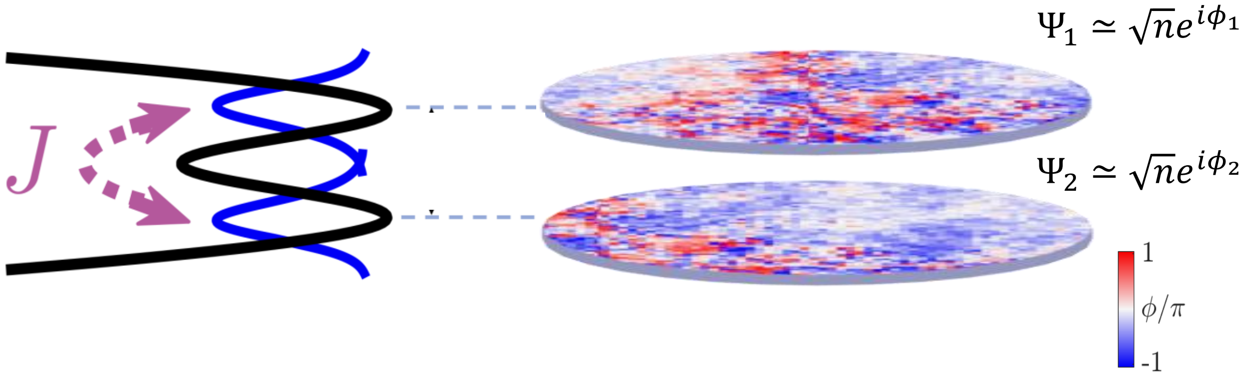
A double-well potential by dressing ultracold atoms with multiple RFs

UNIVERSITY OF OXFORD



Oxford team: Chris Foot (PI),
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Collaborators: Ludwig Mathey (Hamburg), Vijay Singh(Abu Dhabi), Fabian Essler (Oxford)

Benasque, May 2024

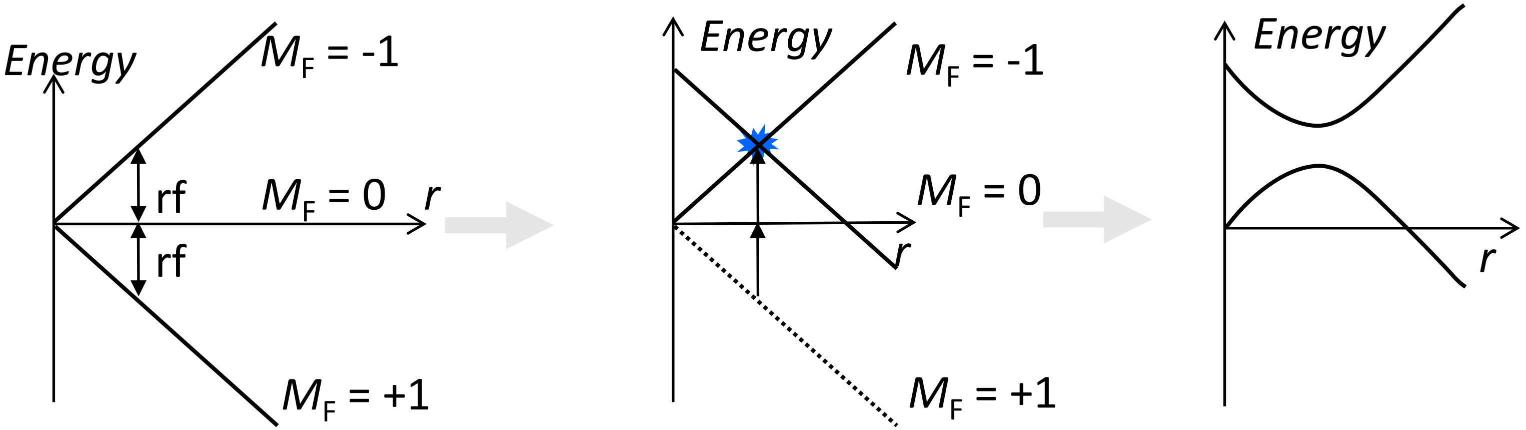
Engineering and Physical Sciences Research Council

Outline

- Double well for RF-dressed atoms: methodology
  - T. Harte, E. Bentine, … , C.J. Foot, PRA 97 013616 (2017)
- Matter-wave interferometry of 2D Bose gases using the double-well potential
  - S. Sunami, V. Singh, …, L. Mathey, C.J. Foot, PRL 128, 250402 (2022)
- Quenching the Kosterlitz-Thouless (BKT) superfluid
  - A. Barker, S. Sunami, … C.J. Foot, NJP 22 103040 (2020)
  - S. Sunami, V. Singh, … L. Mathey, C.J. Foot, Science 382, 443 (2023)
- Experiments with bilayer 2D Bose gases, including effects of disorder
- Ways to improve RF-dressed traps and Outlook

Magnetic trapping potential modified by applied RF radiation.

Eigenenergies for the F=1 hyperfine level of Rb-87 in an inhomogeneous B-field.

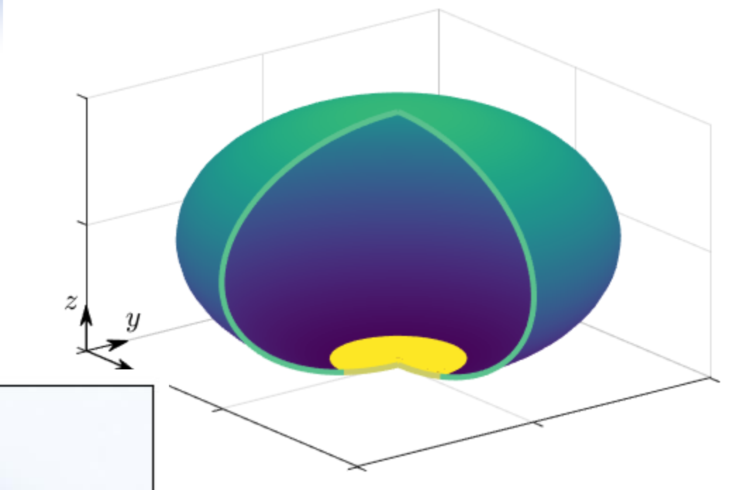


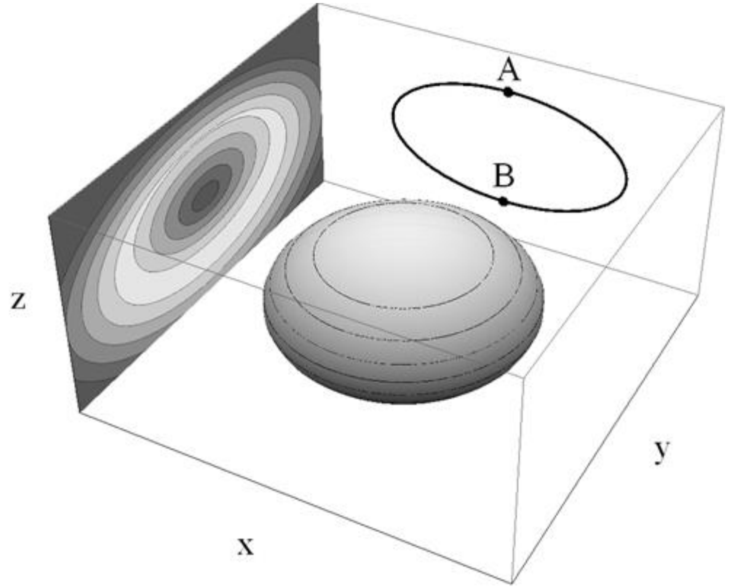
a) Eigenenergies of magnetic potential b) dressed-atom picture c) avoided level crossing

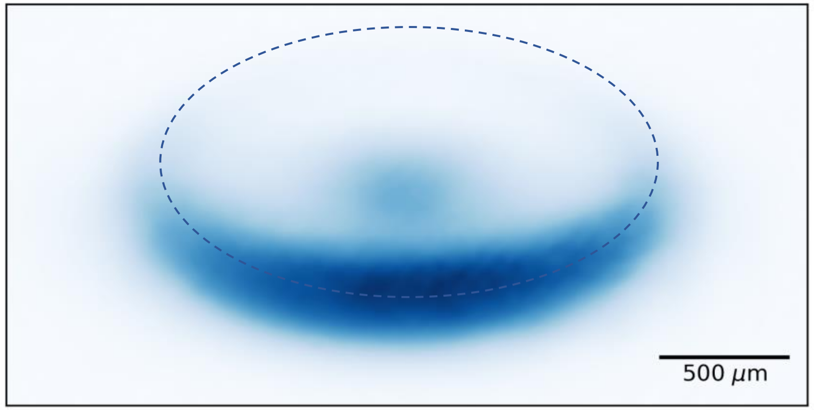
Dressed atoms are confined on a contour of constant magnetic field, $|\mathbf{B}|$ = const. where the RF is resonant with the Zeeman splitting.

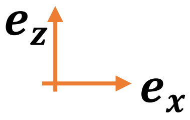
RF-dressed atoms in a quadrupole field

- RF + quadrupole field, $B_{\text{quad}} = B'\left(x\boldsymbol{e_x} + y\boldsymbol{e_y} - 2z\,\boldsymbol{e_z}\right)$
- Resonance occurs on the surface of a spheroid: $hf_{RF} = g_F \mu_B |B_{\text{quad}}|$
- Force of gravity breaks symmetry $\Rightarrow$ atoms accumulate at bottom





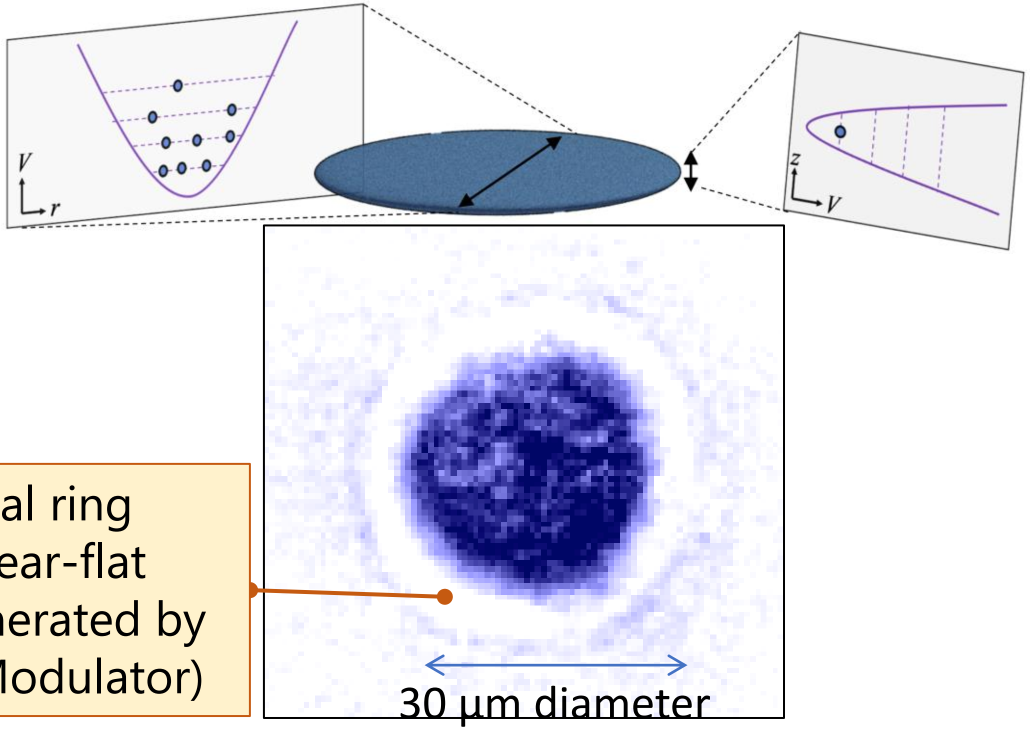




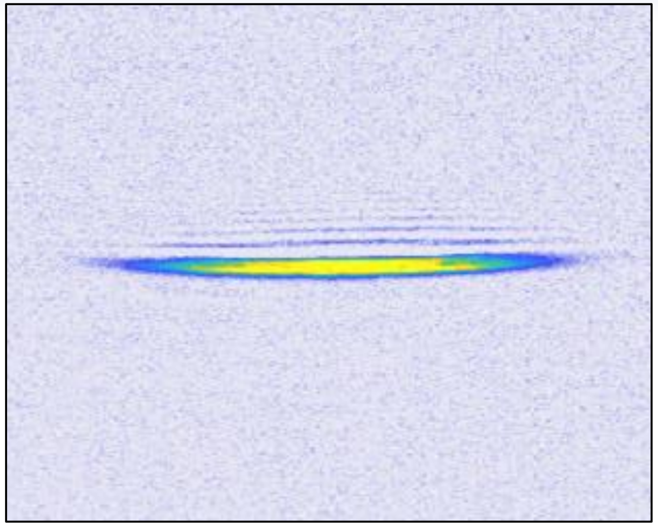
From Abel Beregi, D.Phil. thesis, Oxford (2023)

- Highly anisotropic confinement $\Rightarrow$ quasi-2D potentials: $f_z > 1$ kHz, $f_r < 10$ Hz.

Making a 2D quantum gas in the RF-dressed potential



repulsive optical ring potential for near-flat geometry (generated by Spatial Light Modulator)

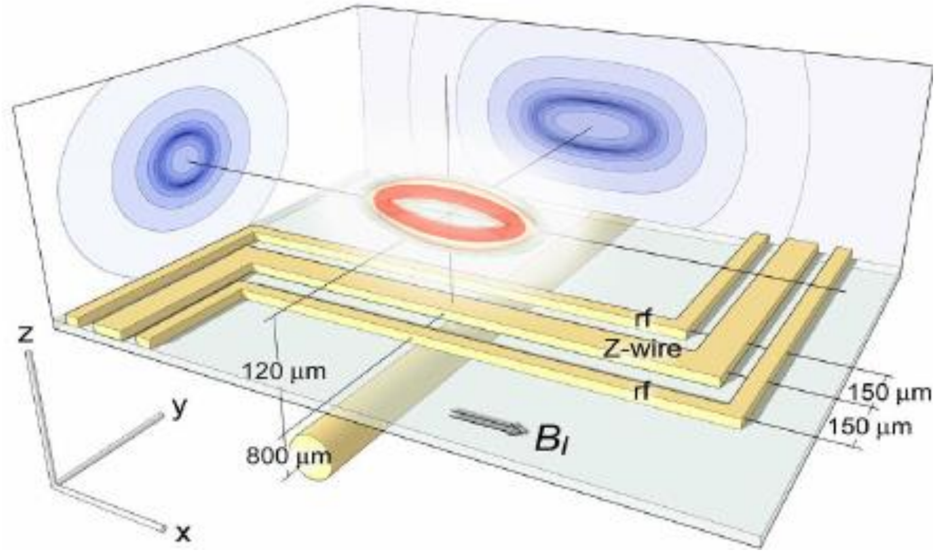


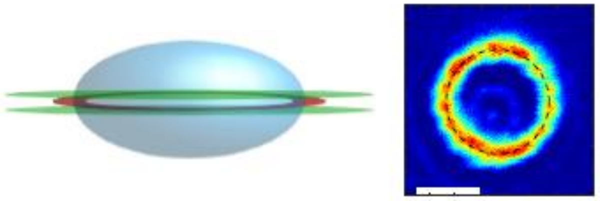
$$\omega_z/2\pi = 1\,\mathrm{kHz}$$
$$\omega_r/2\pi = 11\,\mathrm{Hz}$$

- Cold atomic gas cooled to nanokelvin regime (Temperature = 30 - 50 nK)
- 10^5 rubidium atoms (10 atoms per μm^2)
- Ground state occupied for z-direction, excited states populated in x, y
- Kinematically constrained to 2D, while interaction is 3D (s-wave scattering; ‘quasi-2D’)

Other RF-dressing experiments

Atom chip: 1D confinement

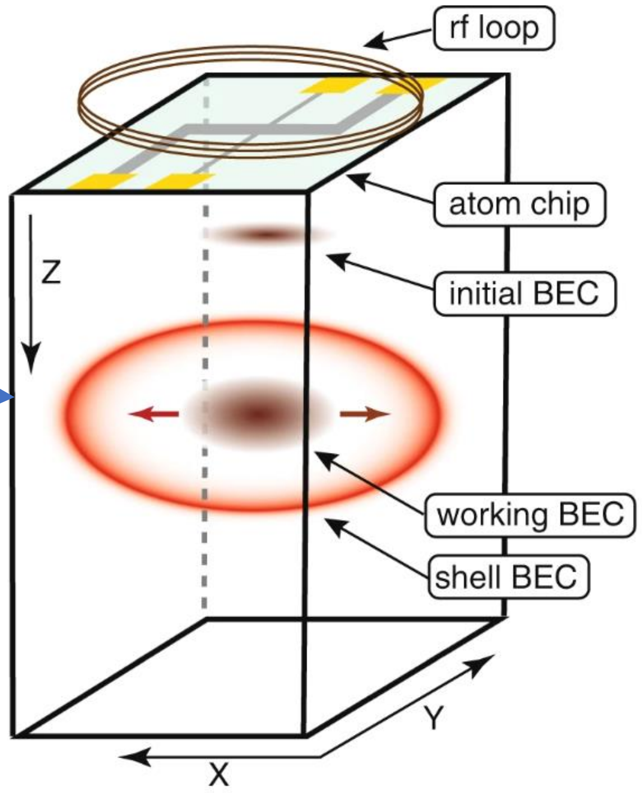




Ring trap.
Hélène Perrin, Romain Dubessy…
Université Sorbonne Paris Nord

NASA experiment. Lundblad et al.
(chip + quad field)

Barry Garraway’s talk



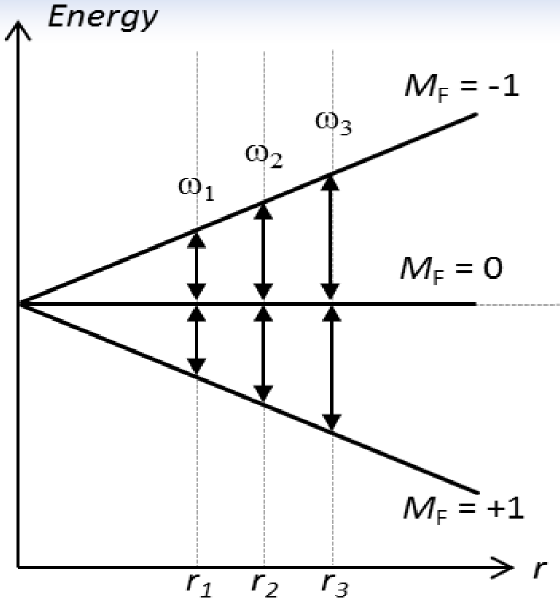
Box traps on an atom chip for one-dimensional quantum gases.
J. van Es, … N van Druten, J. Phys. B: 43,155002 (2010)

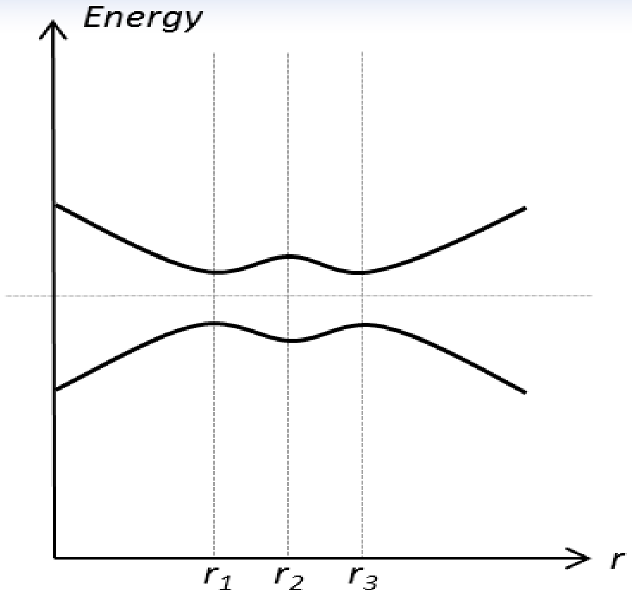
Vienna group – investigation of 1D quantum gases

- *Experimental observation of a generalized Gibbs ensemble*. T. Langen, … J. Schmiedmayer, *Science* **348**, 207 (2015)
- *Experimental characterization of a quantum many-body system via higher-order correlations.* T. Schweigler, … J. Schmiedmayer, *Nature* **545**, 323 (2017).

