Atom Trapping in Subwavelength Superoscillatory Optical Tweezers

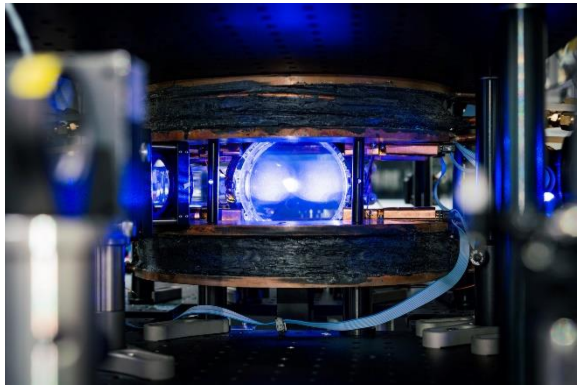
David Wilkowski

Atomtronics, Besnasque, 22 May 2024

Research Activities: An Overview

Quantum Simulation

Non-Abelian geometrical and synthetic gauge fields in ultracold Strontium gas

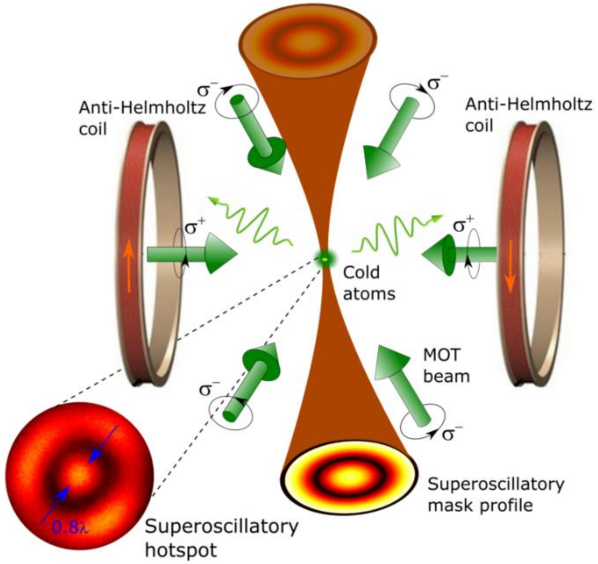


Strontium MOT on 461 nm

Simulation of condensed-matter (spin-orbit coupling) or high energy physics (SU(3))

Atoms with nanophotonics

Use of superoscillatory field for subwavelength optical traps.

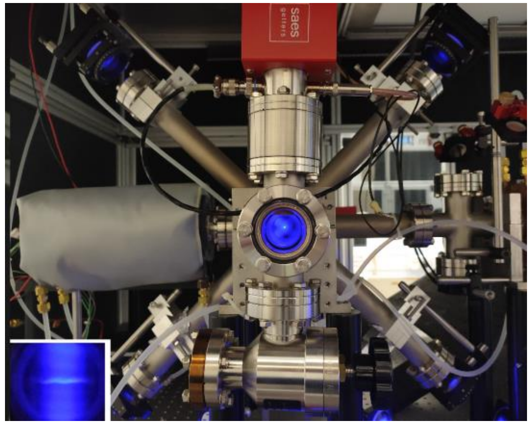


Superoscillatory trap

Controlling and interrogating atoms at the nanometer scale for quantum simulation and computing

Quantum Sensing

Quantum physics coupled to gravitational field



Strontium 2D-MOT on 461 nm

Matter wave interferometry with atomic clock (proper time) and inertial sensing

For more details visit:
https://ultracold.quantumlah.org/
We are hiring PhD and Post-Doc

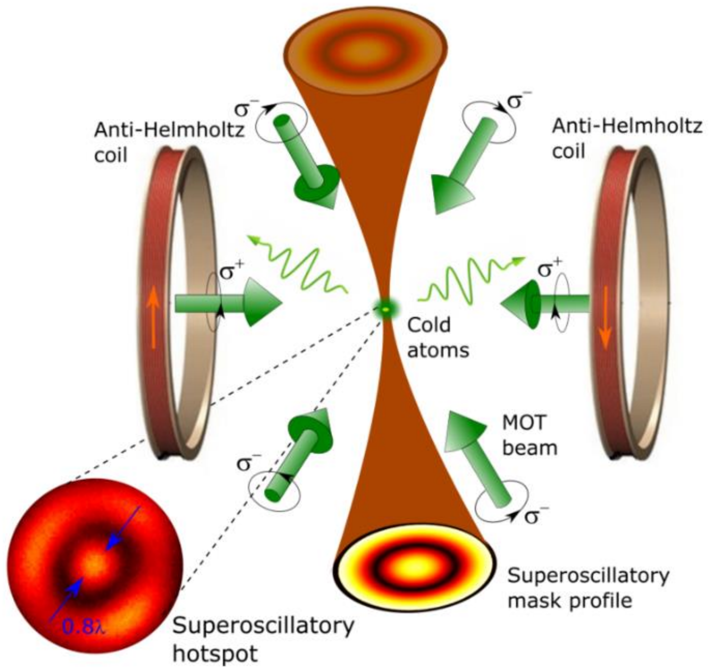
Content

Motivation

Why subwavelength Optical Tweezers?

Trapping Atom in a Superoscillatory Optical Tweezer

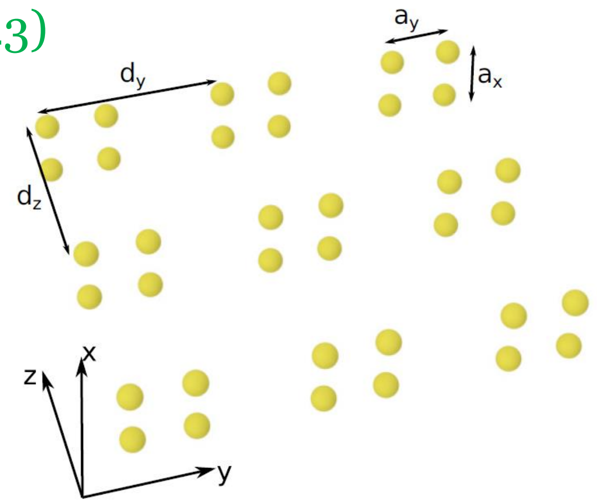
Superoscillation?
Lifetime
Effective Temperature
Trapping Frequency
Toward subwavelength tweezer arrays



H. M. Rivy S. A. Aljunid, E. Lassalle, N. I. Zheludev, D. W., Comm. Phys. **6**, 155 (2023)

Array of Optical Tweezers

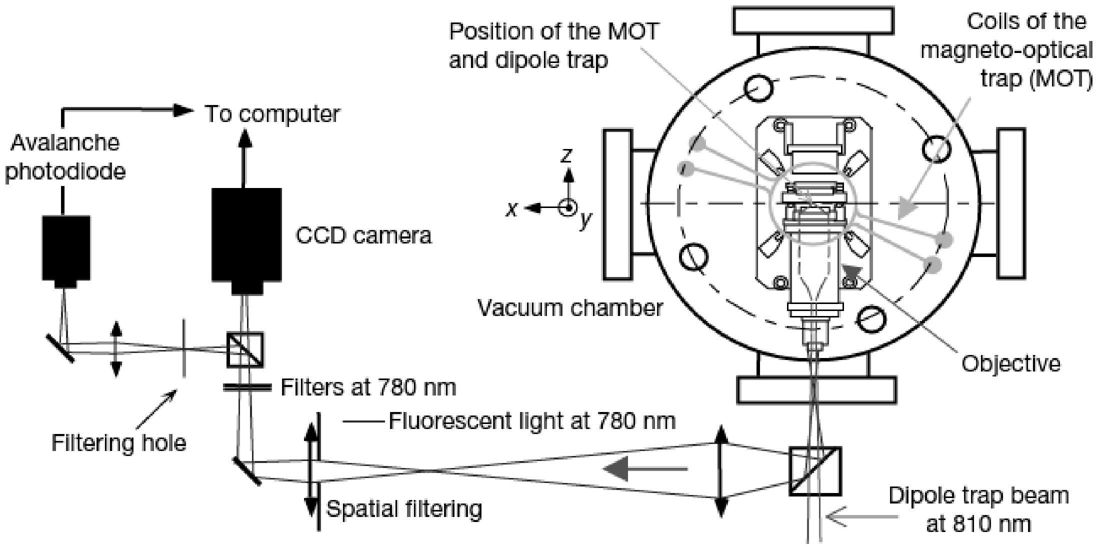
Cooperative metasurfaces



K. E. Ballantine, D. W., and J. Ruostekoski, Phys. Rev. Research **4**, 033242 (2022)

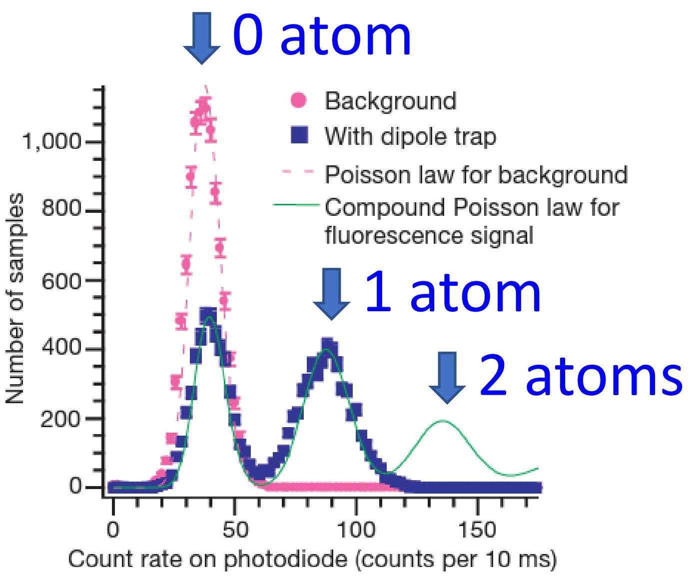
Single Atom in Optical Tweezers

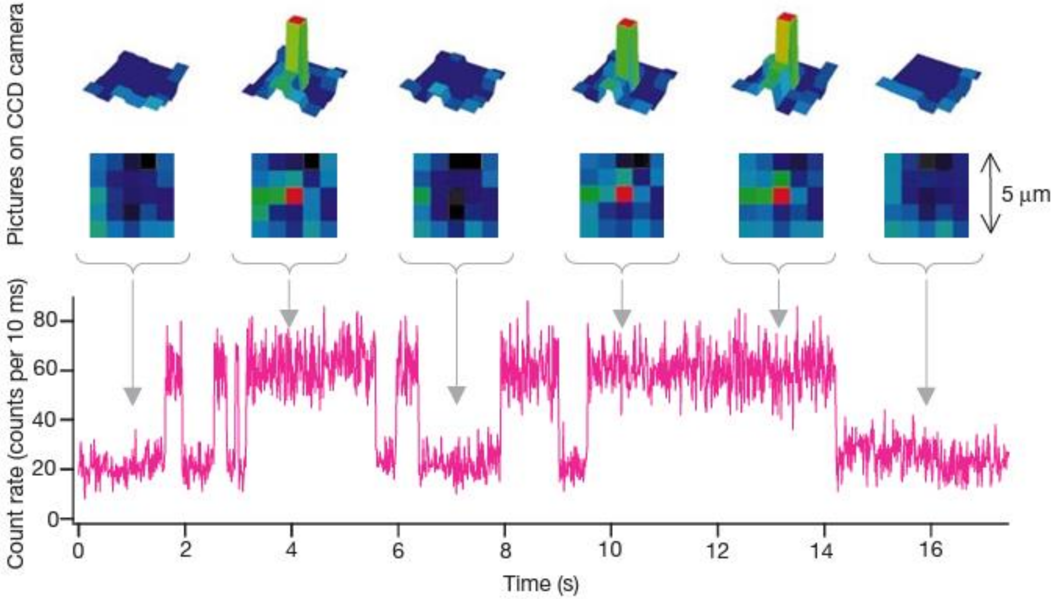
Tightly focused beam



Orsay: N. Schlosser et *al*, Nature **411,** 1024 (2001)

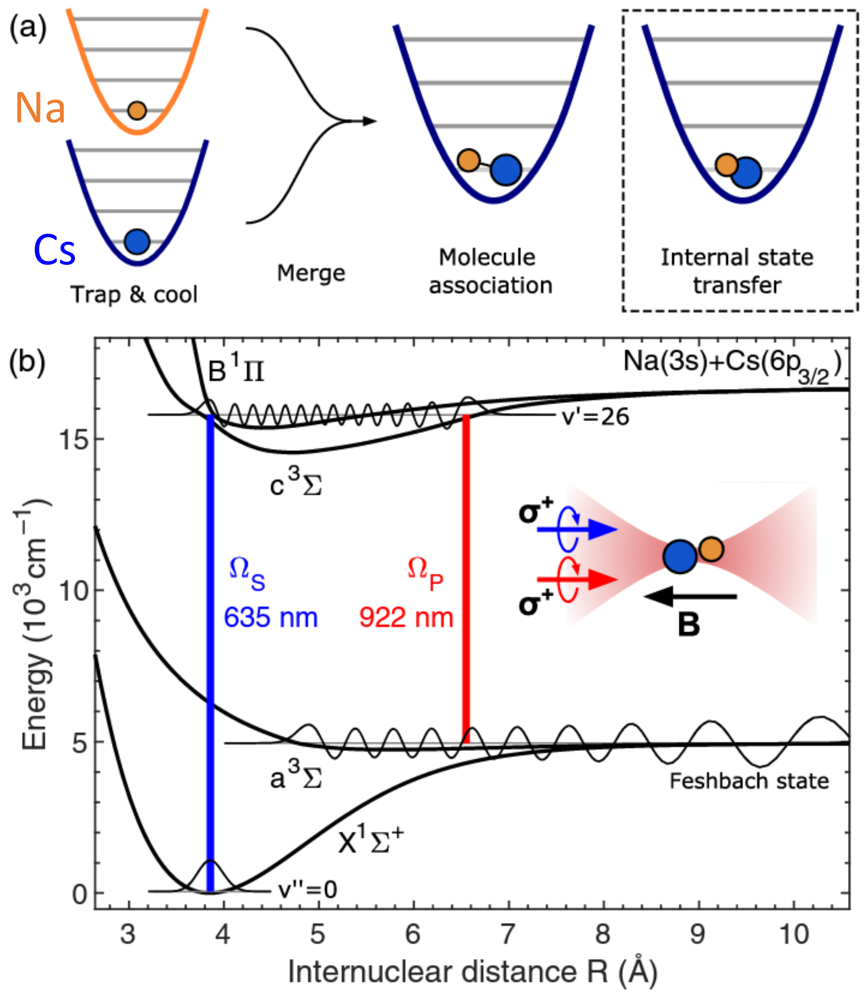
If two atoms are in the trap, there is strong inelastic collisions mediated by the quasi-resonant light





Single Atom in Tweezer: Applications

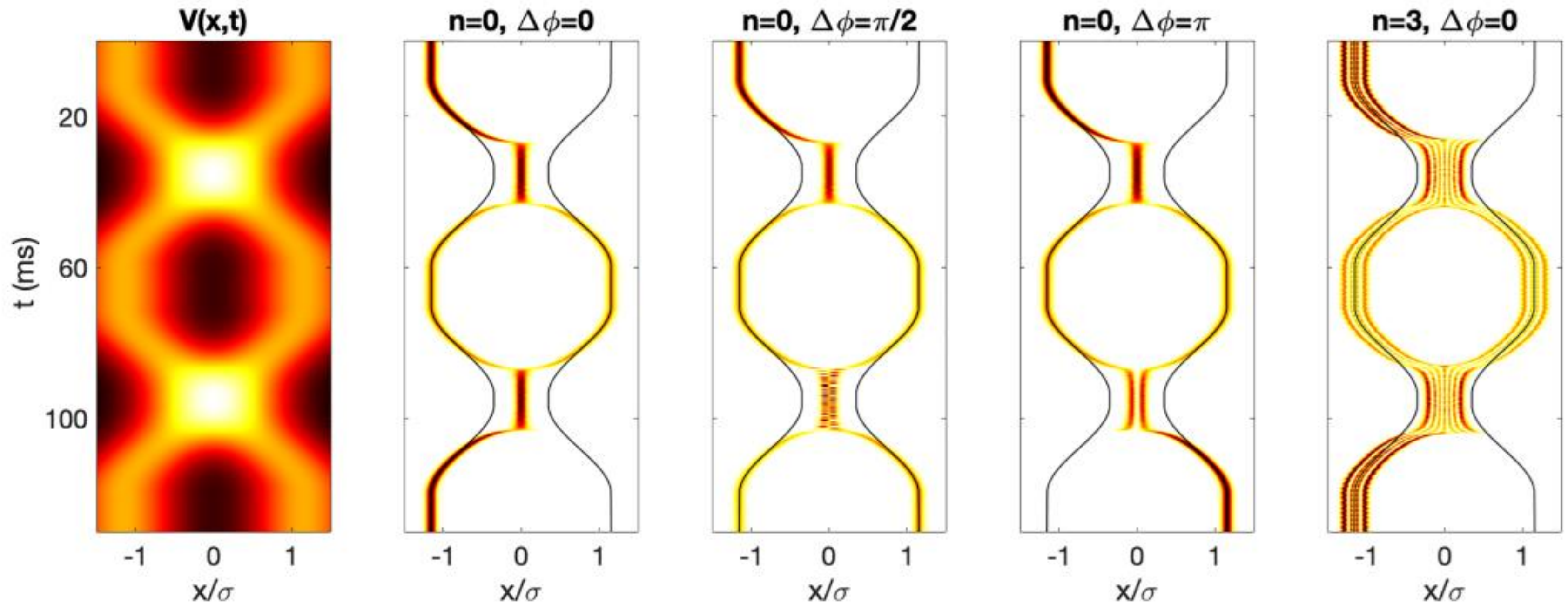
Single molecule Chemistry



Harvard: W. Cairncross et *al*, PRL **126,** 123402 (2021)

