● (Very Quick) review of EW sector of SM

$$\mathcal{L}_\Phi = |D_\mu \Phi|^2 - V(\Phi)$$

Φ = Higgs field, $SU(2)_L$ doublet and with hyperchange $1/2$

$$\Rightarrow V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} G^+ \\ v + h + i G_0 \end{pmatrix}$$

"Would-be" Goldstone Bosons

$$\downarrow$$

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}$$

h = Higgs boson

$v = 246$ GeV (EW scale)

$$\left[v^2 = \frac{\mu^2}{\lambda} \right] \qquad \left[m_h^2 = -\mu^2 + 3\lambda v^2 = 2\lambda v^2 \right]$$

$$\Rightarrow |D_\mu \Phi|^2 = \left| \left(i\partial_\mu - \tfrac{g}{2} \vec{\sigma} \cdot \vec{W}_\mu - \tfrac{g'}{2} Y B_\mu \right) \Phi \right|^2$$

When the Higgs doublet takes a vev, this yields masses for the $SU(2)_L \times U(1)_Y$ gauge bosons & breaks $SU(2)_L \times U(1)_Y \to U(1)_{EM}$

$W_{1\mu}, W_{2\mu} \to$ we define $W_\mu^\pm = \dfrac{W_{1\mu} \mp i W_{2\mu}}{\sqrt{2}}$

$$\frac{g^2 v^2}{8} \cdot 2 W^+ W^- \longrightarrow m_{W^\pm}^2 = \frac{g^2 v^2}{4} \longrightarrow m_W = \frac{g v}{2}$$

$$W_{3\mu}, B_\mu \longrightarrow \frac{v^2}{8}(W_{3\mu}, B_\mu)\begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}\begin{pmatrix} W_{3\mu} \\ B_\mu \end{pmatrix} =$$

we diagonalize !

$$= \frac{1}{2}(Z_\mu, A_\mu)\begin{pmatrix} m_Z^2 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}$$

Zero eigenvalue $\Rightarrow$ 1 massless gauge boson
(photon from unbroken $U(1)_{EM}$)

$$m_Z^2 = \frac{(g^2 + g'^2)\, v^2}{4} \rightarrow m_Z = \frac{\sqrt{g^2 + g'^2}\, v}{2}$$

Eigenstates : $Z_\mu = \dfrac{-g' B_\mu + g W_{3\mu}}{\sqrt{g^2 + g'^2}}$; $A_\mu = \dfrac{g B_\mu + g' W_{3\mu}}{\sqrt{g^2 + g'^2}}$

EW Precision tests (of the SM)

Tests of the consistency of the SM given a set of measurements ($\hat{O}_i$)

Q: Is the SM capable of explaining / accomodating all these measurements?

⇒ Some familiarity with EW SM Lagrangian is assumed (Higgs, EW boson masses, etc.)

⇒ I chose this topic since it is •) Important, •) Useful for BSM research •) Advanced, but tractable, •) Timely (W-boson mass hype!)

⇒ I will mainly follow the (wonderful) TASI Lectures by James Wells (arxiv: hep-ph/0512342), and also Maksymyk, Burgess, London (arxiv: hep-ph/9306267)

$$\mathcal{L}_{SM(EW)} = \bar{L}_L \not{D}_L L_L + \bar{l}_R \not{D}_R l_R + |D_\mu \Phi|^2 - V(\Phi) + \dots \text{(Gauge Kinetic terms, Yukawas...)}$$

$$= \bar{L}_L \gamma^\mu \left(i\partial_\mu - \tfrac{g}{2}\vec{\sigma}\cdot\vec{W}_\mu - \tfrac{g'}{2} Y B_\mu\right) L_L$$

$$+ \bar{l}_R \gamma^\mu \left(i\partial_\mu - \tfrac{g'}{2} Y B_\mu\right) l_R$$

$$+ \left|\left(i\partial_\mu - \tfrac{g}{2}\vec{\sigma}\cdot\vec{W}_\mu - \tfrac{g'}{2} Y B_\mu\right)\Phi\right|^2 - \underbrace{\left(-\mu^2 \Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2\right)}_{V(\Phi)} + \dots$$

