

# A lightning course on Flavor Physics

## II - FCNCs and sensitivity to New Physics

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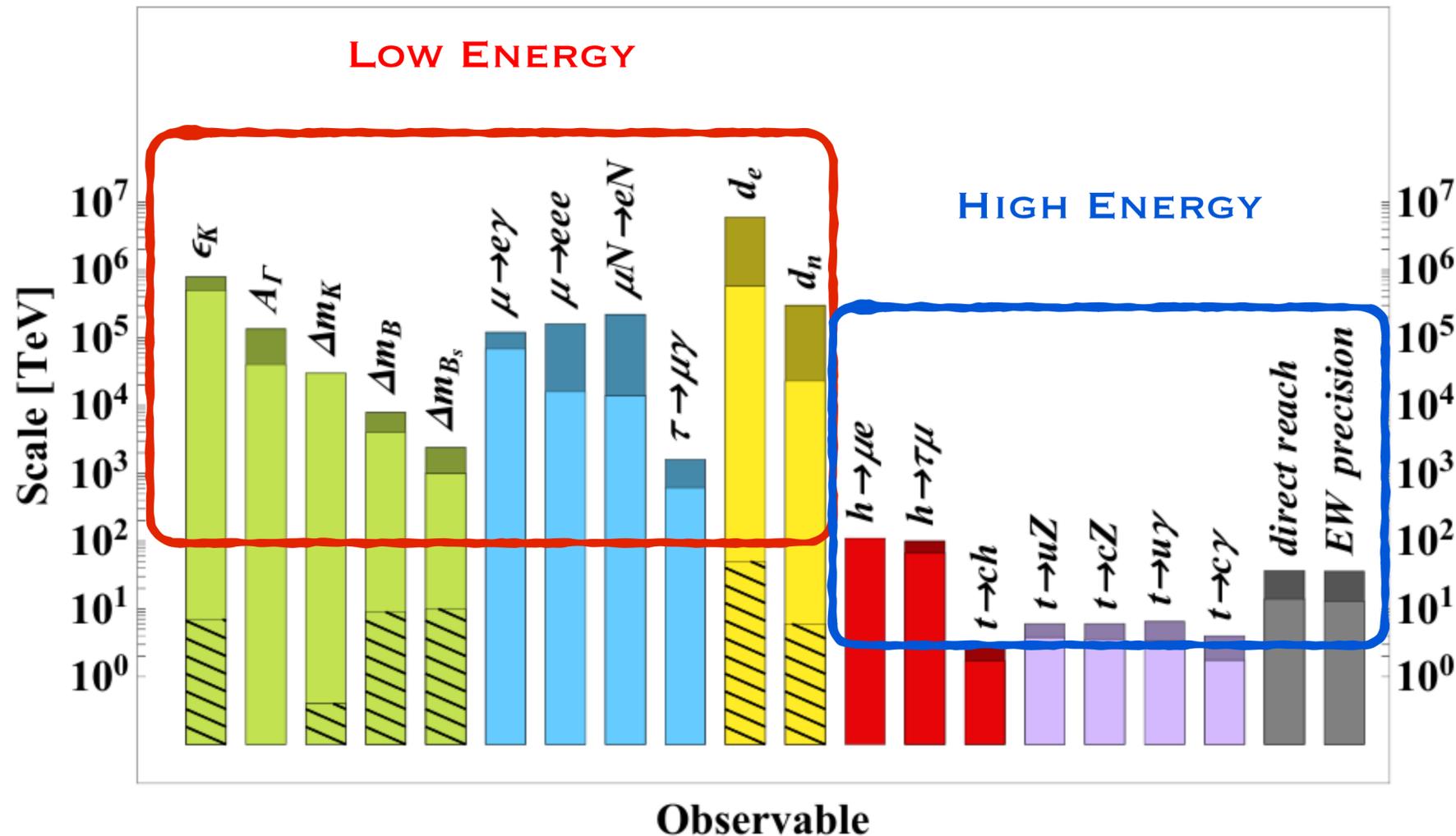


# FCNCs as probes of BSM

- FCNCs can provide null tests of the SM

*Null test: Observational test that would contradict the model*

- FCNCs are very sensitive to BSM with *non-standard* gauge or flavor structure
  - Searching for FCNCs in experiment could herald the discovery of New Physics!
  - Null searches are typically expressed as lower-bounds on mass scales of the putative BSM



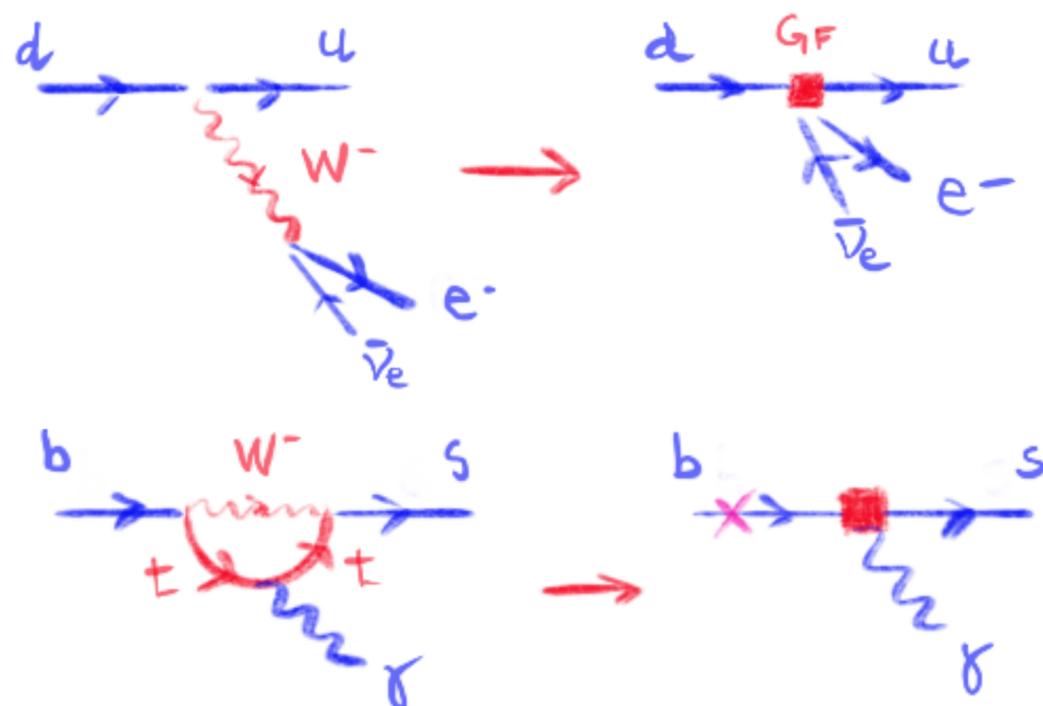
- Flavor physics is mostly a low-E endeavour
- Flavor is sensitive to BSM scales orders of magnitude higher than direct searches or EW precision tests

