A lightning course on Flavor Physics

I - Introduction and flavor structure in the SM

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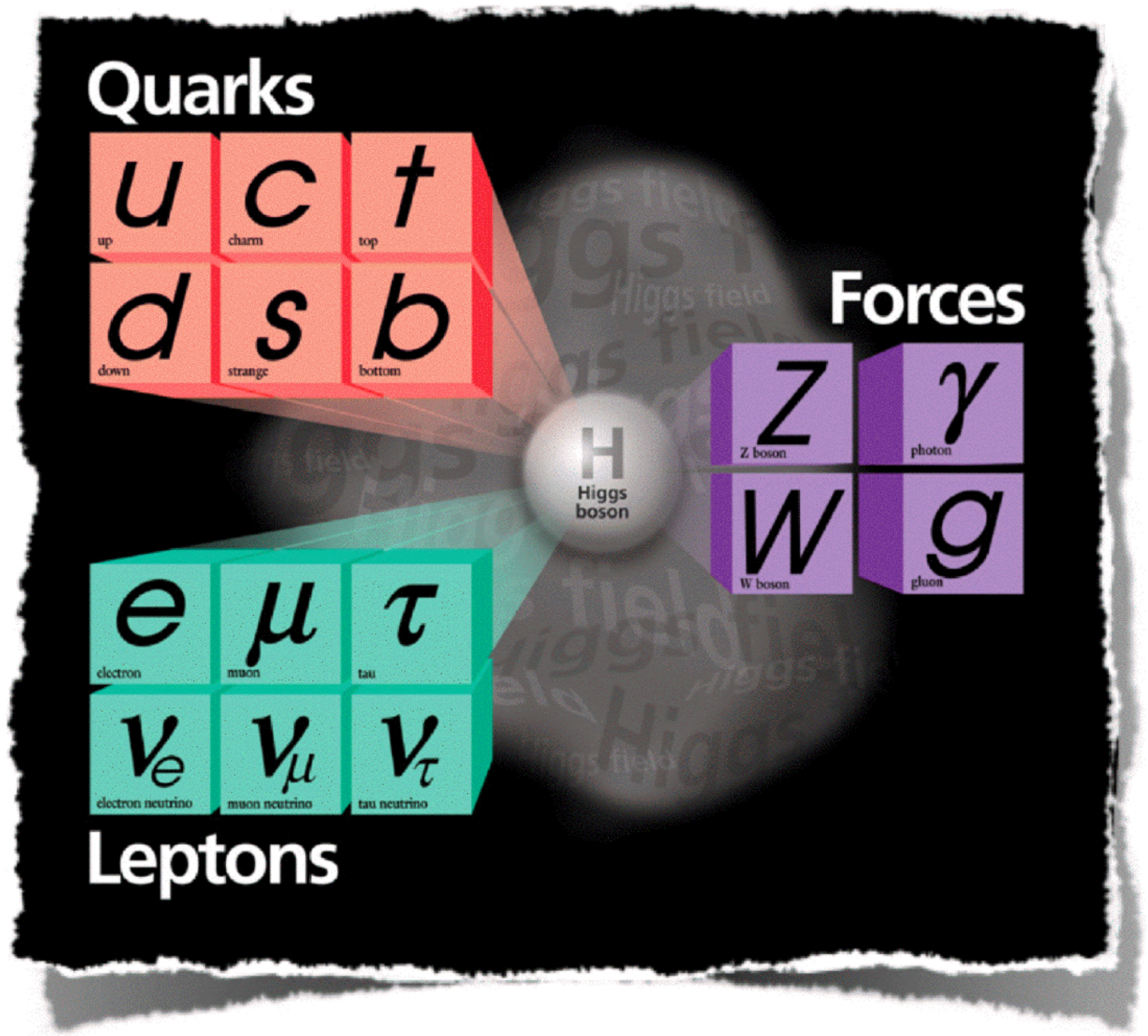
What is Flavor Physics?

"Just as ice cream has both colour and flavour so do quarks"

H. Fritzsch & M. Gell-Mann, 1971

The Standard Model of Particle Physics

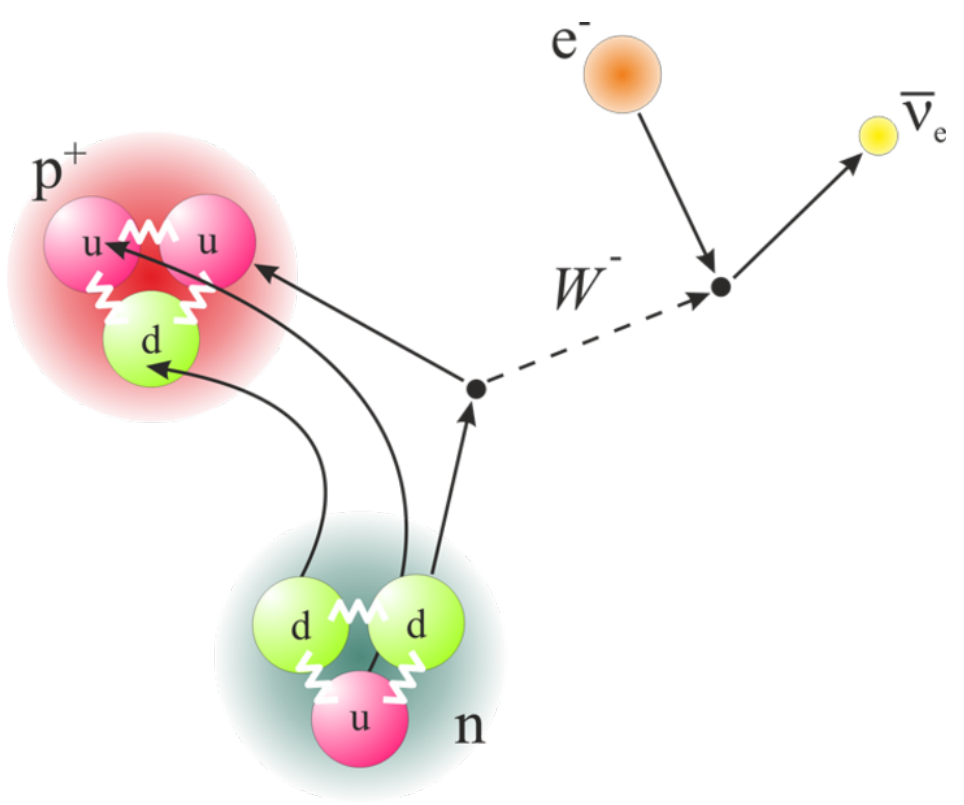


- 3 *almost identical* "families" of matter fields
  - **Up quarks (Q=2/3):** "up", "charm" and "top"
  - **Down quarks (Q=-1/3):** "down", "strange" and "bottom"
  - **Neutral leptons:** Neutrinos
  - **Charged leptons:** Electron, muon and tau
  - **"Identical":** Same gauge couplings (e.g. charges)
  - **"Almost":** Different masses!

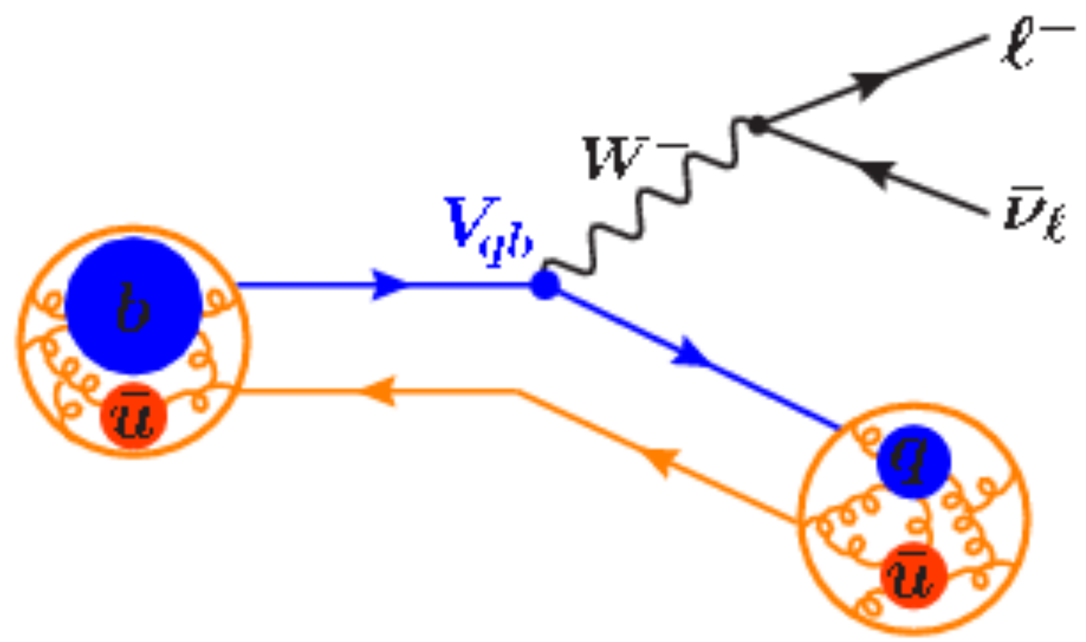
What is Flavor Physics?