Single Atom in Tweezer: Applications

Atom interferometry

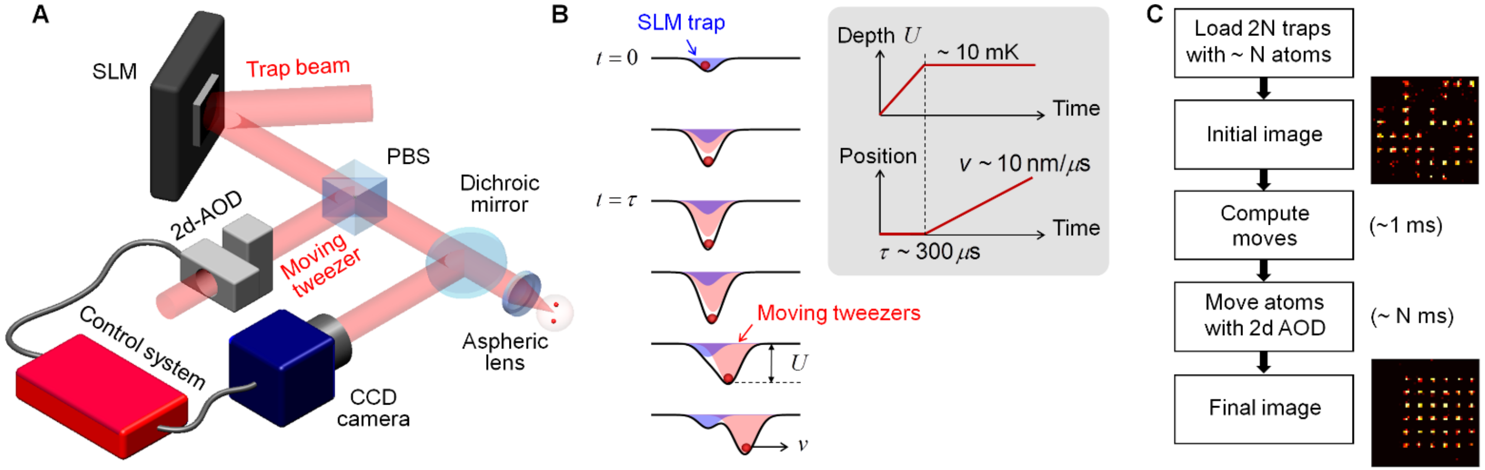


Technion: J. Nemirovsky et *al*, PRR **5**, 043300 (2023)

Y. Sagi's talk next week

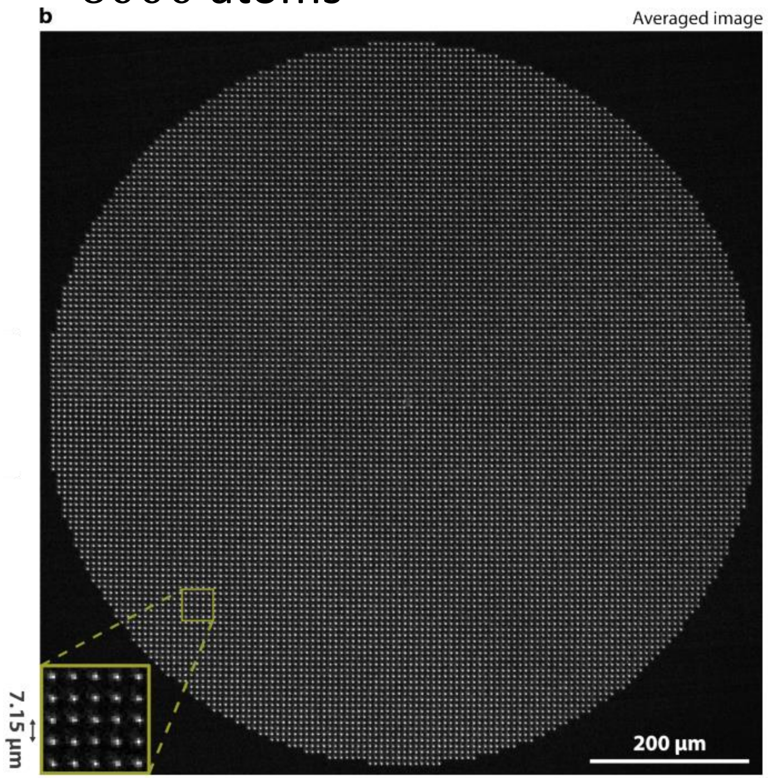
Single Atom in Tweezer: Applications

Array of single atoms

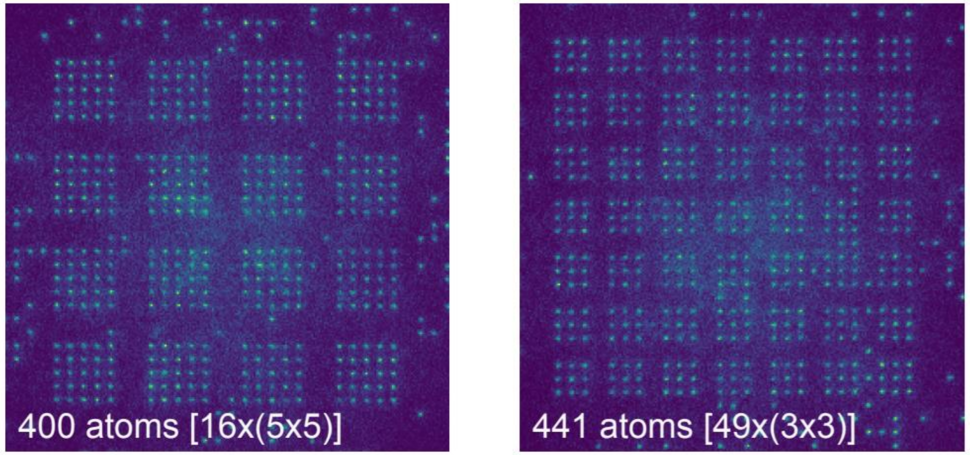


Orsay: D. Barredo et *al*, Science **354,** 1021 (2016)

~6000 atoms



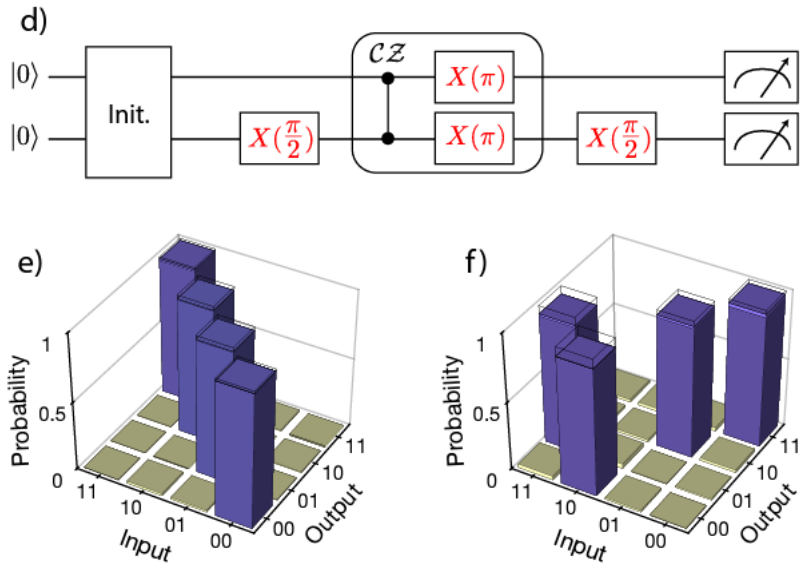
Caltech: J. Manetsch et *al*, arXiv 2403.1202 (2024)

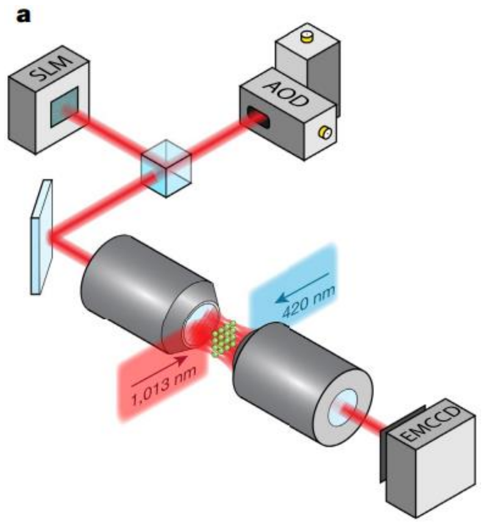


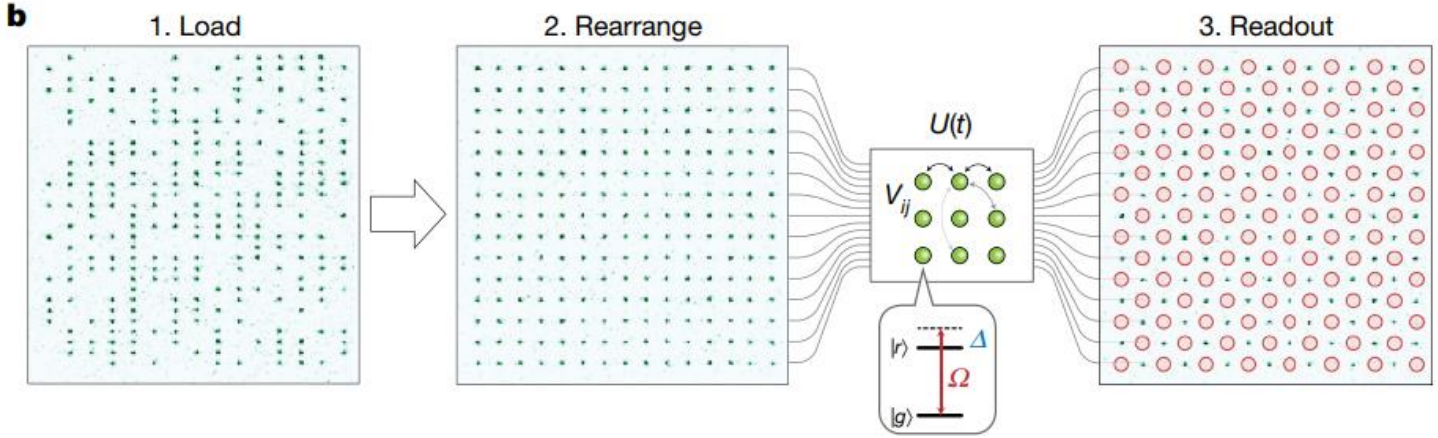
Darmstadt: L. Pause et al, Optica **11,** 222 (2024)

Tweezer arrays: Quantum computer/simulator

Quantum computer/simulator with neutral Rydberg atoms.







CNOT gate, fidelity ≥ 99 %

Harvard:
H. Levine et *al*, PRL **121,** 123603 (2019)

Harvard: S. Ebadi et *al*, Nature **595,** 227 (2021)

Tweezer arrays vs lattice

Tweezer array summary

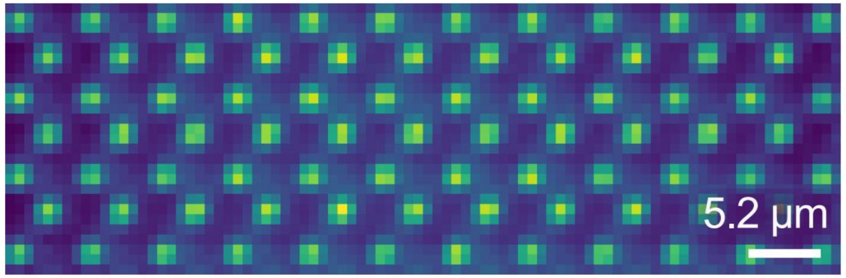
- Single molecule Chemistry
- Sensing
- Quantum simulation and computation

Technical difficulties for tweezer arrays:

- Short period
- Subwavelength spot?

→ Quantum matter simulation
Cooperative effect with metasurface

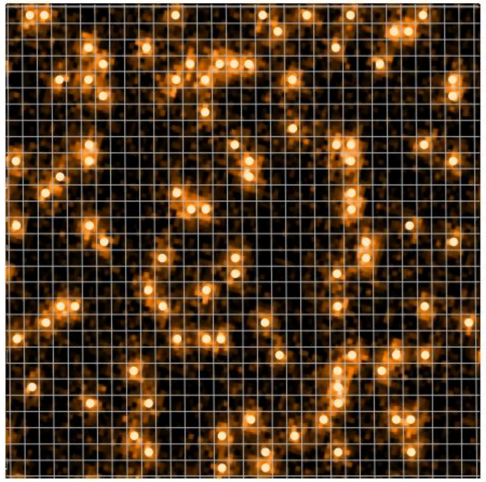
Tweezer array



Birkl's group

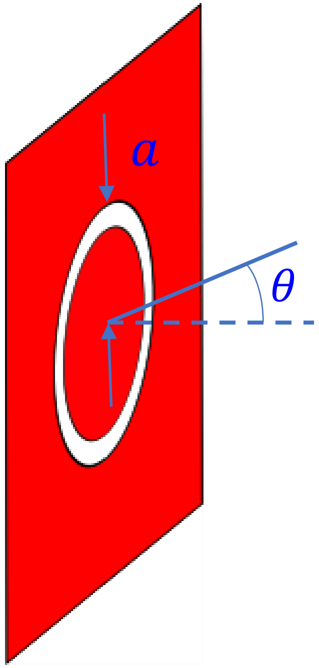
Lattice (Quantum gas microscope)

Distance (541nm)



Zwierlein's group

A Brief History of Superoscillation



Farfield linear optic is limited by diffraction!

- Spherical aperture

$x = 2ka \sin\theta$

a: radius

$$I_a(\theta) \propto a^4 \left[\frac{J_1(x)}{x}\right]^2 \quad \text{(Airy disk)}$$

1^{st} zero: $2ka \sin\theta \approx 1.22\pi$

- Ring aperture

$$I_{da}(\theta) = I_a(\theta) - I_{a-da}(\theta) \propto (ada)^2 {J_0}^2(x)$$

The central lob is below the “standard” diffraction limit

Less optical power in the central lob

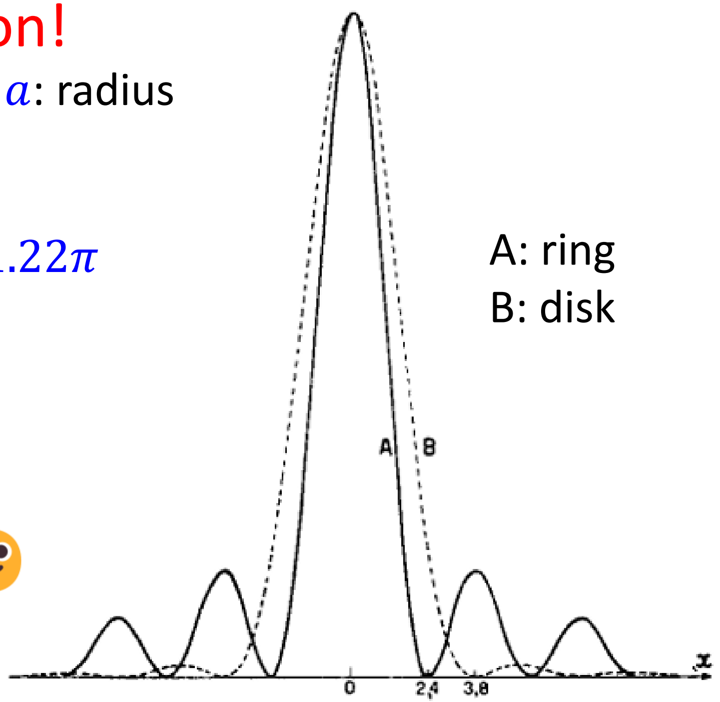


Fig. 3. – Diffraction pattern of a ring-shaped aperture (curve A) and a uniform pupil of equal diameter (curve B).