How do we reach these conclusions and what does this mean?

# The theorist's tool kit: Effective field theories

- Energies involved in hadron decays  $m_h \ll m_W$

Rigorous and systematic expansion in the small parameter  $\epsilon \approx m_h/m_W$  within the **Effective Field Theory (EFT)**



- Modern subnuclear extension of Fermi Theory

- Neutron  $\beta$  decay

$$\mathcal{M}_\beta \approx G_F C_\beta (\bar{u}\gamma^\mu P_L d) (\bar{e}\gamma_\mu P_L \nu)$$

- Extended also to FCNCs

- Radiative  $B$ -meson decays (e.g.  $B^0 \rightarrow K^*\gamma$ )

$$\mathcal{M}_\gamma \approx \frac{e m_b}{4\pi^2} G_F C_\gamma \bar{s}\sigma^{\mu\nu} P_R b F_{\mu\nu}$$

- We work with low-energy effective Lagrangians:

- **Dimensionful constant:** Scale of dynamics that have been *integrated out* -  $G_F \approx 1/m_W^2$

- **Wilson coefficient:** Structure and constants of UV theory -  $C_\beta \approx V_{ud}$ ,  $C_\gamma = V_{tb} V_{ts}^* f(x_t)$

- **Non-renormalizable operators:** with  $d \geq 5$  and composed of dynamical fields at  $E \ll m_W$

# EFT for BSM: Low energies

- EFTs can be also used to parametrize new physics

## Recipe for the construction of the EFT of the BSM

1. List **fields** that can be made *on-shell* at the energies of interest
2. List **gauge symmetries** manifest at the energies of interest
3. Construct all **gauge invariant operators** with these fields up to a given dimension  $d$

**Power counting:** Ordering of the  $\infty$  operators according to power  $n$  in  $(E/\Lambda_{\text{BSM}})^n$

- Only a **finite number of operators** needed for a given precision!

- **EFTs of UV new physics are phenomenologist's 2<sup>nd</sup> best friend**

- **Top-down perspective:** One can readily test a model with low-energy data by *matching* to the EFT
- **Bottom-up perspective:** *All* UV BSM models are subsumed within the EFT

## Two caveats about bottom-up usage of EFT

1. Not *all* combinations of EFT operators can be accommodated by sensible UV models
2. Only grasps the *shapes* of the UV physics

# Low-energy EFT of the CC quark-lepton interactions

- Ingredients for e.g. neutron  $\beta$  decay

1. Degrees of freedom at  $m_n \sim 1$  GeV: *up* and *down* quarks, electron, neutrinos and photons
2. Gauge symmetries:  $SU(3)_c \times U(1)_{EM}$
3. Power counting: Leading contribution at  $n = 2$  (dimension-6 operators)

$$\mathcal{L}_\beta = \frac{4G_F V_{ud}}{\sqrt{2}} \left( C_{LL} (\bar{u} \gamma^\mu P_L d) (\bar{e} \gamma_\mu P_L \nu) + C_{RL} (\bar{u} \gamma^\mu P_L d) (\bar{e} \gamma_\mu P_L \nu) \right. \\ \left. + C_{S_L S_L} (\bar{u} P_L d) (\bar{e} P_L \nu) + C_{S_R S_L} (\bar{u} P_L d) (\bar{e} P_L \nu) + C_{T_L T_L} (\bar{u} \sigma^{\mu\nu} P_L d) (\bar{e} \sigma_{\mu\nu} P_L \nu) \right)$$

- Any UV contribution to  $\beta$  decay must be described by these **5 dimension operators** at leading order
- Operators distinguished by their Lorentz and chiral structure
- If we added **right-handed neutrinos** we would need **5 more operators** (replace  $P_L \rightarrow P_R$  in leptons)

Actually, it is very old technology: Equivalent effective Lagrangians led to the discovery of the  $V - A$  interaction! (see [S. Weinberg's "V-A was the key"](#))

# Two concepts in the EFT

- **Matching:** Procedure by which we transition from a QFT to an EFT with less dynamical d.o.f. at a UV scale  $\Lambda_{\text{NP}}$ 
  - One *integrates out* heavy d.o.f. and the **short distance (UV)** information is frozen in the Wilson coefficients
  - The **long distance (IR)** information is described dynamically at the operator level
- **RGE evolution of operators**
  - EFT is nonrenormalizable *but* can be renormalized order by order
  - **Renormalization Group Equation** provides the renormalization scale evolution of the **Wilson coefficients**

$$\frac{d\vec{C}(\mu)}{d \log \mu} = \gamma^T(\alpha_{\text{em}}, \alpha_s) \vec{C}(\mu)$$

