B and K physics: the role of lattice QCD

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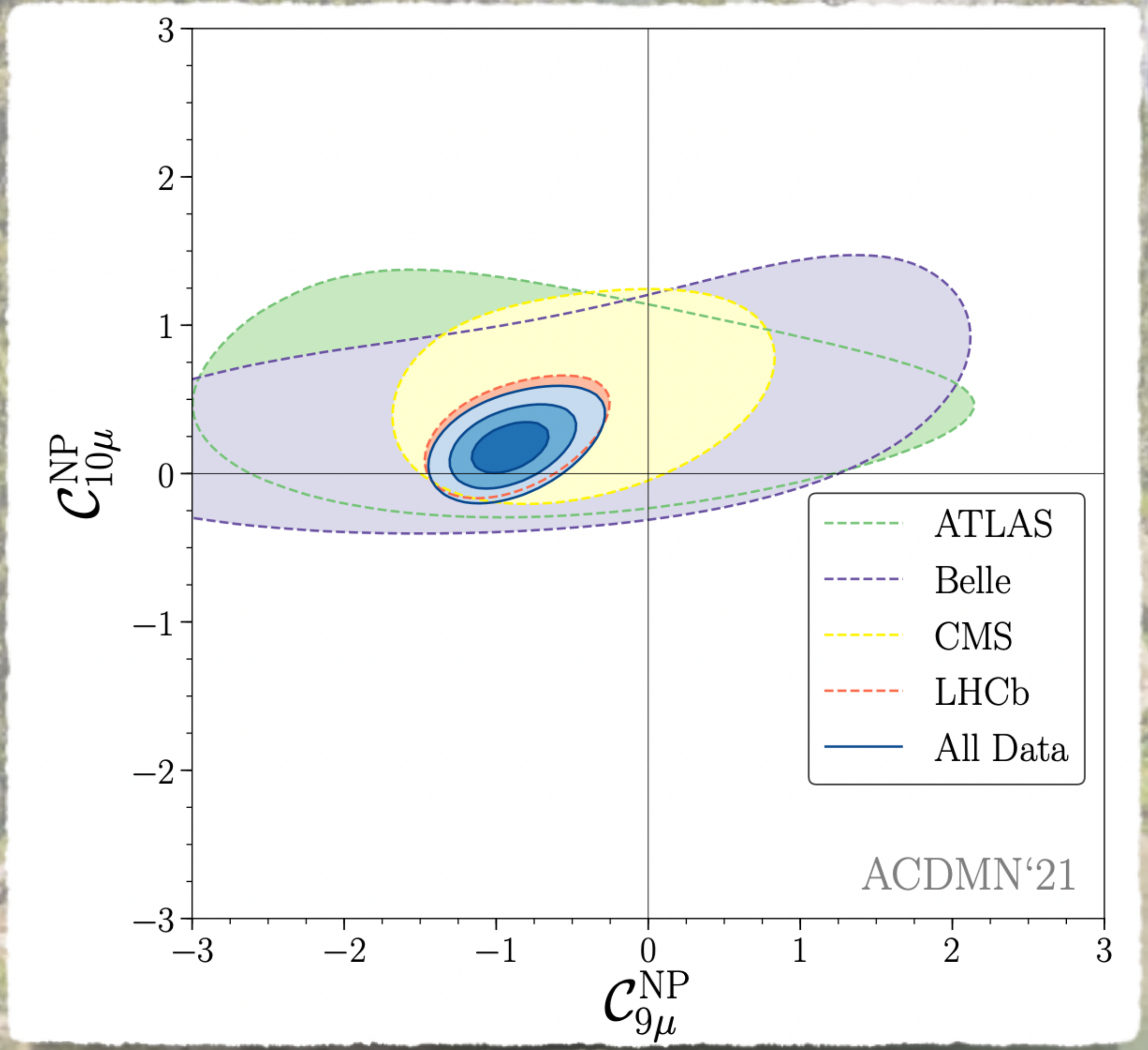
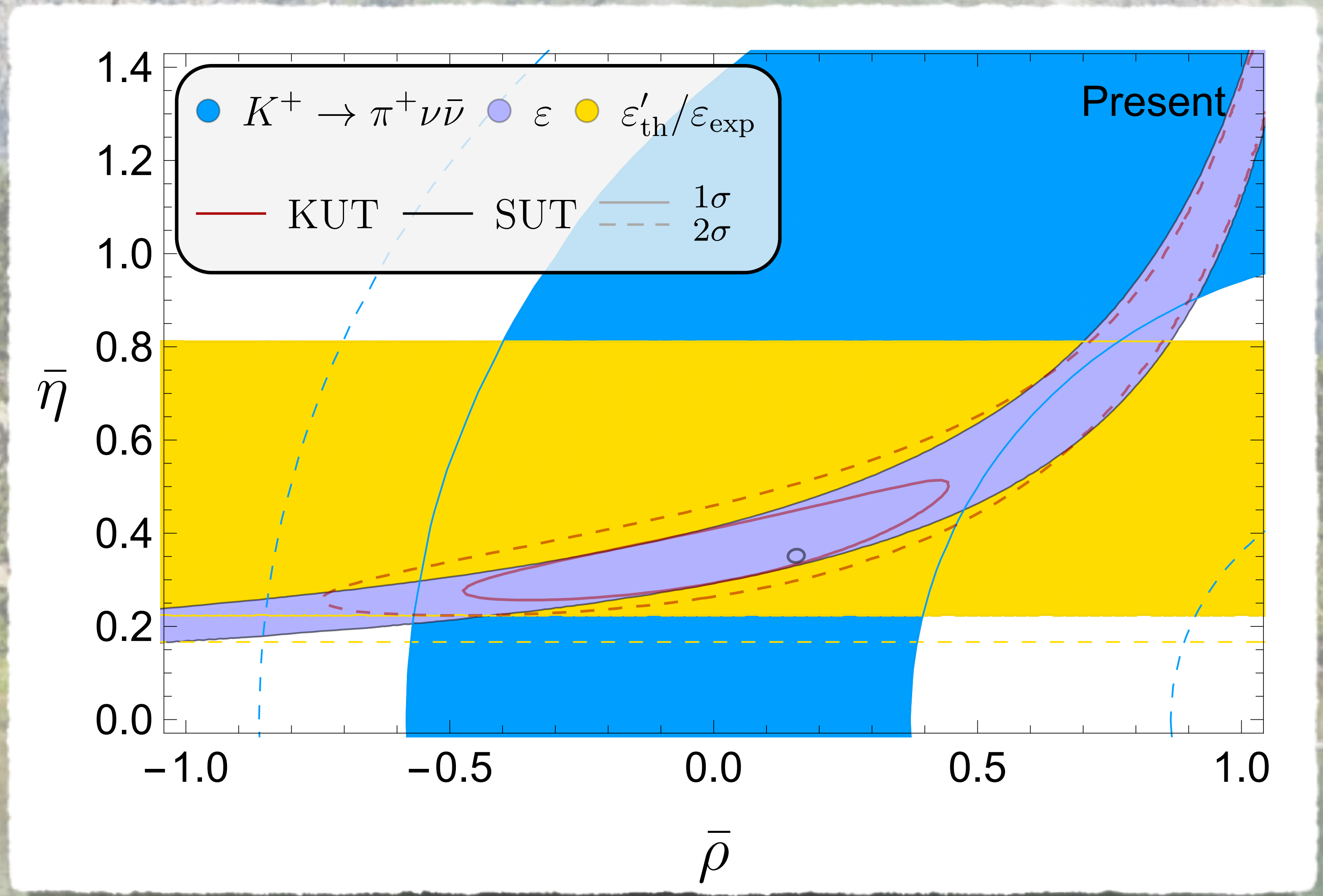
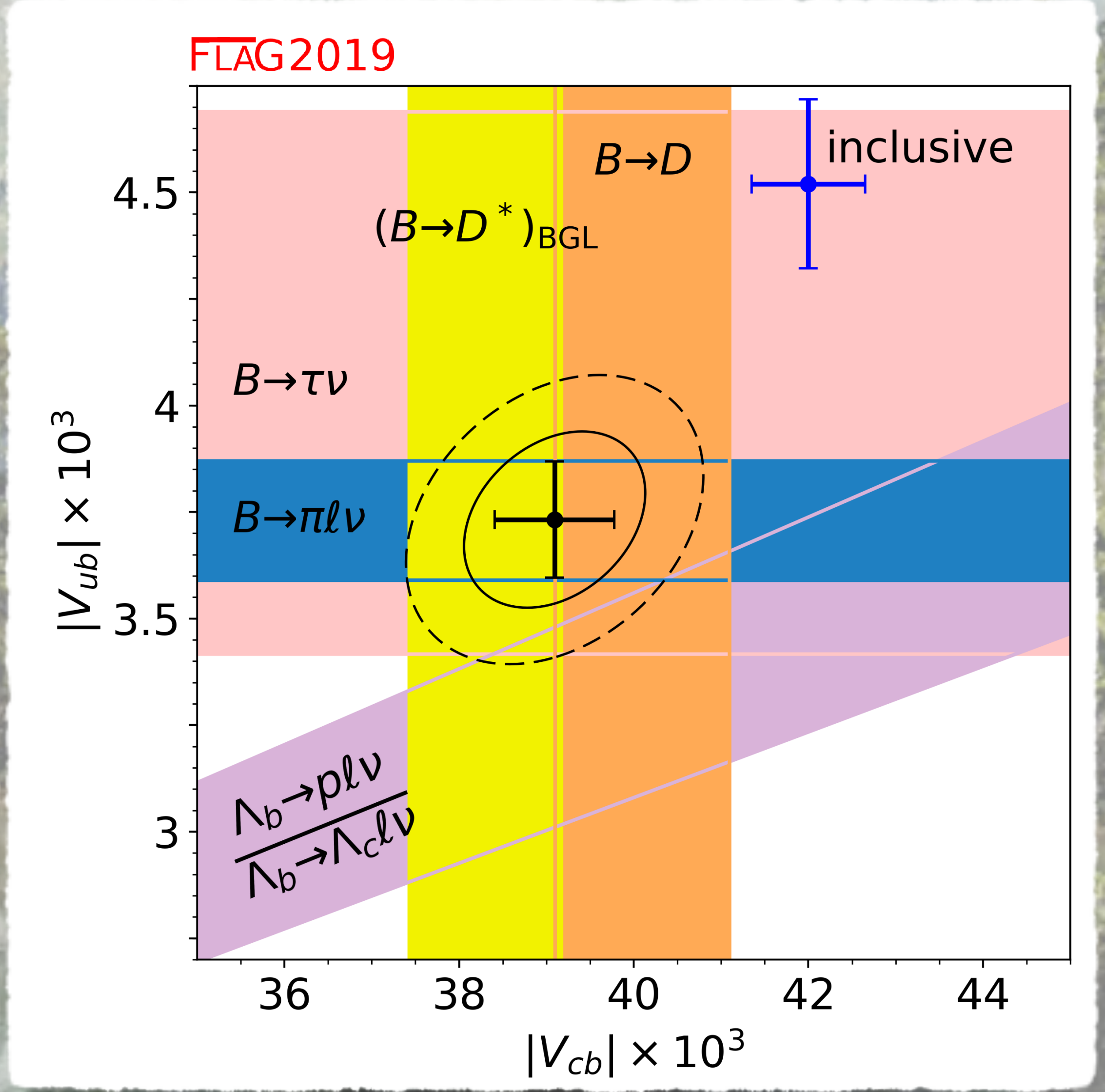