A: ring
B: disk

G. T. Di Francia, Il Nuovo Cimento **9,** 426 (1952)

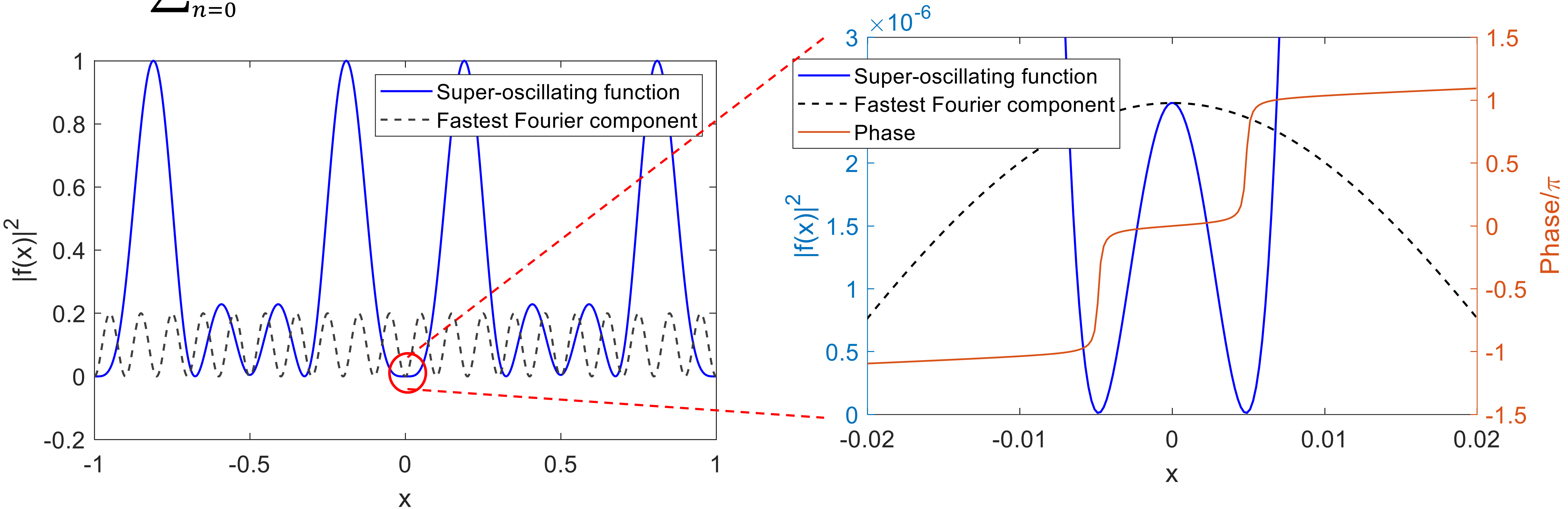
M. Berry:

- A band-limited function could locally oscillate faster than its highest Fourier component → Superoscillation
- No fundamental limitation on the spot size

M. Berry and S. Popescu, JPMG **39,** 6965 (2006)

Superoscillation: 1D periodic signal

$$f(x) = \sum_{n=0}^{5} a_n \exp(i2\pi nx) \qquad \text{with} \qquad a_n = [19.0123\ -2.7348\ -15.7629\ -17.9047\ -1\ 18.4910]$$

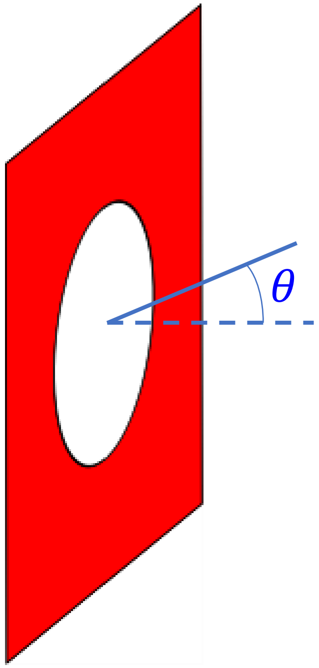


M. Berry:

- A band-limited function could locally oscillate faster than its highest Fourier component → Superoscillation
- No fundamental limitation on the spot size 🙂

M. Berry and S. Popescu, JPMG **39**, 6965 (2006)

Construction of a Superoscillating Spot



Circular Prolate Spheroidal Wave Functions (CPSWFs) are eigenfunctions of the Finite Hankel Transform operator $H_{c,N}$:

$$H_{c,N}(\psi)(x) = \int_0^1 J_N(cxy)\,\psi(y)\,y dy = \gamma\,\psi(x), \qquad x \geq 0$$

where J_N is the N-th order Bessel function of the first kind and c is the bandwidth of the function.

For rotational invariant 2D pattern, we use only $N = 0$ (zero-order Bessel function)

Then, the 2D Fourier transform reduces to the Hankel transform of the radial profile.

Some important properties of CPSWFs:

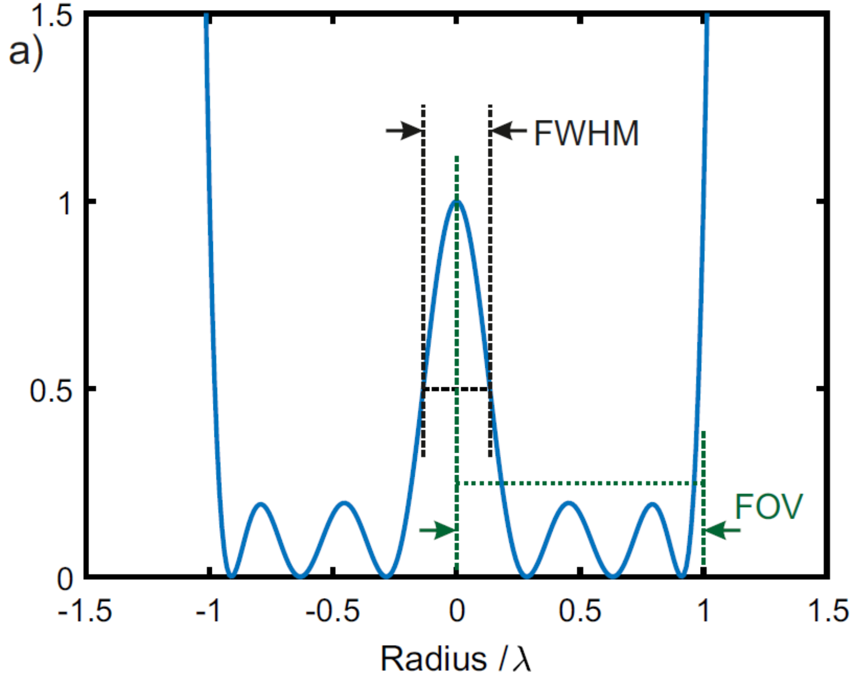
- For any integer $n \geq 0$, the eigenfunctions $\psi_{c,N}^{n}$ are a band-limited function under the Hankel transform.
- The set $\{\psi_{c,N}^{n}, n \geq 0\}$ is an orthogonal basis on the interval $[0, 1]$ and an orthonormal basis on the interval $[0, +\infty)$.

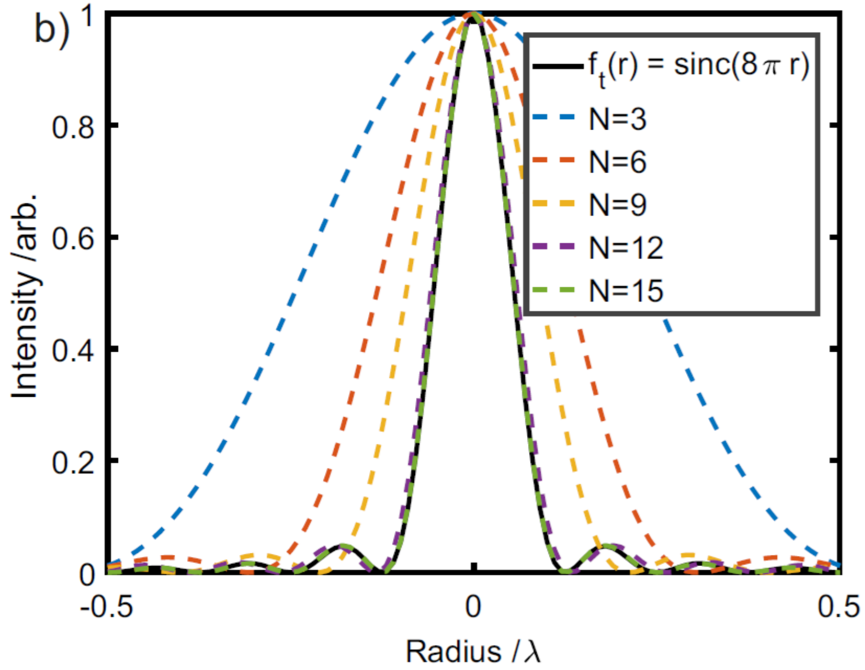
A. Karoui, T. Moumni, Journal of Computational and Applied Mathematics **233**, 315 (2009).

Construction of a Superoscillating Spot

CPSWF

$$f_N(r) = \sum_{i=1}^{N} A_i \psi_i(r) \quad \text{where} \quad A_i = \frac{\int_0^1 f_t(r)\psi_i(r) r dr}{\int_0^1 \psi_i^2(r) r dr} \quad \text{and} \quad f_{N\to\infty}(r) \to f_t(r) = \mathrm{sinc}(ar\pi)$$





$\text{FWHM} \sim 0.1\lambda$
$\text{FOV} = \lambda$
$\text{NA} = 1$

$$\frac{P_{SO}}{P} \sim 2 \times 10^{-42}$$

Not of practical interest!

K. S. Rogers, et al, Opt. Express **26** 8095 (2018).

Construction of a "Useful" Superoscillating Spot

Find a (band-limited) function $f(r)$, and for $I(r) = |f(r)|^2$:

- Minimize the FWHM,
- Maximized the power in the superoscillating spot,

$$\max\left\{\frac{P_{SO}}{P}\right\}$$

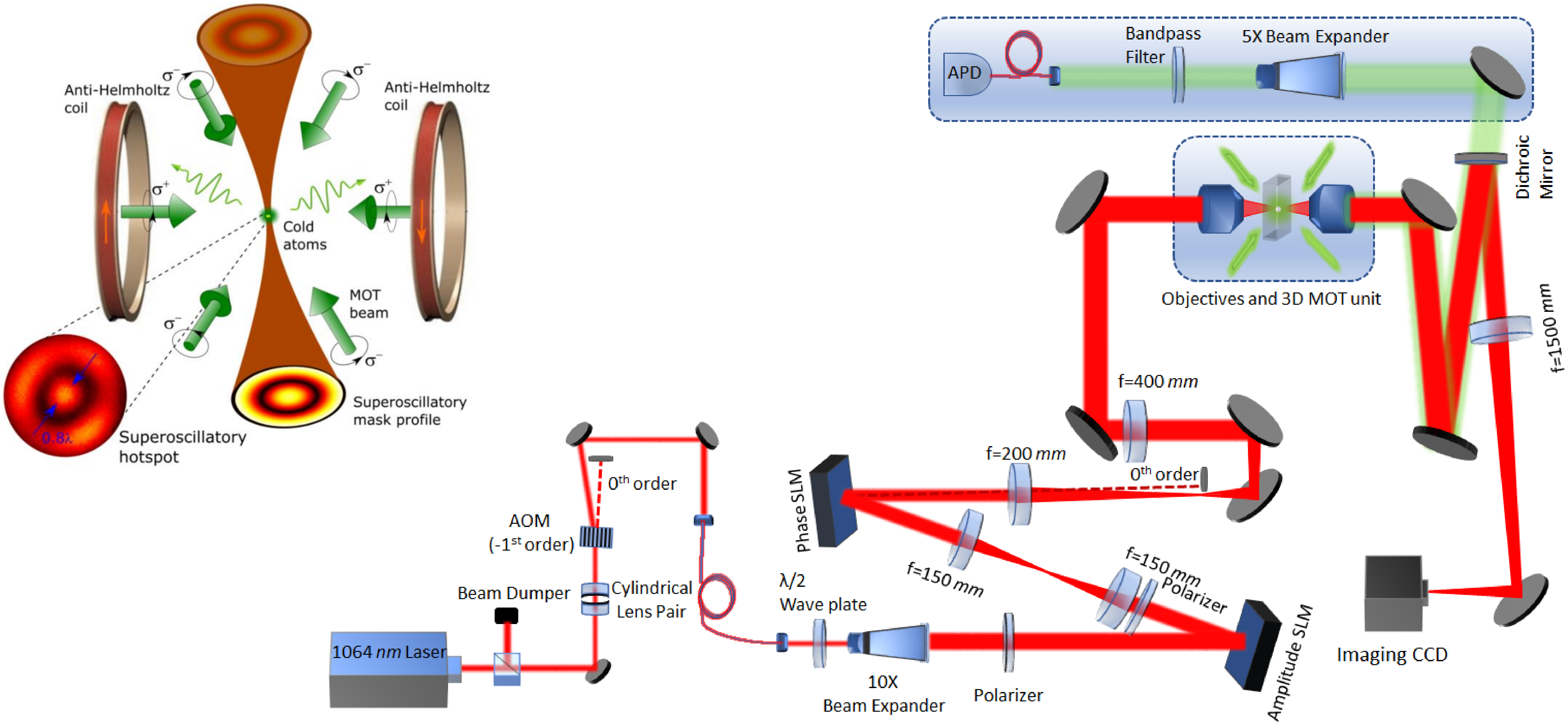
Genetic algorithm: considers the full problem space and find the set of best FWHM

→ takes a long time to run!

A simple (and good enough, at least in our case) optimization method: Two-function optimization.

