

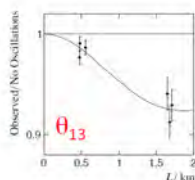
Neutrino physics (phenomenology)

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U.Valencia and IFIC

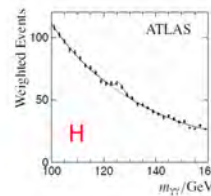
TAE 2022
Benasque - September 12, 2022

~~two~~ 2012 One major discovery ~~ies~~ in particle physics

- A SM-like Higgs boson (ATLAS, CMS)
The key to EWSB and a possible window to



- $\theta_{13} \sim 10^\circ$ (T2K, MINOS, Daya Bay, RENO)
about as large as it could have been !
The door to CP Violation in the leptonic sector



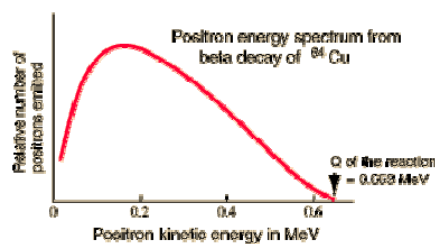
Summer Schools (if existed) were VERY short

β decay was supposed to be a two body decay



$$E_e = \frac{m_n^2 + m_e^2 - m_p^2}{2 m_n}$$

Studies of β decay revealed a continuous energy spectrum.



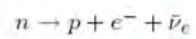
Another anomaly was the fact that the nuclear recoil was not in the direction opposite to the momentum of the electron.

The emission of another particle was a probable explanation of this behaviour, but searches found no evidence of either mass or charge.



...desperate remedy to save the law of conservation of energy...

Neutron Decay:



Fermi postulated a theory for β decay in terms of spinors

$$H_{ew} = \frac{G_F}{\sqrt{2}} \bar{\Psi}_p \gamma_\mu \Psi_n \bar{\Psi}_e \gamma^\mu \Psi_\nu$$

A Dirac field is described by a four component spinor

$$\begin{pmatrix} e_L \\ e_R \\ \hat{e}_L \\ \hat{e}_R \end{pmatrix}$$

Standard Model of Particle Physics

Gauge Theory based on the group:

$$SU(3) \times SU(2) \times U(1)$$

$SU(3) \Rightarrow$ Quantum Chromodynamics

Strong Force (Quarks and Gluons)

$SU_L(2) \times U(1) \Rightarrow$ ElectroWeak Interactions broken to $U_{EM}(1)$

by HIGGS

$$\underline{SU_L(2) \times U_Y(1) \Rightarrow U_{EM}(1)}$$

Force Carriers: W^\pm , Z^0 and γ masses: 80, 91 and 0 GeV

quark, SU(2) doublets: $\begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L$

up-quark, SU(2) singlets: u_R, c_R, t_R

down-quark, SU(2) singlets: d_R, s_R, b_R

lepton, SU(2) doublets: $\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$

neutrino, SU(2) singlets: — — —

charge lepton, SU(2) singlets: e_R, μ_R, τ_R

Electron mass

comes from a term of the form

$$\bar{L}\phi e_R$$

Absence of ν_R

forbids such a mass term (dim 4)

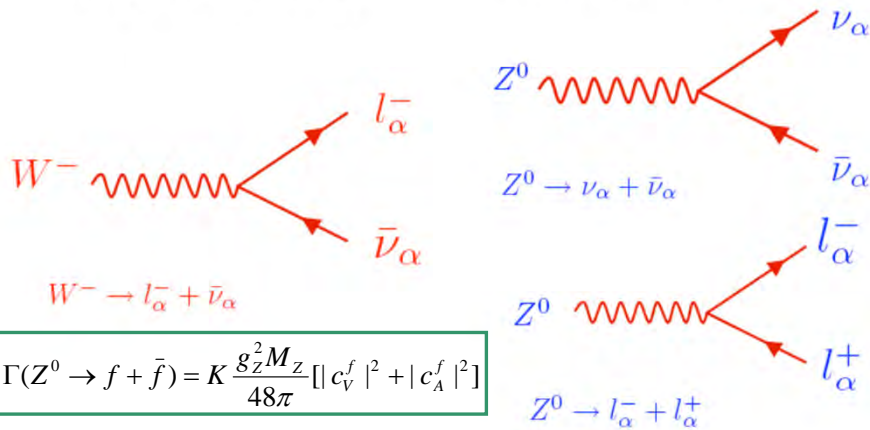
for the Neutrino

Therefore in the SM neutrinos are massless
and hence travel at speed of light.

Interactions:

Charge Current (CC)

Neutral Current (NC)



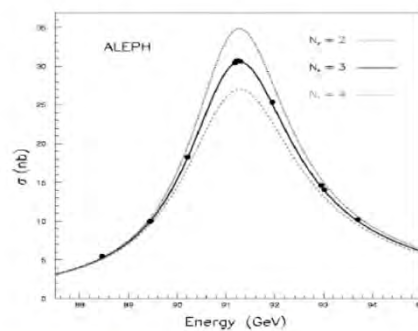
$$\Gamma(Z^0 \rightarrow f + \bar{f}) = K \frac{g_Z^2 M_Z}{48\pi} [|c_V^f|^2 + |c_A^f|^2]$$

$\alpha = e, \mu, \text{ or } \tau$

Invisible width of Z plus other data from LEP:

$$Z^0 \rightarrow \nu \bar{\nu}$$

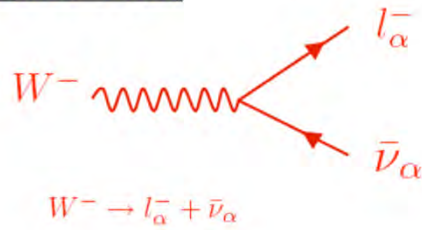
Implies $N_\nu = 2.99 \pm 0.01$



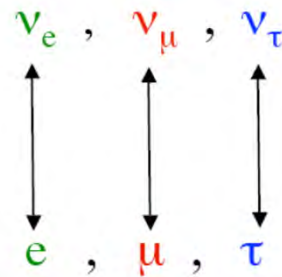
Three Active Neutrinos!!!

Sterile Neutrinos don't couple to Z^0

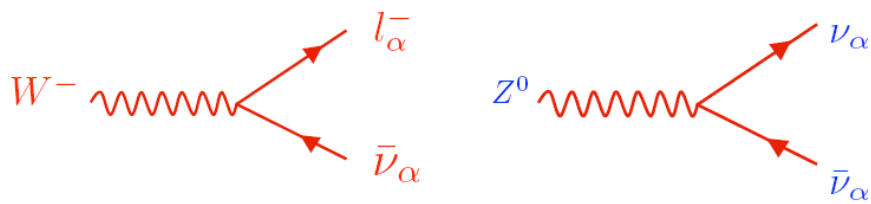
Note That



Implies



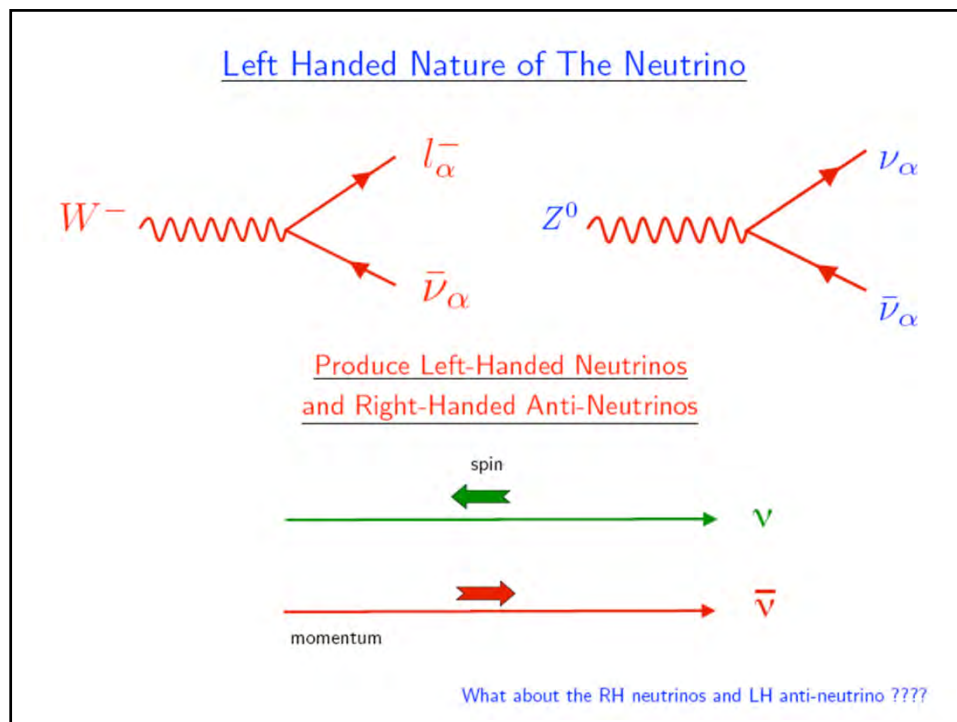
Standard Model



couplings conserve the **Lepton Number L**
defined by—

$$L(\nu) = L(\ell^-) = -L(\bar{\nu}) = -L(\ell^+) = 1.$$

Actually L_e , L_μ , and L_τ
separately



There exist three fundamental and discrete transformations in nature:

- Parity \mathcal{P} $\vec{x} \rightarrow -\vec{x}$
- Time reversal \mathcal{T} $t \rightarrow -t$
- Charge conjugation \mathcal{C} $q \rightarrow -q$

\mathcal{P} , \mathcal{T} and \mathcal{C} are conserved in the classical theories of mechanics and electrodynamics!

$CPT \leftrightarrow$ Lorentz invariance \oplus unitarity: is an essential building block of field theory

CPT : L particle \leftrightarrow R antiparticle

Neutrinos in the MSM are massless and exist only in two states: particle with negative helicity and antiparticle with positive one: **Weyl fermion**

Summary of ν 's in SM:

Three flavors of massless neutrinos

$$W^- \rightarrow l_{\alpha}^- + \bar{\nu}_{\alpha}$$

$$W^+ \rightarrow l_{\alpha}^+ + \nu_{\alpha}$$

$$\alpha = e, \mu, \text{ or } \tau$$

Anti-neutrino, $\bar{\nu}_{\alpha}$, has +ve helicity, Right Handed

Neutrino, ν_{α} , has -ve helicity, Left Handed

ν_L and $\bar{\nu}_R$ are CPT conjugates

massless implies helicity = chirality

Beyond the SM

What if Neutrino have a MASS?

speed is less than c therefore time can pass

and

Neutrinos can change character!!!

What are the stationary states?

How are they related to the interaction states?

NEUTRINO OSCILLATIONS:

Two Flavors

flavor eigenstates \neq mass eigenstates

$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

W's produce ν_μ and/or ν_τ 's

but ν_1 and ν_2 are the states

that change by a phase over time, mass eigenstates.

$$|\nu_j\rangle \rightarrow e^{-ip_j \cdot x} |\nu_j\rangle \quad p_j^2 = m_j^2$$

$\alpha, \beta \dots$ flavor index

$i, j \dots$ mass index

Production:

$$|\nu_\mu\rangle = \cos \theta |\nu_1\rangle + \sin \theta |\nu_2\rangle$$

Propagation:

$$\cos \theta e^{-ip_1 \cdot x} |\nu_1\rangle + \sin \theta e^{-ip_2 \cdot x} |\nu_2\rangle$$

Detection:

$$|\nu_1\rangle = \cos \theta |\nu_\mu\rangle - \sin \theta |\nu_\tau\rangle$$

$$|\nu_2\rangle = \sin \theta |\nu_\mu\rangle + \cos \theta |\nu_\tau\rangle$$

$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

$$P(\nu_\mu \rightarrow \nu_\tau) = |\cos \theta (e^{-ip_1 \cdot x} (-\sin \theta) + \sin \theta (e^{-ip_2 \cdot x} \cos \theta)|^2$$

$$P(\nu_\mu \rightarrow \nu_\tau) = |\cos \theta (e^{-ip_1 \cdot x})(-\sin \theta) + \sin \theta (e^{-ip_2 \cdot x}) \cos \theta|^2$$

$$\text{Same } E, \text{ therefore } p_j = \sqrt{E^2 - m_j^2} \approx E - \frac{m_j^2}{2E}$$

$$e^{-ip_j \cdot x} = e^{-iEt} e^{-ip_j L} \approx e^{-i(Et-EL)} e^{-im_j^2 L/2E}$$

$$P(\nu_\mu \rightarrow \nu_\tau) = \sin^2 \theta \cos^2 \theta |e^{-im_2^2 L/2E} - e^{-im_1^2 L/2E}|^2$$

$$P(\nu_\mu \rightarrow \nu_\tau) = \sin^2 2\theta \sin^2 \frac{\delta m^2 L}{4E}$$

$$\delta m^2 = m_2^2 - m_1^2 \text{ and } \frac{\delta m^2 L}{4E} \equiv \Delta \text{ kinematic phase:}$$

$$P(\nu_\mu \rightarrow \nu_\tau) = |\cos \theta (e^{-ip_1 \cdot x})(-\sin \theta) + \sin \theta (e^{-ip_2 \cdot x}) \cos \theta|^2$$

