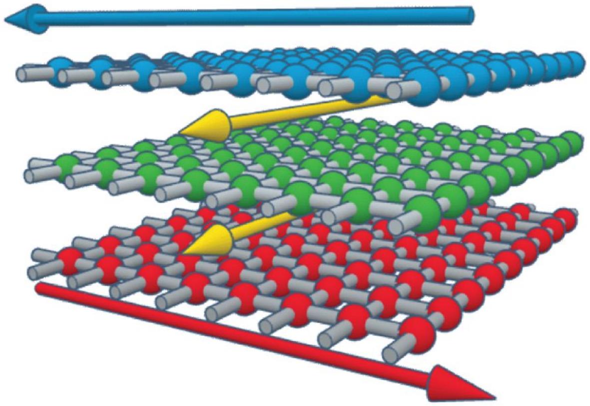
SISSA

Atomtronics 2024

Interaction-enhanced chiral currents in atomic synthetic structures



SciPost Phys. 14, 048 (2023)

Speaker:
Matteo Ferraretto

Outline

Introduction to quantum simulation with **alkaline-earth-like** atoms in optical lattices and SU(N) Hubbard model

SU(N)-breaking Raman processes for the realization of synthetic structures with synthetic magnetic fields

Methods and results: Dynamical Mean Field Theory (DMFT) and interaction-enhanced chiral currents

Outline

Introduction to quantum simulation with alkaline-earth-like atoms in optical lattices and SU(N) Hubbard model

SU(N)-breaking Raman processes for the realization of **synthetic structures** with synthetic magnetic fields

Methods and results: Dynamical Mean Field Theory (DMFT) and interaction-enhanced chiral currents

Outline

Introduction to quantum simulation with alkaline-earth-like atoms in optical lattices and SU(N) Hubbard model

SU(N)-breaking Raman processes for the realization of **synthetic structures** with synthetic magnetic fields

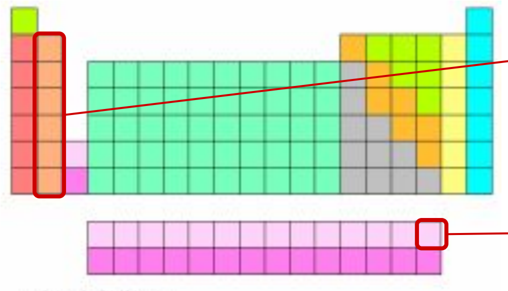
Methods and results: Dynamical Mean Field Theory (DMFT) and **interaction-enhanced chiral currents**

SISSA

Atomtronics 2024

1. Introduction to quantum simulation with alkaline-earth-like atoms in optical lattices and SU(N) Hubbard model

Alkaline-earth-like atoms



Alkaline-earth metals

$^{173}\mathrm{Yb}$

Heavy fermionic Lanthanide with complete f shell (alkaline-earth-like)

Electronic states (Yb)

$6s$ $6p$

$^1S_0$

$$L = 0$$
$$S = 0$$
$$J = 0$$

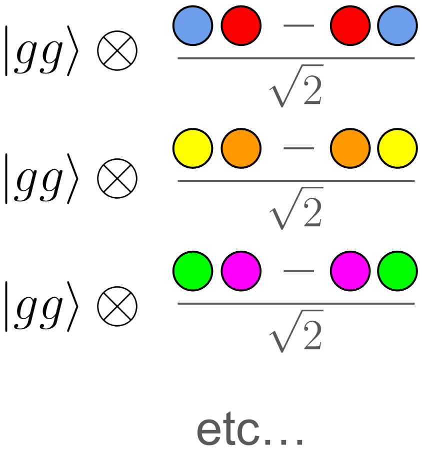
No hyperfine coupling with the nucleus

Nuclear states (Yb)

$I = \frac{5}{2}$ (up to $2I + 1 = 6$ states)

$m = -\frac{5}{2} \quad -\frac{3}{2} \quad -\frac{1}{2} \quad \frac{1}{2} \quad \frac{3}{2} \quad \frac{5}{2}$

Alkaline-earth-like atoms



same **s-wave scattering length**

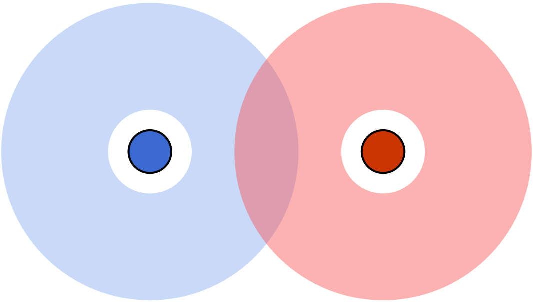
$$a_{gg} \approx 200 a_0$$

consequence of absence of hyperfine coupling

Experimentally measured within a precision of

$$\delta a_{gg} \approx 10^{-9} a_{gg}$$

Origin of **SU(N) symmetry** of interaction

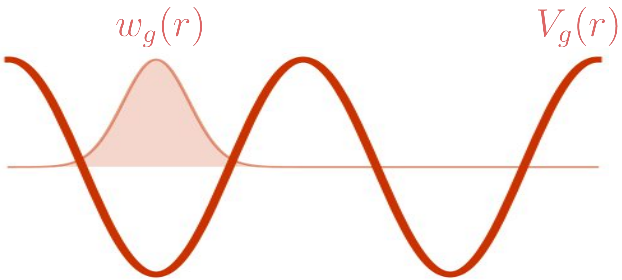


- atoms interact via their electronic clouds;
- these are decoupled from their nuclei;
- nuclear states don’t affect the scattering

Alkaline-earth-like atoms

SU(N) symmetric Hubbard model

$$H = -t \sum_{\langle ij \rangle} \left(c^\dagger_{i\sigma} c_{j\sigma} + \text{h.c.} \right) - \mu \sum_{j\sigma} n_{j\sigma} + \frac{U}{2} \sum_j n_j (n_j - 1)$$



- $t = \int d^3r \, w_g(\vec{r}) \left[-\frac{\hbar^2}{2m} \nabla^2_{\vec{r}} + V_g(\vec{r}) \right] w_g(\vec{r} - \vec{R})$
- $U = \frac{4\pi\hbar^2 a_{gg}}{m} \int d^3r |w_g(r)|^4$

Symmetry group

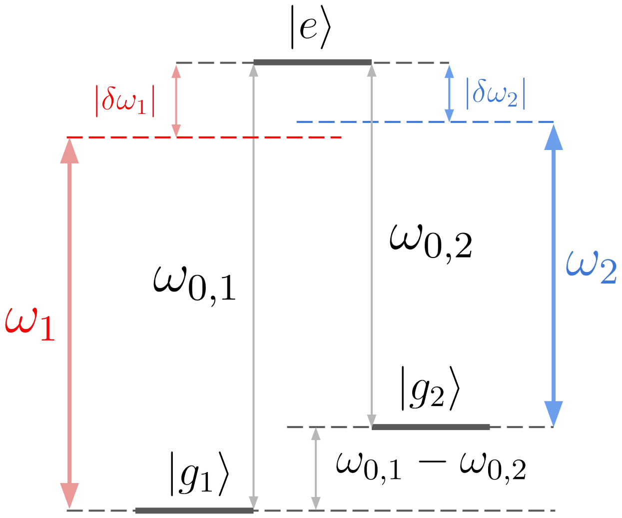
$$U(1) \times SU(N)$$

$S_{\sigma\rho} = \sum_j c^\dagger_{j\sigma} c_{j\rho}$ "flavor" ladder operators

2. SU(N)-breaking Raman processes for the realization of synthetic structures with synthetic magnetic fields

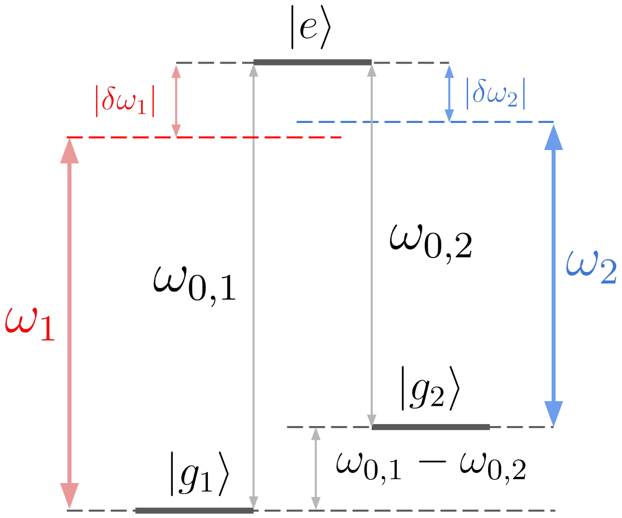
Breaking SU(N): Raman transitions

Three-level atom coupled to 2 detuned **external laser** beams

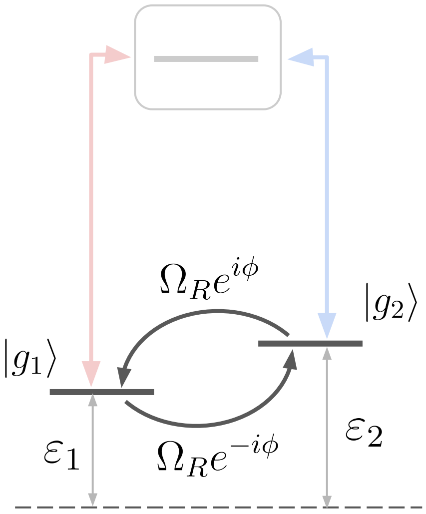


Breaking SU(N): Raman transitions

Three-level atom coupled to 2 detuned **external laser** beams



Effective 2-level dynamics after integrating out fast dynamics

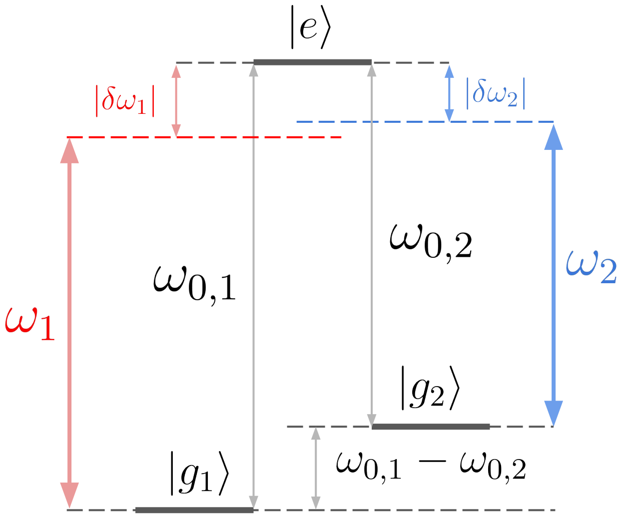


fast oscillation

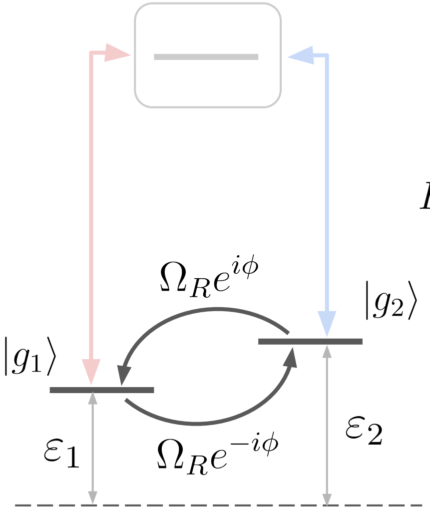
$$|\psi(t)\rangle = \alpha(t)|g_1\rangle + \beta(t)|g_2\rangle + \gamma(t)|e\rangle$$

