Heavy flavor physics with $\mathrm{O}(a)$ improved Wilson quarks

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First LatticeNET workshop on challenges in Lattice field theory
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ALPHA Collaboration

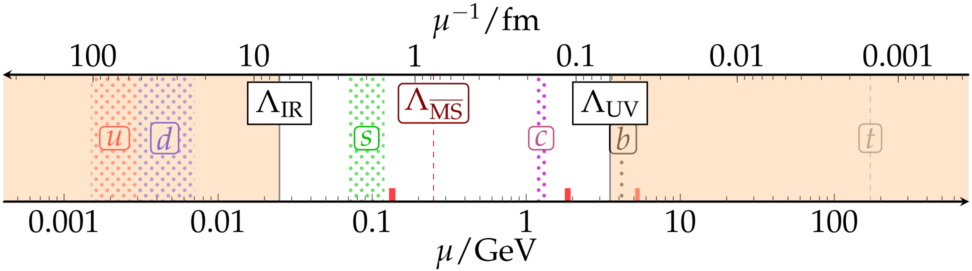
HIM HELMHOLTZ Helmholtz-Institut Mainz

- Indirect search via precision physics in the flavor sector.
- CKM matrix elements can be overconstrained to test the SM.
- Discrepancy between exclusive and inclusive determination of $|V_{\mathrm{cb}}|$:

$$\left|V_{\mathrm{cb}}^{\mathrm{excl}}\right| = (39.1 \pm 0.5) \times 10^{-3}\,, \qquad \left|V_{\mathrm{cb}}^{\mathrm{incl}}\right| = (42.2 \pm 0.8) \times 10^{-3}$$

Lattice QCD provides input for both quantities

- Form factors of semi-leptonic B decays for $\left|V_{\mathrm{cb}}^{\mathrm{excl}}\right|$.
- Charm and bottom quark masses for $\left|V_{\mathrm{cb}}^{\mathrm{incl}}\right|$.



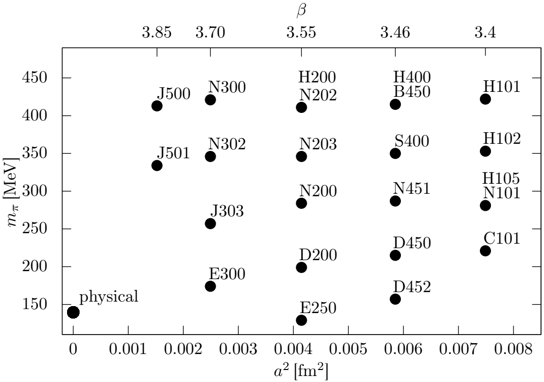
- **Red marks:** Pion, D- and B-meson masses
- $\Lambda_{\text{IR}} = L^{-1}$, inverse lattice extent. Typically $L \gtrsim 4/m_\pi \approx 6\,\text{fm}$
- $\Lambda_{\text{UV}} = a^{-1}$, inverse lattice spacing. Typically $a \lesssim 1/(2m_{\text{D}}) \approx 0.05\,\text{fm}$

Image taken from [1002.1807, Della Morte and Heitger]

- We work with three flavors of $\mathrm{O}(a)$ improved Wilson quarks in the sea.
- Renormalization and improvement pattern of flavor non-diagonal operators in a massless renormalization scheme [hep-lat/0511014, Bhattacharya et al.]:

$$(O^{ij})^{\mathrm{I}}_{\mathrm{R}}(x) = Z_{\mathrm{O}}(\tilde{g}_0^2, a\mu)\left[1 + ab_O(\tilde{g}_0^2)m_{\mathrm{q},ij} + a\bar{b}_O(\tilde{g}_0^2)\mathrm{Tr}[M_{\mathrm{q}}]\right]\left(O^{ij}(x) + a\delta O^{ij}(x)\right)$$

  with the $\mathrm{O}(a)$ improved bare coupling $\tilde{g}_0^2$ and a possible scale dependence $a\mu$.
- Valence $am_{\mathrm{q},ij}$ and sea quark mass $a\mathrm{Tr}\,[M_{\mathrm{q}}]$ dependent cutoff effects.
- Heavy flavor physics on $N_{\mathrm{f}} = 2+1$ ensembles: Non-perturbative determination of b_O to cancel valence quark mass dependent cutoff effects $\mathrm{O}(am_{\mathrm{q}})$.



- $\mathrm{O}(a)$ improved Wilson-clover fermions, Lüscher-Weisz gauge action.
- Five values of $a \in [0.039, 0.087]\,\mathrm{fm}$, a factor of 5 in a^2.
- Open boundary conditions in temporal direction.
- $m_\pi \in [129, 422]\,\mathrm{MeV}$

- Trajectory in this work: $\mathrm{Tr}[M_q] = 2m_\mathrm{l} + m_\mathrm{s} = \mathrm{const.}$ to keep improved coupling $\tilde{g}_0^2 = g_0^2 \left(1 + \frac{1}{N_f} b_g a \mathrm{Tr}[M_q]\right)$ constant up to $\mathcal{O}(a^2)$.

