

# Solving Problems in Atomic Superfluid Rotation using Cavity Optomechanics\*

Postdocs



P. Kumar N. Daloi

Graduate students

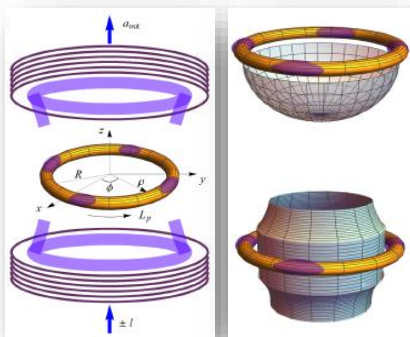


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Faculty



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## Mishkat Bhattacharya

School of Physics and Astronomy  
&  
Future Photon Initiative  
Rochester Institute of Technology



- N. Pradhan et al., Physical Review A **109**, 023524 (2024)  
 N. Pradhan et al., Physical Review Research **6**, 013104 (2024)  
 S. Kalita et al. Physical Review A **107**, 013525 (2023)  
 \*P. Kumar, et al. Physical Review Letters **127**, 113601 (2021)



# AMO Theory group @ RIT



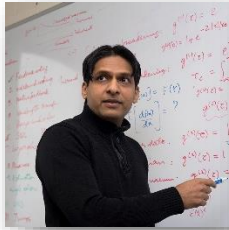
Rochester Institute of Technology:  
School of Physics and Astronomy:

Future Photon Initiative

$T \gg$

20,000 students + 1100 faculty  
~ 40 fulltime faculty,  
PhD Program in Astrophysics  
MS in physics started in 2018  
PhD in Physics started in 2024!!

THEORY  
(RIT)



MB

Prof.  
RIT



Brandon R.

MITRE,  
Princeton



Wenchao Ge

Asst. Prof.  
U Rhode Is



Pardeep K.

Postdoc  
Max Planck  
Erlangen



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KAIST  
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A. Kani

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OIST  
Japan

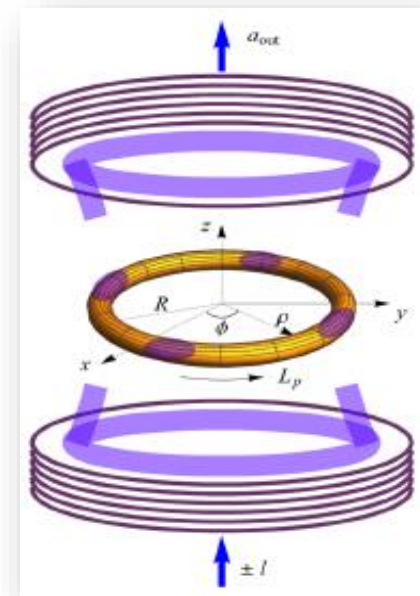


2 Postdoc positions open: [mb6154@gmail.com](mailto:mb6154@gmail.com)



# Outline

- Atomic Superfluid Rotation
- Cavity Optomechanics
- Rotation Sensing
- Conclusion



\*L. Amico et al., Roadmap on Atomtronics,  
*AVS Quantum Sci.* **3**, 039201 (2021)

\*M. Aspelmeyer et al. Cavity Optomechanics  
*Reviews of Modern Physics* **82**, 1155 (2014)

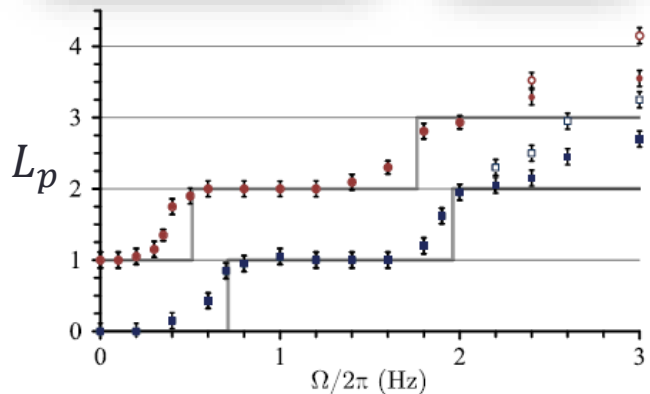
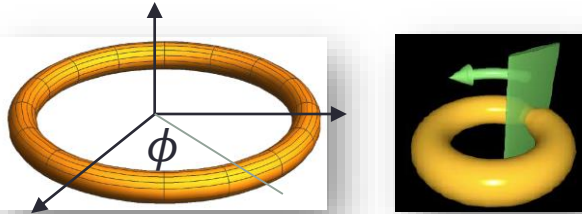


# Quantized Persistent Currents in a Ring BEC



PRL **99**, 260402 (2007)

## Persistent currents

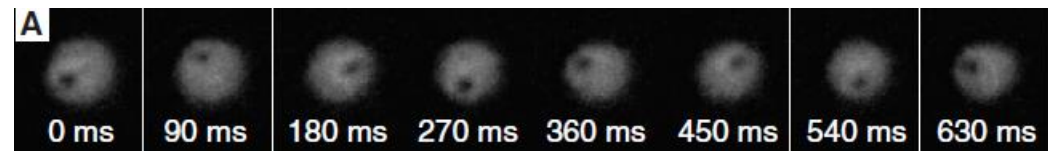


Wright et al., PRL **110**, 025302 (2013)

$$\psi = \sqrt{\frac{n}{2\pi}} e^{iL_p\phi}, L_p = 0, \pm 1, \pm 2, \dots$$

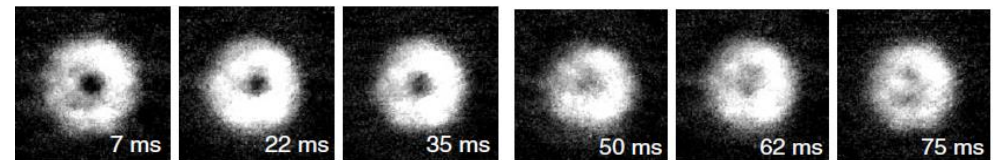
## Why the ring BEC is important

### 1. The ring trap pins the vortex down.



$L_p = 1$  in a harmonically trapped BEC [Science **329**, 1182 (2010)]

### 2. It topologically stabilizes high circulation.



$L_p = 2$  in a harmonically trapped BEC [PRL **93**, 160406 (2004)]

### 3. Metastable currents persist for minutes!

- Beattie et al., PRL **110**, 025301 (2013)



**Supersonic:**  $L_p \approx 350$ : Guo et al., PRL **124**, 025301 (2020)

**Hypersonic:**  $L_p \approx 40,000$ : Pandey et al., Nature **570**, 205 (2019)





# Ring BECs: A Paradigm of Quantum Rotation

## Superfluid hydrodynamics

Z. Mehdi et al., SciPost Phys. **11**, 080 (2021)

J. Polo et al., PRL **123**, 195301 (2019)

K. C. Wright et al., PRL **110**, 025302 (2013)

## Matter wave interferometry

S. Pandey et al., PRL **126**, 170402 (2021)

C. Ryu et al., Nat. Comm. **11**, 3338 (2020)

## Atomtronic circuits

J. Polo et al., PRL **121**, 0904040 (2018)

S. Eckel et al., Nature **506**, 200 (2014)

## Topological defect formation

Aidelsburger et al., PRL **119**, 190403 (2017)