Breaking SU(N): Raman transitions

Three-level atom coupled to 2 detuned **external laser** beams



Effective 2-level dynamics after integrating out fast dynamics



fast oscillation

$$|\psi(t)\rangle = \alpha(t)|g_1\rangle + \beta(t)|g_2\rangle + \gamma(t)|e\rangle$$

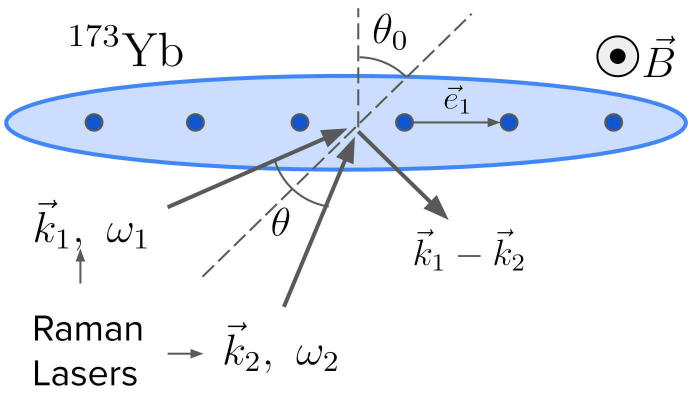
$$H_{\text{eff}} = \begin{pmatrix} \varepsilon_1 & \hbar\Omega_R e^{i\phi} e^{i(\vec{k}_2 - \vec{k}_1)\cdot\vec{r}} \\ \hbar\Omega_R e^{-i\phi} e^{i(\vec{k}_1 - \vec{k}_2)\cdot\vec{r}} & \varepsilon_2 \end{pmatrix}$$

effective Rabi coupling with space-dependent phase

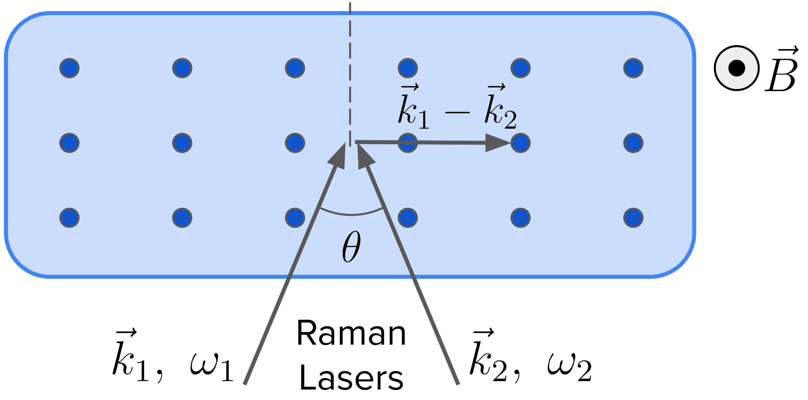
AC light shift

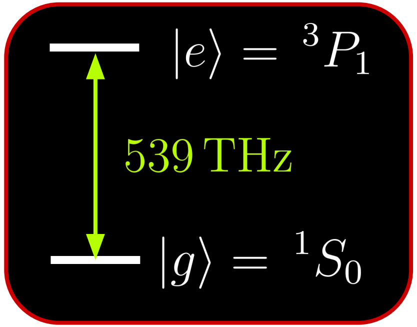
Synthetic dimension with Yb

d = 1



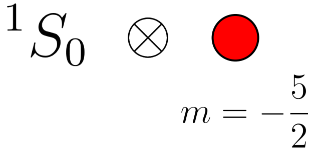
d = 2



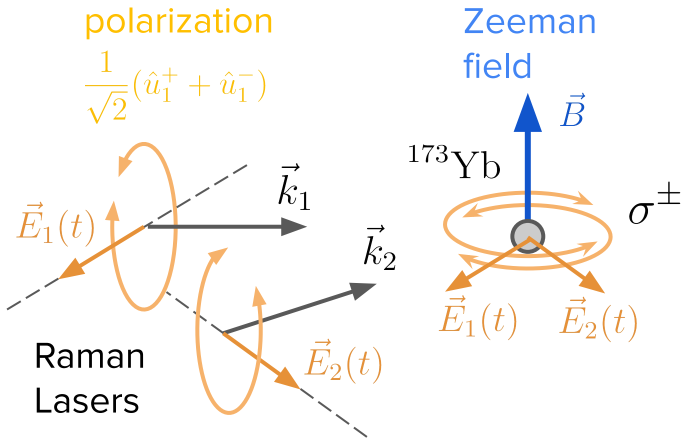


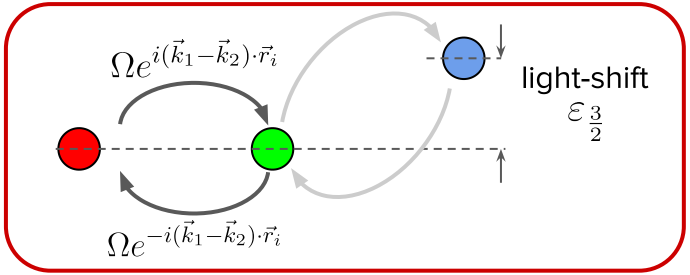
Strong hyperfine coupling (J=1). Breaks SU(*N*)

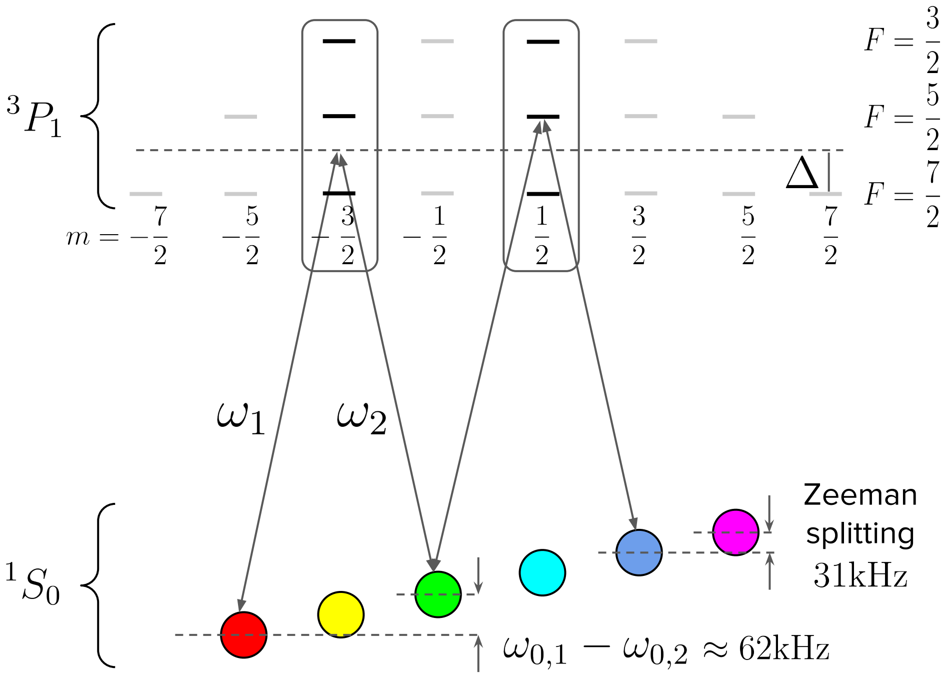
Initial state



Synthetic dimension with Yb



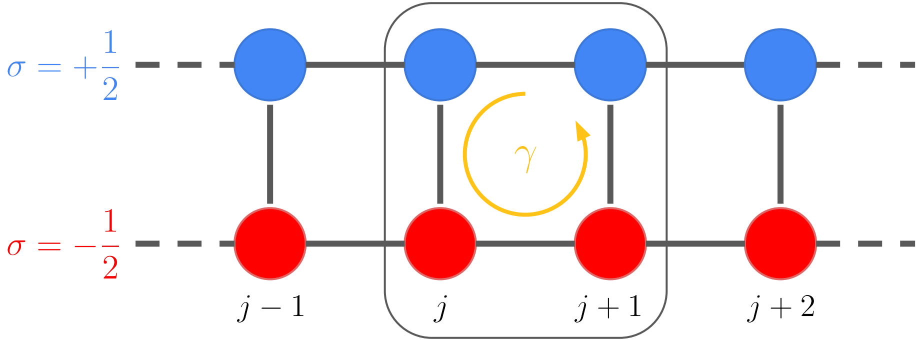




M. Mancini, et al. "Observation of chiral edge states with neutral fermions in synthetic Hall ribbons." *Science* 349.6255 (2015): 1510-1513.