Charm physics

- Charm quark mass is a basic parameter of the Standard Model.
- Meson and quark masses from axial A_μ^{ij} and pseudoscalar P^{ij} quark bilinears.
- Bare quark masses from the PCAC relation:

$$\partial_0 \langle (A_\mathrm{I})_0^{ij} P^{ji} \rangle = 2m_{ij} \langle P^{ij} P^{ji} \rangle + \mathrm{O}(a^2)\,, \qquad \text{where} \quad (A_\mathrm{I})_\mu^{ij} = A_\mu^{ij} + a c_\mathrm{A} \partial_\mu P^{ij}$$

- Measure at two heavy masses around the charm.
- Set the charm quark hopping parameter κ_c from the flavor averaged D-meson mass $m_{\bar{D}} = \frac{2}{3} m_D + \frac{1}{3} m_{D_\mathrm{s}}$ or the connected part of η_c.

*with J. Heitger and F. Joswig

- Compute the renormalized RGI charm quark mass from PCAC masses:

$$M_c^{\mathrm{RGI}} = \frac{M}{\overline{m}(\mu_{\mathrm{had}})} m_{cc',\mathrm{R}}$$
$$\equiv Z_{\mathrm{M}} m_{\mathrm{cc'}} \left[1 + \frac{(b_{\mathrm{A}} - b_{\mathrm{P}})}{Z} a m_{\mathrm{cc'}} + \left(\frac{(\bar{b}_{\mathrm{A}} - \bar{b}_{\mathrm{P}})}{Z r_{\mathrm{m}}} - \frac{(b_{\mathrm{A}} - b_{\mathrm{P}})}{Z} \frac{(r_{\mathrm{m}} - 1)}{r_{\mathrm{m}} N_{\mathrm{f}}} \right) a M_{\mathrm{sum}} \right]$$

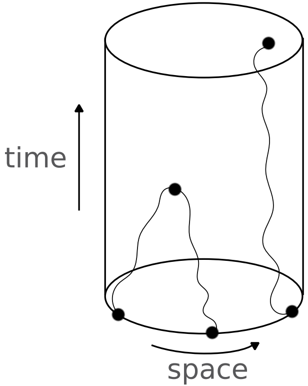
  - $Z_{\mathrm{M}} = M/\overline{m}(\mu_{\mathrm{had}}) \cdot Z_{\mathrm{A}}/Z_{\mathrm{P}}(\mu_{\mathrm{had}})$: [1802.05243, Campos et al.], including Z_{A} from [1808.09236, Dalla Brida et al.]
  - $(b_{\mathrm{A}} - b_{\mathrm{P}})$ and $Z = Z_{\mathrm{m}} Z_{\mathrm{P}}/Z_{\mathrm{A}}$: [1906.03445, de Divitiis et al.]
  - $(r_{\mathrm{m}} - 1)$: [2101.10969, Heitger et al.]
  - $(\bar{b}_{\mathrm{A}} - \bar{b}_{\mathrm{P}}) \propto \mathcal{O}(g_0^4)$, not known non-perturbatively
  - $a M_{\mathrm{sum}} = a m_{12} + a m_{23} + a m_{31} = Z r_{\mathrm{m}} \mathrm{Tr}\left[M_{\mathrm{q}}\right] + \mathrm{O}(a)$

- Alternative definitions via heavy-sea currents and the ratio-difference method [1011.2711, Dürr et al.]. Two definitions of discretized derivative $\tilde{\partial}_0$ in PCAC mass.

Charm physics

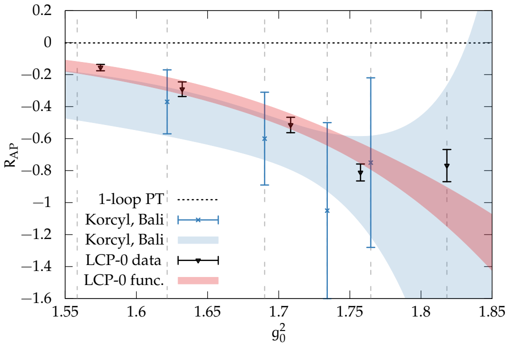
Intermission: Non-perturbative improvement and renormalization

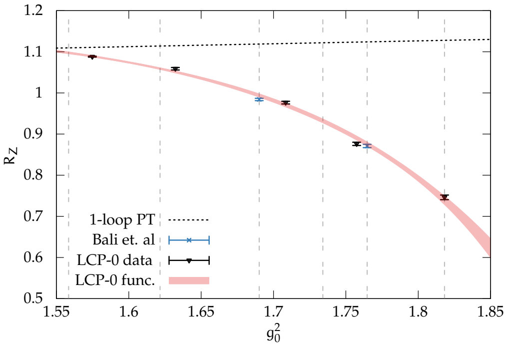
The Schrödinger functional



- ALPHA's tool of choice for renormalization and $\mathrm{O}(a)$ improvement [hep-lat/9207009, Lüscher et al.].
- Dirichlet boundary conditions in time direction. Suited for Monte Carlo simulations and perturbative calculations.
- Boundary conditions and finite volume allow the simulation of **massless quarks** [hep-lat/9312079, Sint].
- Finite volume $\rightarrow$ consider short-ranged quantities.
- Renormalization and improvement from Ward identities.