Only weak interactions *violate* flavor

- **Classic:** Nuclear (neutron) β decay



- **Contemporary:** *B* meson decay



Flavor physics focuses on studying the **implications** of flavor violation in the SM and beyond

- **Structural:** Gauge symmetries, quantum numbers, vacuum structure (**accidental symmetries**).
- **Parametric:** Flavor eventually determined by **13** (out of **18**) **free parameters** of the SM.
  - **Flavor** patterns and **hierarchies.**
  - **Approximate symmetries** (e.g. isospin, $SU(3)_F$, etc)

Why studying Flavor Physics?

1. Deep fundamental questions about nature
    - **Flavor puzzle:** Origin of flavor hierarchies in the SM
    - ***CP* violation:** Origin of the cosmological matter-antimatter asymmetry
2. Very sensitive (indirect) probes/constraints of beyond SM
    - **Flavor-changing neutral currents:** Sensitive up to 1000 TeV!
    - Rapid and revolutionary experimental progress
3. Flavor anomalies and discovery potential
    - *B*-meson lepton-flavor universality anomalies
    - Muon $(g-2)$

Why studying Flavor Physics?

Flavor Physics spearheaded the discovery of the SM when the SM was the New Physics!

- **Nuclear β decay:** Discovery of weak interactions and the neutrinos
- **Rare kaon decays:** Discovery of charm quark
- **Kaon decays:** Discovery of *CP* violation $\rightarrow$ Discovery of 3 generations

PROPOSAL FOR K^o_2 DECAY AND INTERACTION EXPERIMENT

J. W. Cronin, V. L. Fitch, R. Turlay

(April 10, 1963)

I. INTRODUCTION

The present proposal was largely stimulated by the recent anomalous results of Adair et al., on the coherent regeneration of K^o_1 mesons. It is the purpose of this experiment to check these results with a precision far transcending that attained in the previous experiment. Other results to be obtained will be a new and much better limit for the partial rate of $K^o_2 \rightarrow \pi^+ + \pi^-$, a new limit for the presence (or absence) of neutral currents as observed through $K_2 \rightarrow \mu^+ + \mu^-$.

Outline of these lectures and bibliography

First day:

- The origin of flavor mixing in the SM
- The counting of flavor parameters
- The unitarity triangle and *CP* violation in the SM
- FCNCs in the SM and GIM mechanism
- EFTs for flavor
- Dealing with hadronic matrix elements
- One example and tutorial

Second day:

- Brief overview on selected hot topics in flavor physics

- **Bibliography**

  - **Lecture notes:** Grossman&Tanedo - arXiv: 1711.03624
    Grinstein - arXiv: 1501.05283
  - **Books:** Branco, Lavoura & Silva - "CP violation" - Core reference
    Donoghue, Golowich & Holstein "Dynamics of SM" - Phenomenology
    Buras "Gauge Theory of Weak Decays ..." -Detailrd calcs in SM and BSM

Flavor universality of gauge interactions in the SM

- **The gauge interactions of the fermions of family *k***

$$\mathscr{L}_{\text{gauge}} \subset \bar{\psi}^k \left(i\partial_\mu + g X_\mu^A t_k^A \right) \gamma^\mu \psi^k$$

- **SM's gauge group:** $SU(3)_c \times SU(2)_L \times U(1)$

Family-independent quantum numbers

$$Q_L^k \sim (3,2)_{1/6} \qquad u_R^k \sim (3,1)_{2/3} \quad d_R^k \sim (3,1)_{-1/3}$$
$$L_L^k \sim (1,2)_{-1/2} \qquad e_R^k \sim (1,1)_{-1}$$

The gauge interactions in the SM are flavor universal

$\mathscr{L}_{\text{gauge}}$ has a **global accidental** $U(3)^5$ flavor symmetry

Origin of flavor in the yukawa interactions of the SM

$$\mathscr{L}_{\text{yukawa}} = y_u^{kl} \bar{Q}_L^k \tilde{H} u_R^l + y_d^{kl} \bar{Q}_L^k H d_R^l + y_e^{kl} \bar{L}_L^k H e_R^l + \text{h.c.}$$

Matrices with N^2 complex parameters

- **Mass generation in the SM:** $SU(2)_L \times U(1)_Y \xrightarrow{\text{SSB}} U(1)_{\text{EM}}$

$$m_f^{kl} = v_{\text{ew}}\, y_f^{kl}$$

$$\mathscr{L}_{\text{masses}} = m_u^{kl} \bar{u}_L^k u_R^l + m_d^{kl} \bar{d}_L^k d_R^l + m_e^{kl} \bar{e}_L^k e_R^l + \text{h.c.}$$

- **Diagonalization:** *Linear & unitary* field redefinitions commuting with Lorentz and $U(1)_{\text{EM}}$

$$f_L \to L_f f_L \qquad f_R \to R_f f_R$$

Unitary matrix

$$m_u \to L_u^\dagger m_u R_u = \text{diag}\left(m_u, m_c, m_t\right)$$

$$m_d \to L_d^\dagger m_d R_d = \text{diag}\left(m_d, m_s, m_b\right)$$

$$m_e \to L_e^\dagger m_e R_e = \text{diag}\left(m_e, m_\mu, m_\tau\right)$$

9 real parameters

Flavor violation in the charged currents (CC)

$$\mathscr{L}_{\text{gauge}} \supset g\bar{\psi}_L^k \left(T^+W_\mu^+ + T^-W_\mu^-\right)\gamma^\mu\psi_L^k = g\left(\bar{u}_L^k\gamma^\mu d_L^k + \bar{\nu}_L^k\gamma^\mu e_L^k\right)W_\mu^+ + \text{h.c.}$$

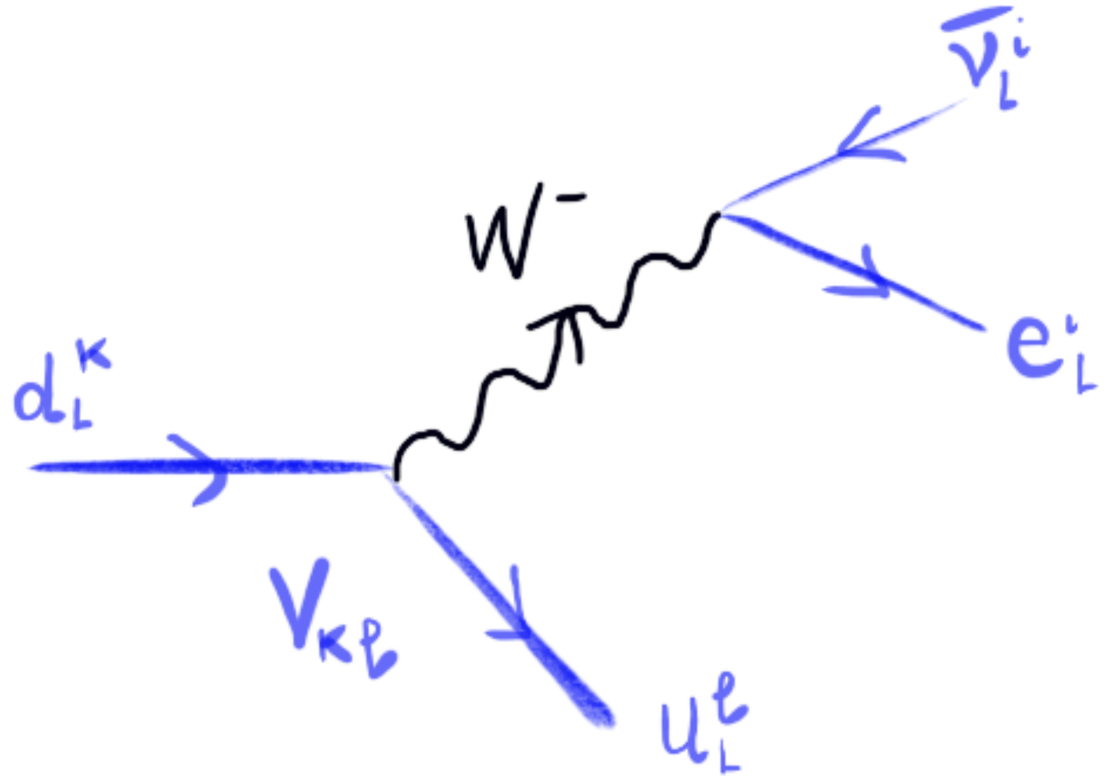
$$Q_L^k = (u_L^k, d_L^k)^T \qquad\qquad L_L^k = (\nu_L^k, e_L^k)^T$$