At this level, 4 Lagrangian parameters fix $\mathcal{L}_{SM(EM)}$: $g, g', \overbrace{v, m_h}^{\lambda,\, \mu^2}$

$\Rightarrow$ In fact, g, g', v control the EW theory

Let's see what this means:

- Set of EW observables (I denote observables/measurements as $\hat{O}$)

$\hat{\alpha}$ (obtained from Thomson limit $q^2 \to 0$ of $\gamma^* \to e^+e^-$ scattering; strength of interaction between photons and electrons in QED; nowadays measured directly via anomalous magnetic moment of electron / photon recoil in atom interferometry)

$\hat{G}_F$ (measured from muon lifetime: $\tau_\mu^{-1} = \frac{\hat{G}_F^2 m_\mu^5}{192\pi^3} F(\rho)\left(1 + \ldots\right)$)

$\rho = m_e^2/m_\mu^2$

$H_i(\rho) \cdot \left(\frac{\hat{\alpha}}{\pi}\right)^i$

$i = 1, \ldots \infty$

$\hat{m}_Z$

$\hat{m}_W$

$\hat{\Gamma}_{e^+e^-}$ (leptonic partial width of Z-boson)

$\hat{s}^2_{eff}$ (effective $\sin^2$ of weak mixing angle; obtained/defined from left-right asymmetry of Z decays to leptons A^{ℓ}_{LR})

$$\hat{A}_{LR} = \frac{\left(\tfrac{1}{2} - \hat{s}^2_{eff}\right)^2 - \hat{s}^4_{eff}}{\left(\tfrac{1}{2} - \hat{s}^2_{eff}\right)^2 + \hat{s}^4_{eff}}$$

Measured values of these quantities: (PDG, 2022)

$\hat{\alpha}^{-1} = 137.035999180(10)$

$\hat{G}_F = 1.1663788(6) \times 10^{-5}\ \mathrm{GeV}^{-2}$

$\hat{m}_Z = 91.1876(21)$ GeV

$\hat{m}_W = 80.377(12)$ GeV

$\hat{s}^2_{eff} = 0.23148(33)$ (LEP/Tevatron)

29 (LHC)

$\hat{\Gamma}_{e^+e^-} = 83.942(85)$ MeV

At tree-level, all these observables can be casted in terms of g, g', v.

We use an alternative set of parameters:

$$e = \frac{g g'}{\sqrt{g^2+g'^2}} \quad , \quad s \left(\equiv \sin\theta\right) = \frac{g'}{\sqrt{g^2+g'^2}} \quad , \quad v$$

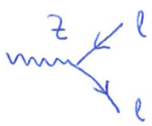
With $g = e/s$, $g' = e/\cos\theta$; $g'/g = \tan\theta$

Let's then do a tree-level analysis of the consistency of the theory:

$$\hat{\alpha} = \frac{e^2}{4\pi} \quad ; \quad \hat{G}_F = \frac{1}{\sqrt{2}\, v^2} \quad ; \quad \hat{m}_Z^2 = \frac{e^2 v^2}{4 s^2 c^2} \left(= \frac{(g^2+g'^2) v^2}{4} \right)$$

$$\hat{m}_W^2 = \frac{e^2 v^2}{4 s^2} \left(= \frac{g^2 v^2}{4} \right) \quad ; \quad \hat{s}_{eff}^2 = s^2$$

$$\hat{\Gamma}_{e^+e^-} = \frac{v}{96\pi} \frac{e^3}{s^3 c^3} \left[\left(-\tfrac{1}{2} + 2 s^2 \right)^2 + \tfrac{1}{4} \right]$$