Synthetic dimension with Yb

(1+1)-dimensions



Two-legs ladder or "heterostructure" with synthetic gauge flux

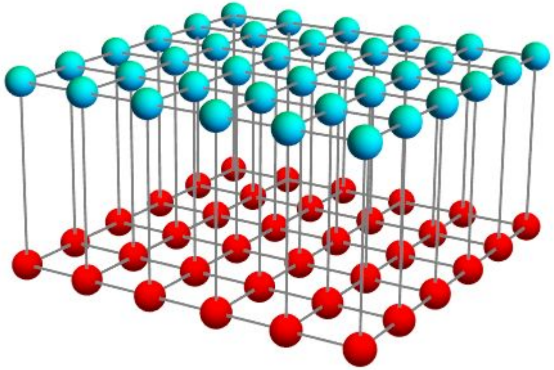
$$H = -t \sum_{\langle ij \rangle, \sigma} (c^{\dagger}_{i\sigma} c_{j\sigma} + \text{h.c.})$$

$$+\Omega \sum_{j,\sigma} e^{i\gamma j} (c^{\dagger}_{j\sigma} c_{j,\sigma+1} + \text{h.c.})$$

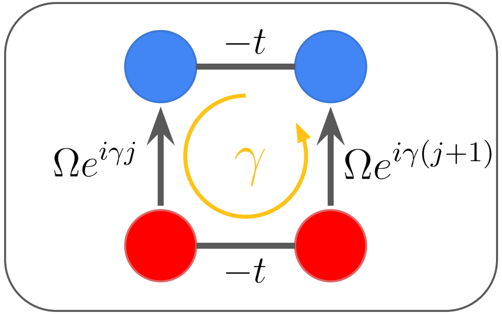
$$+\frac{U}{2} \sum_{j} n_j (n_j - 1) - \mu \sum_{j} n_j$$

Filling factor: 1 particle per site

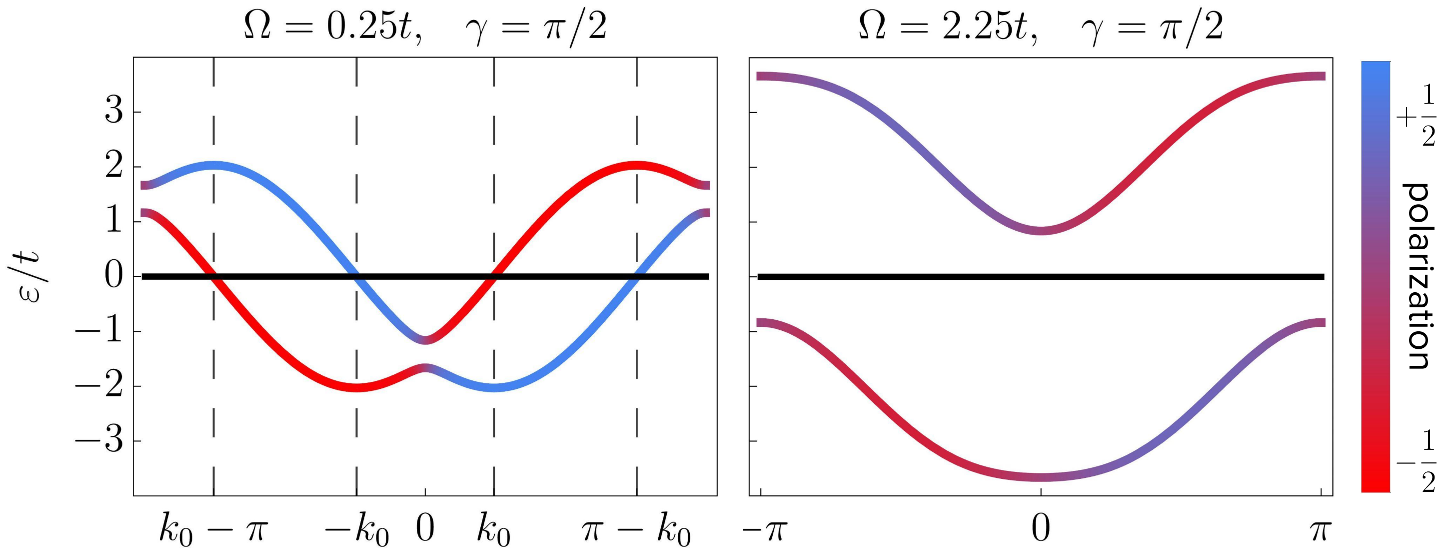
(2+1)-dimensions

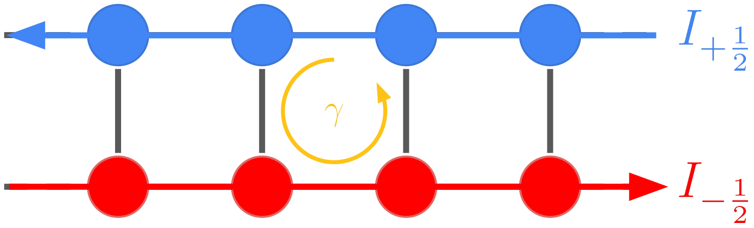


plaquette



Synthetic dimension with Yb

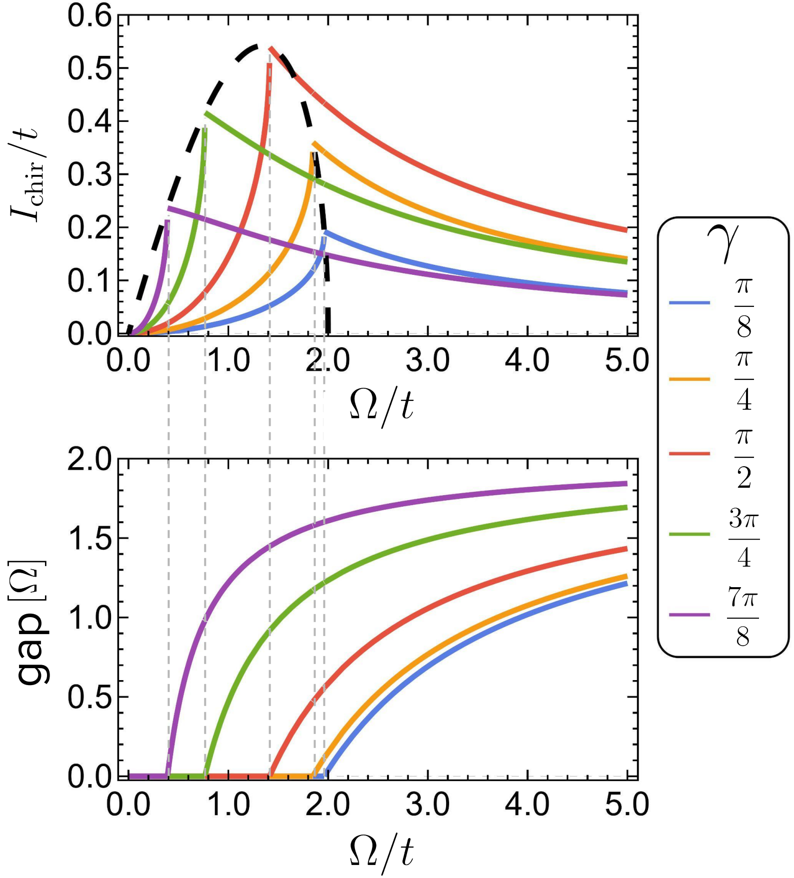


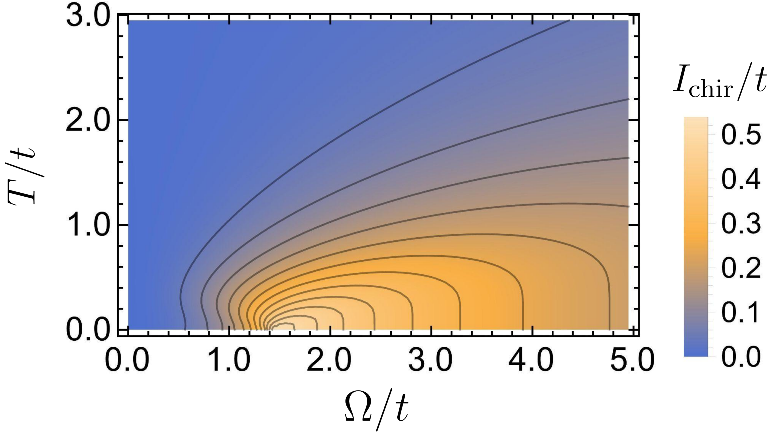


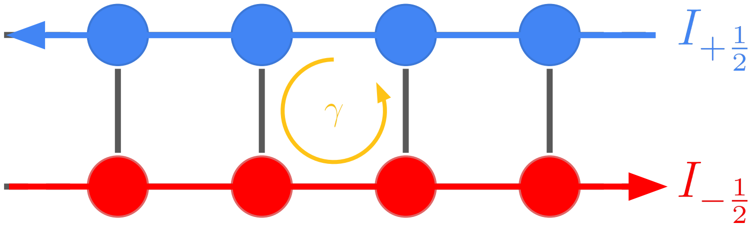
$$I_{a,\sigma} = \frac{2t}{N_{\text{sites}}} \sum_{\vec{k} \in \text{BZ}} \sin\left(\vec{k} \cdot \vec{e}_a + \sigma \gamma_a\right) n_{\vec{k}\sigma}$$