- Missalignment between gauge and *up* and *down* quark mass matrices

$$\mathscr{L}_{\text{CC}} = g\left(V_{\text{CKM}}\right)_{kl}\bar{u}_L^k\gamma^\mu d_L^l W_\mu^+ + g\bar{\nu}_L^k\gamma^\mu e_L^k W_\mu^+ + \text{h.c.}$$

- **The Cabibbo-Kobayashi-Maskawa mixing matrix**

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



Flavor violation occurs because we cannot diagonalize simultaneously the gauge and yukawa interactions

- Neutrinos in the SM are massless and flavor mixing can be rotated away

Parameter counting in the CKM matrix

1. V_{CKM} **is a unitary matrix** (it is the product of 2 unitary matrices)

2. An $N \times N$ unitary matrix is **complex** and is parametrized by N^2 **real numbers**

3. Physics invariant w.r.t. $(2N-1)$ **rephasings** of the quark fields

$$u_L^k \to e^{i\alpha_k}\, u_L^k \qquad d_L^k \to e^{i\beta_k}\, d_L^k$$

Number of independent parameters: $(N-1)^2$

4. How many are **rotation angles** and **complex phases**?

- **Unitary matrix** = Complex extension of **orthogonal matrix** with $N_{\mathrm{angles}} = N(N-1)/2$

An $N-$**dimensional unitary mixing matrix contains ...**

$$N_{\mathrm{angles}} = N(N-1)/2 \qquad N_{\mathrm{phases}} = (N-1)(N-2)/2$$

- **The minimum number of generations needed to generate *CP* violation is 3!**

More about parameter counting and spurions

Elegant symmetry-breaking argument for counting physical parameters

- **Illustration with leptons**

1. $\mathscr{L}_{\text{gauge}}$ in the SM invariant w.r.t. $U(3)_L \times U(3)_e \Rightarrow$ **18 generators**
2. $\mathscr{L}_{\text{yukawa}}$ breaks $U(3)_L \times U(3)_e \to U(1)_e \times U(1)_\mu \times U(1)_\tau \Rightarrow$**3 unbroken generators**
3. **We can use broken generators to rotate away *unphysical parameters* in** $\mathscr{L}_{\text{yukawa}}$

#physical parameters = #total parameters — #broken generators

4. For leptons $18 - 15 = 3$ corresponding to the **3 lepton masses**

   Same analysis leads to 10 physical parameters for quarks (**6 masses, 3 angles, 1 phase**)

- **Spurions:** Pretend yukawa matrices are bifundamentals of the flavor group

  Keep track of flavor violation in the SM and beyond (Minimal flavor violation)

A standard parametrization of CKM

- Phase redefinitions of quarks ⇒ **Set** V_{ud} , V_{us} , V_{cb} **and** V_{tb} **real**

- **The "standard"** ***unitary*** **parametrization** $(s_{ij} = \sin\theta_{ij}\ ,\ c_{ij} = \cos\theta_{ij})$

$$V_{\rm CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}c_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

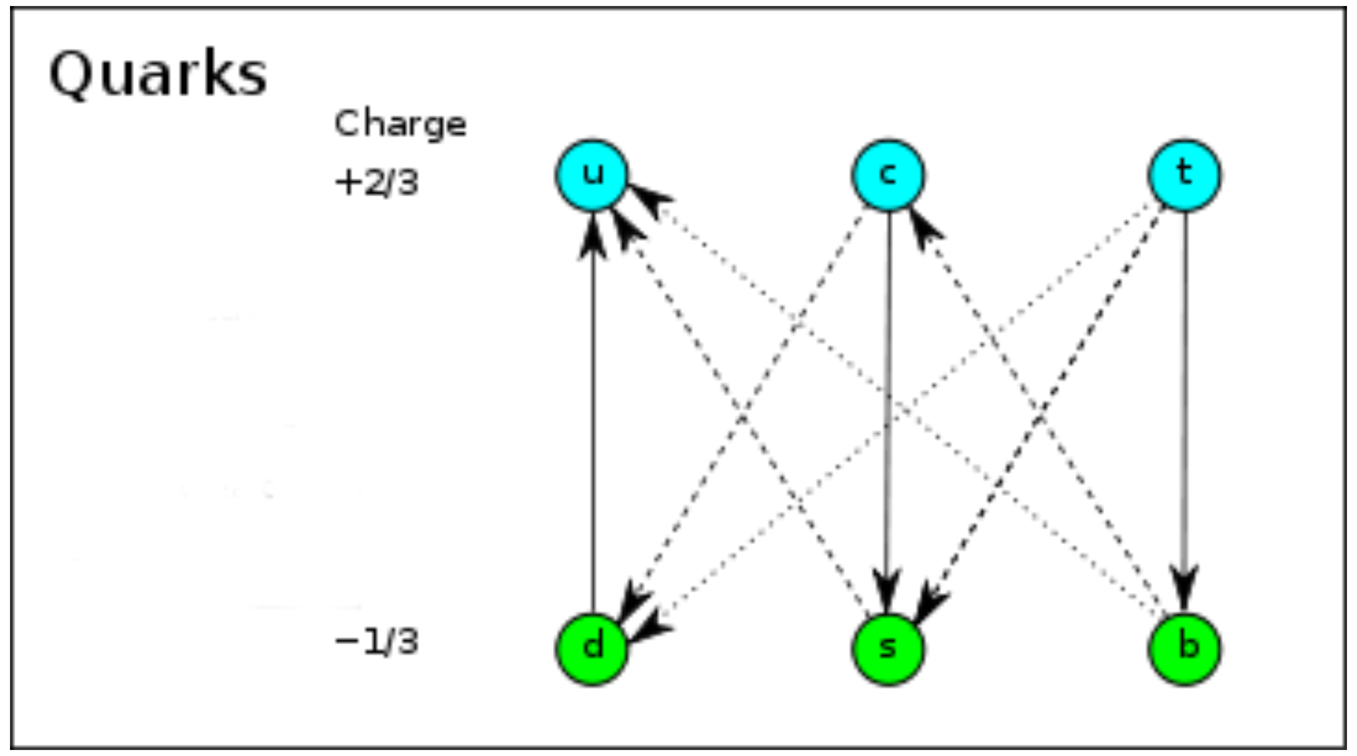
The SM is *defined* when the 3 CKM angles and its 1 phase are **determined experimentally** ...

$$s_{12} = 0.22650(48) \qquad s_{23} = 0.04053(71) \qquad s_{13} = 0.00361(10) \qquad \delta = 68.5(2.6)^\circ$$