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$$P(\nu_\mu \rightarrow \nu_\tau) = \sin^2 \theta \cos^2 \theta |e^{-im_2^2 L/2E} - e^{-im_1^2 L/2E}|^2$$

$$P(\nu_\mu \rightarrow \nu_\tau) = \sin^2 2\theta \sin^2 \left(\frac{\delta m^2 L}{4E} \frac{c^4}{hc} \right)$$

Appearance:

$$P(\nu_\mu \rightarrow \nu_\tau) = \sin^2 2\theta \sin^2 \frac{\delta m^2 L}{4E}$$

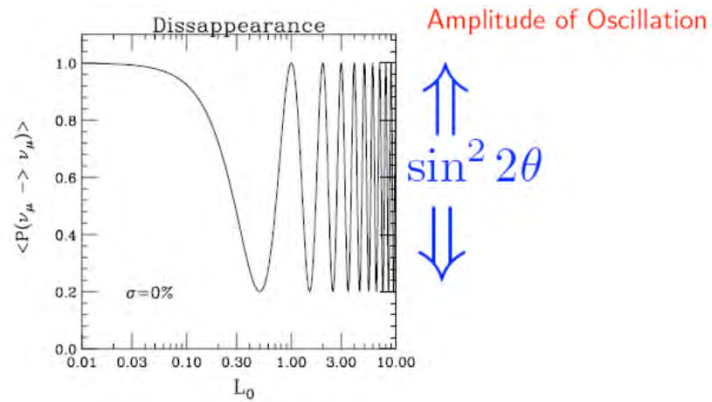
Disappearance:

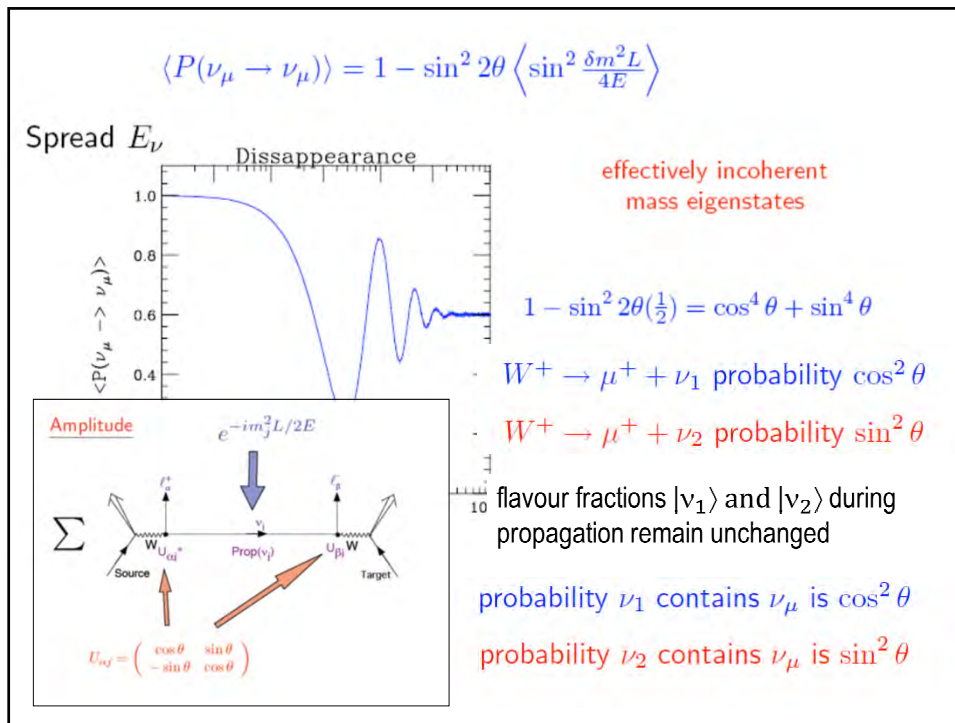
$$P(\nu_\mu \rightarrow \nu_\mu) = 1 - \sin^2 2\theta \sin^2 \frac{\delta m^2 L}{4E}$$

$$P(\nu_\mu \rightarrow \nu_\mu) = 1 - \sin^2 2\theta \sin^2 \frac{\delta m^2 L}{4E}$$

Oscillation Length $L_0 = 4\pi E / \delta m^2$

Fixed E_ν





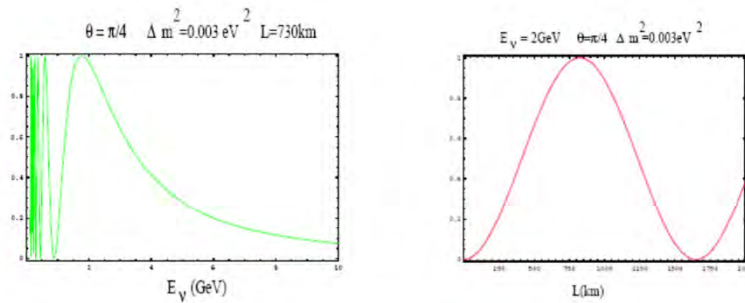
Using the unitarity of the mixing matrix: ($W_{\alpha\beta}^{jk} \equiv [V_{\alpha j} V_{\beta j}^* V_{\alpha k}^* V_{\beta k}]$)

$$P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{k>j} \text{Re}[W_{\alpha\beta}^{jk}] \sin^2 \left(\frac{\Delta m_{jk}^2 L}{4E_\nu} \right) \\ \pm 2 \sum_{k>j} \text{Im}[W_{\alpha\beta}^{jk}] \sin \left(\frac{\Delta m_{jk}^2 L}{2E_\nu} \right)$$

For 2 families: $V_{MNS} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$

$$P_{\alpha\beta} = \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 L}{4E_\nu} \right) \rightarrow \text{appearance}$$

$$P_{\alpha\alpha} = 1 - P_{\alpha\beta} < 1 \rightarrow \text{disappearance}$$



Oscillation probabilities show the expected **GIM** suppression of any flavour changing process: they vanish if the neutrinos are degenerate

Probability for Neutrino Oscillation in Vacuum

$$P(\nu_\alpha \rightarrow \nu_\beta) = |\text{Amp}(\nu_\alpha \rightarrow \nu_\beta)|^2 =$$

$$P_{\alpha\beta} = \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 L}{4E_\nu} \right) \rightarrow \text{appearance}$$

$$P_{\alpha\alpha} = 1 - P_{\alpha\beta} < 1 \rightarrow \text{disappearance}$$

Probability for Neutrino Oscillation in Vacuum

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$$P_{\alpha\beta} = \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 L}{4 E} \right)$$

$$P_{\alpha\alpha} = 1 - P_{\alpha\beta}$$

$$\left(1.27 \frac{\Delta m^2 (eV^2) L(km)}{E(GeV)} \right)$$

L/E becomes crucial !!!

Evidence for Flavor Change:

*** Atmospheric and Accelerator Neutrinos with $L/E = 500 \text{ km/GeV}$

*** Solar and Reactor Neutrinos with $L/E = 15 \text{ km/MeV}$

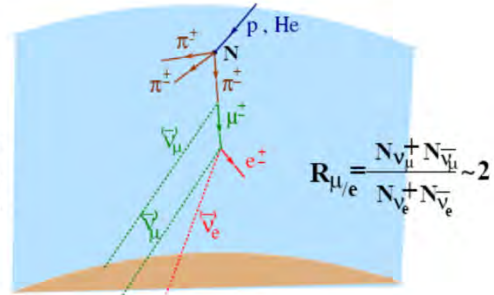
Neutrinos from Stopped muons $L/E = 2 \text{ m/MeV}$ (Unconfirmed)

Atmospheric neutrinos

- Atmospheric neutrinos are produced by the interaction of *cosmic rays* (p, He, \dots) with the Earth's atmosphere:

- 1 $A_{\text{cr}} + A_{\text{air}} \rightarrow \pi^{\pm}, K^{\pm}, K^0, \dots$
- 2 $\pi^{\pm} \rightarrow \mu^{\pm} + \nu_{\mu}$
- 3 $\mu^{\pm} \rightarrow e^{\pm} + \nu_e + \nu_{\mu}$

- at the detector, some ν interacts and produces a **charged lepton**, which is observed.

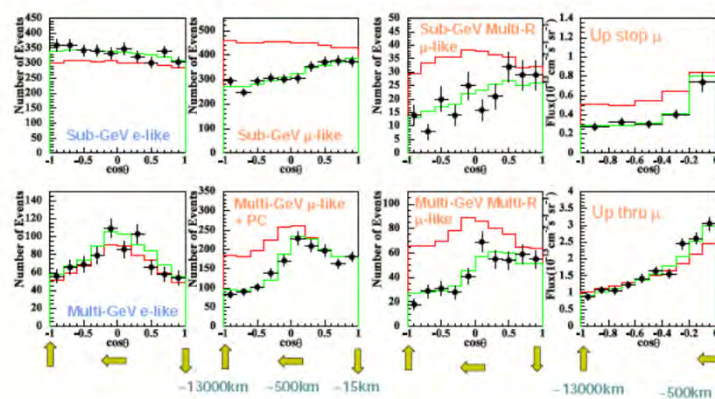


A deficit was observed in the ratio μ/e events: **Soudan2, IMB, Kamiokande**

Zenith angle distributions

$\nu_{\mu} \leftrightarrow \nu_{\tau}$
2-flavor oscillations

Best fit
 $\sin^2 2\theta = 1.0, \Delta m^2 = 2.0 \times 10^{-3} \text{ eV}^2$
Null oscillation



Half of the upward-going, long-distance-traveling ν_μ are disappearing.

Voluminous atmospheric neutrino data are well described by —

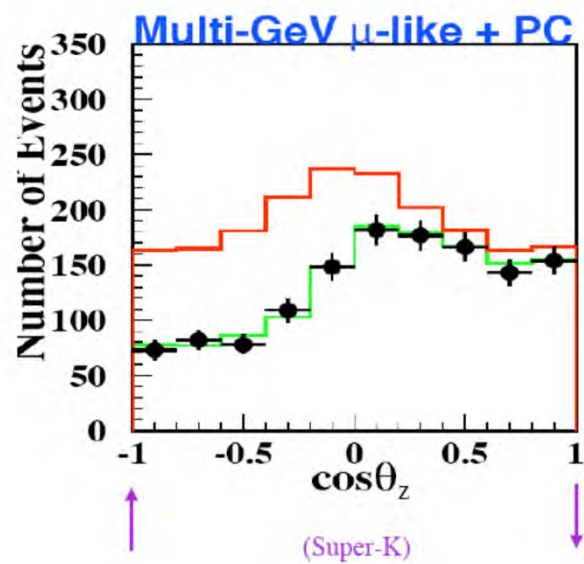
$$\nu_\mu \longrightarrow \nu_\tau$$

with —

$$\Delta m_{\text{atm}}^2 \cong 2.4 \cdot 10^{-3} \text{ eV}^2$$

and —

$$\sin^2 2\theta_{\text{atm}} \cong 1$$



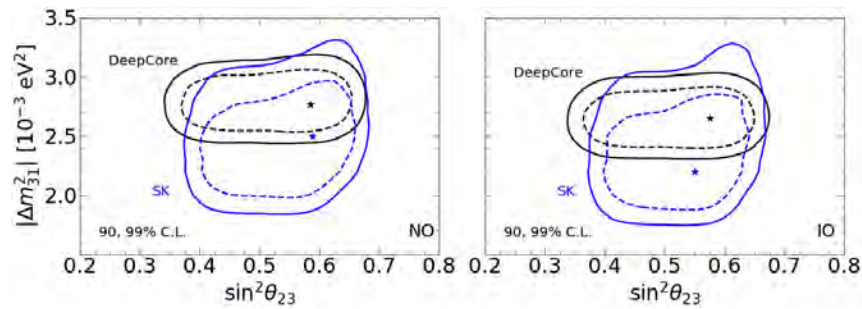
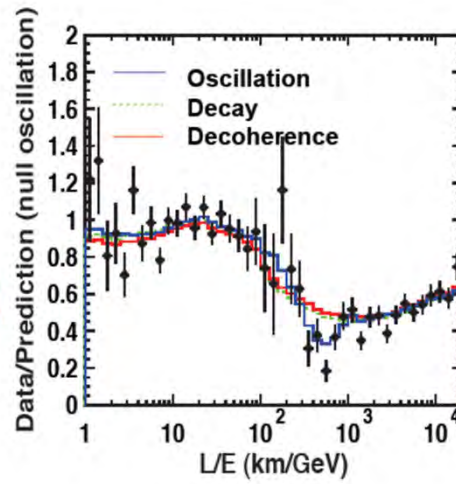
L/E Analysis