$$I_{\text{chir}} = I_{\uparrow} - I_{\downarrow}$$

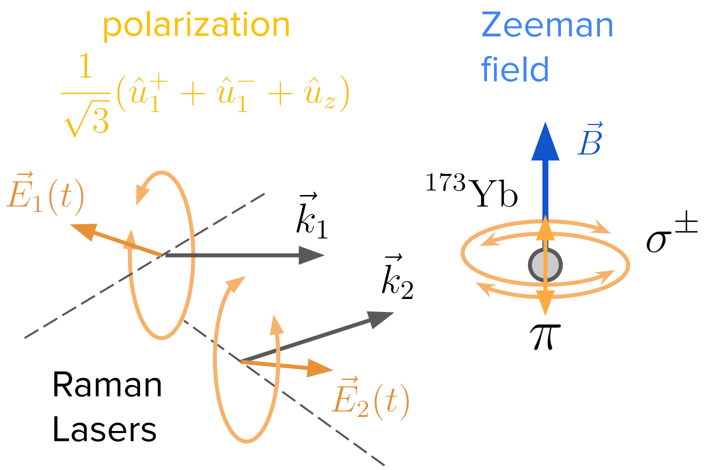
Synthetic dimension with Yb

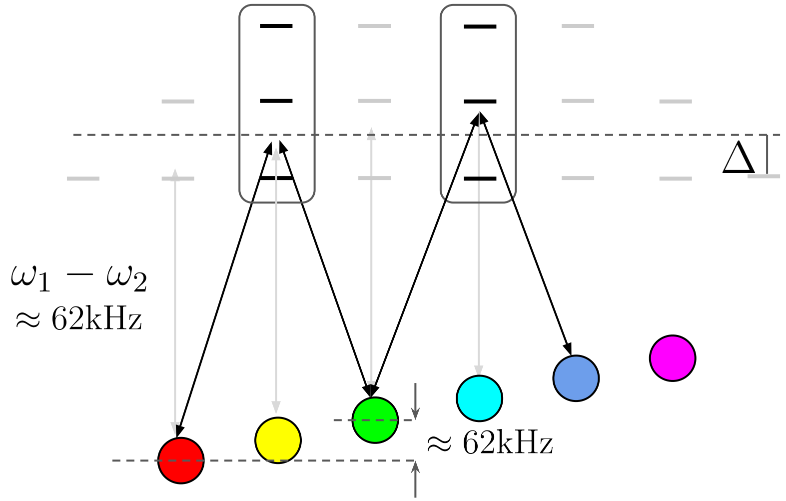


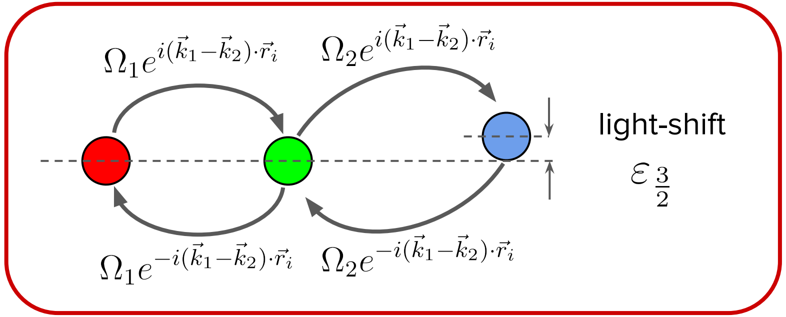


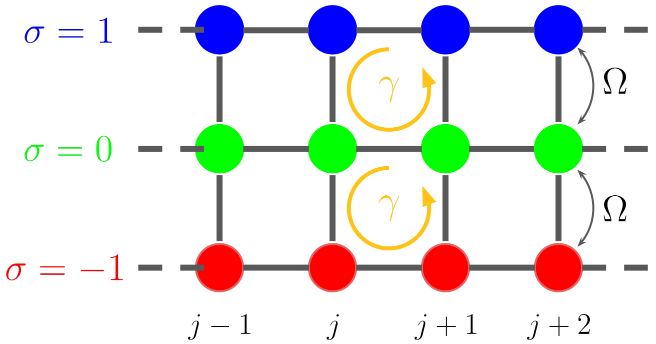


Synthetic dimension with Yb

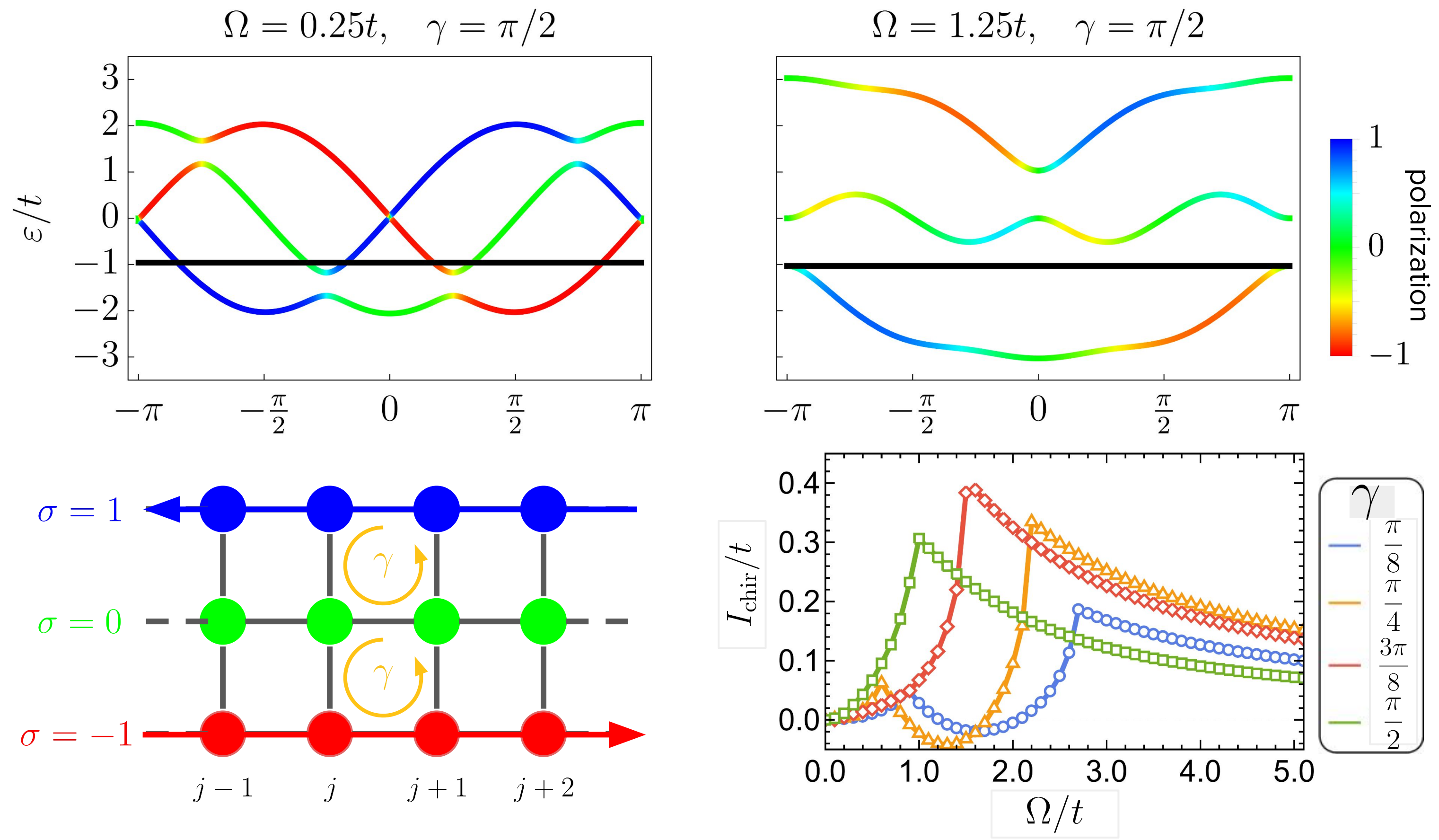








Synthetic dimension with Yb



3. Methods and results: Dynamical Mean Field Theory (DMFT) and interaction-enhanced chiral currents

Dynamical mean field theory (DMFT)

Hartree-Fock MFT: **local** and **static** self-energy:

$$\hat{\Sigma}(\vec{k}, i\omega_n) \approx \hat{\Sigma}$$

DMFT: **local** self-energy with **full dynamical** dependence:

$$\hat{\Sigma}(\vec{k}, i\omega_n) \approx \hat{\Sigma}(i\omega_n)$$

Dynamical mean field theory (DMFT)

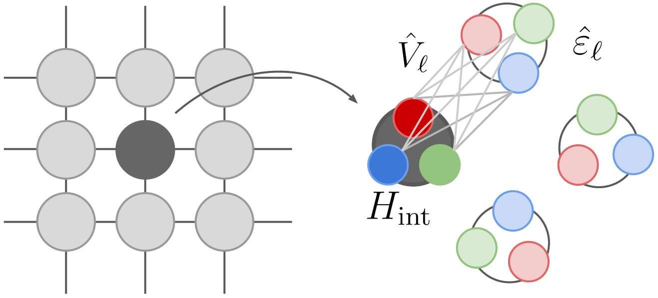
Hartree-Fock MFT: **local** and **static** self-energy:

$$\hat{\Sigma}(\vec{k}, i\omega_n) \approx \hat{\Sigma}$$

DMFT: **local** self-energy with **full dynamical** dependence:

$$\hat{\Sigma}(\vec{k}, i\omega_n) \approx \hat{\Sigma}(i\omega_n)$$

DMFT maps the lattice problem into a self-consistent **Anderson impurity model**



$$H_{\text{AIM}} = \underbrace{\sum_{\ell=1}^{N_{\text{bath}}} \Psi_\ell^\dagger \hat{\varepsilon}_\ell \Psi_\ell}_{\text{local processes of the bath sites}} + \underbrace{\sum_{\ell=1}^{N_{\text{bath}}} (\Psi_\ell^\dagger \hat{V}_\ell \Psi_0 + \text{h.c.})}_{\text{bath – impurity hybridization}} + \underbrace{\Psi_0^\dagger \hat{\varepsilon}_0 \Psi_0 + H_{\text{int}}}_{\text{local processes of the impurity}}$$

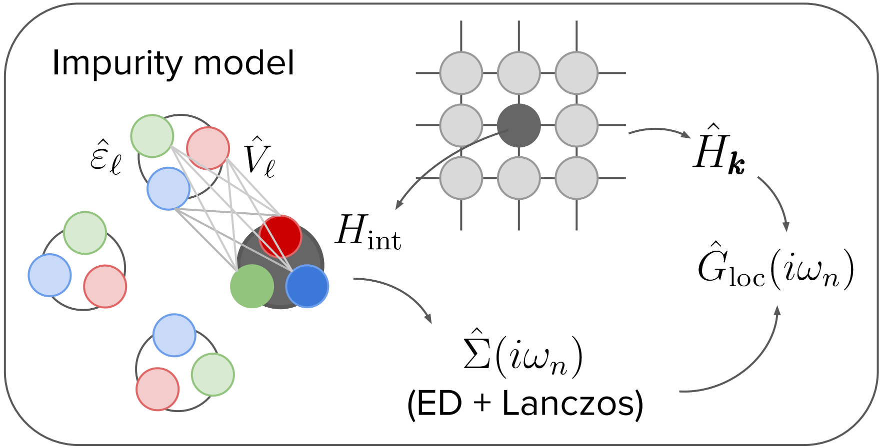
$$\Psi_\ell = \begin{pmatrix} c_{\ell\uparrow} \\ c_{\ell\downarrow} \end{pmatrix}$$

Dynamical mean field theory (DMFT)