Improvement and renormalization from the SF *



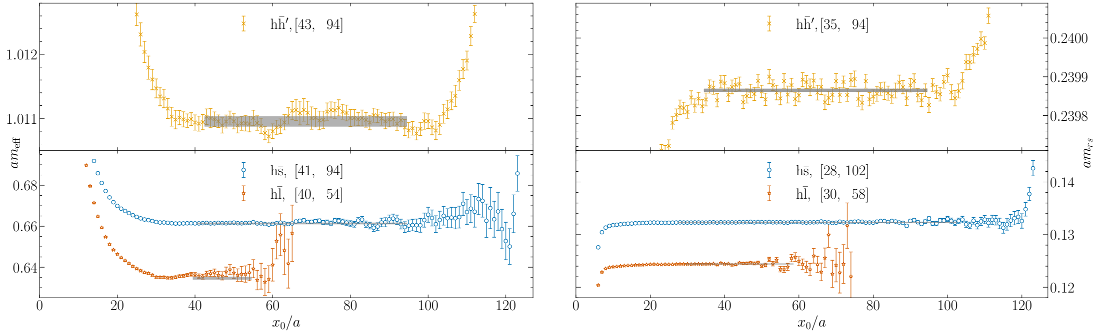


- Non-perturbative determination of $b_\mathrm{A} - b_\mathrm{P}$ and Z [1906.03445, de Divitiis et al.].
- Cut-off effects of $\mathrm{O}(am_{\mathrm{cc'}})$ are canceled via $\frac{(b_\mathrm{A} - b_\mathrm{P})}{Z}\, am_{\mathrm{cc'}}$.

* with G. M. de Divitiis, P. Fritzsch, J. Heitger, C. C. Koester, A. Vladikas

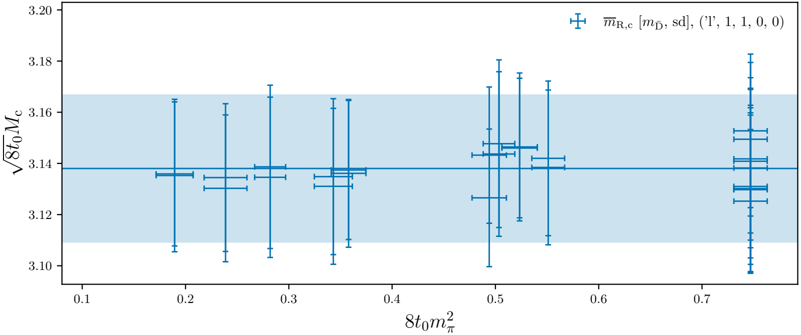
Charm physics

Back to the quark mass

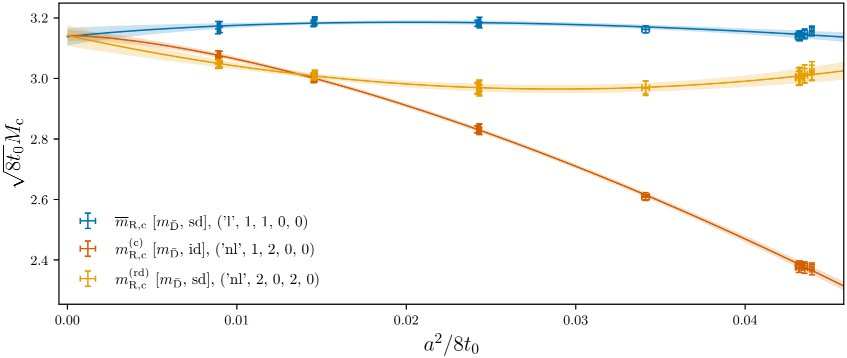


- Obtain bare quark masses with a relative error of $\mathrm{O}(10^{-3})$ to $\mathrm{O}(10^{-5})$.
- Use distance preconditioning [1006.4028, de Divitiis et al.][1701.05502, Collins et al.] to prevent loss of numerical precision at large source-sink separation.

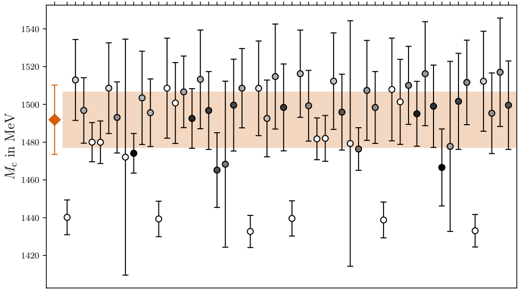
- Extrapolate to physical quark mass and vanishing lattice spacing.
- Measured on ensembles with $m_\pi \geq 200\,\mathrm{MeV}$. No dependence on pion mass resolved.