❖ Oscillation, decay and decoherence models tested

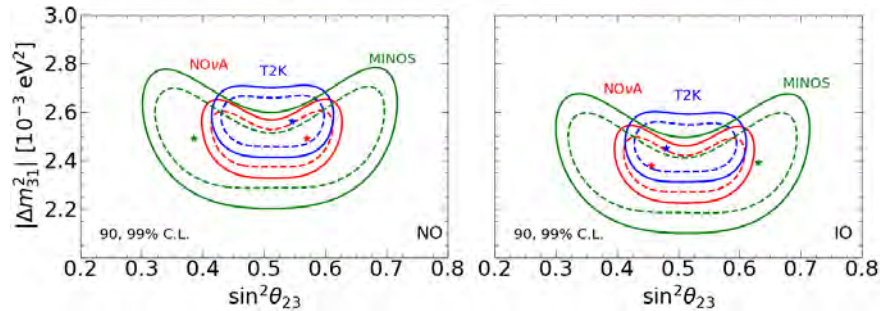
$$\chi^2_{\text{osc}} = 83.9/83$$

$$\chi^2_{\text{dcy}} = 107.1/83, \Delta\chi^2 = 23.2(4.8\sigma)$$

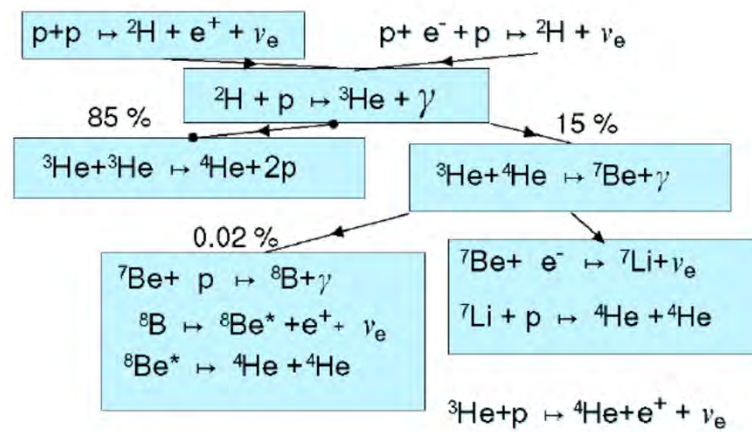
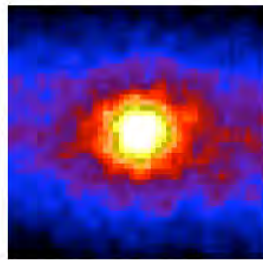
$$\chi^2_{\text{dec}} = 112.5/83, \Delta\chi^2 = 27.6(5.3\sigma)$$



<https://globalfit.astroparticles.es/>



Solar δm^2



Solar Spectrum:

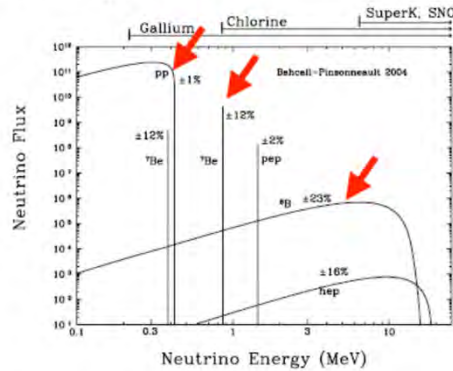
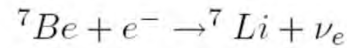


Figure 1. The predicted solar neutrino energy spectrum. The figure shows the energy spectrum of solar neutrinos predicted by the BP04 solar model [22]. For continuum sources, the neutrino fluxes are given in number of neutrinos $\text{cm}^{-2} \text{s}^{-1} \text{MeV}^{-1}$ at the Earth's surface. For line sources, the units are number of neutrinos $\text{cm}^{-2} \text{s}^{-1}$. Total theoretical uncertainties taken from column 2 of table 1 are shown for each source. To avoid complication in the figure, we have omitted the difficult-to-detect CNO neutrino fluxes (see table 1).



$$\phi_{pp} = 5.94(1 \pm 0.01) \times 10^{10} \text{cm}^{-2} \text{sec}^{-1}$$



$$\phi_{{}^7\text{Be}} = 4.86(1 \pm 0.12) \times 10^9 \text{cm}^{-2} \text{sec}^{-1}$$

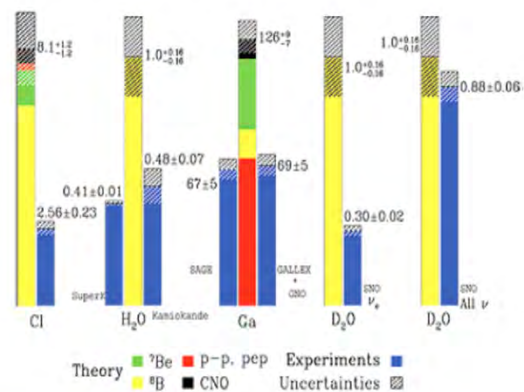


$$\phi_{{}^8\text{B}} = 5.82(1 \pm 0.23) \times 10^6 \text{cm}^{-2} \text{sec}^{-1}$$



Ray Davis & John Bahcall

Total Rates: Standard Model vs. Experiment
Bahcall-Serenelli 2005 [BS05(OP)]



Theory v Exp.

Neutrino Flavor Transitions!!!

Kinematical Phase:

$$\delta m_{\odot}^2 = 8.0 \times 10^{-5} eV^2$$

$$\sin^2 \theta_{\odot} = 0.31$$

$$\Delta_{\odot} = \frac{\delta m_{\odot}^2 L}{4E} = 1.27 \frac{8 \times 10^{-5} eV^2 \cdot 1.5 \times 10^{11} m}{0.1-10 MeV}$$

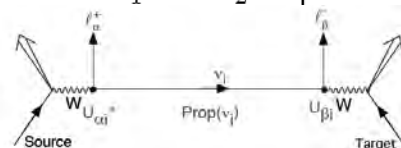
$$\Delta_{\odot} \approx 10^{7 \pm 1}$$

Effectively Incoherent !!!

Vacuum ν_e Survival Probability:

$$\langle P_{ee} \rangle = f_1 \cos^2 \theta_{\odot} + f_2 \sin^2 \theta_{\odot}$$

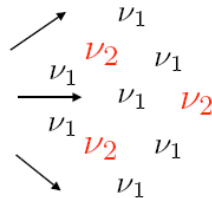
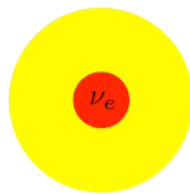
where f_1 and f_2 are the fraction of ν_1 and ν_2 at production.

In vacuum $f_1 = \cos^2 \theta_{\odot}$ 

$$\langle P_{ee} \rangle = \cos^4 \theta_{\odot} + \sin^4 \theta_{\odot} = 1 - \frac{1}{2} \sin^2 2\theta_{\odot}$$

for pp and ${}^7\text{Be}$ this is approximately THE ANSWER.

$$f_1 \sim 69\% \text{ and } f_2 \sim 31\% \text{ and } \langle P_{ee} \rangle \approx 0.6$$

pp and ${}^7\text{Be}$ 

$$f_1 \sim 69\%$$

$$f_2 \sim 31\%$$

$$\langle P_{ee} \rangle \approx 0.6$$

$$f_3 = \sin^2 \theta_{13} < 4\%$$

What about ${}^8\text{B}$?

SNO's CC/NC

CC: $\nu_e + d \rightarrow e^- + p + p$

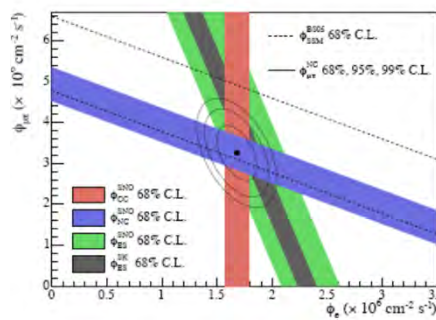
NC : $\nu_x + d \rightarrow \nu_x + p + n$

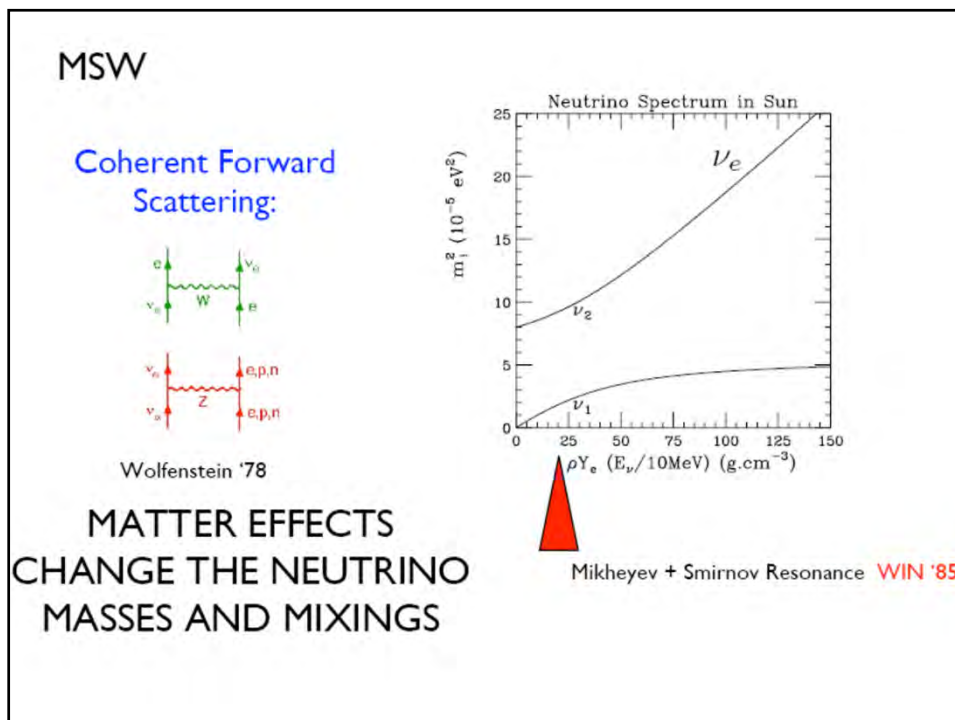
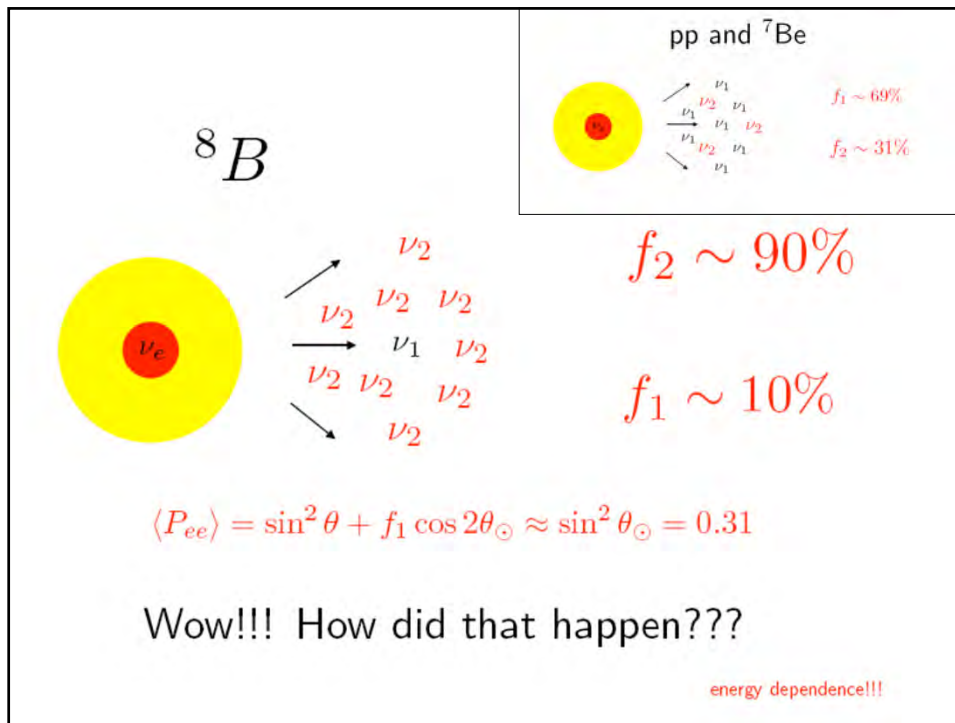
ES: $\nu_\alpha + e^- \rightarrow \nu_\alpha + e^-$

$$\frac{CC}{NC} = \langle P_{ee} \rangle = f_1 \cos^2 \theta_\odot + f_2 \sin^2 \theta_\odot$$

$$f_1 = (\frac{CC}{NC} - \sin^2 \theta_\odot) / \cos 2\theta_\odot$$

$$= (0.35 - 0.31) / 0.4 \approx 10\%$$





Neutrino Evolution:

$$-i\frac{\partial}{\partial t}\nu = H\nu$$

in the mass eigenstate basis

$$\nu = \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \text{ and } H = \begin{pmatrix} \sqrt{p^2 + m_1^2} & 0 \\ 0 & \sqrt{p^2 + m_2^2} \end{pmatrix} \quad E = \sqrt{p^2 + m^2}$$

$$H = (p + \frac{m_1^2 + m_2^2}{4p})I + \frac{1}{4E} \begin{pmatrix} -\delta m^2 & 0 \\ 0 & \delta m^2 \end{pmatrix}$$

$$\delta m^2 = m_2^2 - m_1^2 > 0$$

in the flavor basis

$$\nu \rightarrow U\nu \text{ and } H \rightarrow UHU^\dagger$$

$$\text{where } \nu = \begin{pmatrix} \nu_e \\ \nu_\sigma \end{pmatrix} \text{ and } U = \begin{pmatrix} \cos \theta_\odot & \sin \theta_\odot \\ -\sin \theta_\odot & \cos \theta_\odot \end{pmatrix}$$