K. S. Rogers, et al, Opt. Express **26** 8095 (2018).

Experimental Setup



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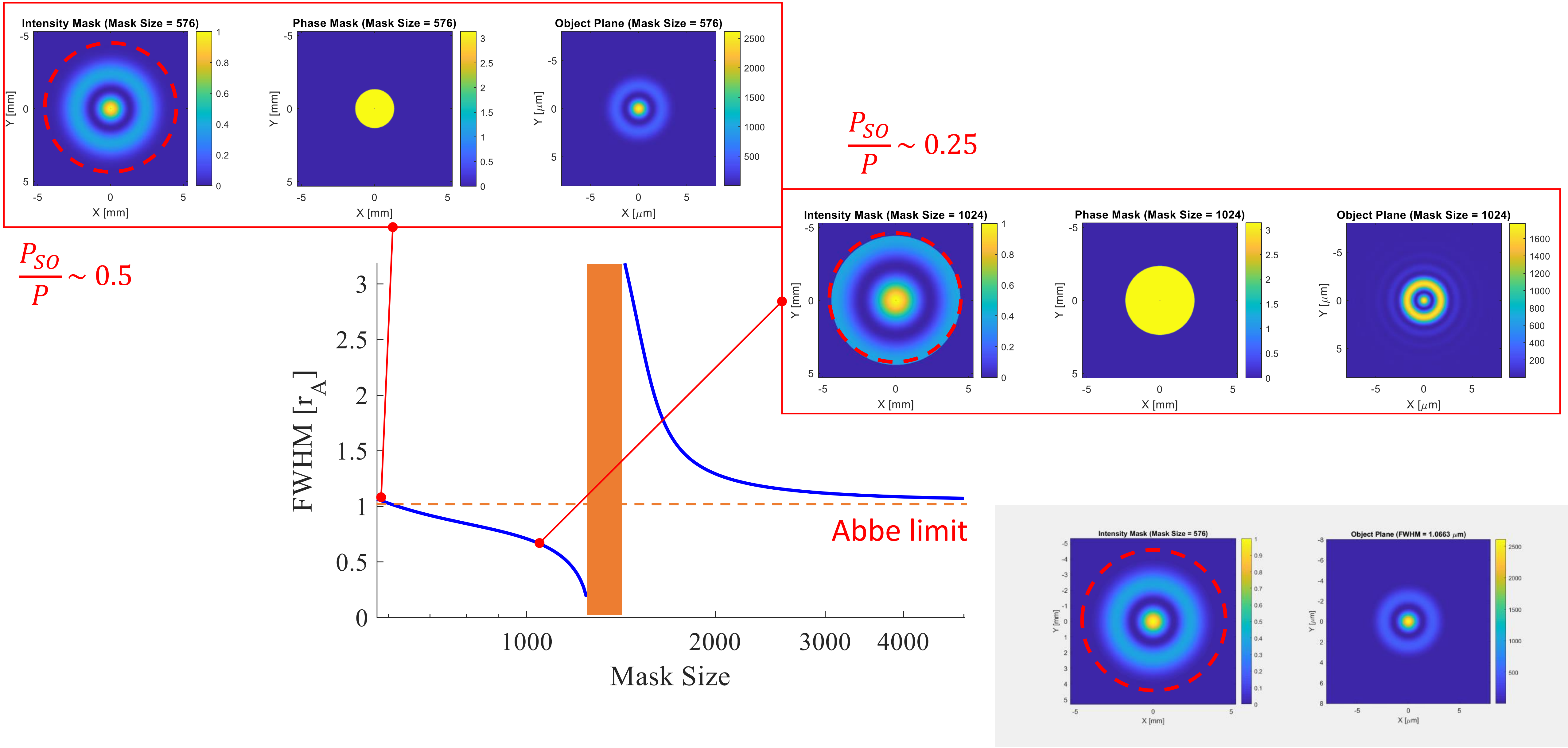
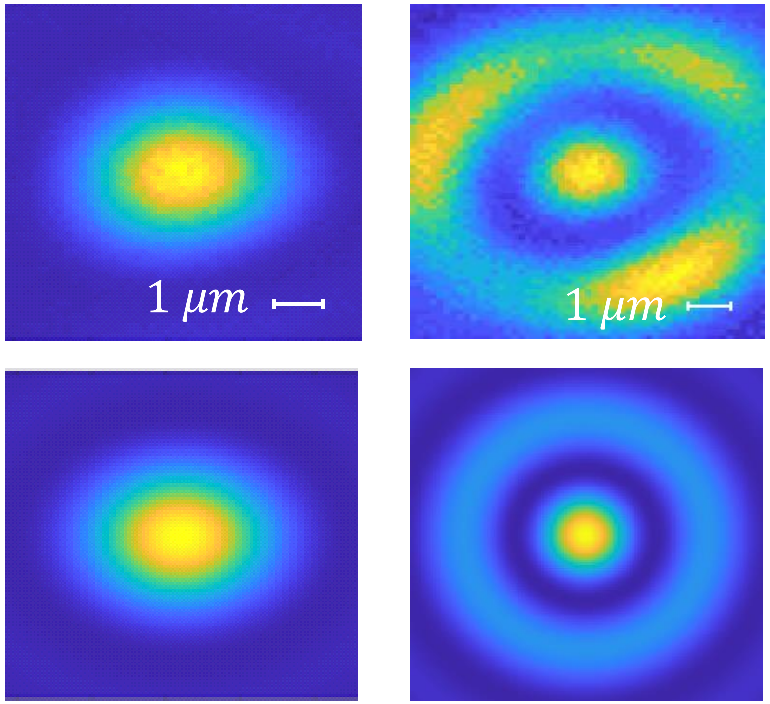
Construction of a Superoscillating Spot



Experimental Setup

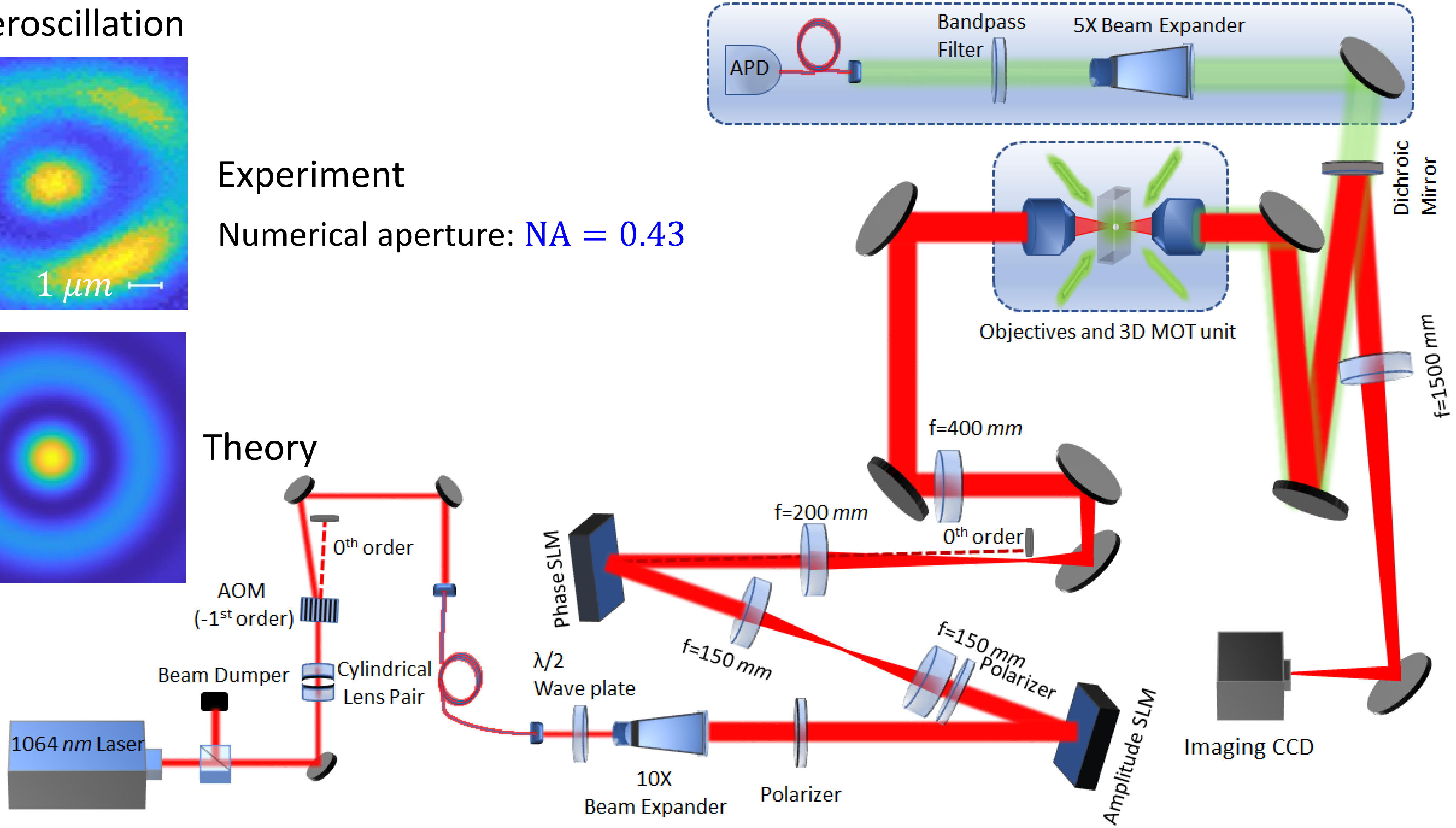
Tophat | Superoscillation



Experiment

Numerical aperture: $\mathrm{NA} = 0.43$

Theory



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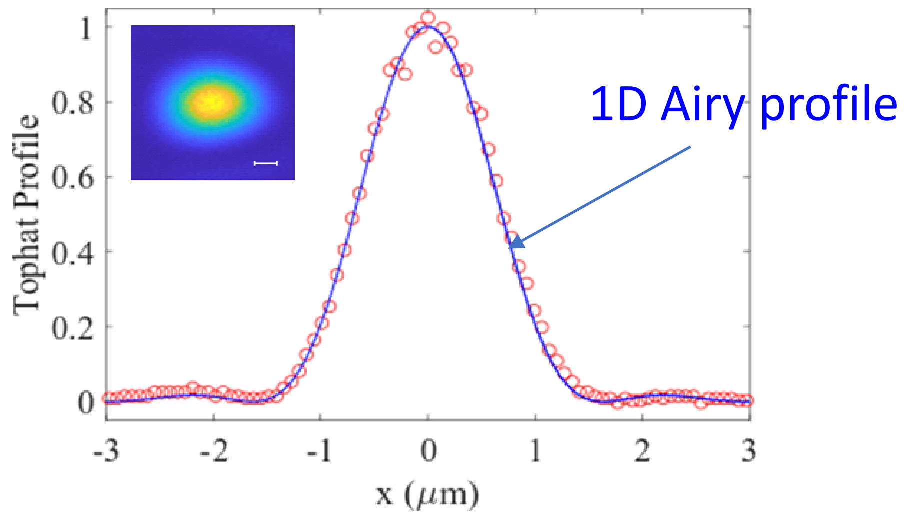
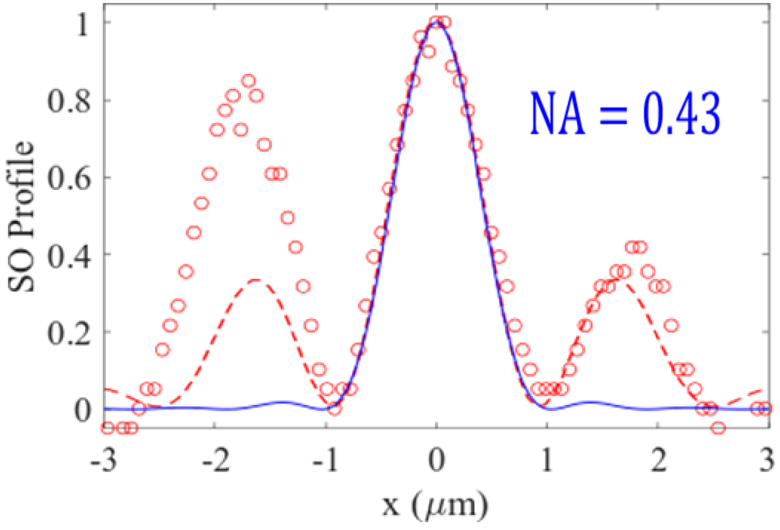
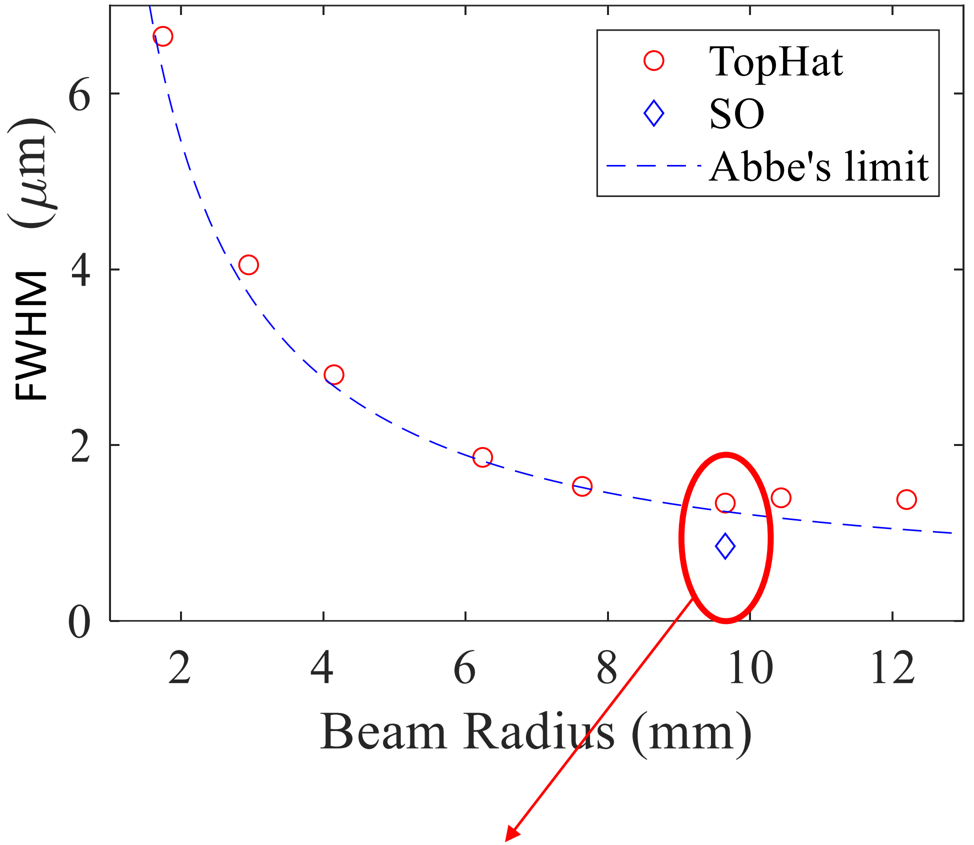
Imaging System Performance







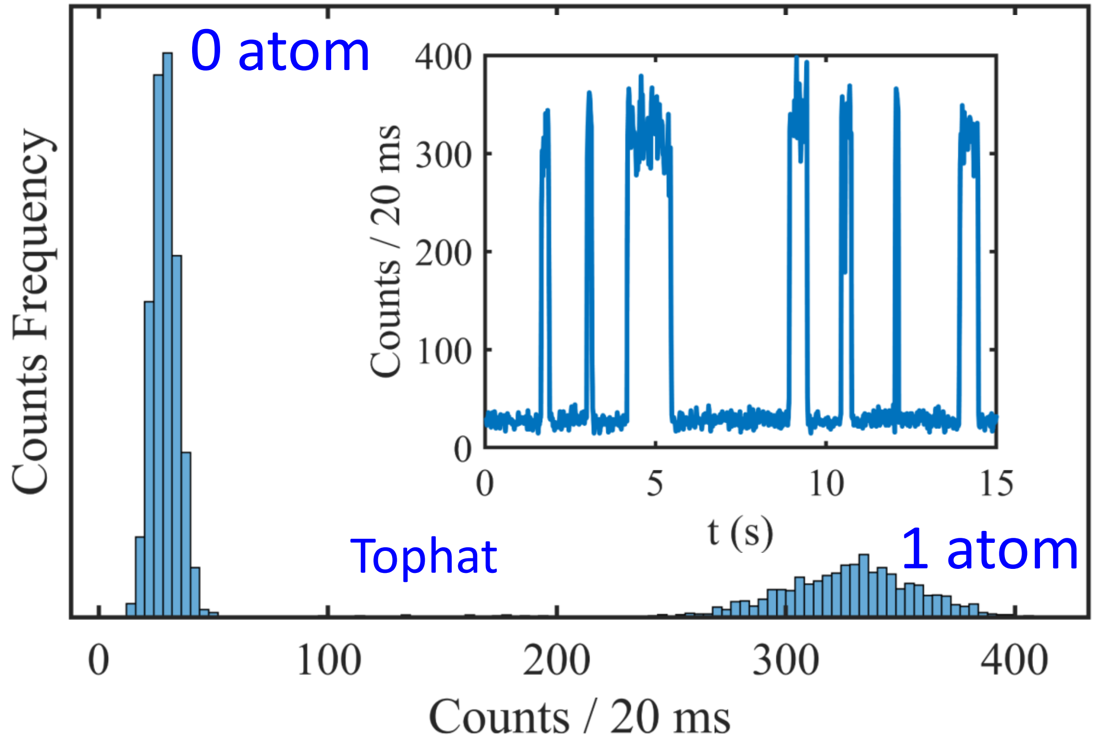
FWHM
$$d_{TH} = 1.34(3)\ \mu m = 1.26(3)\ \lambda = 1.09(3)\ d_A$$
$$d_{SO} = 0.85(3)\ \mu m = 0.80(3)\ \lambda = 0.69(3)\ d_A$$

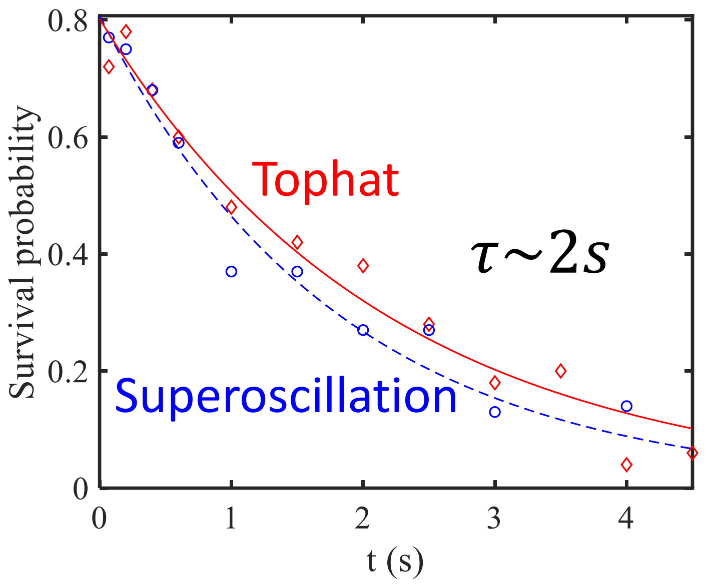
Abbe's limit distance: $d_A = \lambda / 2\mathrm{NA}$

