Dark Matter Problems
Anne Green

1. **Dark matter direct detection**
(problem originally formulated by David Cerdeño)

In Planetary Dependence of Melanoma (Zioutas and Valachovic, 2018), the authors claim to have observed a correlation between the occurrence of melanoma in the human skin and the annual modulation in the dark matter (DM) flux that arrives to the Earth.

- Assume that the DM is a WIMP with mass of order 100 GeV. Calculate the number of expected DM recoils on the human skin over a whole human lifetime. Use the upper limit on the DM-nucleus scattering from recent experiments (LZ for example).
- Consider the possibility of lighter DM particles, so that the number density is larger, and the cross-section less tightly constrained by experiments. How large must the scattering cross section be so as to have a substantial number of interactions?
- Are there any sources of background that can give similar recoils (and that we are exposed to on a daily basis)?
- Based on all of the above, do you think that it is plausible that DM causes skin cancer?

2. **Primordial Black Hole dark matter**

(As we'll see in lecture 3) Primordial Black Holes (PBHs) are black holes that may form in the early Universe. Following the LIGO-Virgo detection of gravitational waves from the mergers of $\sim 10\,M_\odot$ black hole binaries, PBHs have received a lot of attention, in particular as a DM candidate. In this question we'll work through the details of some simple calculations regarding the abundance of PBHs.

- PBHs that form during radiation domination have mass roughly equal to the horizon mass, $M_{\rm H} = tc^3/G$, at the time they form. Show that the fraction of DM in the form of PBHs today, $f_{\rm PBH}$, is related to the initial abundance of PBHs, $\beta \equiv \rho_{\rm PBH}/\rho_{\rm rad}$ where $\rho_{\rm PBH}$ and $\rho_{\rm rad}$ are the densities of PBHs and radiation respectively at the time the PBHs form, by

$$f_{\rm PBH} \sim 10^9 \left(\frac{M}{M_\odot}\right)^{-1/2} \beta\,. \tag{1}$$

  For this rough calculation you can, e.g., assume the universe is matter dominated from radiation-matter equality to the present day, and ignore the variation in the number of relativistic degrees of freedom

- The initial abundance of PBHs is related to the mass variance (the typical size of density perturbations) at horizon crossing, $\sigma(M)$, by

$$\beta(M) \sim {\rm erfc}\left(\frac{\delta_{\rm c}}{\sqrt{2}\sigma(M)}\right)\,, \tag{2}$$

where $\delta_{\rm c} \approx 0.5$ is the threshold for PBH formation.

Calculate the values of $\sigma(M)$ required for PBHs with mass M equal to
i) $1M_{\odot}$,
ii) $10^{15}\,{\rm g}$,
to make up all of the DM ($f_{\rm PBH} = 1$).

What are the implications of the dependence of β on $\sigma(M)$?