## Cosmological simulations

S. Eckel et al., PRX **8**, 021021 (2018)

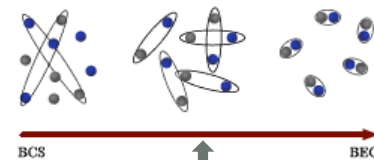
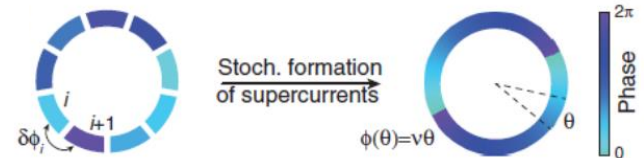
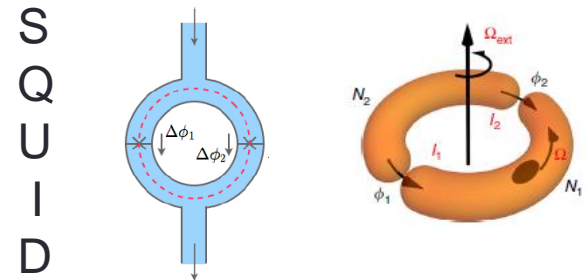
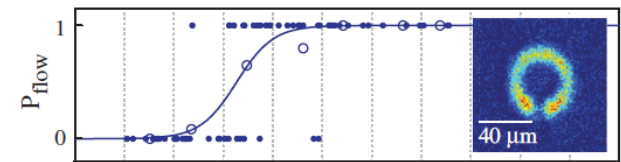
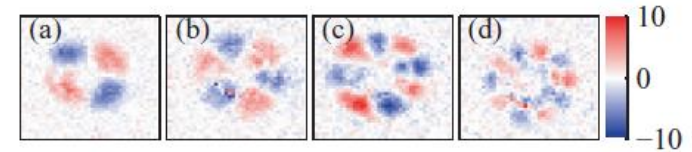
## Fermionic supercurrents (BCS-BEC crossover)

Y. Cai et al. PRL **128**, 150401 (2022)

G. Del Pace et al., Phys. Rev. X **12**, 041037 (2022)

G. Pecci et al., PRR **3**, L032064 (2021)

J. W. Park et al, PRL **121**, 225301 (2018)



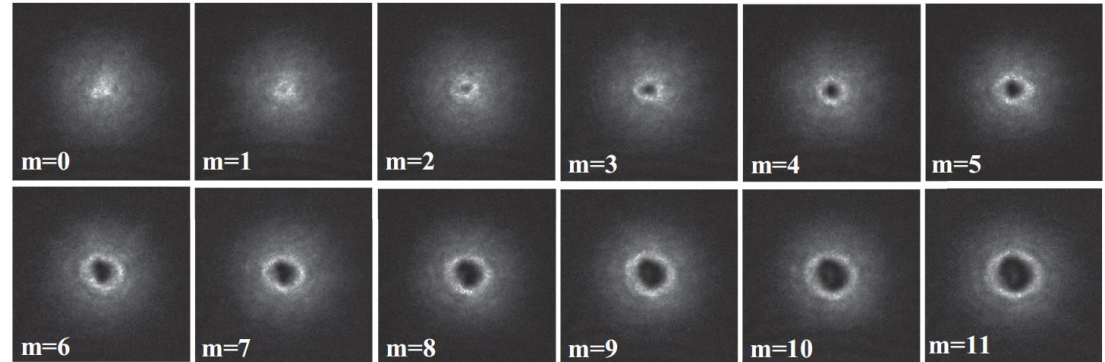
High  $T_c$ , QGP, Holography



# Measuring the Ring BEC Rotation

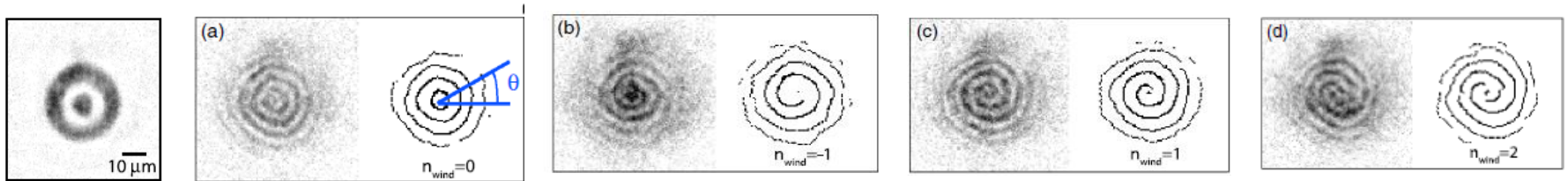
Measure the radius after time of flight expansion

N. Murray et al.,  
Phys. Rev. A **88**, 053615 (2013)  
K. C. Wright et. al,  
PRL **110**, 025302 (2013)



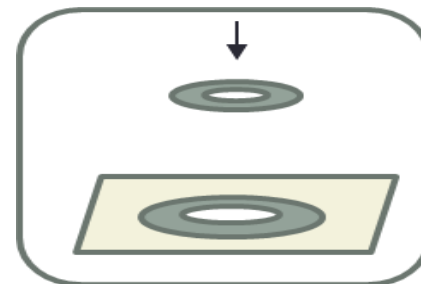
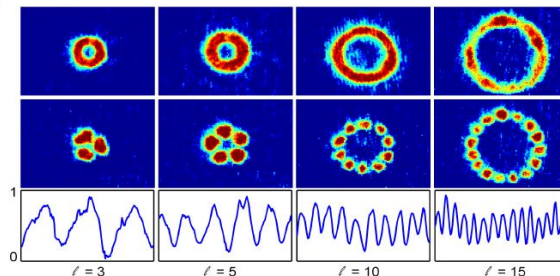
Interfere with central non-rotating BEC

Corman et. al, PRL **113**, 135302 (2014)



More complicated schemes

Moulder et. al, PRA **86**, 013629 (2012)



← Destructive!!

Absorption imaging



# Measuring the Ring BEC rotation

**Required:** A measurement technique that works

1. Non-destructively
- +
2. *In situ*
- +
3. In real time



## Superconductors

- Nat. Comm. **8**, 85 (2017)
- Nature Physics **5**, 651 (2009)
- Nature **397**, 308 (1999)

## Superfluid He

- Nat. Comm. **12**, 2645 (2021)
- Science **366**, 1480 (2019)
- Nature **441**, 588 (2006)

## Simply connected BECs

- *In situ* detection of vortices in BEC, PRA **91**, 023631 (2015)
- Real-time dynamics of single vortex lines in BECs, Science **329**, 1182 (2010)
- Vortex precession in BECs, PRL **85**, 2857 (2000)
- Rotational superradiance... arXiv:2404.1013v1 (2024)

**Aims:**

- Observing system dynamics in real time + *in situ*
  - the ideal aim of experiments
- Following historical precedent
- Coherent and reversible control
  - undoing phase slips
- Storing and retrieve information
  - quantum memory
- Detecting fermionic superfluidity directly without projecting to the BEC regime...

Minimally destructive, Doppler measurement of a Quantized flow in a ring-shaped BEC,  
A. Kumar et al., NJP **18**, 025001 (2016)



# Cavity Optomechanics



PRL 116, 061102 (2016)

PHYSICAL REVIEW LETTERS

12 FEBRUARY 2016

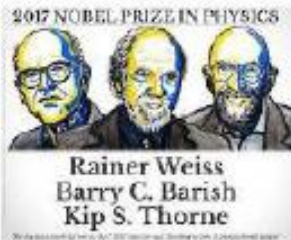


## Observation of Gravitational Waves from a Binary Black Hole Merger

B. P. Abbott *et al.*\*

(LIGO Scientific Collaboration and Virgo Collaboration)

(Received 21 January 2016; published 11 February 2016)



Also...

