Excited States in Nucleon Matrix Elements

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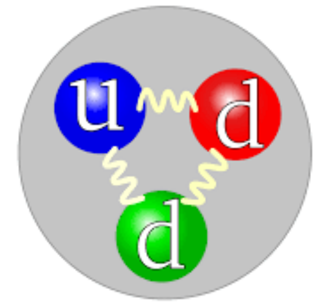

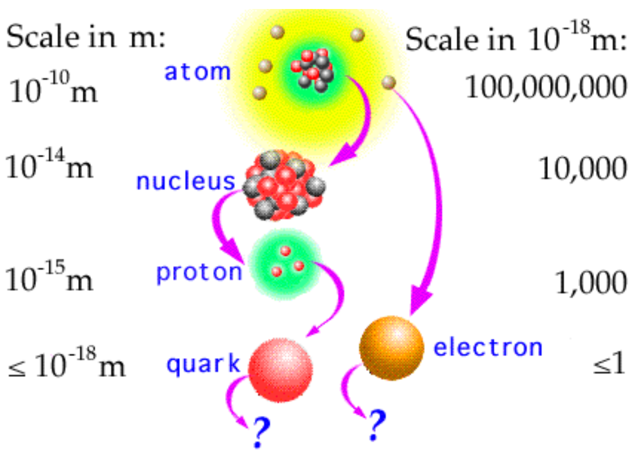

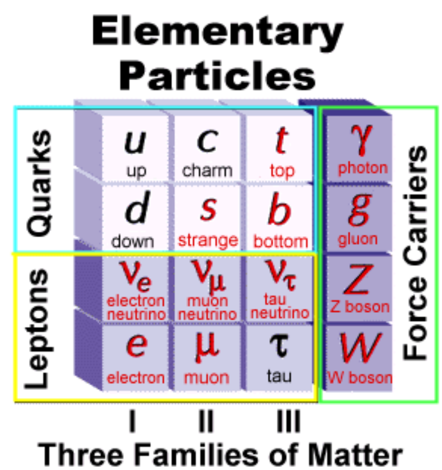

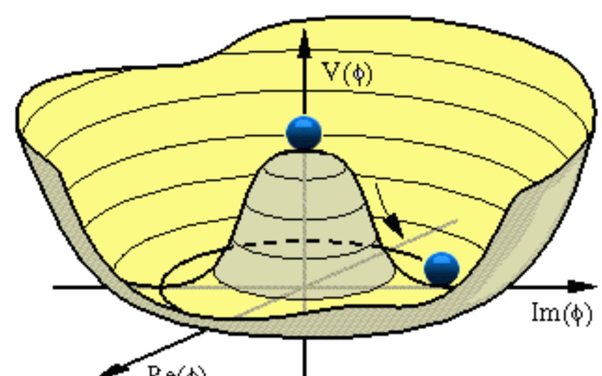

LA-UR: 16-29009, 17-23678, 18-25335, 19-25275

Benasque, 13 September 2022

PNDME Collaboration: **Thirteen 2+1+1-flavor HISQ ensembles = clover-on-HISQ formulation**

NME Collaboration: **Thirteen 2+1-flavor clover ensembles = clover-on-clover formulation**

PNDME and NME members

- Tanmoy Bhattacharya (T-2)
- Vincenzo Cirigliano (T-2 → **INT, UW)**
- Rajan Gupta (T-2)
- Emanuele Mereghetti (T-2)
- Boram Yoon (CCS-7)
- Junsik Yoo (PD: 2022 May –)
- Yong-Chull Jang (PD: 2017-2018)
- Sungwoo Park (PD: 2018-2021)
- Santanu Mondal (PD: 2019-2021)
- Huey-Wen Lin (MSU)
- Balint Joo (ORNL)
- Frank Winter (Jlab)

References

• Charges:	Gupta et al, PRD.98 (2018) 034503
• AFF:	Gupta et al, PRD 96 (2017) 114503
• AFF:	Jang et al, PRL 124 (2020) 072002
• VFF:	Jang et al, PRD 100 (2020) 014507
• $\sigma_{\pi N}$	Gupta et al, PRL 127 (2021) 242002
• d_n from Θ-term	Bhattacharya etal, PRD 103 (2021) 114507
• d_n from qEDM	Gupta et al, PRD 98 (2018) 091501
• Moments of PDFs	Mondal et al, PRD 102 (2020) 054512
• Proton spin:	Lin et al, PRD 98 (2018) 094512
NME	
• Charges, FF:	Park et al, PRD 105 (2022) 054505
• Moments of PDFs	Mondal et al, JHEP 04 (2021) 044

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USQCD
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Lattice QCD is the best-known method for non-perturbative calculations of

- Properties of quarks, gluons and hadrons

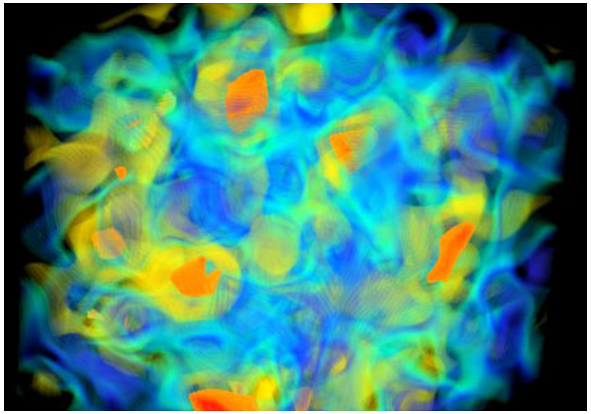
- QCD corrections to weak and electromagnetic processes

- QCD corrections to beyond the standard model processes

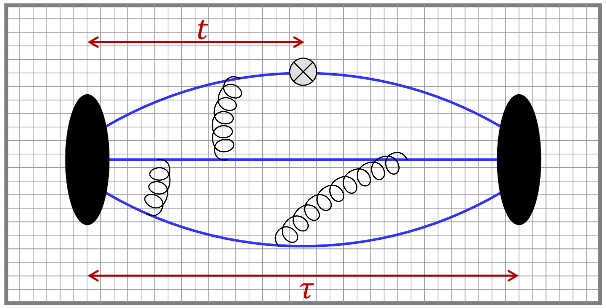
GOAL: Elucidate nucleon structure and decays using large scale simulations of lattice QCD.
Calculate the matrix elements of quark and gluon operators within the nucleon state.

Simulations of LQCD turn a Quantum Field Theory (QCD) into a stocastic computational problem. They provide

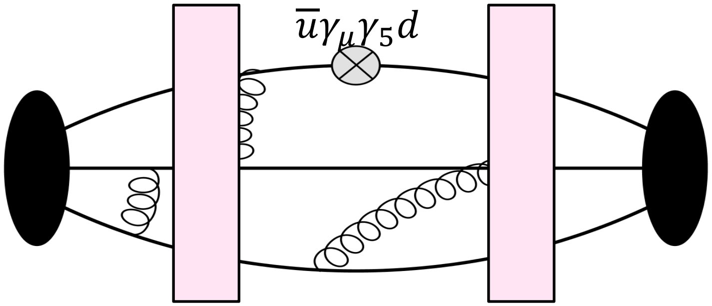
- The quantum vacuum of QCD
  - ensembles of gauge configurations



- Hadrons & interactions put in as external probes
  - N-point correlation functions

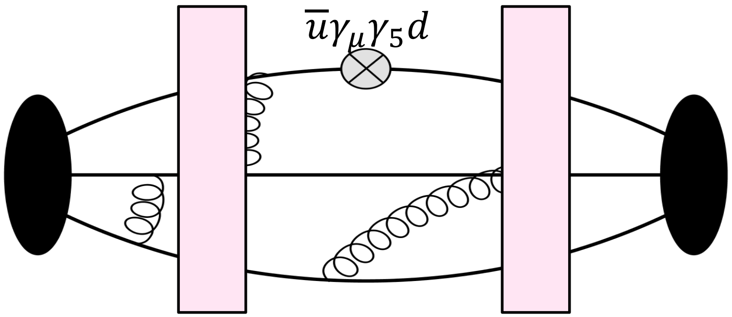


- Quantum wavefunctions of hadronic states
  - Matrix elements: $\langle N(p_f) | O\ (Q^2) | N(p_i) \rangle$



LQCD Methodology

- Generate gauge configurations
- Calculate quark propagator $S_F = \frac{1}{D}$ and construct hadronic states
- Isolate ground state wavefunctions $|N(p_i)\rangle$
- Formulate operators that best probe the physics
  - Low energy effective operators encapsulating SM & BSM physics
  - Examples: Axial, scalar, tensor and vector quark bilinears $(O = \bar{q}\ \Gamma_i\ q)$, …
- Calculate matrix elements: $\langle N(p_f) \big| O\ (Q^2) \big| N(p_i) \rangle$



Ingredients and Challenges

- Generate gauge configurations
  - New ideas: normalizing flows (autocorrelations, topology,)
- Calculate quark propagator $S_F = \frac{1}{D}$ and construct hadron correlators
  - Multigrid is very efficient
- Statistics: signal in correlation functions
  - Signal to noise (S2N) for nucleons degrades exponentially $e^{-(M_N - 1.5 M_\pi)\tau}$
- Isolate ground state wavefunctions $|N(p_i)\rangle$
  - Need to control large excited state contamination in all NME
- Formulate operators that best probe the physics
  - O(a) Improvement, Renormalization, Mixing
- Calculate matrix elements: $\langle N(p_f) | O\ (Q^2) | N(p_i) \rangle$
  - Contractions (improved operators, variational, multihadron states) expensive

Rich Landscape of LQCD calculations

HEP

- ϵ, ϵ' in kaon CP violation
- s, c, b decay form factors
- g-2
- α_s, quark masses

NP

- Nucleon structure, form factors
- Matrix elements within Nuclei
- Parton distribution functions PDFs
- Hadron Spectroscopy

Physics objectives

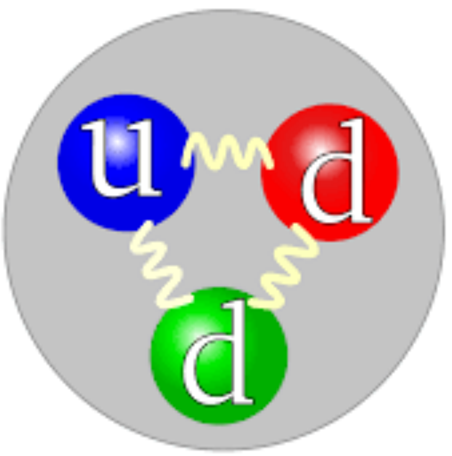
- Charges $g_A, g_S, g_T, \sigma_{\pi N}, \sigma_s$
- Axial & EM form factors
- Neutron electric dipole moment nEDM
- Neutrinoless double beta decay $0\nu\beta\beta$

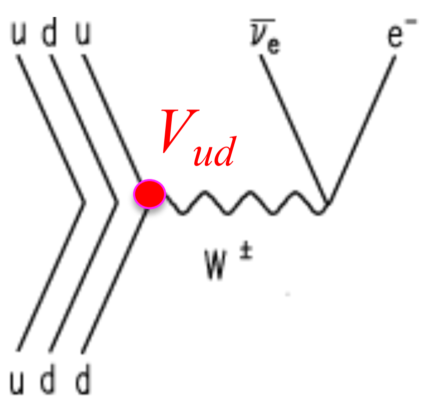
The neutron is a clean but challenging system

Decays weakly ⇒ a stable bound state of QCD

Properties:

- Charges g_A , g_P, g_S , g_T, g_V
- Spin content
  - Quarks
  - Gluons
- EDM
- Form factors
  - Electric, Magnetic
  - Axial
- Distribution functions, moments
  - PDF
  - GPD
- Radiative Corrections to decay





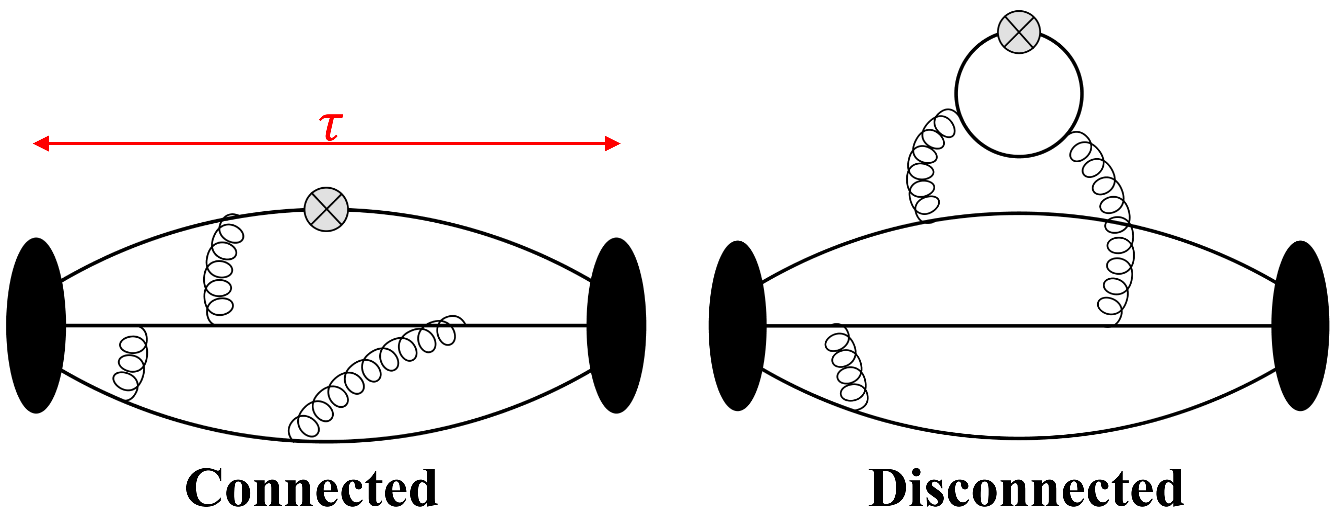
Physics from nucleon matrix elements

- Isovector charges g_A, g_S, g_T $\langle p|\bar{u}\Gamma d|n\rangle$
- Axial vector form factors $\langle p(q)|\bar{u}\gamma_\mu\gamma_5 d(q)|n(0)\rangle$
- Vector form factors $\langle p(q)|\bar{u}\gamma_\mu d(q)|n(0)\rangle$
- Flavor diagonal matrix elements $\langle p|\bar{q}q|p\rangle$
- nEDM: Θ-term, quark EDM, quark chromo EDM, Weinberg operator, 4-quark operators
- $0\nu\beta\beta$
- Generalized Parton Distribution Functions
- Radiative corrections to neutron decay