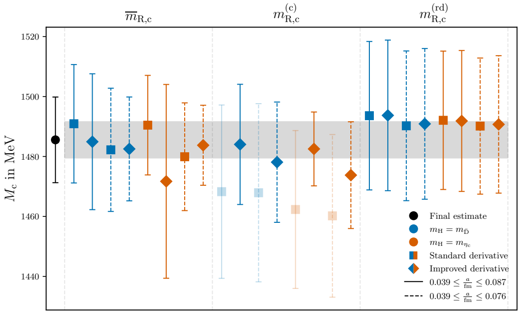


Continuum extrapolation



- We find significant $\mathrm{O}(a^n), n > 2$ contributions for $a > 0.06\,\mathrm{fm}$.
- Different lattice definitions coincide in the continuum limit.

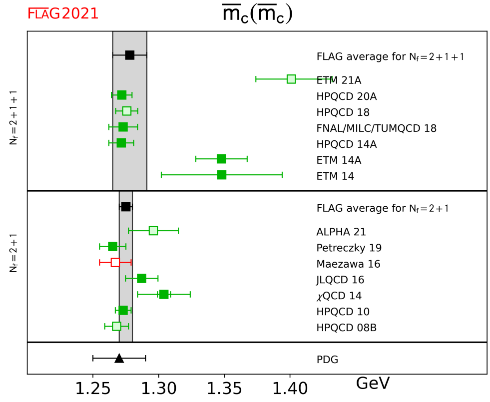




- Quantify systematic uncertainties in each data set via model averaging procedure [2008.01069 , Jay and Neil].
- Combine results from all data sets for final result.

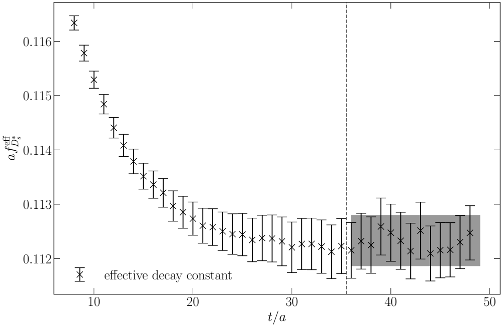
Results for m_c

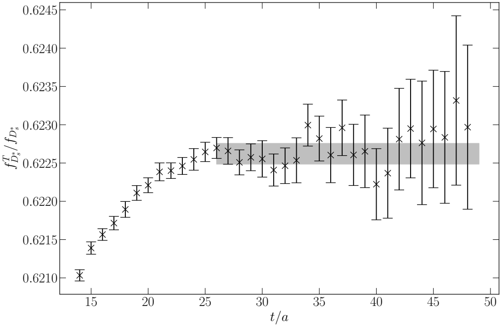
- Our final result is $M_c(N_f = 3) = 1486(14)_{stat}(14)_{RGI}(6)_{sys}$ MeV $= 1486(21)$ MeV.



- Error dominated by external input.
- Convert to $\bar{m}_c(\bar{m}_c)$ for comparison with other groups.
- Ongoing work on CLS ensembles:
  - RQCD with extended set of ensembles and different renormalization scheme [2206.04178, Bouma et al.].
  - Madrid group with twisted-mass on Wilson-clover $\rightarrow$ automatic $O(a)$ improvement of the charm.

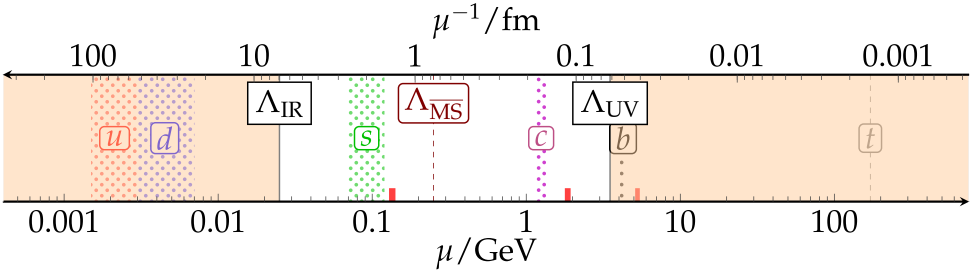
- Work with RQCD collaboration on the determination of $f_{D^*_{(s)}}$ and $f^{\mathrm{T}}_{D^*_{(s)}}$.
- Bare vector decay constants at 0.5% precision.
- Bare ratios $f^{\mathrm{T}}_{D^*_{(s)}}/f_{D^*_{(s)}}$ at sub permille precision, expect dominant contribution from $Z_{\mathrm{T}}(\mu)$ [1910.06759, Chimirri et al.][2111.15325, de Divitiis et al.].





*with G. Bali, S. Collins, J. Heitger, F. Joswig, W. Söldner

Bottom physics



- B-observables cannot be evaluated in large volume at several resolutions:

$$L^{-1} \ll m_\pi \approx 140\,\text{MeV} \ll m_{\text{B}} \approx 5\,\text{GeV} \ll a^{-1}$$

- Bottom physics is a multi-scale problem!
- Employ non-perturbative Heavy Quark Effective Theory (HQET) to compute m_{b}, $f_{B_{(\text{s})}}$, form factors of semi-leptonic B-decays (everything is **preliminary**!).