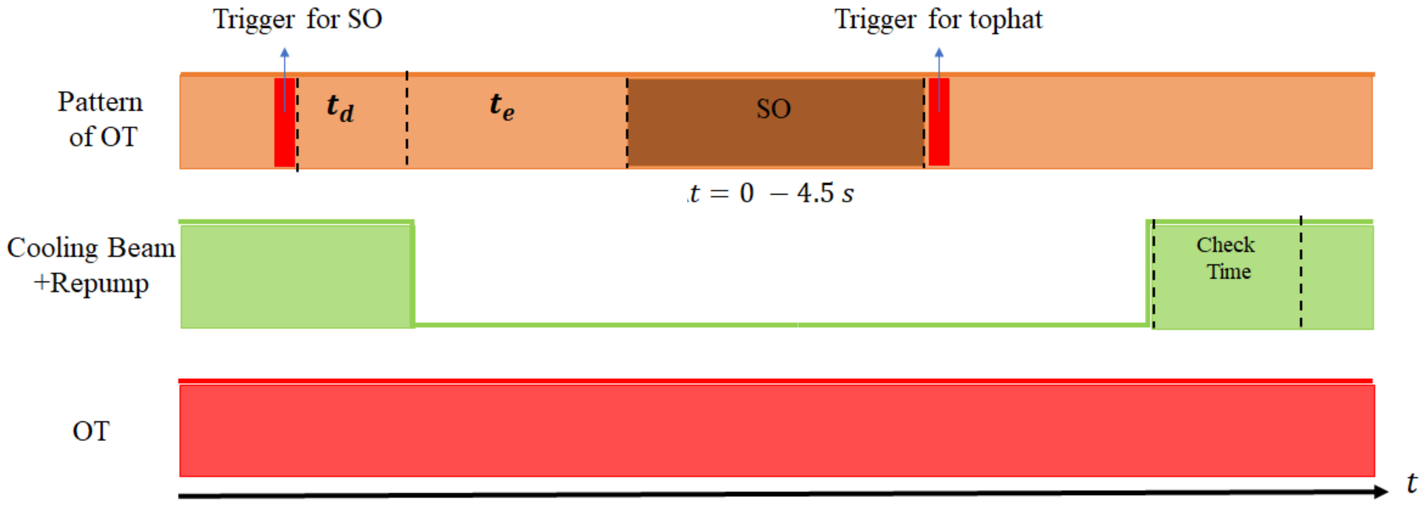
$\mathrm{NA} = 0.43$

The Superoscillation spot (SO) is subwavelength and below the Abbe's limit

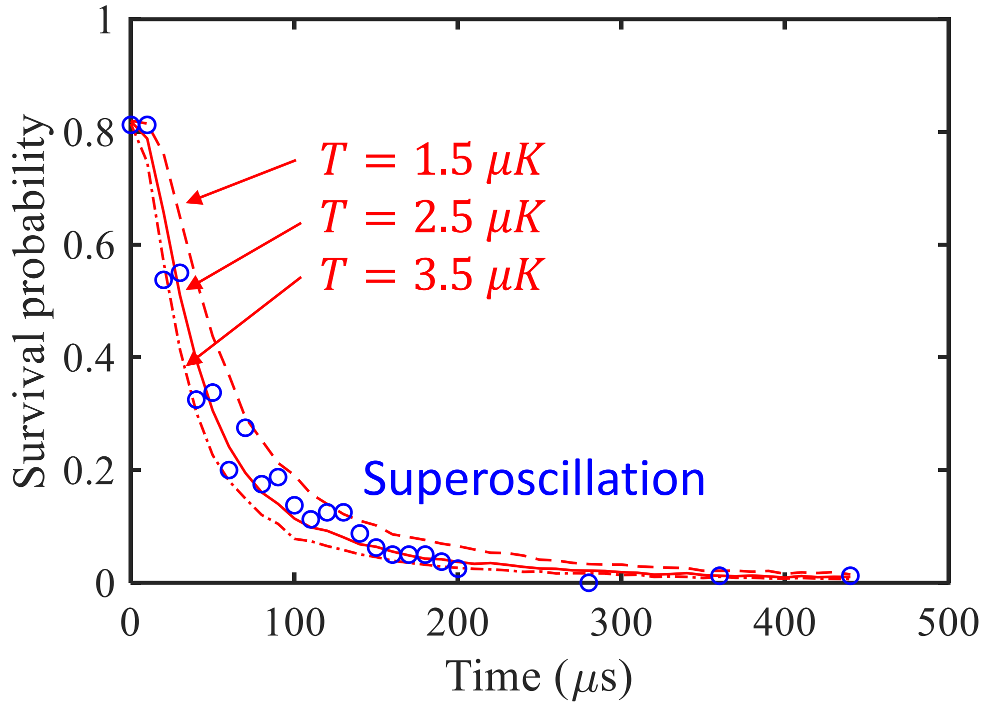
Trapping and Lifetime

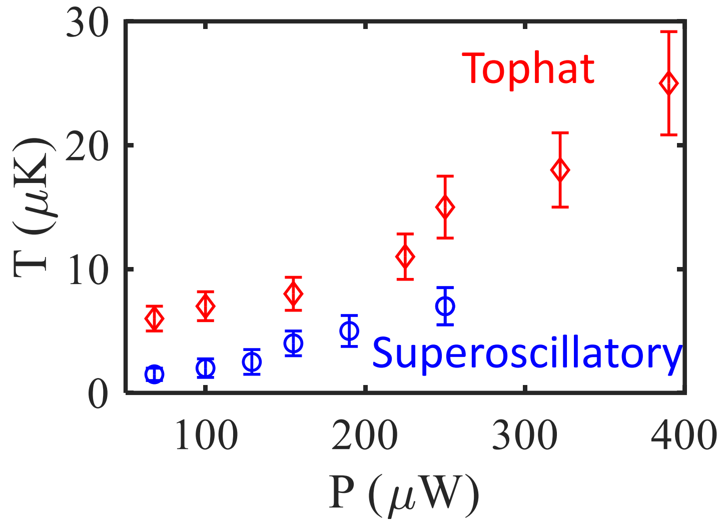






Effective Temperature



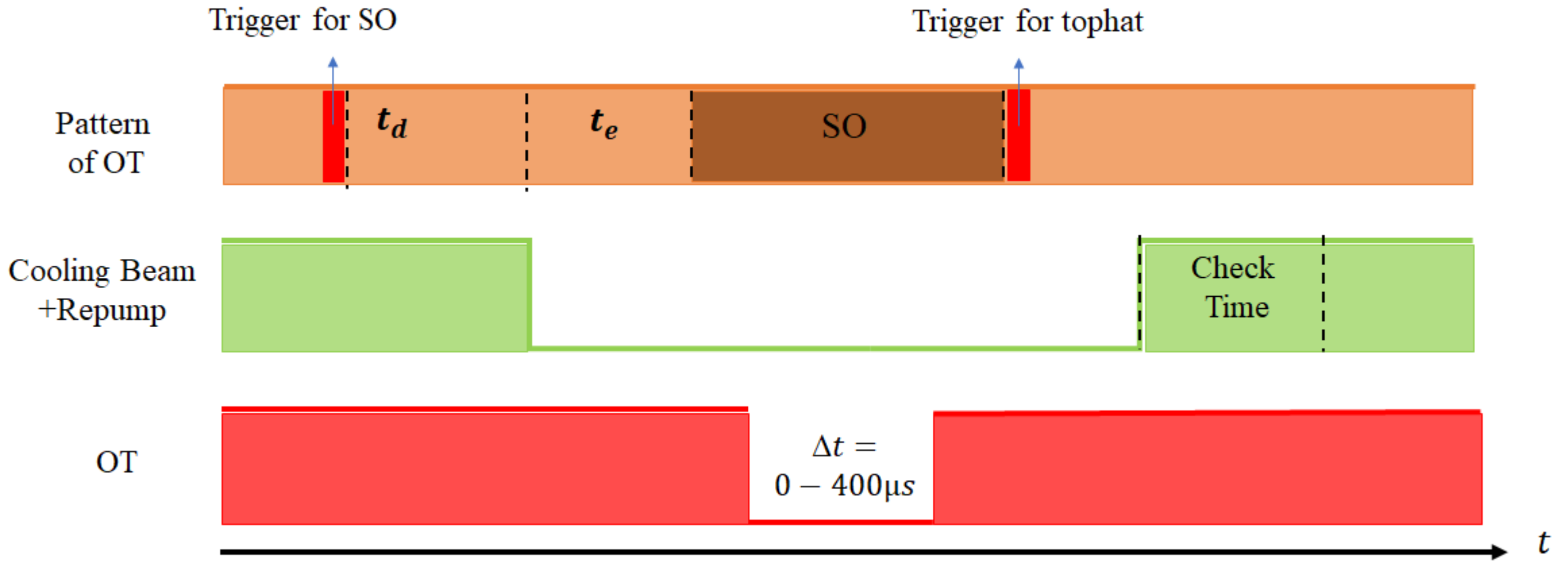


$T_{TH} > T_{SO}$
Because of adiabatic cooling

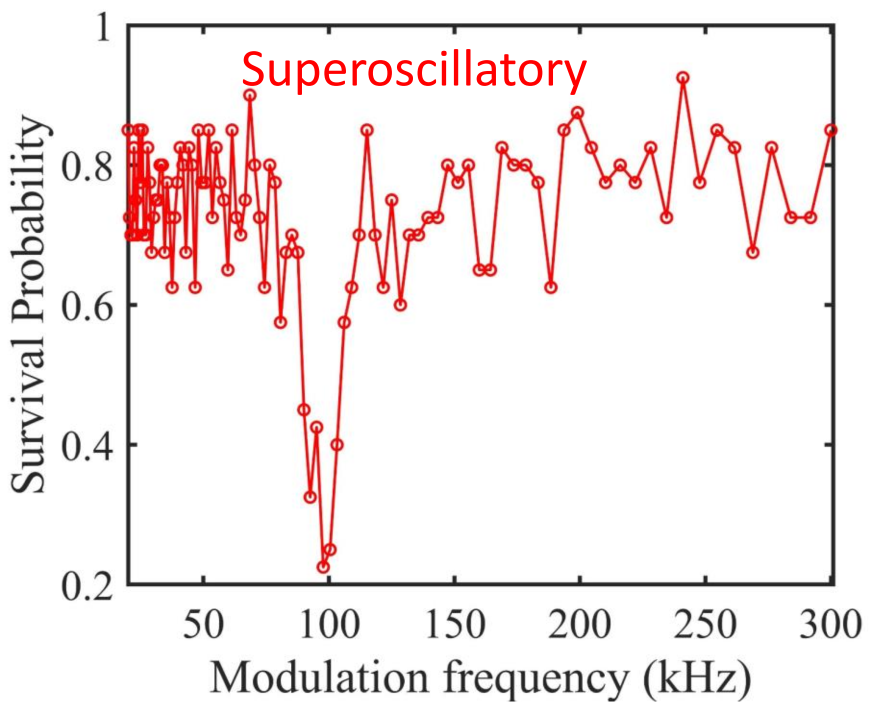
Namely $\nu_{TH} > \nu_{SO}$
ν_X: trapping frequency

The trap is switch off and on during a short time

We model using Maxwell-Boltzmann distribution with an effective temperature T



Trapping Frequency



Total power:
$P_T = 23\ \text{mW}$

$\nu_{SO} \sim 50\ \text{kHz}\ (@\ P_{SO} = 1.1\ \text{mW})$

$\nu_{TH} \sim 80\ \text{kHz}\ (@P_{TH} = 23\ \text{mW})$

$\frac{\nu_{SO}}{\nu_{TH}} \sim 0.6$

$$\nu \propto \frac{\sqrt{U_0}}{d} \propto \frac{\sqrt{I_0}}{d} \propto \frac{\sqrt{P}}{d^2}$$

U_0: Trap depth

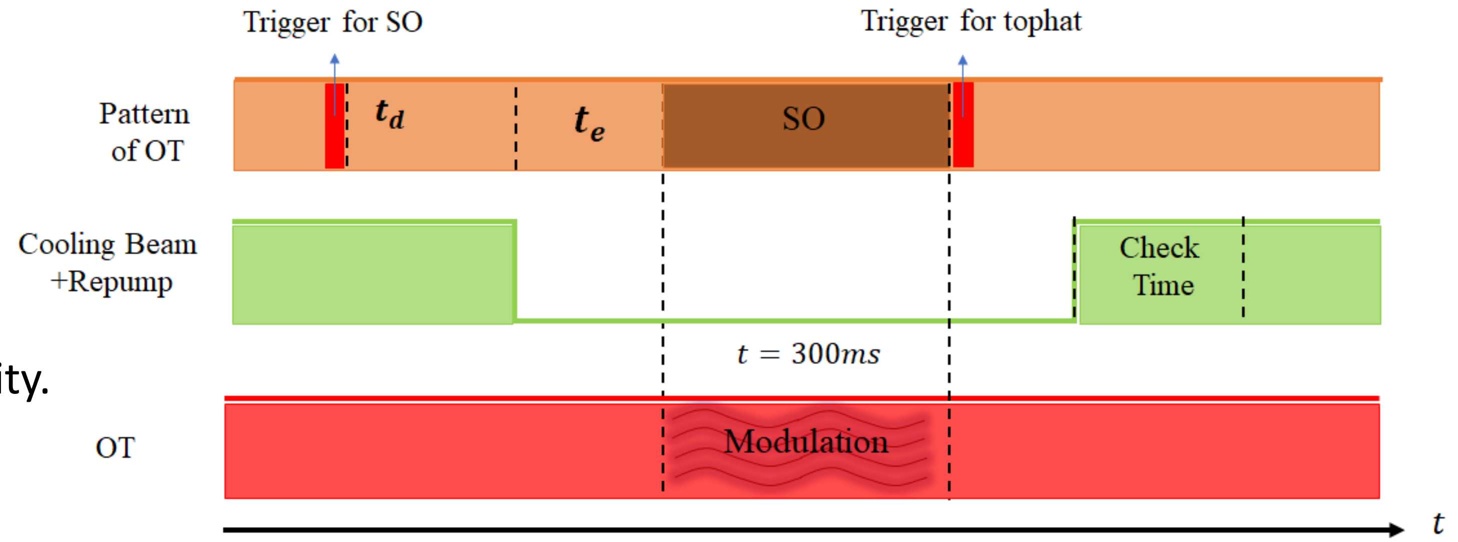
I_0: Peak intensity

$$\left(\frac{\nu_{SO}}{\nu_{TH}}\right)_{\text{theory}} = \sqrt{\frac{P_{SO}}{P_{TH}}\left(\frac{d_{TH}}{d_{SO}}\right)^2} \sim 0.6$$

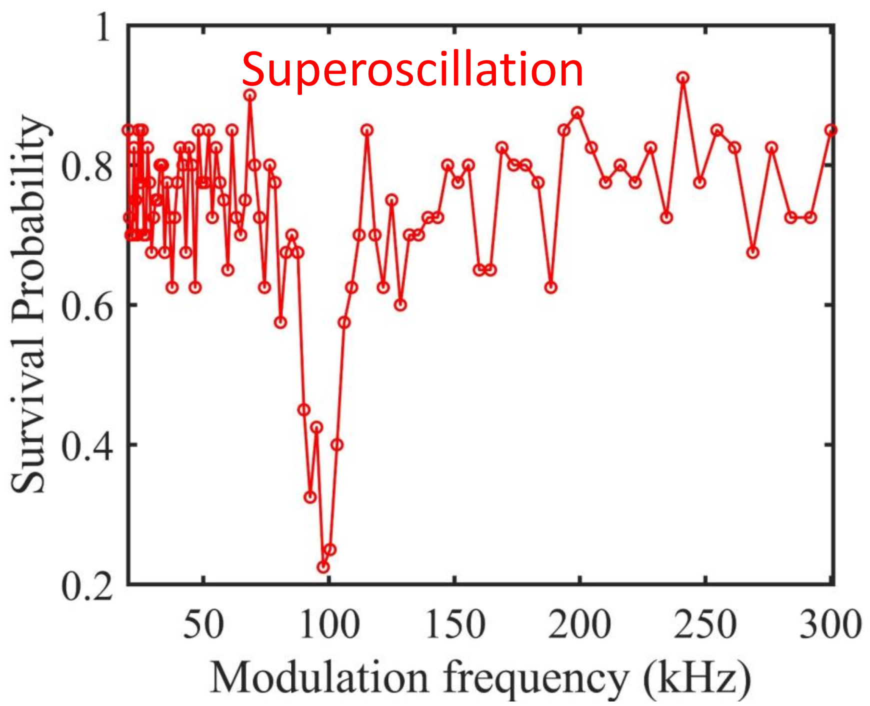
$d_{TH} = 1.09(3)\ d_A$ $d_{SO} = 0.69(3)\ d_A$

The trap is modulated in amplitude.