Lattice Methodology well established

for "connected" and "disconnected" 3-point correlation functions

disconnected contributions are noisier (stochastic method) for the same computational cost and smaller in value



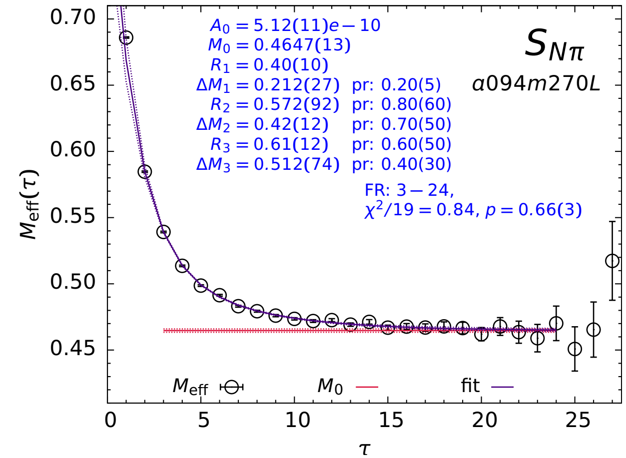
Isoscalar $$g_{A,S,T}^{u+d} = g_{A,S,T}^{u+d,\mathrm{conn}} + 2g_{A,S,T}^{l,disc}$$

Isovector $$g_{A,S,T}^{u-d} = g_{A,S,T}^{u-d,\mathrm{conn}}$$

In the isospin symmetric limit

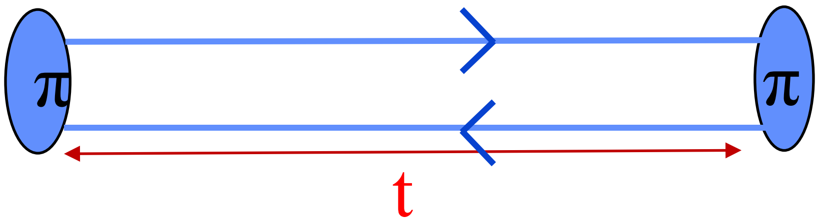
Challenges for Nucleons

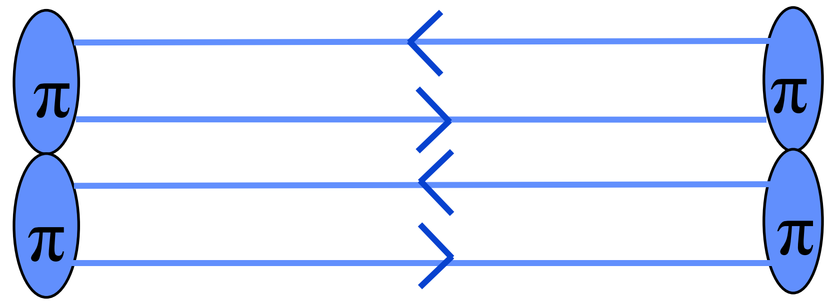
- S2N in all nucleon correlations degrades as $e^{-(M_N - 1.5 M_\pi)\tau}$
  Current extent of the signal
  - 2-pt: τ~ 2fm
  - 3-pt: τ~1.5fm



- $\widehat{N}$ couples to the nucleon, all its excitations and multi-hadron states $N\pi$, $N\pi\pi$, … with the same quantum numbers
- As $\vec{q} \rightarrow 0$, the tower of physical $N\pi$, $N\pi\pi$, … states becomes arbitrarily dense starting at ~1210 MeV
- The excited states that give significant contributions to a given ME are not known *a priori*

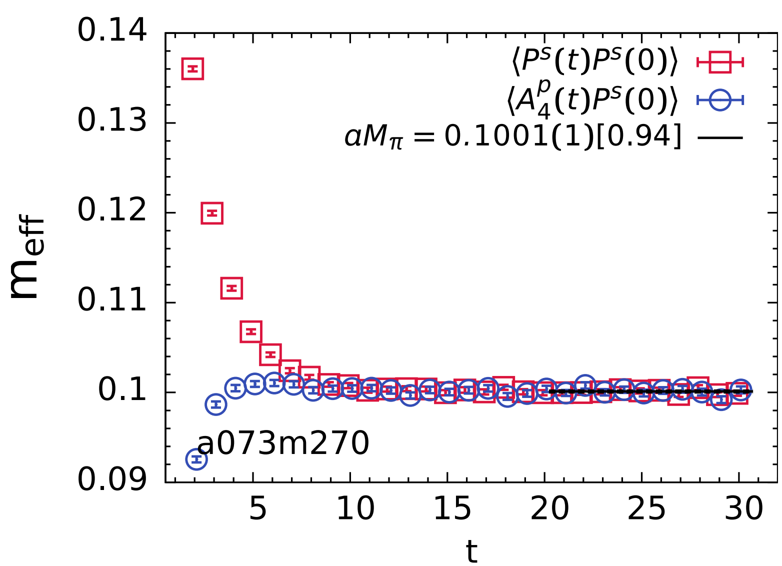
Signal-to-noise (S2N) in pion’s 2-point function





Signal: $\Gamma^2 \sim e^{-E_\pi t}$

Variance: $e^{-2E_\pi t}$

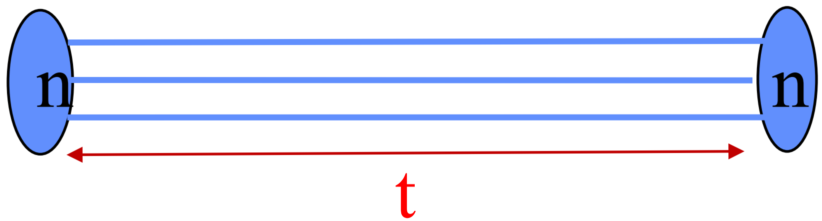


$$M_{eff}(t) = \ln \frac{\Gamma^2(t)}{\Gamma^2(t+1)}$$

S2N is a constant, ie, it does not degrade with *t*

The mass gap is large

Nucleon spectrum from 2-point function $\Gamma^2(t) = \langle\Omega|\overline{N}\ \mathrm{N}|\Omega\rangle$



$$\hat{N} = \epsilon^{abc}\left[q_1^{aT}(x)C\gamma_5\frac{(1\pm\gamma_4)}{2}q_2^b(x)\right]q_1^c(x)$$

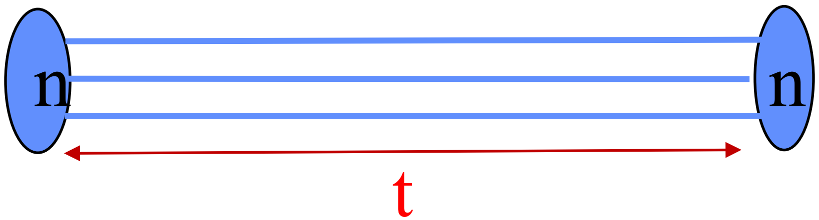
Spectral decomposition has same form as for the pion

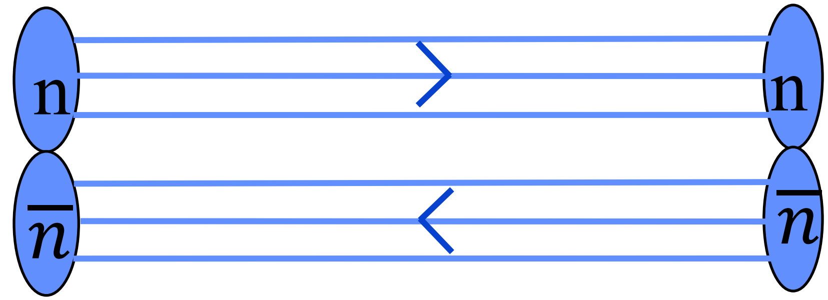
$$\Gamma^2(t) = |A_0|^2 e^{-M_0 t} + |A_1|^2 e^{-M_1 t} + |A_2|^2 e^{-M_2 t} + |A_3|^2 e^{-M_3 t} + \ldots.$$

Fit the data for $\Gamma^2(t)$ versus t to extract

M_0, M_1, … masses of the ground & excited states
A_0, A_1, … corresponding amplitudes for creating/annihilating states

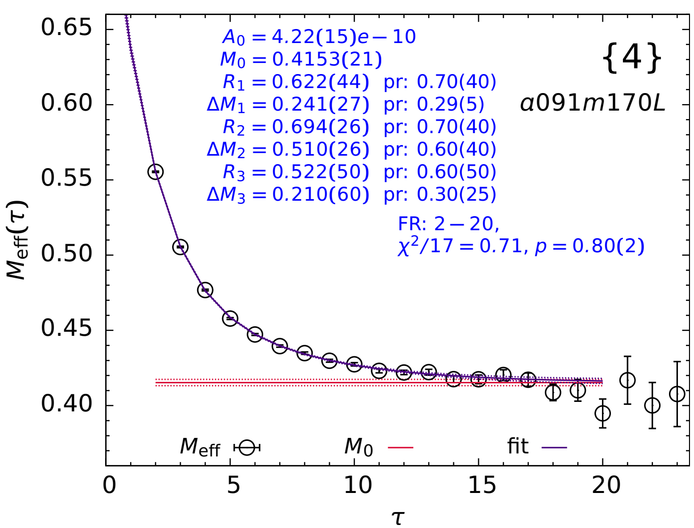
Signal-to-noise in the nucleon 2-point function $\boldsymbol{\Gamma^2}$