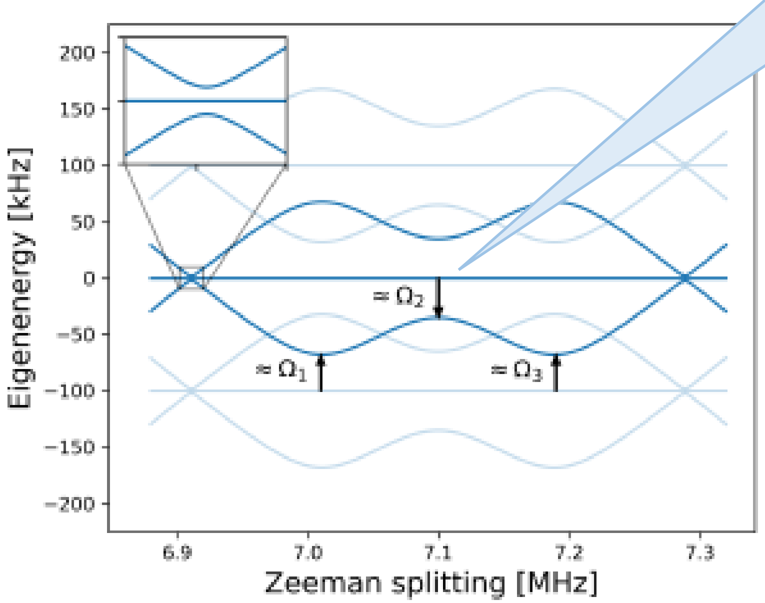
Recent developments in trapping and manipulation of atoms with adiabatic potentials. [Review article]
B. Garraway & H. Perrin, J. Phys. B 49, 172001 (2016)

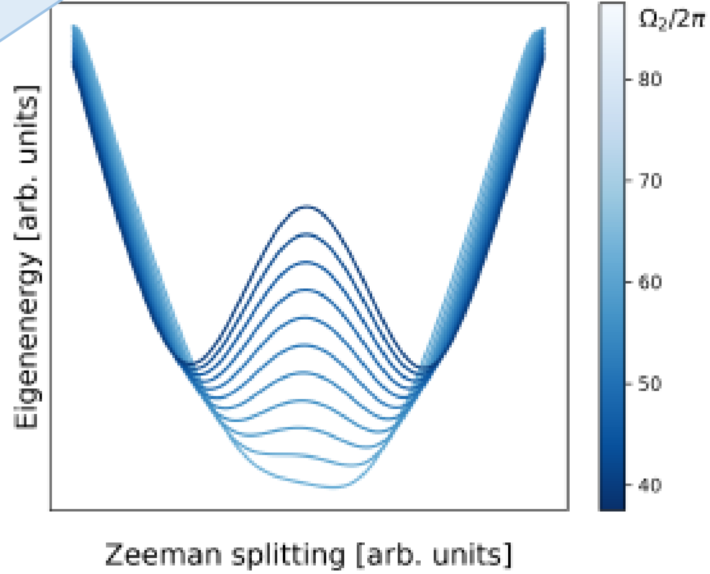
Multiple-RF dressing ⇒ double-well potential





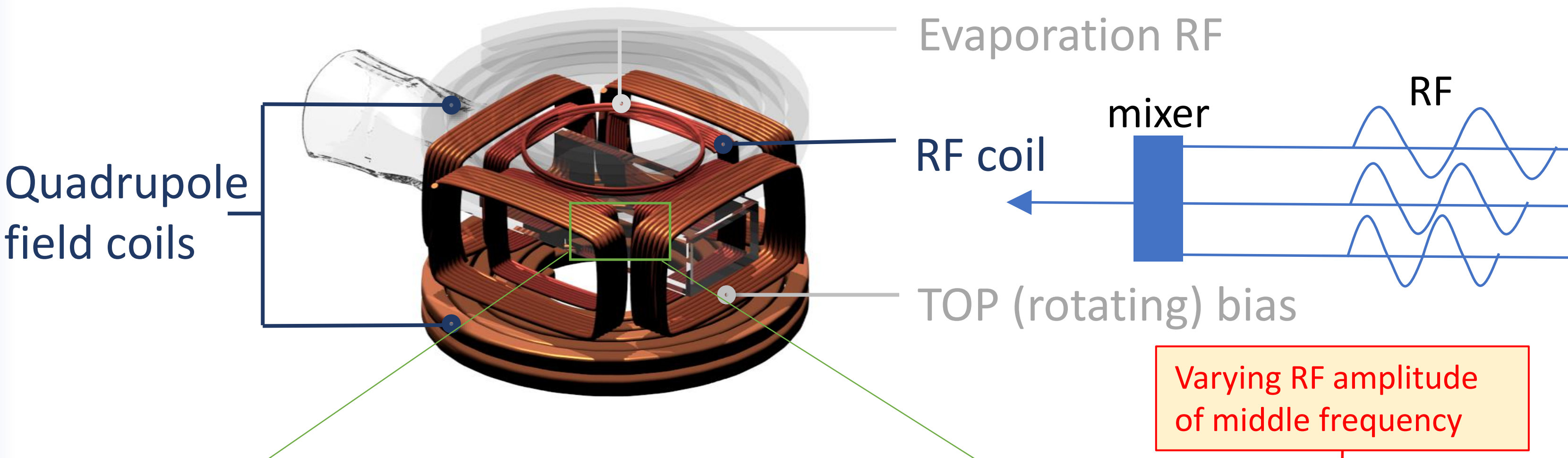
Single → Double well: change amplitude RF at ω_2 (Rabi frequency Ω_2). Precisely controllable.



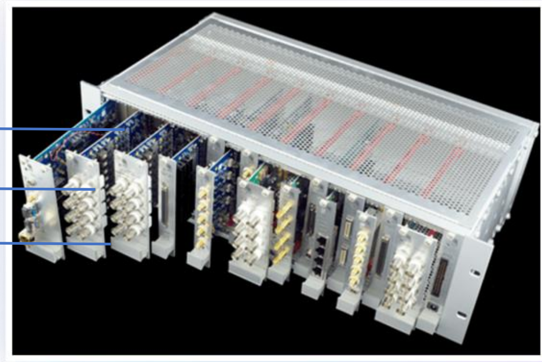


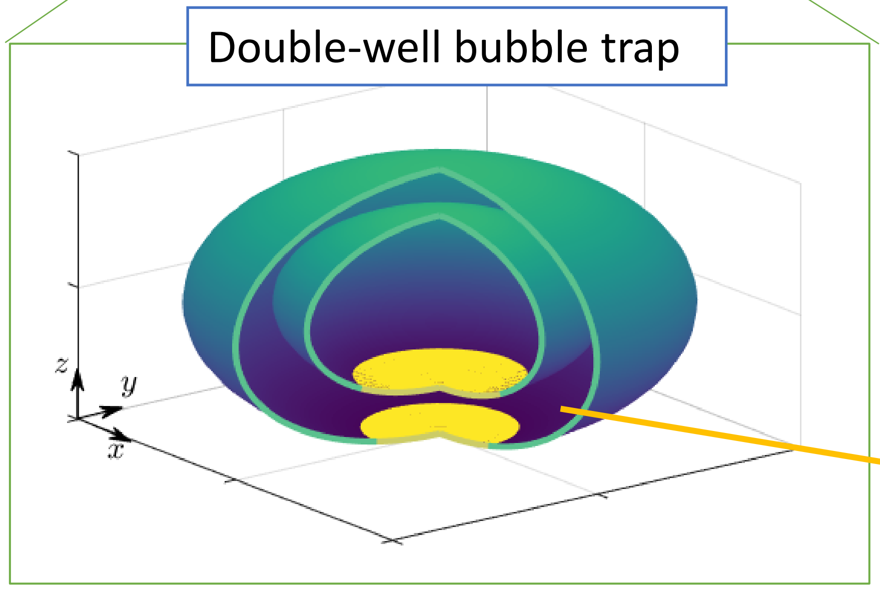
From Abel Beregi D.Phil. thesis, Oxford (2023)

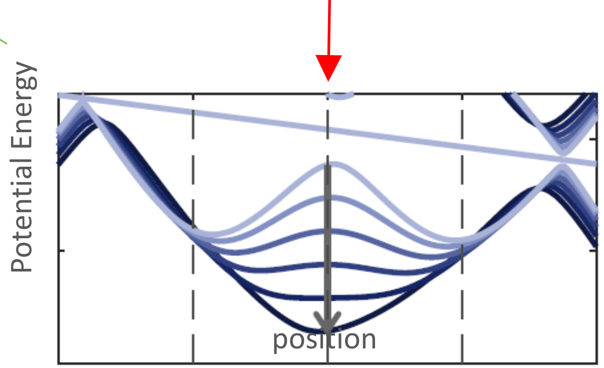
Experimental apparatus: multiple-RF dressed potential

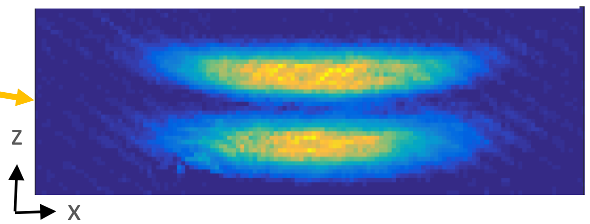


Varying RF amplitude of middle frequency




Double-well bubble trap



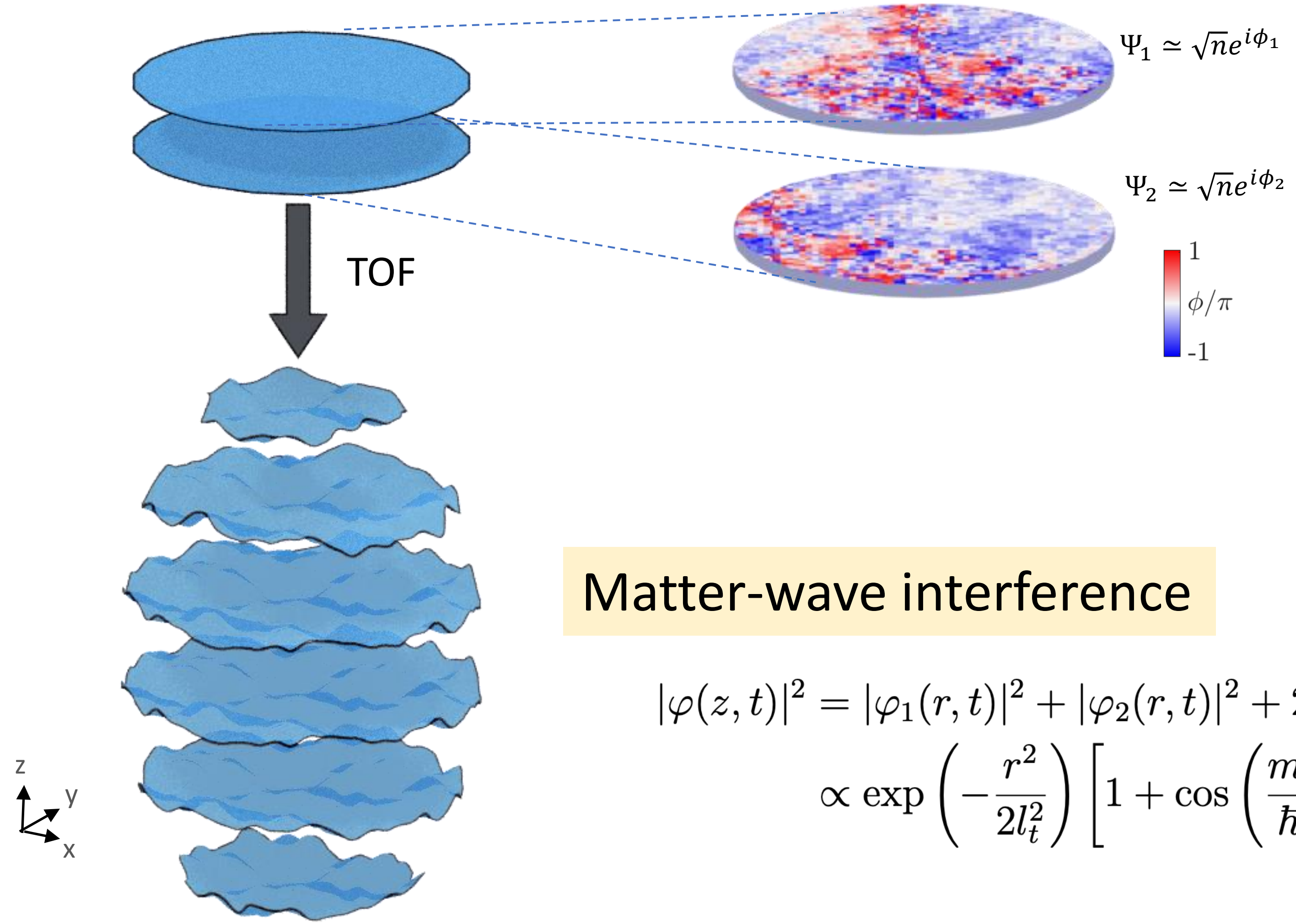


Controllability:

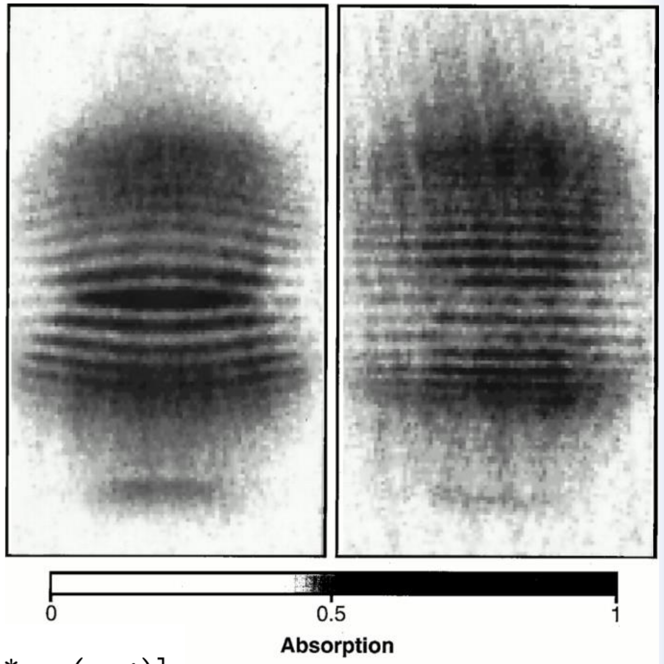
- RF frequencies
  → *z*-position, well separation
- RF amplitudes
  → trap frequency, trap shape
- RF polarizations
  → anisotropy of each cloud

T. Harte,... C. J. Foot.
PRA 97, 013616 (2017)

Matter-wave interferometry



First matter-wave interferometry of BEC @MIT. Science 275, 637 (1997)