Renormalization scale dependence  $\longleftarrow$

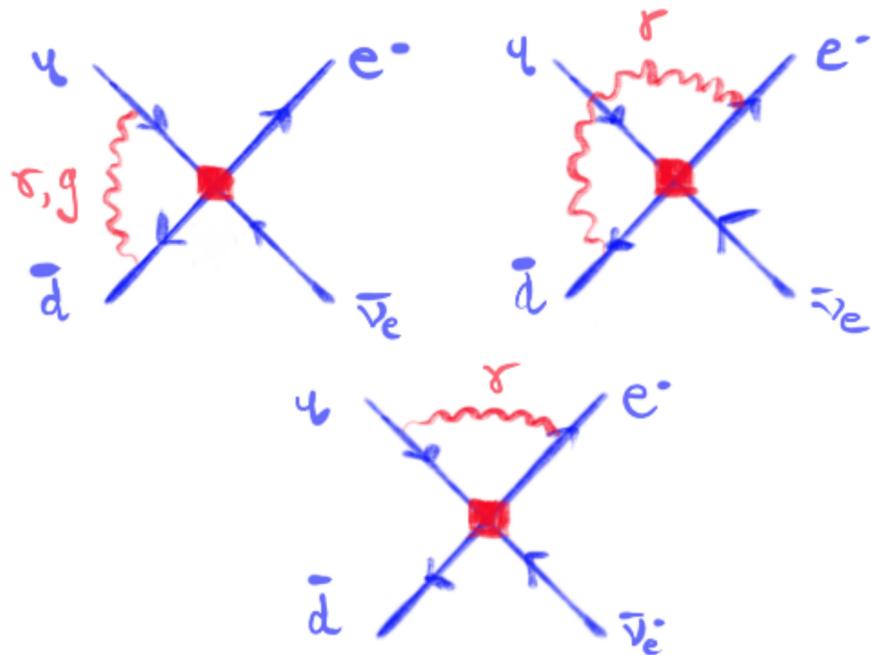
$\xrightarrow{\text{Vector of Wilson coefficients}}$

**Anomalous dimensions matrix**  
Coefficients of UV divergences

- **Matching** provides the initial conditions  $\vec{C}(\Lambda_{\text{NP}})$  for the RGE flow
- Operators suffer **rescaling** and **mixing** when connecting the UV and IR scales! ... Unless symmetry protection!

# RGE in $\beta$ decays and similar transitions

- Corrections at 1-loop given by QED and QCD



$$\vec{C}^T = (C_{LL}, C_{RL}, C_{S_R S_L}, C_{S_L S_L}, C_{T_L T_L})$$

$$\gamma_{\text{QED}}^T = \begin{pmatrix} -2 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & \frac{2}{3} & 4 \\ 0 & 0 & 0 & \frac{1}{12} & -\frac{20}{9} \end{pmatrix}$$

$$\gamma_{\text{QCD}}^T = \text{diag}(0, 0, -4, -4, 4/3)$$

- Final RGE between 2 GeV and Z-pole mass

$$\begin{pmatrix} C_{LL} \\ C_{RL} \\ C_{S_R S_L} \\ C_{S_L S_L} \\ C_{T_L T_L} \end{pmatrix}_{(\mu = 2 \text{ GeV})} = \begin{pmatrix} 1.005 & 0 & 0 & 0 & 0 \\ 0 & 1.005 & 0 & 0 & 0 \\ 0 & 0 & 1.72 & 0 & 0 \\ 0 & 0 & 0 & 1.72 & -0.0242 \\ 0 & 0 & 0 & 0 & 0.825 \end{pmatrix} \begin{pmatrix} C_{LL} \\ C_{RL} \\ C_{S_R S_L} \\ C_{S_L S_L} \\ C_{T_L T_L} \end{pmatrix}_{(\mu = m_Z)}$$

- General comments

- QED rescales all operators and mix scalar-tensor (QED/QCD vectorial  $\Rightarrow$  Do not mix different chiralities!)
- QCD only rescales scalar and tensor (Conservation vector current prevents RGE of vectors!)

RGE needs to be taken into account when connecting our UV models to phenomenology!

# EFT above EW scale: SMEFT

- **SMEFT:** EFT built with the *full* SM gauge group and field content

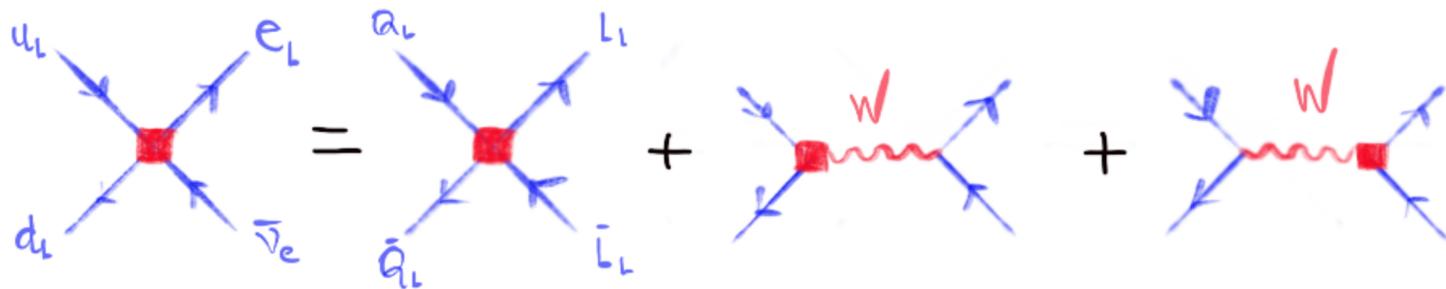
**Example: Current-current operators**

4-fermion -  $\mathcal{O}_{LQ}^{(3)} = \left( \bar{L}_L \gamma^\mu \tau^I L_L \right) \left( \bar{Q}_L \gamma_\mu \tau^I Q_L \right)$