### Accelerometry

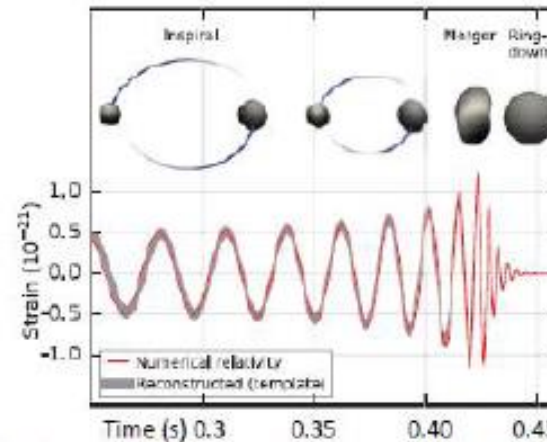
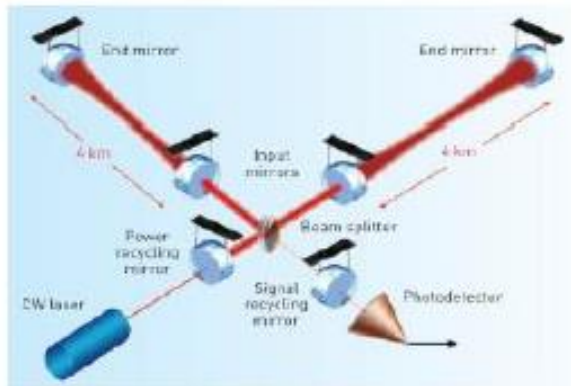
A. G. Krause, M. Winger,  
T. D. Blasius, Q. Lin and  
O. Painter,  
*Nat. Photon.* 6 768 (2012)

### Magnetometry

S. Forstner *et. al.*,  
*PRL* 108 120801 (2012)

### Thermometry

J. Millen *et. al.*,  
*Nat. Nano.* 9 425(2012)



Displacement = Strain x Length of int. arm  
 $\approx 10^{-18} \text{ m}$

→ LIGO is a cavity optomechanical device !!

\*M. Aspelmeyer *et. al*

*Reviews of Modern Physics* **82**, 1155 (2014)

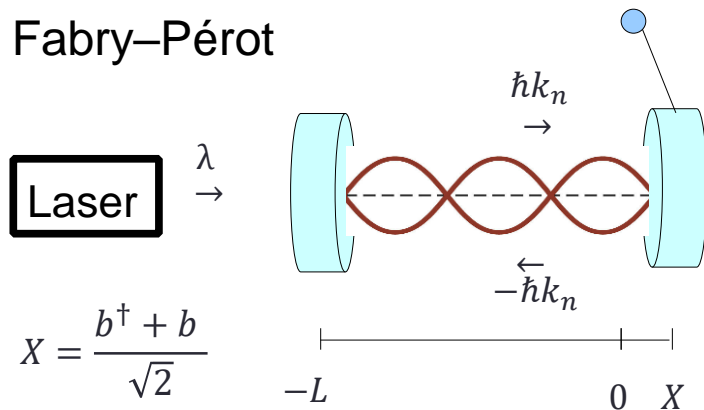
\*S. Bose, K. Jacobs and P. Knight, *PRA* **56**, 4175 (1997)





# The Radiation Pressure interaction

Fabry–Pérot



C. K. Law, PRA **51**, 2537(1995)

$$H = \hbar\omega_c a^\dagger a + \hbar\omega_m b^\dagger b + \hbar G a^\dagger a X + H_E$$

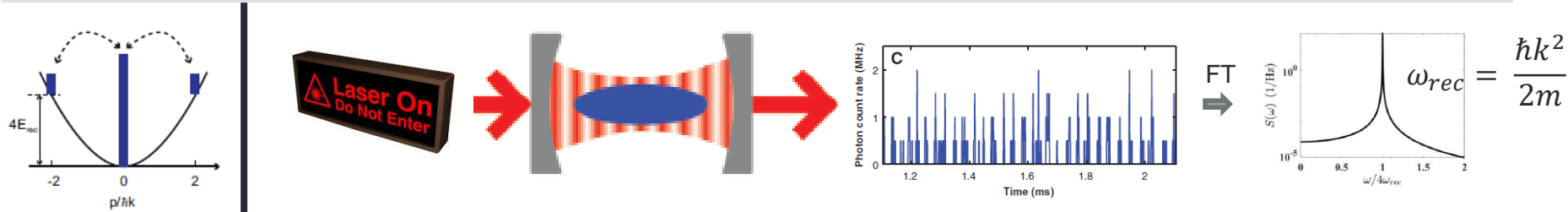
↗ optical mode      ↑ mechanical oscillator      ↑ radiation pressure interaction      ↖ environment

## TOOLBOX FOR MEASURING AND MANIPULATING MECHANICAL MOTION

1. Sub-quantum limit **detection** of mechanical motion..Nature Physics **15**, 745 (2019)
2. **Detection** of quantum back-action..... Science **339**,801(2013)
3. Back-action-evading measurement of motion.....Nat. Comm. **10**, 2086 (2019)
4. **Squeezing** of mechanical quadrature of motion.....Science **349**, 952 (2015)
5. Preparation of quantum mechanical **ground state**.....**MB**...PRL **99**, 073601 (2007)
6. **Storing** and **retrieving** optical information from mechanical elements.....PRL **107**, 133601 (2011)
7. Phonon **lasing**.....PRL **113**, 030802 (2014)
8. **Entangling** multiple mechanical oscillators.....Nature **556**, 478 (2018)



# Nondestructive Measurement of Linear BEC motion



## Cavity Optomechanics with a Bose-Einstein Condensate,

F. Brennecke et al., Science **322**, 235 (2007)

Real-time observation of fluctuations at the driven-dissipative Dicke phase transition, PNAS **110**, 11763 (2013)

## Dynamical Instability of a Bose-Einstein Condensate in an Optical Ring Resonator,

D. Schmidt et al., Phys. Rev. Lett. **112**, 115302 (2014)

## Observing chiral superfluid order by matter-wave interference,

T. Kock et al., Phys. Rev. Lett. **114**, 115301 (2019)

## Entangled light from Bose-Einstein

Condensates, H. T. Ng and S. Bose, New Journal of Physics **11**, 043009 (2009)

## Cavity QED with Quantum Gases

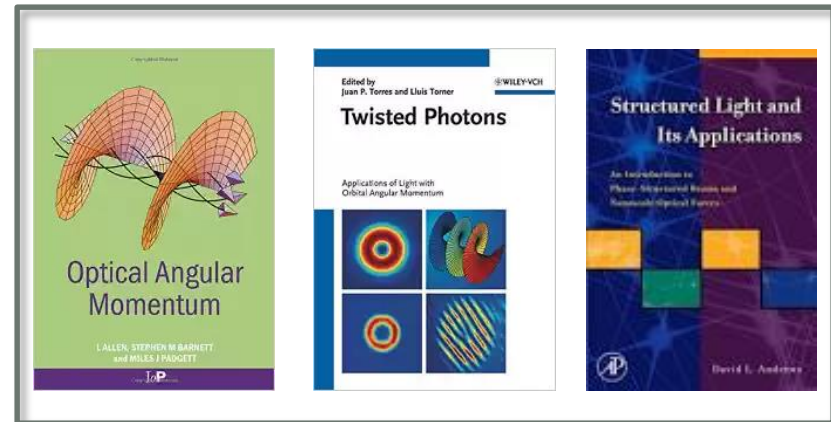
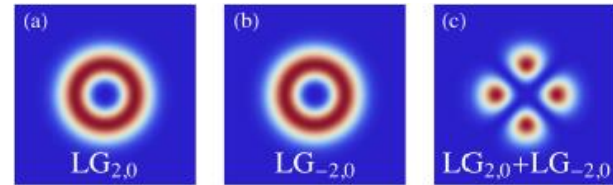
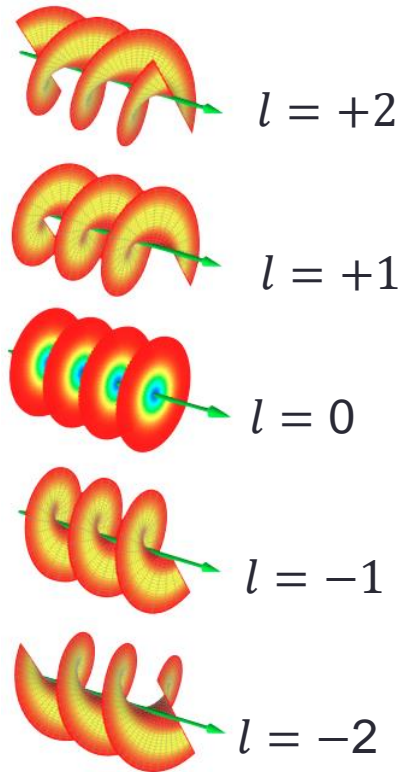
Mivehvar et al., Advances in Physics **70**, 1 (2021)