Image taken from [1002.1807, Della Morte and Heitger]

Heavy Quark Effective Theory

- Integrate out heavy degrees of freedom of QCD Lagrangian for one heavy quark.
- Expand the Lagrangian in powers of $1/m_\mathrm{h}$.
- Describe bottom physics at next-to-leading order in HQET.

$$\mathcal{L}_\mathrm{heavy} = \bar{h}_v D_0 h_v - \omega_\mathrm{kin}\mathcal{O}_\mathrm{kin} - \omega_\mathrm{spin}\mathcal{O}_\mathrm{spin}\,, \qquad \mathcal{O}_\mathrm{kin} = \bar{h}_v \mathbf{D}^2 h_v\,, \quad \mathcal{O}_\mathrm{spin} = \bar{h}_v \sigma\cdot\mathbf{B} h_v$$

HQET on the lattice

- Matrix elements in the effective theory can be computed in large volume.
- Renormalize the effective theory via **non-perturbative** matching to QCD.

HQET AT $\mathrm{O}(1/m_\mathrm{h})$ ON THE LATTICE

- Cannot define the path integral from the HQET action at order $1/m_\mathrm{h}$ (contains dimension five operators $\mathcal{O}_\mathrm{kin}$ and $\mathcal{O}_\mathrm{spin}$).
- Expand the Boltzmann factor in the path integral in powers of the inverse heavy quark mass $1/m_\mathrm{h}$.
- Expectation value of an operator:

$$\begin{aligned}\langle\mathcal{O}\rangle \approx& \frac{1}{Z}\int \mathcal{D}\phi\,\mathcal{O}\left(1 + a^4\sum_x[\omega_\mathrm{kin}\mathcal{O}_\mathrm{kin}(x) + \omega_\mathrm{spin}\mathcal{O}_\mathrm{spin}(x)]\right)\mathrm{e}^{-(S_\mathrm{light}+S^\mathrm{stat}_\mathrm{HQET})} \\ =& \langle\mathcal{O}\rangle_\mathrm{stat} + \omega_\mathrm{kin}a^4\sum_x\langle\mathcal{O}\mathcal{O}_\mathrm{kin}\rangle_\mathrm{stat} + \omega_\mathrm{spin}a^4\sum_x\langle\mathcal{O}\mathcal{O}_\mathrm{spin}\rangle_\mathrm{stat}\,.\end{aligned}$$

- $\mathrm{O}(1/m_\mathrm{h})$ and $\mathrm{O}(a)$ mix on the lattice $\rightarrow$ $\mathrm{O}(a)$ improvement is vital!

- Perturbative matching at order g_0^{2l} leads to power divergences in the coefficients [Nucl.Phys.B 368 (1992) 281-292, Maiani et al.]

$$\Delta c_k \sim g_0^{2(l+1)} a^{-p} \sim a^{-p} \left[\ln(a\Lambda)\right]^{-(l+1)} \overset{a\to 0}{\to} \infty$$

  due to mixing of operators differing in dimensions by p.
  $\rightarrow$ Non-perturbative matching makes HQET well defined on the lattice.

- Hadronic input: Would loose predictivity of HQET.

- Finite-volume matching:
  - Match with continuum limit of lattice QCD observables.
  - Computation with fine resolutions in finite volume.
  - Assumption: Finite-volume effects only due to light degrees of freedom.

- **Finite volume** $L_1 = 0.5\,\mathrm{fm}$: Simulation of QCD with relativistic b-quarks and $N_\mathrm{f} = 3$ massless sea quarks.
- Solve matching equations [hep-lat/0310035, Heitger and Sommer] [1312.1566, Della Morte et al.]:

  $$\lim_{a\to 0} \phi_i^{\mathrm{QCD}}(L_1, M, a) = \phi_i^{\mathrm{HQET}}(L_1, M, a) = \eta_i(L_1, a) + \varphi_i^j(L_1, a)\omega(M, a) + \mathrm{O}(1/m_\mathrm{h}^2)$$

  for matching parameters $\omega(M, a)$.
- 19 parameters $\rightarrow$ 19 equations to define Lagrangian and heavy-light axial and vector currents at $\mathrm{O}(1/m_\mathrm{h})$.

- Valence line of constant physics defined from fixed RGI quark mass,

$$z \equiv L_1 M = L_1 \frac{M}{\overline{m}(1/L_0)} \frac{Z(g_0^2) Z_{\rm A}(g_0^2)}{Z_{\rm P}(g_0^2, L_0)} (1 + b_{\rm m}(g_0^2) a m_{\rm q,h}) a m_{\rm q,h} \,,$$

  based on the subtracted quark mass $am_{\rm q,h} = 1/2(1/\kappa_{\rm h} - 1/\kappa_{\rm cr})$ and $L_0 = L_1/2$.

- Running factor known, dedicated computation of $Z_{\rm A}$, $Z_{\rm P}$, Z and $b_{\rm m}$*.

- Measure in a range of quark masses $0.8 z_{\rm c} < z_{\rm q,h} < 1.2 z_{\rm b}$.