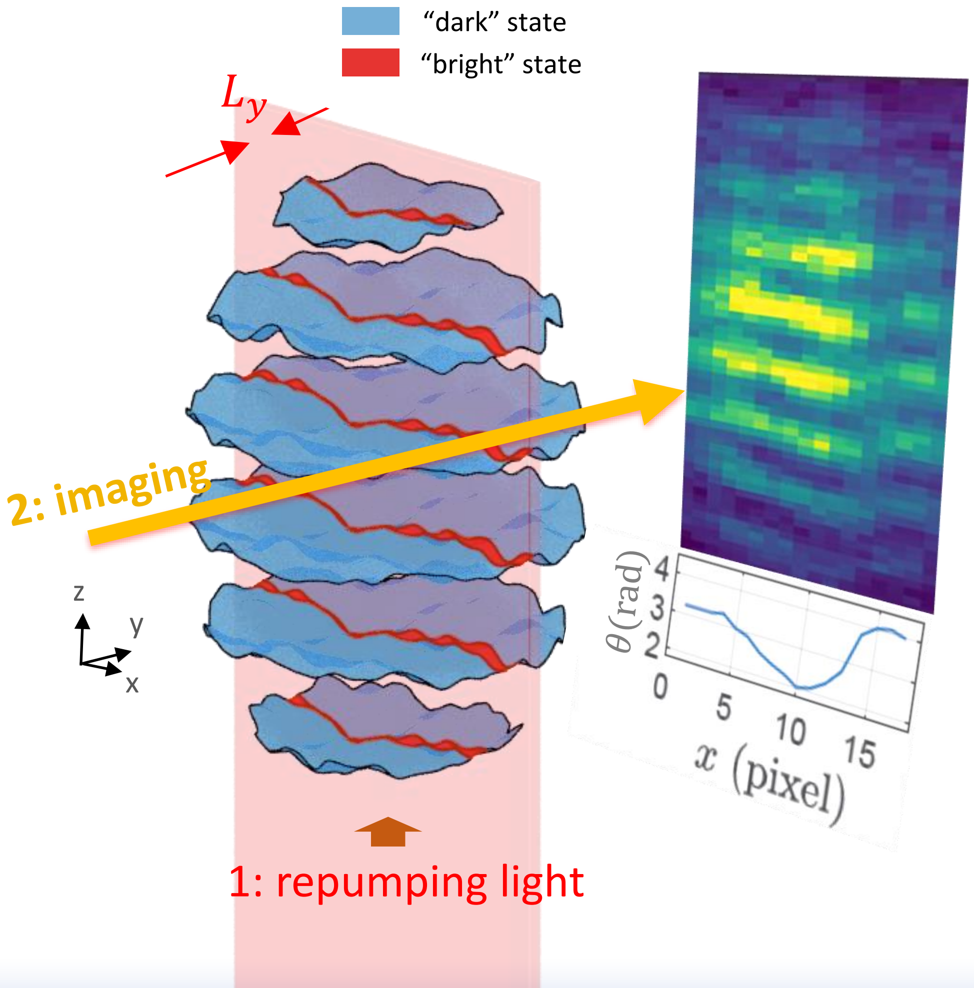


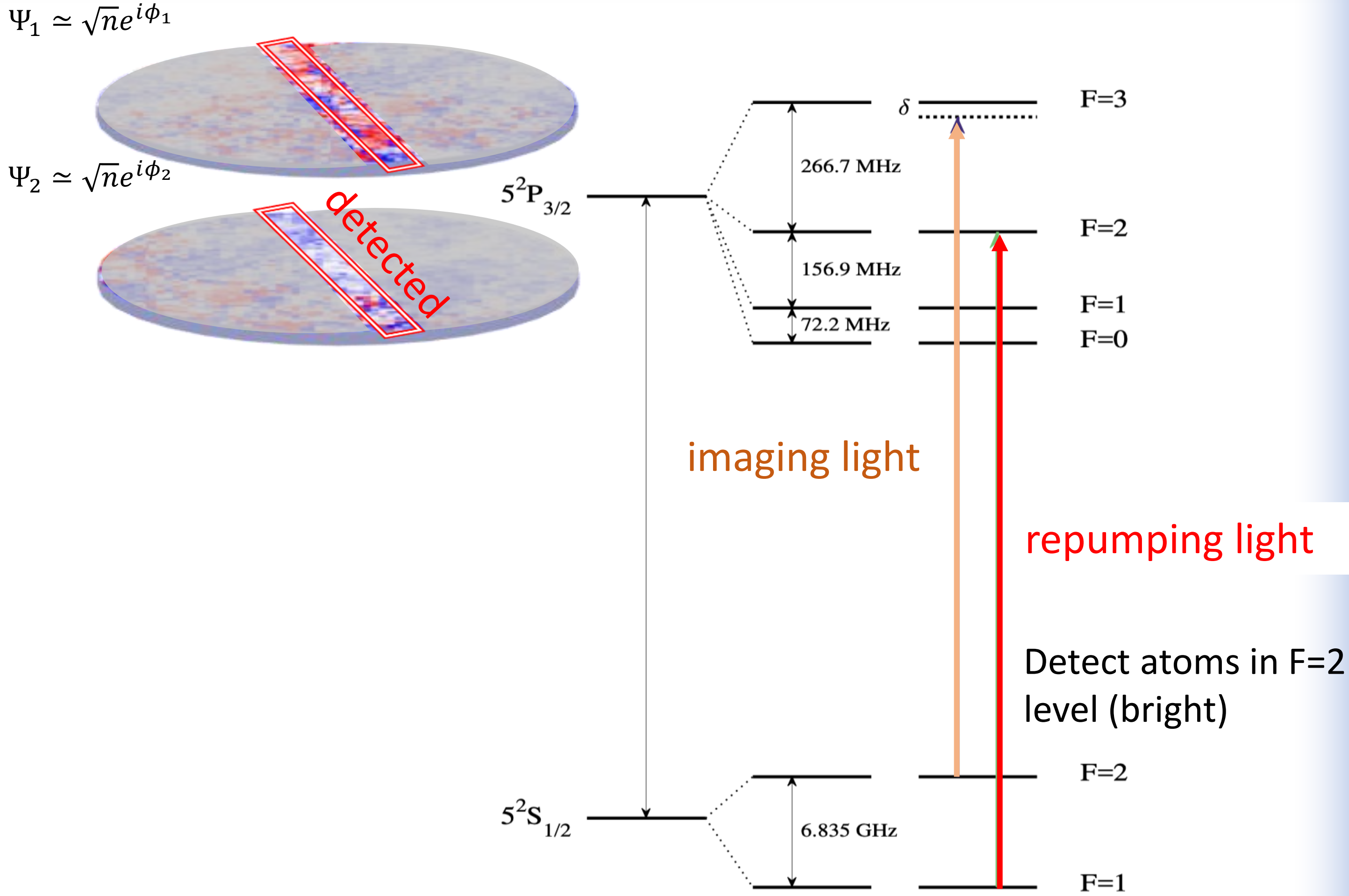
Matter-wave interference

$$\begin{aligned}|\varphi(z,t)|^2 &= |\varphi_1(r,t)|^2 + |\varphi_2(r,t)|^2 + 2\,\mathrm{Re}[\varphi_1(r,t)^*\varphi_2(r,t)],\\ &\propto \exp\left(-\frac{r^2}{2l_t^2}\right)\left[1+\cos\left(\frac{md}{\hbar t}z+\underbrace{\phi_1-\phi_2}\right)\right],\end{aligned}$$

probe the local phase difference

Matter-wave interferometry: detect slice of atoms



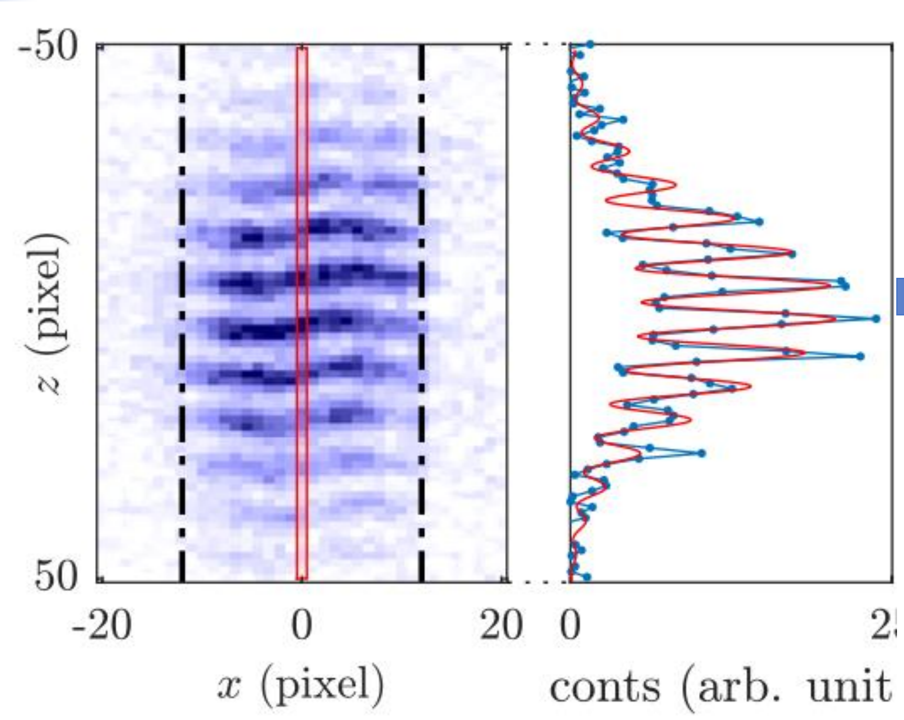


Detect atoms in F=2 level (bright)

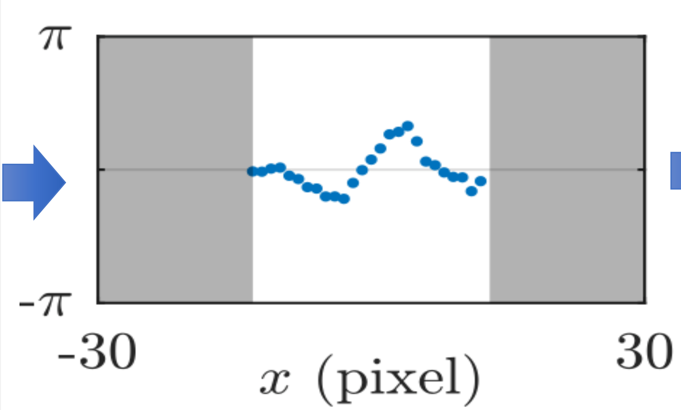
2D Bose gas in F=1 level (dark)

Analyse interference pattern to find the Correlation function

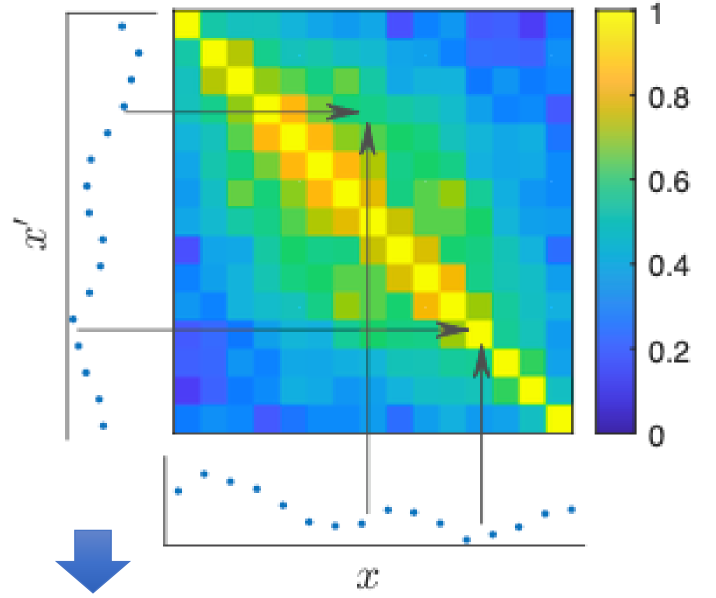
Fit image columns



Fit results



Correlate and average (N>50)



Spatial average: $C(\bar{x}) = \mathrm{Re}[\langle e^{i[\theta(x)-\theta(x-\bar{x})]}\rangle]$

relative phase correlation function

$$C(\boldsymbol{r},\boldsymbol{r}') = \frac{\langle \Psi_1(\boldsymbol{r})\Psi_2^\dagger(\boldsymbol{r})\Psi_1^\dagger(\boldsymbol{r}')\Psi_2(\boldsymbol{r}')\rangle}{\langle|\Psi_1(\boldsymbol{r})|^2\rangle\langle|\Psi_2(\boldsymbol{r}')|^2\rangle} \underset{J=0}{\propto} g_1(\boldsymbol{r},\boldsymbol{r}')^2$$

c.f. Vienna group's experiments on double-well **1**D Bose gas

Physics of a 2D Bose gas

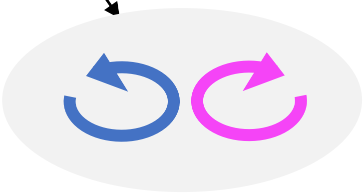
Single vortex has Energy proportional to $\ln(R)$, where R is system size.

$$E_v \approx \frac{\hbar^2 n_s}{2m} \int_\xi^R \frac{1}{r^2} 2\pi r dr = \frac{\hbar^2 n_s \pi}{m} \ln\left(\frac{R}{\xi}\right)$$

J = phase stiffness of wave function
ξ = vortex core size (healing length)

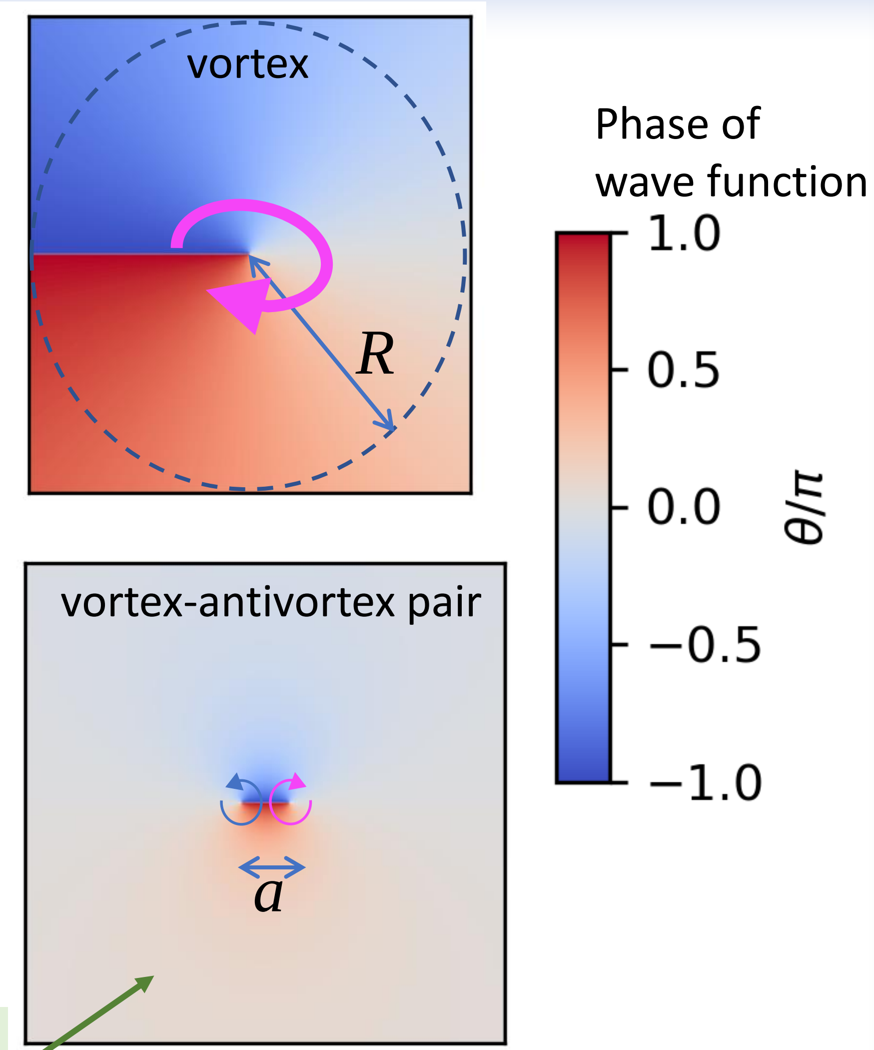
$$\equiv \pi J \ln\left(R/\xi\right)$$

Vortex-antivortex pairs have lower energy cost to create.

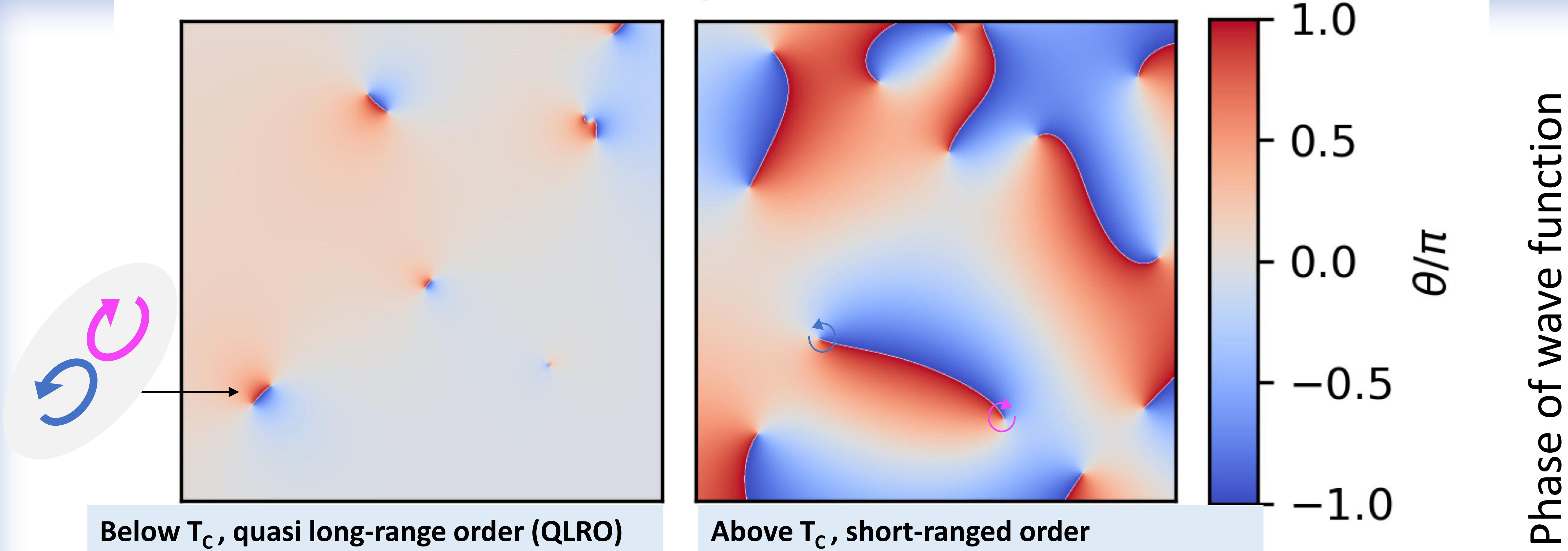


$$E_{\mathrm{pair}} = 2\pi J \ln\left(a/\xi\right)$$

no flow at large distances for pair:
cancellation of opposite circulations



Role of vortices in the BKT transition



Below T_c, quasi long-range order (QLRO)

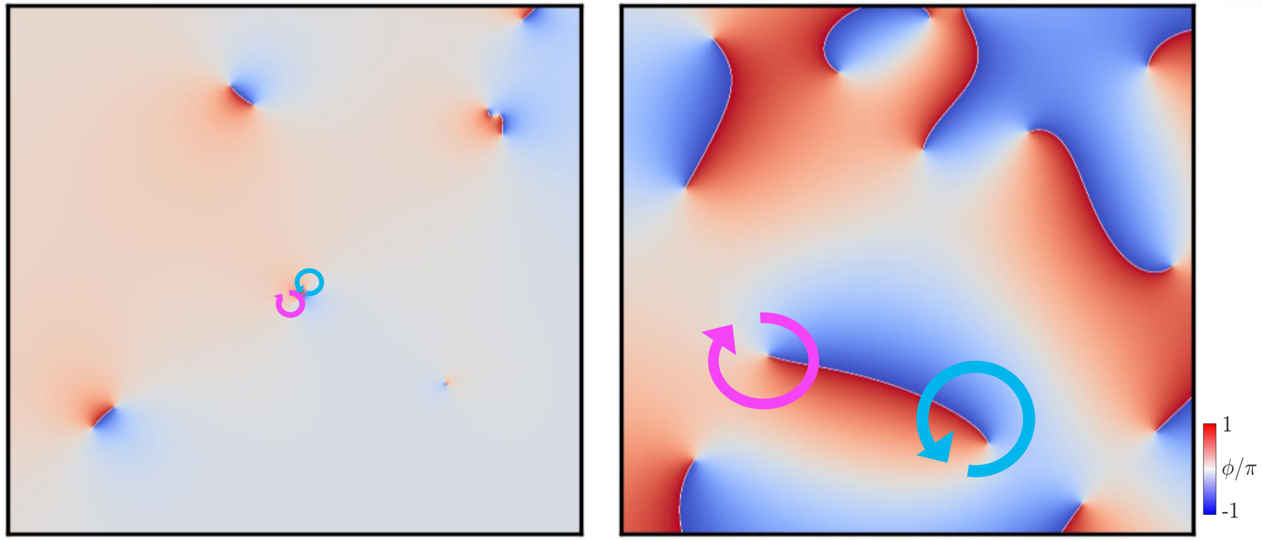
- tightly bound pairs of vortices have little effect on the phase field at distances larger than their characteristic size.