If the modulation frequency $= 2\nu_{SO,TH}$,
we have heating and losses due to parametric instability.



Trapping Frequency



Total power:
$P_T = 23$ mW

$\nu_{SO} \sim 50$ kHz (@ $P_{SO} = 1.1$ mW)
$\nu_{TH} \sim 80$ kHz (@$P_{TH} = 23$ mW)

$\frac{\nu_{SO}}{\nu_{TH}} \sim 0.6$

$$\nu \propto \frac{\sqrt{U_0}}{d} \propto \frac{\sqrt{I_0}}{d} \propto \frac{\sqrt{P}}{d^2} \qquad \left(\frac{\nu_{SO}}{\nu_{TH}}\right)_{\text{theory}} = \sqrt{\frac{P_{SO}}{P_{TH}}\left(\frac{d_{TH}}{d_{SO}}\right)^2} \sim 0.6$$

U_0: Trap depth

I_0: Peak intensity

The trap is modulated in amplitude.

If the modulation frequency $= 2\nu_{SO,TH}$,
we have heating and losses due to parametric instability.

What about atom confinement ?

Zero point energy wavefunction spread $\propto \frac{1}{\sqrt{\nu}}$ (Harmonic approx.)

Power limited case: The Tophat is the right choice

Intensity limited case: The Superoscillatory is the right choice

H. M. Rivy S. A. Aljunid, E. Lassalle, N. I. Zheludev, D. W., Comm. Phys. **6**, 155 (2023)

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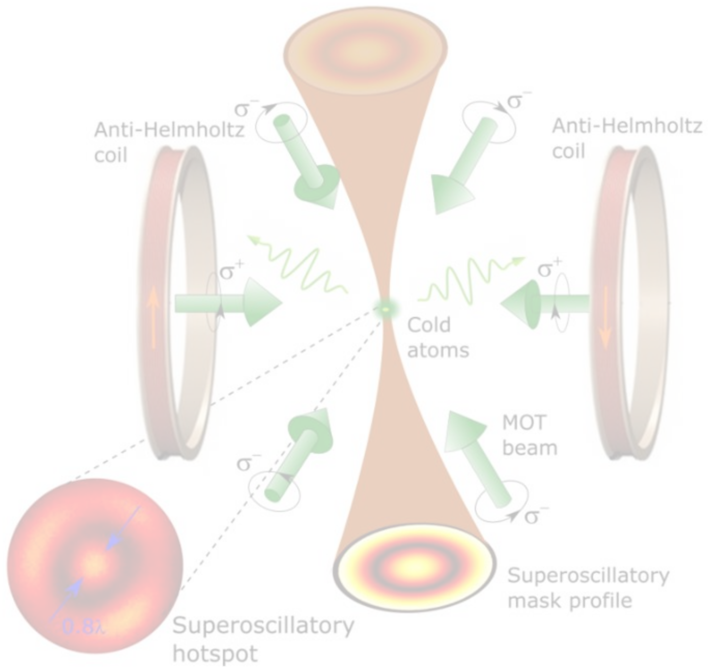
Effective Temperature
Trapping Frequency
Toward subwavelength tweezer arrays



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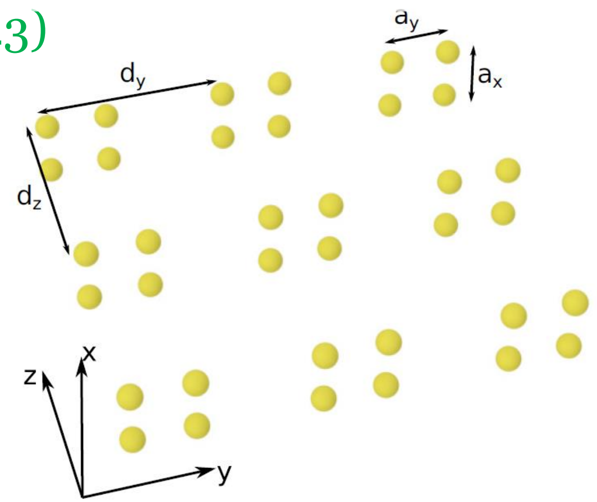
Array of Optical Tweezers

Cooperative metasurfaces



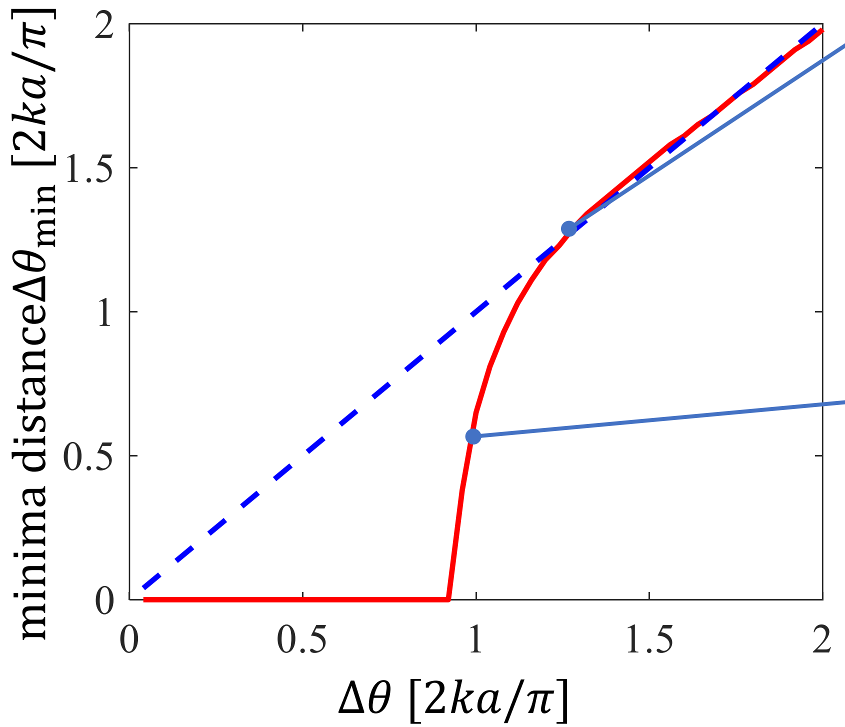
K. E. Ballantine, D. W., and J. Ruostekoski, Phys. Rev. Research **4**, 033242 (2022)

Two Incoherent Spots Case

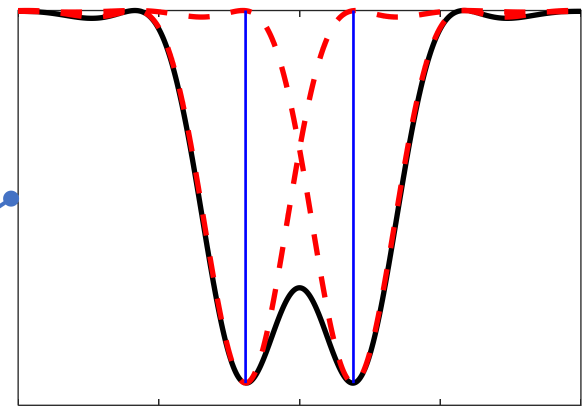
Let's consider Two incoherent Airy discs, giving two potential minima.

Question: What angle separation between the two minima can be achieved?

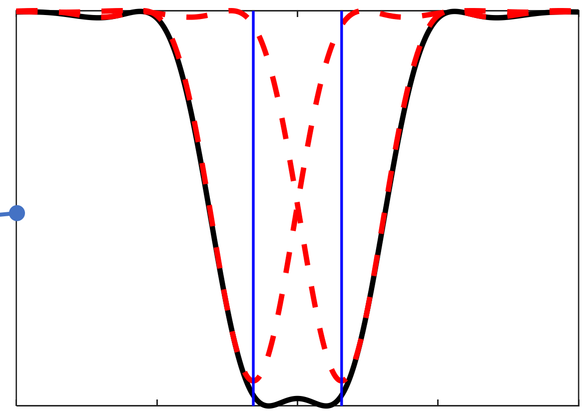
Answer: The two minima can be at an arbitrary small angle separation



Rayleigh Limit $2ka\sin\Delta\theta = 1.22\pi$



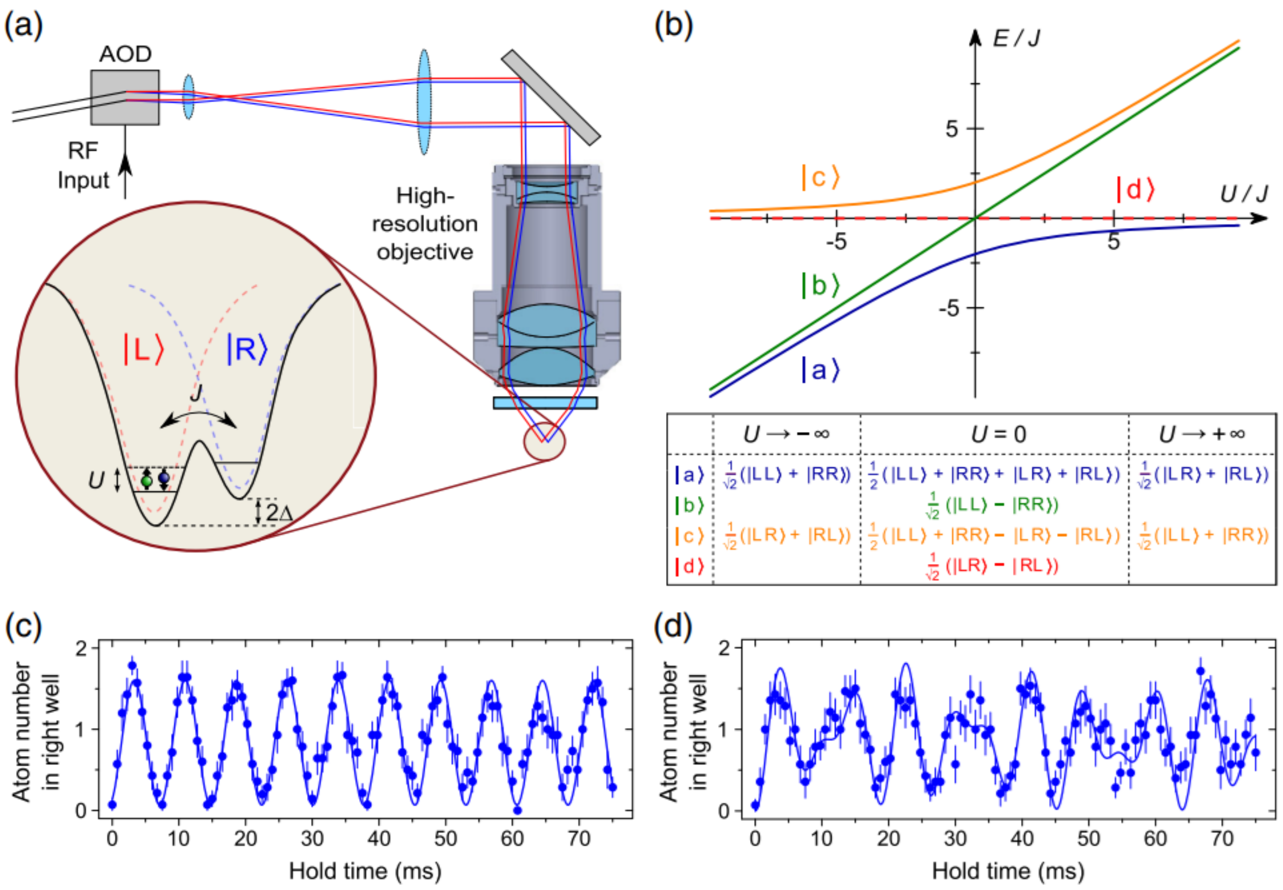
Abbe Limit $2ka\sin\Delta\theta = \pi$



Can't be generalized to more than two spots!

Double-well Experiment

Jochim's Group: Two-fermions in double well.



S. Murmann et al, PRL **114**, 080402 (2015)

Coherent Trap Array Preparation

We found a (band-limited) function $f(r)$, So $I(r) = |f(r)|^2$ gives a single spot (Superoscillation or not)

$$f(r) \xrightarrow[\text{Hankel trans. + SLM encoding}]{} h(r_s)$$

A $N \times M$ trap array in the xy-plane is performed adding phase gradient in the Fourier plane as such

$$h(r_s) \sum_{n,m}^{N,M} e^{i(nkx_s + mky_s + \varphi_{n,m})}$$

Leading to an extra amplitude and phase pattern.

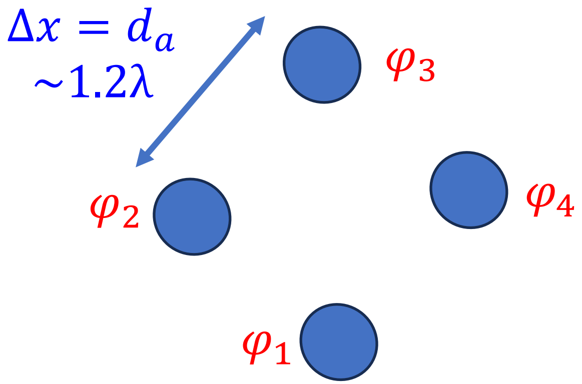
The minimal spot separation Δx shall correspond to a full wrapping of the phase across the pupil entrance, so $2ak = 2\pi$ leading to $\Delta x = \frac{\lambda}{2\mathrm{NA}} = d_a$

→ Small distance with strong overlap → Interference shall play a crucial role

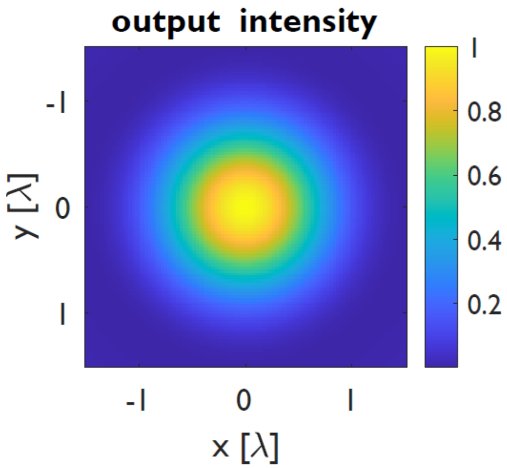
Relative Spot phase: Tophat

Tophat illumination with 4 spots

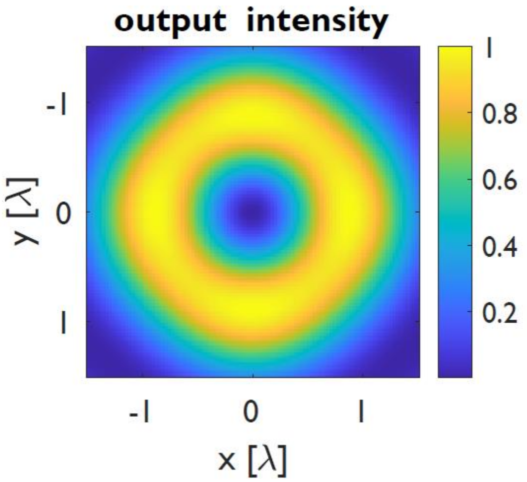
$\mathrm{NA} = 0.43$



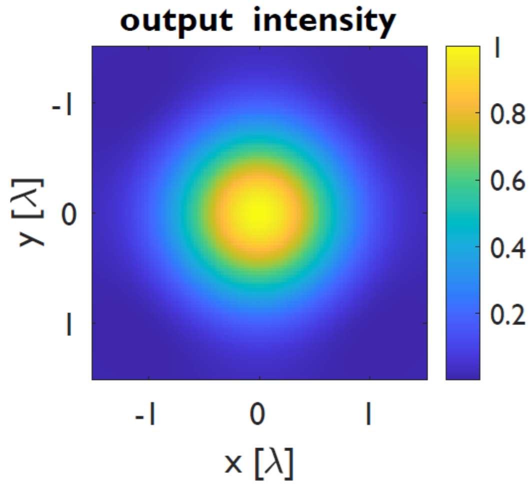
$\varphi_1 = \varphi_2 = \varphi_3 = \varphi_4$