First LatticeNET workshop on challenges in Lattice field theory
Sep 11-17 2022
Centro de Ciencias de Benasque Pedro Pascual

Outline

- $b \to s\ell\ell$ anomalies
- Semileptonic B decays
- Kaon physics ($\varepsilon'_K/\varepsilon_K$, $K \to \pi\nu\bar{\nu}$ modes)

$b \to s\ell\ell$

- The effective Hamiltonian responsible for $b \to q$ (q=d,s) transitions in the SM is:

$$\mathcal{H}_{\rm eff} = -\frac{4G_F}{\sqrt{2}} V_{tb}V_{tq}^* \left[\sum_{i=1}^{10} C_i Q_i + \underbrace{\frac{V_{ub}V_{uq}^*}{V_{tb}V_{tq}^*}}_{\lambda_q} \sum_{i=1}^{2} C_i (Q_i - Q_i^u) + \sum_{i=3}^{6} C_{iQ} Q_{iQ} + C_b Q_b + C_{\nu\nu} Q_{\nu\nu} \right]$$

- λ_q contributions are relevant only for $b \to d$ transitions and yield large CP asymmetries ($\lambda_s = -0.0074 + 0.020\, i$, $\lambda_d = -0.036 - 0.43\, i$)

- Phenomenologically important operators are:

$$Q_9 = \frac{\alpha_{\rm em}}{4\pi} (\bar{q}_L \gamma_\mu b_L) \sum (\bar{\ell} \gamma^\mu \ell)$$
$$Q_{10} = \frac{\alpha_{\rm em}}{4\pi} (\bar{q}_L \gamma_\mu b_L) \sum (\bar{\ell} \gamma^\mu \gamma_5 \ell)$$

$B \to (K^{(*)}, \pi, X_s, X_d, \ldots)\ell\ell$

$B_s \to \phi\mu\mu$, $\Lambda_b \to \Lambda\ell\ell$

$$Q_7 = \frac{e}{16\pi^2} (\bar{q}_L \sigma^{\mu\nu} b_R) F_{\mu\nu}$$

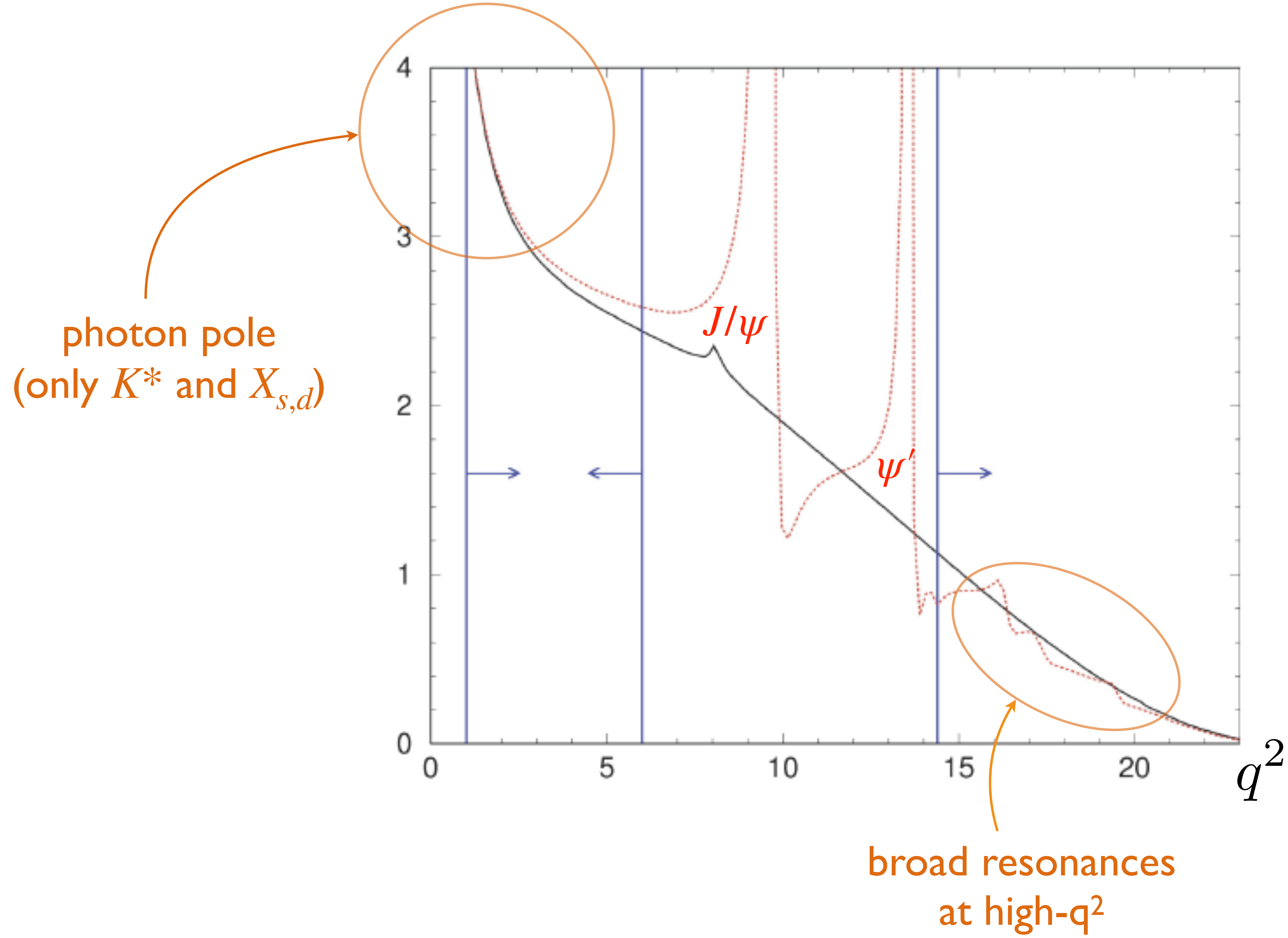
$B \to (K^*, K_1, \rho, X_s, X_d, \ldots)\gamma$

$$Q_2 = (\bar{q}_L \gamma_\mu c_L)(\bar{c}_L \gamma_\mu b_L)$$

charmonium resonances:
$B \to (K^{(*)}, \pi, X_s, X_d, \ldots)(\psi_{cc} \to \ell\ell)$

$b \to s\ell\ell$

- Typical spectrum:



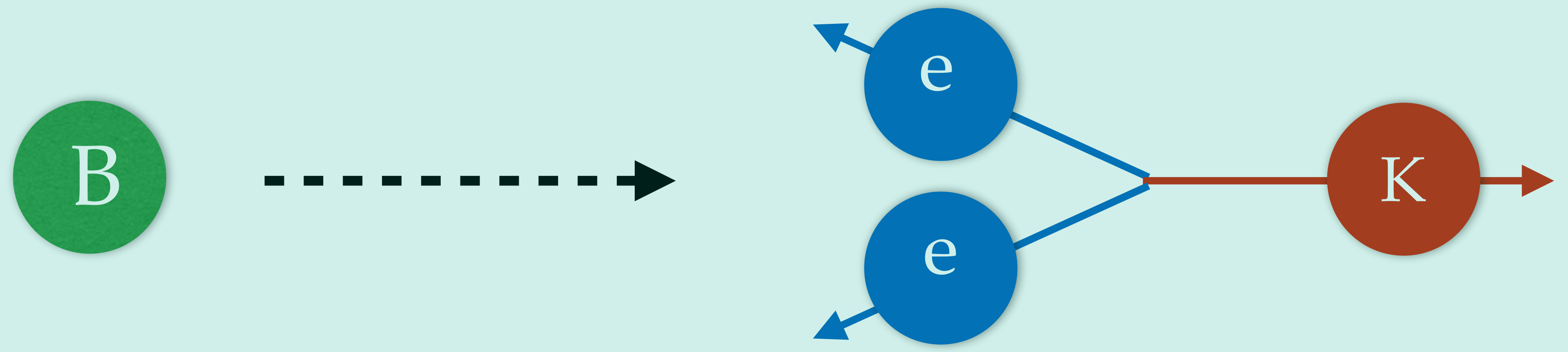
- Intermediate charmonium resonances contribute via:
  $B \to (K, K^*, X_s)\ \psi_{\bar{c}c} \to (K, K^*, X_s)\ \ell^+\ell^-$
- Contributions of J/ψ and ψ' have to be dropped
- Theory at low-q^2 and high-q^2 presents different challenges