Signal: $\Gamma^2 = e^{-E_N t}$

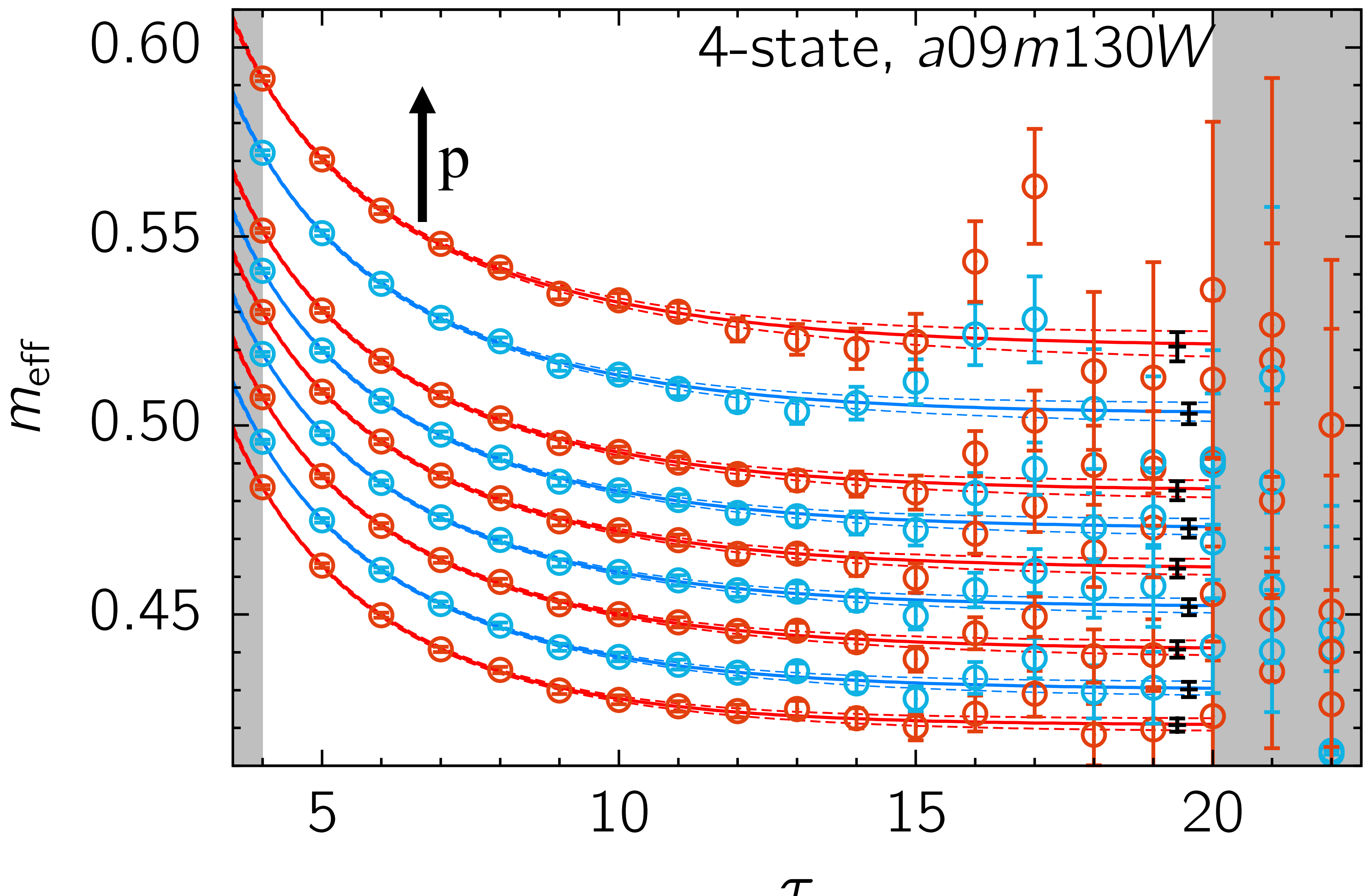
Variance: $e^{-3E_\pi t}$



$$M_{eff}(t) = \ln \frac{\Gamma^2(t)}{\Gamma^2(t+1)}$$

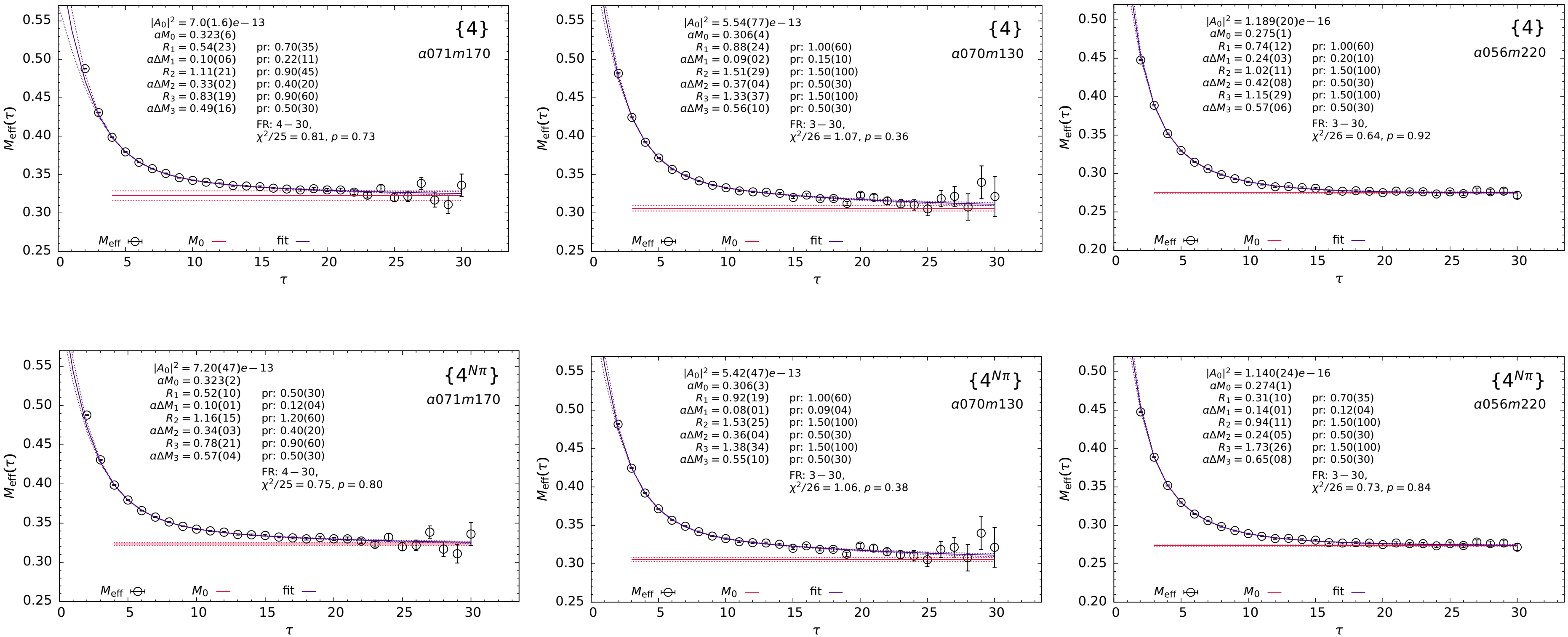
- The S2N degrades exponentially $e^{-(M_N - 1.5M_\pi)t}$
- To resolve a *small* mass gap *($M_1 - M_0$) requires* large t

4-state fits for $\Gamma^2(\vec{p}, t)$



Signal degrades with t and p

4-state fits to 2-point function



a = 0.071 fm
M_π = 170 MeV
$72^3 \times 192$ lattices

a = 0.070 fm
M_π = 130 MeV
$96^3 \times 192$ lattices

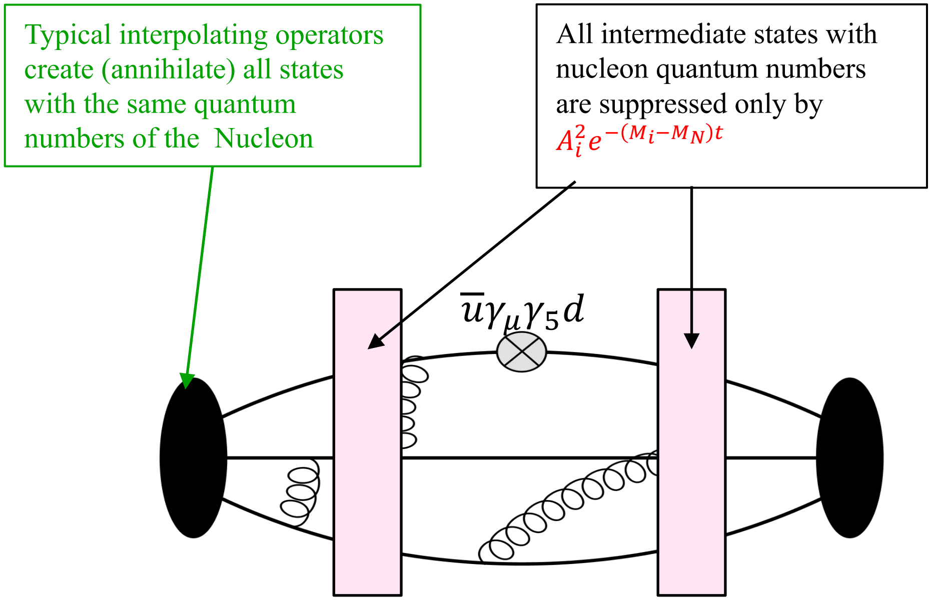
a = 0.056 fm
M_π = 220 MeV
$72^3 \times 192$ lattices

Surprise: ΔM_1 consistent with $N\pi$

Large region of $E_{i>0}$ values give similar χ^2/dof

Excited states in correlation functions

Challenge: To get the matrix elements within ground state of hadrons (nucleons), the contributions of all excited states must be removed.



Towers of multihadron states

$N(\vec{p})\pi(-\vec{p})$

$N(0)\pi(\vec{p})\pi(-\vec{p})$

$N(\vec{p})2\pi(-\vec{p})$

$\cdots$

+ radial excitations

- Which excited states make significant contributions to a given matrix element?
- What are their energies in a finite box?

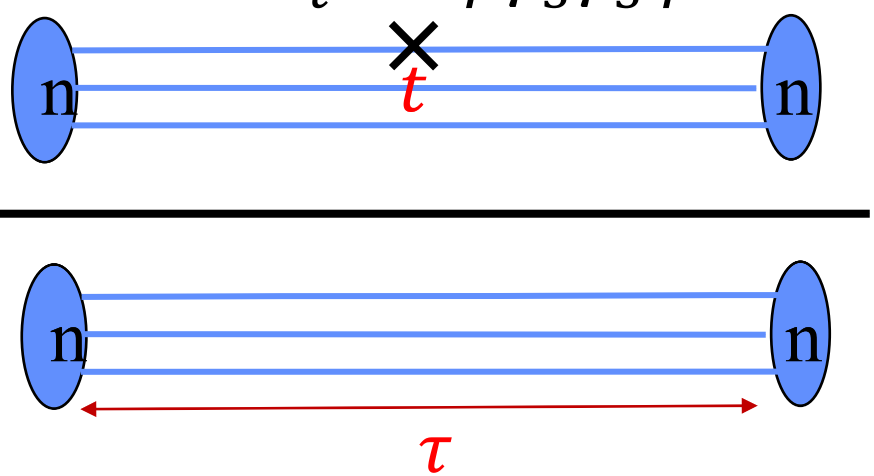
Calculating Nucleon Charges

$$\Gamma^2 = \sum_i A_i^* A_i \, e^{-E_i \tau} \qquad \Gamma^3 = \sum_{i,j} A_i^* A_j \langle N_i | O | N_j \rangle e^{-E_i t} \, e^{-E_j(\tau - t)}$$

$$\frac{\Gamma^3}{\Gamma^2} = \frac{\langle \Omega | \overline{N} \, \mathrm{A}_\mu \, \mathrm{N} | \Omega \rangle}{\langle \Omega | \overline{N} \, \mathrm{N} | \Omega \rangle} \to \langle N(p_f) | A_\mu \, (Q^2) | N(p_i) \rangle \to \boldsymbol{g_A}$$

$$O_t = \bar{\psi} \gamma_3 \gamma_5 \psi$$

$$\frac{\Gamma^3}{\Gamma^2} =$$

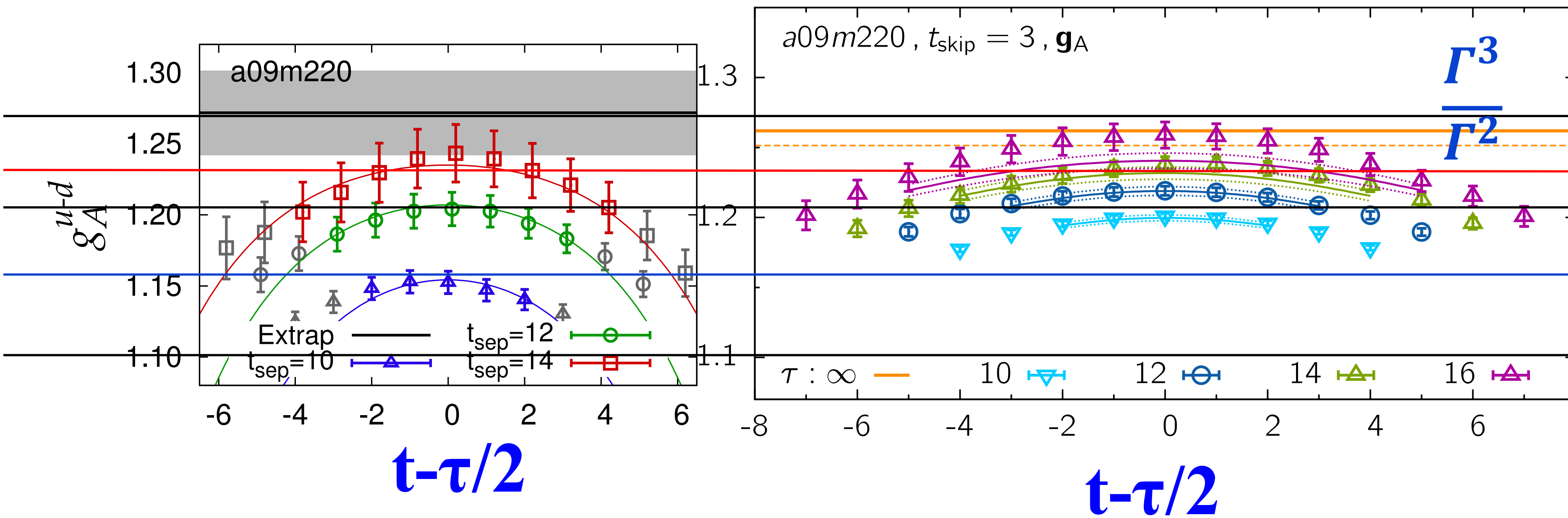


$$\to \boldsymbol{g_A}$$

Fits to Γ^3 taking the ΔE_i from Γ^2 "work"

What is current industry standard

- Better smearing reduces ESC
- Higher statistics (10K➞100K) with TSM
- 4-5 values of source-sink separations $\tau \lesssim 1.5$ fm
- 4-state fits to 2-point functions, 3-state fits to 3-point functions
- Full covariance error matrix



Yoon et al, PRD 93 (2016) 114506; Gupta et al, PRD98 (2018) 034503

Homework: Which excited states contribute and how to determine their energies?