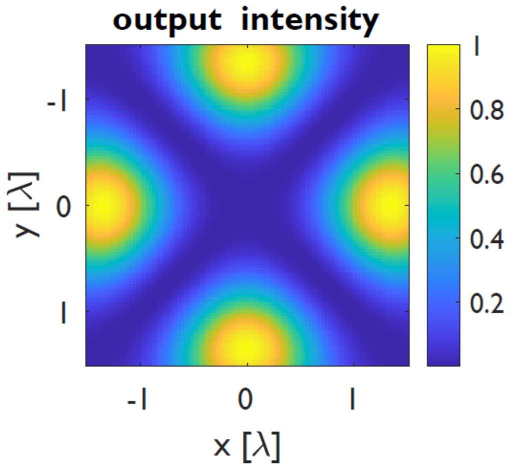
$\varphi_2 = \varphi_1 + \frac{\pi}{2}, \varphi_3 = \varphi_2 + \frac{\pi}{2}, \varphi_4 = \varphi_3 + \frac{\pi}{2}$



$\varphi_1 = \varphi_3, \varphi_2 = \varphi_4 = \varphi_1 + \frac{\pi}{2}$



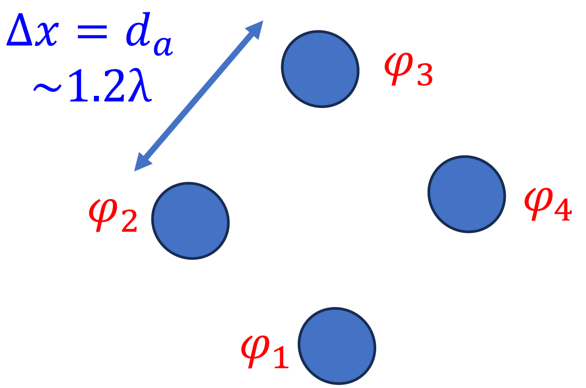
$\varphi_2 = \varphi_1 + \pi, \varphi_3 = \varphi_2 + \pi, \varphi_4 = \varphi_3 + \pi$

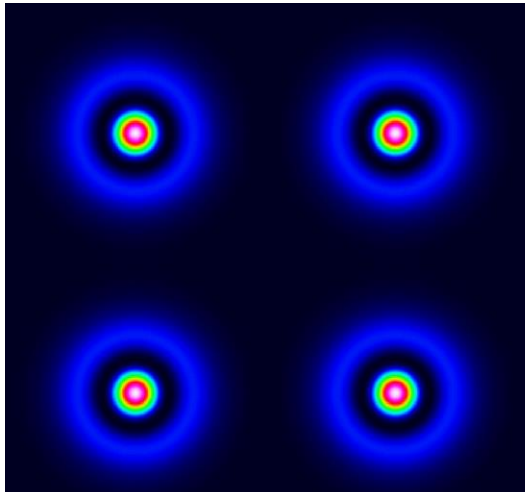


Relative Spot phase: Tophat

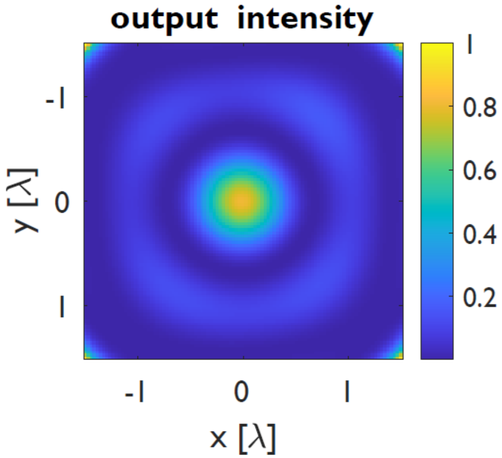
Superoscillation illumination with 4 spots

Size: $0.8\lambda \sim 0.7 d_A$ $\quad$ $\mathrm{NA} = 0.43$

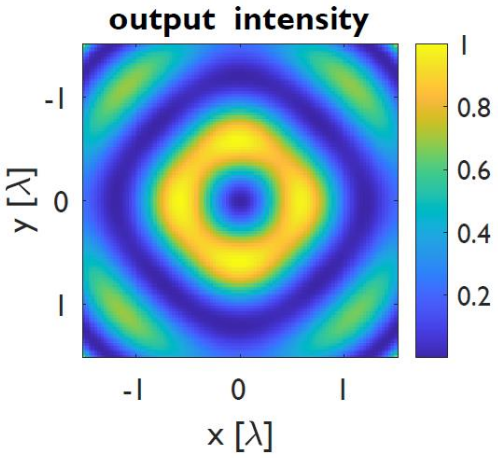




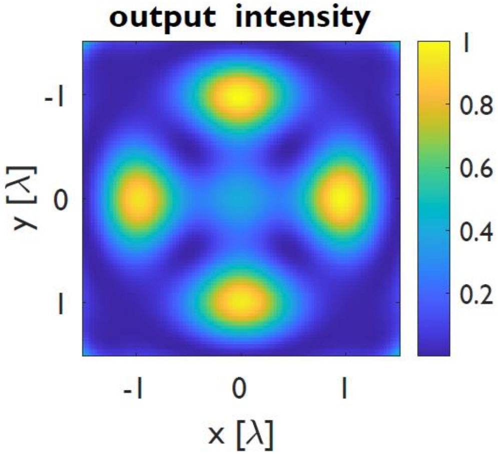
$\varphi_1 = \varphi_2 = \varphi_3 = \varphi_4$



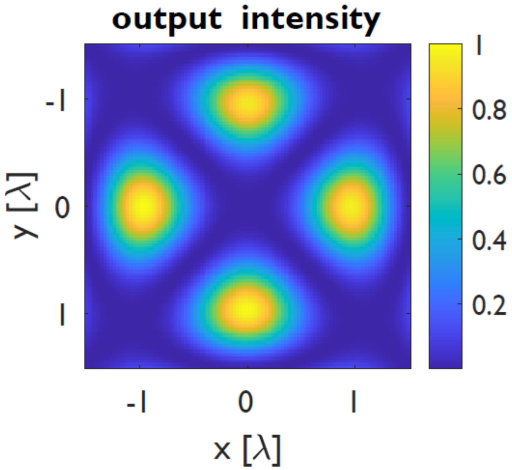
$\varphi_2 = \varphi_1 + \frac{\pi}{2}, \varphi_3 = \varphi_2 + \frac{\pi}{2}, \varphi_4 = \varphi_3 + \frac{\pi}{2}$



$\varphi_1 = \varphi_3, \varphi_2 = \varphi_4 = \varphi_1 + \frac{\pi}{2}$



$\varphi_2 = \varphi_1 + \pi, \varphi_3 = \varphi_2 + \pi, \varphi_4 = \varphi_3 + \pi$



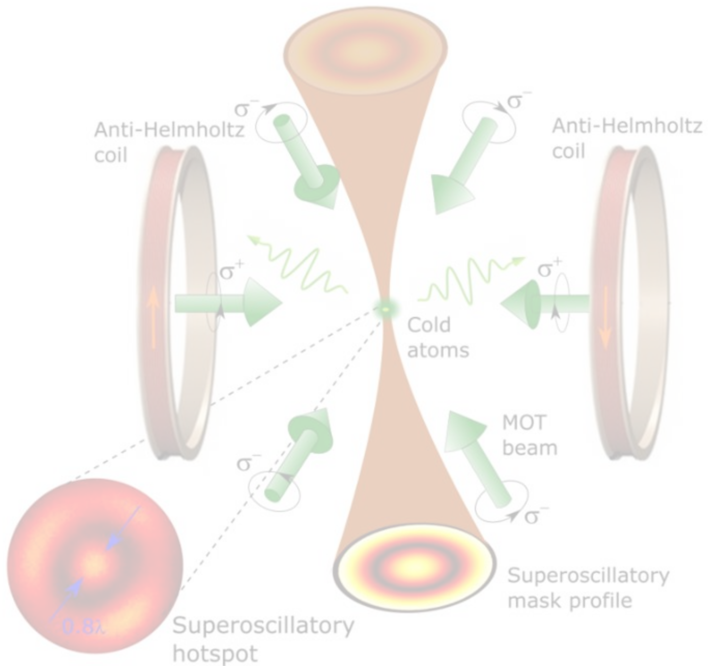
Content

Motivation

Why subwavelength Optical Tweezers?

Trapping Atom in a Superoscillatory Optical Tweezer

Superoscillation?
Lifetime
Effective Temperature
Trapping Frequency
Toward subwavelength tweezer arrays

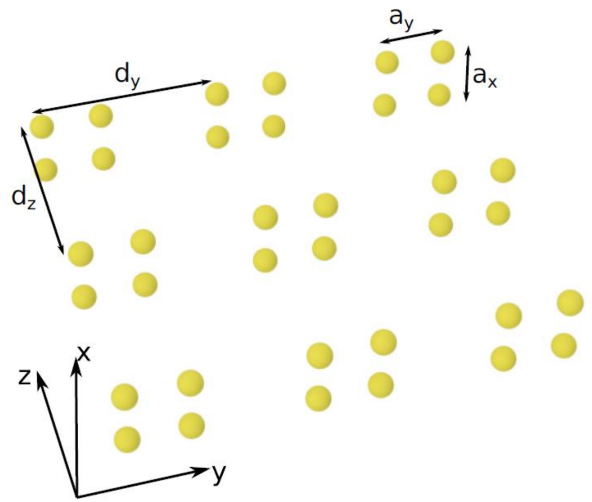


H. M. Rivy S. A. Aljunid, E. Lassalle, N. I. Zheludev, D. W., Comm. Phys. **6**, 155 (20

Array of Optical Tweezers

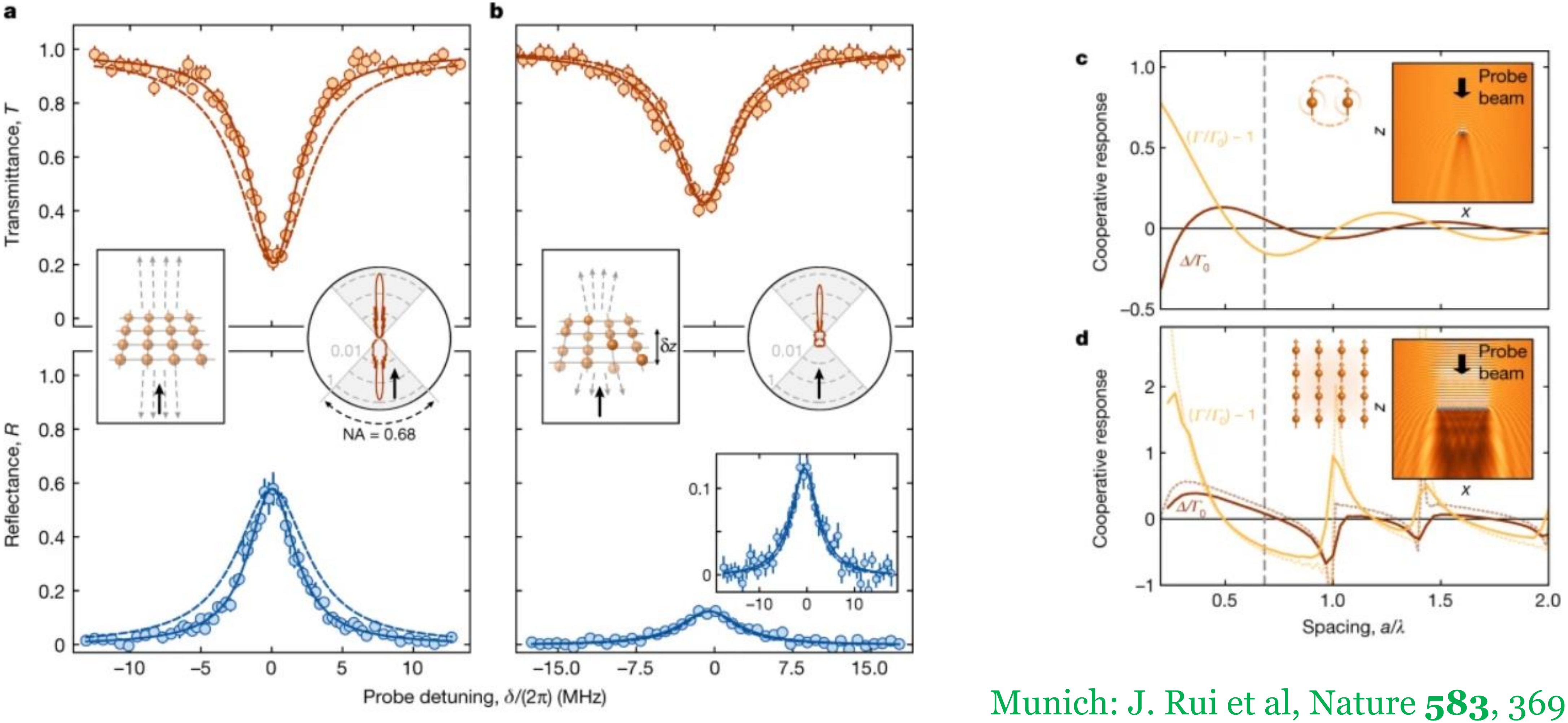
Cooperative metasurfaces

K. E. Ballantine, D. W., and J. Ruostekoski, Phys. Rev. Research **4**, 033242 (2022)



Cooperative metasurfaces with Mott Insulator

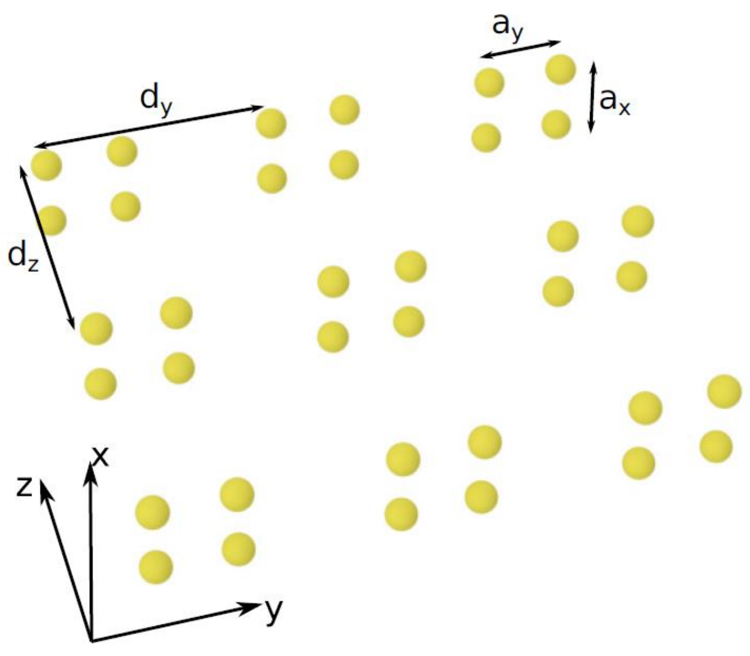
Lossless cooperative quantum metasurfaces.