Initial guess for $\{\hat{\varepsilon}_\ell,\, \hat{V}_\ell\}$

If converged, exit

Impurity solver

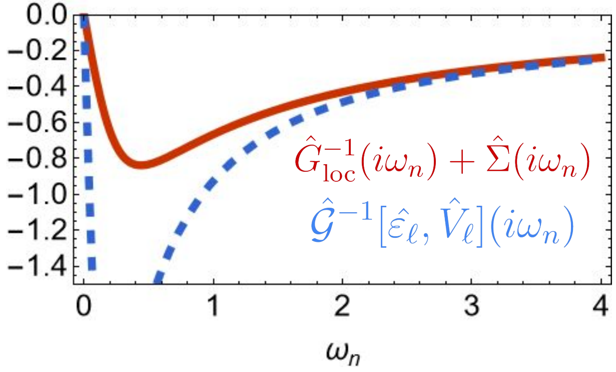


Self Consistency

$$\hat{\mathcal{G}}^{-1}(i\omega_n) = \hat{G}^{-1}_{\mathrm{loc}}(i\omega_n) + \hat{\Sigma}(i\omega_n)$$

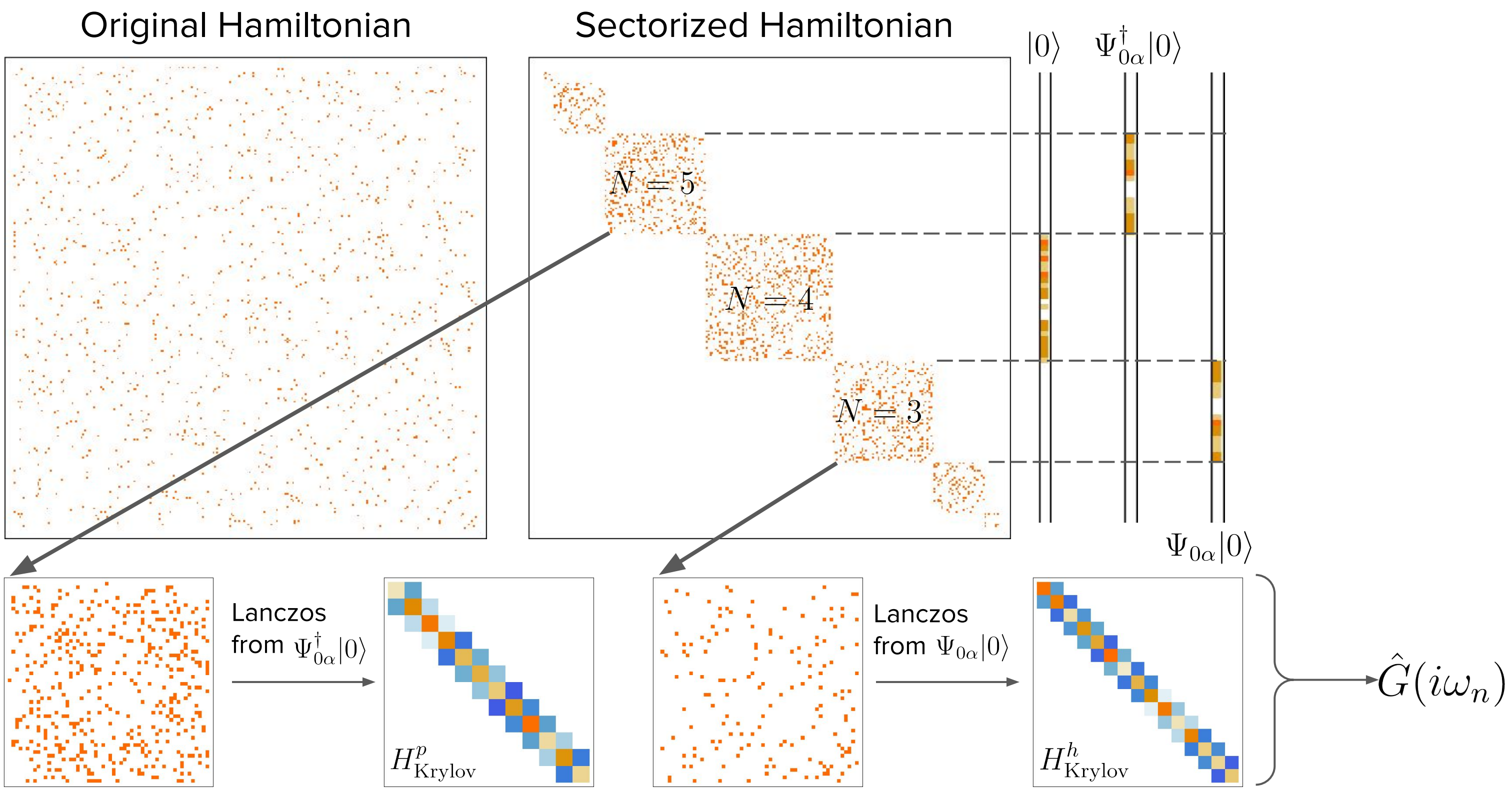
Minimize their distance and update $\{\hat{\varepsilon}_\ell,\, \hat{V}_\ell\}$

Iteration: 1

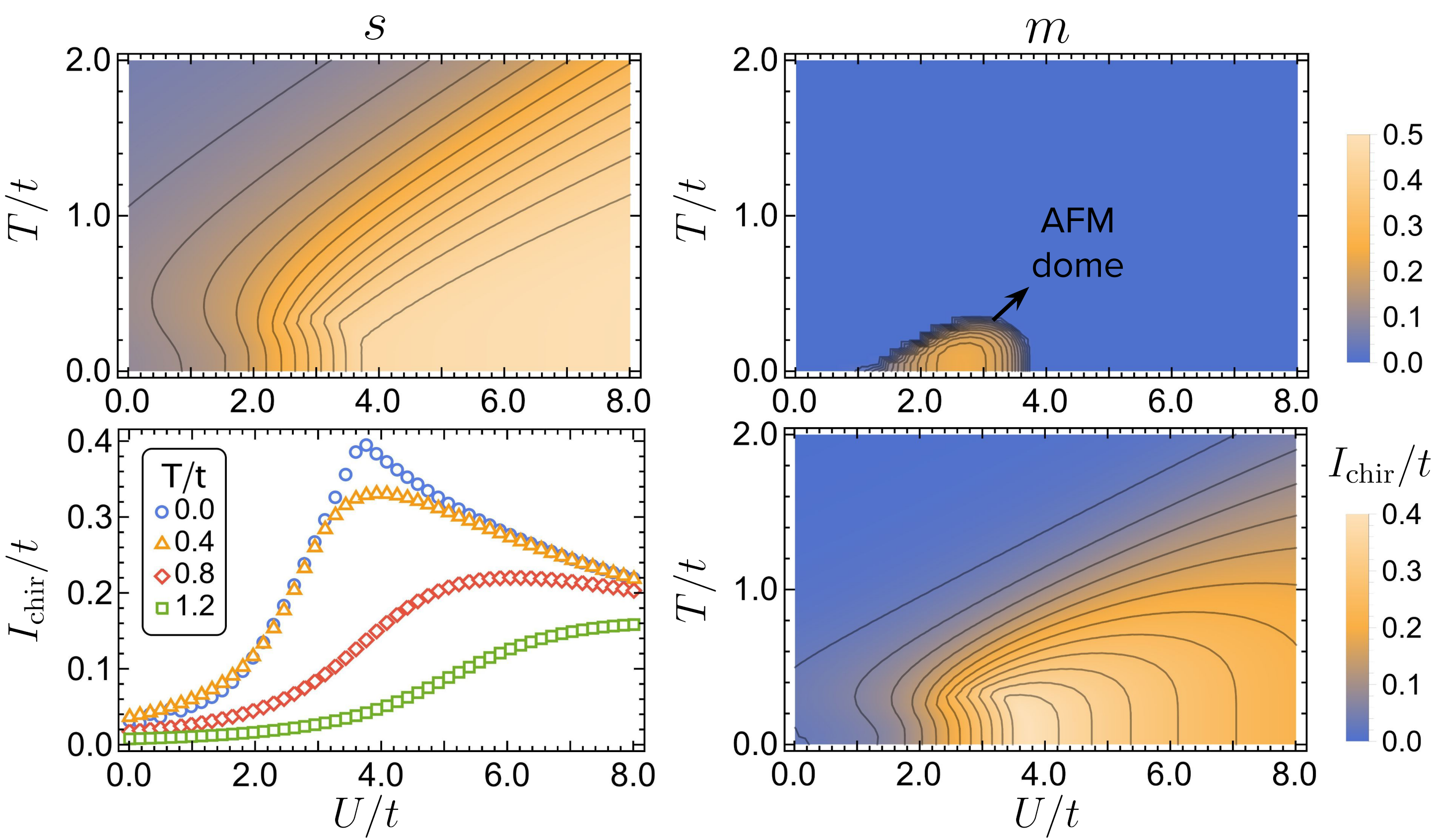


$$\hat{G}_{\mathrm{loc}}(i\omega_n) = \int d\varepsilon D(\varepsilon) \left[i\omega_n + \mu - \varepsilon - \hat{\Sigma}(i\omega_n) \right]^{-1}$$

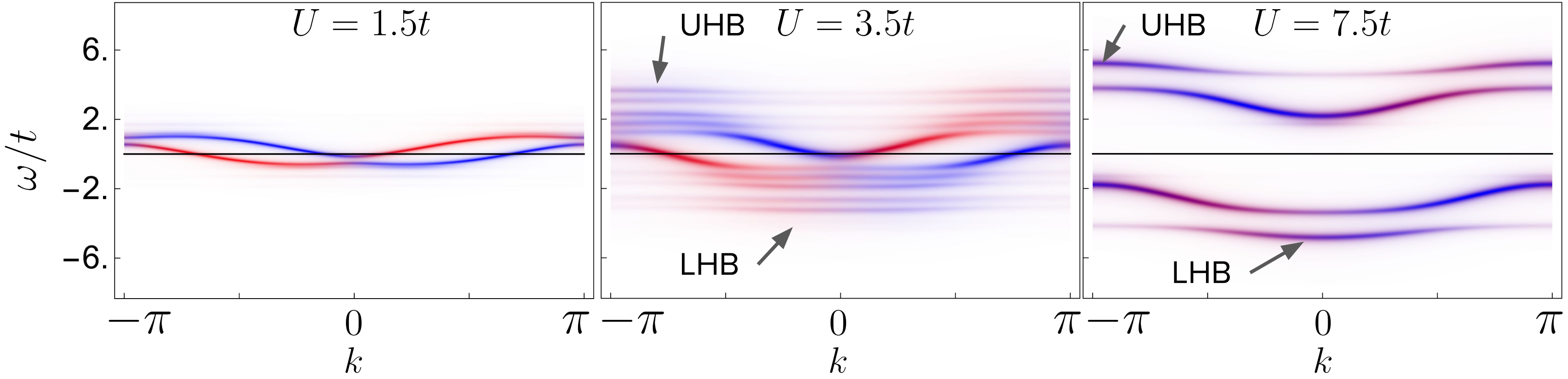
Dynamical mean field theory (DMFT)