$b \to s\ell\ell$: exclusive modes

- The central problem is the calculation of matrix elements:

$$\langle K^{(*)}\ell\ell|O(y)|B\rangle \approx \langle K^{(*)}|T\, J_\mu^{em}(x)\, O(y)|B\rangle$$

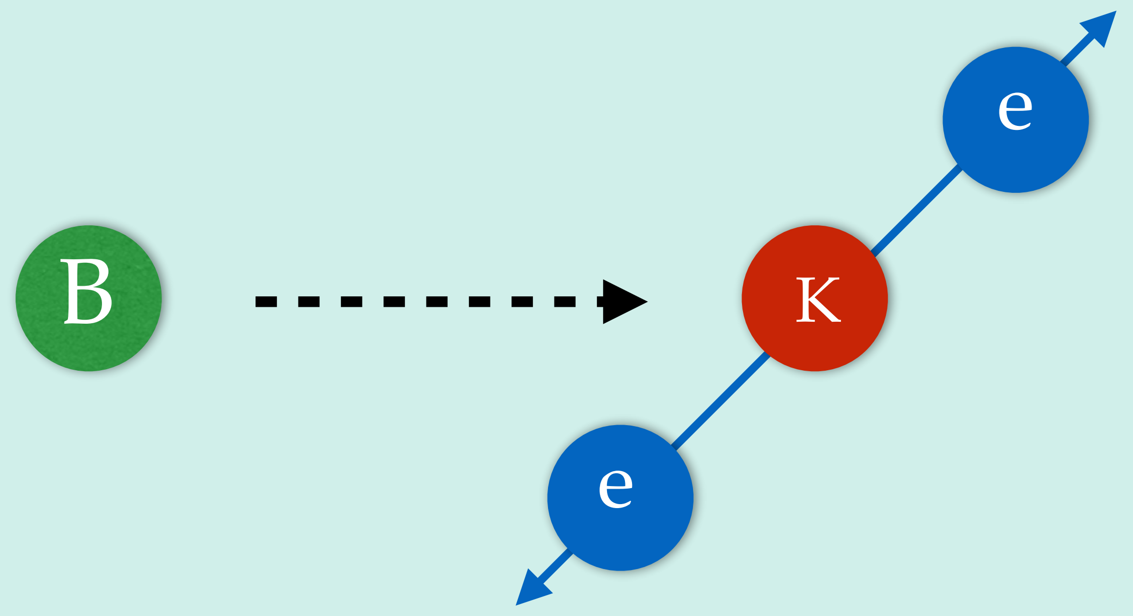
- At low-q^2 the $K^{(*)}$ has large energy:



The large energy of the $K^{(*)}$ introduces three scales: m_b^2, Λm_b and Λ^2

$$\langle K^{(*)}|TJ_\mu^{em}(x)O(y)|B\rangle$$
$$\sim C \times [\,\text{Form Factor} + \phi_B \star J \star \phi_{K^{(*)}}] + O\left(\Lambda/m_b\right)$$

- At high-q^2 the $K^{(*)}$ does not recoil:



We can write an OPE in $1/q^2 \sim 1/m_b^2$ and we have two scales: m_b^2 and Λ^2

$$\langle K^{(*)}|TJ_\mu^{em}(x)O(y)|B\rangle$$
$$\sim C \times [\,\text{Form Factor}] + O\left(\Lambda/m_b\right)$$

dynamical suppression (e.g. end-point overlap)

local

$b \to s\ell\ell$: exclusive modes

- Amplitude for a generic exclusive decay (here we show $B \to K\ell\ell$ at low-q^2):

$$A(B \to K\ell\ell) \sim C_7\, f_T + C_9\, f_+ + C_{10}\, f_+ + \sum_{i\neq 7,9,10} C_i \langle K|TJ_{\rm em}Q_i|B\rangle$$

exact

$$\sim C_7\, f_T + C_9\, f_+ + C_{10}\, f_+ + \sum_{i\neq 7,9,10} C_i \left[A_i^T\, f_T + A_i^+\, f_+ + \phi_B \otimes H_i \otimes \phi_K + O\left(\frac{\Lambda_{\rm QCD}}{m_b}\right)\right]$$

$C_{7,9}^{\rm eff}(q^2)$

non-factorizable corrections

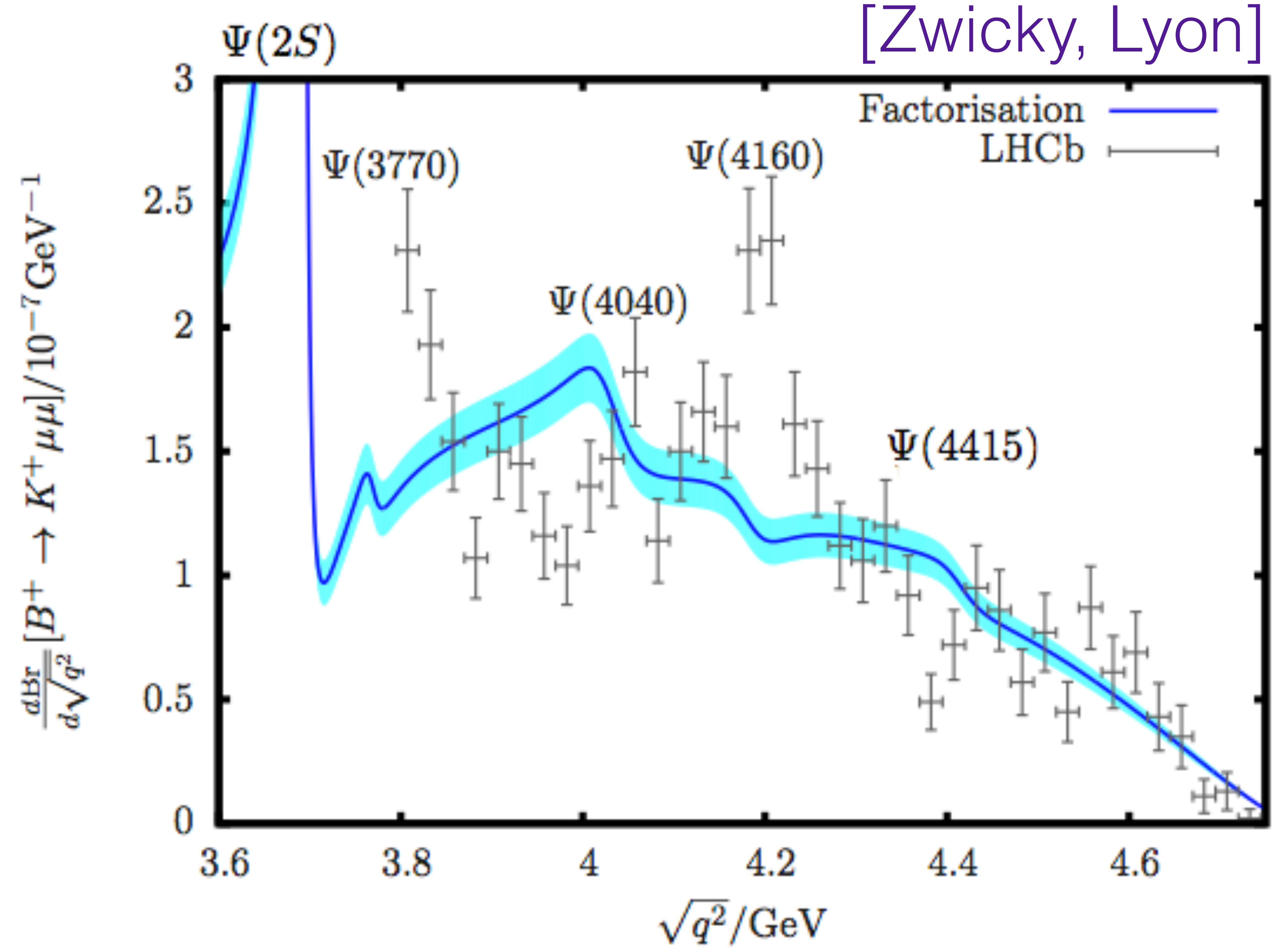
power corrections

- Inputs are form factors (at low and high-q^2), Light-Cone Distribution Amplitudes (LCDA) for strange and bottom mesons (and baryons)
- Every term in the amplitude not proportional to $C_{7,9,10}$ receives $O(10\%)$ power corrections which can only by fitted/parameterized
- See Chris Bouchard's talk

$b \to s\ell\ell$: exclusive modes

- At high-q^2 an OPE in $1/q^2$ allows to write the amplitude entirely in terms of form factors (up to power corrections)
- There are remaining issues related to presence of several broad charmonium resonances:

[Zwicky, Lyon]



