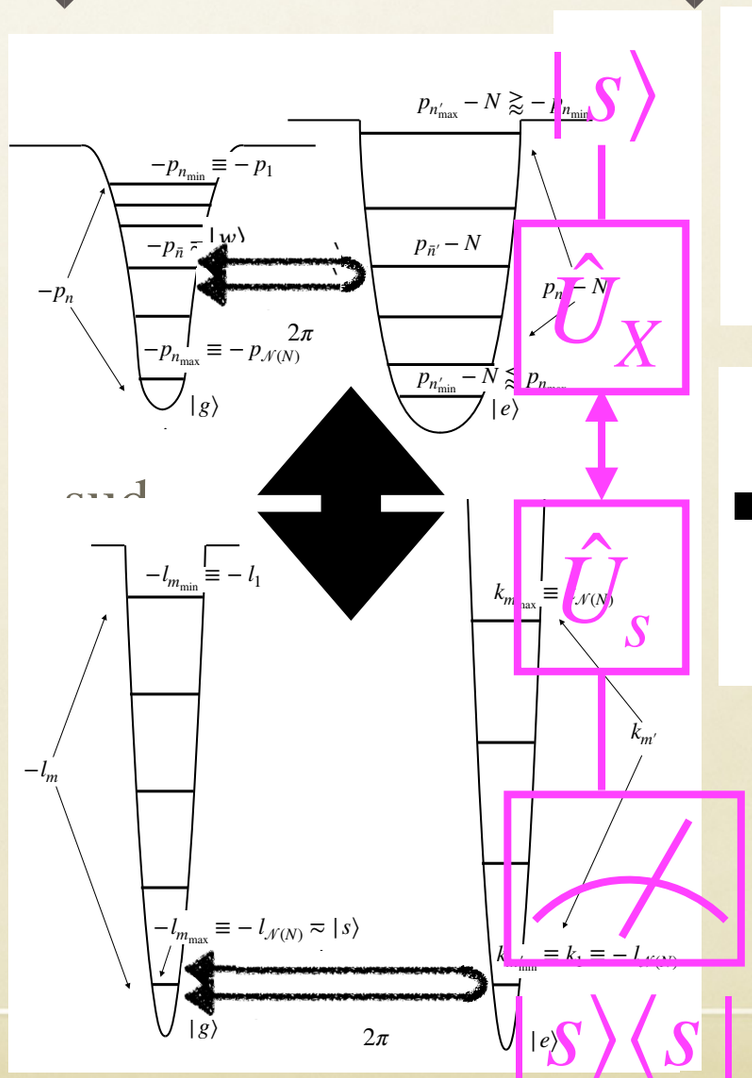
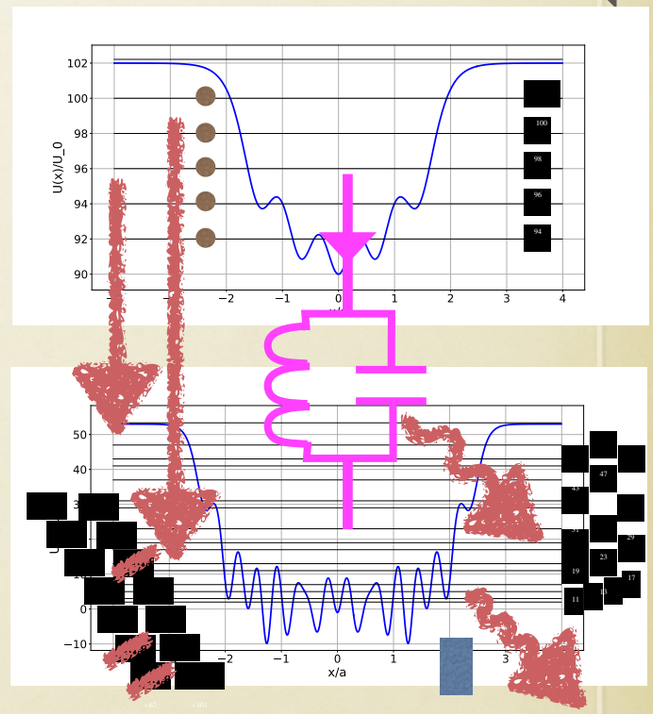








SciPost Submission

Resonance Cascades and Number Theory

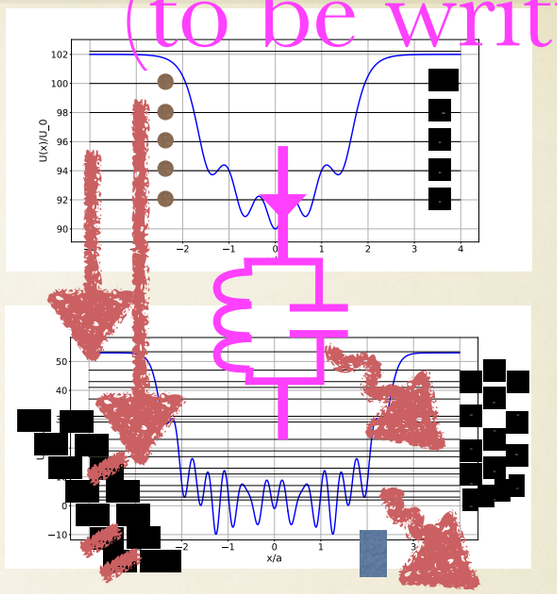
Oleksandr V. Marchukov[1], Maxim Olshanii[2*]