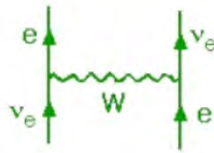
and therefore in flavor basis $0 < \theta_\odot < \frac{\pi}{2}$

$$H = \frac{\delta m^2}{4E} \begin{pmatrix} -\cos 2\theta_\odot & \sin 2\theta_\odot \\ \sin 2\theta_\odot & \cos 2\theta_\odot \end{pmatrix}$$

$$\text{i.e. } \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}_{mass} \Rightarrow \frac{\delta m^2}{4E} \begin{pmatrix} -\cos 2\theta_\odot & \sin 2\theta_\odot \\ \sin 2\theta_\odot & \cos 2\theta_\odot \end{pmatrix}_{flavor}$$

Coherent Forward Scattering:

$$\text{dimensions } [G_F N_e] = M^{-2} L^{-3} = M$$

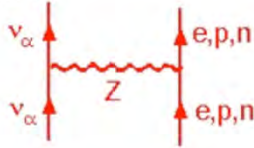


$$\pm \sqrt{2} G_F N_e \delta_{ee}$$

N_e is number density of electrons

$+$ $(-)$ for neutrinos (anti-neutrinos)

Wolfenstein '78



Same for all active flavors,
therefore overall phases

$$\begin{pmatrix} +\sqrt{2} G_F N_e & 0 \\ 0 & 0 \end{pmatrix} \rightarrow \frac{G_F N_e}{\sqrt{2}} I_2 + \frac{1}{2} \begin{pmatrix} +\sqrt{2} G_F N_e & 0 \\ 0 & -\sqrt{2} G_F N_e \end{pmatrix}$$

Including Matter Effects in the Flavor Basis:

$$H_{flavor} = \frac{1}{4E_\nu} \begin{pmatrix} -\delta m^2 \cos 2\theta_\odot + 2\sqrt{2} G_F N_e E_\nu & \delta m^2 \sin 2\theta_\odot \\ \delta m^2 \sin 2\theta_\odot & \delta m^2 \cos 2\theta_\odot - 2\sqrt{2} G_F N_e E_\nu \end{pmatrix}$$

Diagonalize by identifying with

$$H_{flavor} = \frac{1}{4E_\nu} \begin{pmatrix} -\delta m_N^2 \cos 2\theta_\odot^N & \delta m_N^2 \sin 2\theta_\odot^N \\ \delta m_N^2 \sin 2\theta_\odot^N & \delta m_N^2 \cos 2\theta_\odot^N \end{pmatrix}$$

Masses and Mixings in MATTER: δm_N^2 and θ_\odot^N

$$\begin{aligned} \delta m_N^2 \cos 2\theta_\odot^N &= \delta m^2 \cos 2\theta_\odot - 2\sqrt{2} G_F N_e E_\nu \\ \delta m_N^2 \sin 2\theta_\odot^N &= \delta m^2 \sin 2\theta_\odot \end{aligned}$$

Notice:

- (1) Possible zero when $\delta m^2 \cos 2\theta_\odot = 2\sqrt{2} G_F N_e E_\nu$
- (2) the invariance of the product $\delta m^2 \sin 2\theta_\odot$

ν_e disappearance in Looooong Block of Lead:

$$1 - P(\nu_e \rightarrow \nu_e) = \sin^2 2\theta_{\odot}^N \sin^2 \Delta_N$$

$$\Delta_N = \frac{\delta m_N^2 L}{4E}$$

same form as vacuum

The Solution:

$$\delta m_N^2 = \sqrt{(\delta m^2 \cos 2\theta_{\odot} - 2\sqrt{2}G_F N_e E_{\nu})^2 + (\delta m^2 \sin 2\theta_{\odot})^2}$$

$$\sin^2 \theta_{\odot}^N = \frac{1}{2} \left(1 - \frac{(\delta m^2 \cos 2\theta_{\odot} - 2\sqrt{2}G_F N_e E_{\nu})}{\delta m_N^2} \right) \quad \theta_{\odot}^N > \theta_{\odot}$$

Quasi-Vacuum: $2\sqrt{2}G_F N_e E_{\nu} \ll \delta m^2 \cos 2\theta_{\odot}$ pp and ${}^7\text{Be}$

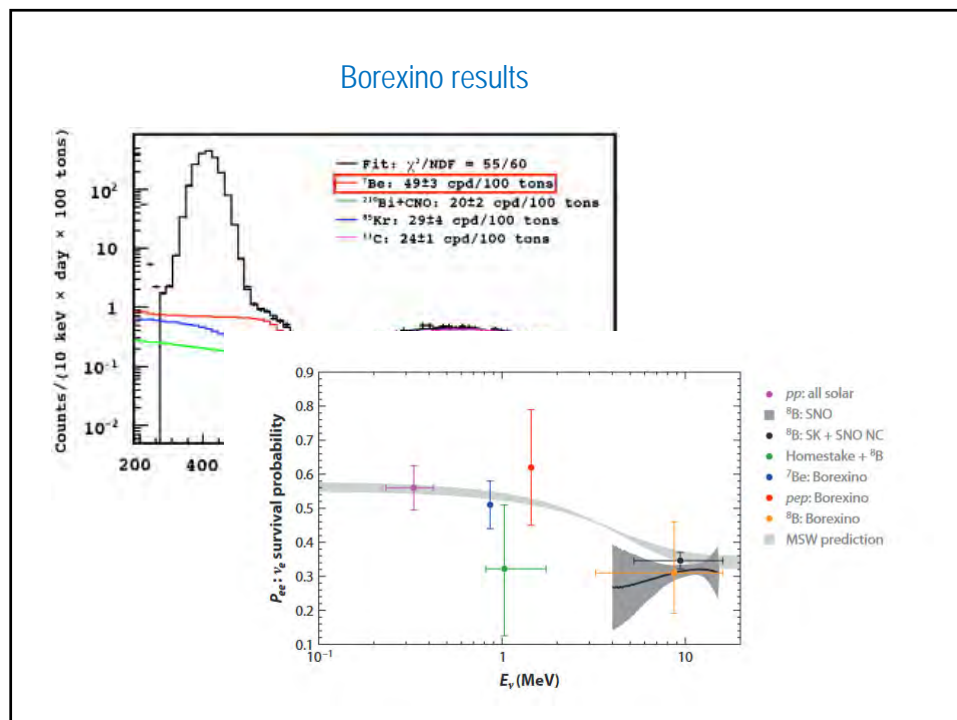
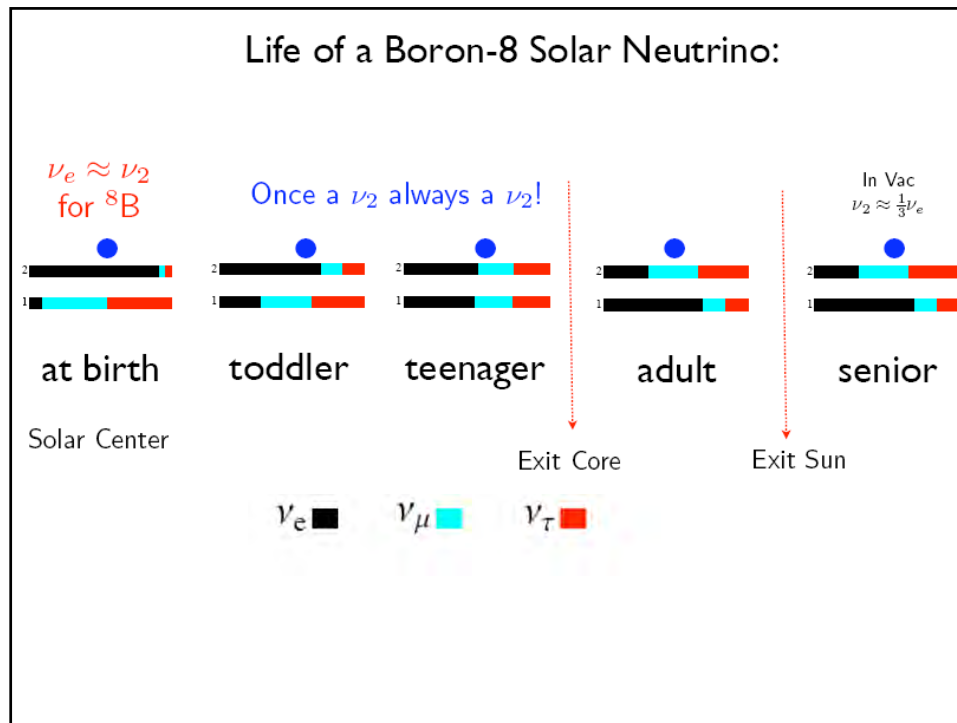
$$\delta m_N^2 = \delta m^2 \text{ and } \theta_{\odot}^N = \theta_{\odot}$$

Resonance (Mikheyev + Smirnov '85): $2\sqrt{2}G_F N_e E_{\nu} = \delta m^2 \cos 2\theta_{\odot}$

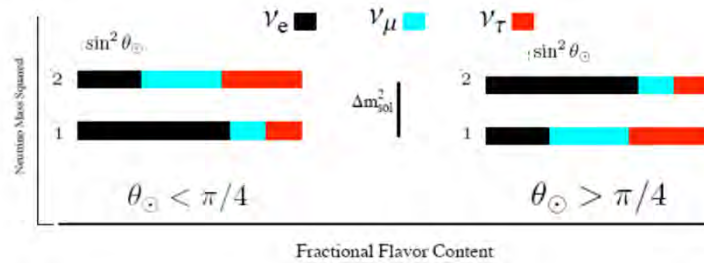
$$\delta m_N^2 = \delta m^2 \sin 2\theta_{\odot} \text{ and } \theta_{\odot}^N = \pi/4$$

Matter Dominated: $2\sqrt{2}G_F N_e E_{\nu} \gg \delta m^2 \cos 2\theta_{\odot}$

$$\delta m_N^2 \rightarrow 2\sqrt{2}G_F N_e E_{\nu} \text{ and } \theta_{\odot}^N \rightarrow \pi/2 \quad {}^8\text{B}$$



Solar Pair Mass Hierarchy:



Who cares ?
SNO does !!!

for neutrino in matter
 $\theta_{\odot}^N > \theta_{\odot}$

$$\langle P_{ee} \rangle = \cos^2 \theta_{\odot}^N \cos^2 \theta_{\odot} + \sin^2 \theta_{\odot}^N \sin^2 \theta_{\odot} = \frac{1}{2} + \frac{1}{2} \cos 2\theta_{\odot}^N \cos 2\theta_{\odot}$$

if $\theta_{\odot} < \pi/4$
 $\langle P_{ee} \rangle \geq \sin^2 \theta_{\odot}$

if $\theta_{\odot} > \pi/4$
 $\langle P_{ee} \rangle \geq \frac{1}{2}(1 + \cos^2 2\theta_{\odot}) \geq \frac{1}{2}$

SNO: $\langle P_{ee} \rangle_{day} = 0.347 \pm 0.038$

**Solar Hierarchy
Determined !!!**

Day/Night Asymmetry:

$$\sin^2 \theta_{\odot} \rightarrow \sin^2 \theta_{\oplus} = \sin^2 \theta_{\odot} + \frac{1}{2} \sin^2 2\theta_{\odot} \left(\frac{A_{\oplus}}{\delta m_{\odot}^2} \right) \text{ in the earth.}$$

$A=2(D-N)/(D+N)$ expected to be few %

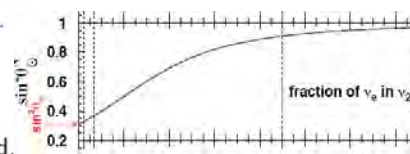
	Amplitude fit		separate D, N: (D-N)/((D+N)/2)
	Δm	Δm	
SK-I	$-2.0 \pm 1.8 \pm 1.0\%$	$-1.9 \pm 1.7 \pm 1.0\%$	$-2.1 \pm 2.0 \pm 1.3\%$
SK-II	$-4.4 \pm 3.8 \pm 1.0\%$	$-4.4 \pm 3.6 \pm 1.0\%$	$-5.5 \pm 4.2 \pm 3.7\%$
SK-III	$-4.2 \pm 2.7 \pm 0.7\%$	$-3.8 \pm 2.6 \pm 0.7\%$	$-5.9 \pm 3.2 \pm 1.3\%$
SK-IV	$-3.6 \pm 1.6 \pm 0.6\%$	$-3.3 \pm 1.5 \pm 0.6\%$	$-4.9 \pm 1.8 \pm 1.4\%$
comb	$-3.3 \pm 1.0 \pm 0.5\%$	$-3.1 \pm 1.0 \pm 0.5\%$	$-4.1 \pm 1.2 \pm 0.8\%$
non-zero signal	3.0σ	2.8σ	2.8σ

Spectral Distortion:

A characteristic of matter effects is that
the Fraction of ν_2 is energy dependent.