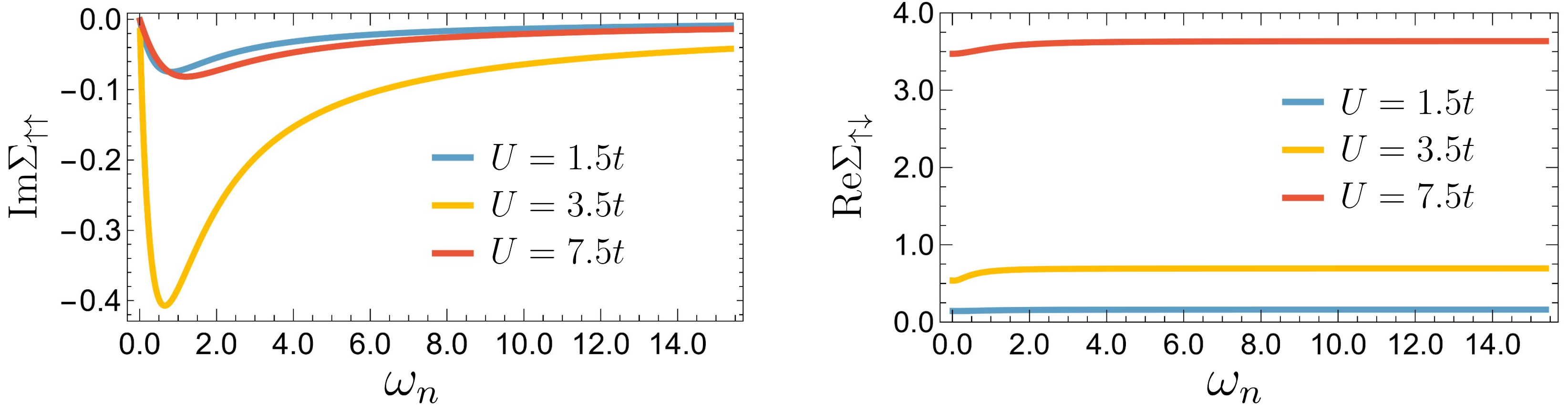


Chiral current and the interaction

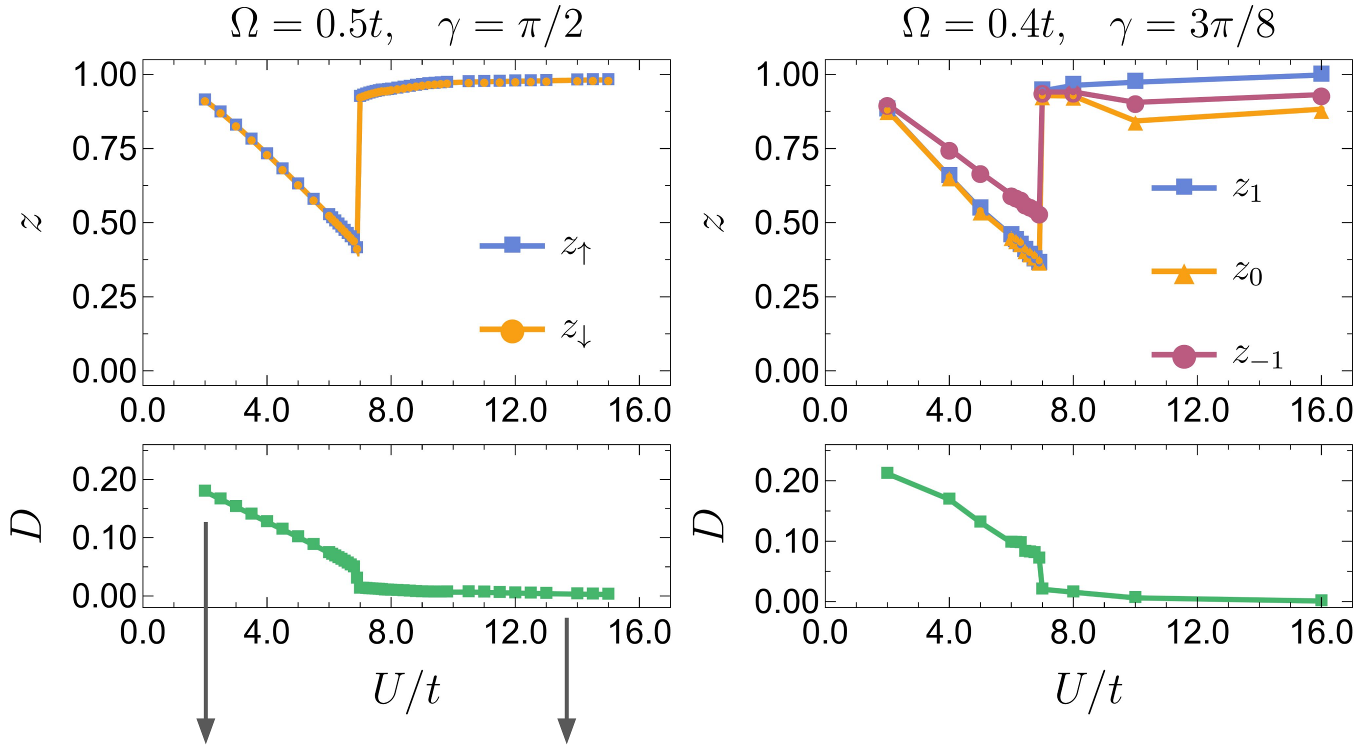


Chiral current and the interaction





Chiral current and the interaction



“metal”

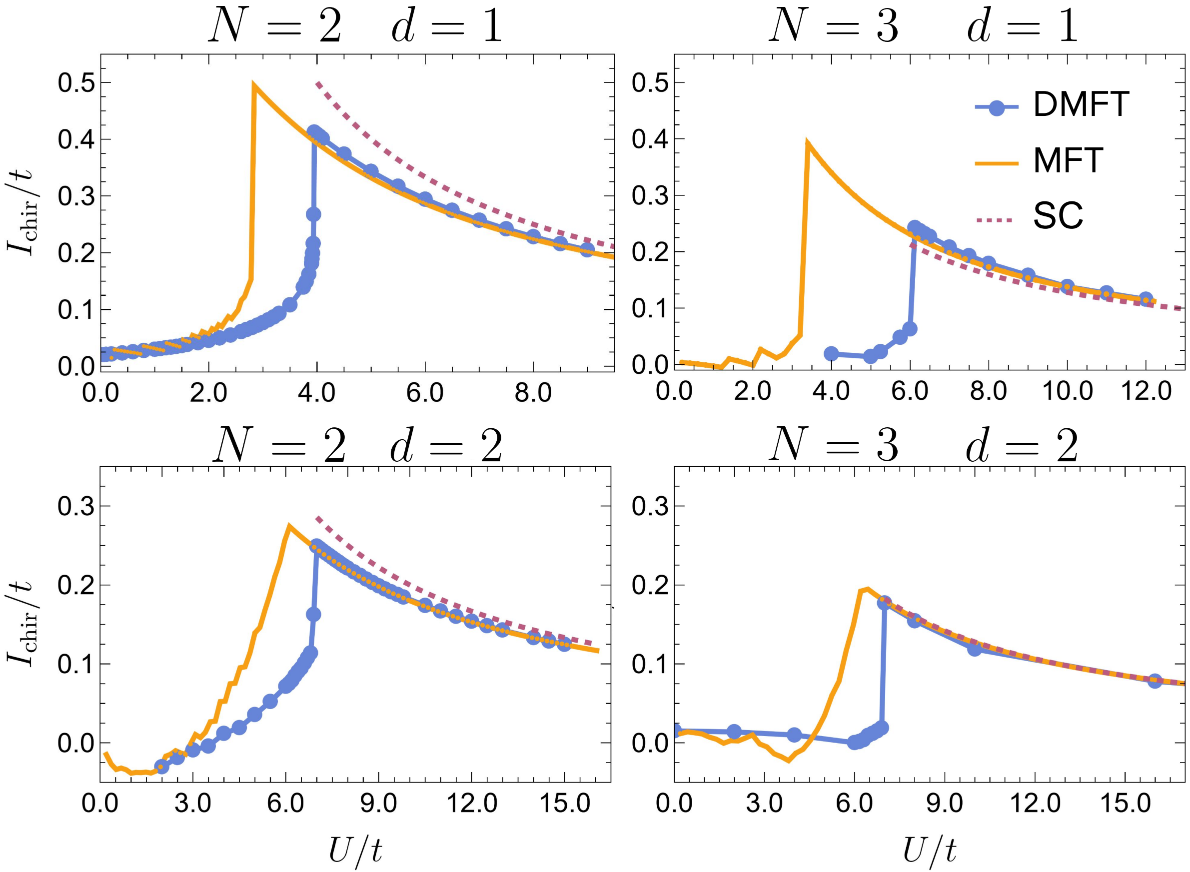
“insulator”

quasiparticle weight

$$z_\sigma = \left(1 - \left.\frac{\partial \Sigma_{\sigma\sigma}(i\omega_n)}{\partial i\omega_n}\right|_{i\omega_n \to 0}\right)^{-1}$$