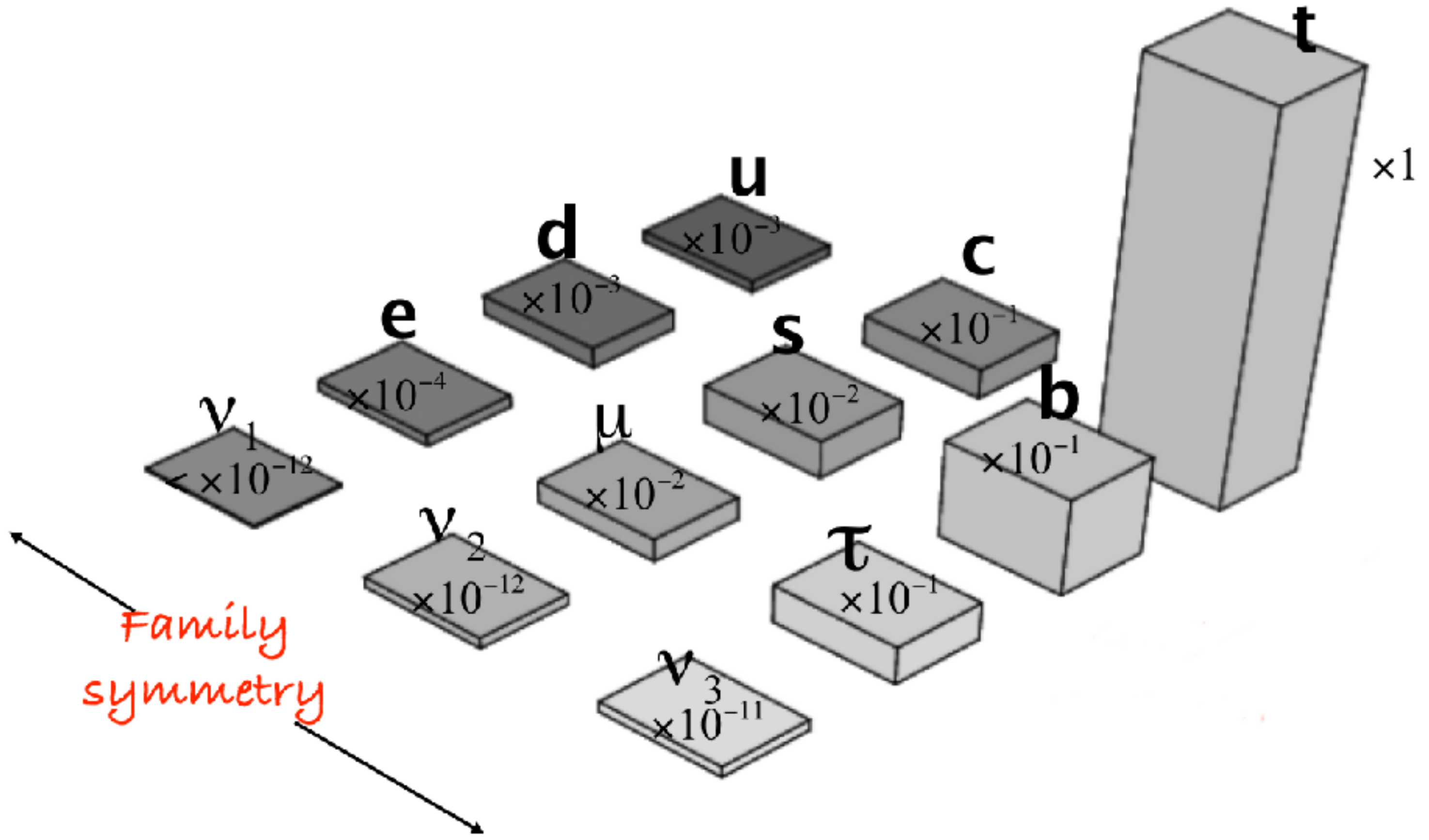
- The quark mixing matrix is **hierarchical!**
    - Compare to the **anarchical** neutrino sector (PMNS matrix) ... **What´s going on?**

Flavor hierarchies and the flavor puzzle

- Flavor transitions



- Masses



Flavor puzzle: **Origin of patterns and hierarchies in the values of the flavor parameters**

- **Portal to BSM physics!**
  - Horizontal symmetries (Froggatt-Nielsen), extra dimensions (Randall-Sundrum), tree-loop hierarchies (Weinberg), clockwork mechanism, etc
- **Essential for our existence! - Anthropic principle**
  - Stability of matter (*up* and *down* quark masses) & stability of vacuum (top-quark mass)
- **Origin of *CP* violation? - Connection to baryogenesis**
  - Why 3 families?

Complex phases and CP violation

- The SM is a **chiral theory** $\Rightarrow$ The SM violates **parity (*P*)** and **charge conjugation** (*C*)
- However the SM **does not necessarily** violate *CP*

The SM violates *CP* because the nontrivial CKM phase is not 0 or π

$$\left.\begin{aligned} \mathscr{L}_{\text{toy}} &= y_{ij}\,\bar{\chi}_i\psi_j\,S + y_{ij}^*\,\bar{\psi}_j\chi_i\,S^\dagger \\ (CP)\mathscr{L}_{\text{toy}}(CP)^\dagger &= y_{ij}\,\bar{\psi}_j\chi_i\,S^\dagger + y_{ij}^*\,\bar{\chi}_i\psi_j\,S \end{aligned}\right\} \quad \Rightarrow \quad \boxed{\mathscr{L}_{\text{toy}} = (CP)\mathscr{L}_{\text{toy}}(CP)^\dagger \quad \Longleftrightarrow \quad y_{ij}^* = y_{ij}}$$

- Unambiguous (**rephasing invariant**) measure of *CP* violation in the SM

Jarlskog invariant

$$J = \mathrm{Im}\left(V_{ij}V_{kl}V_{il}^*V_{kj}^*\right)$$

- In the standard CKM parametrization $J = c_{12}s_{12}^2c_{13}^2s_{13}c_{23}s_{23}\sin\delta$

  All mixing angles must be nonzero for *CP* violation
- *CP* violation *is in* the SM but *not explained by* the SM

CKM hierarchies in practice: Wolfenstein parametrization

- **Expose the CKM hierarchies explicitly**

Small parameter: **Cabibbo angle**

$$\lambda \equiv s_{12} = 0.22650(48)$$

- **Define ...**

$$s_{23} \equiv A\,\lambda$$

$$s_{13}e^{i\delta} \equiv A\lambda^3(\rho - i\eta)$$

... and expand in λ!

First 2 families subspace

$$V_{\rm CKM} = \begin{pmatrix} \boxed{\begin{matrix} 1-\lambda^2/2 & \lambda \\ -\lambda & 1-\lambda^2/2 \end{matrix}} & \begin{matrix} A\lambda^3(\rho - i\eta) \\ A\lambda^2 \end{matrix} \\ \begin{matrix} A\lambda^3(1-\rho-i\eta) & -A\lambda^2 \end{matrix} & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$$A = 0.826(12) \qquad \rho = 0.152(14) \qquad \eta = 0.357(10)$$

- The Wolfenstein parametrization is **not exactly unitary**
- Mixing first two families is unitary (and independent of 3rd family) up to $\mathcal{O}(\lambda^2)$

The unitary triangle(s)

V_{CKM} is unitary

1. Row(column) unitarity: $|V_{i1}|^2 + |V_{i2}|^2 + |V_{i3}|^2 = 1$
2. Off-diagonal unitarity: $V_{i1}V_{j1}^* + V_{i2}V_{j2}^* + V_{i3}V_{j3}^* = 0$

- **2. is a null sum of complex vectors $\Rightarrow$ Unitarity triangles**

  1st and 3rd columns give triangle with all sides of same $\mathcal{O}(\lambda^3)$
  - Three (**rephasing invariant**) angles (**directly observable!**)