## Typical parameters

Atom number	$N$	$10^5$
Trap frequency	$\omega$	200 Hz
Atomic linewidth	$\gamma_0$	$2\pi \times 3 \text{ MHz}$
<b>Detuning</b>	$\Delta$	$10^4 \gamma_0$
Beam waist	$\omega_0$	$25 \mu\text{m}$
TF radius	$R$	$5 \mu\text{m}$
Cavity linewidth	$\kappa$	$2\pi \times 1.5 \text{ MHz}$
Coupling	$g_0$	$2\pi \times 10 \text{ MHz}$
<b>Cavity photon no.</b>	$\bar{n}$	1
Photon count rate	$S$	1 MHz
Recoil frequency	$\omega_{rec}$	$2\pi \times 4 \text{ kHz}$



# Beams with Orbital Angular Momentum

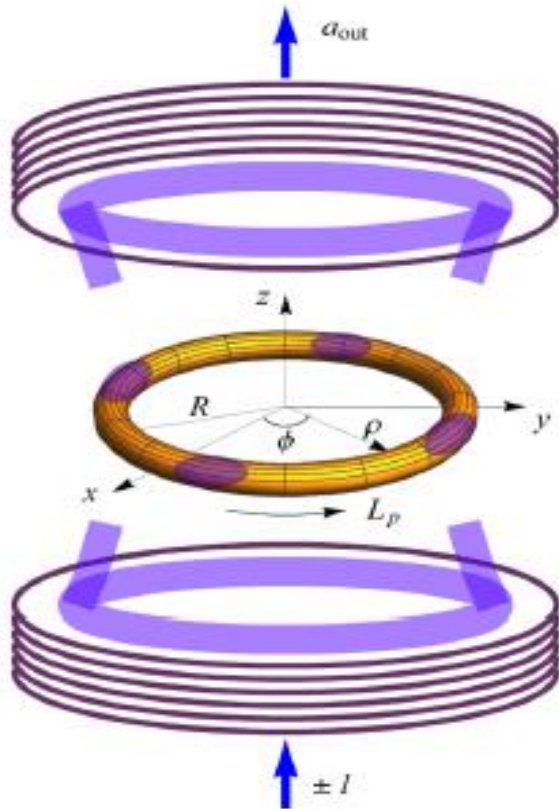


Mirhosseini et al., NJP **17**, 033033 (2015)  
 Leach et al., Science **329**, 662 (2010)

M. Padgett, Optics Express **25**, 11265 (2017)  
 MB and P. Meystre, PRL **99**, 153603 (2007)  
 H. Shi and MB, J. Phys. B **49**, 153001 (2016)

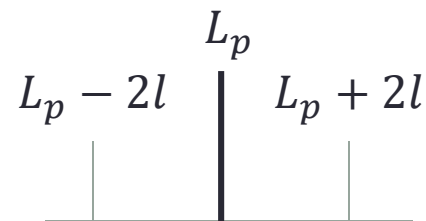
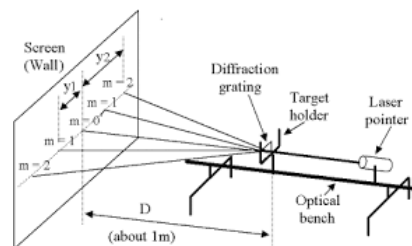


# Ring BEC in a cavity: Detecting $L_p$



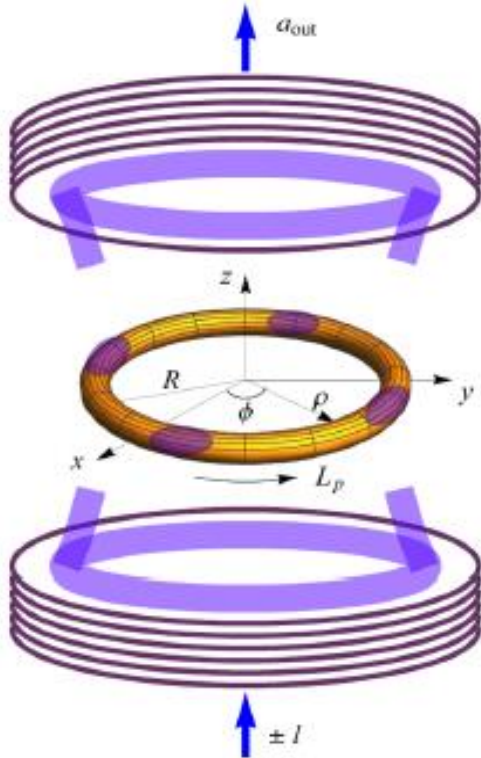
## Measurement Scheme

1. We start with a trapped ring BEC in a cavity.
2. The BEC has winding number  $L_p$ .
3. We drive the cavity with beams carrying OAM  $\pm l\hbar$  far detuned from any atomic transition
4. The BEC matter wave Bragg diffracts from the angular lattice:  $L_p \rightarrow L_p \pm 2l$
5. These matter waves beat to modulate the BEC density.
6. These modulations are picked up by the light transmitted by the cavity.
7. Cavity transmission yields  $L_p$ .





# Ring BEC in a cavity: Detecting $L_p$



**Hamiltonian:** 
$$H = \int_0^{2\pi} \Psi^\dagger(\phi) \left[ -\frac{\hbar^2}{2I} \frac{d^2}{d\phi^2} + \hbar U_0 \cos^2(l\phi) a^\dagger a \right] \Psi(\phi) d\phi$$

$$+ \frac{g}{2} \int_0^{2\pi} \Psi^\dagger(\phi) \Psi^\dagger(\phi) \Psi(\phi) \Psi(\phi) d\phi$$

$$- \hbar \Delta_0 a^\dagger a - i\hbar \eta (a - a^\dagger),$$

$$I = mR^2; U_0 = \frac{g_a^2}{\Delta_a}; g_a = E_0 \langle e|d|g \rangle; \eta = \sqrt{P_{in} \gamma_0 / \hbar \omega_0},$$

$$g = 2\hbar \omega_p a_{Na} / R$$

**Ansatz:** 
$$\Psi(\phi) = \frac{e^{iL_p\phi}}{\sqrt{2\pi}} c_p + \underbrace{\frac{e^{i(L_p+2l)\phi}}{\sqrt{2\pi}} c_+ + \frac{e^{i(L_p-2l)\phi}}{\sqrt{2\pi}} c_-}_{\text{Sidemodes}}$$