Above T_c, short-ranged order

- unbound vortices at random positions 'scramble' the phase: absence of long-range order. No net circulation.

The insight of Kosterlitz and Thouless was that a transition can occur between a superfluid phase containing vortex pairs and a normal phase with individual 'unbound' vortices.

BKT transition: Berezinskii (1972), Kosterlitz & Thouless (1972)



BKT transition:

- ○ Infinite-order phase transition
- ○ Universal jump of superfluid density
- ○ Vortex binding-unbinding mechanism
- ○ Quasi-order with power-law correlation

- Initial observation with liquid He [Bishop 1978]:
- Cold-atom experiments: Paris, Cambridge, Oxford, Seoul, Chicago, Purdue, Heidelberg, …

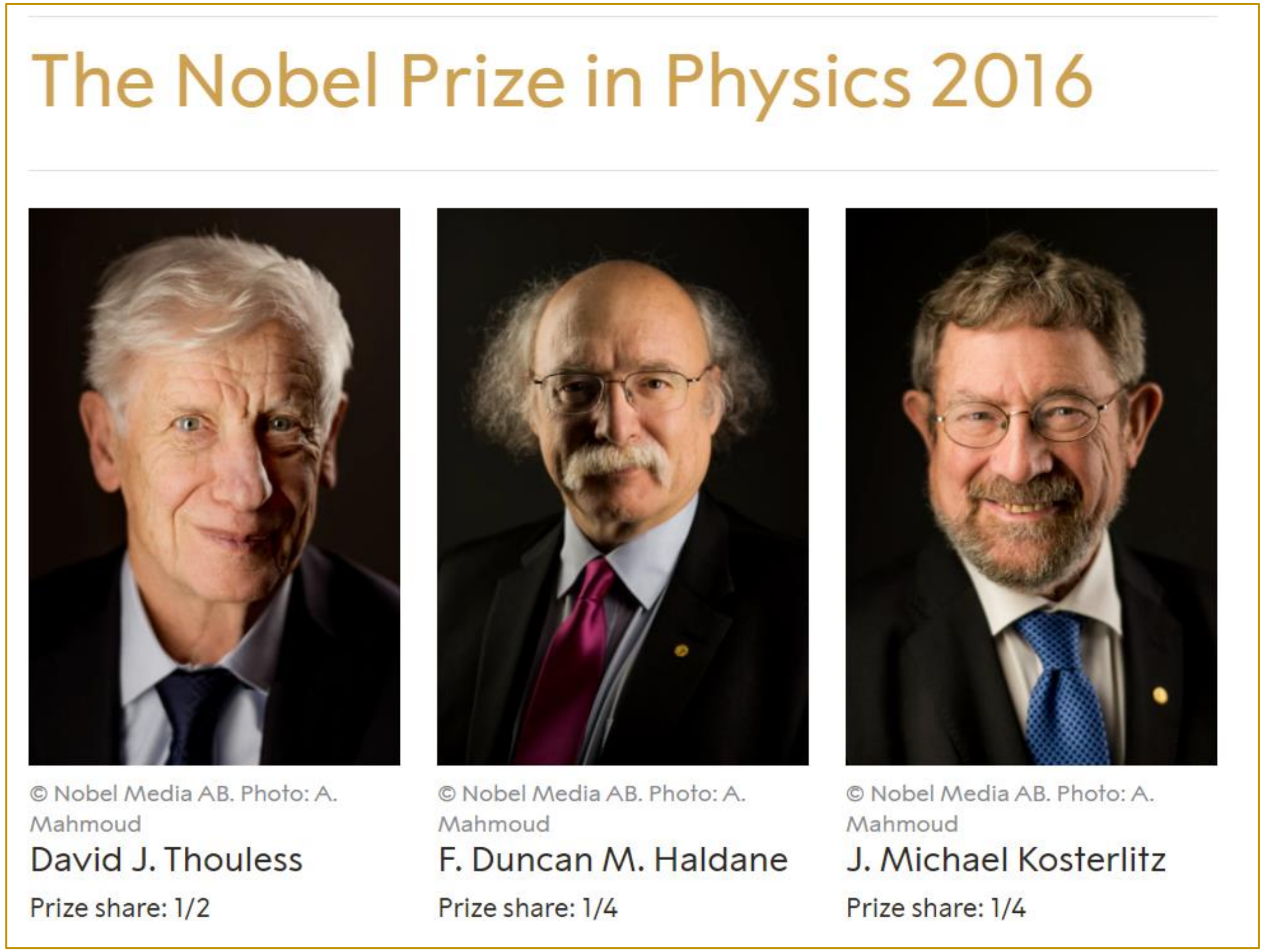
Temperature → T

Universal jump in superfluid density

		T_c		
Superfluid density	finite	4	0	$\mathcal{D}_{SF} = n_{SF}\lambda_{th}^2$
Correlation function	Power-law $\propto r^{-\eta}$		Exponential $\propto e^{-r/r_0}$	$g_1(r)$
Free vortices	none		finite probability	n_v

Kosterlitz-Thouless transition

Also known as the Berezinskii-Kosterlitz-Thouless transition



The Nobel Prize in Physics 2016

© Nobel Media AB. Photo: A. Mahmoud
David J. Thouless
Prize share: 1/2

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F. Duncan M. Haldane
Prize share: 1/4

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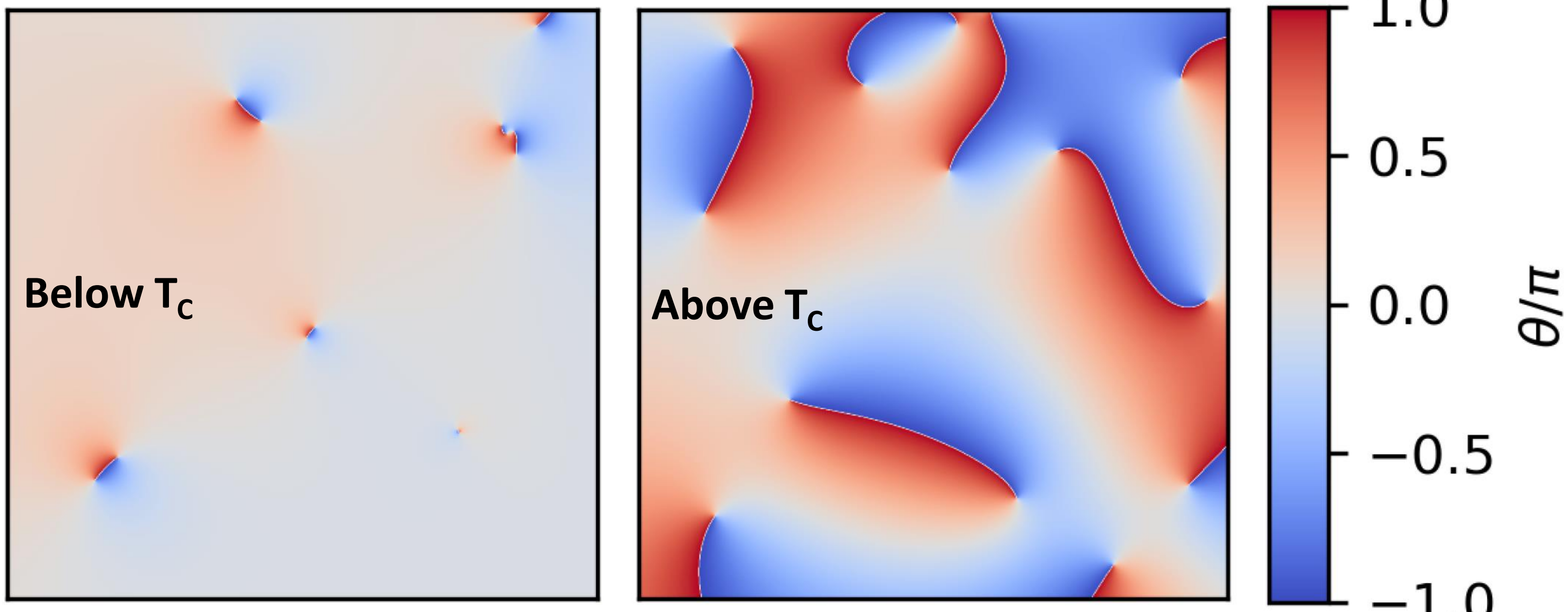
J. Michael Kosterlitz
Prize share: 1/4

- for theoretical discoveries of topological phase transitions and topological phases of matter.
- The KT-transition does not break any symmetry, something that was completely new and unexpected – it should not occur according to Mermin-Wagner Theorem.
- Different to long-range order and superfluidity with ‘condensation’.
- The BKT phase transition does not rely on spontaneous symmetry breaking.

The BKT transition – reminder of previous slide



Below T_c, tightly bound pairs of vortices.

Above T_c, unbound vortices at random positions.

$$g_1(\mathbf{r}) = \langle \mathbf{S}(\mathbf{r}) \cdot \mathbf{S}(0) \rangle$$
$$= \left\langle e^{\mathrm{i}[\theta(\mathbf{r})-\theta(0)]} \right\rangle \propto (\xi/r)^{\eta} \sim r^{-\eta}$$

algebraic ('slow') decay of correlations
⇔ quasi long-range order (QLRO)

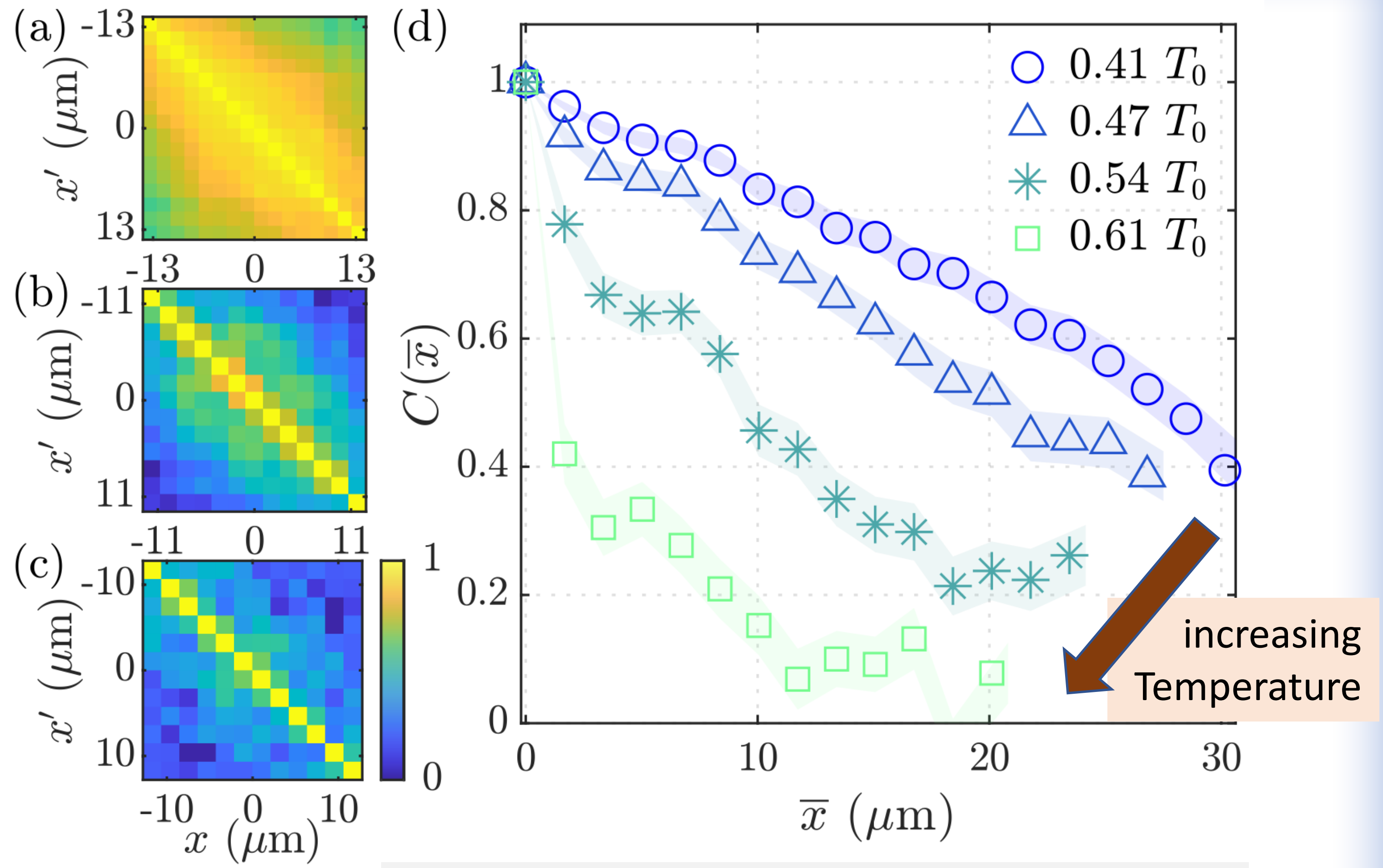
$$g_1(\mathbf{r}) = \langle \mathbf{S}(\mathbf{r}) \cdot \mathbf{S}(0) \rangle$$
$$= \left\langle e^{\mathrm{i}[\theta(\mathbf{r})-\theta(0)]} \right\rangle \propto e^{-r/\xi}$$

exponentially decaying correlations
('fast' decay)

Correlation function in harmonic trap

Phase correlation function

$$
\begin{aligned}
C(\boldsymbol{r},\boldsymbol{r}') &:= \frac{\langle \Psi_1(\boldsymbol{r})\Psi_2^\dagger(\boldsymbol{r})\Psi_1^\dagger(\boldsymbol{r}')\Psi_2(\boldsymbol{r}')\rangle}{\langle|\Psi_1(\boldsymbol{r})|^2\rangle\langle|\Psi_2(\boldsymbol{r}')|^2\rangle} \\
&\simeq \frac{\langle \Psi^\dagger(\boldsymbol{r})\Psi(\boldsymbol{r}')\rangle^2}{\langle|\Psi(\boldsymbol{r})|^2\rangle\langle|\Psi(\boldsymbol{r}')|^2\rangle} = \frac{g_1(\boldsymbol{r},\boldsymbol{r}')^2}{n_{2\mathrm{D}}^2} \\
&= \boxed{\langle e^{i\theta(\boldsymbol{r})-i\theta(\boldsymbol{r}')}\rangle}
\end{aligned}
$$



increasing Temperature

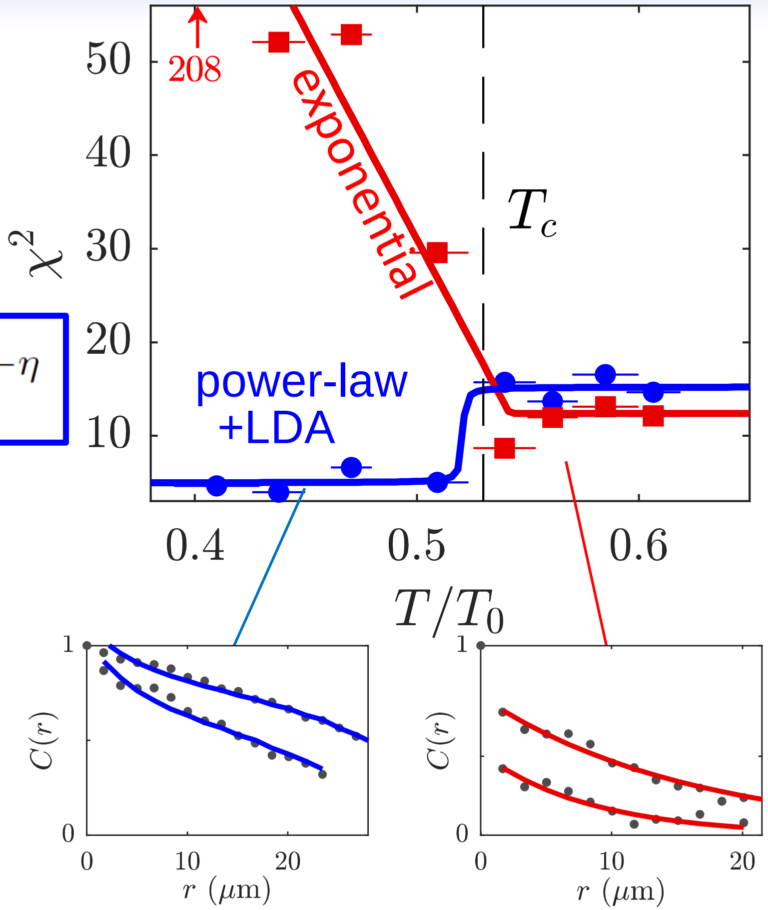
Scaled temperatures T / T_0
where quantum degeneracy occurs at T_0 .