Smaller at smaller E.

Implies an increase in P_{ee} near threshold.



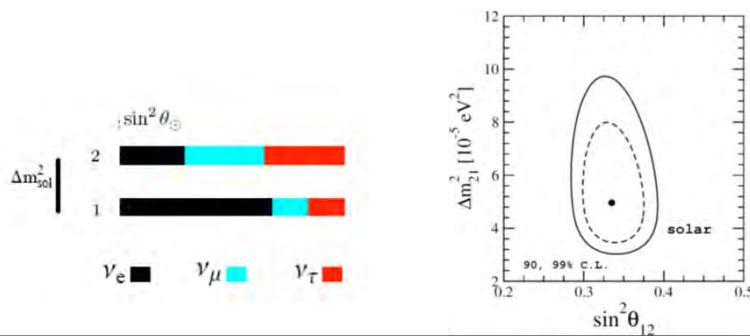
Summary:

The low energy pp and ${}^7\text{Be}$ Solar Neutrinos exit the sun as two thirds ν_1 and one third ν_2 due to (quasi-) vacuum oscillations.

$$f_1 = 65 \pm 2\%, f_2 = 35 \pm 2\% \text{ with } P_{ee} \approx 0.56$$

The high energy ${}^8\text{B}$ Solar Neutrinos exit the sun as "PURE" ν_2 mass eigenstates due to matter effects.

$$f_2 = 91 \pm 2\% \text{ and } f_1 = 9 \pm 2\% \text{ with } P_{ee} \approx 0.35.$$



Testing solar neutrino oscillations with reactors

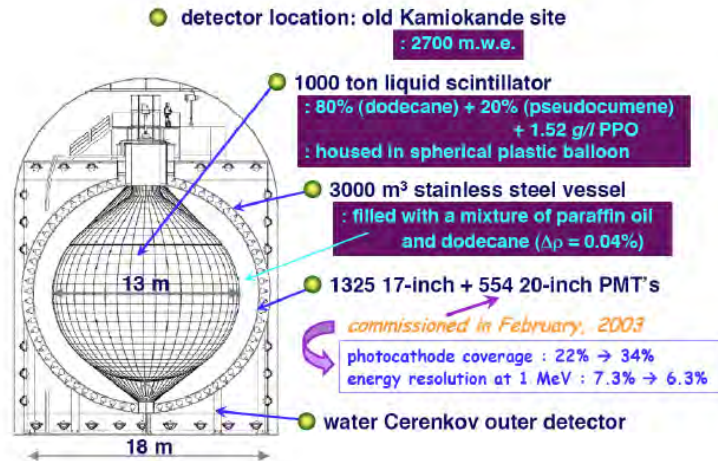
$$1 - P(\nu_e \rightarrow \nu_e) = \sin^2 2\theta_\odot \sin^2 \Delta$$

$$\Delta = \frac{\delta m^2 L}{4E}$$

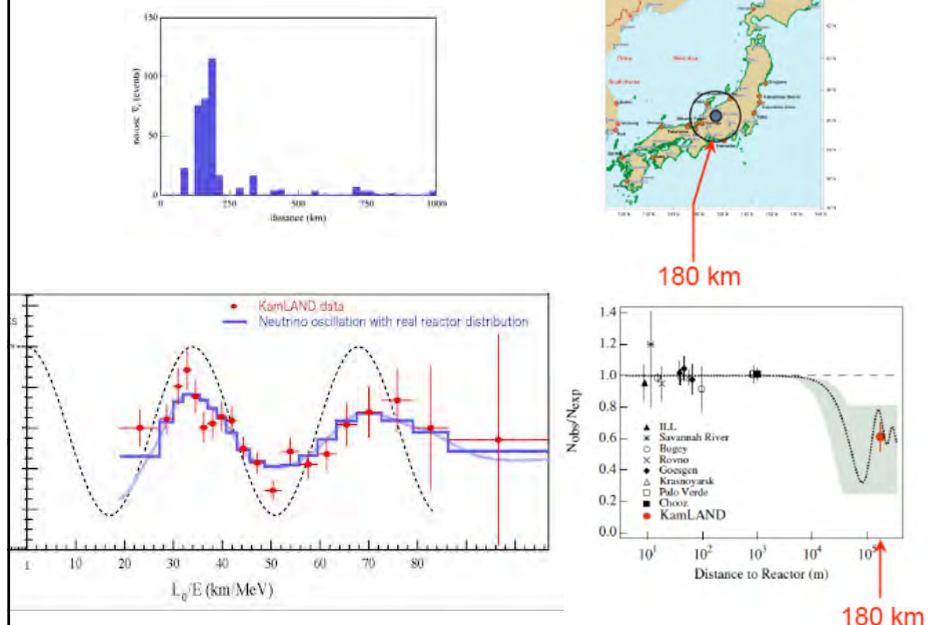
10^{-5} eV^2
 $10^5 \text{ m} = 100 \text{ km}$
 1 MeV

Reactor Neutrinos

KamLAND Detector



expected no-oscillation neutrino event rate at KamLAND



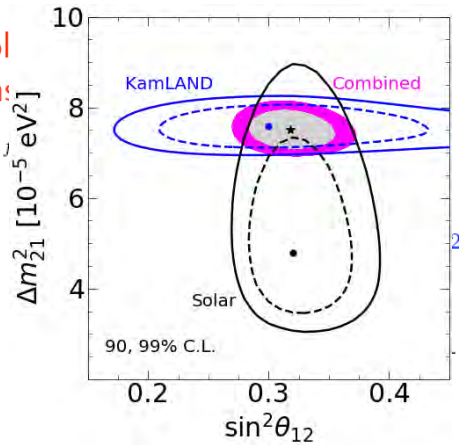
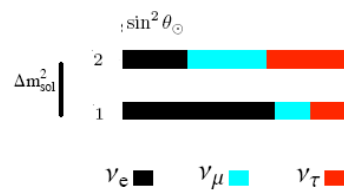
Summary:

The low energy pp and ${}^7\text{Be}$ Solar Neutrinos exit the sun as two thirds ν_1 and one third ν_2 due to (quasi-) vacuum oscillations.

$$f_1 = 65 \pm 2\%, f_2 = 35 \pm 2\% \text{ with } P_{ee} \approx 0.56$$

The high energy ${}^8\text{B}$ Sol
"PURE" ν_2 mass eigenstate

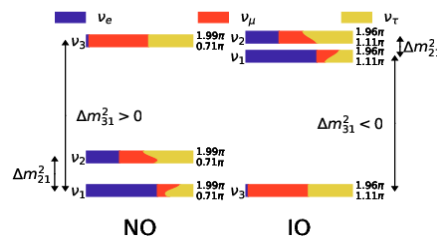
$$f_2 = 91 \pm 2\% \text{ and } f_1 = 9\% \text{ and } f_3 = 0\%$$

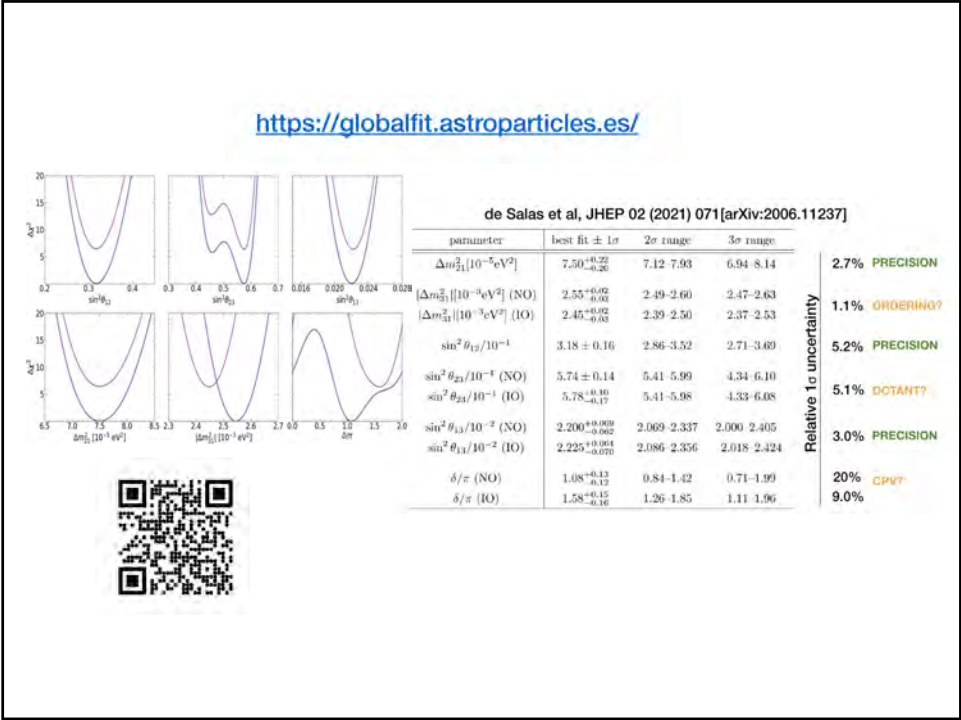


Three-neutrino oscillations

Neutrino mixing matrix

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$





Parameter	Main contribution	Other contributions
θ_{12}	SOL	KamLAND
θ_{13}	REAC	ATM+LBL and SOL+KamLAND
θ_{23}	ATM+LBL	-
δ_{CP}	LBL	ATM
Δm_{21}^2	KamLAND	SOL
$ \Delta m_{31}^2 $	LBL+ATM+REAC	-
MO	LBL+REAC and ATM	-

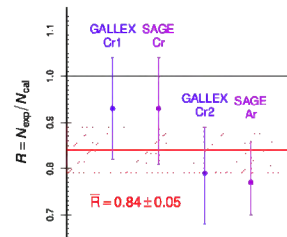
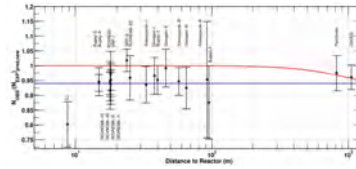
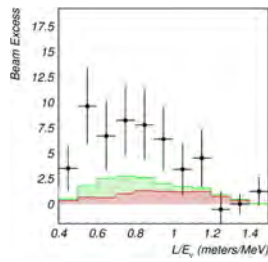
SOL: Solar

ATM: Armtopsheric neutrinos

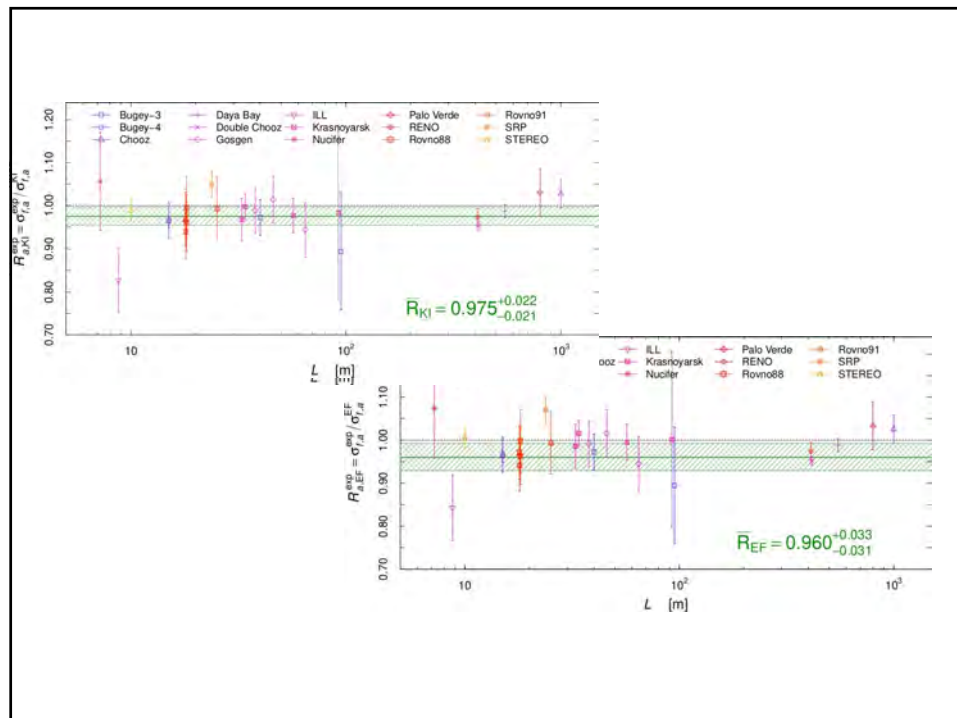
LBL: Long baseline accelerator experiments

REAC: Short-baseline reactor experiments

Anomalies



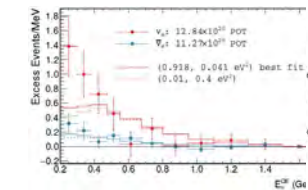
Need extra states !!!