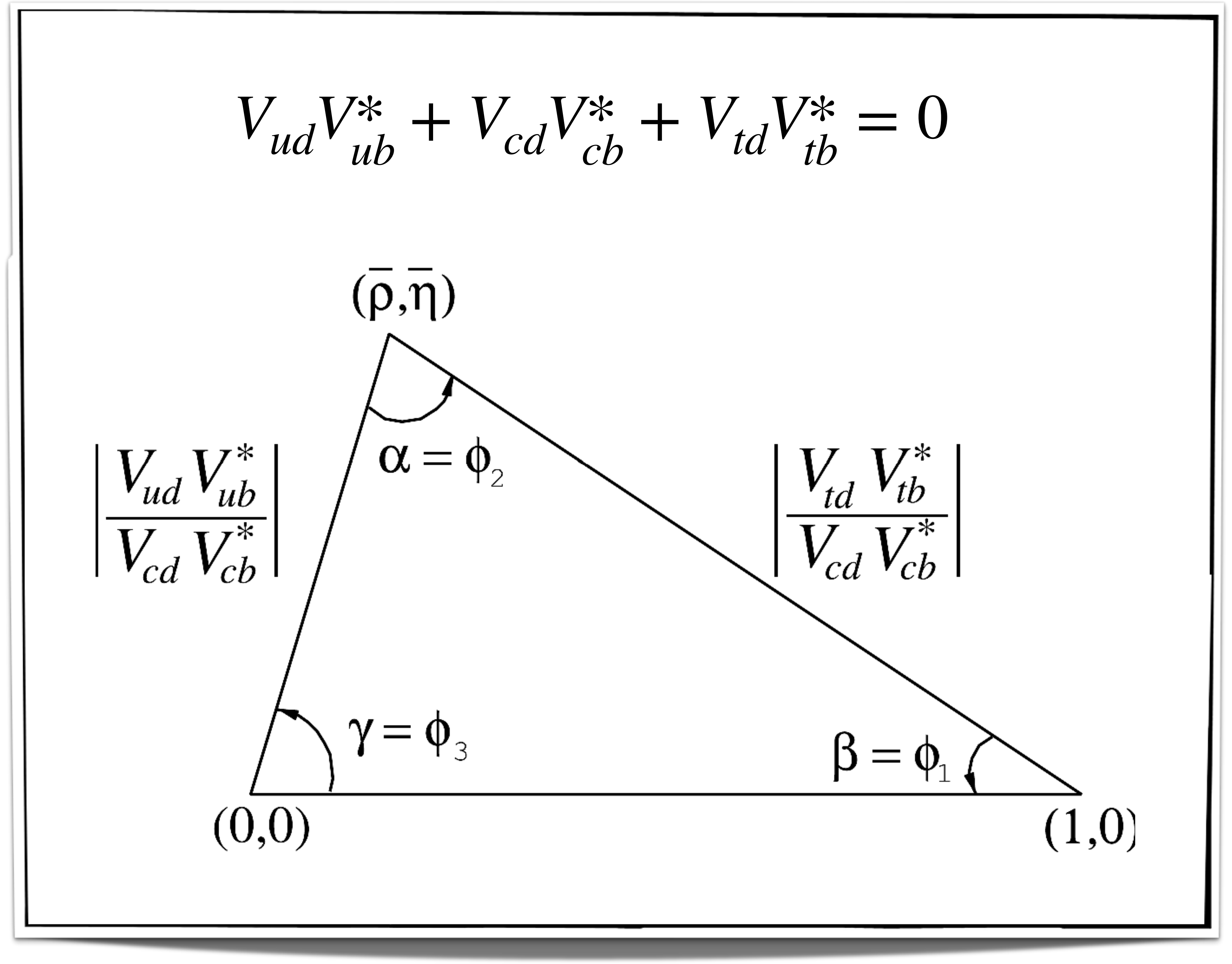
$$\beta = \phi_1 = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right) \qquad \alpha = \phi_2 = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right) \qquad \gamma = \phi_3 = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

  - The apex is fixed by a redefinition:

$$s_{13}e^{i\delta} = A\lambda^3(\rho + i\eta) \equiv A\lambda^3(\bar\rho + i\bar\eta)\frac{\sqrt{1 - A^2\lambda^4}}{\sqrt{1-\lambda^2}\left(1 - A^2\lambda^4(\bar\rho + i\bar\eta)\right)}$$