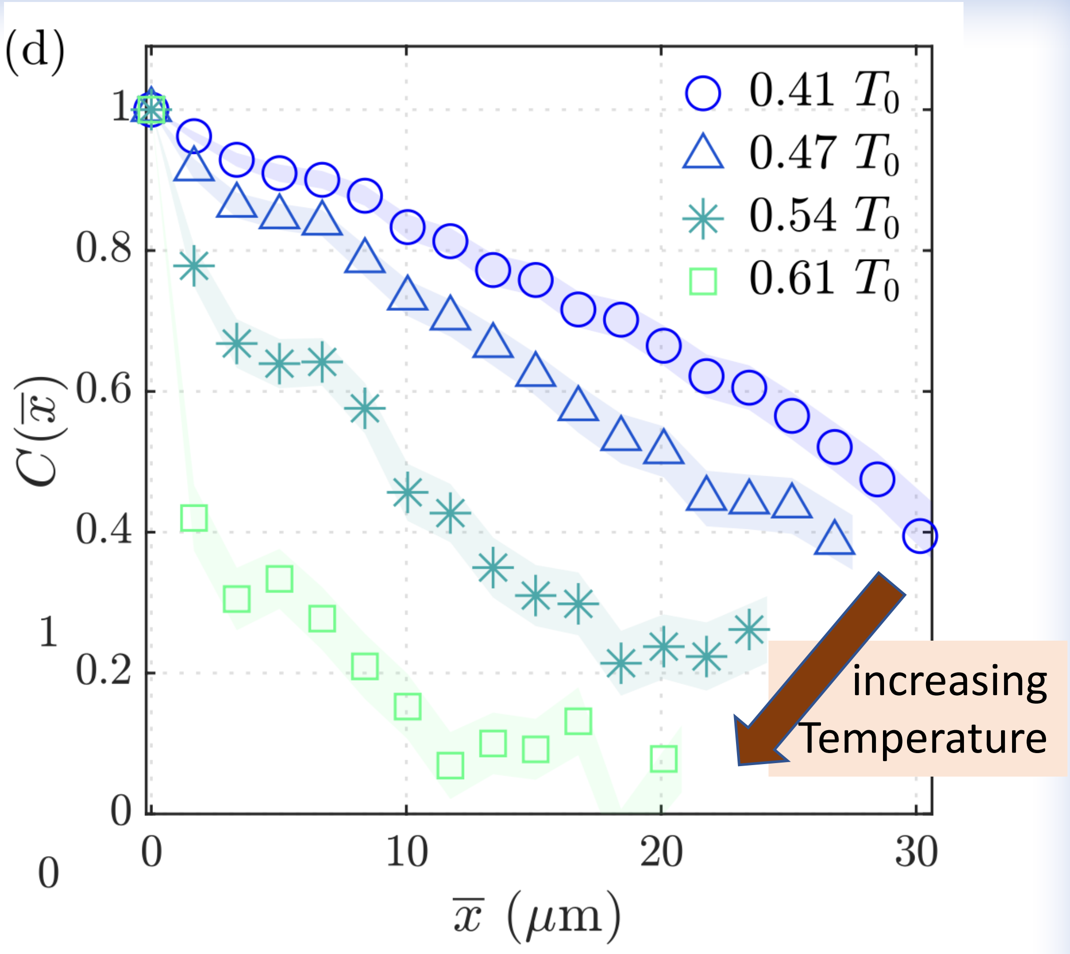
Sunami *et al.*, Phys. Rev. Lett. 128, 250402 (2022)

Correlation function in harmonic trap

$$\left\langle e^{\mathrm{i}[\theta(\mathbf{r})-\theta(0)]}\right\rangle \propto e^{-r/\xi}$$

$$\left\langle e^{\mathrm{i}[\theta(\mathbf{r})-\theta(0)]}\right\rangle \propto (\xi/r)^{\eta} \sim r^{-\eta}$$



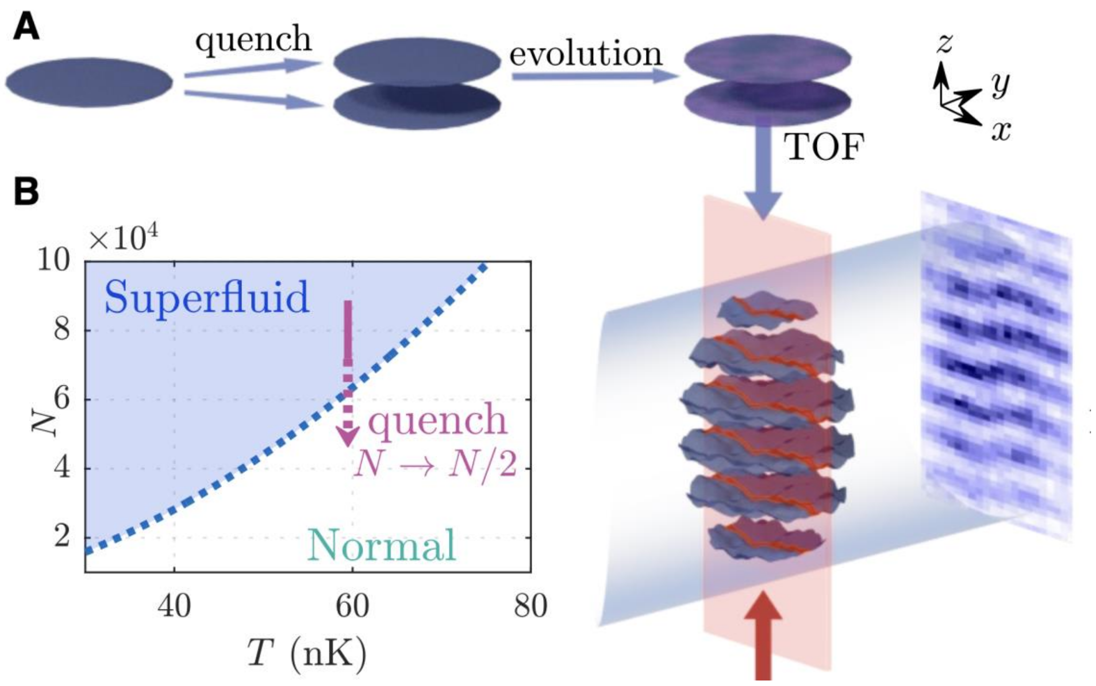


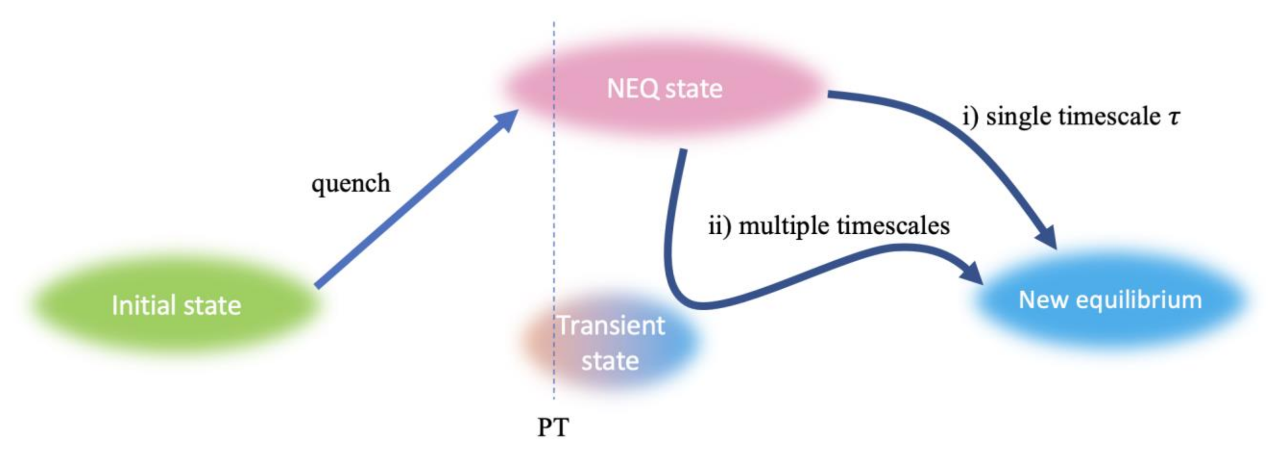
increasing Temperature

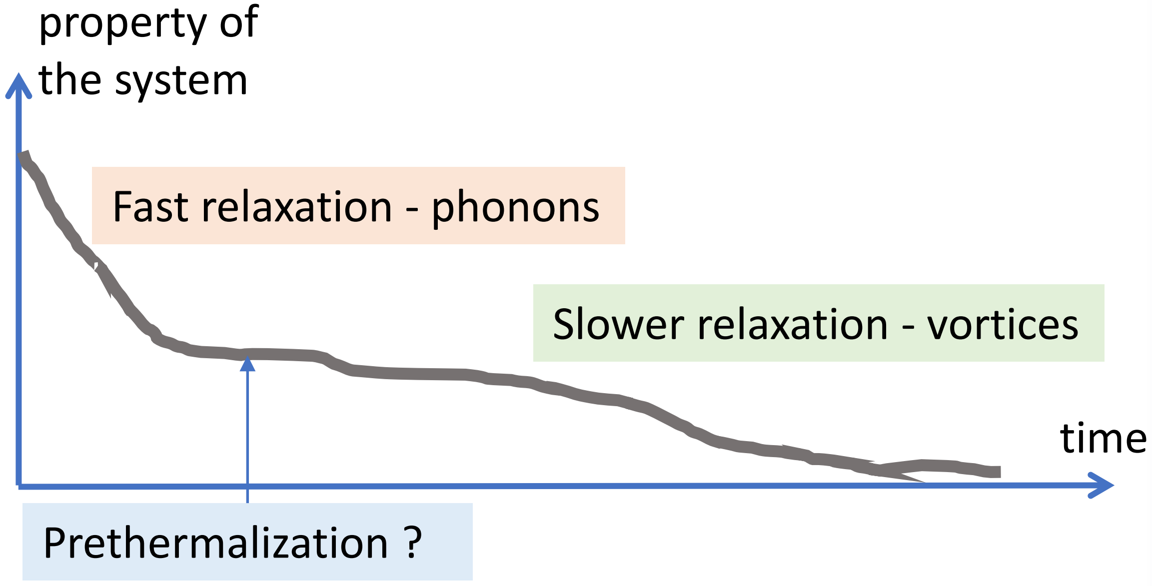
Scaled temperatures T / T_0 ; quantum degeneracy at T_0 .

Observation of the BKT Transition in a 2D Bose Gas via Matter-Wave Interferometry. S. Sunami, V. Singh, D.Garrick, A. Beregi, A.Barker, K.Luksch, E.Bentine, L. Mathey & C.J. Foot, Phys. Rev. Lett. 128, 250402 (2022).

Quenching the 2D quantum gas→ bilayer





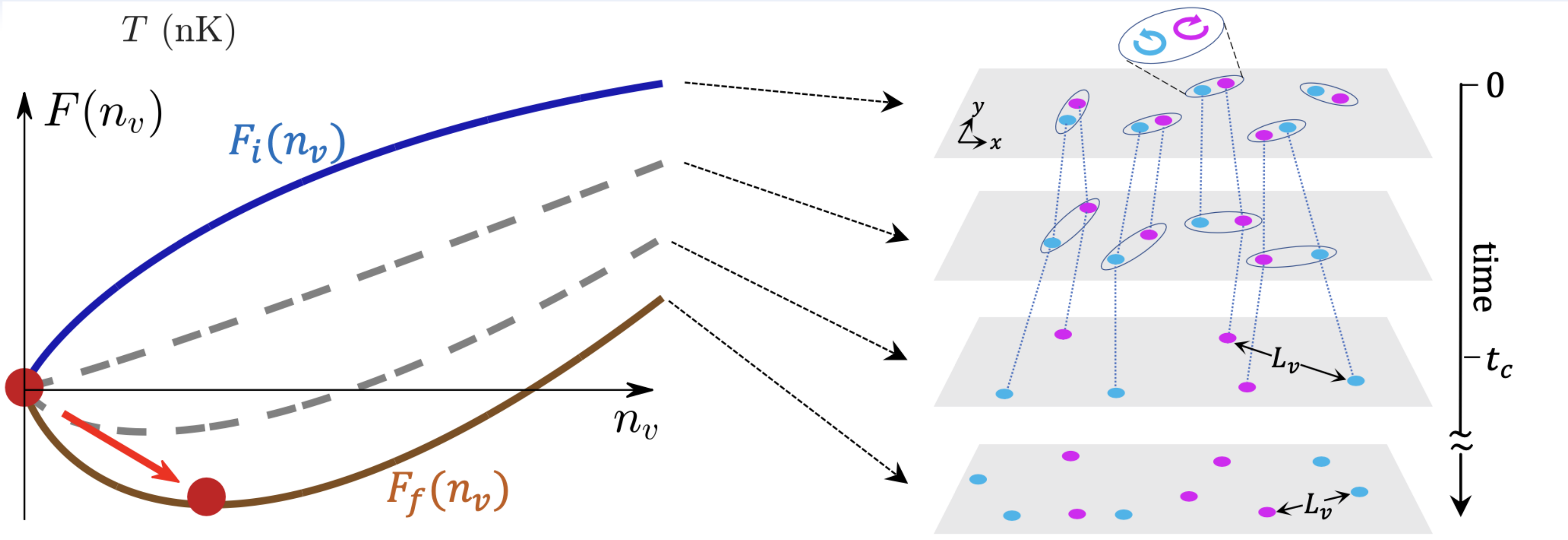


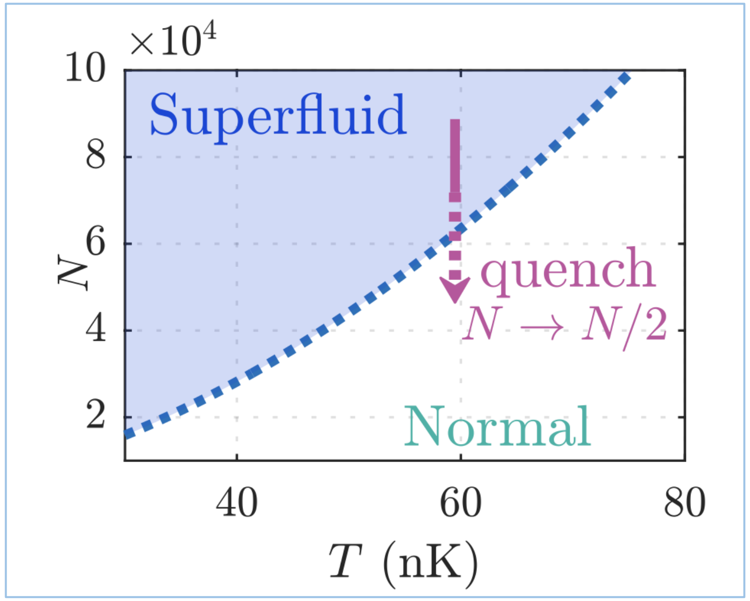
Fast relaxation - phonons

Slower relaxation - vortices

Prethermalization ?

Vortex-unbinding dynamics





Reverse Kibble-Zurek mechanism:

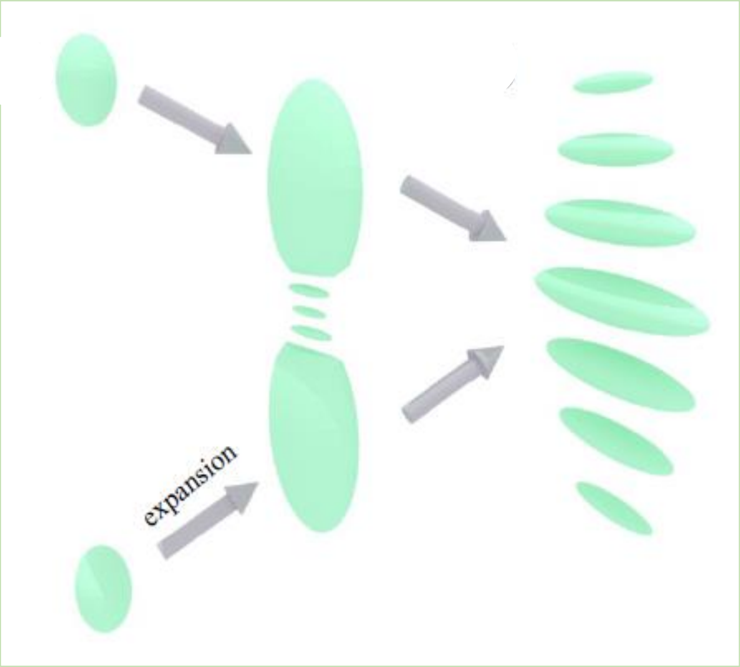
- ordered phase → disordered phase (vortices)

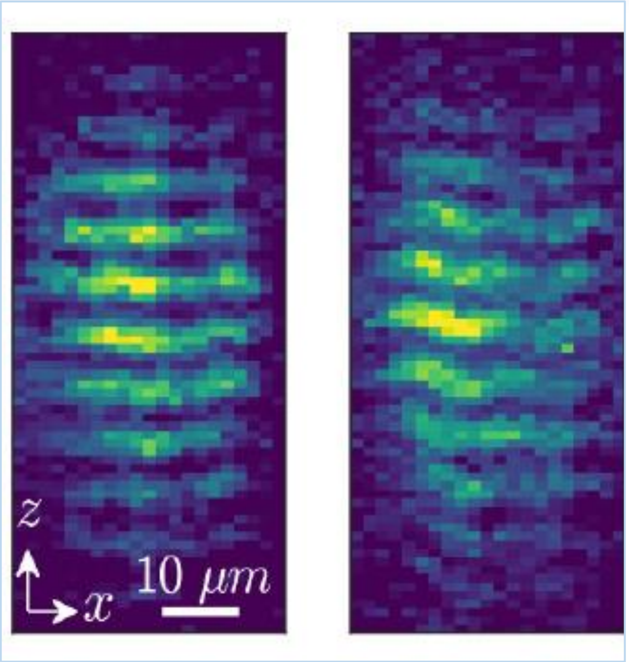
Kibble-Zurek mechanism:

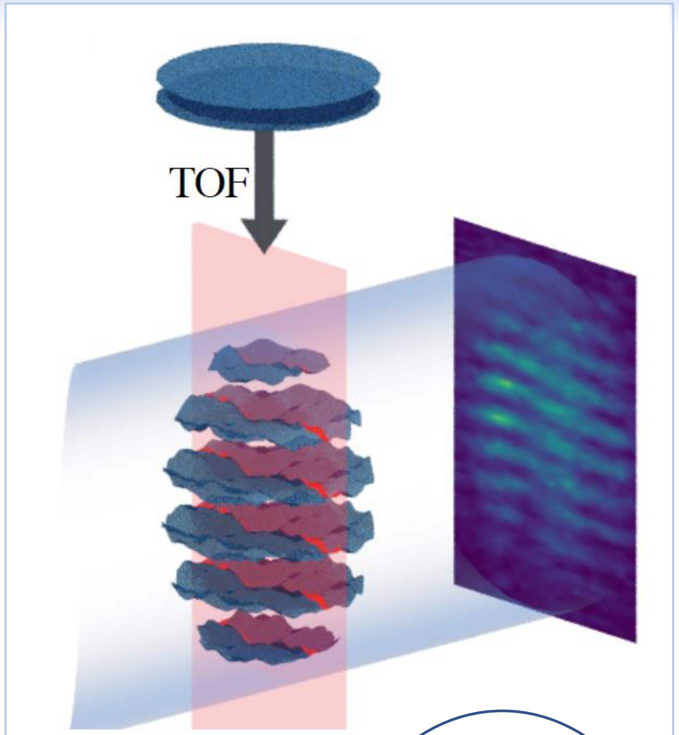
- formation of vortices after quench of temperature

[proposed mechanism for cosmic strings in early universe.]