↓

Persistent current Sidemodes

**Normalization:** 
$$c_p^\dagger c_p + c_+^\dagger c_+ + c_-^\dagger c_- = N$$

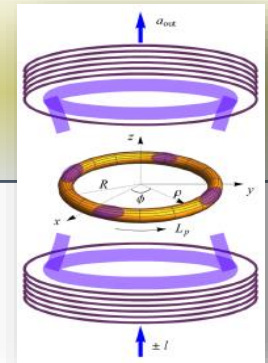
**Recognize:**  $c_p \sim \sqrt{N}$  is a macroscopically occupied mode.

$$H = \hbar \omega_c c^\dagger c + \hbar \omega_d d^\dagger d + \hbar [G(X_c + X_d) - \tilde{\Delta}] a^\dagger a - i\hbar \eta (a - a^\dagger) + \hbar g \tilde{F}$$

$$\omega_{c,d} = \frac{\hbar(L_p \pm 2l)^2}{2I} \quad \left( c = \frac{c_p^\dagger c_+}{\sqrt{N}}, d = \frac{c_p^\dagger c_-}{\sqrt{N}}, G = \frac{U_0 \sqrt{N}}{2\sqrt{2}}, \tilde{\Delta} = \Delta_0 - \frac{U_0 N}{2} \right)$$



# Detecting $L_p$ : Equations of Motion



## Collisionless Hamiltonian ( $g = 0$ )

$$H = \hbar\omega_c c^\dagger c + \hbar\omega_d d^\dagger d + \hbar[G(X_c + X_d) - \tilde{\Delta}]a^\dagger a - i\hbar\eta(a - a^\dagger)$$

## (Quantum Langevin) Equations of motion

$$\begin{aligned}\ddot{X}_c + \gamma_m \dot{X}_c + \omega_c^2 X_c &= -\omega_c G a^\dagger a + \omega_c \epsilon_c \\ \ddot{X}_d + \gamma_m \dot{X}_d + \omega_d^2 X_d &= -\omega_d G a^\dagger a + \omega_d \epsilon_d \\ \dot{a} &= i[\tilde{\Delta} - G(X_c + X_d)]a - \frac{\gamma_o}{2}a + \sqrt{\gamma_o}a_{in}\end{aligned}$$

## Heuristic argument (neglecting noise and damping)

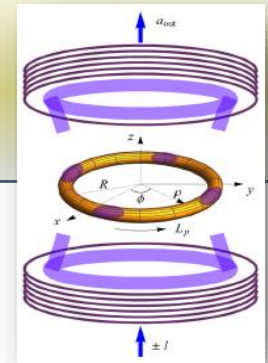
For weak optical driving,  $X_{c,d}$  will be modulated at  $\omega_{c,d}$ .  
And therefore  $a$  will also.

→ So the cavity output will show two peaks at  $\omega_{c,d}$ .



# Detecting $L_p$ : Experimental Parameters

1. Mass of Sodium atom	$m = 23 \text{ amu}$
2. Number of atoms	$N \sim 1.2 \times 10^4$
3. BEC radius	$R = 12 \mu\text{m}$
4. Trap frequencies	$\omega_{\rho,z} = 2\pi \times 4 \times 10^{1-2} \text{ Hz}$
5. Temperature	$T = 10 - 20 \text{ nK}$
6. Chemical potential	$\frac{\mu}{\hbar} = 200 - 500 \text{ Hz}$
7. Winding number of BEC	$L_p = 1 - 5$
8. OAM of light	$l = 1 - 15$
9. Atom velocity	$v_a \sim 0.3 \text{ mm/s}$
10. Sound velocity	$v_c \sim 1.5 \text{ mm/s}$
11. Healing length	$\xi = 4 \mu\text{m}$
12. Atomic detuning	$\Delta_a = 2\pi \times 4.7 \text{ GHz}$
13. Mechanical damping	$\gamma_m \sim 2\pi \times 1 \text{ Hz}$
14. Cavity linewidth	$\gamma_0 = 2\pi \times 1.3 \text{ MHz}$
15. Cavity detuning	$\Delta' = 0 - 500 \text{ Hz}$
16. Cavity finesse	$\sim 10^4$
17. Optomechanical coupling	$G = 2\pi \times 260 \text{ kHz}$
18. Input power	$P_{in} \sim 10 \text{ fW}$
19. Angular lattice recoil	$\leq 1 \text{ KHz}$



P. Kumar et al.,  
PRL **127**,  
113601 (2021)

S. Kalita et al.,  
PRA **107**,  
013525 (2023)



# Detecting $L_p$ : Noise Spectrum

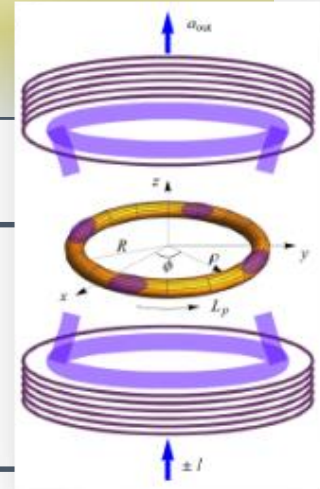
Linear response  $\mathcal{M} \rightarrow \mathcal{M}_s + \delta\mathcal{M}$  for  $\mathcal{M} = X_c, X_d, a$

Phase quadrature

$$P = \frac{i(a_{out} - a_{out}^\dagger)}{\sqrt{2}}$$

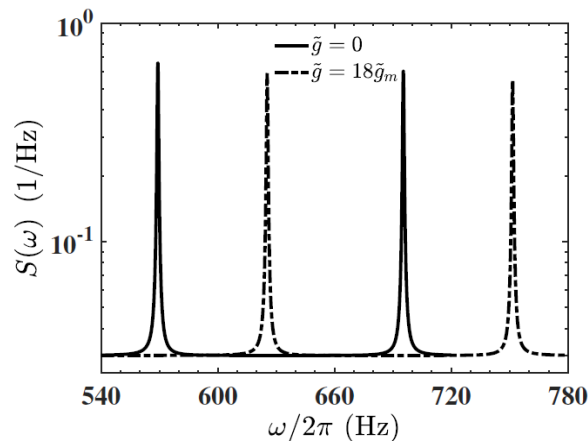
Input-output relation

$$a_{out} = -a_{in} + \sqrt{\gamma_0} a$$



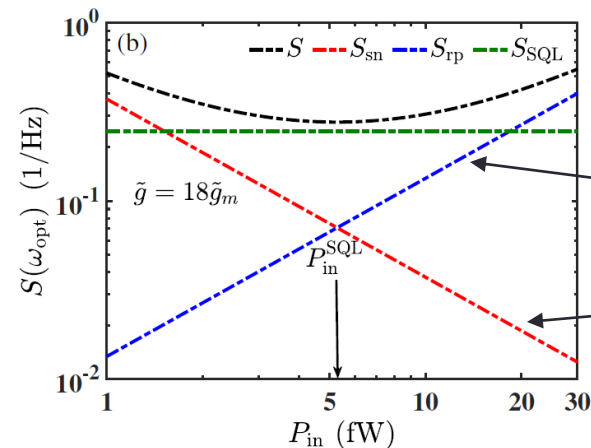
## Output Noise Spectrum

$$S(\omega) = \int_{-\infty}^{+\infty} dt e^{i\omega t} \langle \delta P(t) \delta P(t) \rangle$$



$$\omega_c - \omega_d \cong \frac{4lL_p \hbar}{I}$$

$$S(\omega) = S_{sn}(\omega) + S_{rp}(\omega) + S_{th}(\omega)$$



Backaction noise

Shot noise



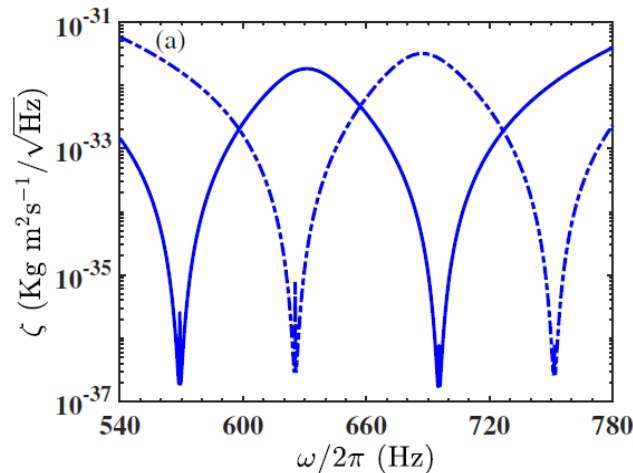
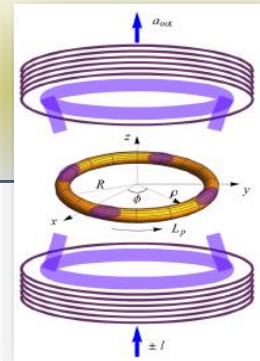