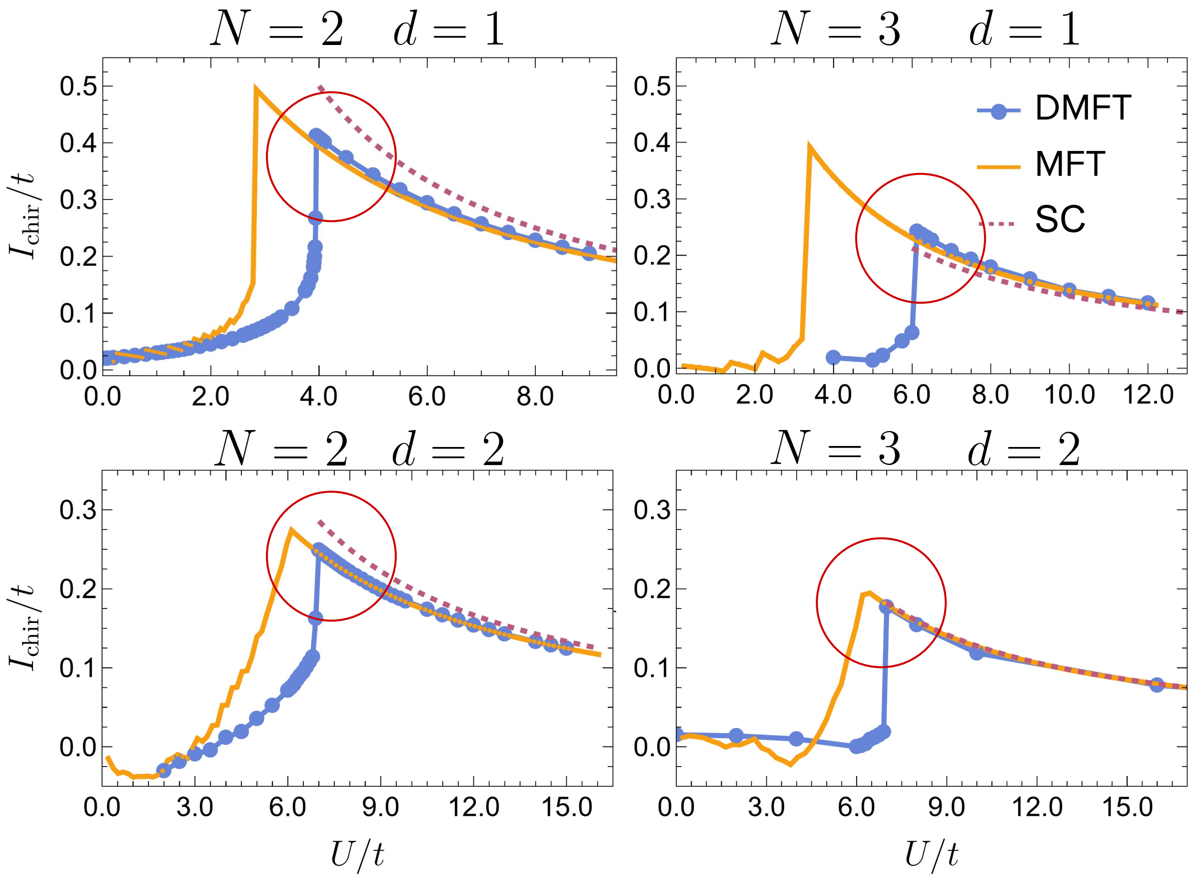
double occupancy

Chiral current and the interaction



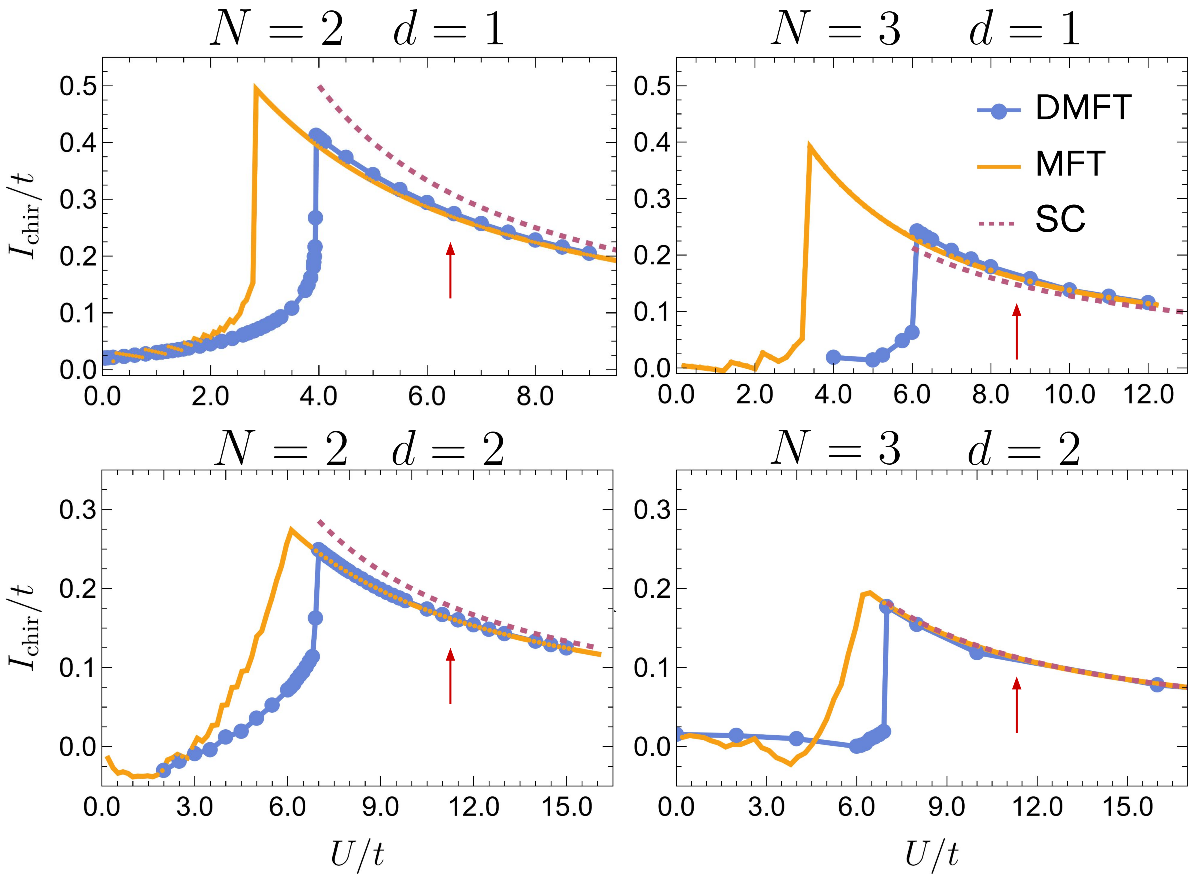
- Similar behavior for different N and different d
- Peak at the phase transition
- Hyperbolic tail in the insulator
- Qualitative agreement with static mean field; quantitative differences at the transition

Chiral current and the interaction



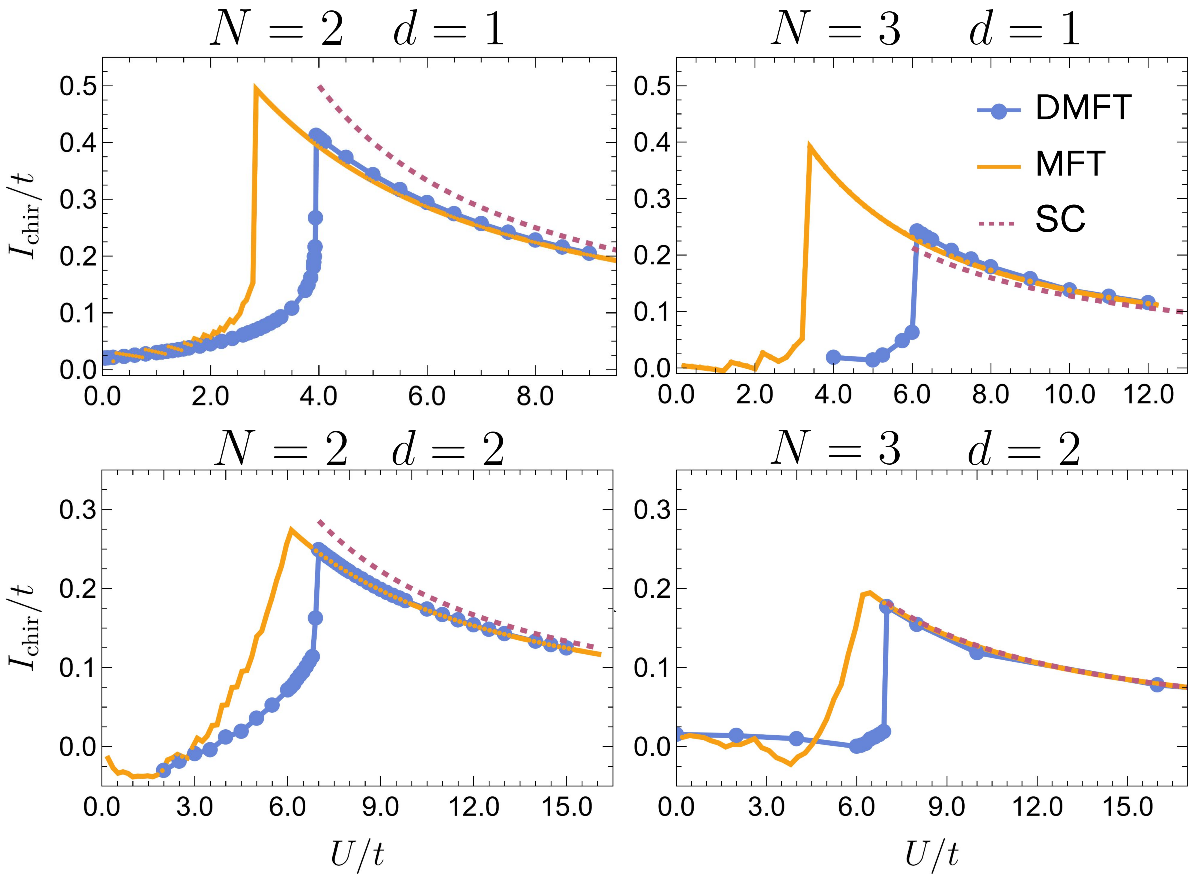
- Similar behavior for different N and different d
- Peak at the phase transition
- Hyperbolic tail in the insulator
- Qualitative agreement with static mean field; quantitative differences at the transition

Chiral current and the interaction



- Similar behavior for different N and different d
- Peak at the phase transition
- Hyperbolic tail in the insulator
- Qualitative agreement with static mean field; quantitative differences at the transition

Chiral current and the interaction



- Similar behavior for different N and different d
- Peak at the phase transition
- Hyperbolic tail in the insulator
- Qualitative agreement with static mean field; quantitative differences at the transition

Effective spin model

Strong Coupling limit

Hamiltonian

$$H_{\text{eff}} = \frac{4t^2}{U} \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j - \sum_i \vec{B}_i \cdot \vec{S}_i$$