(this comes from  $\frac{ie}{sc} \gamma_\mu \left[(T_3^l - Q_e s^2) P_L - Q_e s^2 P_R \right]$

and $\Gamma_{e\bar{e}} \propto \frac{|\vec{p}_f|}{m_Z^2} \int |\overline{\mathcal{M}}|^2 \, d\Omega$)

$$\Gamma_{e\bar{e}} \propto \frac{e^2}{s^2 c^2} \cdot m_Z \left[\left(\tfrac{1}{2} - s^2 \right)^2 + s^4 \right] = \frac{1}{4} \frac{e^3}{s^3 c^3} v \left[\left(-\tfrac{1}{2} + 2 s^2 \right)^2 + \tfrac{1}{4} \right]$$

Consistency of theory $\equiv$ Can we reproduce (fit) all experimental results with a suitable choice of input parameters g, g', v?

- We can build a $$\chi^2(e,s,v) = \sum_{\substack{i \\ \text{(observables)}}} \frac{(\hat{O}_i - O_i(e,s,v))^2}{(\Delta\hat{O}_i)^2}$$

and assess the goodness-of-fit

- Alternatively, we can fix e, s, v using the measured values for $\hat{\alpha}$, $\hat{G}_F$ and $\hat{m}_Z$ (the three best measured observables) and use these to derive tree-level theoretical predictions for m_W, s^2_{eff} and $\Gamma_{e^+e^-}$

"Predict observables in terms of other observables"

$$e^2 = 4\pi\hat{\alpha}$$

$$v^2 = \hat{G}_F^{-1}/\sqrt{2}$$

$$s^2 = \frac{1}{2} - \frac{1}{2}\sqrt{1 - 4\underbrace{\frac{\pi\hat{\alpha}}{\sqrt{2}\,\hat{G}_F\,\hat{m}_Z^2}}_{\hat{x}}}$$

(comes from $s^2c^2 = \dfrac{e^2v^2}{4\hat{m}_Z^2} = \dfrac{\pi\hat{\alpha}}{\sqrt{2}\,\hat{G}_F\,\hat{m}_Z^2}$)

Then:

$$m_W^2(e,s,v) \longrightarrow m_W^2(\hat{\alpha}, \hat{G}_F, \hat{m}_Z) = \hat{m}_Z^2\left(\frac{1}{2} + \frac{1}{2}\sqrt{1-4\hat{x}}\right)$$

$$= \frac{\pi\sqrt{2}\,\hat{G}_F^{-1}\,\hat{\alpha}}{\left(1 - \sqrt{1-4\hat{x}}\right)}$$

$$s^2_{eff}(e, s, v) \to s^2_{eff}(\hat{\alpha}, \hat{G}_F, \hat{m}_Z) = \frac{1}{2} - \frac{1}{2}\sqrt{1 - 4\frac{\pi\hat{\alpha}}{\sqrt{2}\,\hat{G}_F\,\hat{m}_Z^2}}$$

$$\Gamma_{e^+e^-}(\hat{\alpha}, \hat{G}_F, \hat{m}_Z) = \frac{\sqrt{2}\,\hat{G}_F\,\hat{m}_Z^3}{12\pi}\left[\left(\frac{1}{2} - \sqrt{1 - 4\frac{\pi\hat{\alpha}}{\sqrt{2}\,\hat{G}_F\,\hat{m}_Z^2}}\right)^2 + \frac{1}{4}\right]$$

The predictions for m_W, s^2_{eff} and $\Gamma_{e^+e^-}$ are:

$m_W = 80.939$ GeV $\longrightarrow$ $\sim 47\,\sigma$ away from $\hat{m}_W$

$s^2_{eff} = 0.21215$ $\longrightarrow$ $\sim 59\,\sigma$ away from $\hat{s}^2_{eff}$

$\Gamma_{e^+e^-} = 84.842$ MeV $\sim 11\,\sigma$ away from $\hat{\Gamma}_{e^+e^-}$