# Detecting $L_p$ : Sensitivity

$$\zeta = \frac{S(\omega)}{\partial S(\omega)/\partial \Lambda} \sqrt{t_{meas}}$$

$$\Lambda = \hbar L_p$$

$$t_{meas} = \frac{\gamma_0}{8(a_s G)^2} \sim 60 \text{ ms} \ll 1 \text{ min}$$



Number of atoms  
in the sidemodes  
 $\sim 10$

$\sim$  Real time

Minimally  
destructive

$10^{-3} \text{ Hz}/\sqrt{\text{Hz}} \rightarrow$  **Thousand times more sensitive than existing methods.**

Kumar et al., New Journal of Physics **18**, 025001 (2016)

Ragole and Taylor, PRL **117**, 203002 (2016)

S. Safaei et al, PRA **100**, 013621 (2019)



# Matter wave (bipartite) entanglement

## Logarithmic negativity

$$\mathcal{E}_{\mathcal{N}} = \max[0, -\ln(2\sigma_-)]$$

$$\sigma_- = 2^{-1/2} \left[ \Sigma - \sqrt{\Sigma^2 - 4\det(V_{sub})} \right]^{1/2}$$

$$\Sigma = \det A + \det B - 2\det C$$

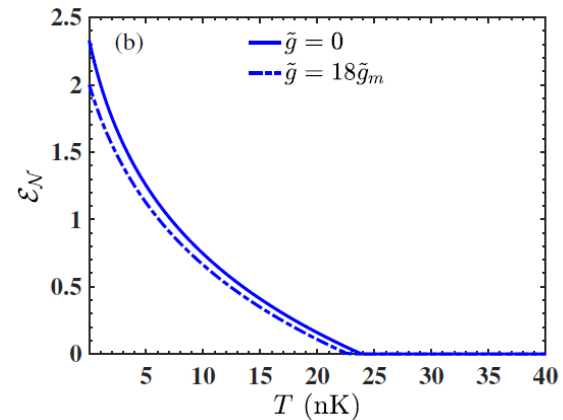
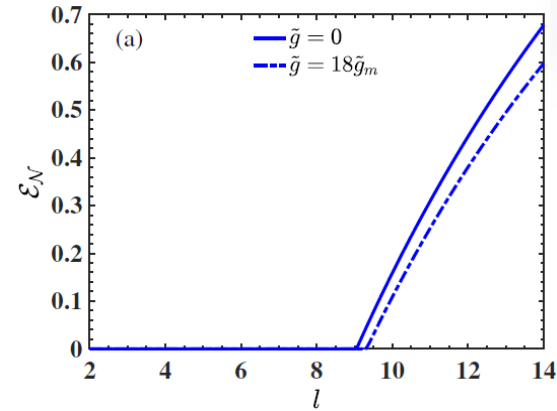
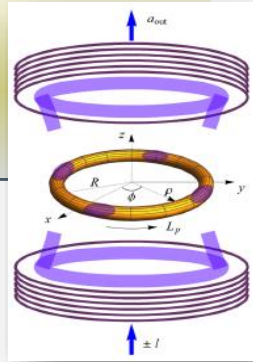
$$V_{sub} = \begin{pmatrix} A & C \\ C^T & B \end{pmatrix}$$

$$V_{X_c X_c} = \langle \delta X_c \delta X_c \rangle$$

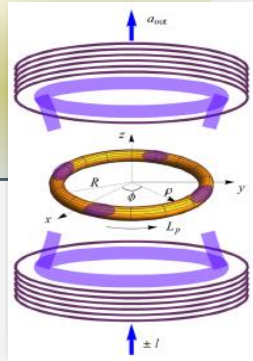
$$FV + VF^T = -D$$

$$F = \begin{pmatrix} 0 & \Omega_c & 0 & 0 & 0 & 0 \\ -\Omega_c & -\gamma_m & -\frac{A}{\Omega_c} & 0 & -\frac{\tilde{\omega}_c G_r}{\Omega_c} & -\frac{\tilde{\omega}_c G_i}{\Omega_c} \\ 0 & 0 & 0 & \Omega_d & 0 & 0 \\ \frac{A}{\Omega_d} & 0 & -\Omega_d & -\gamma_m & -\frac{\tilde{\omega}_d G_r}{\Omega_d} & -\frac{\tilde{\omega}_d G_i}{\Omega_d} \\ G_i & 0 & G_i & 0 & -\frac{\gamma_e}{2} & -\Delta' \\ -G_r & 0 & -G_r & 0 & \Delta' & -\frac{\gamma_o}{2} \end{pmatrix}$$

$$D = \text{diag} \left( 0, \gamma_m(2n_c + 1), 0, \gamma_m(2n_d + 1), \frac{\gamma_o}{2}, \frac{\gamma_o}{2} \right)$$



# Stochastic Gross-Pitaevskii treatment



$$(i - \Gamma) \frac{d\psi}{d\tau} = \left[ -\frac{d^2}{d\phi^2} + \frac{U_0}{\omega_\beta} |\alpha|^2 \cos^2(l\phi) - \mu + 2\pi \frac{\gamma}{N} |\psi|^2 \right] \psi + \xi(\phi, \tau)$$

$$i \frac{d\alpha}{d\tau} = \left\{ -\left[ \Delta_c - U_0 \langle \cos^2(l\phi) \rangle_\tau + i \frac{\gamma_0}{2} \right] \alpha + i\eta \right\} \omega_\beta^{-1} + i\sqrt{\gamma_0} \omega_\beta^{-1} a_{in}$$

**Scales:**

$$\hbar\omega_\beta = \frac{\hbar^2}{2mR^2}, \quad \tau = \omega_\beta t, \quad \gamma = \frac{gN}{2\pi\hbar\omega_\beta}$$

**Noise correlations:**

$$\langle \xi(\phi, \tau) \xi^*(\phi, \tau) \rangle = 2\Gamma T \delta(\phi - \phi') \delta(\tau - \tau')$$

$$\langle a_{in}(\tau) a_{in}^*(\tau') \rangle = \omega_\beta \delta(\tau - \tau')$$

- Reminders:**
1.  $\omega_{c,d} \ll \gamma_0$  (bad cavity limit)
  2.  $(\mu, k_B T) < \hbar(\omega_r, \omega_z)$  (1D BEC)  $\rightarrow$
  3.  $U_0 |\alpha|^2 < \mu$  (permeable barrier)

\*PRA 74,  
023617  
(2006)