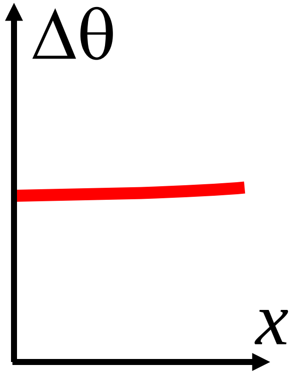
Experimental measurement of phase detects phonons and vortices

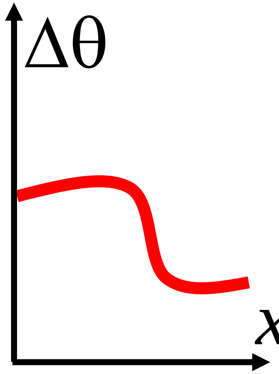


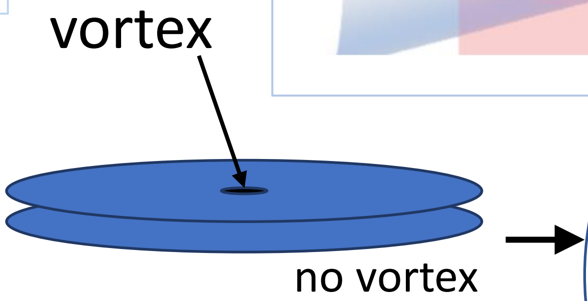


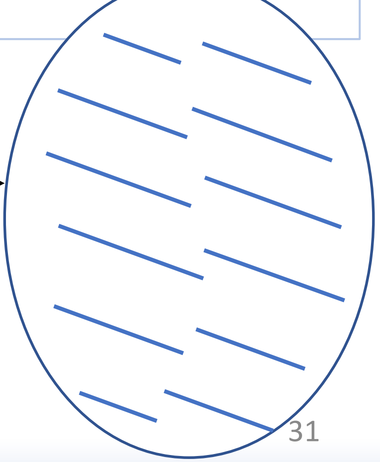


phonons – ‘waviness’ of fringes
vortices – phase discontinuity

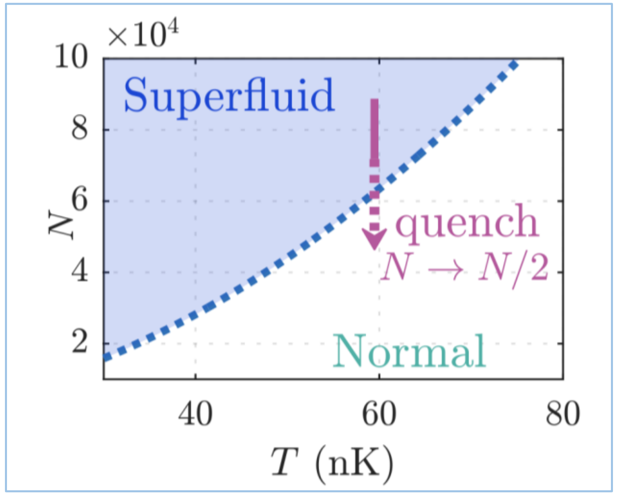


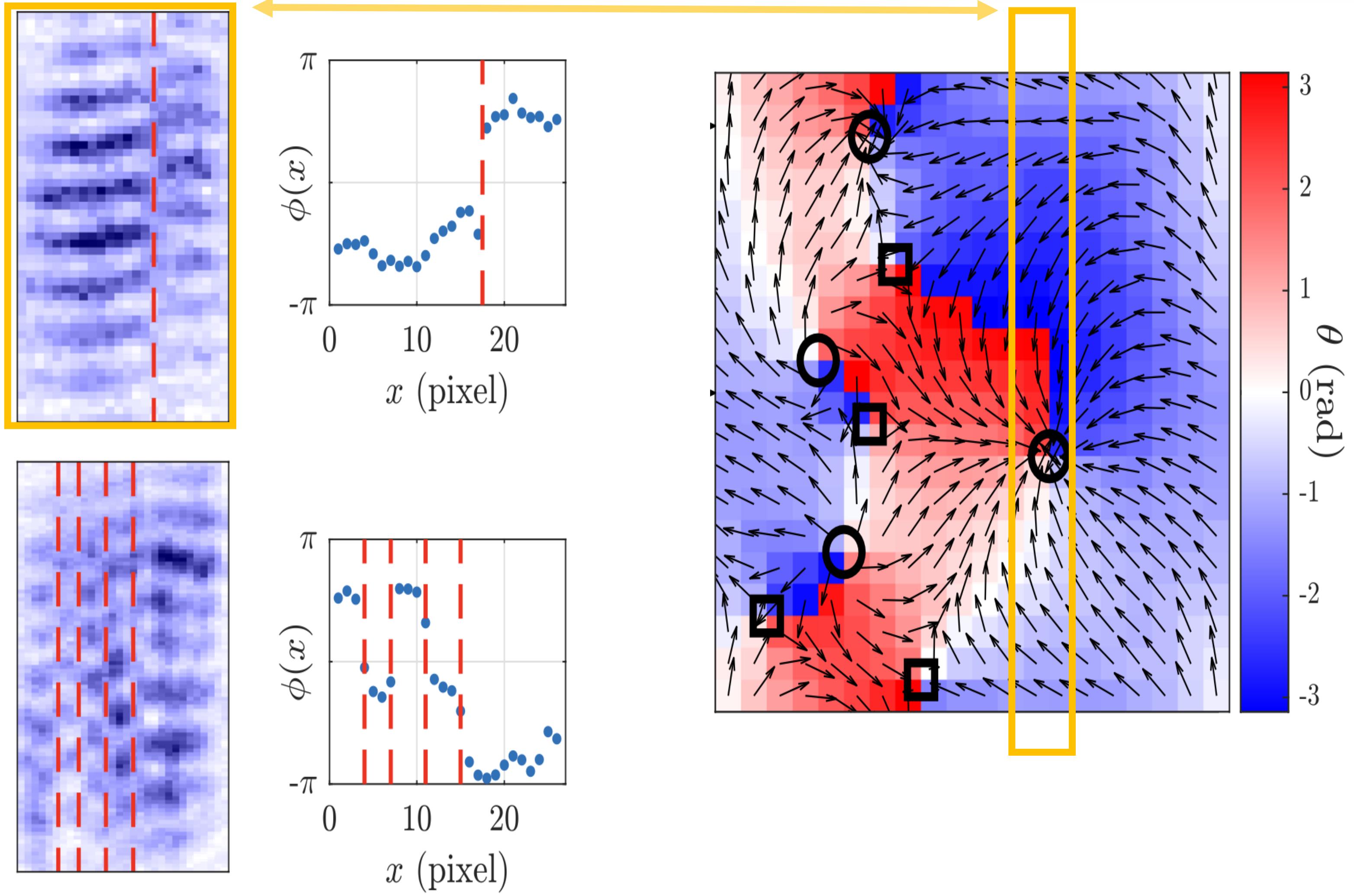




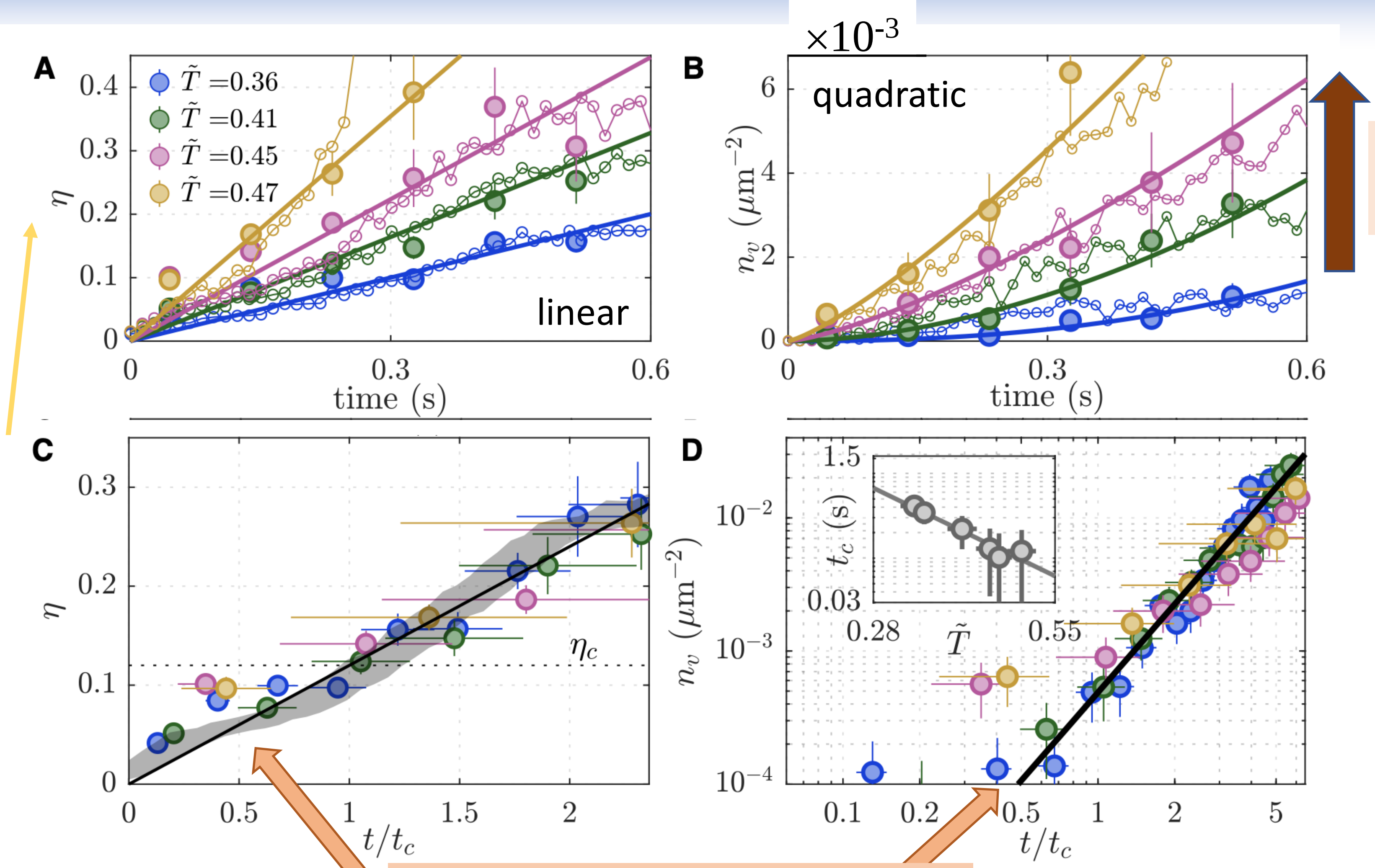


Vortex detection



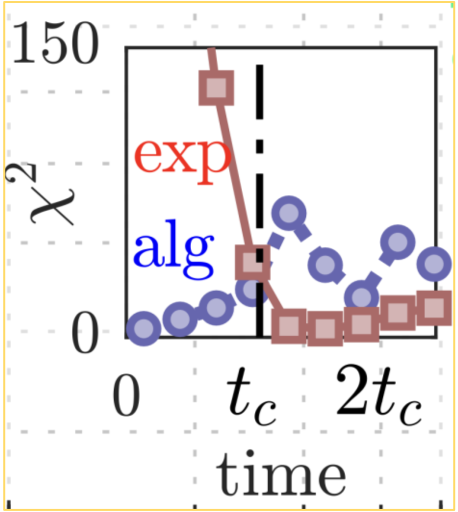


Dynamics after quench



increasing Temperature

superheated superfluid



t_c = time when system makes the transition

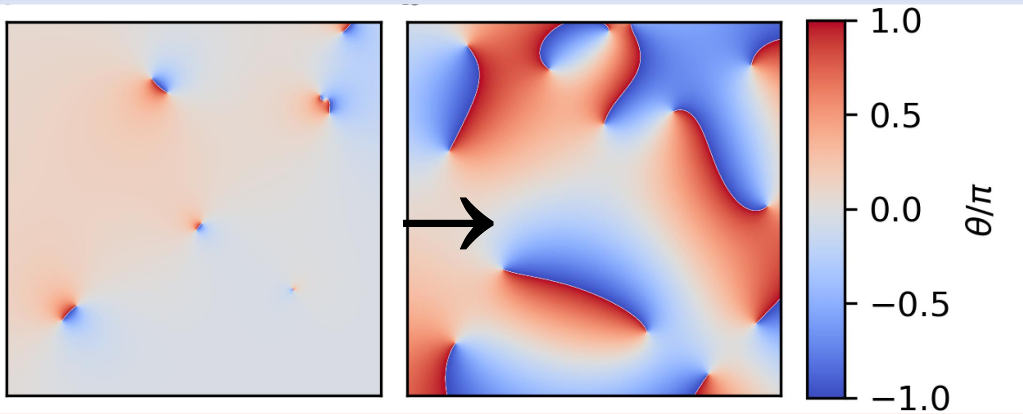
Define fugacity, *g*

Free energy,

$$F = E_\text{v} - TS_\text{v}$$
$$= E_\text{core} + (\pi J - 2k_\text{B}T)\ln(R/\xi)$$

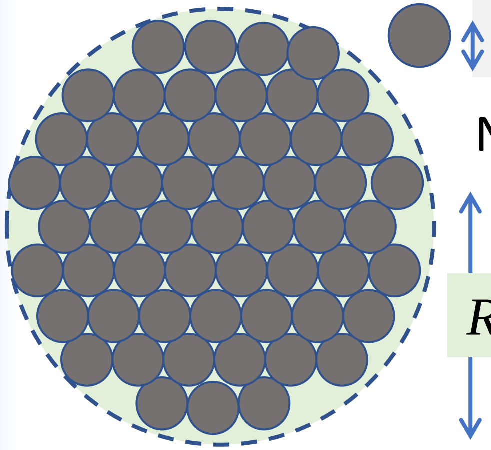
Entropy of vortex,

$$S_v = k_\text{B}\ln(\Omega) = 2k_\text{B}\ln(R/\xi)$$



BKT transition: **Below T_c**, tightly bound vortex pairs → **Above T_c**, unbound vortices at random positions.

- At the KT transition temperature, the vortex pairs dissociate so that individual vortices proliferate.



ξ = vortex core size

Number of microstates, $\Omega = \dfrac{\pi R^2}{\pi\xi^2}$

R = system size

$p_i = n_v\xi^2$ = probability of vortex at x_i

$$\boxed{p_i \propto e^{-\beta E_i} = g \;= \text{fugacity}}$$

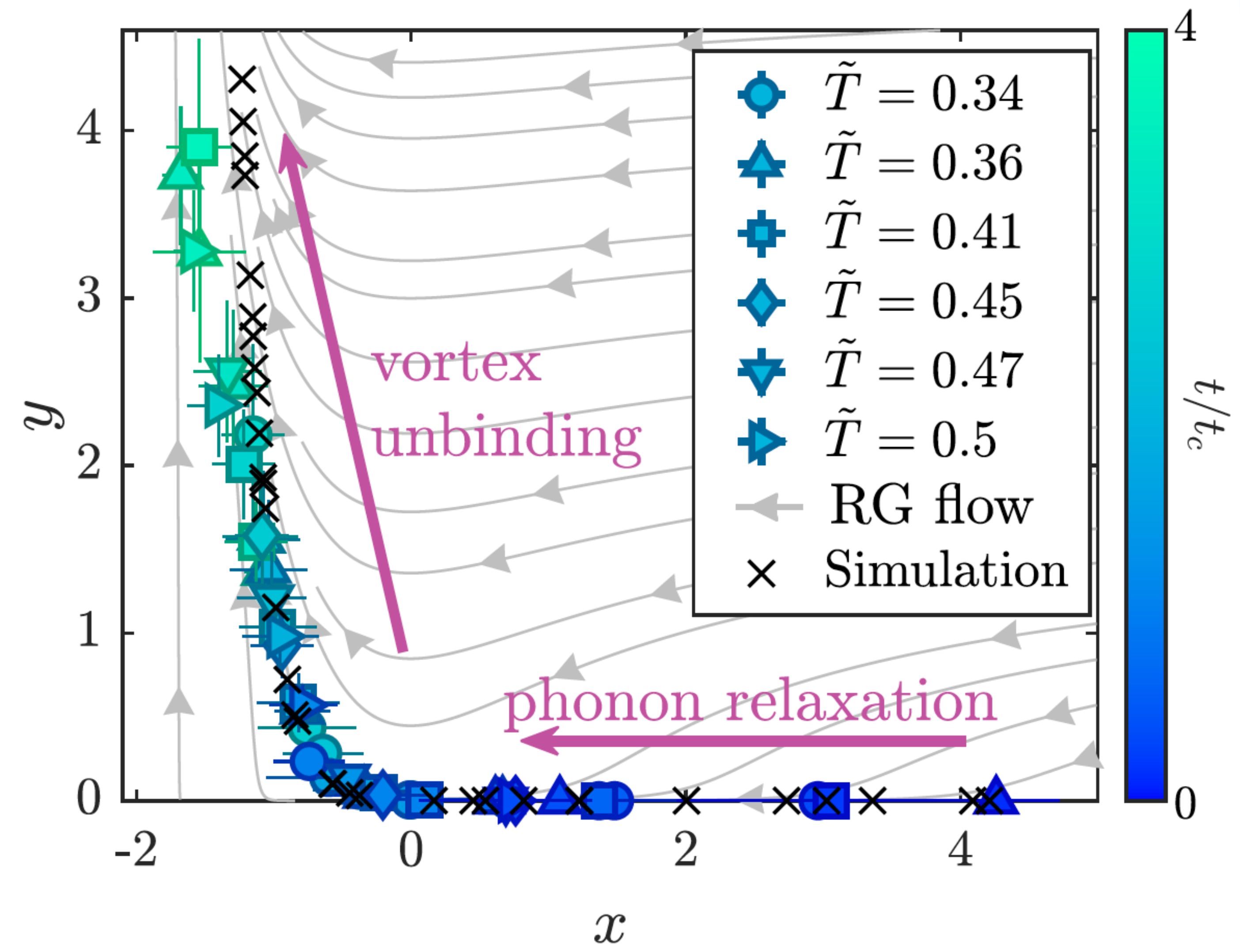
Total probability of finding a vortex in the system. (high for $F < 0$.)

$$P_\text{vortex} \propto \sum_{\text{vortex at } x_i} e^{-\beta E_i}$$

$$P_\text{vortex} \propto \Omega e^{-\beta E_i} = e^{\ln(\Omega) - \beta E_i}$$

$$P_\text{vortex} \propto e^{-\beta F}$$

Time-dependent Renormalization Group picture: L. Mathey & A. Polkovnikov (2010)



L. Mathey & A. Polkovnikov. Phys. Rev. A **81**, 033605 (2010)

$$x = \frac{1}{2\eta} - 2$$

$$y = \sqrt{2}\pi g_v$$

fugacity $\propto n_v$, vortex number density, for large η

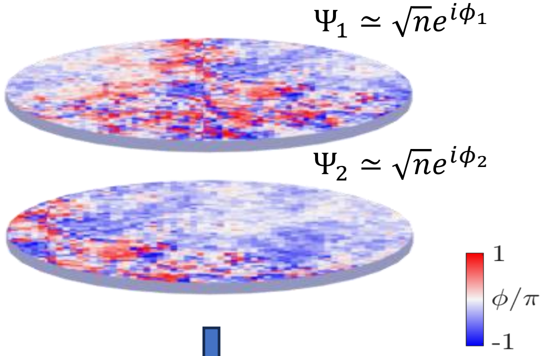
$$\begin{cases} \dfrac{dx}{dt} = -\dfrac{(x+2)^3 y^2}{8t} \\ \dfrac{dy}{dt} = -\dfrac{xy}{t} \end{cases}$$

Scaling: condenses the information exchange between experiment and theory (analytical theory & numerics) into a small set of universal numbers.