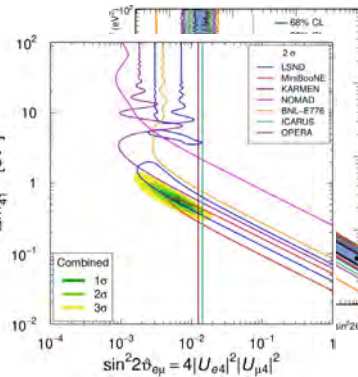
MiniBooNE

MiniBooNE was built to check the LSND results with a different baseline, but similar L/E

MiniBooNE has no near det



MiniBooNE sees an excess at $\sim 5\sigma$ at low energies



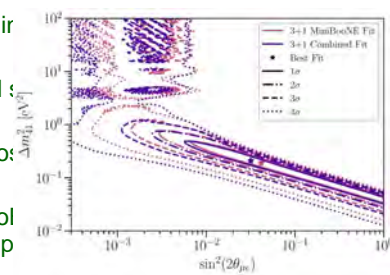
MicroBooNE

MicroBooNE was built to check the Mir results!

Looking for signals using several final channels

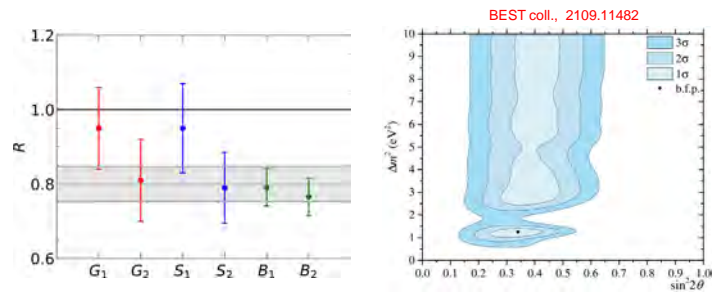
The collaboration did not perform an oscillation analysis

A combined analysis shows that MicroBooNE not exclude the region of parameter space preferred by MiniBooNE

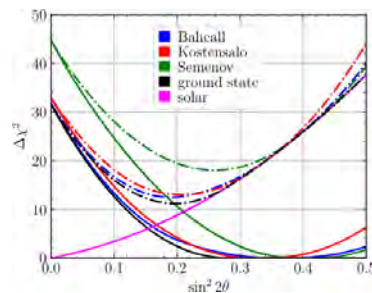


2201.01724

The Gallium anomaly



The Gallium anomaly is now at more than 5σ significance



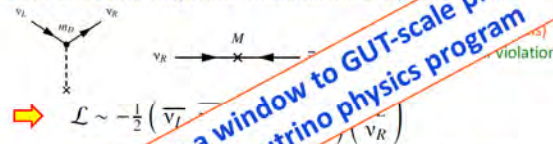
Berryman et al, 2111.12530, JHEP 2022

Can not be explained due to cross section mistakes

a connection to BSM physics

★ Is there a connection to the GUT scale?

- If both Dirac and Majorana mass terms are present



$$\Rightarrow \mathcal{L} \sim -\frac{1}{2} \left(\bar{\nu}_L \gamma^\mu \nu_L + \bar{\nu}_R \gamma^\mu \nu_R \right) - m_D \bar{\nu}_L \nu_R - \frac{1}{2} M \bar{\nu}_R \nu_R$$

- The seesaw mechanism provides physical "mass eigenstates" that are those for which the mass matrix is diagonal

Neutrinos may provide a window to GUT-scale physics
argues for a precision neutrino physics program

$$m_\nu \approx \frac{m_D^2}{M} + \text{heavy RH neutrino } m_N \approx M$$

$m_D \sim m_\ell$ to get to right range of small neutrino masses:

$$M \sim 10^{12} - 10^{16} \text{ GeV}$$

The Known Unknowns

★ Next generation Long-Baseline experiments (such as DUNE) can address three of these questions:

- Are neutrinos Dirac or Majorana ?
 - Is there a connection to the GUT scale?
- Are there light sterile neutrino states ? Breaks 3-flavor paradigm
 - No clear theoretical guidance on mass scale, M , ...
- What is the neutrino mass hierarchy ?
 - An important question in flavor physics, e.g. CKM vs. PMNS



- Is CP violated in the leptonic sector ?
 - Are vs key to understanding the matter-antimatter asymmetry?

We determined that $m(K_L) > m(K_S)$ by

- Passing kaons through matter (regenerator)
- Beating the unknown $\text{sign}[m(K_L) - m(K_S)]$ against the known $\text{sign}[\text{reg. ampl.}]$

We will determine the $\text{sign}(\Delta m^2_{31})$ by

- Passing neutrinos through matter (Earth)
- Beating the unknown $\text{sign}(\Delta m^2_{31})$ against the known $\text{sign}[\text{forward } \nu_e e \rightarrow \nu_e e \text{ ampl.}]$

$$L \approx \frac{2\pi}{G_F n_e} \approx 1.16 \cdot 10^4 \text{ km} \left(\frac{1.69 \cdot 10^{24} \text{ cm}^3}{n_e} \right)$$

The Known Unknowns

★ Next generation Long-Baseline experiments (such as DUNE) can address three of these questions:

- Are neutrinos Dirac or Majorana ?
 - Is there a connection to the GUT scale?
- Are there light sterile neutrino states ? Breaks 3-flavor paradigm
 - No clear theoretical guidance on mass scale, M , ...
- What is the neutrino mass hierarchy ?
 - An important question in flavor physics, e.g. CKM vs. PMNS



- Is CP violated in the leptonic sector ?
 - Are ν s key to understanding the matter-antimatter asymmetry?

In principle, it is straightforward

- ★ CPV \Rightarrow different oscillation rates for ν s and $\bar{\nu}$ s

$$P(\nu_\mu \rightarrow \nu_e) - P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = 4s_{12}s_{13}c_{13}^2s_{23}c_{23}\sin\delta \quad \leftarrow \text{vacuum osc.}$$

$$\times \left[\sin\left(\frac{\Delta m_{21}^2 L}{4E}\right) \times \sin\left(\frac{\Delta m_{23}^2 L}{4E}\right) \times \sin\left(\frac{\Delta m_{31}^2 L}{4E}\right) \right]$$

- ★ Requires $\{\theta_{12}, \theta_{13}, \theta_{23}\} \neq \{0, \pi\}$
 - now know that this is true, $\theta_{13} \approx 9^\circ$
 - but, despite hints, don't yet know "much" about δ
- ★ So "just" measure $P(\nu_\mu \rightarrow \nu_e) - P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$?
- ★ Not quite, there is a complication...

Neutrino Oscillations in Matter

- ★ Accounting for this potential term, gives a Hamiltonian that is **not diagonal** in the basis of the mass eigenstates

$$\mathcal{H} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix} = i \frac{d}{dt} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix} = \begin{pmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_3 \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix} + V|\nu_e\rangle \quad \leftarrow \text{ME}$$

- ★ Complicates the simple picture !!!!

$$P(\nu_\mu \rightarrow \nu_e) - P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) =$$

ME	$\frac{16A}{\Delta m_{31}^2} \sin^2\left(\frac{\Delta m_{31}^2 L}{4E}\right) c_{13}^2 s_{13}^2 s_{23}^2 (1 - 2s_{13}^2)$
ME	$-\frac{2AL}{E} \sin\left(\frac{\Delta m_{31}^2 L}{4E}\right) c_{13}^2 s_{13}^2 s_{23}^2 (1 - 2s_{13}^2)$
CPV	$-8 \frac{\Delta m_{21}^2 L}{2E} \sin^2\left(\frac{\Delta m_{31}^2 L}{4E}\right) \sin\delta \cdot s_{13}c_{13}^2 c_{23}s_{23}c_{12}s_{12}$

with $A = 2\sqrt{2}G_F n_e E = 7.6 \times 10^{-5} \text{eV}^2 \cdot \frac{\rho}{\text{g cm}^{-3}} \cdot \frac{E}{\text{GeV}}$

Experimental Strategy

EITHER:

- ★ Keep L small (~ 200 km): so that matter effects are insignificant

- First oscillation maximum:

$$\frac{\Delta m_{31}^2 L}{4E} \sim \frac{\pi}{2} \Rightarrow E_\nu < 1 \text{ GeV}$$

- Want high flux at oscillation maximum

⇒ **Off-axis beam**: narrow range of neutrino energies

OR:

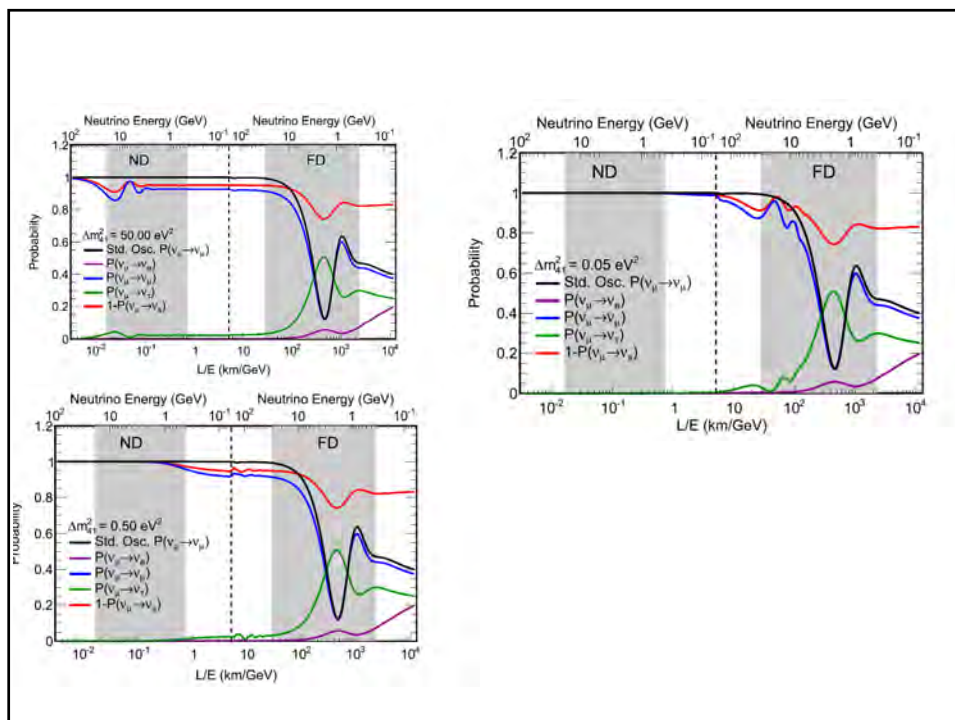
- ★ Make L large (> 1000 km): measure the matter effects (i.e. **MH**)