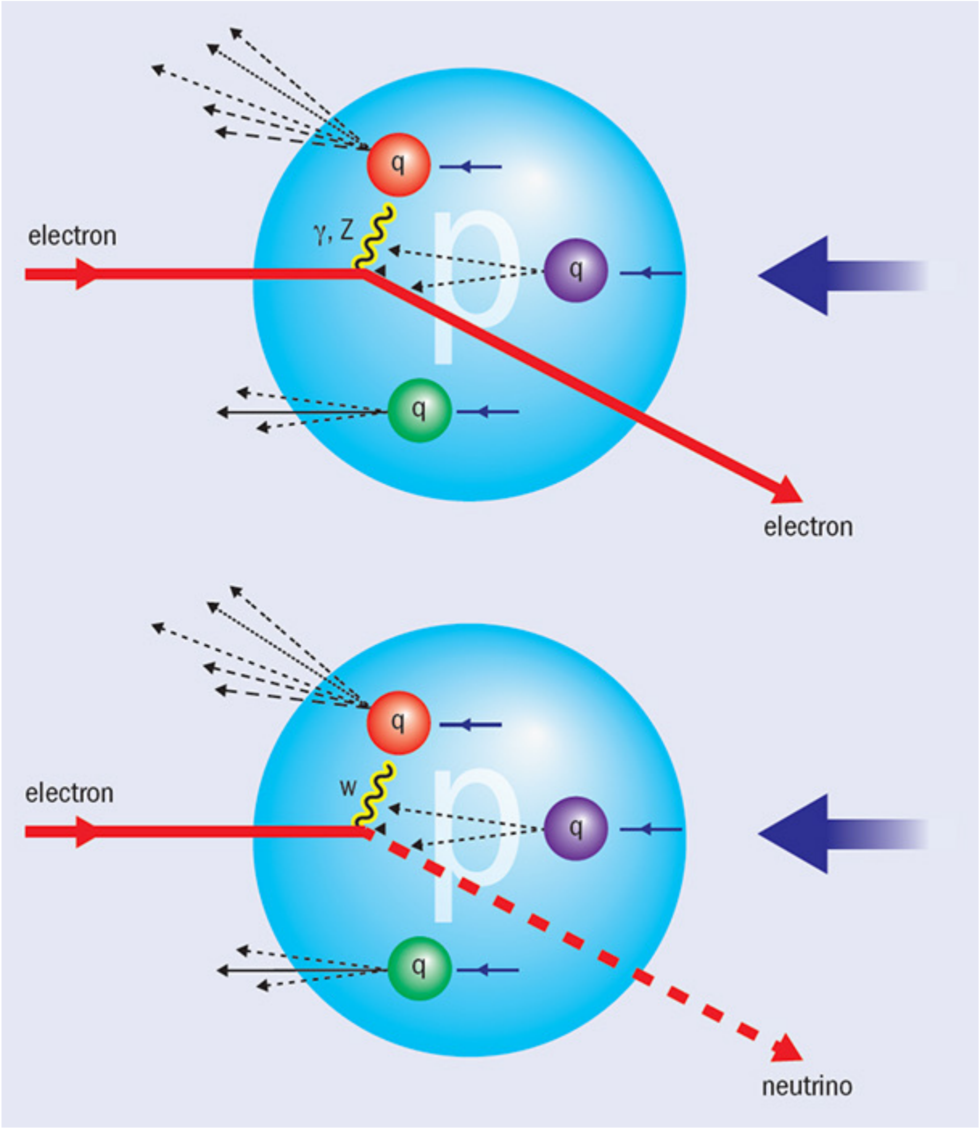
- $\widehat{N}$ couples to all excited states with nucleon quantum numbers
  - $N_p \pi_{-p}$
  - $N_0 \pi_0 \, \pi_0$
  - ……
  - *N(1410)*
- The spectrum of $N_p \pi_{-p}$, … becomes dense as $p \to 0$
- What are energies of these multihadron states in a finite box?
  - Only the $\boldsymbol{\Delta E_i}$ are needed, not the A_i
- Which of these states contribute to a given ME?
- Fits to 2-point function give large $\mathbf{\Delta E_i}$ (>*N(1410)*)
- 4-state fits to $\Gamma^2(t)$ give a large region in E_i with similar χ^2
- 3-state fits to $\Gamma^3(t)$ with these E_i work (χ^2 reasonable)
- 3-state fits to $\Gamma^3(t)$ with $\mathrm{E}_1 = E_{N\pi}$ work (χ^2 reasonable)

4 Examples: ESC in the determination of

- Axial vector form factors $\boldsymbol{G_A}$, $\boldsymbol{\widetilde{G}_P}$, $\boldsymbol{G_P}$
- The axial charge $\boldsymbol{g_A}$
- Contribution of the Θ-term to the neutron EDM
- The pion-nucleon sigma term $\boldsymbol{\sigma_{\pi N}}$

$$\Gamma^3 = \sum_{i,j} A_i^* A_j \langle N_i | O | N_j \rangle e^{-E_i t} \, e^{-E_j(\tau - t)}$$

Lepton-nucleon scattering



$G_E(Q^2)$

$G_M(Q^2)$

$G_A(Q^2)$

$\widetilde{G_P}(Q^2)$

The ν-n differential cross-section:

GOAL: High precision results for axial, electric and magnetic form factors versus Q^2 needed for determining x-section of (ν, e, μ) scattering off nuclei

$$\frac{d\sigma}{dQ^2}\begin{pmatrix} \nu_l + n \to l^- + p \\ \bar{\nu}_l + p \to l^+ + n \end{pmatrix}$$

$$= \frac{M^2 {G_F}^2 cos^2\theta_c}{8\pi {E_\nu}^2}\left\{A(Q^2) \pm B(Q^2)\frac{(s-u)}{M^2} + C(Q^2)\frac{(s-u)^2}{M^4}\right\},$$

$$\begin{aligned} A(Q^2) &= \frac{(m^2+Q^2)}{M^2}\Big[(1+\tau)F_A^2 - (1-\tau)F_1^2 + \tau(1-\tau)F_2^2 + 4\tau F_1 F_2 \\ &\quad -\frac{m^2}{4M^2}\left((F_1+F_2)^2 + (F_A+2F_P)^2 - 4\left(1+\frac{Q^2}{4M^2}\right)F_P^2\right)\Big], \\ B(Q^2) &= \frac{Q^2}{M^2}F_A(F_1+F_2), \\ C(Q^2) &= \frac{1}{4}(F_A^2 + F_1^2 + \tau F_2^2). \end{aligned}$$

$\langle N A_\mu N \rangle \to$ linear combination of F_A, $\tilde{F}_P$

$\langle NPN \rangle \to G_P$

$\langle N V_\mu N \rangle \to G_E$, G_M

F_A = axial form factor

$\tilde{F}_P$ = induced pseudoscalar

$G_E = F_1 - \tau F_2$ Electric

$G_M = F_1 + F_2$ Magnetic

$\tau = Q^2/4M^2$

$M = M_n = M_p \approx 939$ MeV

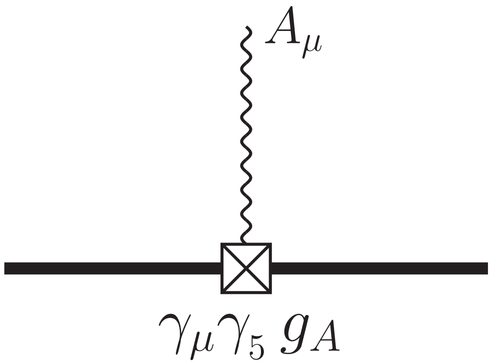
$m = M_\pi$

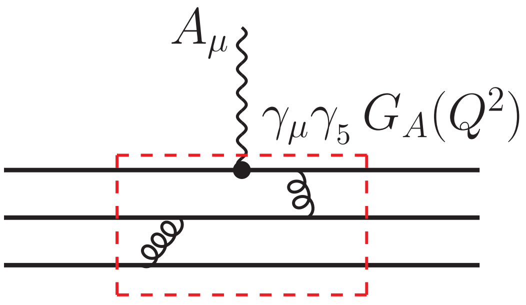
Cohesive strategy for (e, μ, ν)-Z scattering
☛5 Form Factors, g_A, μ, g_p^*

- $G_E(Q^2)$ Electric
- $G_M(Q^2)$ Magnetic
- $G_A(Q^2)$ Axial
- $\tilde{G}_P(Q^2)$ Induced pseudoscalar
- $G_P(Q^2)$ Pseudoscalar
- All 5 form factors are calculated together
- Precise experimental data exist for $G_E(Q^2)$ and $G_M(Q^2)$
- Axial ward identity relates $G_A(Q^2)$, $\tilde{G}_P(Q^2)$, $G_P(Q^2)$

- $G_E(Q^2 = 0)$ $= 1$ Conserved vector charge
- $G_M(Q^2 = 0)$ $= \mu = 4.7058$ Magnetic moment
- $G_A(Q^2 = 0)$ $= g_A = 1.277(2)$ Axial charge
- $\tilde{G}_P(Q^2 = 0.88 m_\mu^2) = g_p^* = 8.06(55)$ Induced pseudoscalar charge

Axial-vector form factors





$\propto 1/Q^4$

Calculate the 3 form factors on the lattice

- Axial: G_A
- Induced pseudoscalar: $\tilde{G}_P$
- Pseudoscalar: G_P

ground state matrix elements decomposed into G_A, $\tilde{G}_P$, G_P

$$\langle N(p_f)|A^\mu(q)|N(p_i)\rangle = \bar{u}(p_f)\left[\gamma^\mu G_A(q^2) + q_\mu \frac{\tilde{G}_P(q^2)}{2M}\right]\gamma_5 u(p_i)$$

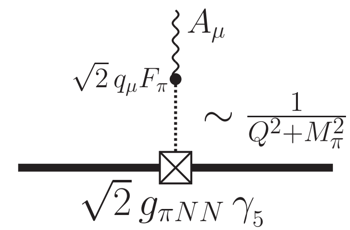
$$\langle N(p_f)|P(q)|N(p_i)\rangle = \bar{u}(p_f) G_P(q^2)\gamma_5\, u(p_i)$$

PCAC relation $\partial_\mu A_\mu = 2mP$ implies G_A, $\widetilde{G}_P$, G_P must satisfy

$$2\widehat{m}G_P(Q^2) = 2M_N G_A(Q^2) - \frac{Q^2}{2M_N}\tilde{G}_P(Q^2)$$

Pion pole-dominance (PPD) hypothesis states

$$\widetilde{G_P}(Q^2) = G_A(Q^2)\left[\frac{4M_N^2}{Q^2 + M_\pi^2}\right]$$



If pion pole-dominance holds
⇒ there is only one independent form factor

Goldberger-Trieman relation at $Q^2 = 0$

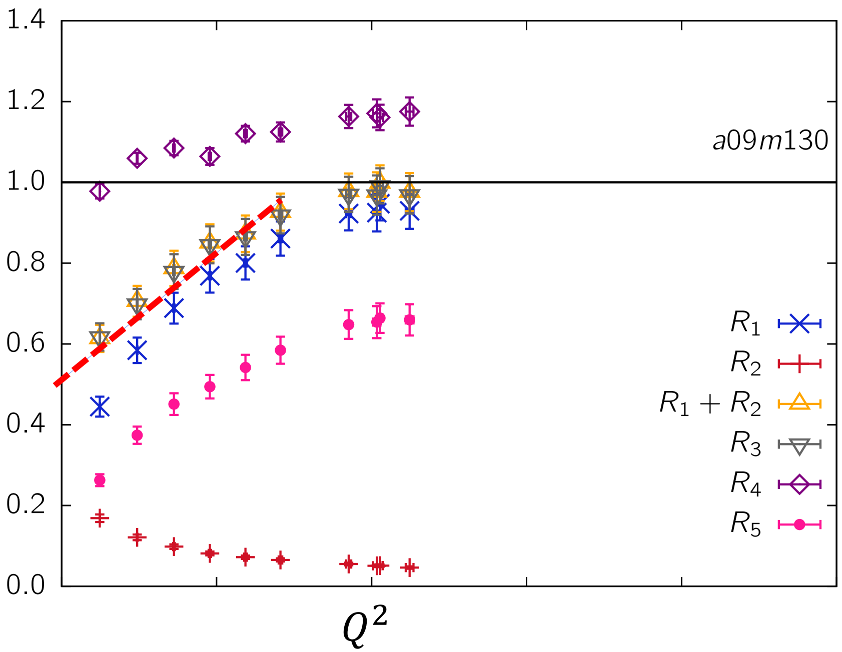
$$F_\pi \; g_{\pi NN} = M_N \, g_A$$

2017: Showed axial form factors with E_i from Γ^2 violate PCAC

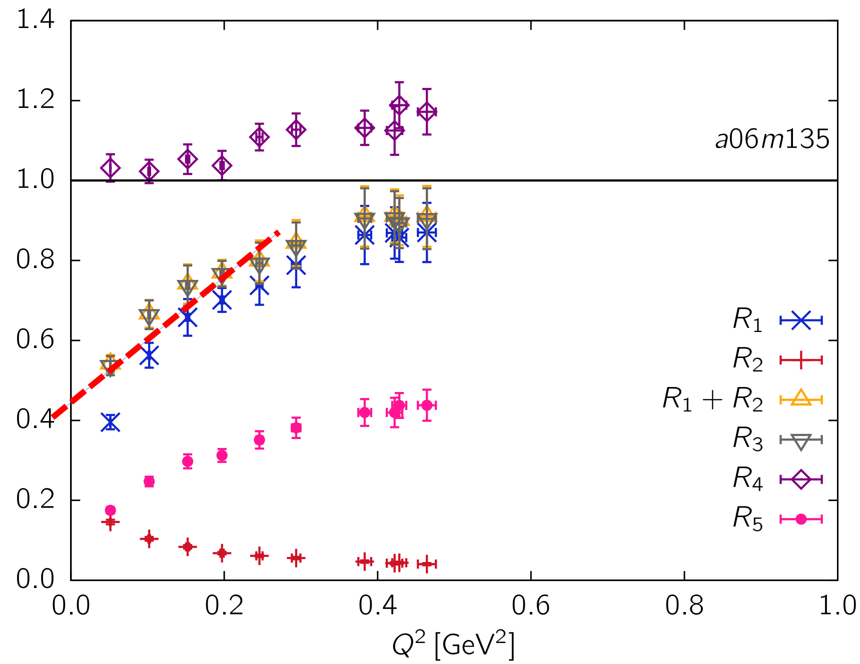
Gupta et al, PRD 96 (2017) 114503

$$\text{PCAC: } R_1 + R_2 = \frac{\hat{m}\, G_P}{M_N G_A} + \frac{Q^2 \tilde{G}_P}{4M_N^2} = 1$$