Munich: J. Rui et al, Nature **583**, 369 (2020)

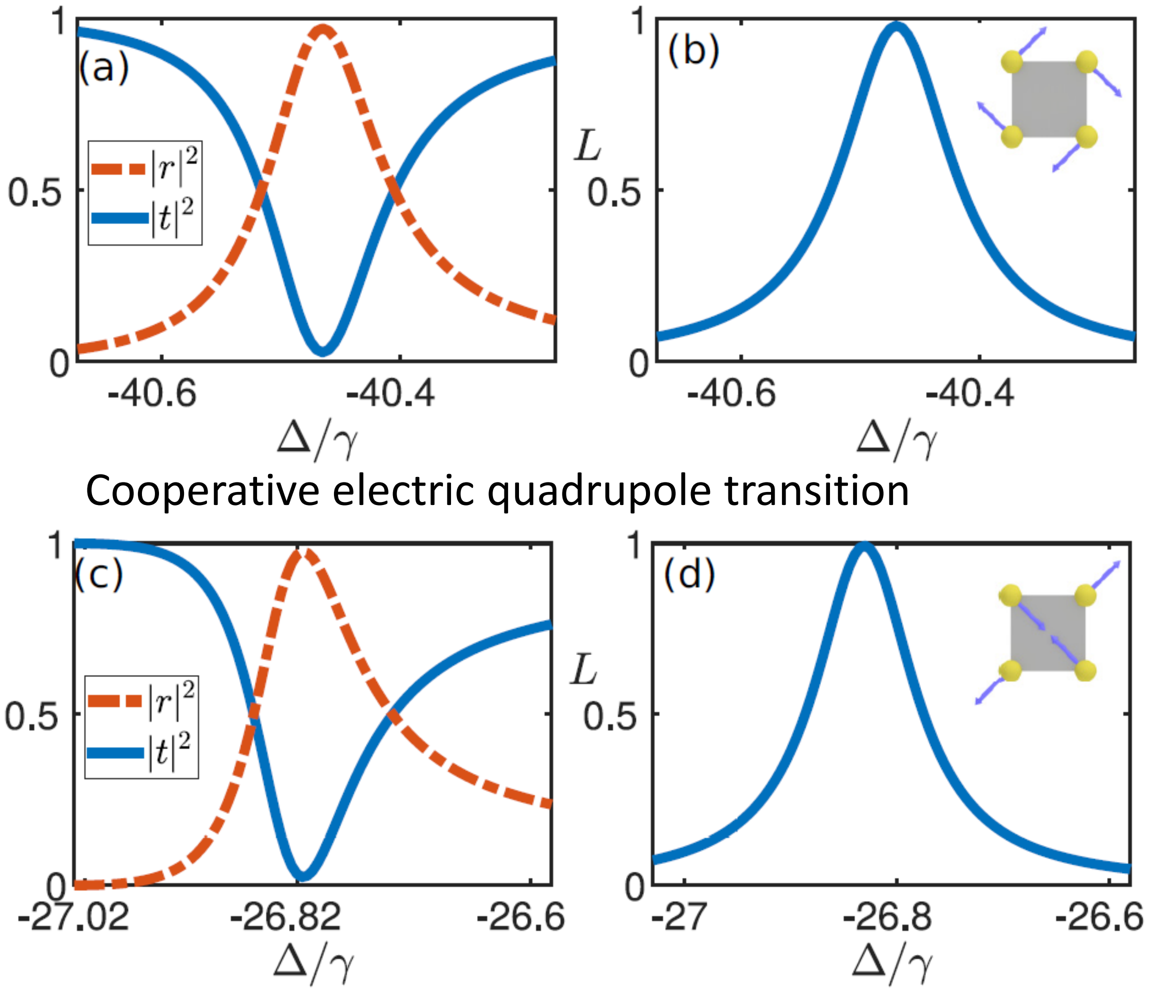
OT arrays: Cooperative Multipole Excitation

Atomic bilayer with square unit cells



$a_{x,y} < d_{y,z} < \lambda$ → cooperative effects

Cooperative magnetic dipole transition

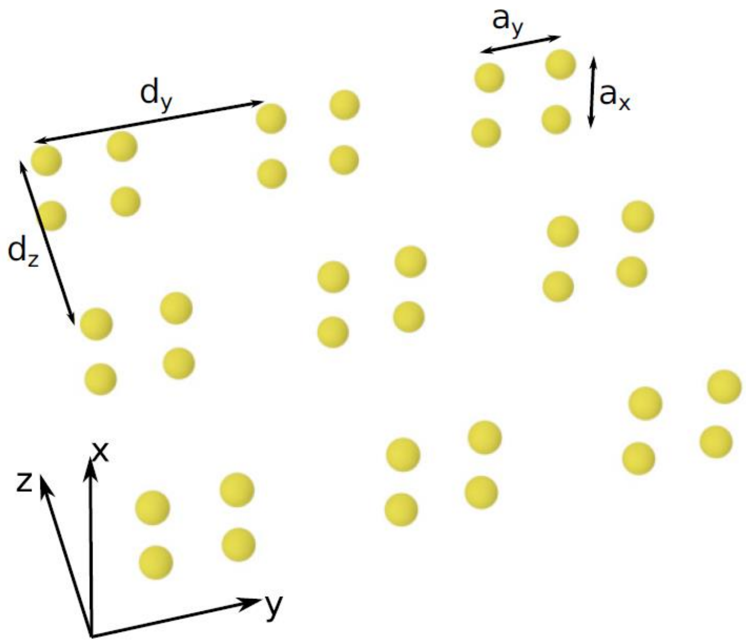


Cooperative electric quadrupole transition

K. E. Ballantine, D. W., and J. Ruostekoski, Phys. Rev. Research **4**, 033242 (2022)

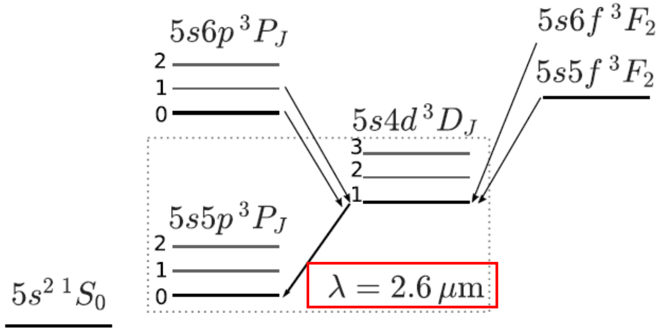
OT arrays: Cooperative multipole excitation

Atomic bilayer with square unit cells



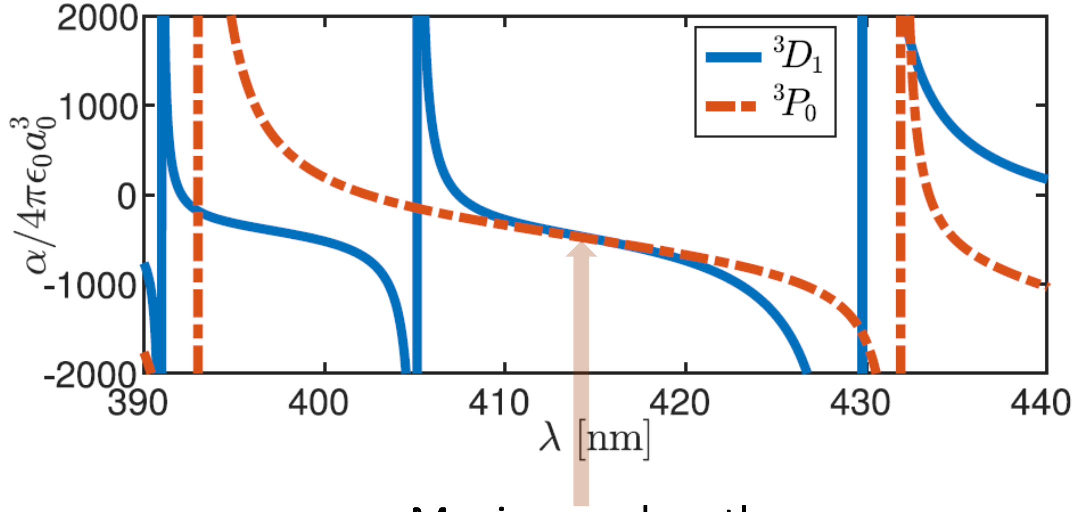
$a_{x,y} < d_{y,z} < \lambda \; \rightarrow$ cooperative effects

Energy spectrum



Strontium atom

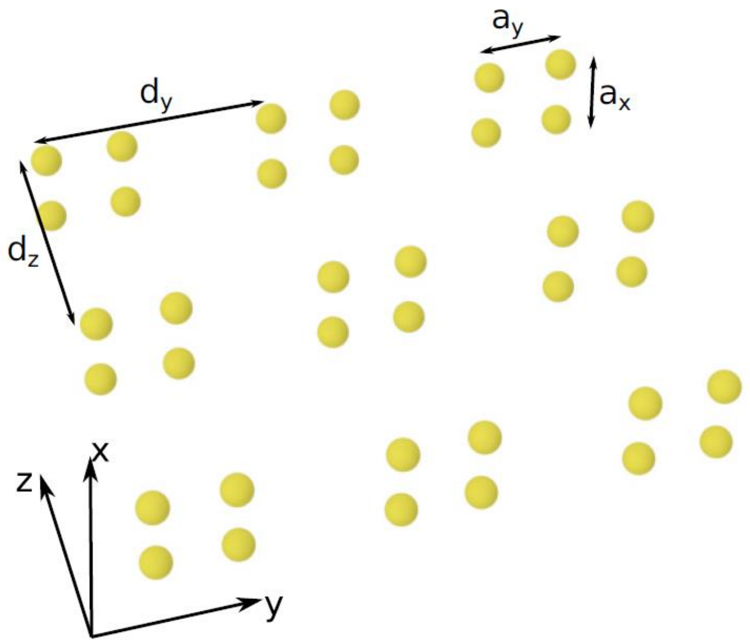
Polarizability in the “blue” region



Magic wavelength

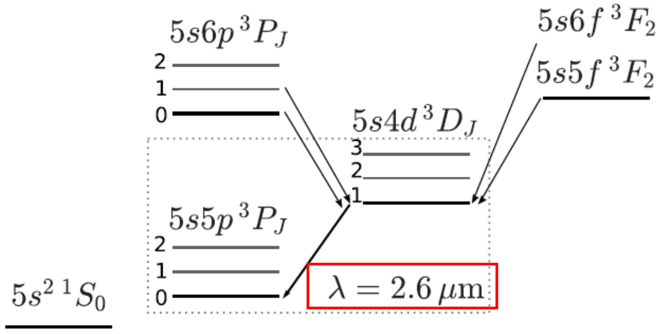
OT arrays: Coincidence of Resonances

Atomic bilayer with square unit cells



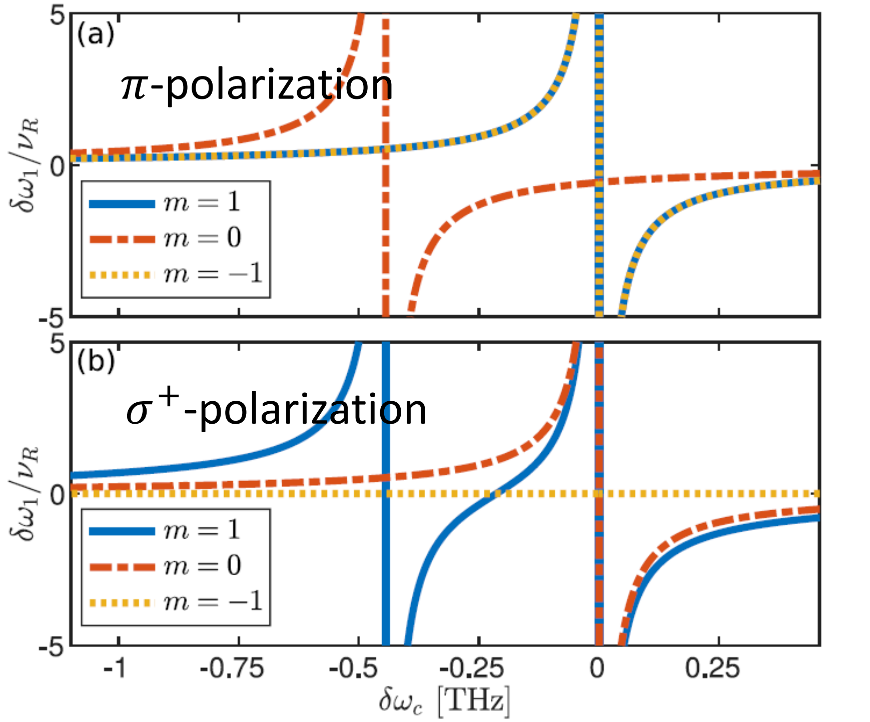
$a_{x,y} < d_{y,z} < \lambda$ → cooperative effects

Energy spectrum



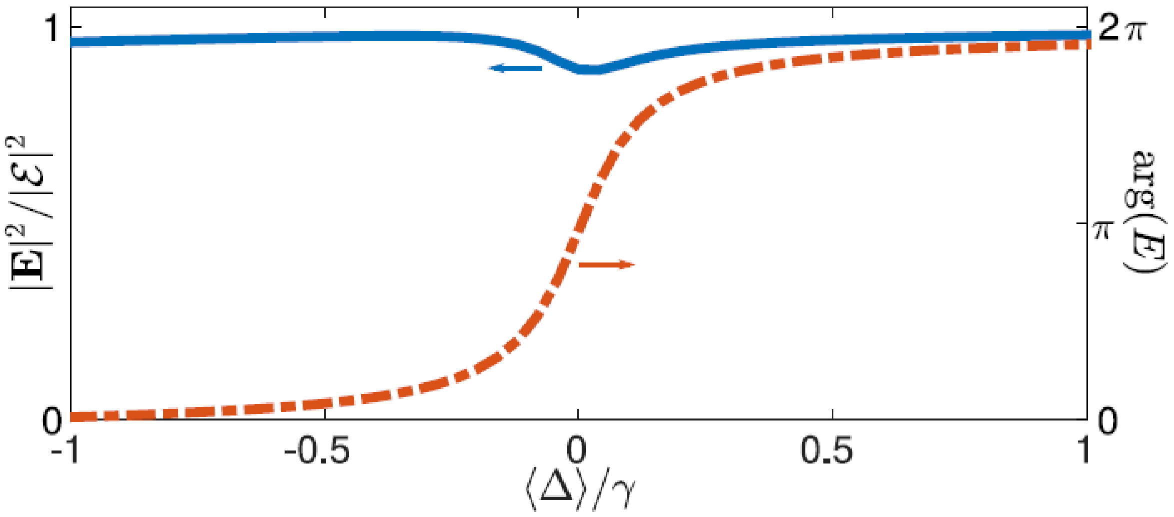
Strontium atom

Light shift at λ~639 nm

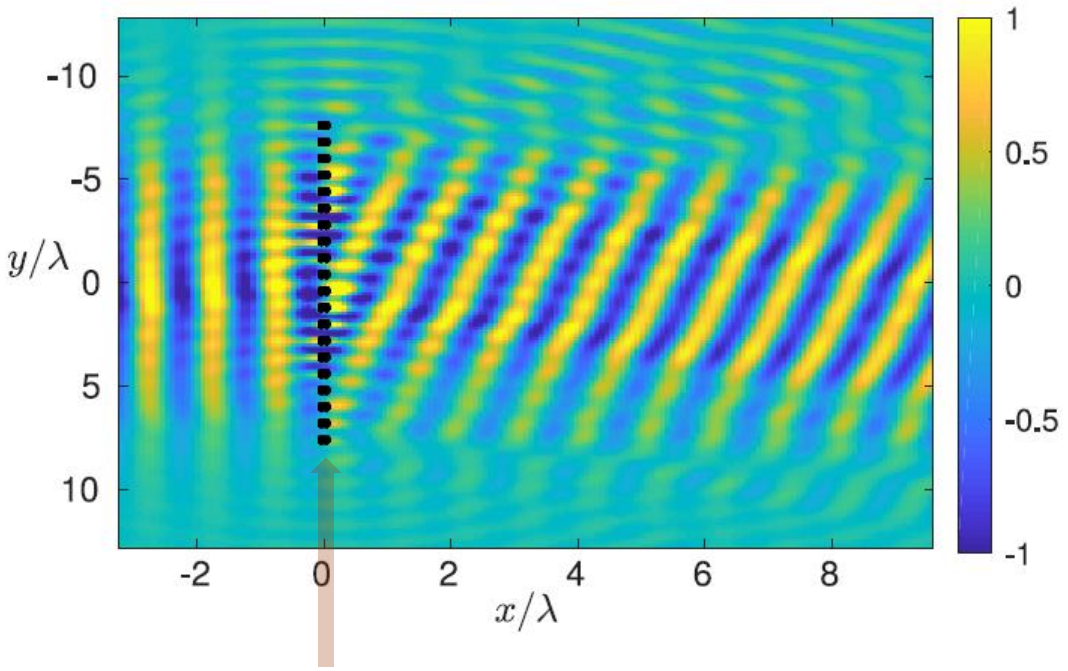


OT arrays: Huygens' Surface

Huygens' surface



Beam steering



Phase gradient

K. E. Ballantine, D. W., and J. Ruostekoski, Phys. Rev. Research **4**, 033242 (2022)

Conclusion

- We trap a single atom in a superoscillation spot

- $d_{SO} = 0.85(3)\ \mu m = 0.80(3)\ \lambda = 0.69(3)\ d_A$

  The trap is subwavelength and below the Abbe's limit

- The confinement is characterized by the trapping frequency

  → Intensity limited case: The Superoscillation OT is the right choice

  H. M. Rivy S. A. Aljunid, E. Lassalle, N. I. Zheludev, D. W., Comm. Phys. **6**, 155 (2023)

  → Toward subwavelength tweezer arrays

  - → Quantum computing and simulation
  - → Cooperative metasurface

  K. E. Ballantine, D. W., and J. Ruostekoski, Phys. Rev. Research **4**, 033242 (2022)

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