Vertex corrections -

$$\left\{ \begin{array}{l} \mathcal{O}_{HQ}^{(3)} = \left( H^\dagger D_\mu^I H \right) \left( \bar{Q}_L \gamma^\mu \tau^I Q_L \right) \\ \mathcal{O}_{HL}^{(3)} = \left( H^\dagger D_\mu^I H \right) \left( \bar{L}_L \gamma^\mu \tau^I L_L \right) \\ \mathcal{O}_{Hud}^{(3)} = \left( \tilde{H}^\dagger D_\mu H \right) \left( \bar{u}_R \gamma^\mu d_R \right) \end{array} \right.$$

- At scales  $\approx v_{ew}$  we need to *switch* EFTs!



$$\begin{aligned} C_{LL}^{ij,\alpha\beta} &\simeq - [\bar{C}_{LQ}^{(3)} \cdot V]^{ij,\alpha\beta} + [\bar{C}_{HQ}^{(3)} \cdot V]^{ij} + [\bar{C}_{HL}^{(3)}]^{i\alpha\beta} \\ C_{RL}^{ij,\alpha\beta} &\simeq [\bar{C}_{Hud}^{(3)}]^{ij} \delta^{\alpha\beta} \end{aligned}$$

- **More symmetry in SMEFT:** There are relations among low-energy EFT Wilson coefficients!

There are no lepton non-universal  $C_{RL}$  interactions! (only quark vertex corrections)

**Operators live in flavor space!**

1. The  $\bar{C}$  are tensors in flavor space of Wilson coefficients
2. Anomalous dimensions have to be re-calculated in SMEFT above  $\approx v_{ew}$

# Imposing a flavor ansatz: Minimal Flavor violation

- SMEFT is a full-fledged flavored EFT
  - There are **59** flavor-diagonal operator vs. **2499** operators in the full flavor SMEFT at dim-6!

It is often practical to impose flavor ansatz in  $\bar{C}$  to guide the model building

- **Minimal Flavor Violation (MFV)**: All the flavor violation in SM+BSM stems from *just* the SM Yukawas
  - One can implement MFV in the EFT using the spurion analysis

Impose an additional global symmetry  $\mathcal{G} = U(3)_Q \times U(3)_u \times U(3)_d$  to  $\mathcal{L}_{\text{SMEFT}}$

$$Q_L \sim (3,1,1), \quad u_R \sim (1,3,1), \quad d_R \sim (1,1,3), \quad y_u \sim (3,\bar{3},1), \quad y_d \sim (3,1,\bar{3})$$

- MFV is useful because it transfers the flavor component of the GIM suppression to BSM

**Example:** Contribution to the FCNC  $b \rightarrow s\gamma$

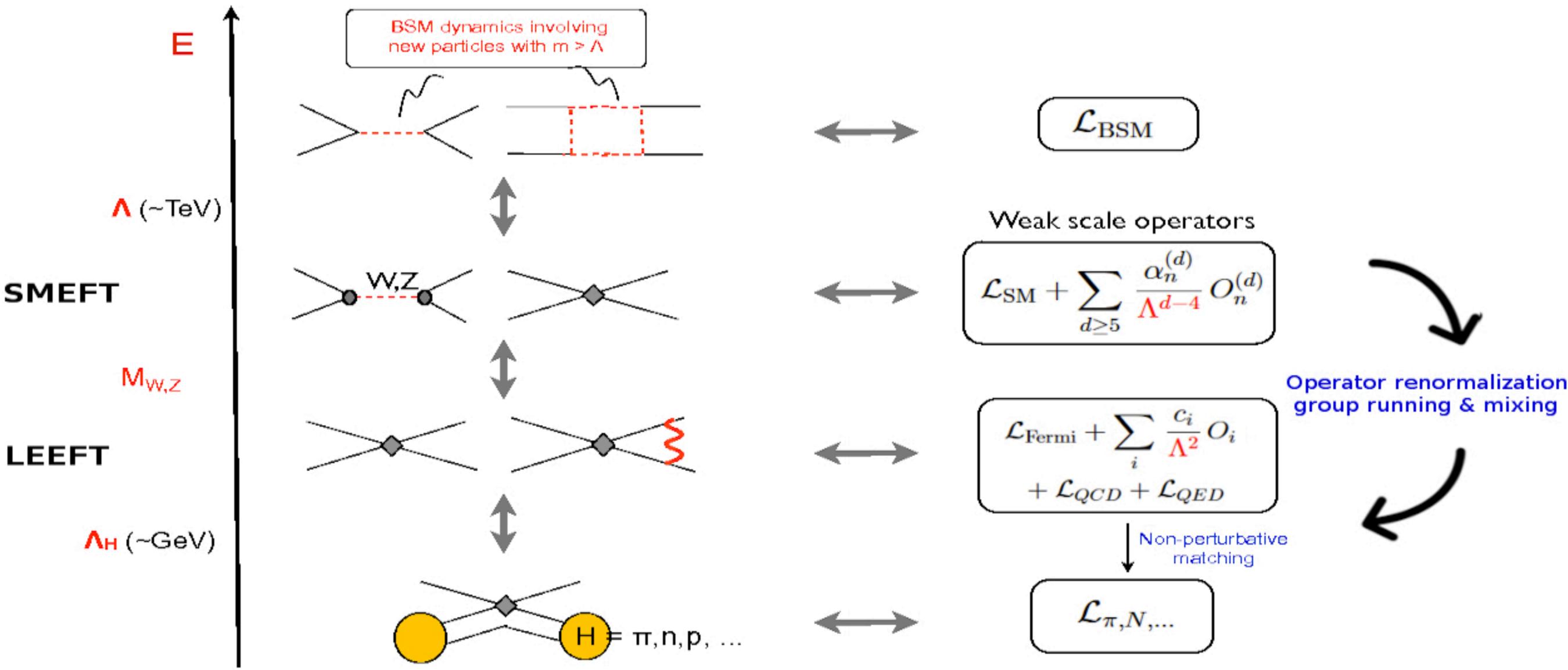
$$\frac{e \bar{c}}{\Lambda_{\text{NP}}^2} F_{\mu\nu} \bar{Q}_L \sigma^{\mu\nu} \underbrace{y_u y_u^\dagger y_d}_{\text{small}} b_R \Rightarrow \frac{e \bar{c}}{\Lambda_{\text{NP}}^2} F_{\mu\nu} \begin{pmatrix} \bar{U}_L \\ \bar{D}_L V^\dagger \end{pmatrix} \sigma^{\mu\nu} m_u^2 V m_d b_R$$

$y_d$  alone is to small

Same yukawa suppression as in the SM!