PCAC violated if one uses the spectrum from 2-point function

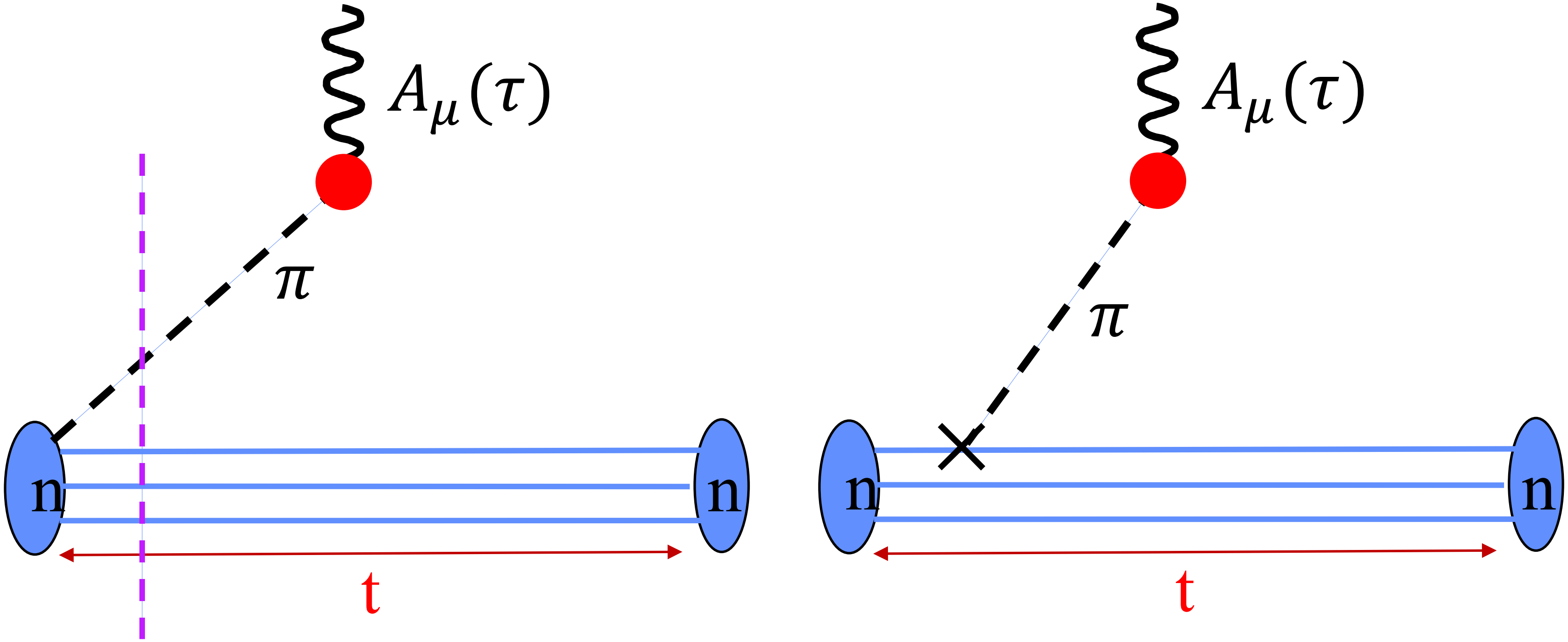


$a = 0.087\ fm$
$M_\pi = 138\ MeV$



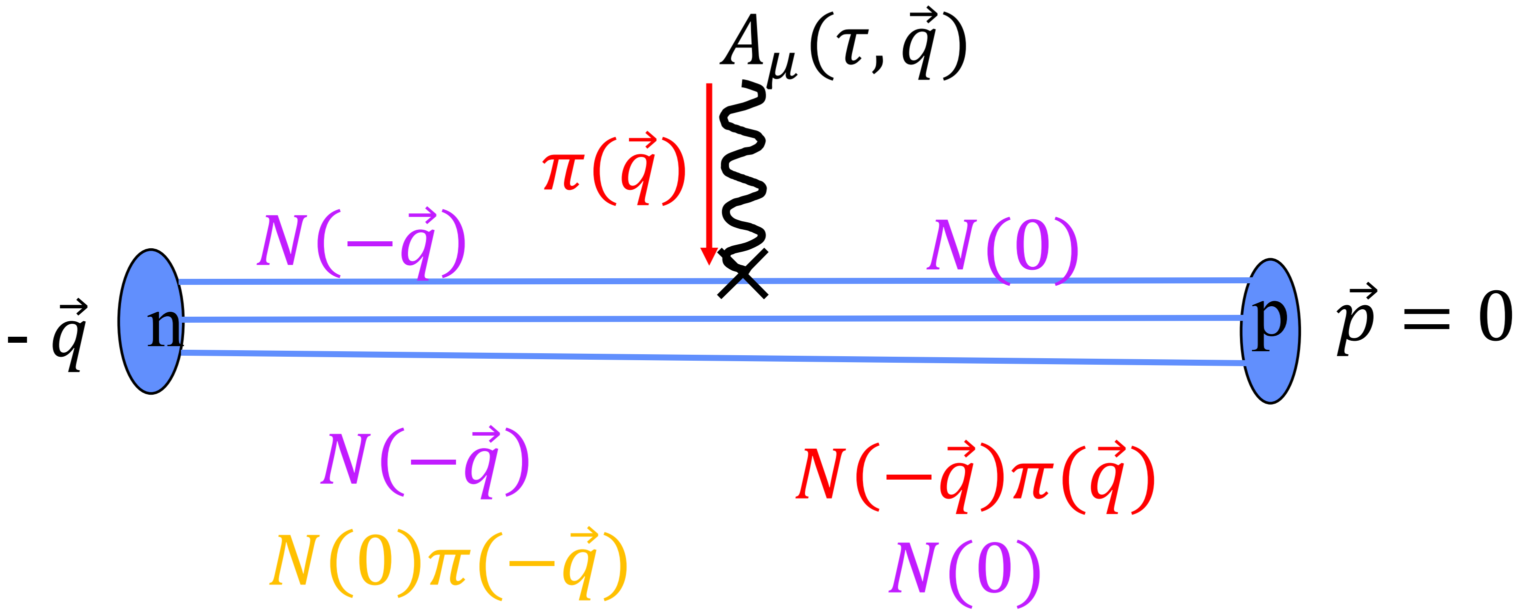
$a = 0.057\ fm$
$M_\pi = 136\ MeV$

$N\pi$ state couples in the axial channel

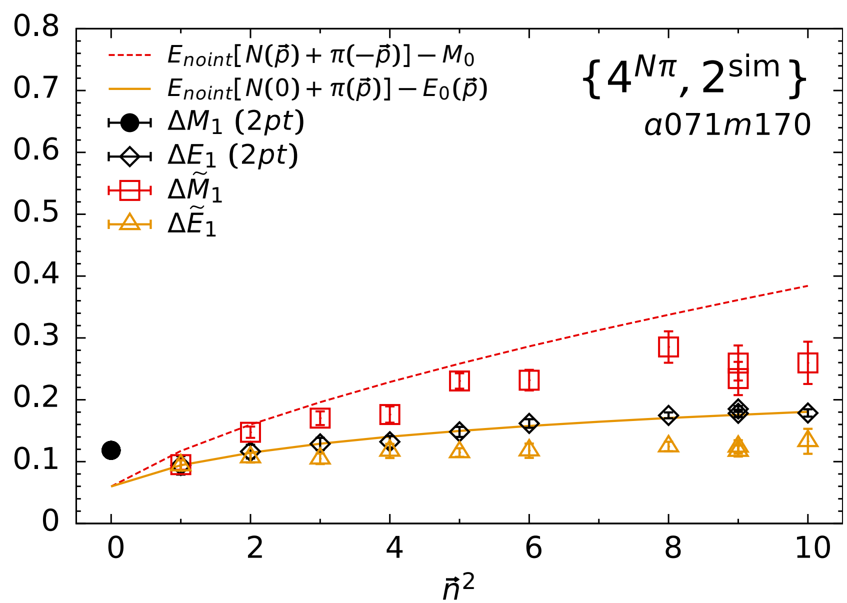


Enhanced coupling to $N\pi$ state: Since the pion is light, the vertex ● can be anywhere in the lattice 3-volume

$N\pi$ state in the axial channel



Extracted mass gaps match above picture



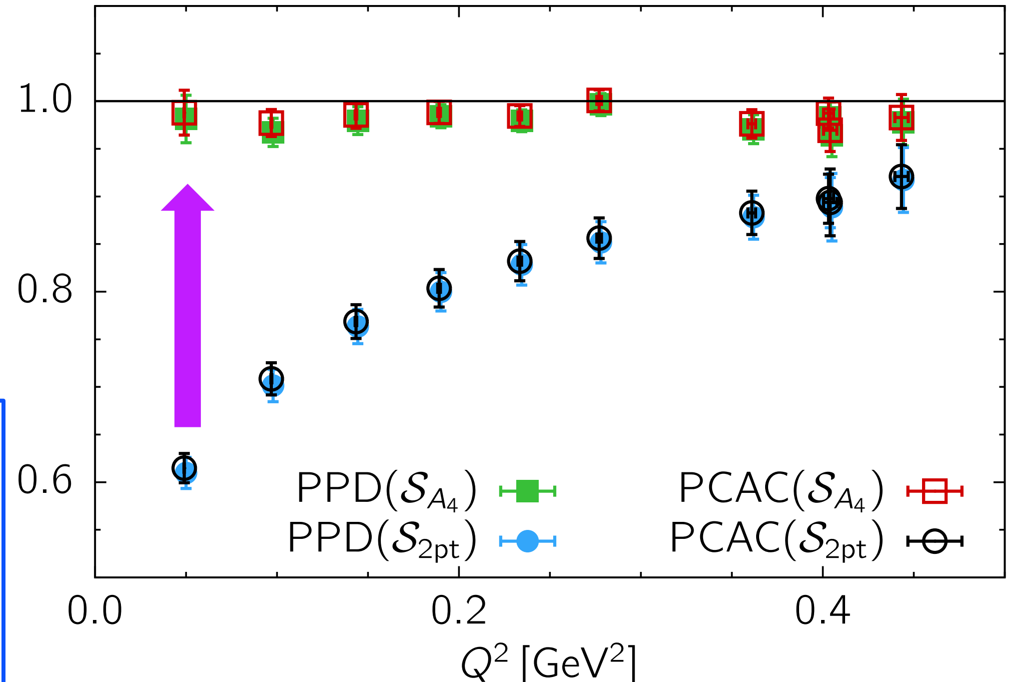
2019: Resolution of PCAC and PPD

Jang et al, PRL 124 (2020) 072002

On including low mass $N_{p=0}\pi_p$ and $N_p\pi_{-p}$ excited states neglected in previous works, we showed PCAC and PPD are satisfied

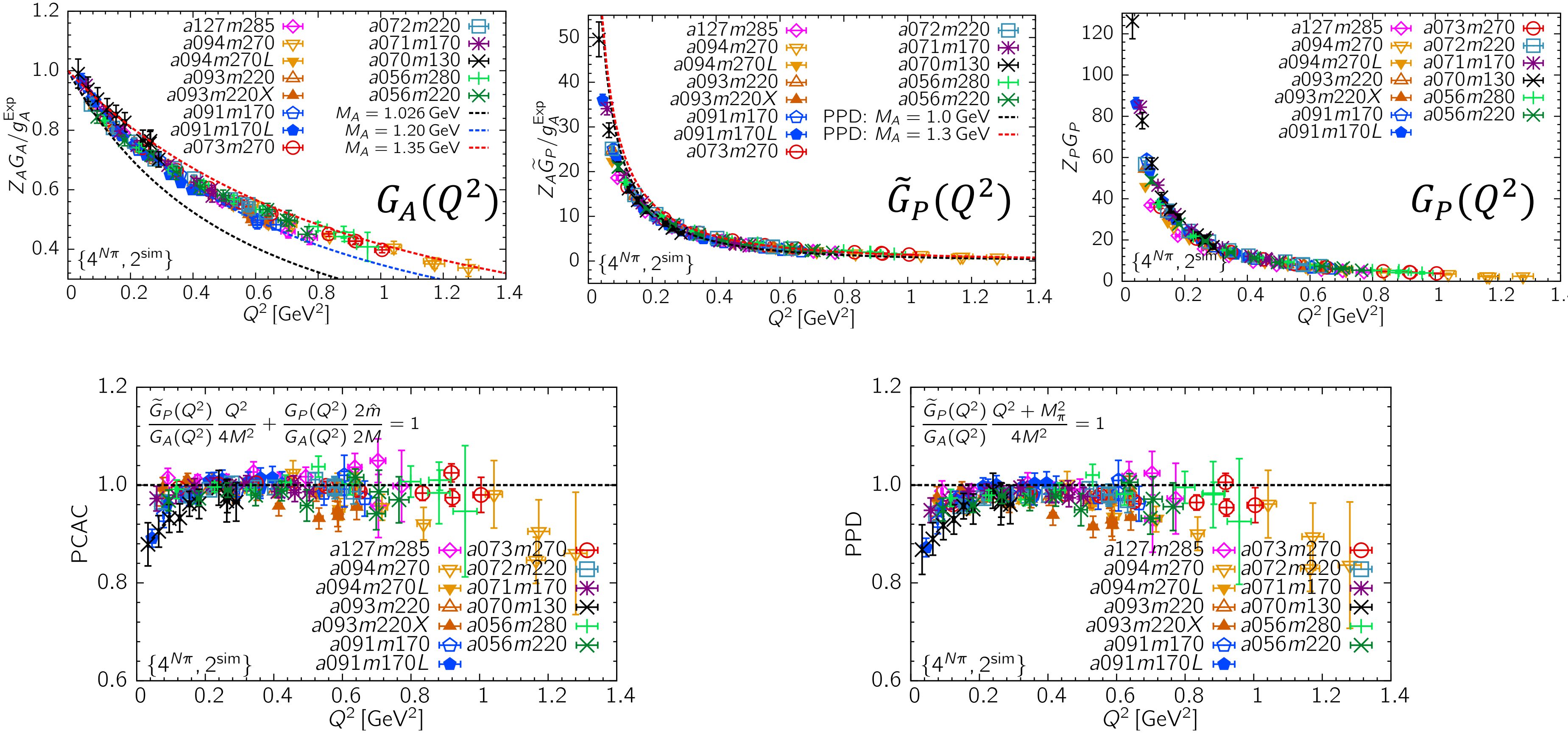
$$\frac{\hat{m}G_P}{M_N G_A} + \frac{Q^2\tilde{G}_P}{4M_N^2 G_A} = 1$$

The E_1 of $N_{p=0}\pi_p$ and $N_p\pi_{-p}$ are determined from fits to the $\langle N A_4 N\rangle$ 3-point axial correlation function!!