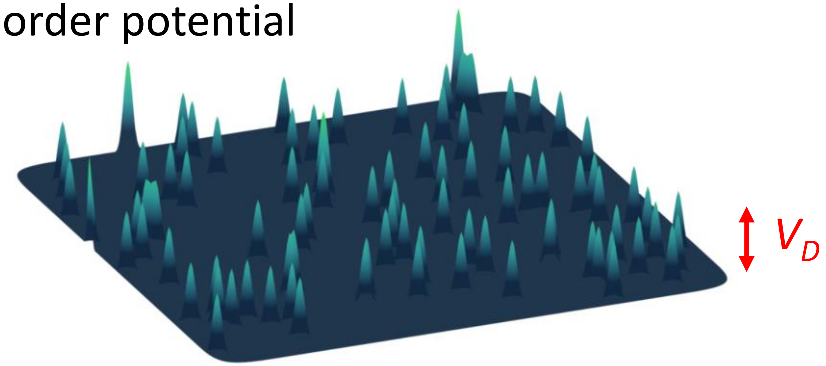
The supercritical state relaxes to a disordered state by dynamical vortex unbinding. This dynamically suppressed vortex proliferation constitutes a reverse Kibble-Zurek effect.

Disorder-induced superfluid transition: ongoing work

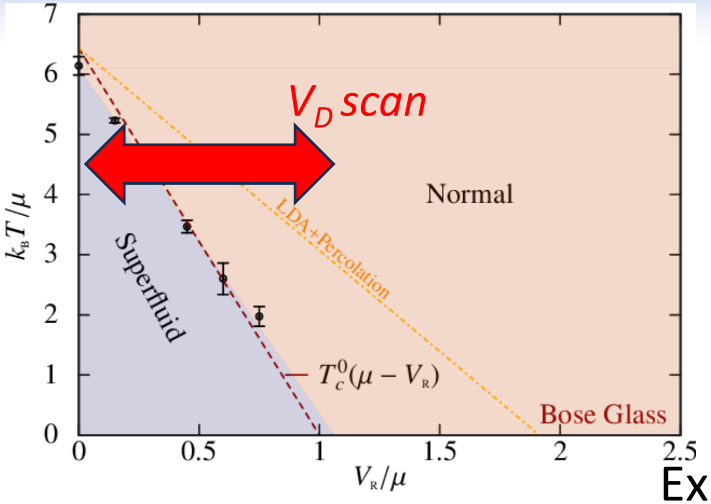
bilayer quantum gas (J=0)



Disorder potential



Disorder potential



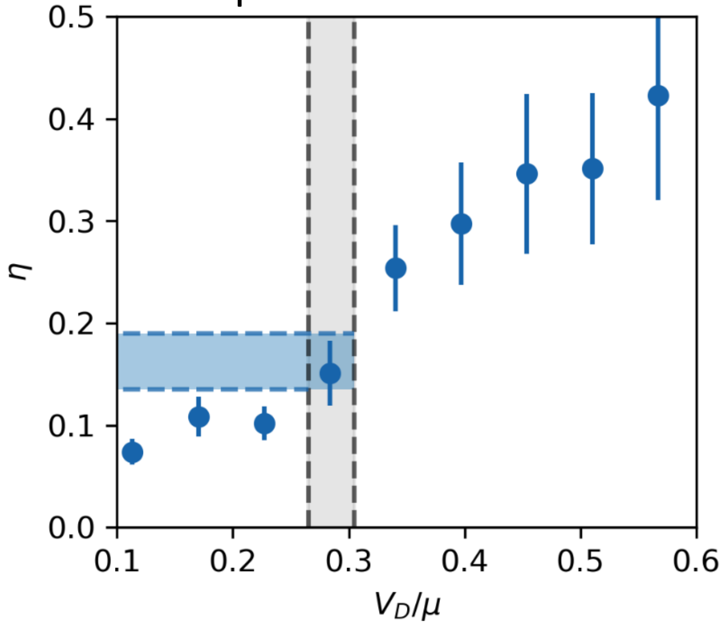
Theory: Carleo, Boeris, Holzmann, Sanchz-Palencia, PRL 111, 050406 (2013)

See also Bourdel (2011, 2012)

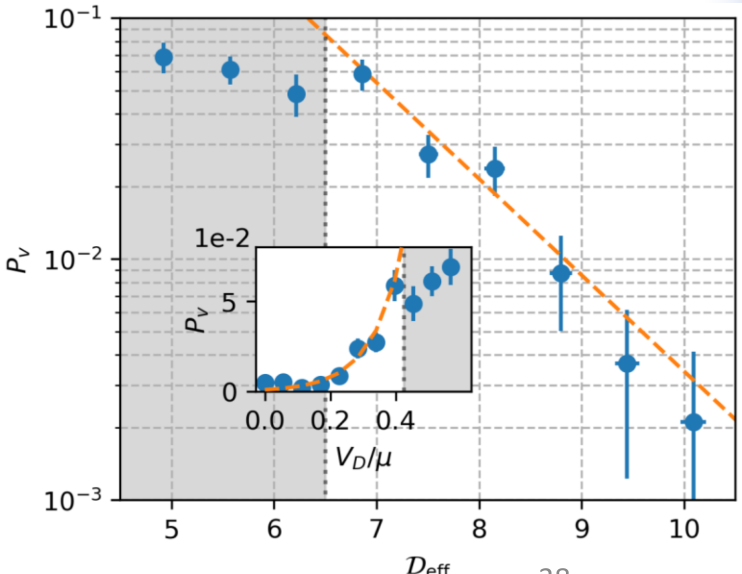
Abel Beregi, D.Phil. thesis, Oxford (2023)

Experiment:

eta = power law of correl.



Probability of detecting a vortex



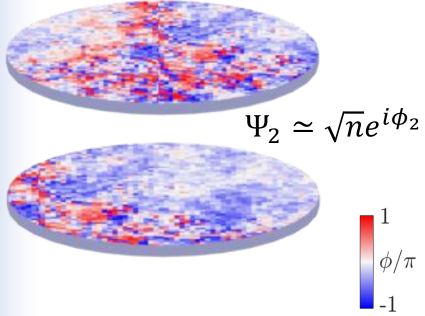
A. Beregi et al., in preparation

Probing a bilayer system: measure both relative and common phase

bilayer

$\Psi_1 \simeq \sqrt{n}e^{i\phi_1}$

$\Psi_2 \simeq \sqrt{n}e^{i\phi_2}$



Antisymmetric mode

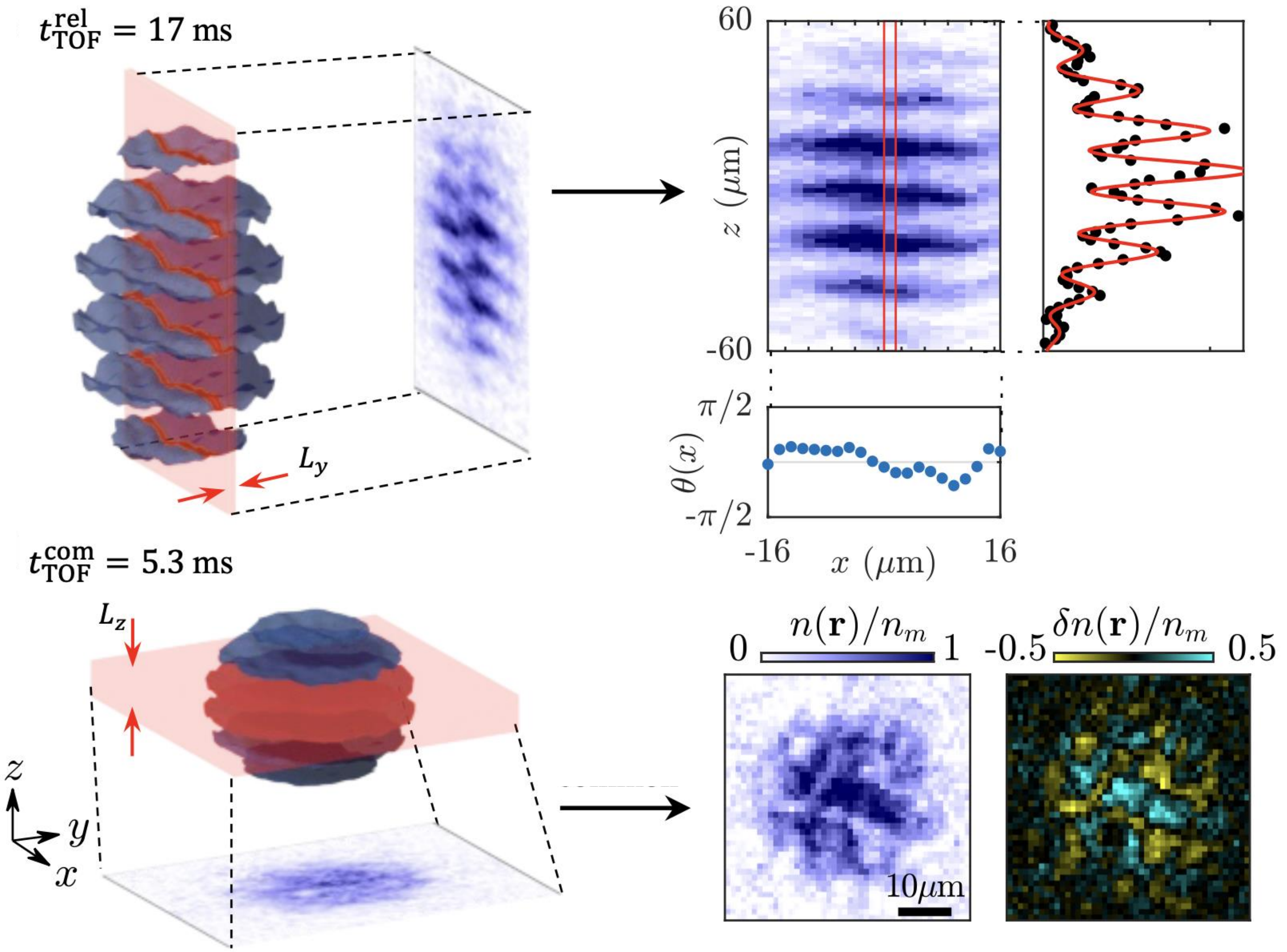
Relative phase

$$\phi_1 - \phi_2$$

$$\phi_1 + \phi_2$$

Common phase

Symmetric mode



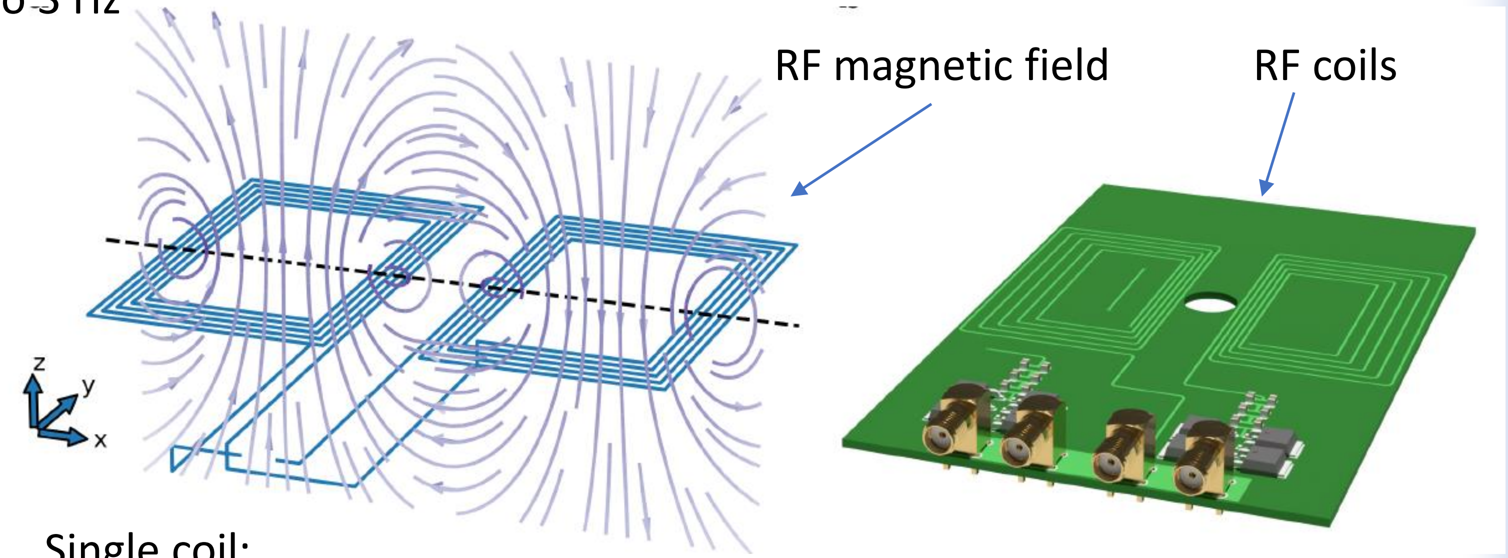
RF-dressed traps with flat potential (uniform atomic density)

Reduce radial trapping frequency, e.g. from 10 to 3 Hz
→ 'flatter potentials'

- anisotropy of 1 : 300 (for Rb-87).
- density variation < 10% across cloud.

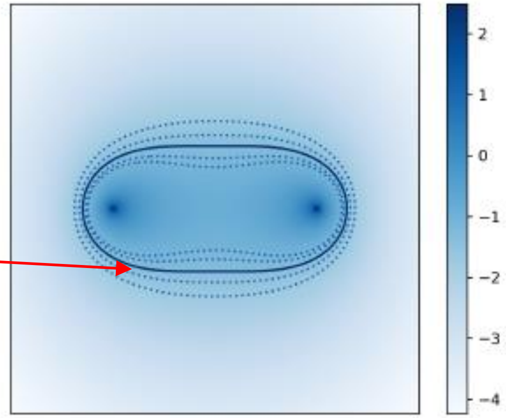
Two possible methods:

- Increase radio-frequency,
  e.g. from 7 MHz to 21 MHz using coils
  with higher self-resonant frequency.



- Magnetic field from a single coil
  – flat contour

Single coil:
isomagnetic contours



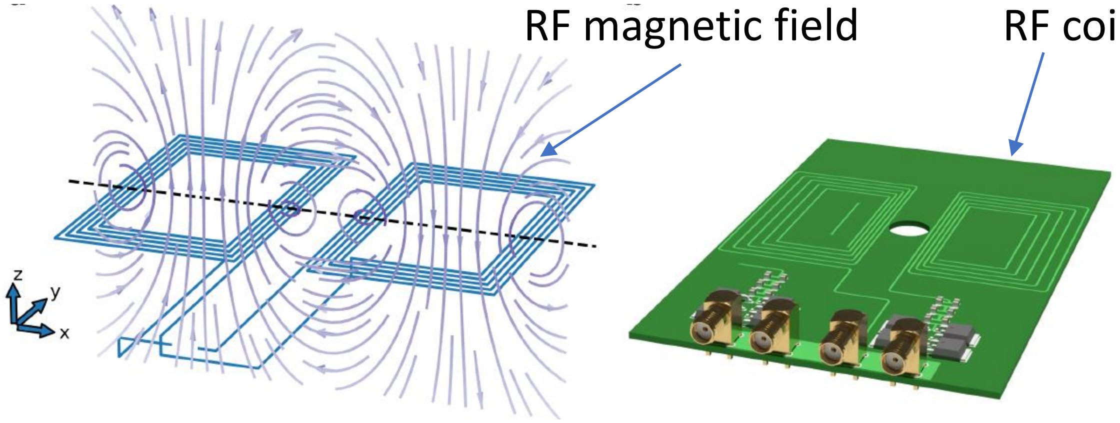
From Abel Beregi, D.Phil. thesis, Oxford (2023)

RF-dressed traps with flatter potentials (more uniform atomic density)

Two possible methods:

- Increase radio-frequency.

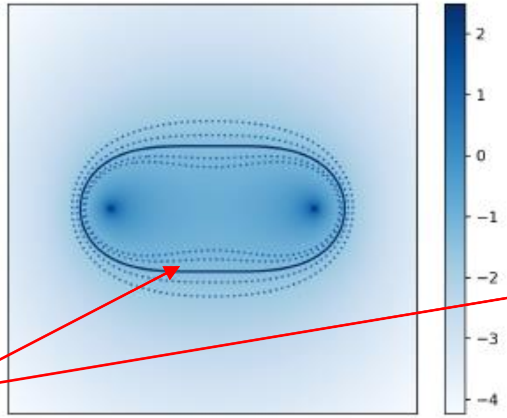
- Magnetic field with flat contours



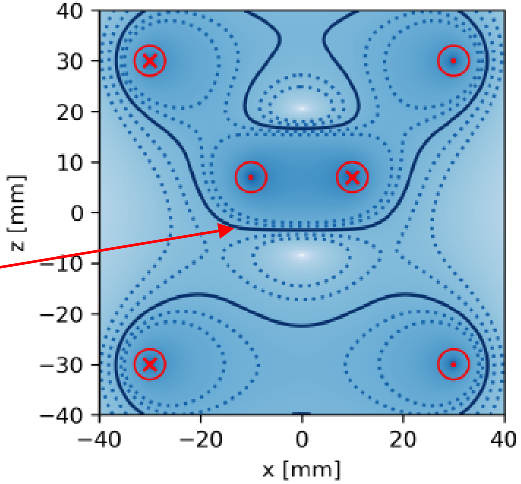
RF magnetic field

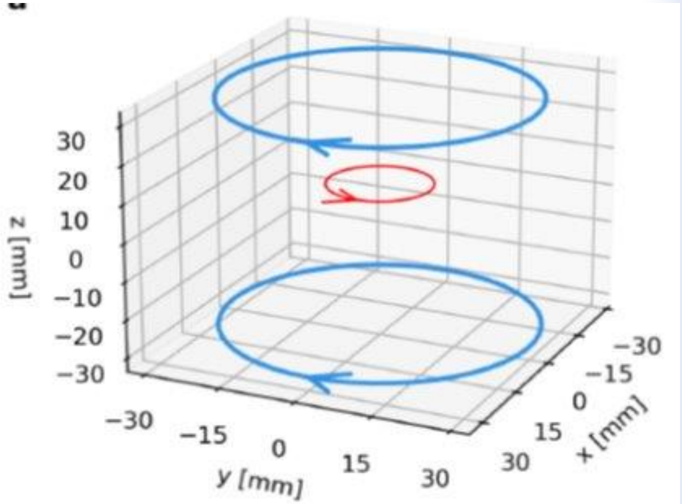
RF coils

Single coil:
isomagnetic contours



Single coil + Helmholtz pair (to independently control magnitude and vertical gradient)





From Abel Beregi
D.Phil. thesis,
Oxford (2023)

Contour flat,
(on the length scale of the atom cloud).