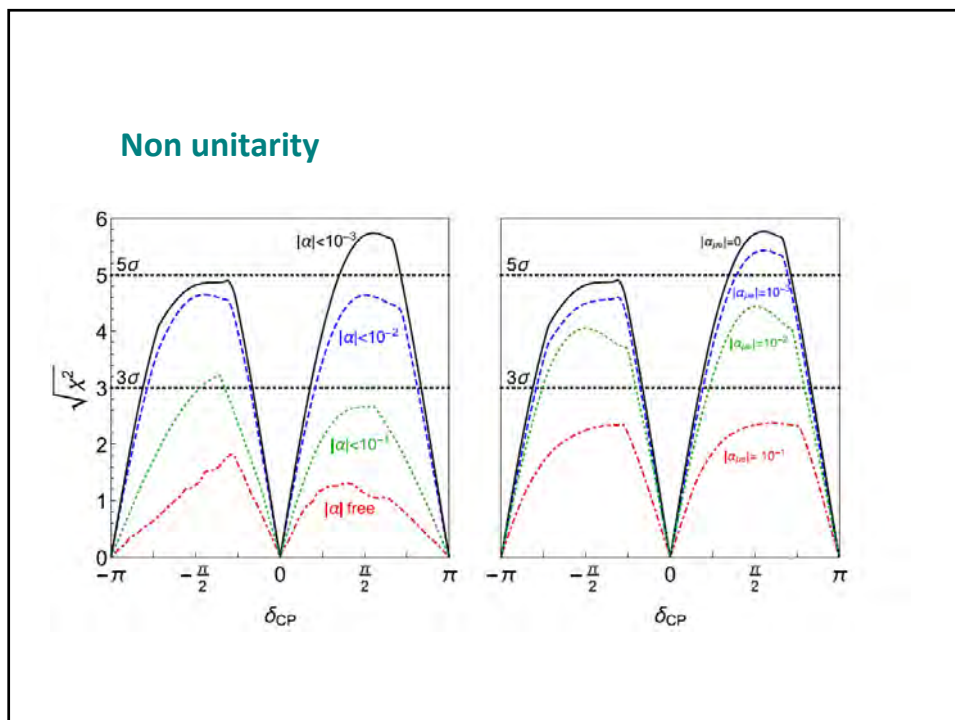
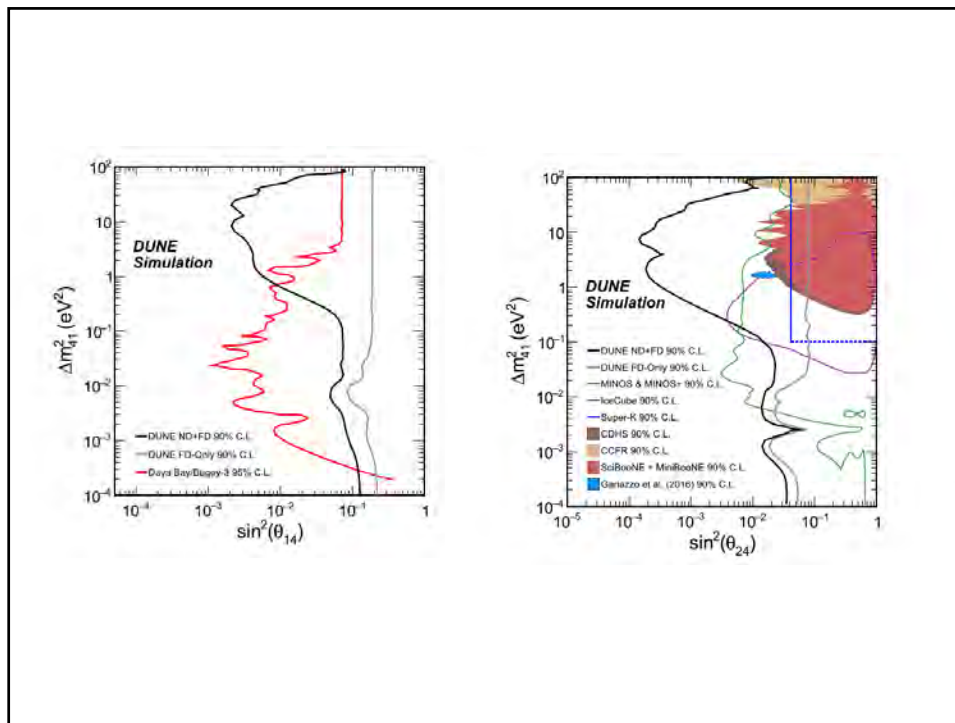
- First oscillation maximum:

$$\frac{\Delta m_{31}^2 L}{4E} \sim \frac{\pi}{2} \Rightarrow E_\nu > 2 \text{ GeV}$$

- **Unfold CPV from Matter Effects through E dependence**

⇒ **On-axis beam**: wide range of neutrino energies





Neutrinos,
In and Beyond the Standard Model:

NEUTRINO MASS:

$$\delta m_{atm}^2 = 2.7_{-0.3}^{+0.4} \times 10^{-3} eV^2$$

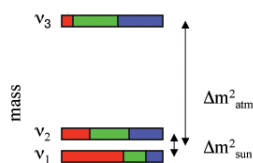
$$L/E = 500 \text{ km/GeV}$$

$$\delta m_{solar}^2 = 8.0 \pm 0.4 \times 10^{-5} eV^2$$

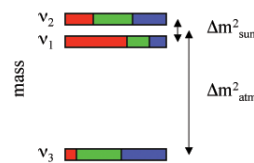
$$L/E = 15 \text{ km/MeV}$$



$$m_{\nu}^{Heavy} > \sqrt{\delta m_{atm}^2} = 50 \text{ meV}$$

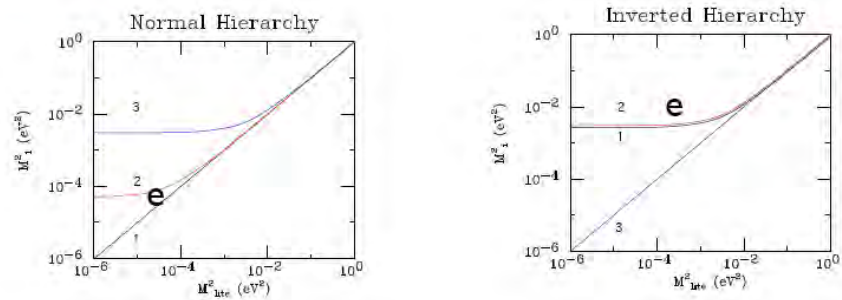


Normal mass hierarchy

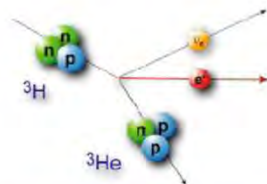
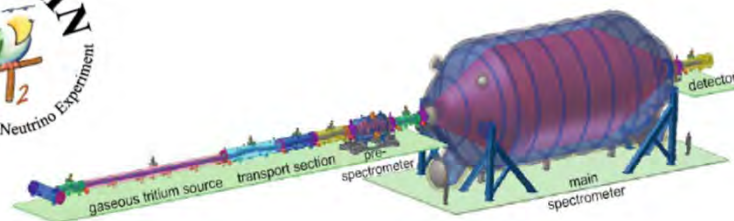


Inverted mass hierarchy

Masses:



States 1 and 2 are ν_e rich.



Requirements:

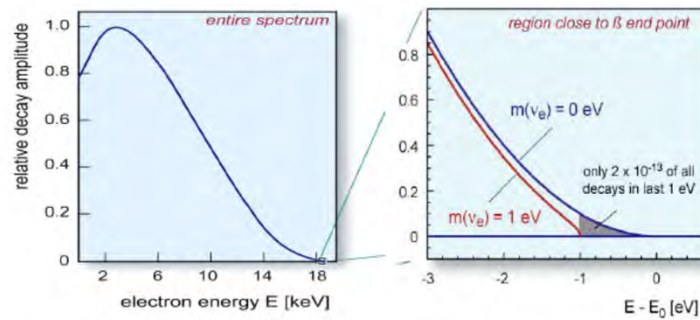
- Strong source
- Excellent energy resolution
- Small endpoint energy E_0
- Long term stability
- Low background rate

KATRIN Task:

Investigate Tritium endpoint with sub-eV precision

KATRIN Aim:

Improve m_ν sensitivity $10 \times (2\text{eV} \rightarrow 0.2\text{eV})$



Decay Rate:

$$|\langle {}^3\text{He} + e^- + \bar{\nu} | T | {}^3\text{H} \rangle|^2 \sim pE(E_0 - E) \sum_k |U_{ek}|^2 \sqrt{(E_0 - E)^2 - m_k^2}$$

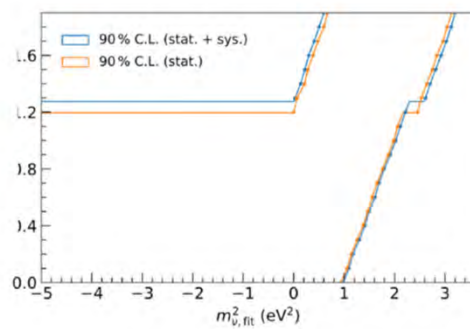
if ν 's quasi-degenerate: $m_1 \approx m_2 \approx m_3$

$$|\langle {}^3\text{He} + e^- + \bar{\nu} | T | {}^3\text{H} \rangle|^2 \sim pE(E_0 - E) \sqrt{(E_0 - E)^2 - m_\nu^2}$$

- KATRIN upper limit on neutrino mass:

LT $m(\nu) < 1.1 \text{ eV (90\% CL)}$

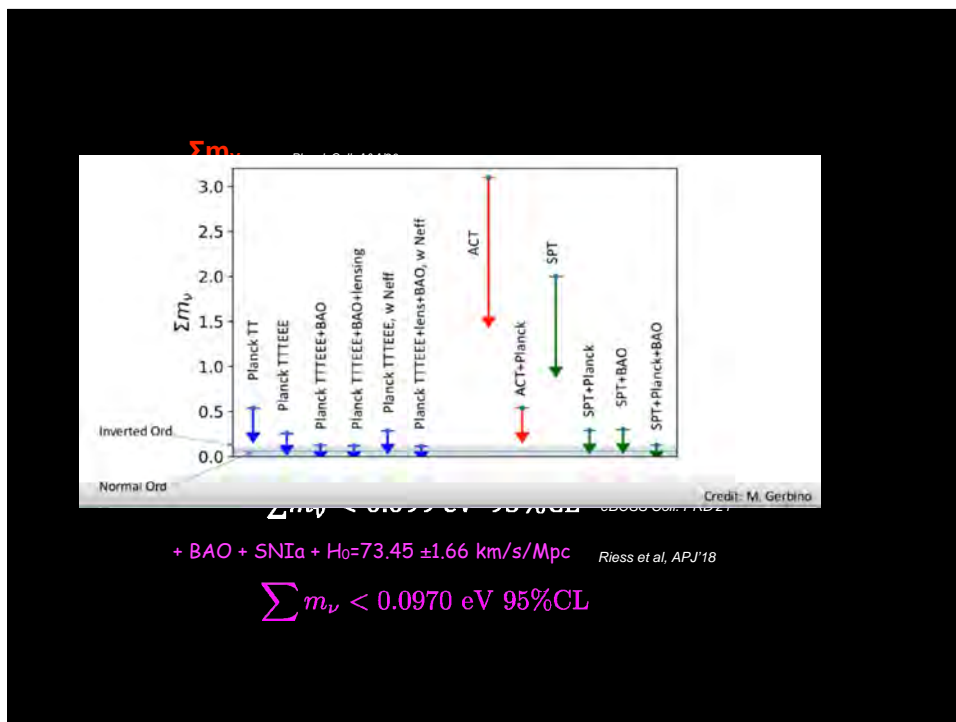
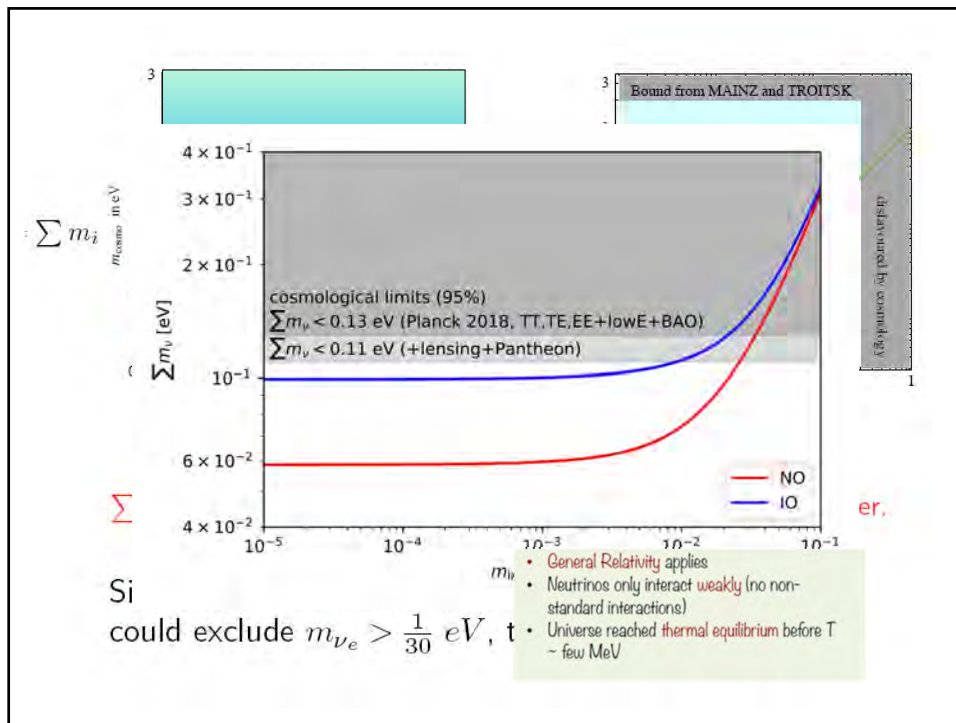
FC $m(\nu) < 0.8 \text{ eV (90\% CL)}$
 $< 0.9 \text{ eV (95\% CL)}$