Impact
$G_A \sim$ 2—8 %
$\tilde{G}_P \sim$ 25—35 %
$G_P \sim$ 25—35 %

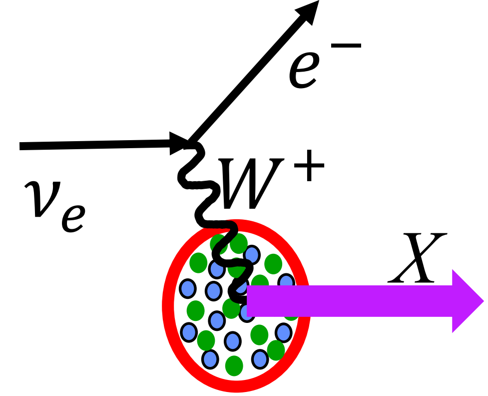
Axial Form Factors (unpublished)



$N\pi$ state needed to satisfy PCAC. Impact on $\tilde{G}_P$ and $\tilde{G}_P$ is large

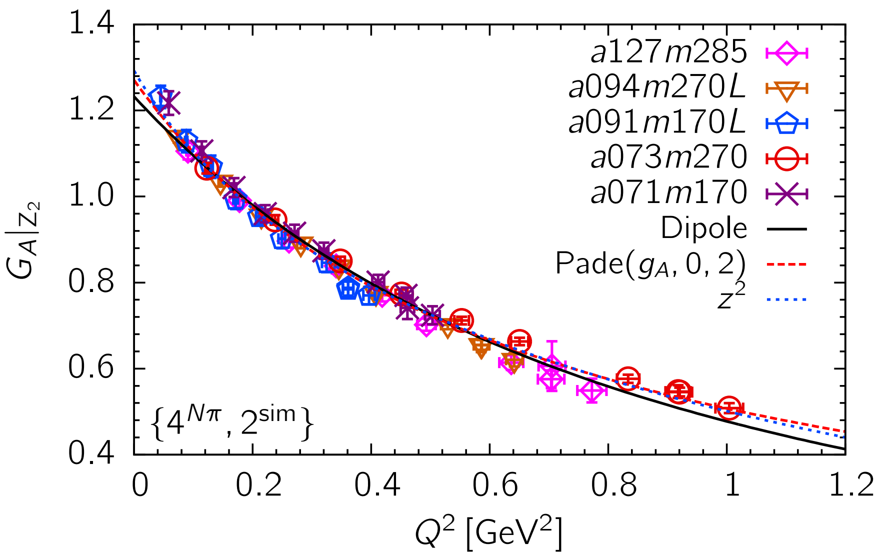
- $G_A \sim$ 2—8 %
- $\tilde{G}_P \sim$ 25—35 %
- $G_P \sim$ 25—35 %

Axial Form Factor: $G_A(Q^2)$



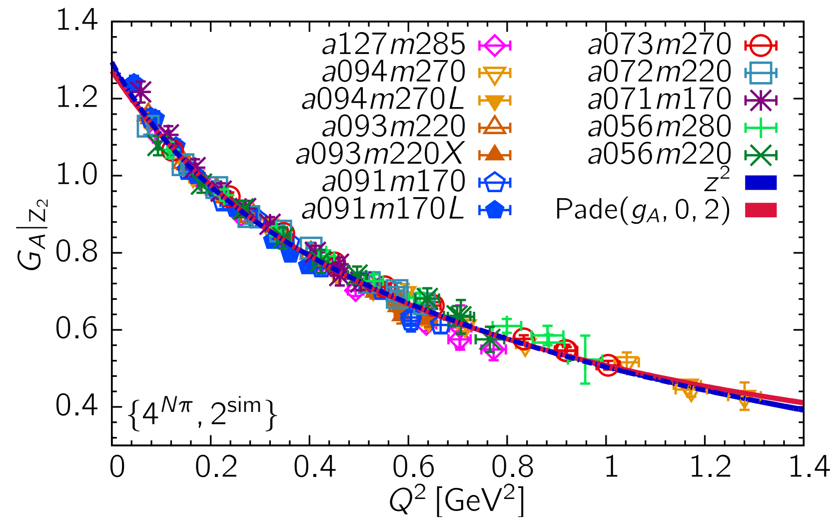
NME Collaboration: 2+1 clover ensembles

7 ensembles PRD 105 (2022) 054505



$$G_A(Q^2) = \frac{g_A = 1.270(11)}{1 + 5.36(20)\frac{Q^2}{4M_N^2} - 0.22(81)\left(\frac{Q^2}{4M_N^2}\right)^2}$$

12 ensembles (preliminary)



$$G_A(Q^2) = \frac{g_A = 1.272(8)}{1 + 5.36(15)\frac{Q^2}{4M_N^2} - 0.22(57)\left(\frac{Q^2}{4M_N^2}\right)^2}$$

$$\langle r_A^2 \rangle = 0.361(13)\ \mathrm{fm}^2$$

Electric & Magnetic form factors

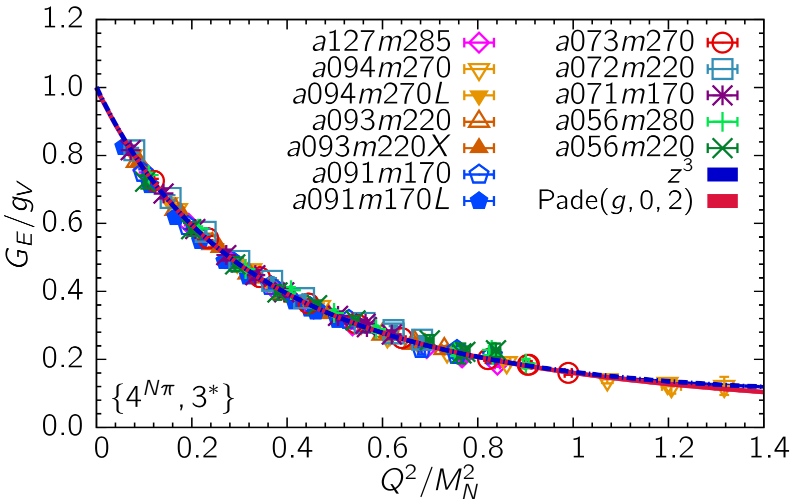
Matrix Elements of $V_\mu \rightarrow$ Dirac (F_1) and Pauli (F_2) form factors

$$\langle N(p_f) | V^\mu(q) | N(p_i) \rangle = \bar{u}(p_f) \left[\gamma^\mu F_1(q^2) + \sigma^{\mu\nu} q_\nu \frac{F_2(q^2)}{2M} \right] u(p_i)$$

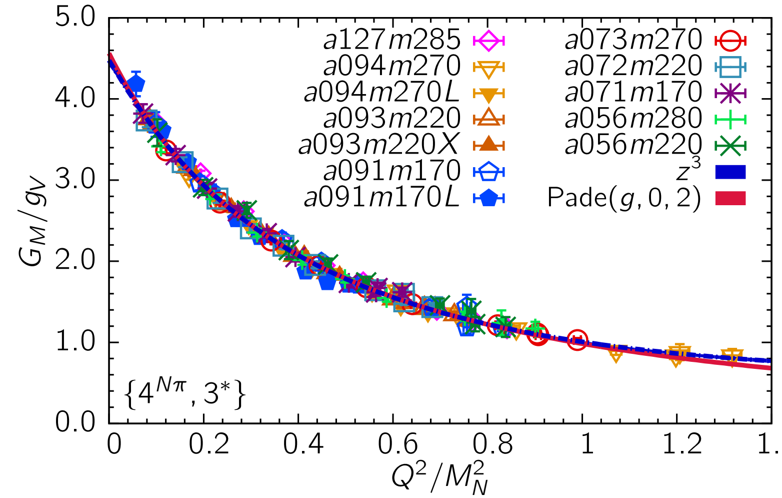
Define Sachs Electric (G_E) and Magnetic (G_M) form factors

$$G_E(q^2) = F_1(q^2) - \frac{q^2}{4M^2} F_2(q^2), \qquad G_M(q^2) = F_1(q^2) + F_2(q^2)$$

Electric & Magnetic FF with 12 ensembles



Electric



Magnetic

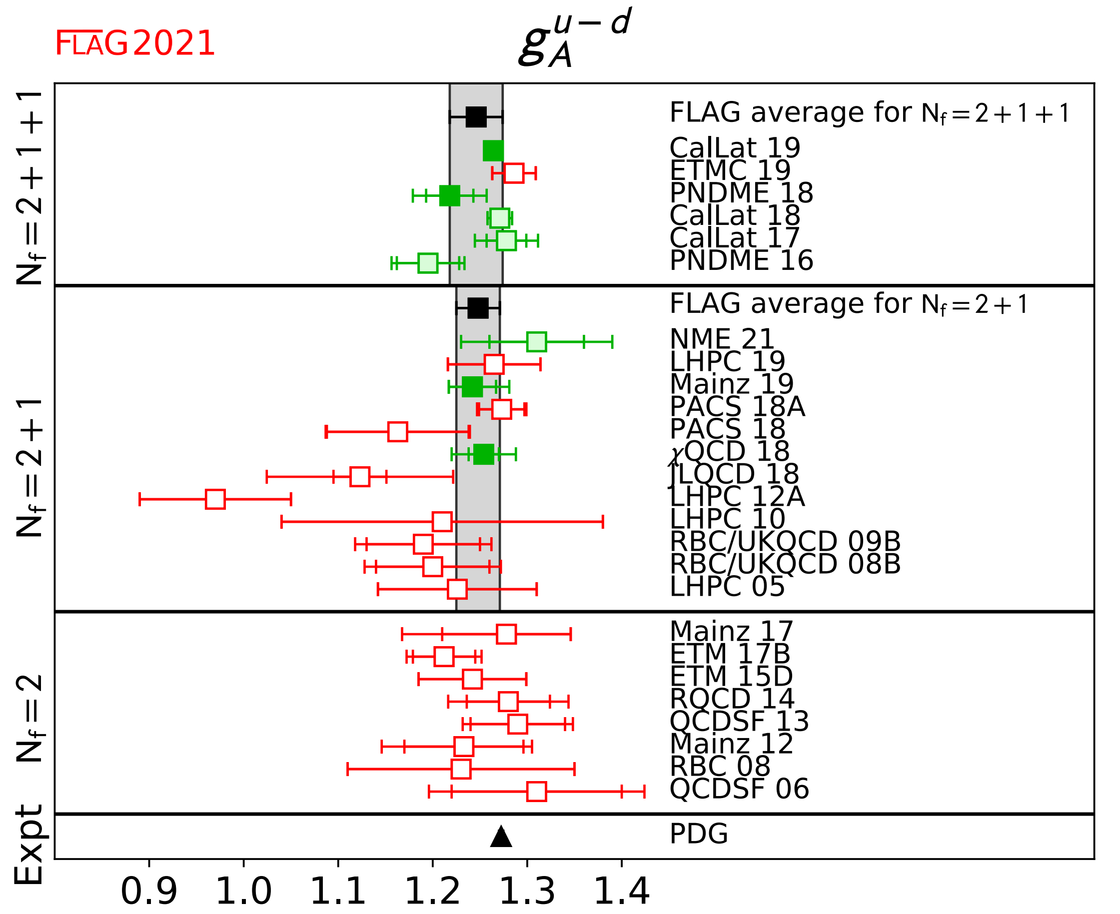
- The extraction of electric and magnetic form factors is insensitive to the details of the excited states
- Vector meson dominance → $N\pi\pi$ state should contribute (some evidence)
- The form factors do not show significant dependence on the lattice spacing or the quark mass
- Good agreement with the Kelly curve. Validates the lattice methodology
- Improve precision and get data over larger range of parameter values $\{a, m_{u,d}, M_\pi L\}$

PRD 105 (2022) 054505

The axial charge

$$g_A \equiv Z_A \langle P | \bar{u} \gamma_5 \gamma_3 d | N \rangle$$

A fundamental low energy parameter of QCD that enters in the analysis of weak interactions of nucleons and nuclei