# Gross-Pitaveskii treatment: Persistent Current

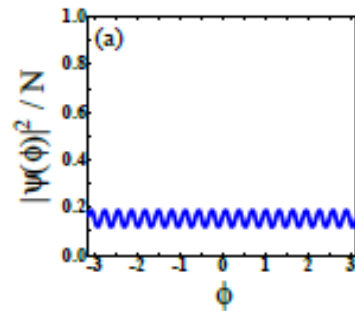
$$\psi(\phi) = \sqrt{\frac{N}{2\pi}} e^{iL_p\phi}$$

Atomic interactions are weakly **repulsive**

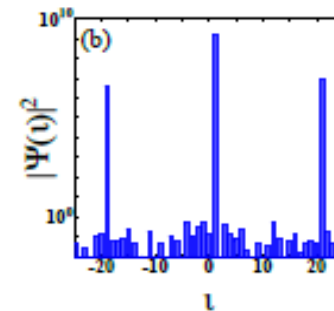
$^{23}\text{Na}$

Ryu et al.,  
PRL **99**, 260401 (2007)

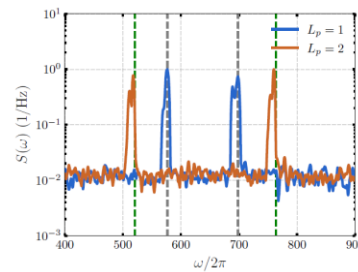
Condensate density



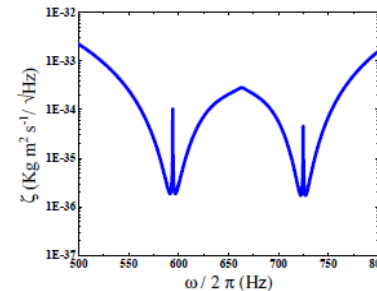
OAM content



Cavity transmission

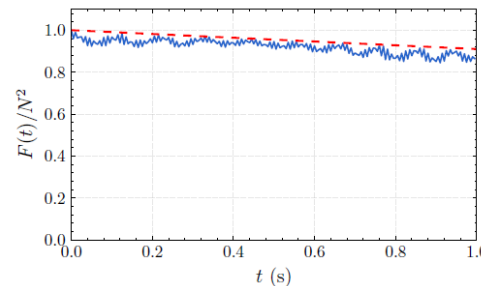


Measurement sensitivity

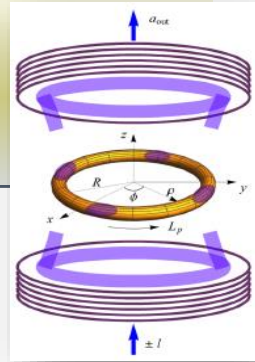


Condensate density autocorrelation

$$F(t) = \int_0^{2\pi} [\psi^*(\phi, t)\psi(\phi, 0)]^2 d\phi$$



\*N. Pradhan et al.,  
Physical Review Research **6**,  
013104 (2024)





# Gross-Pitaveskii treatment: Superposition

$$\psi(\phi) = \sqrt{\frac{N}{4\pi}} (e^{iL_{p1}\phi} + e^{iL_{p2}\phi})$$

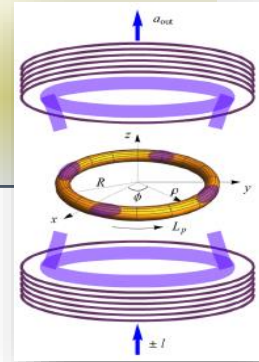
Aghamalyan et al  
2015 New J. Phys. 17 045023

Atomic interactions are weakly **repulsive**

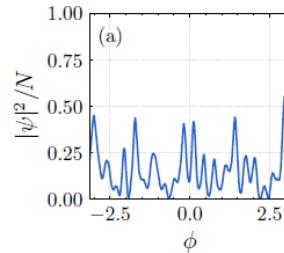
<sup>23</sup>Na

Ryu et al.,  
PRL **99**, 260401 (2007)

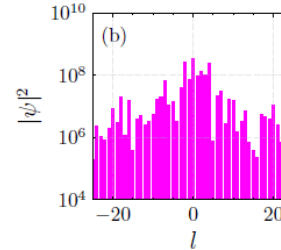
\*N. Pradhan et al.,  
Physical Review Research **6**,  
013104 (2024)



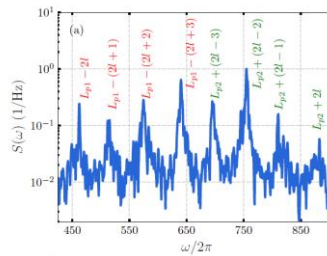
Condensate density



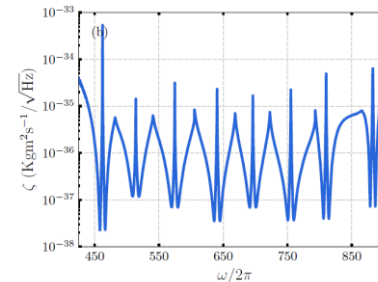
OAM content



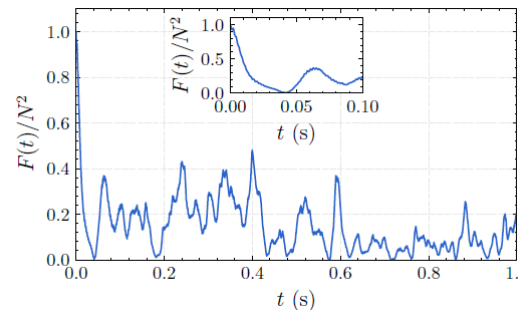
Cavity transmission



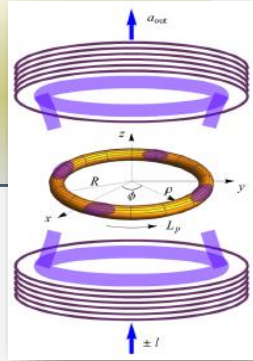
Measurement sensitivity



Condensate density autocorrelation



# Gross-Pitaveskii treatment: Bright Soliton



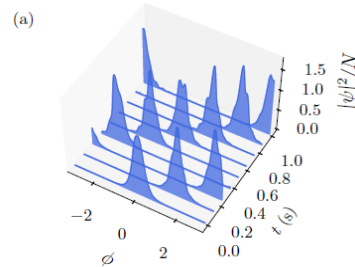
$$\psi(\phi) = \sqrt{\frac{N}{\sqrt{\pi}}} e^{-\phi^2/2} e^{iL_p\phi}$$

$$L_p = 1$$

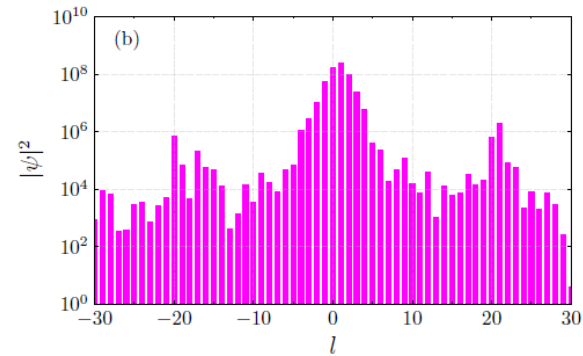
Atomic interactions are **attractive**



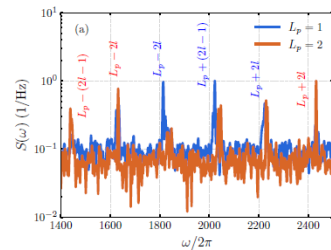
Condensate density



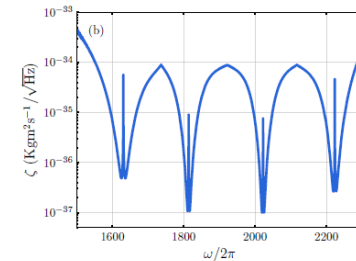
OAM content



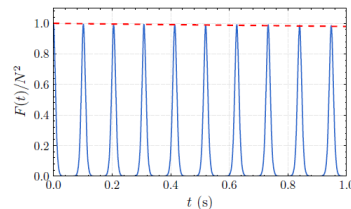
Cavity transmission



Measurement sensitivity



Condensate density autocorrelation



\* Helm et al.,  
PRL **114**,  
134101 (2015)