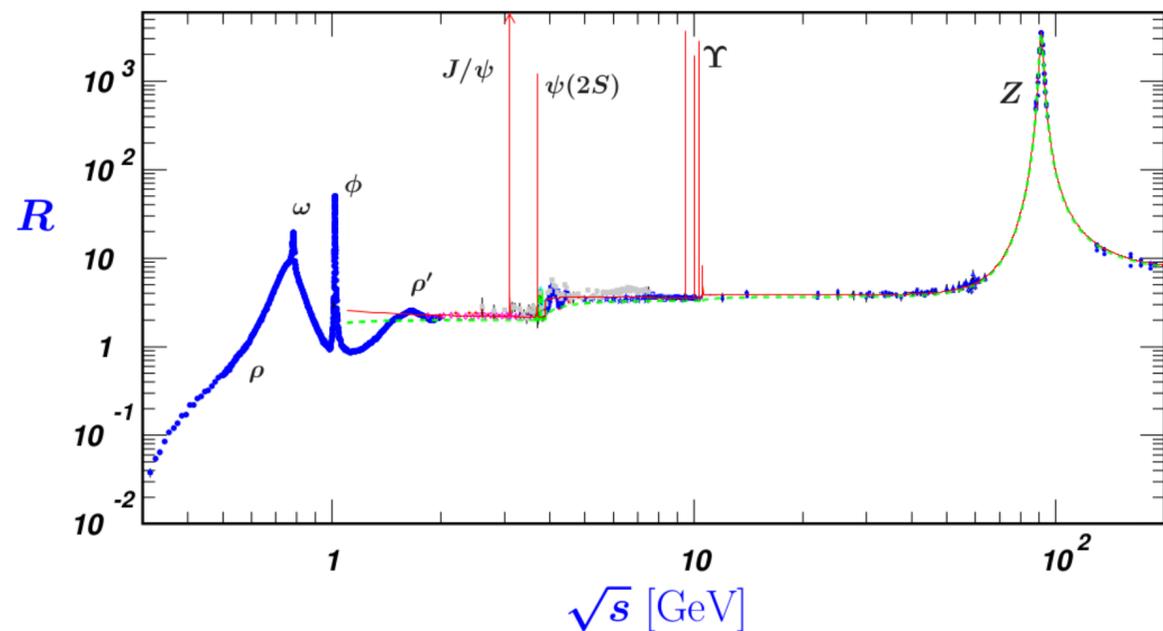
$$C_\gamma = \frac{e \bar{c}}{\Lambda_{\text{NP}}^2} m_b y_t^2 V_{ts}^* V_{tb}$$

# Summary of the EFT procedure



# Low-energy: The realm of the hadrons

- QCD confines around and below energies  $\sim \Lambda_{\text{QCD}} \approx 200 \text{ MeV}$

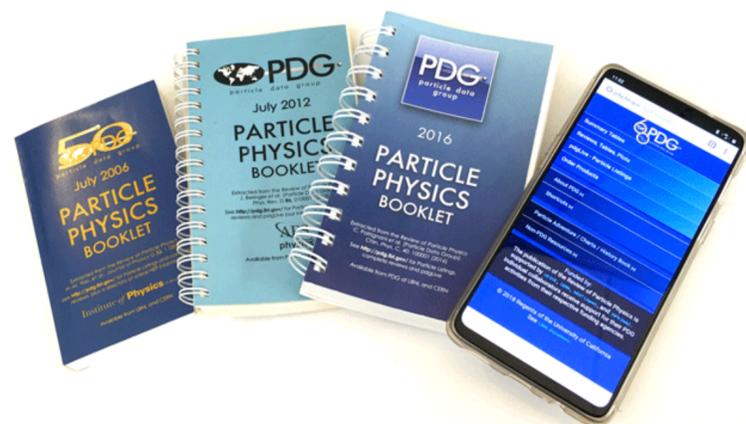


Baryons $qqq$ and Antibaryons $\bar{q}\bar{q}\bar{q}$						Mesons $q\bar{q}$					
Baryons are fermionic hadrons. There are about 120 types of baryons.						Mesons are bosonic hadrons. There are about 140 types of mesons.					
Symbol	Name	Quark content	Electric charge	Mass $\text{GeV}/c^2$	Spin	Symbol	Name	Quark content	Electric charge	Mass $\text{GeV}/c^2$	Spin
$p$	proton	$uud$	1	0.938	1/2	$\pi^+$	pion	$u\bar{d}$	+1	0.140	0
$\bar{p}$	anti-proton	$\bar{u}\bar{u}\bar{d}$	-1	0.938	1/2	$K^-$	kaon	$s\bar{u}$	-1	0.494	0
$n$	neutron	$udd$	0	0.940	1/2	$\rho^+$	rho	$u\bar{d}$	+1	0.770	1
$\Lambda$	lambda	$uds$	0	1.116	1/2	$B^0$	B-zero	$d\bar{b}$	0	5.279	0
$\Omega^-$	omega	$sss$	-1	1.672	3/2	$\eta_c$	eta-c	$c\bar{c}$	0	2.980	0

- Only the proton is (almost) really stable!

The thousands of different decay modes of these hundreds of particles are a precious source of information

The PDG is phenomenologist's 1<sup>st</sup> best friend!



- Branching fraction of a decay channel  $i$  of a hadron  $h$

$$\text{Br}_i = \Gamma_i / \Gamma_h = \tau_h \Gamma_i$$

- Only hadrons whose main decay channel is weak

1. Flavor violations !

2. Sensitivity to  $E \gtrsim m_W$  !

# Connecting to the observables of the hadronic world

- Our Lagrangians are written in terms of quarks and our observables in terms of hadrons!