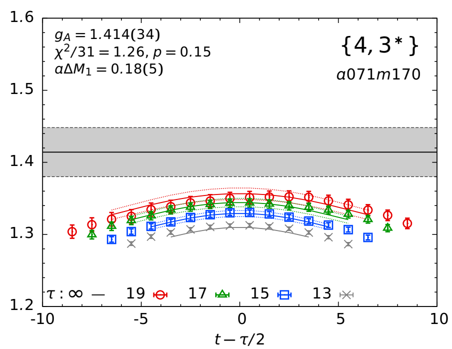
ESC in the extraction of axial charge

Spectrum from Γ^2

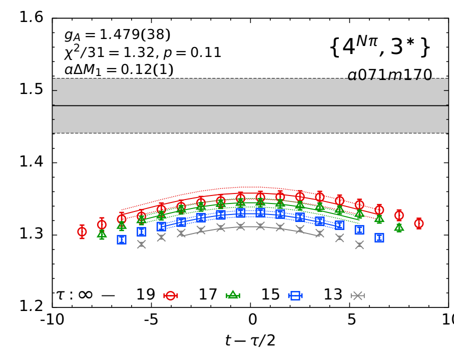
$N\pi$ included in fits (via A_4 or priors)

g_A (Forward ME)

1.23 ⇕ NO



1.29 ⇕ YES



$g_A = G_A(Q^2 \to 0)$

AFF $G_A(Q^2 \to 0)$ do not satisfy PCAC

AFF $G_A(Q^2 \to 0)$ with $N\pi$ satisfy PCAC

The pion-nucleon sigma term

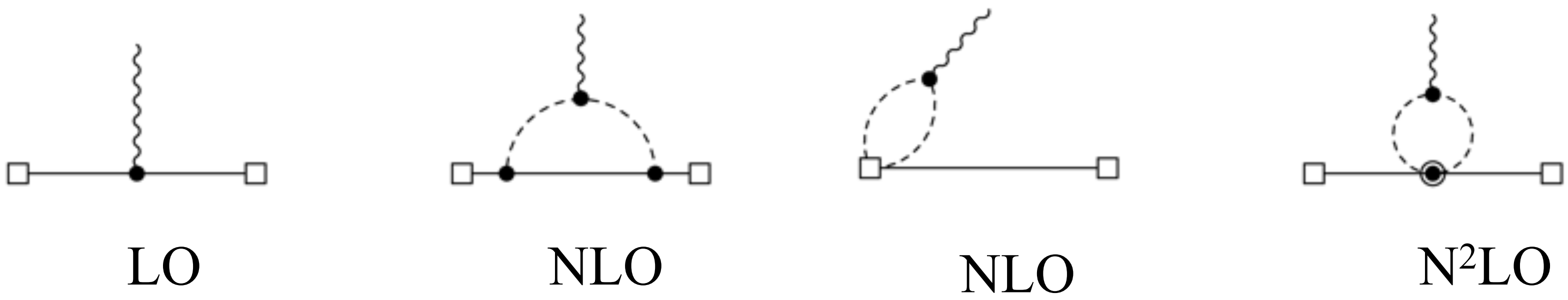
$$\sigma_{\pi N} \equiv m_{ud} g_S^{u+d} \equiv m_{ud} \langle N | \bar{u}u + \bar{d}d | N \rangle$$

- Fundamental parameter of QCD that quantifies the amount of the nucleon mass generated by u and d quarks.
- g_S^2: enters in cross-section of dark matter with nucleons
- Important input in the search of BSM physics

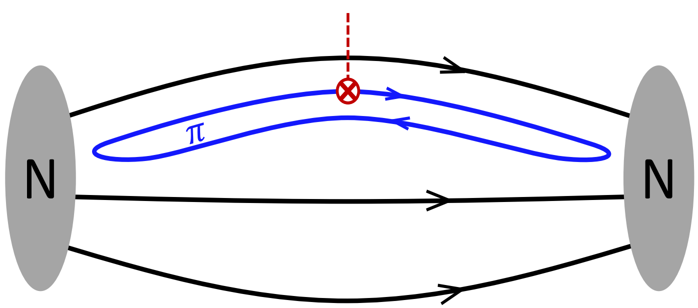
PRL 127 (2021) 242002; e-Print: 2105.12095
Rajan Gupta, Sungwoo Park, Martin Hoferichter, Emanuele Mereghetti, Boram Yoon, Tanmoy Bhattacharya

χPT analysis shows $N(\vec{k})\pi(-\vec{k})$ and $N(0)\pi(\vec{k})\pi(-\vec{k})$ states give significant contributions.

Coupling of S to $\pi\pi$ is large

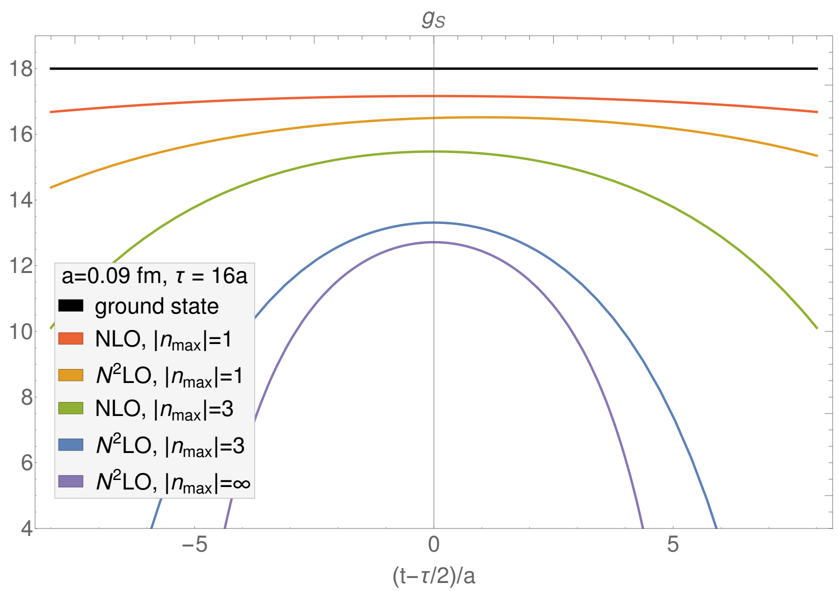


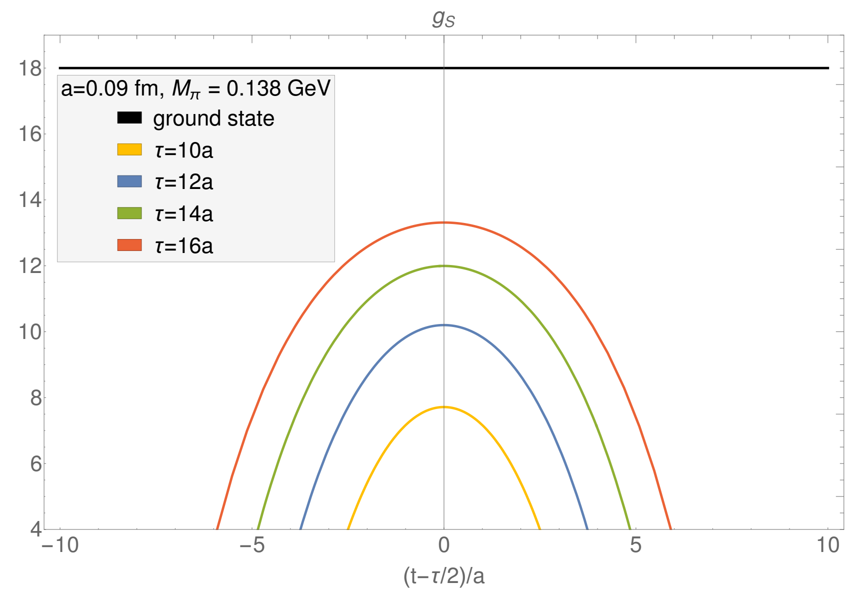
Why disconnected contribution is large



PRL 127 (2021) 242002

g_S: ESC from $N\pi$ / $N\pi\pi$ in N^{2}LO χPT





Different truncations (χPT order and $\vec{p}$)

N^{2}LO χPT estimates for $\tau = 10,12,14,16$

Estimates for the $a \approx 0.09 fm$; $M_\pi \approx 135 MeV$ ensemble assuming the asymptotic value is 18

The NLO and N^{2}LO ESC can each reduce $\sigma_{\pi N}$ at a level of 10 MeV

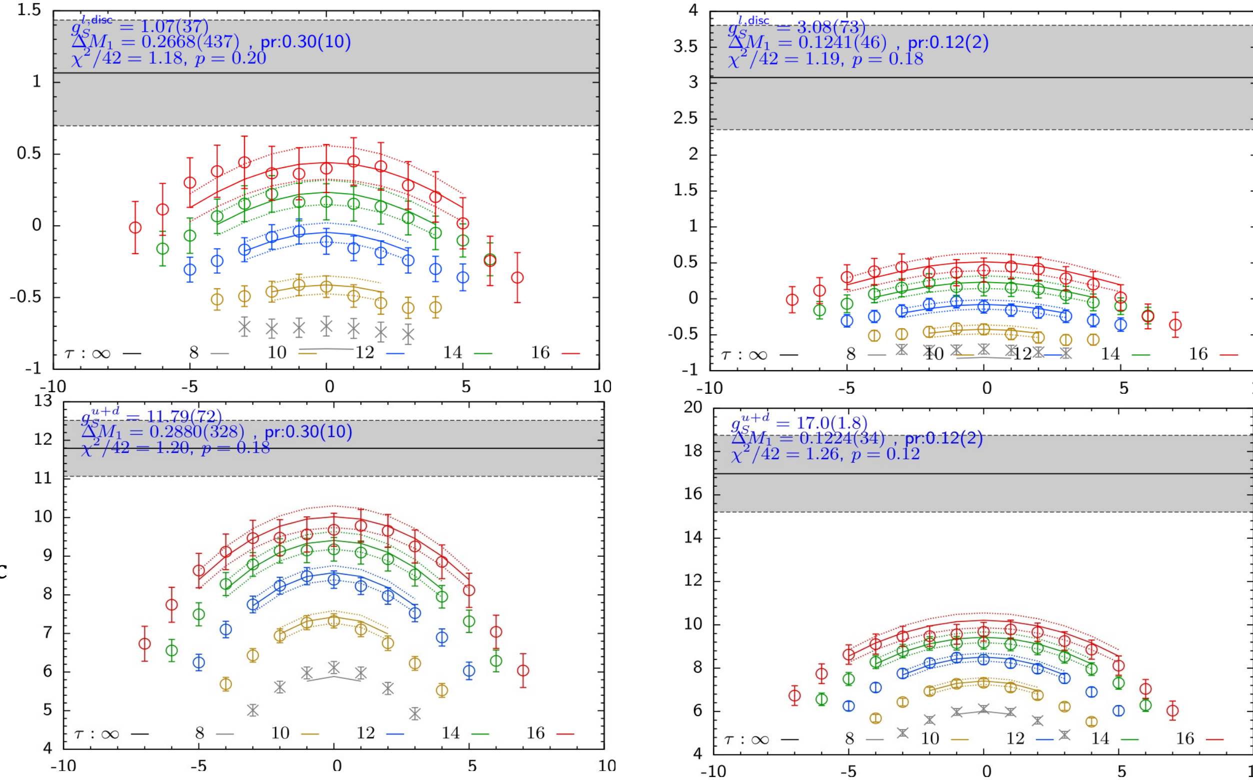
Including the Δ as an explicit degree of freedom does not change the conclusions

- e-Print: 2105.12095 [hep-lat]

Excited-state effects large and results very sensitive to $N\pi$ / $N\pi\pi$ states

Fits without $N\pi/N\pi\pi$ $(M_1 \approx 1.6\ GeV)$ with $N\pi$ / $N\pi\pi$ $(M_1 \approx 1.2\ GeV)$

$g_S^{l,\text{disc}}$



$g_S^{u+d} = g_S^{u+d,\text{conn}} + 2g_S^{l,\text{disc}}$

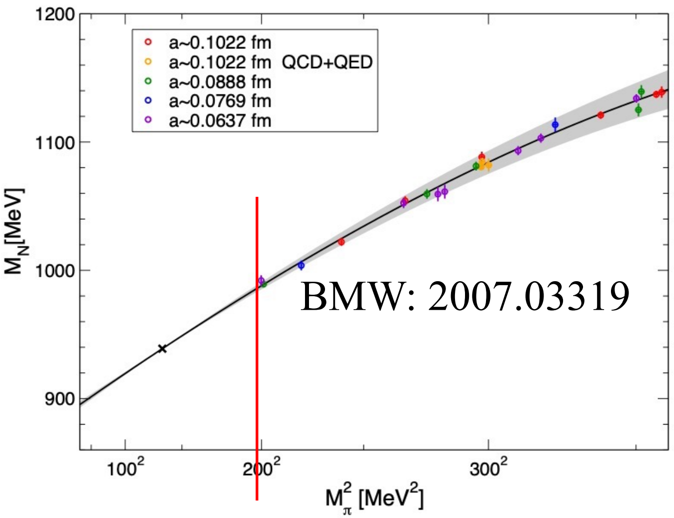
$\sigma_{\pi N} = m_l g_S^{u+d} \sim 40$ MeV $\sigma_{\pi N} = m_l g_S^{u+d} \sim 60$ MeV

PRL 127 (2021) 242002

The Feynman-Hellman method

$$\sigma_{\pi N} \equiv m_{ud} g_S^{u+d} \equiv \frac{m_{ud} \partial M_N}{\partial m} \equiv \frac{m \partial M_\pi^2}{\partial m} \frac{\partial M_N}{\partial M_\pi^2}$$