Disagreement is driven by the high precision of experimental measurements (m_W measured to $\frac{1}{10000}$ precision, s^2_{eff} to $\frac{1}{2000}$ precision, Γ_{ee} to $\frac{1}{1000}$)

Need higher precision from theory! $\rightarrow$ Loop corrections

"EW precision observables"

A full 1-loop renormalization of the SM is a complicated task! We will focus here on the class of 1-loop corrections appearing only via gauge boson self-energies

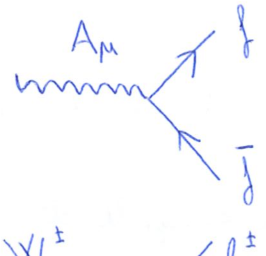
Its importance also lies on the fact that these are the relevant corrections (in BSM theories) if:

1) The gauge group is the SM (EW) one: no new gauge bosons apart from $W^{\pm}$, Z, γ

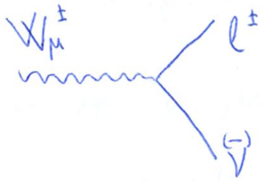
2) The BSM couplings to light SM fermions are suppressed compared to its couplings to SM gauge bosons

(also, a good argument that J. Wells gives for the dominance of self-energy corrections over vertex corrections is that any charged particle (non-singlet) will contribute to self-energies, while in BSM theories we only expect $O(1)$ particles to couple to specific light fermions; thus the sum over all contributions to  dominates over $O(1)$ diagrams )

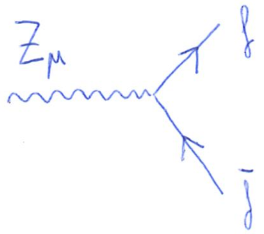
The relevant Feynman rules for our analysis are:



$$i Q_f e \gamma_\mu$$

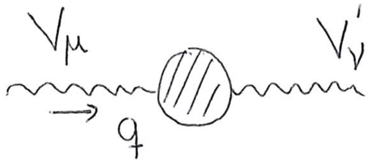


$$\frac{ie}{\sqrt{2}\, S} \gamma_\mu P_L$$



$$\frac{ie}{SC} \gamma_\mu \left[\left(T_f^3 - Q_f S^2 \right) P_L - Q_f S^2 P_R \right]$$

1-loop gauge boson self-energies are of the form



$$\Pi_{VV'}(q^2)\, g_{\mu\nu} - \Delta_{VV'}(q^2)\, q_\mu q_\nu$$

The $\Delta_{VV'} q_\mu q_\nu$ will always vanish when contracted with a light SM fermion current $J^\mu = \bar{f} \gamma^\mu f$ (in the limit $m_f \to 0$)

$$q_\mu J^\mu = \bar{f} \gamma^\mu q_\mu f = \bar{f} m_f f = 0$$

Dirac eq:

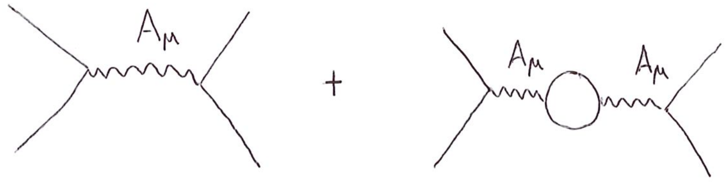
$$(\not{q} - m_f) f = 0$$

So, here we only need to keep the $\Pi_{VV'}(q^2)\, g^{\mu\nu}$ part of the self-energy

We have $\Pi_{\gamma\gamma}(0) = 0$ from $U(1)_{EM}$ gauge invariance, and also we can set $\Pi_{\gamma Z}(0) = 0$ (the BSM contribution to $\Pi_{\gamma Z}(0)$ will always be zero by $U(1)_{EM}$ gauge invariance under our hypothesis 1), and in the SM only $W^{\pm}$ loops contribute to it; it can be neglected (not doing it would make the discussion below more complicated and would lead to the same results, essentially)

1-loop corrections (from self-energies)