Interactions:  $\mathcal{L}(u, d, s, c, b, e, \nu, G, F)$       Asymptotic states:  $|\pi^\pm, \pi^0, K^\pm, D^\pm, B^\pm, p, n, \Lambda, \dots\rangle$

- By *asymptotic* we mean hadrons with long life times (  $\tau_{\text{weak}} \approx \overbrace{10^{-8}}^{\text{Kaons}} - \overbrace{10^{-12}}^{\text{B-mesons}}$  s vs.  $\tau_{\text{EM}} \approx \overbrace{10^{-17}}^{\pi^0}$  s OR  $\tau_{\text{strong}} \approx \overbrace{10^{-24}}^{\rho\text{-resonance}}$  s )
- Observables defined in terms of matrix elements

$$\mathcal{M} \sim \langle e', \nu', \dots; H'_1, H'_2, \dots | \overbrace{\mathcal{O}_\ell \times \mathcal{O}_q}^{\mathcal{L}} | e, \nu, \dots; H_1, H_2, \dots \rangle \quad \text{with Observables} \sim |\mathcal{M}|^2$$

- **Factorization:** Wick's theorem *typically* leads to factorization of matrix element

$$\mathcal{M} \sim \langle e', \nu', \dots | \mathcal{O}_\ell | e, \nu, \dots \rangle \times \langle H'_1, H'_2, \dots | \mathcal{O}_q | H_1, H_2, \dots \rangle$$

Perturbative matrix element



Hadronic matrix element



- **Hadronic matrix elements:** Encapsulate all the nonperturbative-QCD information of the transition

Very difficult to compute! They limit our capacity to learn about short distances

# Determinations of the hadronic brown muck

- **General strategy:**

1. Parametrize the matrix element and **exploit discrete and Lorentz symmetries**

2. **Exploit approximate symmetries of QCD** in perturbative (EFT) expansions

Relations among hadronic elements that **increase predictability** in a robust framework

- Isospin ( $m_d \approx m_u$ ) and  $SU(3)_F$  ( $m_u \approx m_d \approx m_s$ ) in light quarks - **Chiral Perturbation Theory**

- Heavy-quark symmetry ( $m_{c,b} \gg \Lambda_{\text{QCD}}$ ) - **Heavy quark effective theory**

3. **Measure or calculate** hadronic matrix elements

- **Lattice QCD** - systematic approximation to nonperturbative QCD from a discrete and finite space-time

- **QCD sum rules, quark models, Ads/CFT**, etc ... Allow to estimate semi-analytically

**Example:** Leptonic pion decay  $\pi^- \rightarrow e^- \bar{\nu}$

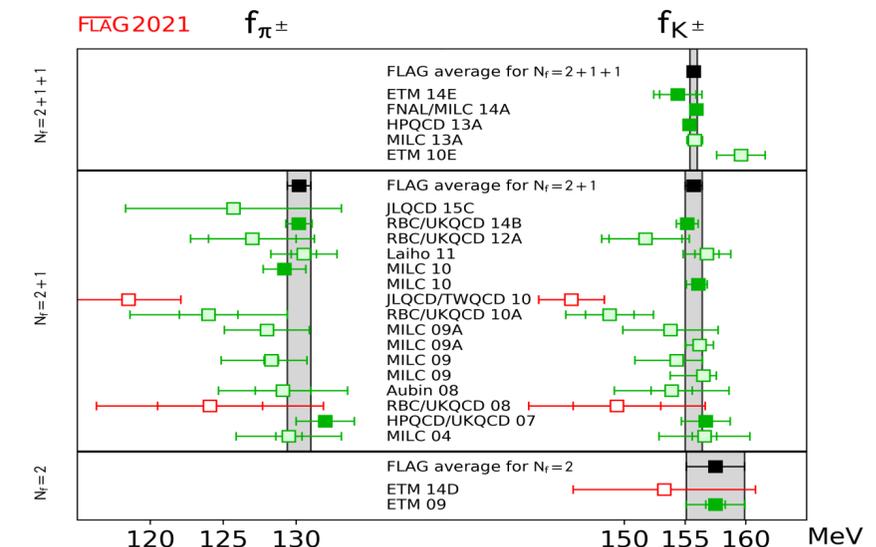
$$\langle 0 | \bar{u} \gamma^\mu \gamma_5 d | \pi^-(p) \rangle = -i p^\mu f_\pi$$

- $f_\pi$  is the pion decay constant  $f_\pi \simeq 130 \text{ MeV}$

- **Parity invariance:** Vector & Scalar are 0!

- **Lorentz invariance:** Tensor is 0!

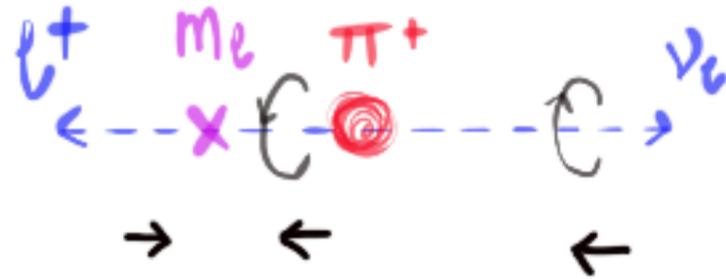
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# More about the leptonic pion decay

- **Chiral suppression:** In the chiral limit  $m_\ell \rightarrow 0$  the amplitude vanishes!

$$\mathcal{M} = \langle \ell^+ \nu_\ell | \mathcal{L}_{SM} | \pi^+ \rangle = \frac{4G_F V_{ud}}{\sqrt{2}} \langle \ell^+ \nu_\ell | \bar{\nu}_L \gamma_\mu P_L \ell | 0 \rangle \langle 0 | \bar{d} \gamma^\mu P_L u | \pi^+ \rangle = \frac{G_F f_\pi V_{ud}}{\sqrt{2}} m_\ell \bar{\nu}_\ell P_R \ell$$



$$\begin{aligned} \text{Br}(\pi^+ \rightarrow \mu^+ \nu_\mu) &= 99.98770(4) \% \\ \text{Br}(\pi^+ \rightarrow e^+ \nu_e) &= 1.230(4) \times 10^{-4} \end{aligned}$$

- **Pseudoscalar operator:** Contribution of  $\bar{d} \gamma_5 u$ ?