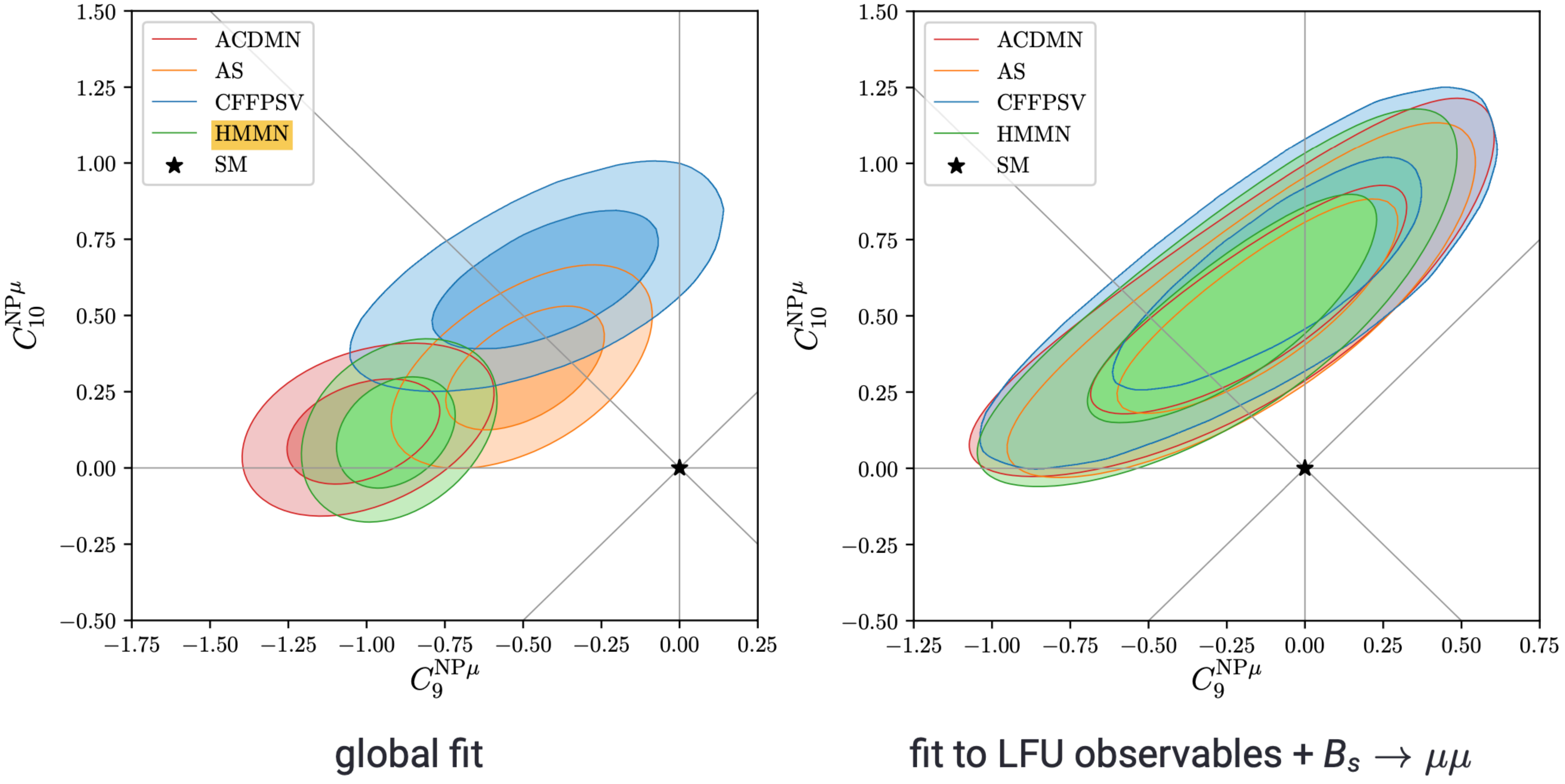
- Nonetheless, low-q^2 observables dominate fits because of the **larger statistics** (about an order of magnitude larger BR)

$b \to s\ell\ell$: global fits

- Experimental inputs:
  - $B \to X_s\gamma$ $\Longleftarrow$ fixes C_7
  - $B \to X_s\ell\ell$
  - $B_{s,d} \to \ell\ell$ $\Longleftarrow$ constrains C_{10}
  - $B \to K^*\gamma$
  - $B \to K\ell\ell$ ($\mathcal{B}_\mu$, R_K, angular ob.)
  - $B \to K^*\ell\ell$ ($\mathcal{B}_\mu$, R_{K^*}, angular ob.)
  - $B_s \to \phi\mu\mu$ ($\mathcal{B}$, angular ob.)
  - $\Lambda_b \to \Lambda\mu\mu$ ($\mathcal{B}$, angular ob.)
- Theoretical inputs:
  - High-q^2 FF from lattice if available
  - Low-q^2 FF usually from a combined z-fit to lattice and LCSR results
  - Moments of final state light meson/baryon LCDA from lattice if available or various asymptotic estimates
  - Moments of b-hadron LCDA from HQET
  - Power corrections are parameterized
- Many fitter groups obtain somewhat different results based on various assumptions

$b \to s\ell\ell$: global fits

[Capdevila, Fedele, Neshatpour, Stangl, Flavour Anomaly Workshop, 20 October 2021]



- **ACDMN** (Algueró, Capdevila, Descotes-Genon, Matias, Novoa-Brunet)
- **AS** (Altmannshofer, Stangl) - **Flavio**
- **CFFPSV** (Ciuchini, Fedele, Franco, Paul, Silvestrini, Valli) - **HEPfit**
- **HMMN** (Hurth, Mahmoudi, Martínez-Santos, Neshatpour) - **SuperIso**

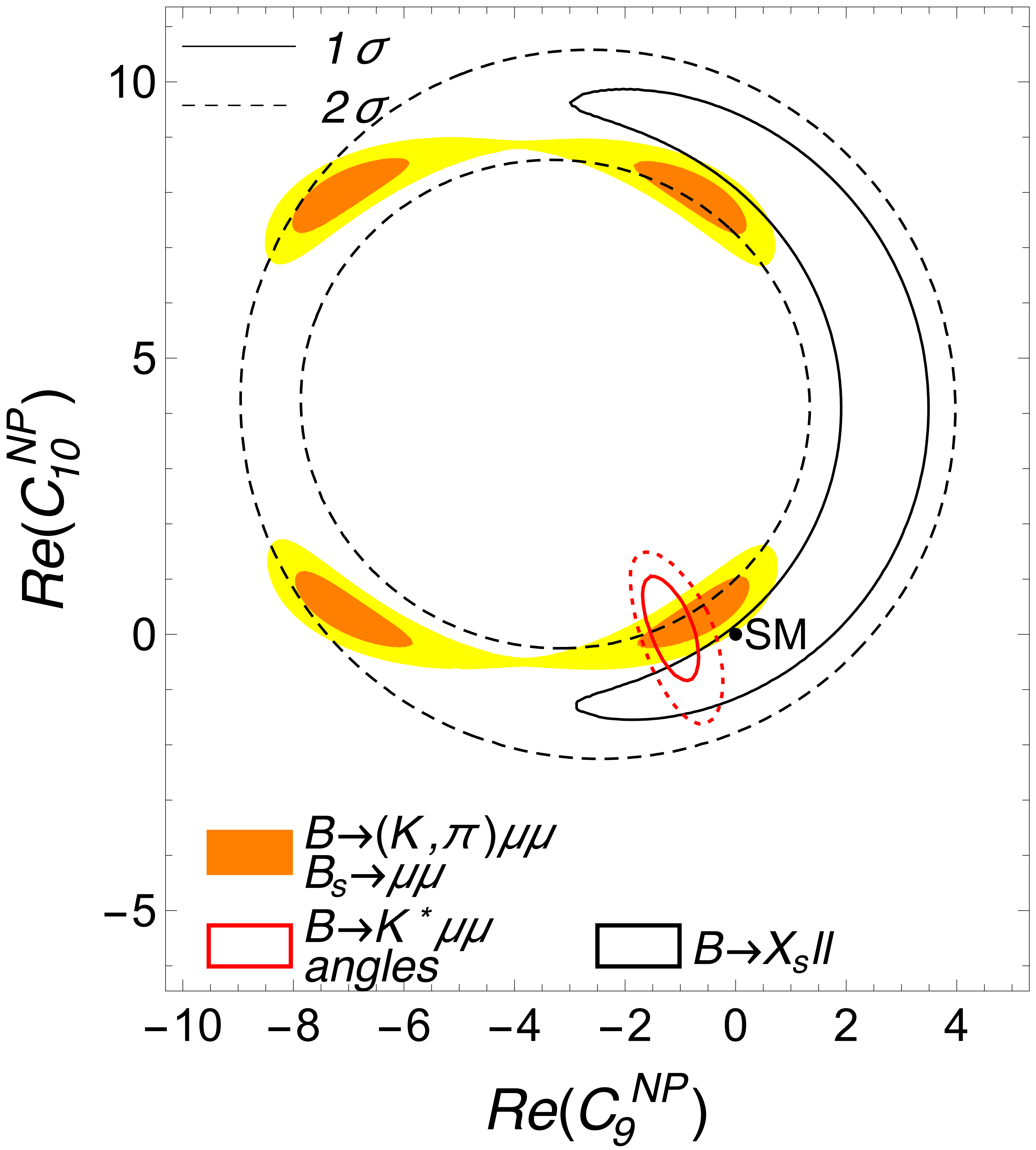
- Flavio, HEPfit and SuperIso are public codes