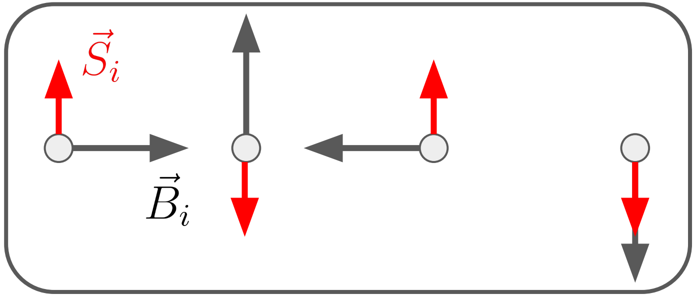
$$\vec{B}_j = 2\Omega(-\cos(\gamma j), \sin(\gamma j), 0)$$

Chiral current

$$I_{\text{chir}} = -\frac{8t^2}{L^d U} \underbrace{\sum_{\langle ij \rangle} \left(\vec{S}_i \times \vec{S}_j \right)_z}$$

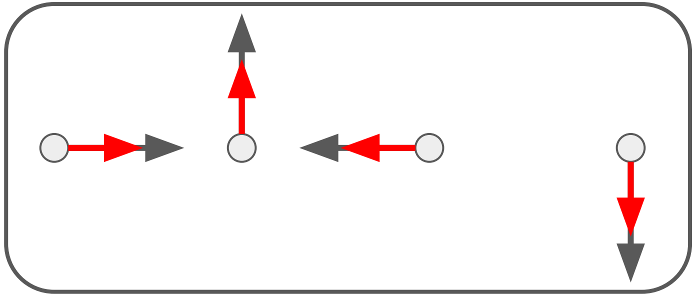
fixed w.r.t. U

$\Omega \ll \frac{4t^2}{U}$



xy - plane

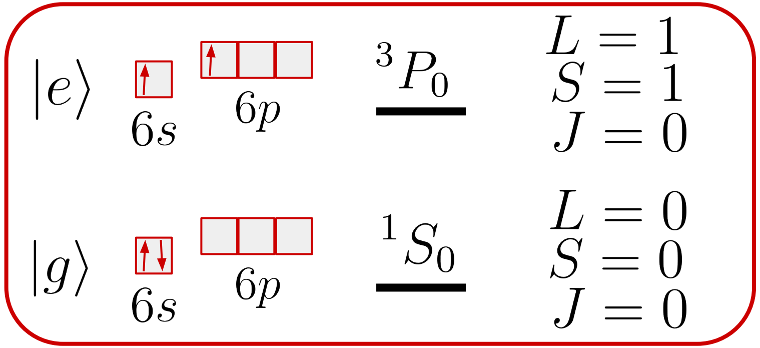
$\Omega \gg \frac{4t^2}{U}$



xy - plane

Outlook

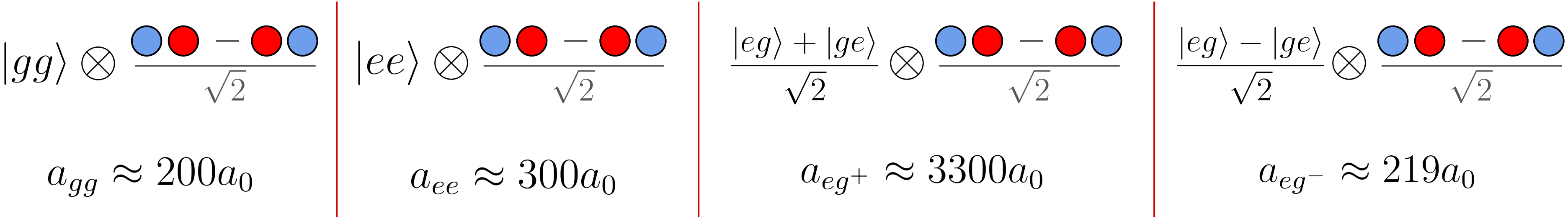
Electronic states (Yb)



Electronic state = "orbital index"

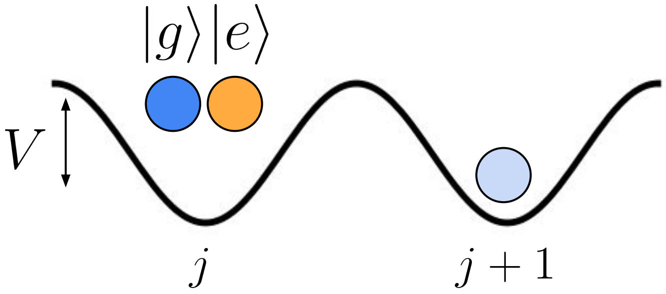
Nuclear state = "flavor index"

Interaction channels



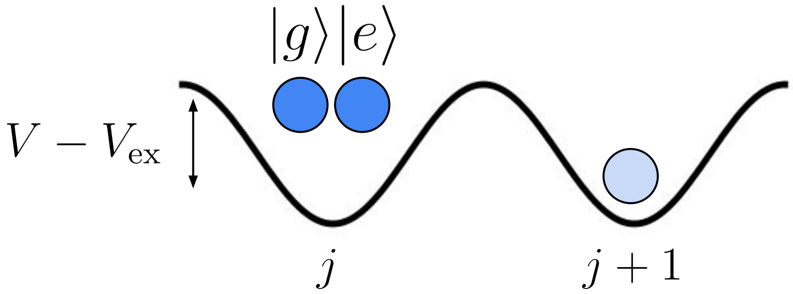
Outlook

$$H = -\sum_{\langle ij\rangle,a,\sigma} t_{ij}^{aa}(c^\dagger_{ia\sigma}c_{ja\sigma} + \mathrm{h.c.}) - \mu \sum_{j,a,\sigma} n_{ja\sigma} + \sum_{j,a} \frac{U_a}{2} n_{ja}(n_{ja}-1)$$
$$+ V \sum_{j,a\neq b,\sigma>\rho} n_{ja\sigma}n_{jb\rho} + (V - V_{\mathrm{ex}}) \sum_{j,a<b,\sigma} n_{ja\sigma}n_{jb\sigma} - V_{\mathrm{ex}} \sum_{j,a\neq b,\sigma>\rho} c^\dagger_{ja\sigma}c_{ja\rho}c^\dagger_{jb\rho}c_{jb\sigma}$$



Outlook

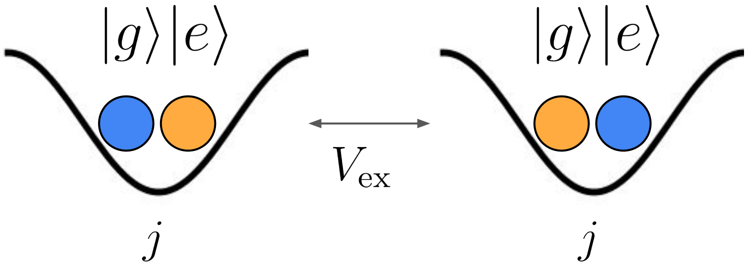
$$H = -\sum_{\langle ij\rangle,a,\sigma} t_{ij}^{aa}(c_{ia\sigma}^\dagger c_{ja\sigma} + \text{h.c.}) - \mu \sum_{j,a,\sigma} n_{ja\sigma} + \sum_{j,a} \frac{U_a}{2} n_{ja}(n_{ja}-1)$$
$$+ V \sum_{j,a\neq b,\sigma>\rho} n_{ja\sigma} n_{jb\rho} + (V - V_{\text{ex}}) \sum_{j,a<b,\sigma} n_{ja\sigma} n_{jb\sigma} - V_{\text{ex}} \sum_{j,a\neq b,\sigma>\rho} c_{ja\sigma}^\dagger c_{ja\rho} c_{jb\rho}^\dagger c_{jb\sigma}$$



Outlook

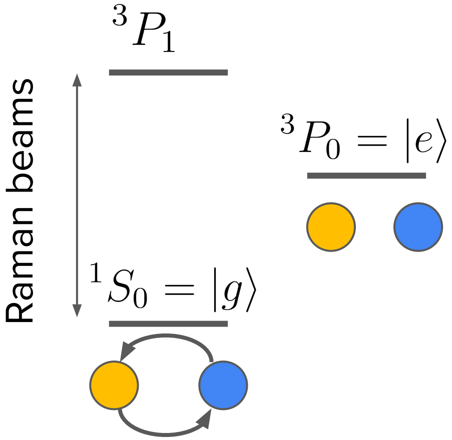
$$H = -\sum_{\langle ij\rangle,a,\sigma} t_{ij}^{aa}(c_{ia\sigma}^{\dagger}c_{ja\sigma} + \text{h.c.}) - \mu\sum_{j,a,\sigma} n_{ja\sigma} + \sum_{j,a}\frac{U_a}{2}n_{ja}(n_{ja}-1)$$

$$+ V\sum_{j,a\neq b,\sigma>\rho} n_{ja\sigma}n_{jb\rho} + (V - V_{\text{ex}})\sum_{j,a<b,\sigma} n_{ja\sigma}n_{jb\sigma} - V_{\text{ex}}\sum_{j,a\neq b,\sigma>\rho} c_{ja\sigma}^{\dagger}c_{ja\rho}c_{jb\rho}^{\dagger}c_{jb\sigma}$$

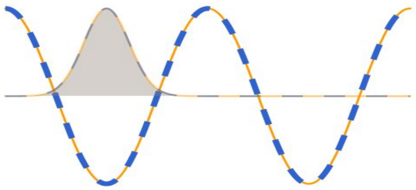


Outlook

$$H = -\sum_{\langle ij\rangle,a,\sigma} t_{ij}^{aa}(c_{ia\sigma}^{\dagger}c_{ja\sigma} + \text{h.c.}) - \mu\sum_{j,a,\sigma} n_{ja\sigma} + \sum_{j,a}\frac{U_a}{2}n_{ja}(n_{ja}-1)$$
$$+ V\sum_{j,a\neq b,\sigma>\rho} n_{ja\sigma}n_{jb\rho} + (V - V_{\text{ex}})\sum_{j,a<b,\sigma} n_{ja\sigma}n_{jb\sigma} - V_{\text{ex}}\sum_{j,a\neq b,\sigma>\rho} c_{ja\sigma}^{\dagger}c_{ja\rho}c_{jb\rho}^{\dagger}c_{jb\sigma}$$



$$\lambda_{\text{magic}} = 759\text{nm} \quad \begin{cases} t_g = t_e \\ U_g = U_e \end{cases}$$



SISSA

Atomtronics 2024

SciPost Phys. 14, 048 (2023)

Thank you!

Collaborators:

Andrea Richaud, Massimo Capone, Leonardo Fallani, Lorenzo Del Re