**Current algebra (PCAC):**  $\partial_\mu (\bar{d} \gamma^\mu \gamma_5 u) = i(m_d + m_u) \bar{d} \gamma_5 u \Rightarrow \langle 0 | \bar{d} \gamma_5 u | \pi^+ \rangle \equiv f_P = i f_\pi \frac{m_\pi^2}{m_d + m_u}$

- Pseudoscalar operator is chirally flipping  $\Rightarrow$  **Not chirally suppressed!**

$$\Gamma_{\ell 2} = \frac{G_F^2 |V_{ud}|^2 f_\pi^2}{8\pi} m_\pi m_\ell^2 \overbrace{\left(1 - \frac{m_\ell^2}{m_\pi^2}\right)^2}^{\text{Phase space}} \left| C_{LL} - C_{RR} - \frac{m_\pi^2}{m_e(m_d + m_s)} (C_{S_R S_L} - C_{S_L S_L}) \right|^2$$

## Physical results

**SM:**  $|V_{ud}| = 0.97438(12)$

**BSM-Vector:**  $\Lambda_{LL} \approx 1 \text{ TeV}$

**BSM-Scalar:**  $\Lambda_{S_L S_L} \approx 1000 \text{ TeV!}$

# Form factors

- **Hadron → Hadron' transitions:** Hadronic matrix elements depend on Lorentz scalar  $q^2 = (p - p')^2$

- Can be **parametrized** in terms of  $q^2$ -dependent functions called **form factors**

- **Meson → Meson form factors**

$$\langle \pi^0(p') | \bar{s} \gamma_\mu d | K^+(p) \rangle = f_+(q^2)(p_\mu + p'_\mu) + f_-(q^2)(p_\mu - p'_\mu), \quad \langle \pi^0(p') | \bar{s} \sigma_{\mu\nu} d | K^+(p) \rangle = f_T(q^2)(p_\mu p'_\nu - p'_\mu p_\nu)$$

- No **axial** form factors! **Scalar** obtained with **current algebra**

- **Baryon → Baryon form factors**

↙ Only in BSM!

$$\langle B_2(p') | \bar{d} \gamma_\mu u | B_1(p) \rangle = \bar{u}_2(p') \left[ f_1(q^2) \gamma_\mu + i \frac{f_2(q^2)}{m_{B_1}} \sigma_{\mu\nu} q^\nu + \frac{f_3(q^2)}{m_{B_1}} q_\mu \right] u_1(p)$$

$$\langle B_2(p') | \bar{d} \gamma_\mu \gamma_5 u | B_1(p) \rangle = \bar{u}_2(p') \left[ g_1(q^2) \gamma_\mu + i \frac{g_2(q^2)}{m_{B_1}} \sigma_{\mu\nu} q^\nu + \frac{g_3(q^2)}{m_{B_1}} q_\mu \right] \gamma_5 u_1(p)$$

- **Both** vector and axial form factors! (pseudo)scalar with current algebra and **3 more tensor** form factors for BSM

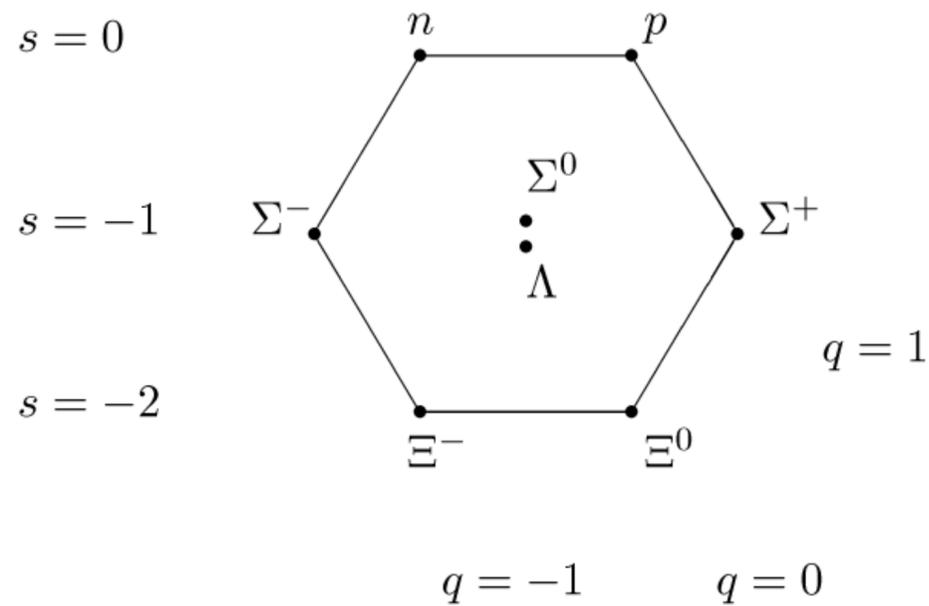
Total of **9 form factors!**

## Important points about form factors and friends

1. Hadronic complexity can lead to extended sensitivity to UV physics
2. Use every trick under your sleeve to get rid of them!