1 Technische Universität Darmstadt, Institut für Angewandte Physik, Hochschulstraße 4a, 64289 Darmstadt, Germany
2 Department of Physics, University of Massachusetts Boston, Boston Massachusetts 02125, USA
maxim.olchanyi@umb.edu

February 9, 2024

(to be resubmitted)

(to be written)



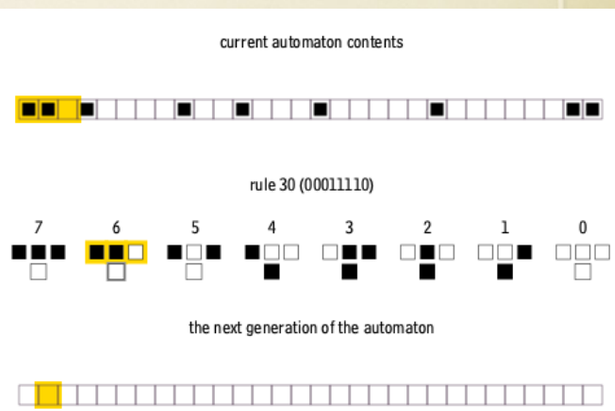
Achieving quantum advantage in a search for a minimal Goldbach partition with driven atoms in tailored potentials

Oleksandr V. Marchukov[1], Andrea Trombettoni[2,3], Giuseppe Mussardo[3], and Maxim Olshanii[4]

arXiv:2404.00517

[1]Technische Universität Darmstadt, Institut für Angewandte Physik, Hochschulstraße 4a, 64289 Darmstadt, Germany
[2]Department of Physics, University of Trieste, Strada Costiera 11, I-34151 Trieste, Italy
[3]SISSA and INFN, Sezione di Trieste, Via Bonomea 265, I-34136 Trieste, Italy
[4]Department of Physics, University of Massachusetts Boston, Boston, Massachusetts 02125, USA

(in refereeing)



Quantum dynamics of atoms in number-theory-inspired potentials

Oleksandr V. Marchukov,[1] Benjamin Carruthers,[2] Harry Kendell,[3,4] Joanna Ruhl,[5] Benz De Mitchell Pierre,[6] Catalin Zara,[6] Donatella Cassettari,[2] Carrie A. Weidner,[4] Andrea Trombettoni,[7,8] Maxim Olshanii,[5] and Giuseppe Mussardo[8]

sche Universität Darmstadt, Institut für Angewandte Physik, Hochschulstraße 4a, 64289 Darmstadt, Germany
PA School of Physics & Astronomy, University of St. Andrews, North Haugh, St. Andrews KY16 9SS, UK
[3] *Quantum Engineering Centre for Doctoral Training, University of Bristol, Bristol BS8 1FD, UK*
[4] *Quantum Engineering Technology Laboratories, H. H. Wills Physics Laboratory and Department of Electronic Engineering, University of Bristol, Bristol BS8 1FD, UK*
[5] *De... University of Massachusetts Boston, Boston Massachusetts 02125, USA*
... University of Massachusetts Boston, Boston Massachusetts 02125, USA
... University of Trieste, Strada Costiera 11, I-34151 Trieste, Italy
... Sezione di Trieste, Via Bonomea 265, I-34136 Trieste, Italy

(Dated: May 20, 2024)

(to be submitted to PRA)

SciPost Submission

Rule 60 cellular automaton, Mersenne numbers, and the Newman-Moore spin lattice

Jonás Carmona-Pírez[1], Adrian J. Peguero[2], Konstantinos Sfairopoulos[3], Maxim Olshanii[2], Joanna Ruhl[2*]

1 Technical Advisory Subdirectorate of Information Management (STAGI), Andalusian Health Service (SAS), 41071 Seville, Spain
2 Department of Physics, University of Massachusetts Boston, Boston Massachusetts 02125, USA
3 School of Physics and Astronomy, University of Nottingham, Nottingham, NG7 2RD, UK and Centre for the Mathematics and Theoretical Physics of Quantum Non-Equilibrium Systems, University of Nottingham, Nottingham, NG7 2RD, UK
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May 24, 2024