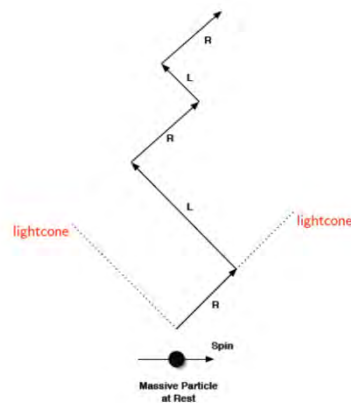
- 1000 days of measurements at
 nominal pd ($5 \cdot 10^{17}$ molecules cm^{-2})
 3 tritium campaigns (65 days each)
 per calendar year

sensitivity $m(\nu_e) = 0.2 \text{ eV (90\% CL)}$

0.35 eV (5σ)



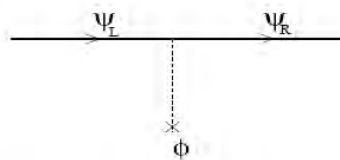
What is Fermion Mass ???



A mass can be thought of as a $L \leftrightarrow R$ transition:

$$m \bar{\psi}_L \psi_R + h.c.$$

In the SM fermion masses originate in the interaction with the Higgs field:



$$\lambda_f \bar{\psi}_L \Phi \psi_R + h.c. \rightarrow m_f = \lambda_f v$$

Fermion Masses:

	electron	positron	
Left Chiral	e_L	\bar{e}_R	$SU(2) \times U(1)$
Right Chiral	e_R	\bar{e}_L	$U(1)$

CPT: $e_L \leftrightarrow \bar{e}_R$ and $e_R \leftrightarrow \bar{e}_L$

Mass couples L to R:

e_L to e_R AND also \bar{e}_R to \bar{e}_L Dirac Mass terms.

Mass couples L to R:

$$P^2 = M^2, \quad P \cdot S = 0 \quad \text{and} \quad S^2 = -1$$

$$u(P, S) = \frac{(1 + \gamma_5)}{2} u\left(\frac{P + MS}{2}\right) + e^{i\phi} \frac{(1 - \gamma_5)}{2} u\left(\frac{P - MS}{2}\right)$$

right massless left massless

A coupling of e_L to \bar{e}_R OR e_R to \bar{e}_L would be (Majorana) mass term but this violates conservation of electric charge!

Seesaw / Dirac Neutrinos / Light Sterile Neutrinos

	Nu	CPT:	Anti-Nu	
Left Chiral	ν_L	\Leftrightarrow	$\bar{\nu}_R$	
	\Uparrow		\Uparrow	Dirac Masses
Right Chiral	ν_R	\Leftrightarrow	$\bar{\nu}_L$	
		Majorana Masses		

Coupling of

- ν_L to ν_R AND $\bar{\nu}_R$ to $\bar{\nu}_L$ are the Dirac masses.
- ν_L to $\bar{\nu}_R$ forbidden by weak isospin.
- ν_R to $\bar{\nu}_L$ allowed and coefficient is unprotected. ($\rightarrow M$)

$$\begin{array}{ccc}
 \nu_L \text{ to } \bar{\nu}_R & & \nu_L \text{ to } \nu_R \\
 \swarrow & & \swarrow \\
 \left(\begin{array}{cc} 0 & m_D \\ m_D & M \end{array} \right) & & \\
 \nwarrow & & \nwarrow \\
 \bar{\nu}_R \text{ to } \bar{\nu}_L & & \nu_R \text{ to } \bar{\nu}_L
 \end{array}$$

Two Majorana neutrinos
with masses m_D^2/M and M

Seesaw:
Yanagida, Gell-man-
Ramond-Slansky

- Coupling of ν_R to $\bar{\nu}_L$ allowed and coefficient is unprotected. ($\rightarrow M$)

Also applies to sterile neutrinos.

Light Sterile Neutrinos and/or Dirac Neutrinos Unexpected!!!

The consequences of this alternative are profound:

- Physics beyond the SM at a scale M !
- Majorana fermions carry no conserved charge: L is violated !

$$\nu_L \rightarrow e^{i\alpha} \nu_L$$

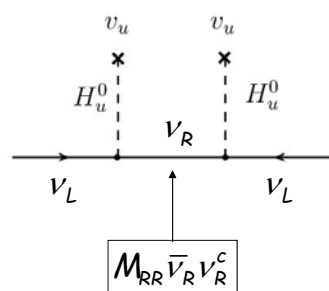
does not leave the Majorana mass term invariant.

→ Most welcome for **baryogenesis**: a mechanism to understand the matter-antimatter asymmetry in the Universe emerges naturally

→ Most welcome by **string theory**: it is difficult to get global $U(1)$ charges conserved

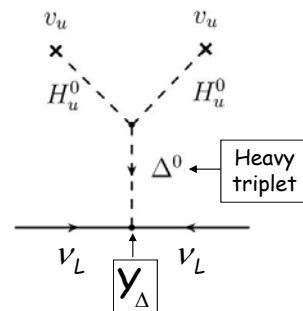
Types of see-saw mechanism

Type I see-saw mechanism



$$m_{LL}^I \approx -m_{LR} M_{RR}^{-1} m_{LR}^T$$

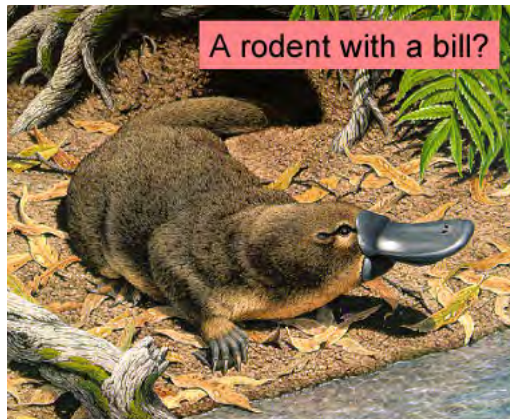
Type II see-saw mechanism



$$m_{LL}^{II} \bar{\nu}_L \nu_L^c \approx Y_{\Delta} \frac{v_u^2}{M_{\Delta}}$$

Naturalness may be over rated ...

Does this look natural ??



How Can We Demonstrate That $\bar{\nu}_i = \nu_i$?

We assume neutrino **interactions** are correctly described by the SM. Then the **interactions** conserve L ($\nu \rightarrow \ell^-$; $\bar{\nu} \rightarrow \ell^+$).

An Idea that Does Not Work
[and illustrates why most ideas do not work]

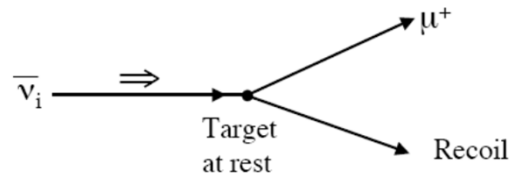
Produce a ν_i via—



Give the neutrino a Boost:
 $\beta_\pi(\text{Lab}) > \beta_\nu(\pi \text{ Rest Frame})$



The SM weak interaction causes —



$\nu_i = \bar{\nu}_i$ means that $\nu_i(h) = \bar{\nu}_i(h)$.

helicity

If $\nu_i \Rightarrow = \bar{\nu}_i \Rightarrow$,
our $\nu_i \Rightarrow$ will make μ^+ too.

Minor Technical Difficulties

$$\beta_{\pi}(\text{Lab}) > \beta_{\nu}(\pi \text{ Rest Frame})$$

$$\Rightarrow \frac{E_{\pi}(\text{Lab})}{m_{\pi}} > \frac{E_{\nu}(\pi \text{ Rest Frame})}{m_{\nu}}$$

$$\Rightarrow E_{\pi}(\text{Lab}) > 10^4 \text{ TeV} \quad \text{if } m_{\nu} \sim 1 \text{ eV}$$

Fraction of all π -decay that get helicity flipped

$$\approx \left(\frac{m_{\nu}}{E_{\nu}(\pi \text{ Rest Frame})} \right)^2 \sim 10^{-16} \quad \text{if } m_{\nu} \sim 1 \text{ eV}$$

For Majorana Neutrinos

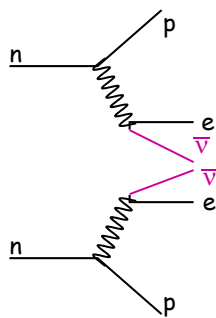


Not Observed

Allowed

BUT Suppressed by $\frac{m_\nu^2}{E^2} \sim 10^{-20}$!!!

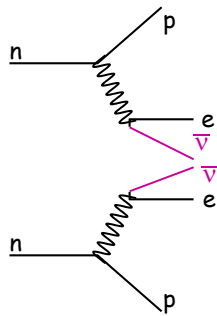
➤ How we can find out ?



SM double weak process

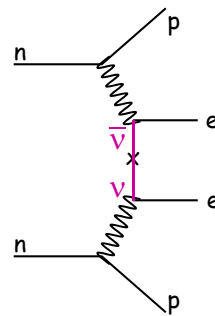
4 body decay: continuous
spectrum for the e
energy sum

➤ How we can find out ?



SM double weak process

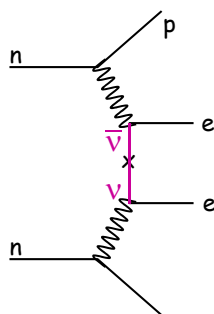
4 body decay: continuous spectrum for the e energy sum



Only allowed for Majorana ν

2 body decay: e energy sum is a delta

$\bar{\nu}_i$ is emitted (RH + $\mathcal{O}(m_i/E)$ LH)



$\text{Amp}[\nu_i \text{ contribution}] \sim m_i$

$$\text{Amp}[0\nu\beta\beta] \propto \left| \sum m_i U_{ei}^2 \right|$$

effective mass

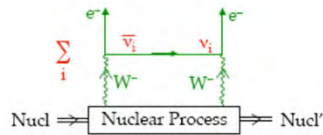
Neutrinoless double beta decay

- Most sensitive (terrestrial) probe of the absolute neutrino mass
- Unique way of proving Majorana nature of ν
- If Majorana ν is the only mechanism, \implies



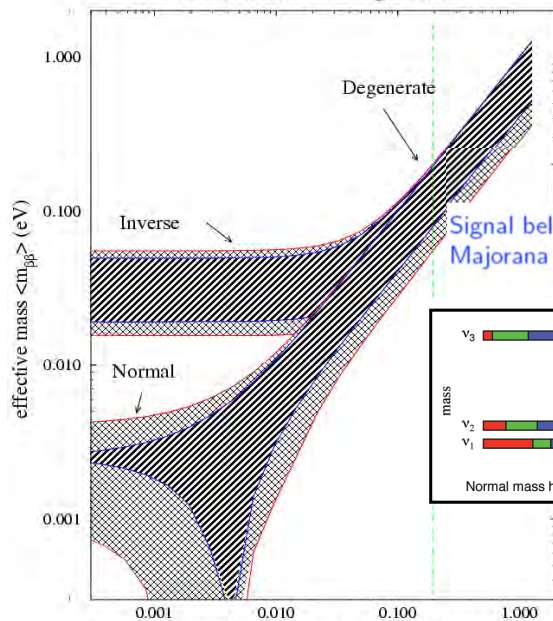
$$\langle m \rangle_{\beta\beta} \equiv \left| \sum_{i=1}^3 m_i U_{ei}^2 \right|$$

$$= \left| m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{2i\beta} + m_3 s_{13}^2 e^{2i(\gamma-\delta)} \right|$$



Effective neutrino mass in $0\nu\beta\beta$ decay

LMA solution, crosshatched region with errors



$$m_{\beta\beta} = \left| \sum m_i U_{ei}^2 \right|$$

dividing point $m_{\beta\beta} \approx 10\text{ meV}$

Signal below $\sim 10\text{ meV}$ would imply Majorana and Normal Hierarchy!