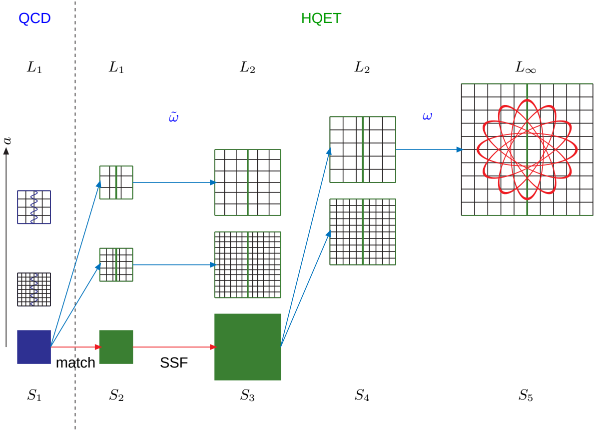
*with P. Fritzsch and J. Heitger

Step-scaling

- Need to know matching parameters in large volume!
- Step-scaling [Nucl. Phys. B359 (1991) 221, Lüscher et al.] offers an approach to multi-scale problems:
  - Determination of $\Lambda_{\rm QCD}$ [1706.03821, ALPHA].
  - Running of quark masses up to RGI point [1802.05243, ALPHA].
  - **B-physics** [hep-lat/0310035, ALPHA].
- Introduce external scale via spatial lattice extent $\mu = 1/L$.
- Investigate scale evolution via step-scaling functions,

$$\sigma(O(L)) = O(sL)\,,$$

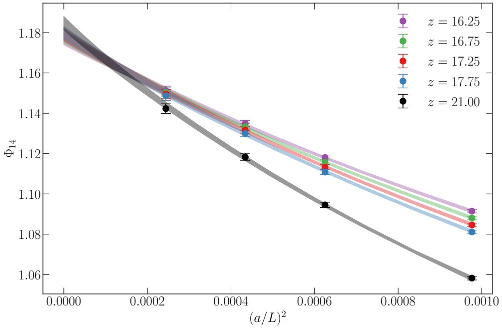
where $s = 2$ is a conventional choice.



- QCD observables with relativistic b quarks in finite volume at $L_1 = 0.5\,\text{fm}$.
- Match with corresponding HQET observables.
- Evolve HQET parameters to large volumes via step-scaling $L_1 \to 2L_1 \to L_{\text{CLS}}$.
- Generated $\mathrm{O}(25)$ SF ensembles with volumes up to $L^4 = 64^4$ and 100k MDU.

Image taken from [1001.4783, Blossier et al.]

*with P. Fritzsch, J. Heitger, H. Simma, R. Sommer



- Look at matching observable

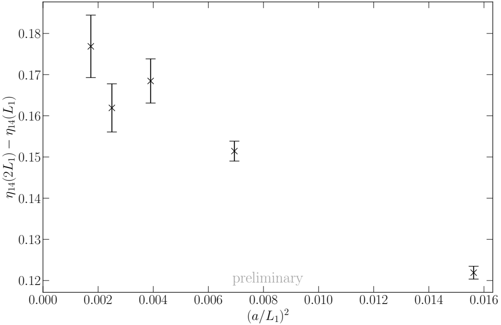
$$\phi_{V_0}^{\mathrm{QCD}}(L_1) = \frac{F_{V_0}^{B\to\pi}(T/2)}{\sqrt{F_1^{\pi}(T)F_1^{B}(T)}}\,,$$

needed to determine $Z_{\mathrm{V}_0}^{\mathrm{HQET}}$.

← QCD results close to $z_{\mathrm{b}} \sim 17$ at four lattice spacings $0.0078\,\mathrm{fm} < a < 0.0156\,\mathrm{fm}$ and mass-continuum fit.

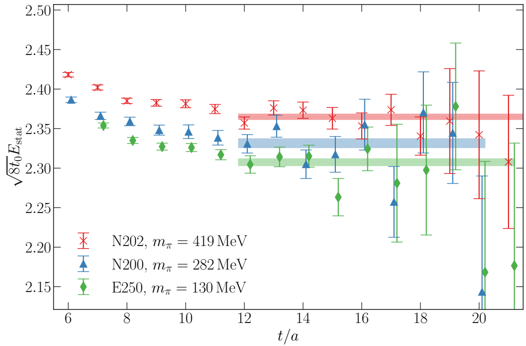
- Solve matching equations for matching parameters $\omega(M,a)$:

$$\lim_{a\to 0}\phi_i^{\mathrm{QCD}}(L_1,M,a) = \phi_i^{\mathrm{HQET}}(L_1,M,a) = \eta_i(L_1,a) + \varphi_i^j(L_1,a)\omega(M,a) + \mathrm{O}(1/m_{\mathrm{h}}^2)$$



$\leftarrow$ Step-scaling function of HQET observable η_{V_0} from $0.5\,\mathrm{fm}$ to $1\,\mathrm{fm}$ at five lattice spacings $0.021\,\mathrm{fm} < a < 0.063\,\mathrm{fm}$

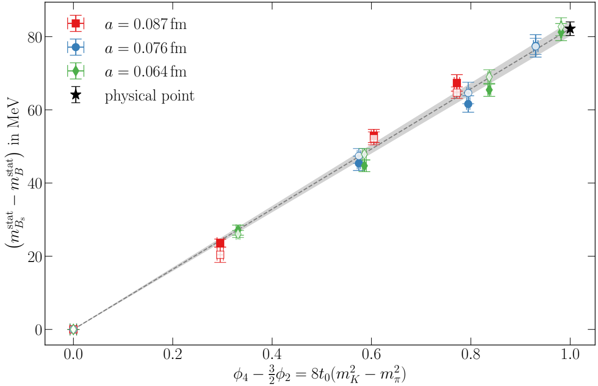
- Next step: contact with large-volume CLS ensembles.
- Measurements done, analysis ongoing.