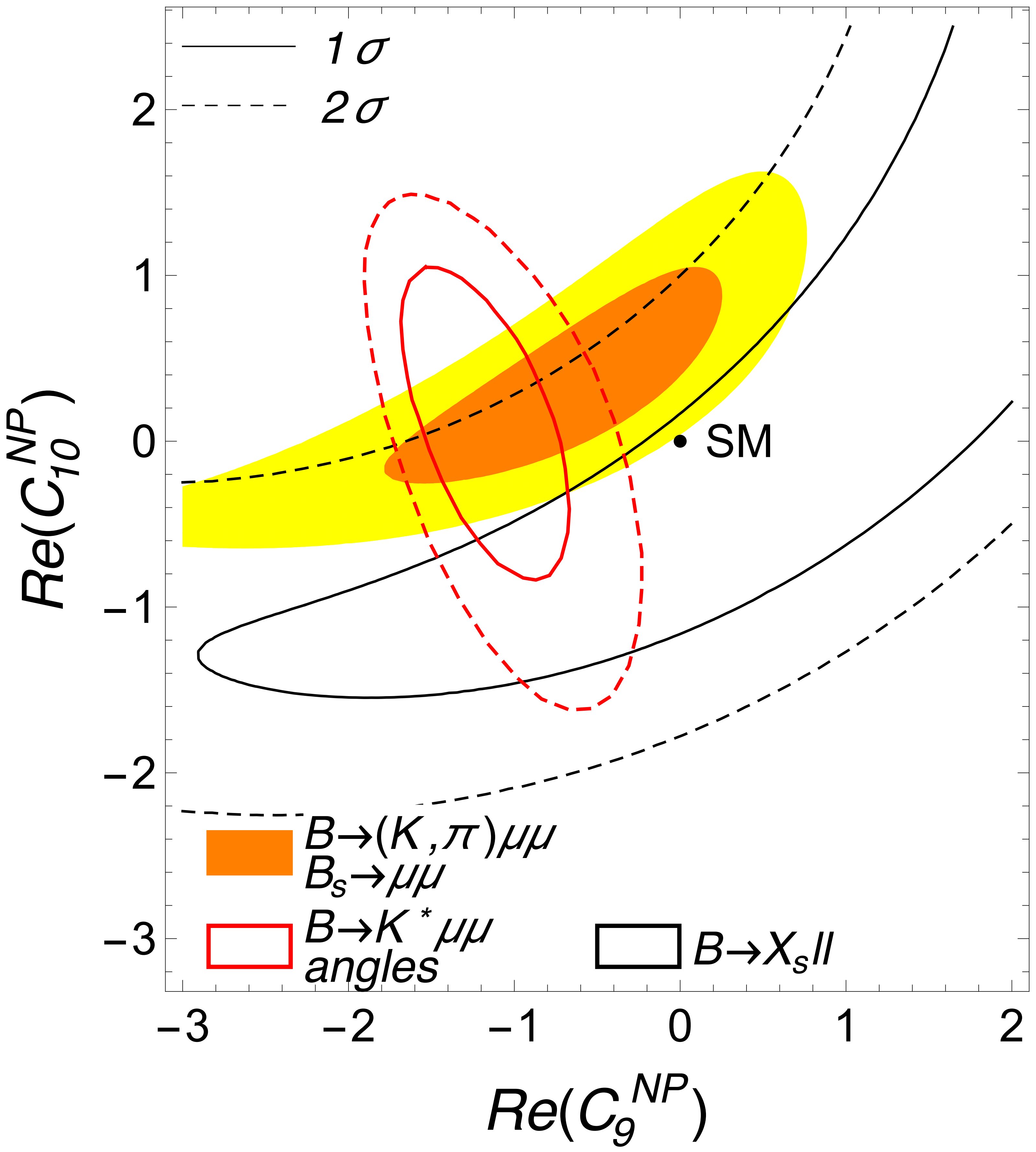
- Obvious agreement on LFU violating observables (theoretically clean)
- Inter-group tensions in the global fit somewhat disturbing

$b \to s\ell\ell$: global fits

- "Lattice-only" fit

[1510.02349; Fermilab/MILC, EL]





- See **Chris Bouchard**'s talk for an update on $B \to K$ form factors from Fermilab/MILC and their implications for phenomenology

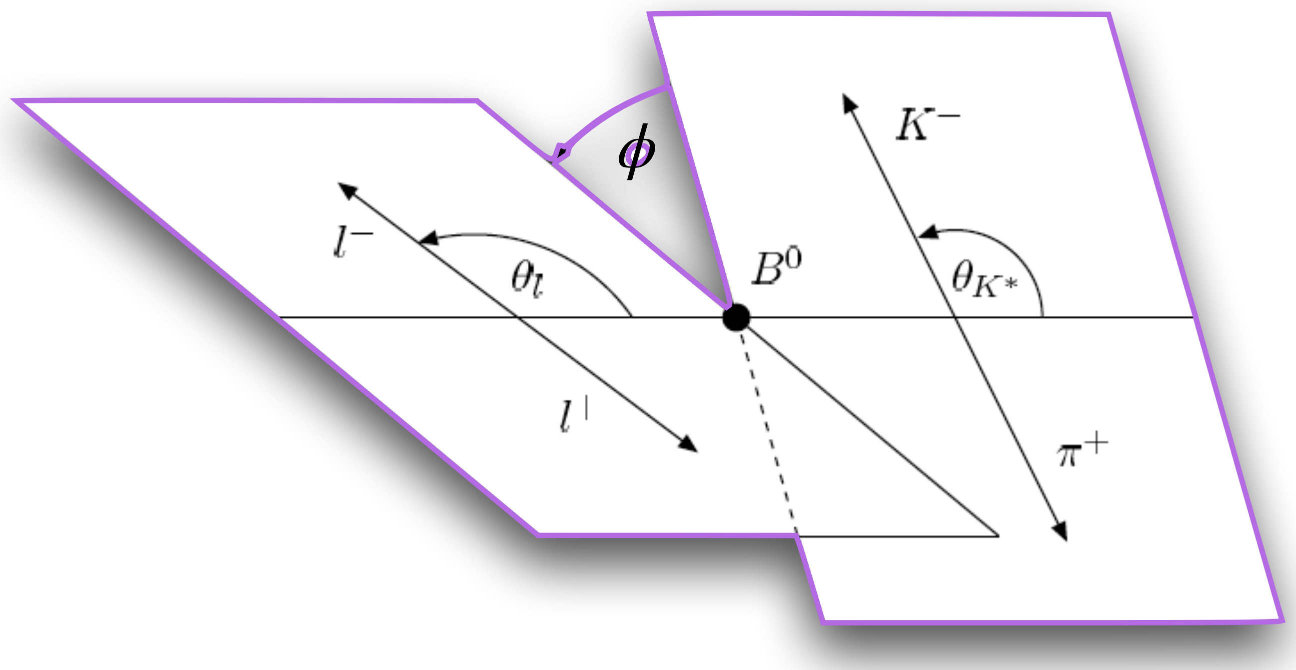
- Note the essential role of $B \to K^*\ell\ell$ observables (especially asymmetries) in "establishing" the anomaly.

- $B \to K^*\ell\ell$ fit results taken from [1503.06199; Altmannshofer, Straub]

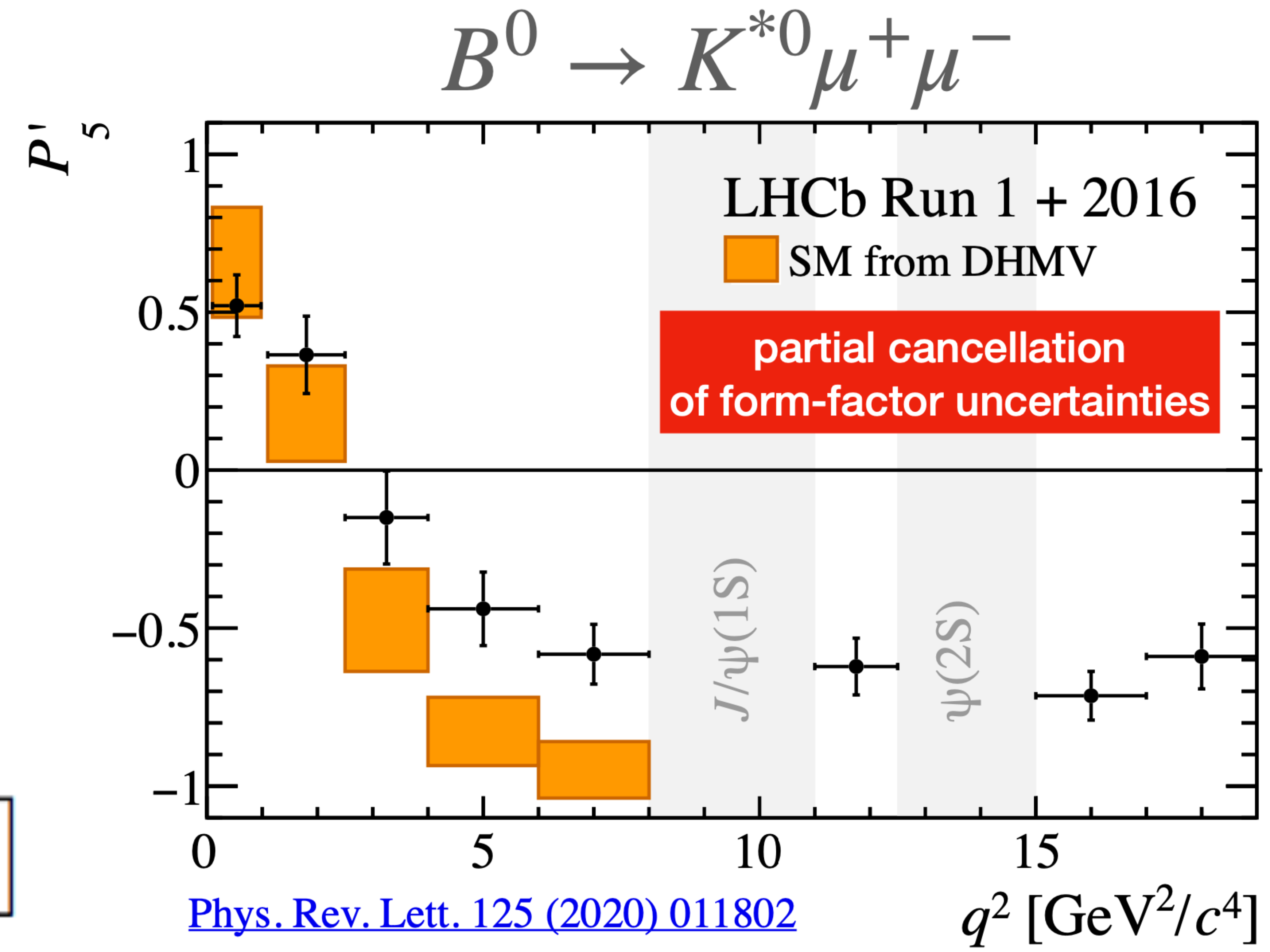
$b \to s\ell\ell$: global fits

- There are "clean observables" and there are "clean observables!"
- Ratios like P_5' are "clean" because all form factor dependence cancels at leading order in α_s and at leading power in $1/m_b$ *as long as* not only the amplitude but also the FF's are factorized; thus exposing the observable to potentially larger power corrections.