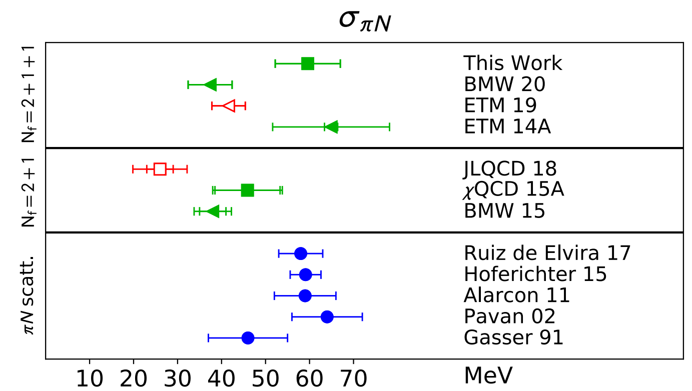
- Both M_N and $\sigma_{\pi N}$ have similar χPT structure. The starting point is the χPT expansion for M_N
- $\boldsymbol{N(\vec{k})\pi(-\vec{k})}$, $\boldsymbol{N(0)\pi(\vec{k})\pi(-\vec{k})}$, … state contribution increases as $M_\pi \to 135$ MeV
- Any fit to M_N versus M_π^2 should be done using data in a range where the mass gap of these excited states is lower than N(1440)
- Our work suggest this is $M_\pi \leq 200$ MeV



Resolved Tension Between Lattice QCD and Phenomenology

FLAG Reports 2019, 2021:

- Lattice results ~40 MeV
- Phenomenology favors ~60 MeV



Post FLAG 2021 results

BMW (arXiv:2007.03319) $\sigma_{\pi N}$ = 37.4(5.1) MeV (FH)

ETM (PRD **102**, 054517) $\sigma_{\pi N}$ = 41.6(3.8) MeV (Direct)

LANL Results: PRL 127 (2021) 242002; e-Print: 2105.12095

- Without including $\boldsymbol{N(\vec{k})\pi(-\vec{k})}$ and $\boldsymbol{N(0)\pi(\vec{k})\pi(-\vec{k})}$ states: = 41.9 (4.9) MeV
- Including $\boldsymbol{N(\vec{k})\pi(-\vec{k})}$ and $\boldsymbol{N(0)\pi(\vec{k})\pi(-\vec{k})}$ states: = 59.7 (7.3) MeV

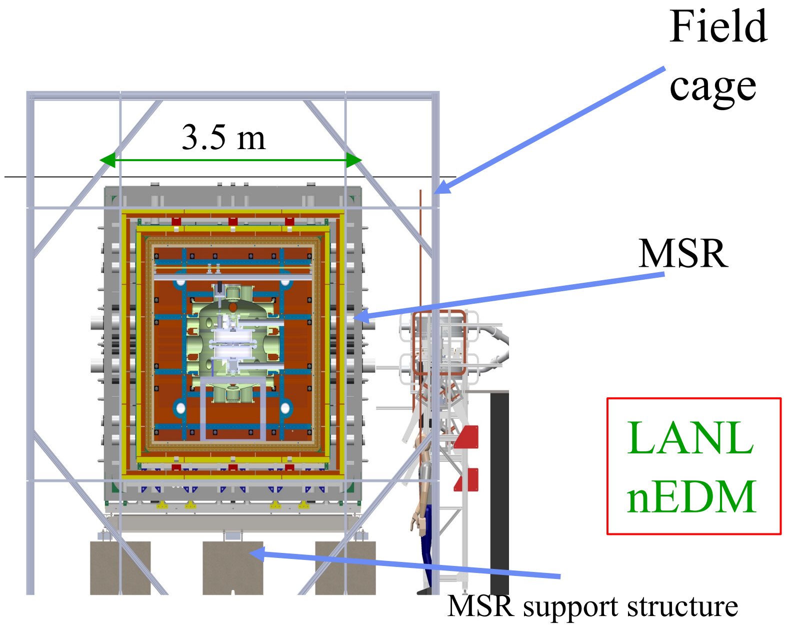
Contributions to Neutron Electric Dipole Moment (nEDM)

- $d_n = \sum_{O_i} \epsilon_i X_i$ where the sum is over all CPV operators O_i with coupling ϵ_i
- X_i are the CPV part of matrix elements of O_i within the neutron ground state
- 5 kinds of operators: Θ-term, qEDM, chromoEDM, Weinberg, 4-fermion
- Measure d_n and calculate X_i using LQCD $\Longrightarrow$ constrain ϵ_i (BSM models)

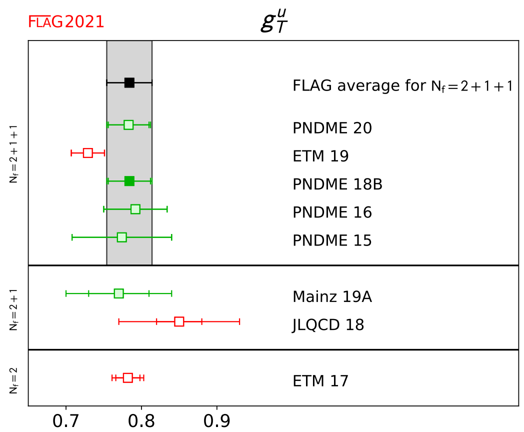
If $d_n \gtrsim 10^{-28}\ e\ cm \rightarrow$ CP violation in quark sector large enough for many BSM to remain viable.

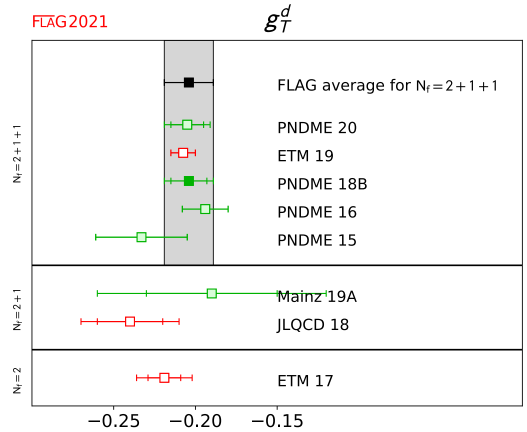
Baryogenesis a viable mechanism for observed matter-antimatter asymmetry in the universe.

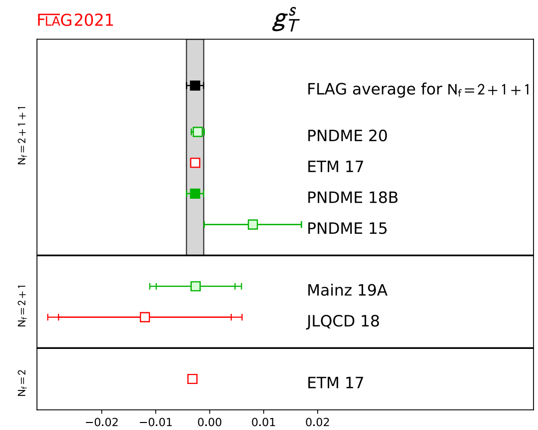
Need X_i to constrain BSM theories



Contribution of the quark EDM to neutron EDM



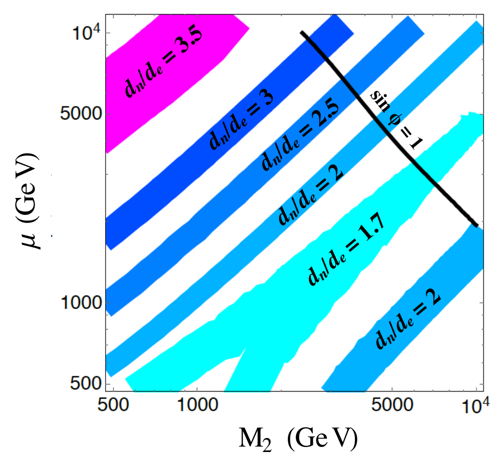




LANL (PNDME) results:

- $g_T^u = 0.784(28)(10)$
- $g_T^d = \text{-}0.204(11)(10)$
- $g_T^s = \text{-}0.0027(16)$

Constrains on the parameter space of split SUSY model



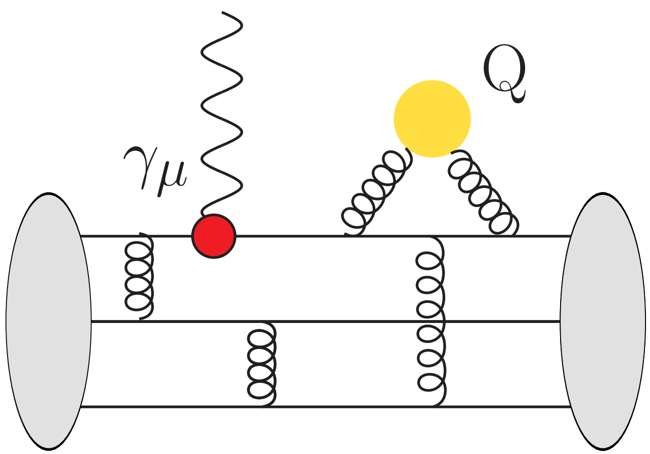
Update provided by Sungwoo Park at Lat2022

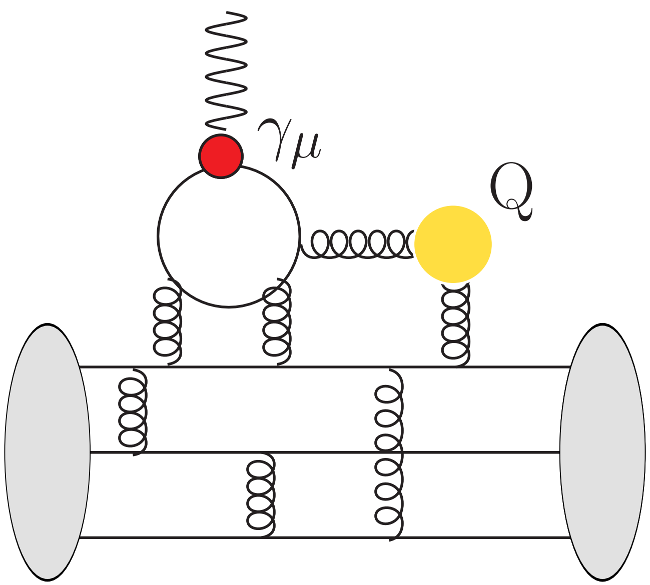
PRD 98 (2018) 091501

Contribution of the Θ-term to the neutron EDM

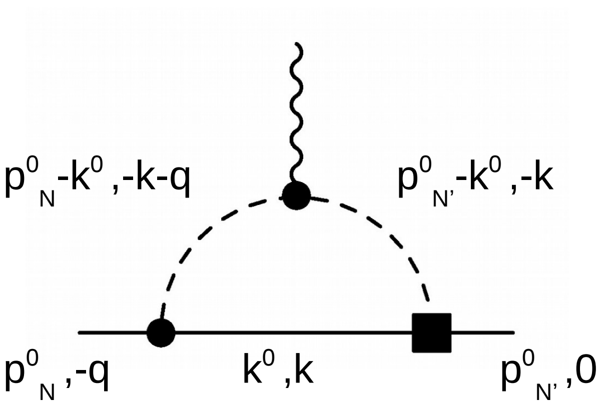
- $L_{\Theta} = \Theta \frac{iG^{a}_{\mu\nu}\tilde{G}^{a}_{\mu\nu}}{32\pi^{2}}$ is a D=4 interaction that violates P and T (CP if CPT holds)
- This D=4 term is allowed in the standard model
- $\Theta \sim 10^{-10}$ (from $d_n \leq 1.8 \times 10^{-26}$ e cm) is unnaturally small
- Axion field proposed to explain this small coupling

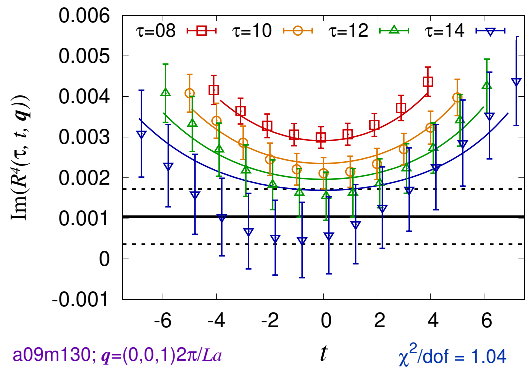
Lattice calculation of X_{Θ}: $d_n = \Theta\, X_{\Theta} \equiv \Theta \left\langle N \left| \frac{L_{\Theta}}{\Theta} \right| N \right\rangle_{CPV}$

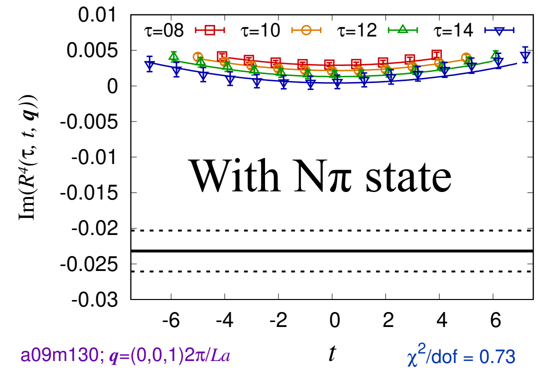




Excited state effect can be large







- Value of X_Θ is small and the signal is poor
- χPT indicates that Nπ states contribute
- Result for d_n is very sensitive to excited states included in the analysis. Much larger d_n with Nπ states!!

No robust result yet from LQCD: PRD 103 (2021) 114507

Outlook

- Statistics: Generating gauge configurations
  - New ideas: normalizing flows (autocorrelations, topology,)
- Signal in correlation functions
  - Signal to noise (S2N) for nucleons degrades exponentially $e^{-(M_N - 1.5 M_\pi)\tau}$
  - Contour deformation to remove noise from imaginary part
- Isolate ground state wavefunctions $|N(p_i)\rangle$
  - Variational method (GEVP)