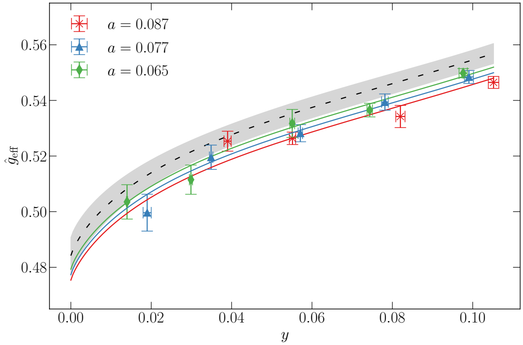
- Measurements at $\mathrm{O}(1/m_\mathrm{h})$ on CLS ensembles ongoing.
- HYP smeared static quark lines for exponential noise reduction [hep-lat/0307021, Della Morte et al.].
- Smeared quark fields and GEVP extraction of energies and matrix elements [0902.1265, Blossier et al.].
- Extraction of static, as well as higher-order kinetic and spin contributions.
- B-meson mass from large volume results and matching parameters:

$$m_B = m_\mathrm{bare} + E^\mathrm{stat} + \omega_\mathrm{kin} E^\mathrm{kin} + \omega_\mathrm{spin} E^\mathrm{spin} + \mathrm{O}(1/m_\mathrm{h}^2)$$

*with A. Gérardin, J. Heitger, H. Simma, R. Sommer



- Splitting $m_{B_\mathrm{s}}^\mathrm{stat} - m_B^\mathrm{stat} = 82.2(1.9)_\mathrm{stat}$ well defined without matching parameters.
- PDG value $m_{B_\mathrm{s}}^\mathrm{phys} - m_B^\mathrm{phys} = 87.42(14)$ allows estimate of $\mathrm{O}(1/m_\mathrm{h})$ effects.
- Three resolutions $a \geq 0.064\,\mathrm{fm}$ so far: cutoff effects are small.



- Low energy constant $\hat{g}$ of Heavy Meson χPT related to the $B^*B\pi$ coupling with **static heavy** and **chiral light** quarks.
- Precision of lattice computations limited by chiral extrapolation (chiral log!).

← Chiral-continuum extrapolation in $y = \frac{m_\pi^2}{8\pi^2 f_\pi^2}$ and a^2.

- Previous works at $m_\pi \geq 270\,\mathrm{MeV}$.

- Measurements are done, analysis to be finished. Aim at 2% precision.
- Will be of great use in (our) chiral extrapolations of lattice results and phenomenological calculations.

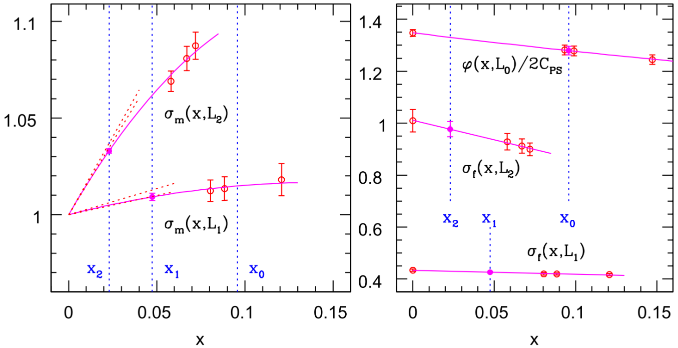
Outlook: The step-scaling method for B-physics *

- There is an alternative route to employ step-scaling [hep-lat/0305018, de Divitiis et al.]: Scale finite-volume relativistic heavy quark observables with $am_{\rm h} \leq am_{\rm b}$:

$$O(m_{\rm h}, L_\infty) = O(m_{\rm h}, L_1)\frac{O(m_{\rm h}, L_2)}{O(m_{\rm h}, L_1)} \cdots \frac{O(m_{\rm h}, L_N)}{O(m_{\rm h}, L_{N-1})}$$

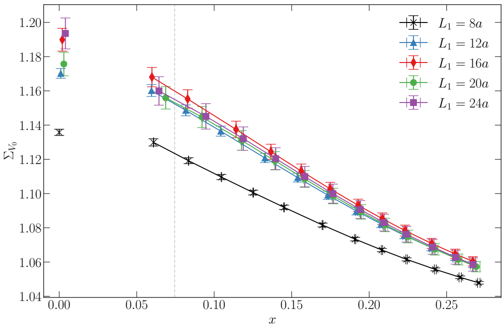
- As the lattice size is doubled:
  - The affordable lattice spacings increase.
  - The simulated maximal heavy quark mass decreases.
  - Need to extrapolate to obtain $m_{\rm h} \to m_{\rm b}$ for $L_2 \to L_{\rm CLS}$.
- Combination with non-perturbative static HQET turns extrapolations to interpolations with enhanced precision [0710.2229, Guazzini et al.].