$\boxed{\alpha}$ (Probe Coulomb potential at $q^2 \to 0$; can discard non-QED contributions)



$$= -i \left.\frac{4\pi\alpha^{th}}{q^2}\right|_{q^2\to 0} = \frac{-ie^2}{q^2}\left[1 + \frac{\Pi_{\gamma\gamma}(q^2)}{q^2}\right]\Bigg|_{q^2\to 0}$$

We define $\Pi'_{\gamma\gamma}(0) = \lim\limits_{q^2\to 0} \dfrac{\Pi_{\gamma\gamma}(q^2)}{q^2}$ $\left(= \left.\dfrac{d\,\Pi_{\gamma\gamma}(q^2)}{dq^2}\right|_{q^2\to 0}\right)$

Then $\alpha^{th} = \dfrac{e^2}{4\pi}\left(1 + \Pi'_{\gamma\gamma}(0)\right)$

$\boxed{G_F}$

Muon decay amplitude is proportional to $\frac{G_F}{\sqrt{2}}$

$$\Rightarrow \quad \frac{G_F^{th}}{\sqrt{2}} = \underbrace{\frac{g^2}{8 m_W^2}}_{\frac{1}{2v^2}} \left[1 + i\,\Pi_{WW}(q^2) \frac{-i}{q^2 - m_W^2} \right] \Bigg|_{q^2 \to 0}$$

$$= \frac{1}{2v^2} \left[1 - \frac{\Pi_{WW}(0)}{m_W^2} \right]$$

$\boxed{m_Z}$

Gauge boson masses get shifted by their corresponding self-energies

$$= \frac{-i}{q^2 - m_Z^2} + \left(\frac{-i}{q^2 - m_Z^2} \right)^2 i\,\Pi_{ZZ}(m_Z^2) + \left(\frac{-i}{q^2 - m_Z^2} \right)^3 \left(i\,\Pi_{ZZ}(m_Z^2) \right)^2 + \dots$$

$$= \frac{-i}{q^2 - m_Z^2} \underbrace{\sum_{\ell=0}^{\infty} \Bigg(\underbrace{\frac{\Pi_{ZZ}(m_Z^2)}{q^2 - m_Z^2}}_{x} \Bigg)^{\ell}}_{\frac{1}{1-x}} = \frac{-i}{q^2 - m_Z^2} \, \frac{1}{1 - \frac{\Pi_{ZZ}(m_Z^2)}{q^2 - m_Z^2}}$$

$$= \frac{-i}{q^2 - m_Z^2 - \Pi_{ZZ}(m_Z^2)}$$

Then, $$\left(m_Z^{th}\right)^2 = \frac{e^2 v^2}{4 s^2 c^2} + \Pi_{ZZ}(m_Z^2)$$

$\boxed{m_W}$

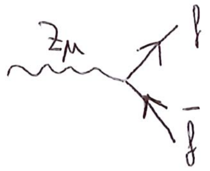
$$\left(m_W^{th}\right)^2 = \frac{e^2 v^2}{4 s^2} + \Pi_{WW}(m_W^2)$$

$\boxed{s^2_{eff}}$

We recall that we had defined s^2_{eff} in terms of the A^ℓ_{LR} asymmetry: the Z-pole production cross-section asymmetry of leptons produced from left-polarized e^+e^- collisions vs leptons produced from right-polarized collisions.

$$A^\ell_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}$$

At tree-level, and for a Feynman rule



$i\gamma_\mu(C_L P_L + C_R P_R)$

$$A^\ell_{LR} = \frac{C_L^2 - C_R^2}{C_L^2 + C_R^2}$$

We have $C_L = \frac{e}{sc}\left(T^3_e - Q_e s^2\right)$; $C_R = -\frac{e}{sc} Q_e s^2$

$$A^\ell_{LR} = \frac{\left(\frac{1}{2} - s^2\right)^2 - s^4}{\left(\frac{1}{2} - s^2\right)^2 + s^4}$$