$$\frac{1}{\mathrm{d}(\Gamma+\bar\Gamma)/\mathrm{d}q^2}\frac{\mathrm{d}^3(\Gamma+\bar\Gamma)}{\mathrm{d}\vec\Omega}\bigg|_{\mathrm{P}} = \frac{9}{32\pi}\Big[\tfrac{3}{4}(1-F_{\mathrm{L}})\sin^2\theta_K + F_{\mathrm{L}}\cos^2\theta_K + \tfrac{1}{4}(1-F_{\mathrm{L}})\sin^2\theta_K\cos 2\theta_l - F_{\mathrm{L}}\cos^2\theta_K\cos 2\theta_l + S_3\sin^2\theta_K\sin^2\theta_l\cos 2\phi + S_4\sin 2\theta_K\sin 2\theta_l\cos\phi + S_5\sin 2\theta_K\sin\theta_l\cos\phi + \tfrac{4}{3}A_{\mathrm{FB}}\sin^2\theta_K\cos\theta_l + S_7\sin 2\theta_K\sin\theta_l\sin\phi + S_8\sin 2\theta_K\sin 2\theta_l\sin\phi + S_9\sin^2\theta_K\sin^2\theta_l\sin 2\phi\Big]$$



$$P'_{i=4,5,6,8} = \frac{S_{j=4,5,7,8}}{\sqrt{F_{\mathrm{L}}(1-F_{\mathrm{L}})}}$$



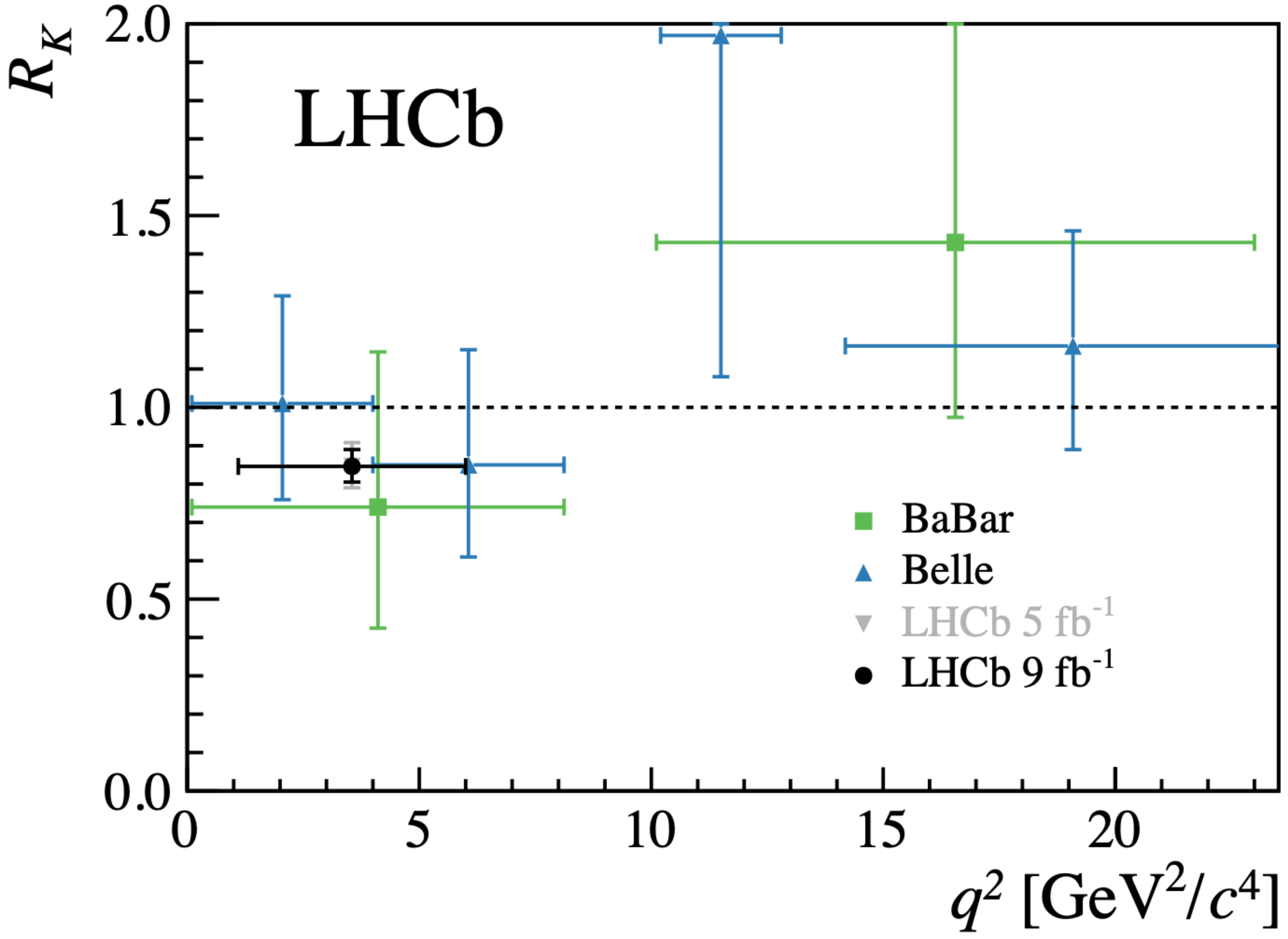
Phys. Rev. Lett. 125 (2020) 011802

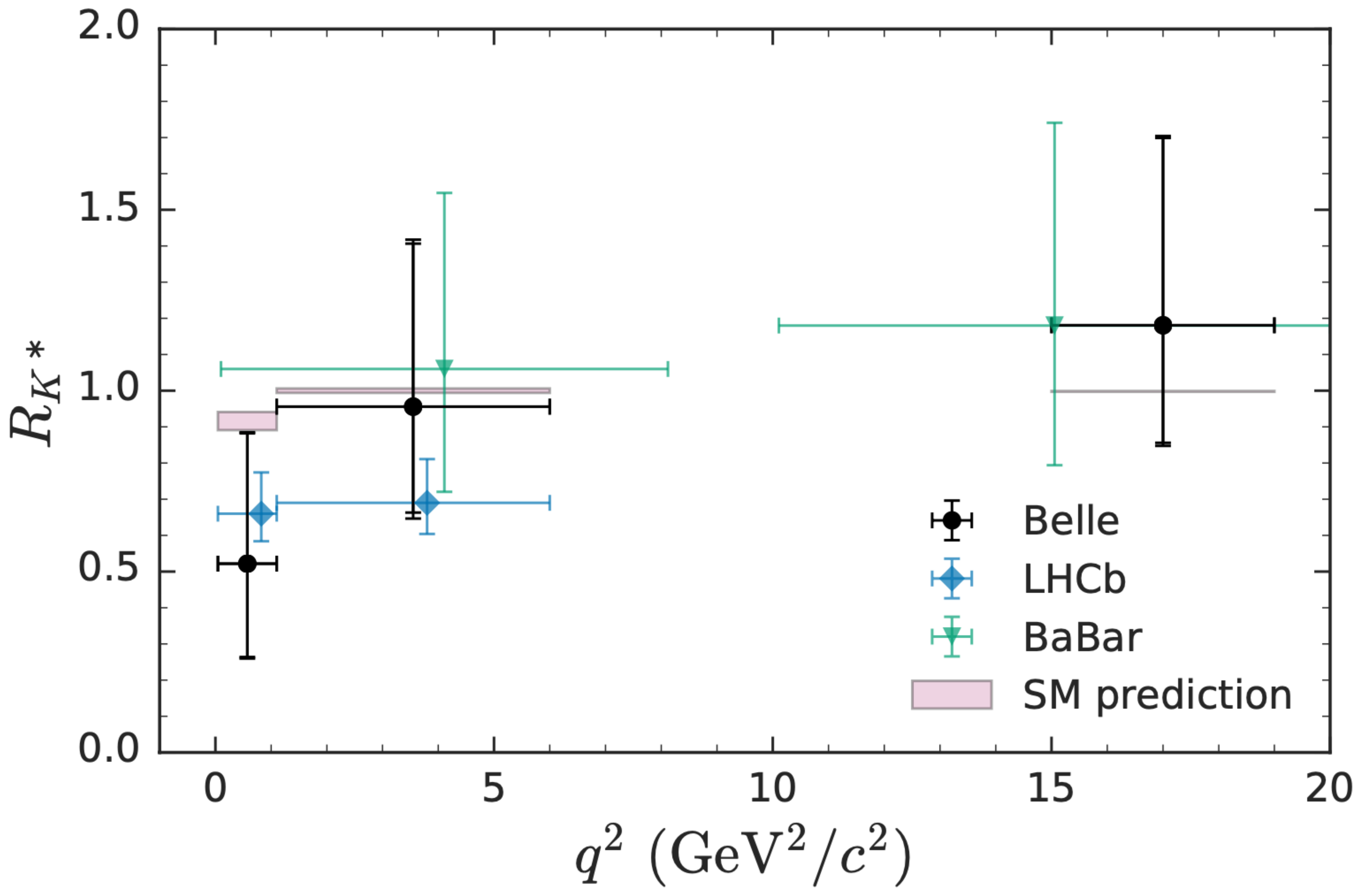
[Lisovskyi, talk@Lattice22]

$b \to s\ell\ell$: global fits

- There are "clean observables" and there are "clean observables!"
- Ratios like R_K are truly clean and phenomenological implications of observing Lepton Flavor Universality Violation would be really staggering!

$$R_K = \frac{\mathcal{B}(B^+ \to K^+\mu\mu)}{\mathcal{B}(B^+ \to K^+ee)} = 1 + \mathcal{O}(10^{-4})$$