* with A. Conigli, J. Frison, P. Fritzsch, A. Gérardin, J. Heitger, G. Herdoíza, C. Pena, H. Simma, R. Sommer



- Quark mass dependence parameterized by $x = 1/(L_1 m_{\rm PS}(m_{\rm h}, L))$.
- *Left:* Step-scaling of the mass - static constraint of the form $1 + \sigma_{\rm m}^{\rm stat} x$.
- *Right:* Step-scaling of decay constant - static result at $x = 0$.

Image taken from [0710.2229, Guazzini et al.]



- Work in progress: Step-scaling and large-volume measurements ($L_2 = L_{\text{CLS}}$).

$\leftarrow$ Step scaling function

$$\Sigma_{V_0}(L_1/a) = \frac{\phi_{V_0}(2L_1/a)}{\phi_{V_0}(L_1/a)}$$

for the time-component of the vector current from $0.5\,\text{fm}$ to $1\,\text{fm}$ at five lattice spacings $0.021\,\text{fm} < a < 0.063\,\text{fm}$

- Good parametrization up to $1.2m_\text{b}$.

- Static HQET observables at $x = 0$ from the HQET project.
- CLS: Heavy quark measurements with $m_\text{c} \leq m_\text{h} \leq 2.5m_\text{c}$ are on track at the $\text{SU}(3)_\text{f}$-symmetric point.

Charm physics

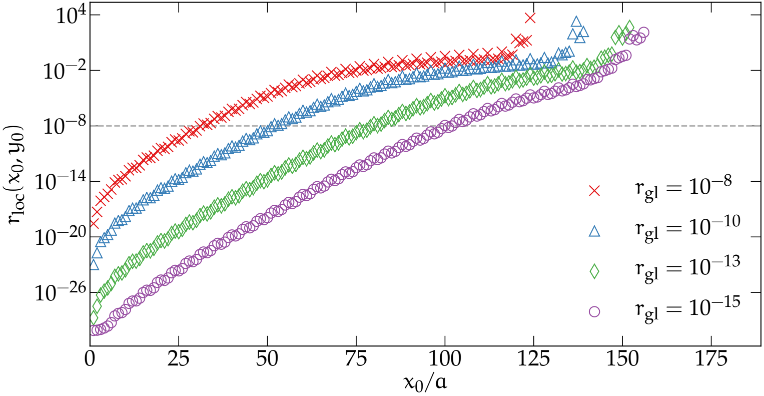
- Symanzik improvement allows reliable extraction of charm observables.
- Competitive determination of Standard Model parameters.
- $N_\mathrm{f} = 2+1+1$ non-trivial for $\mathrm{O}(a)$ improved Wilson [2002.02866, Höllwieser et al.].

Bottom physics

- Non-perturbative HQET on the lattice removes systematic uncertainties from large cutoff effects in B-physics observables.
- High setup cost $\rightarrow$ hope to have first results soon.
- Work on two approaches with different systematic effects.

Backup

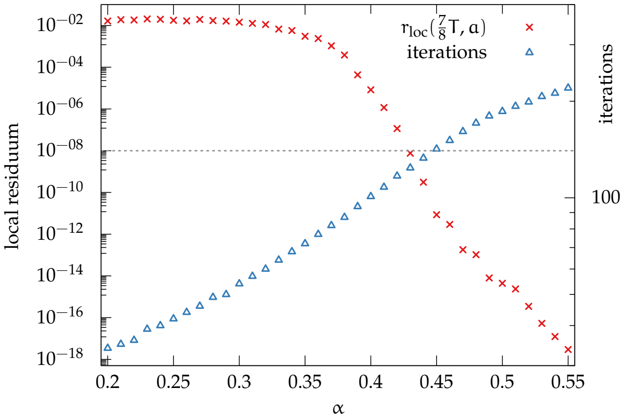
Technical challenges with heavy quarks



$$r_{\text{loc}}(x_0, y_0) = \frac{|D(x_0, y_0) S_{\text{h}}(y_0) - \eta(x_0)|}{|S_{\text{h}}(y_0)|}$$

- Quality of the solution deteriorates quickly for large source-sink separations (shown here: J501).
- Especially problematic if sources are placed close to the boundary.
- Charm-charm correlation functions: small but finite plateau regions.

Distance Preconditioning



- We employ **Distance Preconditioning** [1006.4028, de Divitiis et al.]
- SAP-preconditioned GCR solver [1701.05502, Collins et al.], [T. Korzec, mesons]
- Figure: tuning for H400 at $y_0/a = 84$

Preconditioned system:

$$(PDP^{-1})(PS) = (P\eta), \quad P = \mathrm{diag}(p_i), \quad p_i = \exp(\alpha\,|y_0 - x_0|)$$