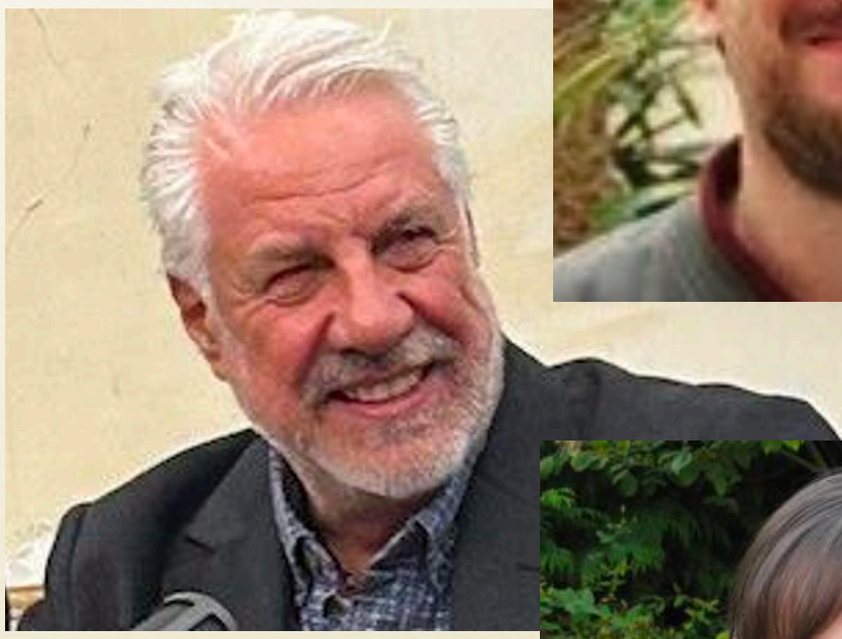
(to be submitted)

Collaborators/coauthors

Oleksandr Marchukov

Andrea Trombettoni

Giuseppe Mussardo

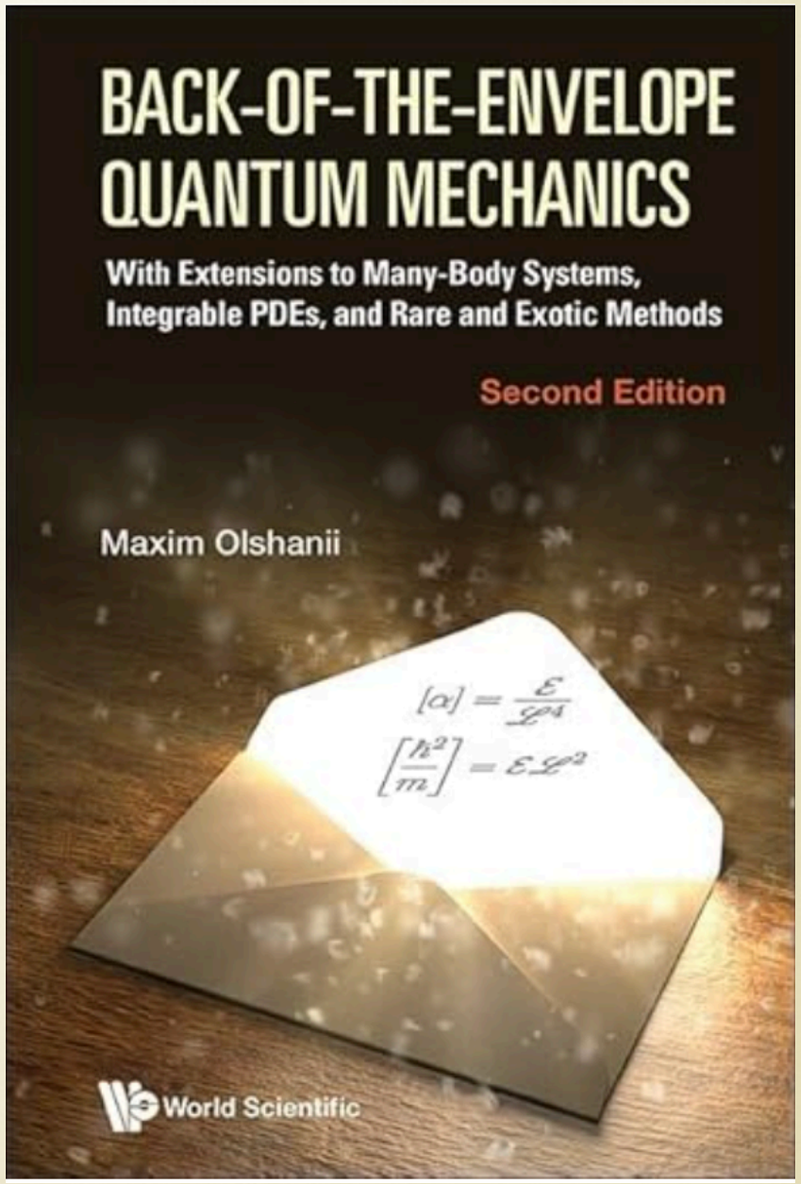


Donatella Cassettari

A shameless ad

New chapters, things you've never heard of:

- Bohlin-Arnold-Vassiliev map
- Shrauner's Method of Power Indices
- Exact solutions from the scale invariance alone, at $E = 0$, after Newton
- Turbiner-Ushveridze's quasi-integrability



... and things little known:

- QM-SUSY
- Connected ladders of solutions from either continuous scale invariance or self-similar tilings

Second edition is out

Support by:

Thank you!