  and is **rephasing invariant!**

$$\bar\rho + i\bar\eta = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}$$

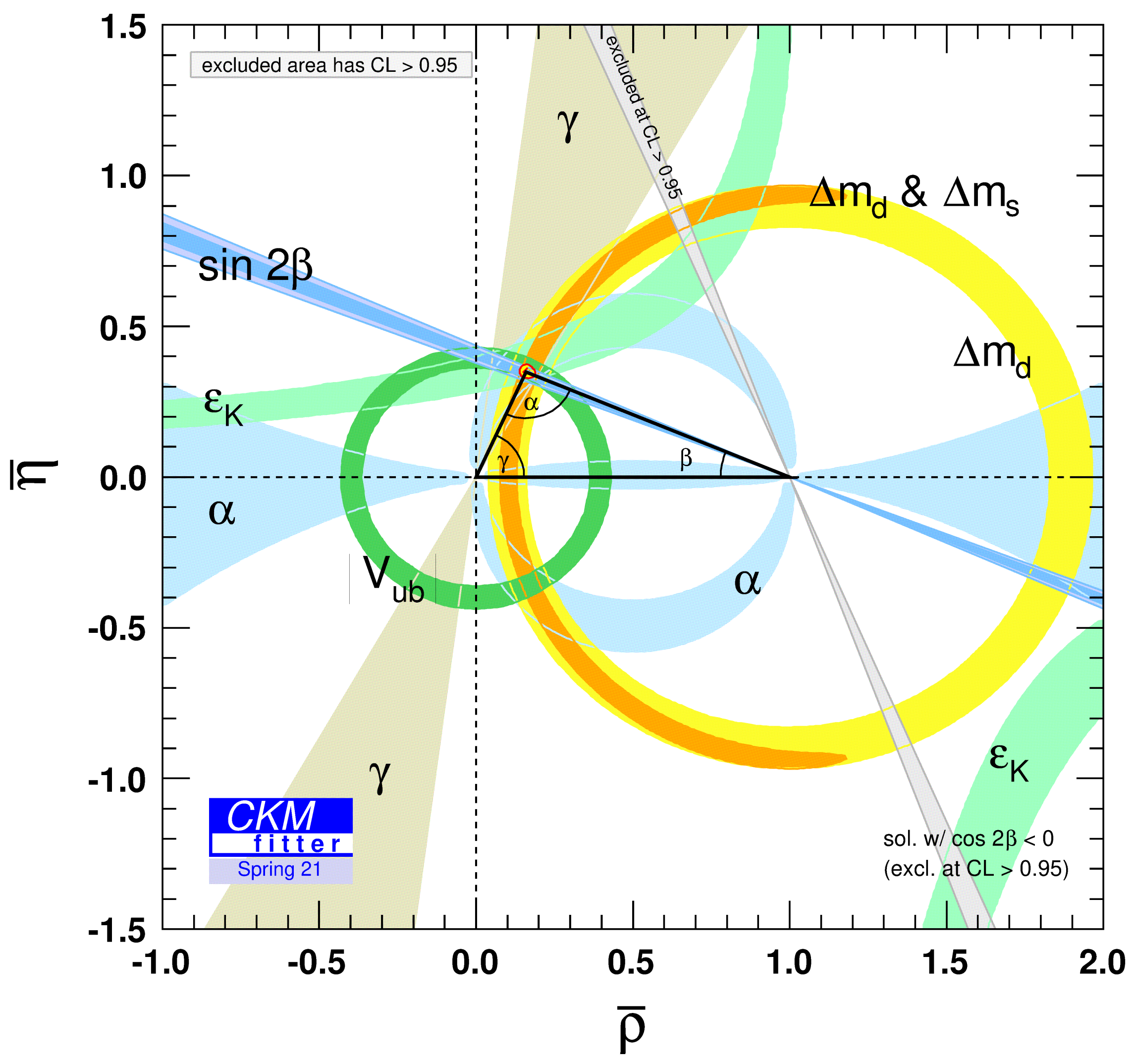


Experimental constraints in the unitary triangle

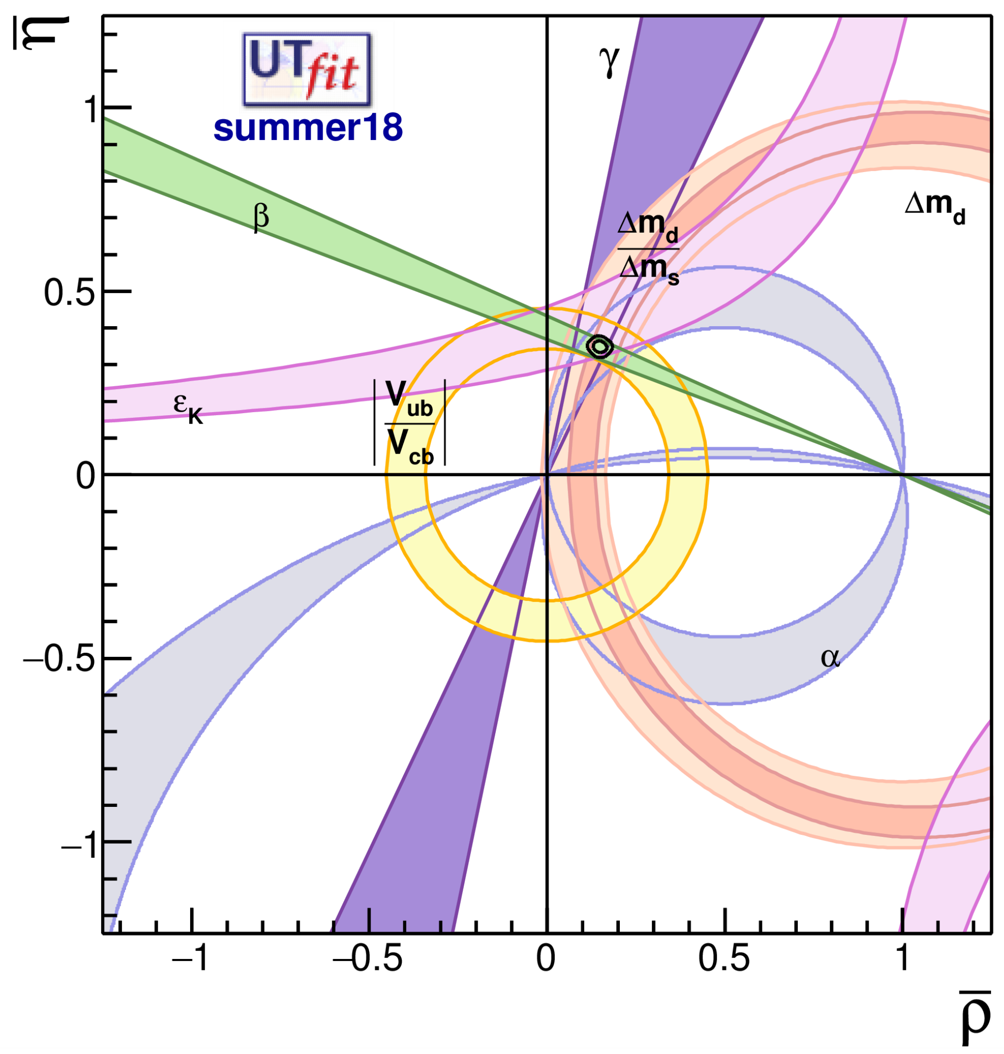
- **The existence of a unitary triangles (non-zero angles) is a signal of *CP*-violation**

Two collaborations perform updated fits to the CKM parameters

- **CKMfitter** - frequentist analysis
  ckmfitter.in2p3.fr
  - Conservative with uncertainties (*Rfit*)



- **UTfit** - bayesian analysis
  www.utfit.org
  - Includes fits with BSM (EFT) parameters

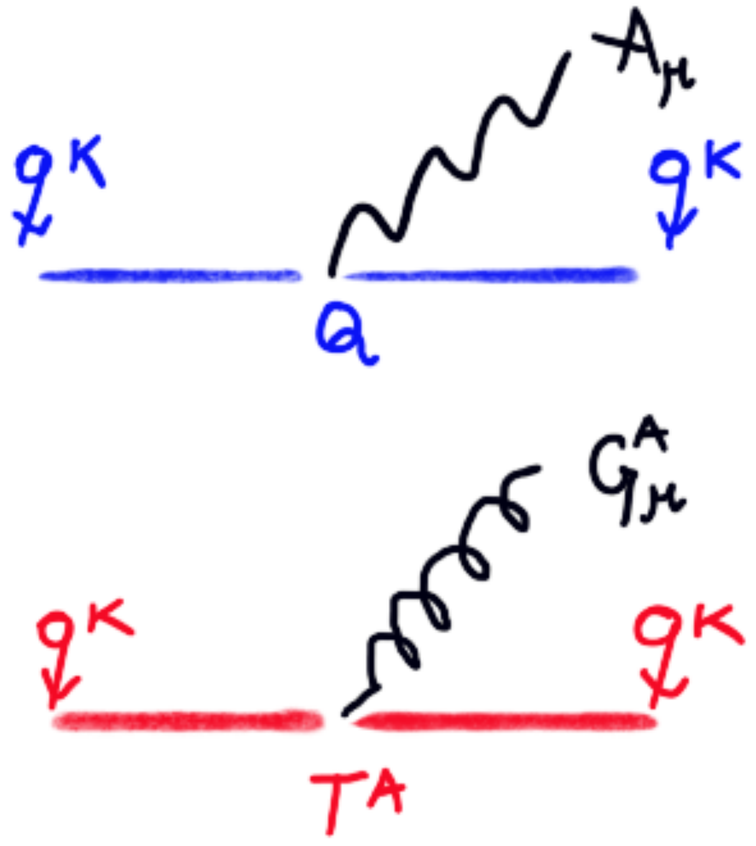


UT triangle and the Jarslkog invariant

- **Geometric interpretation:** $\mathrm{Area}_{\mathrm{UT}} = J/2$
- ***CP* violation small in SM *because of small mixing*:** $J_{\mathrm{SM}} \approx \lambda^6 A^2 \eta = 3.00(12) \times 10^{-5}$

Neutral currents at tree level in the SM: Photon, gluon and Higgs

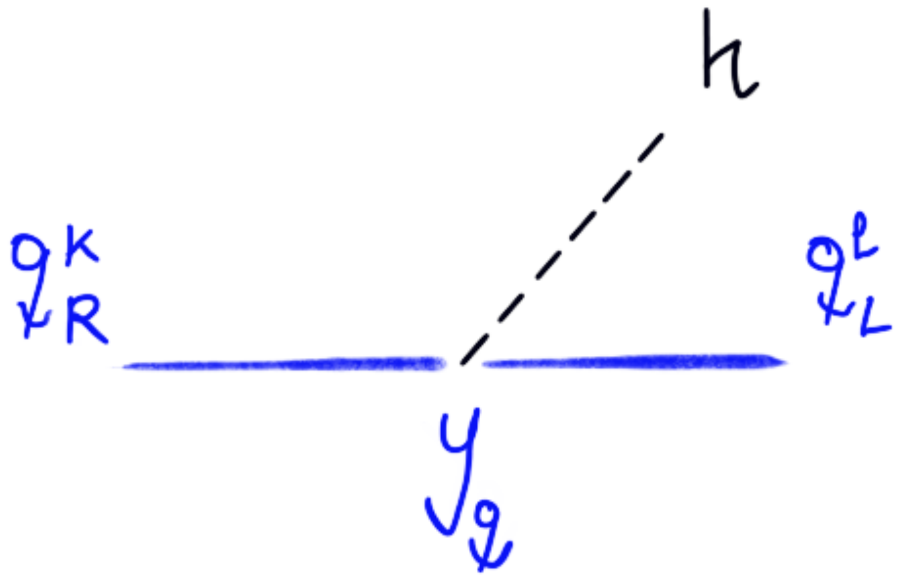
- **QED (photons) and QCD (gluons):** Couplings **diagonal in flavor space** (same charges/reps)



$$\boxed{\textbf{CKM unitarity: } V^\dagger V = \mathbf{1}}$$

$$J^\mu_{\mathrm{EM}} = e\,Q_q\,\bar{q}^k\gamma^\mu\,(\mathbf{1})_{kl}\,q^l \to e\,Q_q\,\bar{q}^k\gamma^\mu\,(V_q^\dagger)_{kj}\,(V_q)_{jl}\,q^l = J^\mu_{\mathrm{EM}}$$