# Exploiting $SU(3)_F$ : e.g. hyperon FFs

- **Hyperon decays:** Form factors related by  $SU(3)_F$  symmetry



- Semileptonic decays  $B_1 \rightarrow B_2 e \bar{\nu}_e$  triggered by  $s \rightarrow d e^- \bar{\nu}_e$  decays

- **5 decay channels:**  $\Lambda p, \Sigma^- n, \Xi^- \Lambda, \Xi^- \Sigma^0, \Xi^0 \Sigma^+$

- Only two  $SU(3)_F$ -singlets if  $\mathcal{O}_{\text{EFT}}$  lives in the octet

$$\langle B_a | \mathcal{O}_c | B_b \rangle = F_{\mathcal{O}} f_{cab} + D_{\mathcal{O}} d_{cab} \quad \text{Reduced ME}$$

Group theory factors

- Vector FFs **predicted** in terms of  $p$  and  $n$  EM charges!

- Axial FFs **predicted** by axial  $np$  coupling and one extra channel

- Further  $SU(3)_F$  expansions involve baryon-mass differences  $\Delta$ !

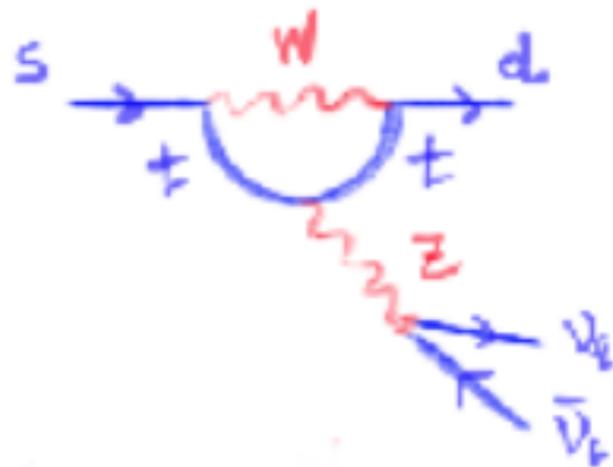
Rates of the **5 decay channels** depend only on **2 numbers!** (@ LO in  $SU(3)_F$ )

$$\Gamma \simeq \frac{G_F^2 |V_{us}|^2 \Delta^5}{60\pi^3} [f_1(0)^2 + 3g_1(0)^2]$$

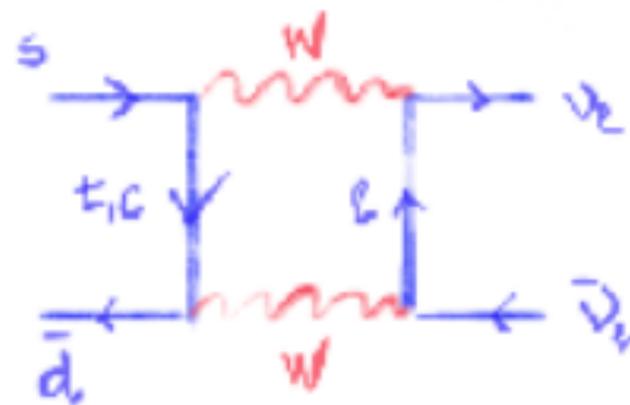
# Flavor physics at work: $K^+ \rightarrow \pi^+ \nu \bar{\nu}$

- Prototypical very-rare kaon decay:  $\Delta S = 1$  FCNC

Penguin diagram



Box diagram

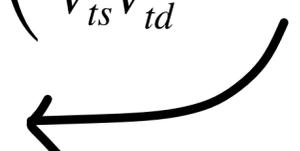


- Effective Lagrangian

$$\mathcal{L}_{\text{SM}} = -\frac{4G_F}{\sqrt{2}} V_{ts} V_{td}^* \frac{\alpha}{2\pi} \sum_{\ell} C_{\nu\ell} (\bar{d} \gamma^\mu P_L d) (\bar{\nu}_\ell \gamma_\mu \nu_\ell)$$

Wilson Coefficient:  $C_{\nu\ell} = \frac{1}{s_w^2} \left( \frac{V_{cs} V_{cd}^*}{V_{ts} V_{td}^*} X_c^\ell + X_t \right)$

Inami-Lin function



- Relevant form factors related by isospin to CCs

$$\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = \frac{\alpha^2 |V_{ts} V_{td}^*|^2 \text{Br}(K^+ \rightarrow \pi^0 e^+ \nu_e)}{2\pi^2 |V_{us}|^2} \sum_{\ell} |C_{\nu\ell}|^2$$

$$\simeq \frac{\alpha^2}{2\pi^2} \lambda^{10} \sum_{\ell} |C_{\nu\ell}|^2$$

## Physical results

$$\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{\text{SM}} = 8.55(4) \times 10^{-11}$$

$$\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{\text{expt}} = 1.14(36) \times 10^{-10}$$

$\Lambda_{\text{NP}} \gtrsim 100 \text{ TeV!}$

# Flavor at work: Tutorial

- Let's connect a UV model to a  $\text{Br}(K^- \rightarrow \pi^- \nu \bar{\nu})$  and derive a bound on its mass scale
- We have a  $Z'$  boson of mass  $m_{Z'}$  that is coupled to the SM with

$$\mathcal{L} \supset \left( g_{ij}^Q \bar{Q}_L^i \gamma^\mu Q_L^j + g^L \bar{L}_L^\alpha \gamma^\mu L_L^\alpha \right) Z'_\mu$$

$g^Q$  is a matrix in general real matrix in flavor space and  $g_L$  a universal coupling for leptons.

## Calculate

1. Matching of the UV model to an SMEFT operator.
2. Match the SMEFT operator to the operator in low-energy EFT ( $C_{\nu_\ell}^{\text{SM}} \simeq 10$ )

$$\mathcal{L}_{\text{EFT}} \supset -\frac{4G_F}{\sqrt{2}} V_{ts} V_{td}^* \frac{\alpha}{2\pi} \sum_{\ell} C_{\nu_\ell} (\bar{d} \gamma^\mu P_L d) (\bar{\nu}_\ell \gamma_\mu \nu_\ell)$$

3. *Estimate* the lower bound on  $m_{Z'}$  given by  $\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{\text{expt}} = 1.14(36) \times 10^{-10}$
4. How does this bound change if we impose MFV?