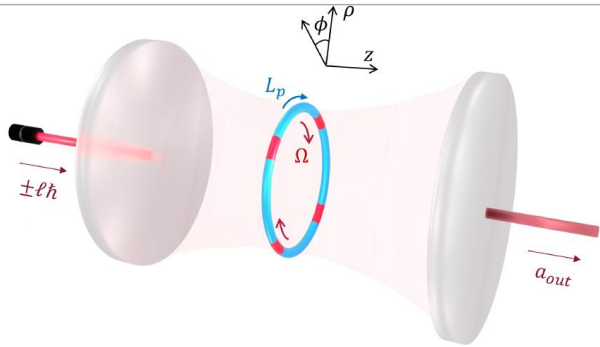
\* Cai et al.,  
PRL **128**,  
150401 (2022)

\* Pace et al.,  
PRX **12**,  
041037 (2022)

\*N. Pradhan et al.,  
Physical Review Research **6**,  
013104 (2024)



# Detecting Chirality: Lattice Rotation



## Lattice rotation

$$\cos^2(l\phi) \rightarrow \cos^2(l\phi + \Omega t)$$

## Sidemode frequencies

$$\omega_{c,d}(\Omega) = \omega_{\beta} \left( L_p \pm 2\ell - \frac{\Omega'}{2} \right)^2$$

$$\omega'_{c,d} = [\omega_{c,d} (\omega_{c,d} + 4\tilde{g}N)]^{1/2}$$

$$\Omega < \frac{v_s}{2\pi R}$$

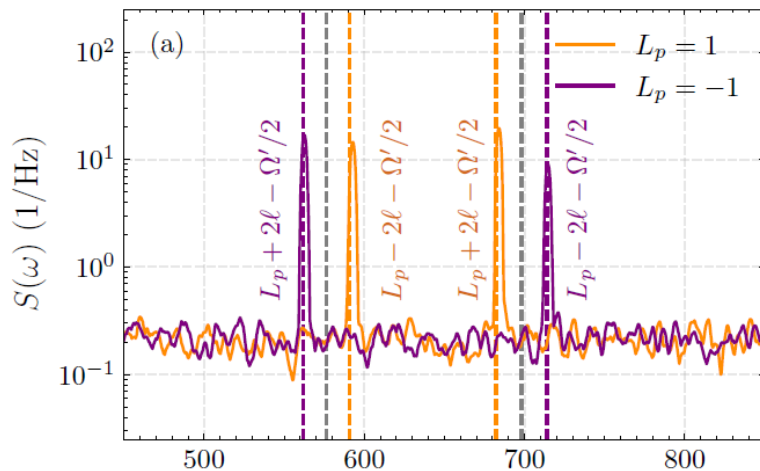
$$v_s = \sqrt{\mu/m}$$

Non-interacting

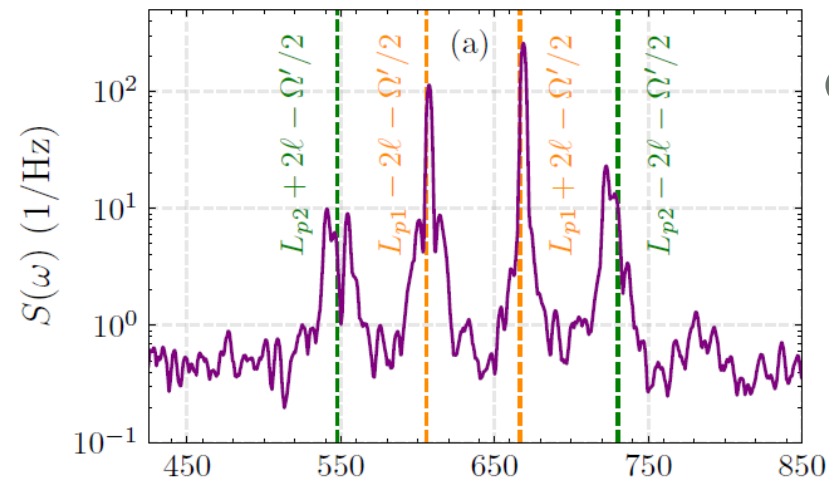
Interacting

\*N. Pradhan et al.,  
Physical Review A **109**,  
023524 (2024)

## Rotational eigenstate



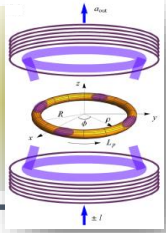
## Counter-rotating superposition



$$\Omega' = 0.5$$

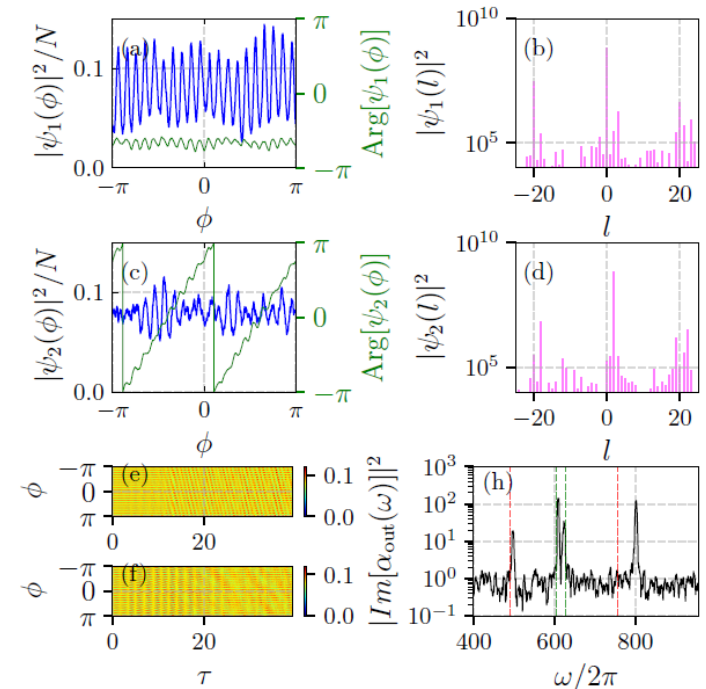
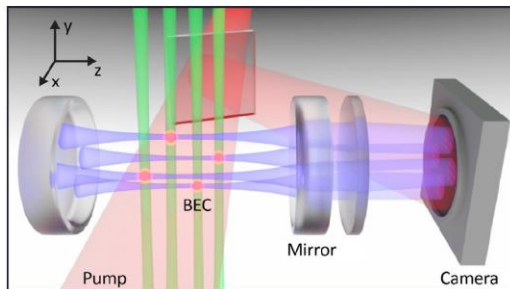


# Future Work: Creating a Stir



- Andreev-Bashkin effect (superfluid drag) →
- Detecting supercurrents in the BCS-BEC crossover
- Atomic memory for photons carrying OAM
- Entangling, squeezing, synchronizing rotating matter waves
- Regenerative (laser) action
- Sub - SQL measurements ([arXiv:2402.19123](https://arxiv.org/abs/2402.19123))

\*Coherent and reversible interface between optics and atomic superfluid rotation.



Marsh et al.,  
PRX 11, 021048(2021)

THANKS !!!



CAREER