- **Yukawa interactions (higgs):** Couplings **aligned** with the mass basis

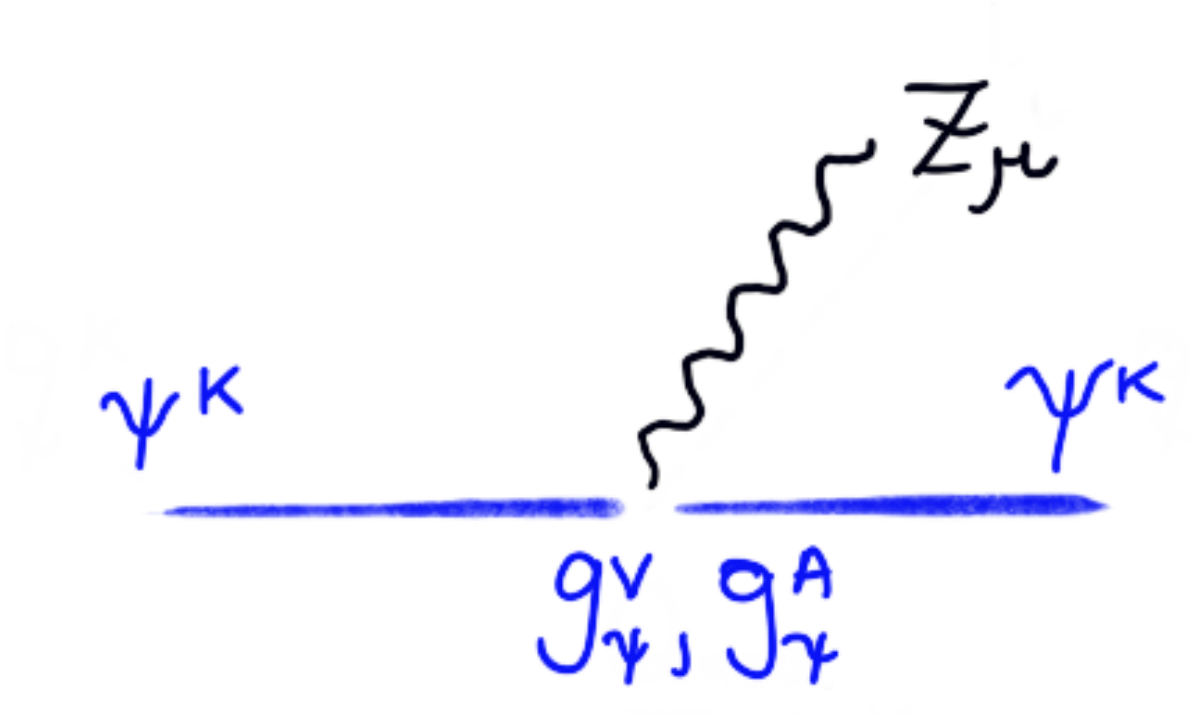


$$\boxed{\textbf{SSB in the SM:} \quad H^T \to \begin{pmatrix} 0 & v + \frac{h}{\sqrt{2}} \end{pmatrix}}$$

$$\bar{Q}^k_L H\,(y_d)_{kl}\,d^l_R \to \bar{Q}^k_L\,(m_d)_{kl}\,d^l_R \left(1 + \frac{h}{v\sqrt{2}}\right)$$

Neutral currents at tree level in the SM: The Z boson

- **Weak charges:** Couplings of the Z also **diagonal in flavor space**



$$J_Z^\mu = -\frac{e}{2s_w^2}\,\bar{\psi}^k\left(g_V^\psi \gamma_\mu + g_A^\psi \gamma^\mu \gamma_5\right)\psi^k$$

$$\boxed{g_V^{\psi_k} = T_3^{(\psi_k)} - 2s_w^2 Q_\psi \qquad g_A^{\psi_k} = T^{(\psi_k)}}$$

  - What is relevant here is that all *up*-like fermions and all *down*-like fermions have the same **weak isospin**
  - **Before 1970 hadrons were thought composed exclusively of *u*, *d* and *s* quark**
    with CC interactions rotated by 2×2 Cabibbo mixing: $J^\mu_{\mathrm{CC}} = \bar{u}(1-\gamma_5)\left(\cos\theta_C d + \sin\theta_C s\right)$
  - If $(u,d)^T$ is iso-doublet and s isosinglet $\Rightarrow$ **There must be tree-level neutral $\Delta S = 1$ decays**
  - PDG (Particle Data Group):

CC: $\mathrm{Br}(K_L \to \pi^+ e^- \bar{\nu}) = 40.55(11)\,\%$

NC: $\mathrm{Br}(K_L \to \mu^+ \mu^-) = 6.84(11) \times \mathbf{10^{-9}}$

Flavor changing neutral currents (FCNC) are suppressed!

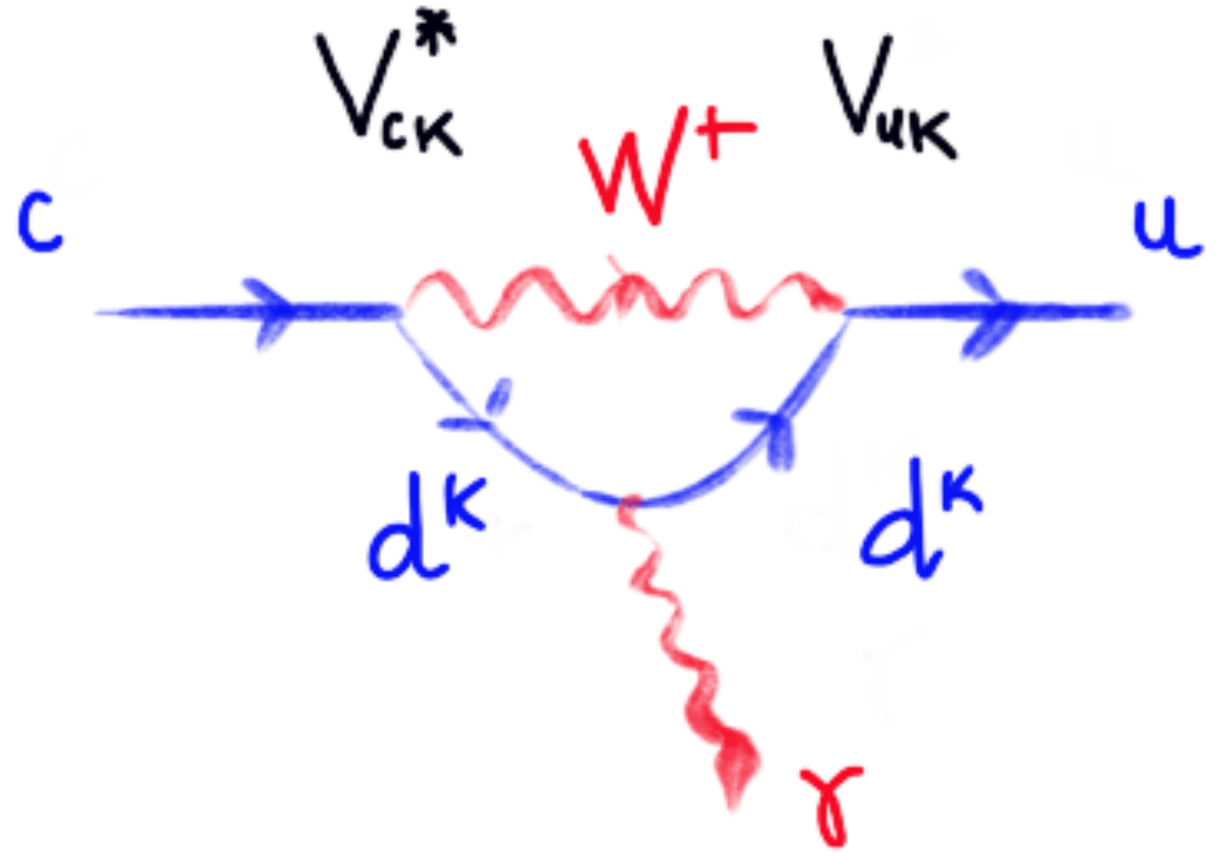
There must be a 4th quark (**charm**)!