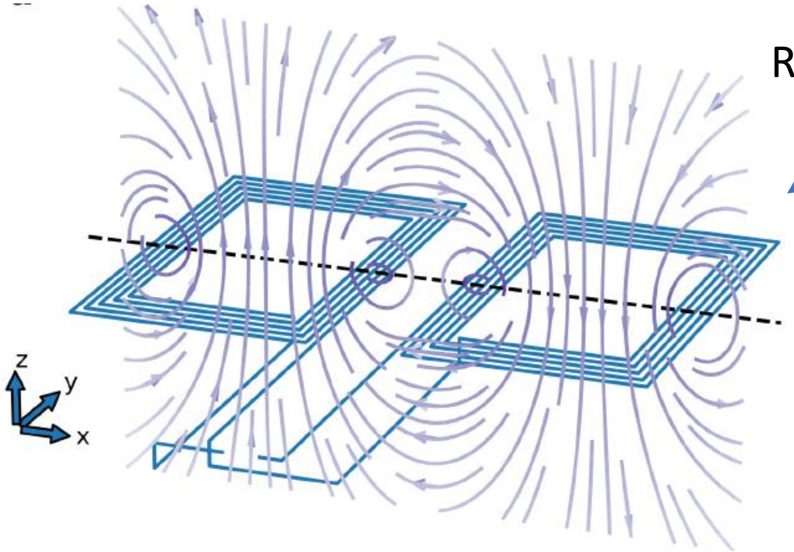
RF-dressed traps – future improvements to give longer lifetimes/ lower temperatures

- The lifetime of RF-dressed trapped atoms is many seconds, with a minimum Rabi freq. much less than magneto-static traps.

- Loss mechanism is not fully characterised – may be electrical noise. It is higher than the predicted intrinsic loss by non-adiabatic transitions (Landau-Zener).

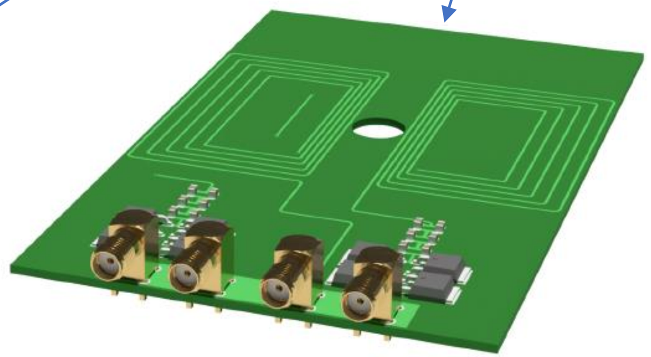
- Lifetime may improve with increasing radio-frequency, for the same double-well spacing.

- Magnetic field with flat contours gives gravity compensation over wide regions (> 10mm).
  $\Rightarrow$ Good for atom interferometry, e.g. see Cass Sackett's talk at this meeting, or other Atomtronics applications
- RF-dressing works very well for Rb-87 – good alternative to squashing between light sheets. (Being extended, elsewhere, to other species.)

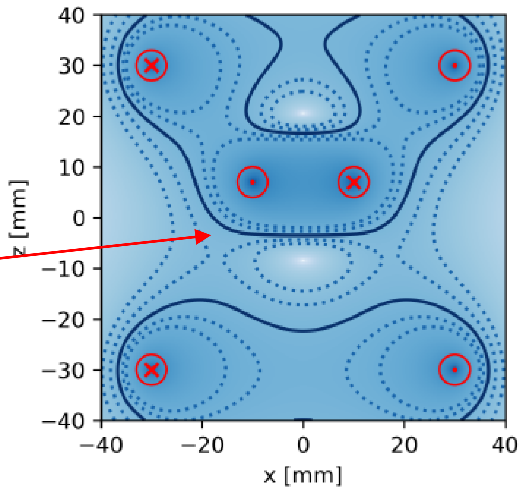


RF magnetic field

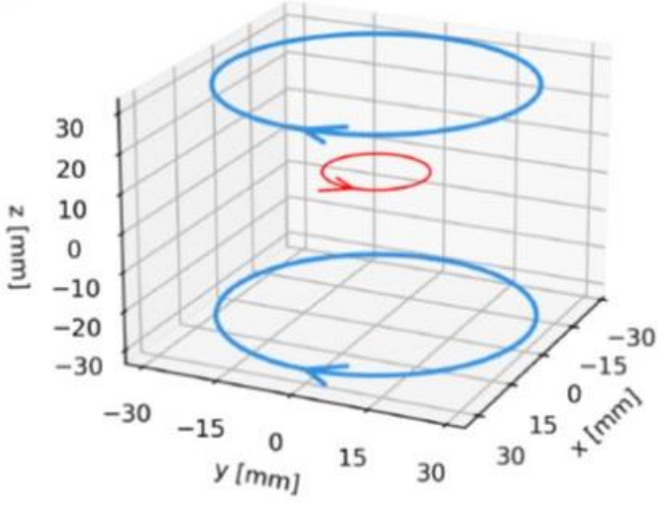
RF coils



Contours: single coil + Helmholtz pair

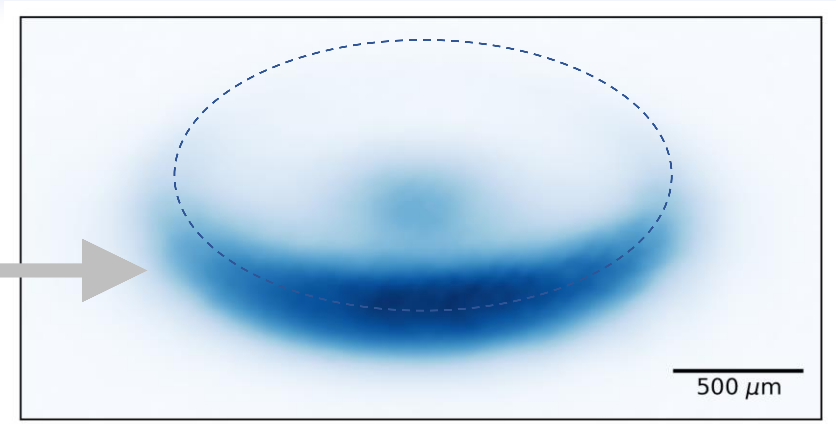


flat contour



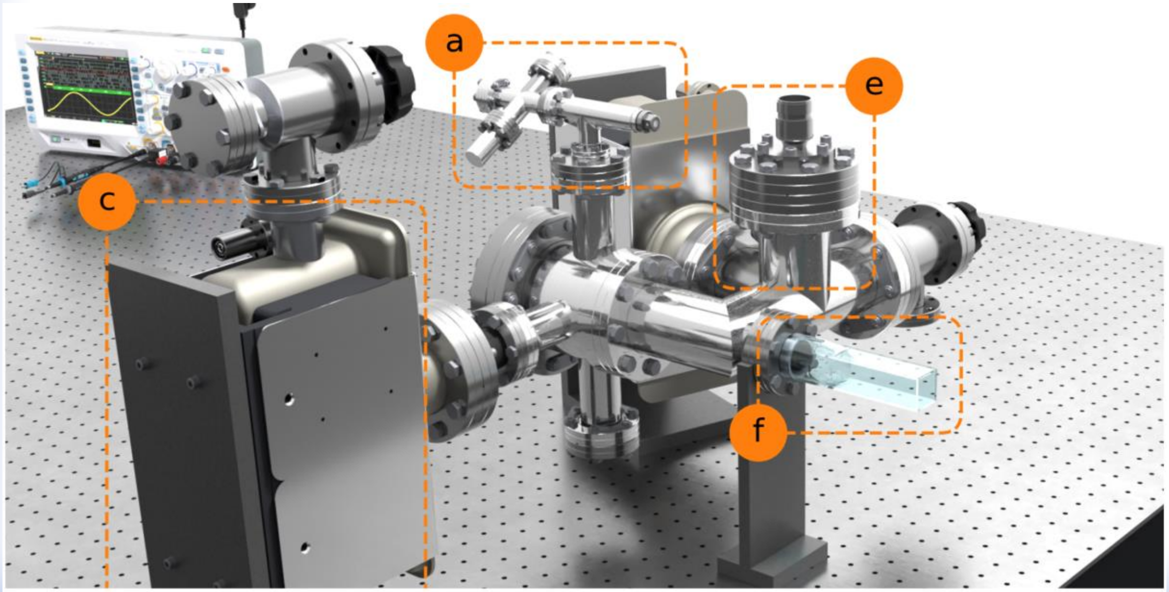
Simplified loading scheme from MOT directly into RF-potential

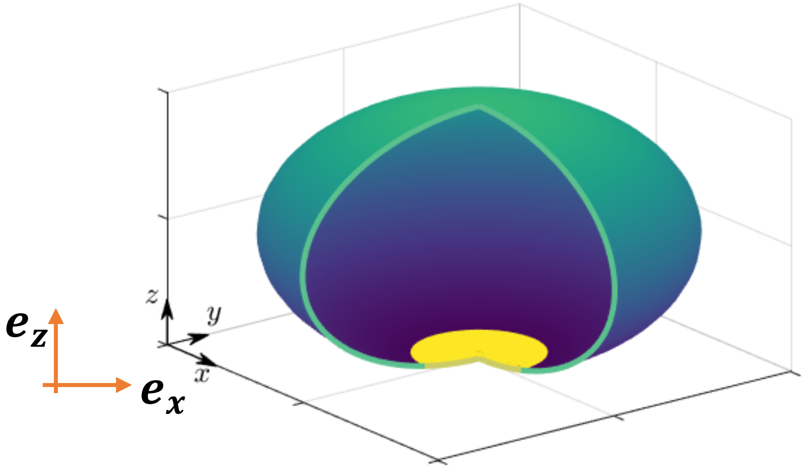
- $> 10^9$ atoms in compressed MOT cloud
- 8×10^7 in RF-dressed potential
- $> 10^5$ after evaporation to quantum degeneracy



Abel Beregi, D.Phil. thesis, Oxford (2023)

- Highly anisotropic confinement $\Rightarrow$ quasi-2D potentials: $f_z > 1$ kHz, $f_r < 10$ Hz.





Summary

2D quantum gases

- Introduction to the physics of 2D quantum gases
- Experiment: making multi-layer 2D traps (RF-dressed potentials)
    - T. Harte, E. Bentine, ... , C.J. Foot, PRA 97 013616 (2017)
- New tool: matter-wave interferometry of 2D quantum gases
    - S. Sunami, V. Singh, ..., L. Mathey, C.J. Foot, PRL 128, 250402 (2022)
- Quenching the Kosterlitz-Thouless (BKT) superfluid
    - A. Barker, S. Sunami, ... C.J. Foot, NJP 22 103040 (2020)
    - S. Sunami, V. Singh, ... L. Mathey, C.J. Foot, Science 382, 443 (2023)

      Universal scaling of the dynamic BKT transition in quenched 2D Bose gases.
- Ways to improve RF-dressed traps and Outlook

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Ultracold Quantum Matter Lab

Engineering and Physical Sciences Research Council

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Dr Shinichi Sunami

Dr Abel Beregi

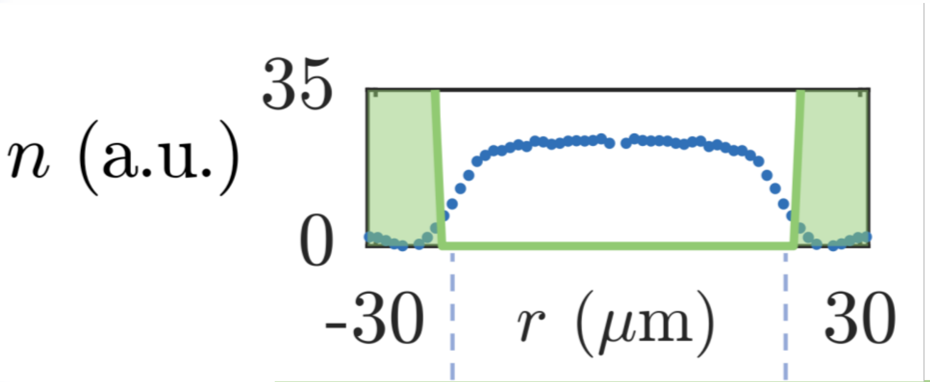
Dr Charu Mishra

Prof. Chris Foot

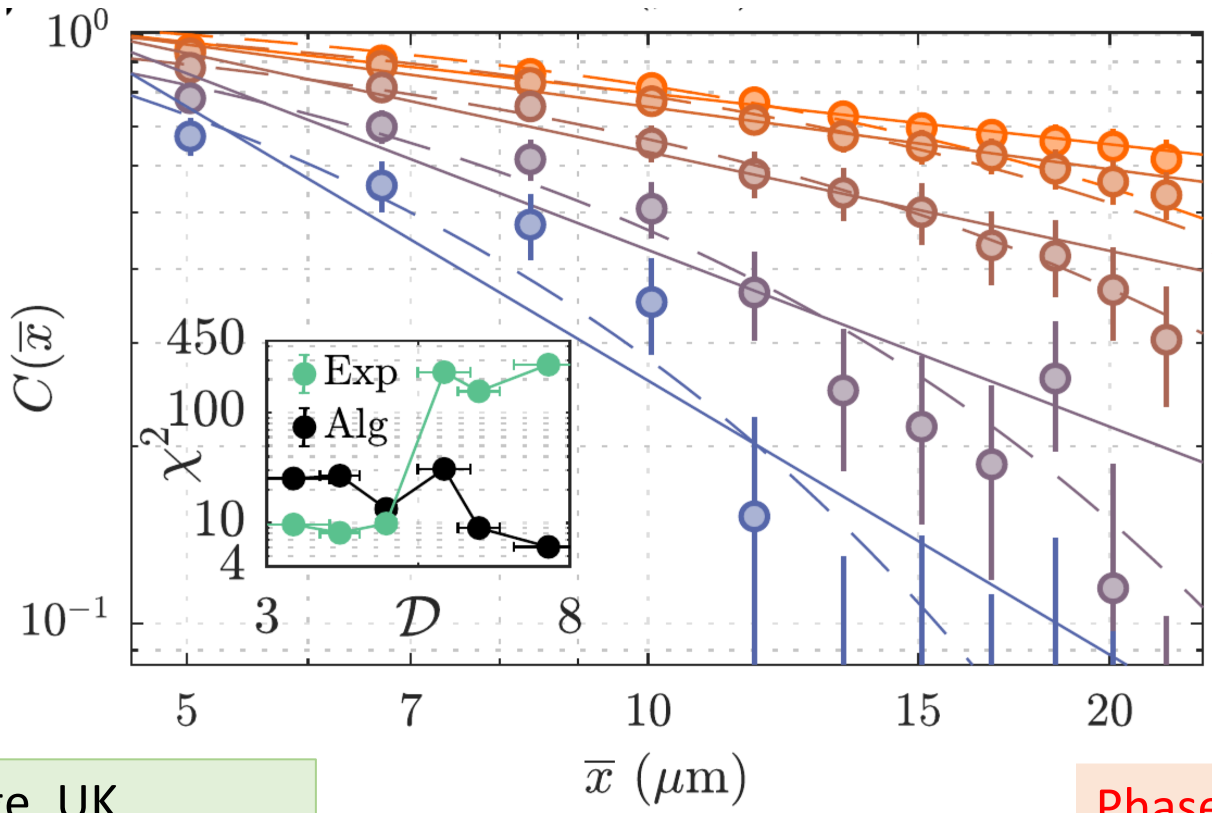
Erik Rydow

Oscar Chang

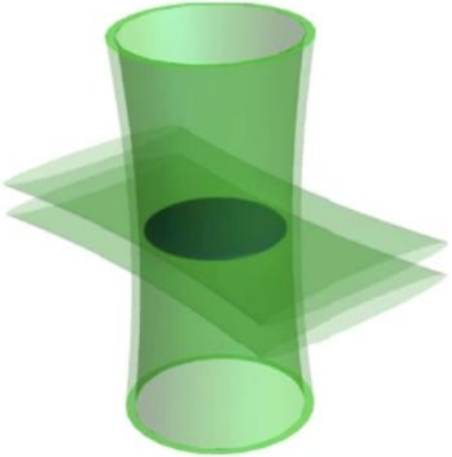
Correlation function in uniform potential



Magnetic trap + optical box
= near-homogeneous system

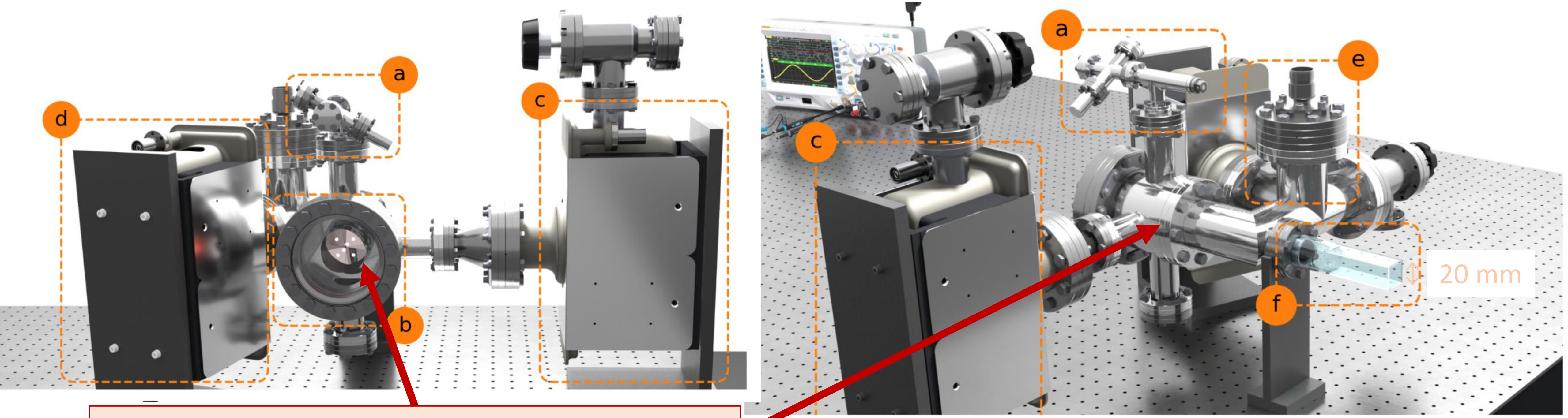


Phase-space density



University of Cambridge, UK.
Quantum gases in optical boxes
N. Navon, R.P. Smith & Z. Hadzibabic
Nat. Phys. 17, 1334 (2021)

New apparatus in Oxford Physics, using Rb-87 atoms



Pyramidal Magneto-Optical Trap (MOT)