Glashow, Iliopoulos & Maiani (GIM) 1970

Flavor-changing neutral currents (FCNC) in the SM

- The GIM mechanism
  - In the SM, FCNCs occur only at **1-loop level**!
  - In addition, they receive a **flavor suppression**

Take the $\Delta C = 1$ **neutral transition** $c \to u\gamma$



$$\text{Amplitude} \approx \frac{e\,g^2}{4\pi^2\,m_W^2} \sum_k V^*_{ck} V_{uk} f(m_k^2/m_W^2)$$

- The loop function can be Taylor expanded

$$f(m_k^2/m_W^2) = a + b m_k^2/m_W^2 + \ldots$$

- CKM unitarity!

$$\text{Amplitude} \approx \frac{e\,g^2}{4\pi^2\,m_W^2} \left(V^*_{cs} V_{us} \frac{m_s^2 - m_d^2}{m_W^2} + V^*_{cb} V_{ub} \frac{m_b^2 - m_d^2}{m_W^2} \right)$$

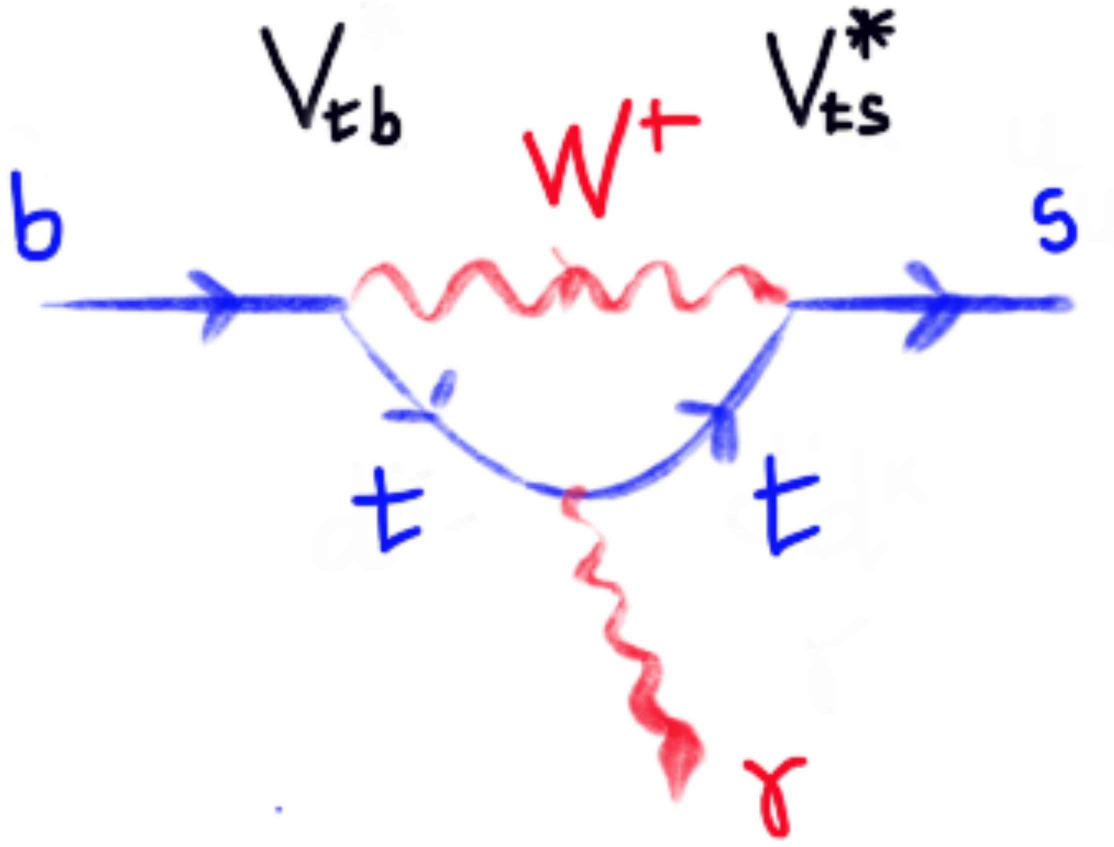
$$\approx \frac{e g^2}{4\pi^2\,m_W^2} \lambda^5 y_b^2$$

- The GIM mechanism is a consequence of CKM unitarity at loop level
- It implies suppression of FCNCs by small yukawas and/or small mixing angles

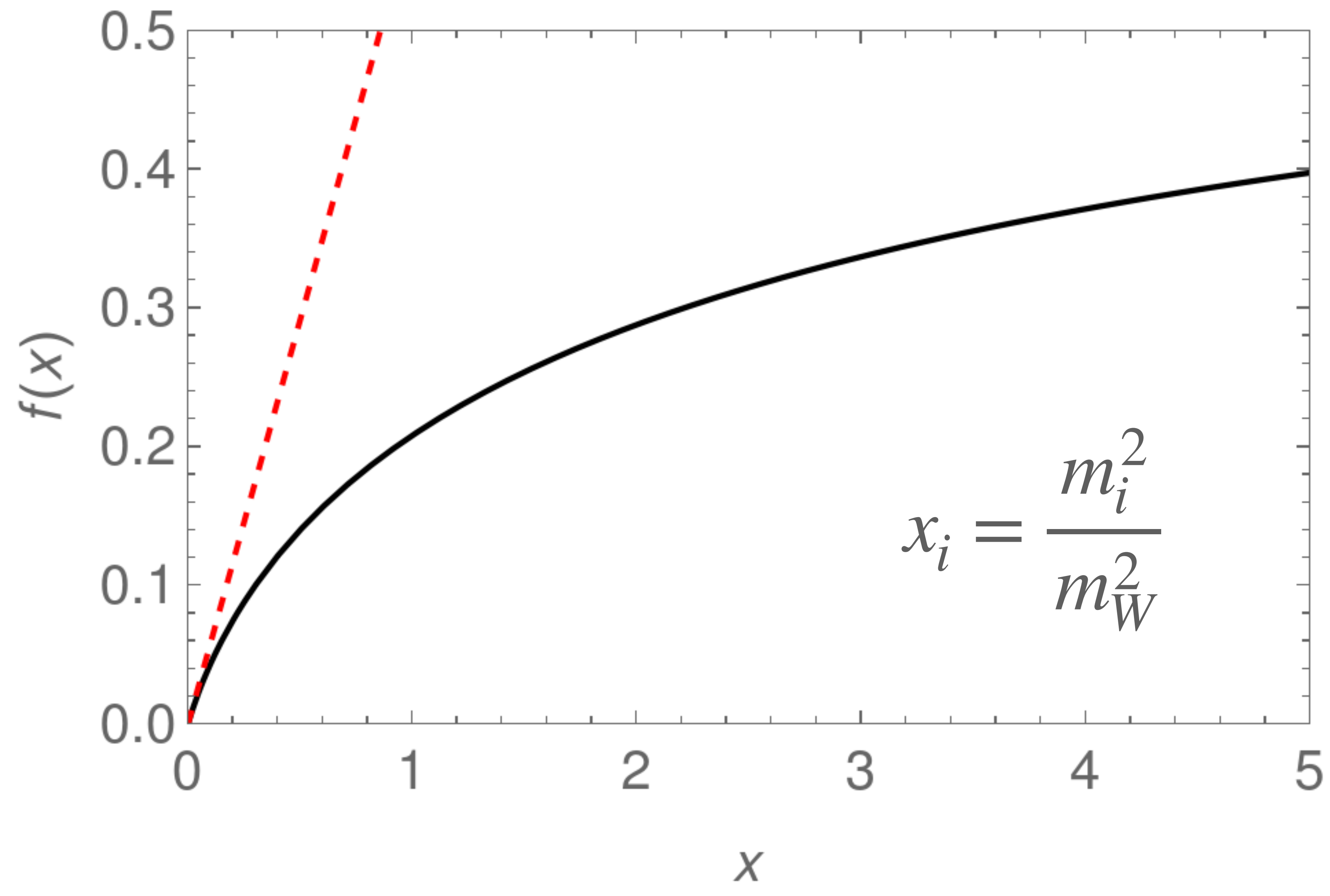
The role of the top-quark in the FCNCs

- **FCNCs in the *down*-quark sector**
  - Sensitive to *up*-quarks $\Rightarrow$ **Prominence of top yukawa**
  - $m_W \lesssim m_t$: Suppression to be revisited

Take now the **neutral *down* quark transition** $b \to s\gamma$



$$\text{Amplitude} \approx \frac{e\,g^2}{4\pi^2 m_W^2} \overbrace{V_{tb}V^*_{ts}}^{\lambda^2} f\left(\frac{m_t^2}{m_W^2}\right)$$



Inami-Lin function(s)

$$f(x) = -\frac{8x^3 + 5x^2 - 7x}{12(1-x)^3} + \frac{x^2(2-3x)}{2(1-x)^4}\log x$$

$$\approx \frac{7}{12}x + \mathcal{O}(x^2)$$

- $f(x)$ linear in x close to 0 and $\mathcal{O}(1)$ for x_t
- $f(x) \to 2/3$ at $x \to \infty$