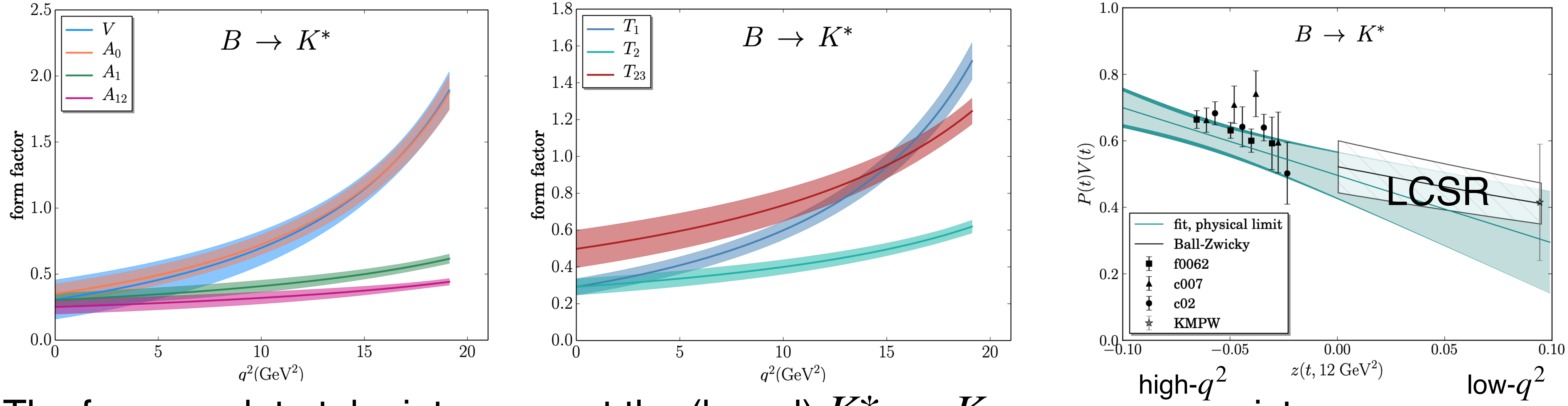




[Capdevila, Fedele, Neshatpour, Stangl, Flavour Anomaly Workshop, 20 October 2021]

$b \to s\ell\ell$: exclusive lattice wish list

- $B \to K^*$ form factors (also $B_s \to \phi$)
- Existing calculation from: Horgan, Liu, Meinel, Wingate [1310.3722, 1501.00367]



- The framework to take into account the (broad) $K^* \to K\pi$ resonance exists:

[587] M.T. Hansen and S.R. Sharpe, *Multiple-channel generalization of Lellouch-Luscher formula*, *Phys. Rev.* **D86** (2012) 016007 [1204.0826].

[588] R.A. Briceno and Z. Davoudi, *Moving multichannel systems in a finite volume with application to proton-proton fusion*, *Phys. Rev.* **D88** (2013) 094507 [1204.1110].

[589] [HS 14] J. J. Dudek, R.G. Edwards, C.E. Thomas and D.J. Wilson, *Resonances in coupled $\pi K - \eta K$ scattering from quantum chromodynamics*, *Phys. Rev. Lett.* **113** (2014) 182001 [1406.4158].

[590] R.A. Briceño, M.T. Hansen and A. Walker-Loud, *Multichannel 1 → 2 transition amplitudes in a finite volume*, *Phys. Rev. D* **91** (2015) 034501 [1406.5965].

[591] R.A. Briceño and M.T. Hansen, *Multichannel 0 → 2 and 1 → 2 transition amplitudes for arbitrary spin particles in a finite volume*, *Phys. Rev. D* **92** (2015) 074509 [1502.04314].

$b \to s\ell\ell$: exclusive lattice wish list

- $B \to K^*$ form factors (also $B_s \to \phi$)
- Having the 7 form factors at hand is essential to properly discuss "clean observables" like P_5'
- Another very interesting use of the form factors is to test FF factorization and "establish" the size of power corrections:

$$\frac{m_B}{m_B + m_K} f_T = f_+ \left[1 + \tilde{\alpha}_s C_F \left(\log \frac{m_b^2}{\mu^2} + 2L\right)\right] - \frac{\pi}{N_c} \frac{f_B f_K}{E} \alpha_s C_F \overbrace{\int \frac{d\omega}{\omega} \Phi_{B,+}(\omega)}^{\lambda_{B,+}^{-1}} \int_0^1 \frac{du}{\bar{u}} \Phi_K(u)$$

$$\frac{m_B}{m_B + m_{K^*}} V(q^2) = \frac{m_B + m_{K^*}}{2E} A_1(q^2) = T_1(q^2) = \frac{m_B}{2E} T_2(q^2) = \xi_\perp(E),$$

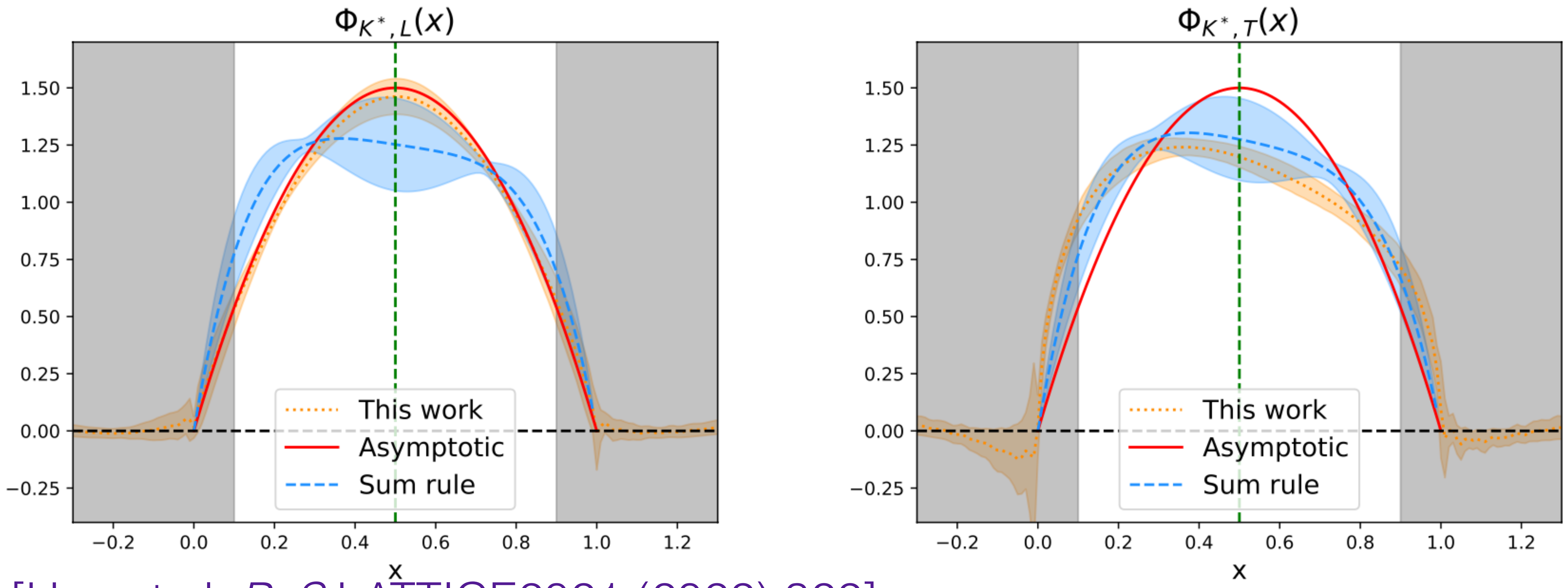
$$\frac{m_{K^*}}{E} A_0(q^2) = \frac{m_B + m_{K^*}}{2E} A_1(q^2) - \frac{m_B - m_{K^*}}{m_B} A_2(q^2) = \frac{m_B}{2E} T_2(q^2) - T_3(q^2) = \xi_\parallel(E)$$

From 7 to 2 form factors!

$$P_5' = \frac{C_{10}(C_{9\perp} + C_{9\parallel})}{\sqrt{(C_{9\parallel}^2 + C_{10}^2)(C_{9\perp}^2 + C_{10}^2)}} + O(\alpha_s, \Lambda/m_b)$$

$b \to s\ell\ell$: exclusive lattice wish list

- Mesons LCDA
- Light mesons LCDAs (π, K, ρ, K^*, ϕ) have known asymptotic ($\mu \to \infty$) behavior
  - First few Gegenbauer moments have been calculated (e.g. RBC/UKQCD 1011.5906)
  - Whole LCDA can be obtained using the LaMET:



[Hua et al, *PoS* LATTICE2021 (2022) 322]

  - Whole function is more important for vector mesons rather than pseudoscalar

$b \to s\ell\ell$: exclusive lattice wish list

- Mesons LCDA
- B meson LCDAs, $\phi_B^{\pm}(\omega,\mu)$, are poorly known (besides their RG running and a constraint in the Wandzura-Wilczek limit)
  - The first negative moment of ϕ_B^+ (λ_B^{-1}) can be extracted by future Belle II measurements of $B \to \gamma\ell\nu$
  - Determination from first principles would be extremely useful:

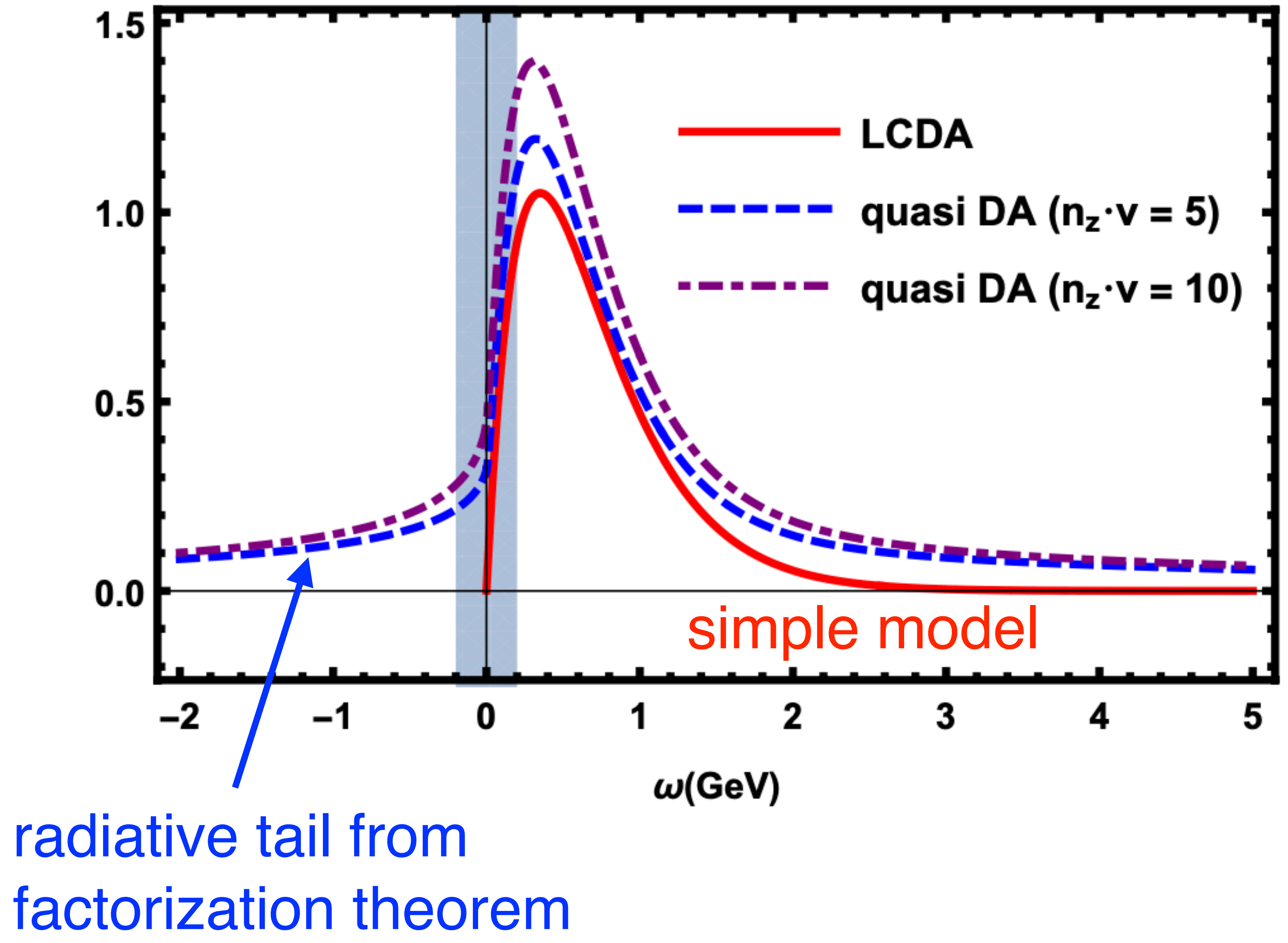
[330] H. Kawamura and K. Tanaka, *Coordinate-space calculation of radiative corrections to the B-meson distribution amplitudes: light-cone vs. static distributions*, *PoS* **RADCOR2017** (2018) 076.

[331] W. Wang, Y.-M. Wang, J. Xu and S. Zhao, *B-meson light-cone distribution amplitude from Euclidean quantities*, *Phys. Rev. D* **102** (2020) 011502 [1908.09933].

[332] S. Zhao and A.V. Radyushkin, *B-meson Ioffe-time distribution amplitude at short distances*, *Phys. Rev. D* **103** (2021) 054022 [2006.05663].

[Di Canto, Meinel, 2208.05403]

[Wang et al., 1908.09933]

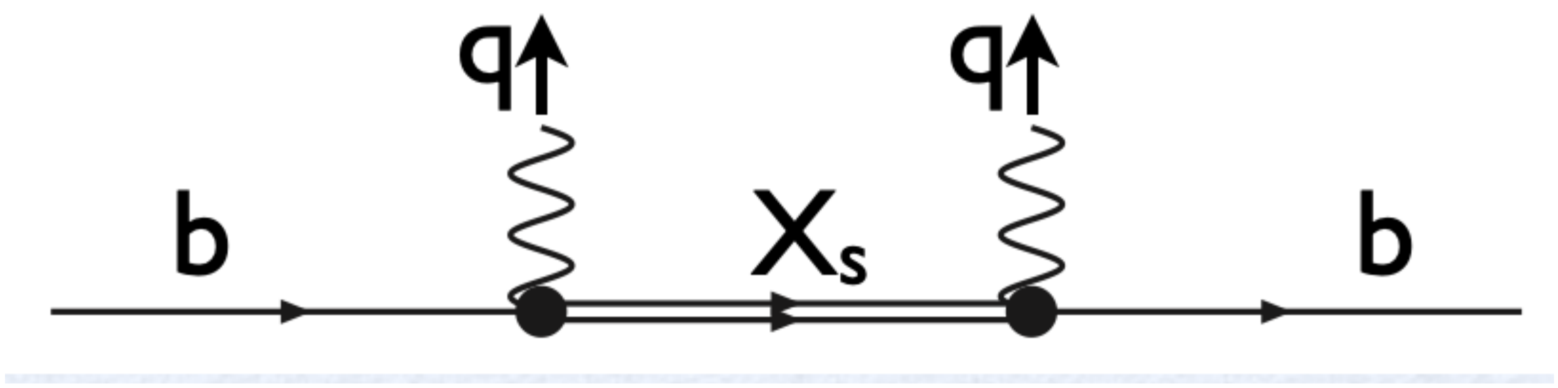


$b \to s\ell\ell$: inclusive

- Standard OPE with many asterisks:

$$\Gamma\left[\bar{B} \to X_s \ell^+\ell^-\right] = \Gamma\left[\bar{b} \to X_s \ell^+\ell^-\right] + O\left(\frac{\Lambda^2_{QCD}}{m_b^2}, \frac{\Lambda^3_{QCD}}{m_b^3}, \frac{\Lambda^2_{QCD}}{m_c^2}, \ldots\right)$$

* OPE breaks down near $q^2 \sim m_b^2$



$$p^2_{X_s} = (p_b - q)^2 = m_b^2 + q^2 - 2m_b q_0 < m_b^2 + q^2 - 2m_b\sqrt{q^2} = \left(m_b - \sqrt{q^2}\right)^2$$

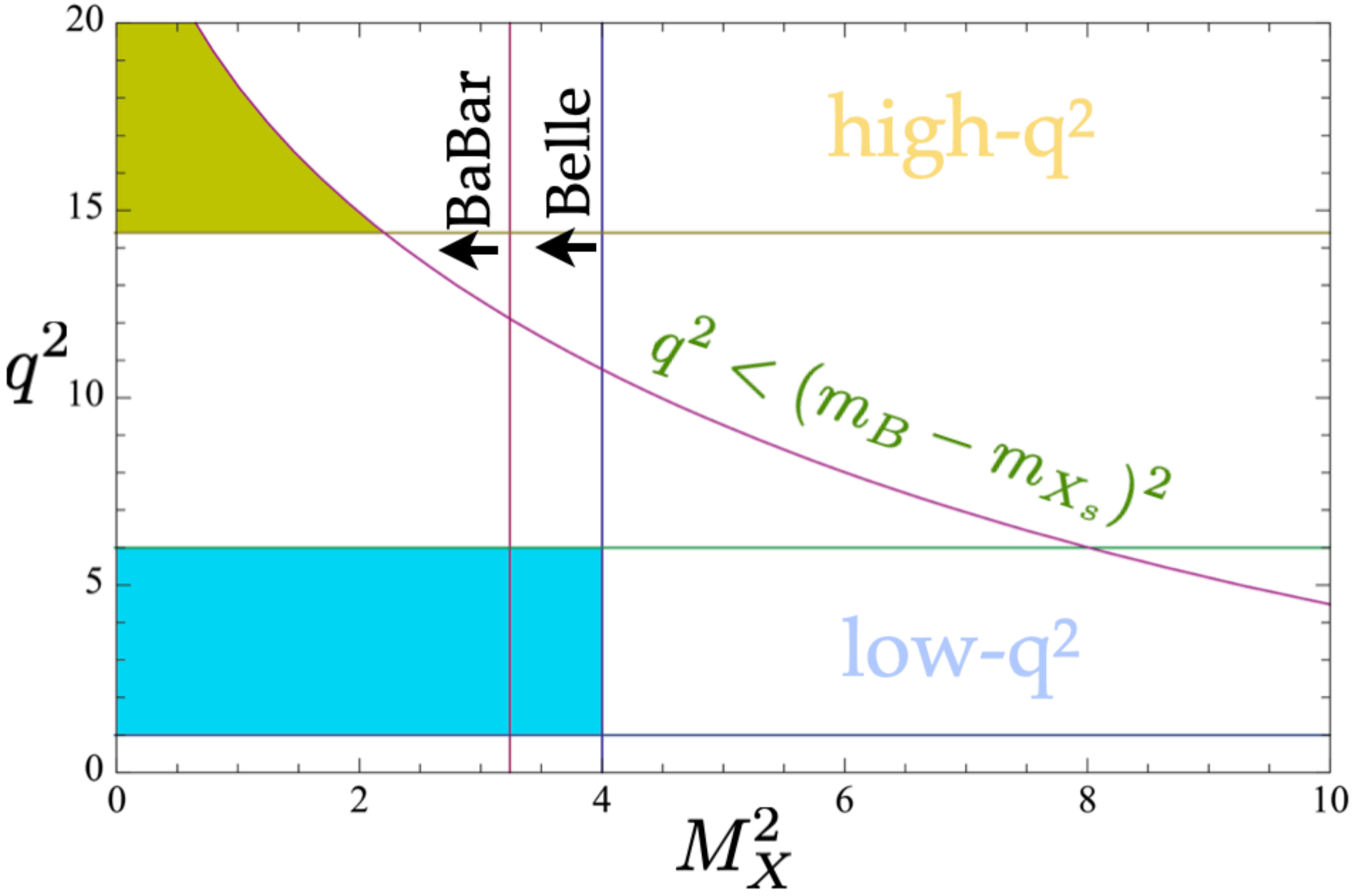
Solution: normalize to $b \to u\ell\nu$ with same q^2 cut: $\mathcal{R}(s_0) = \frac{\int_{\hat{s}_0}^1 d\hat{s} \frac{\mathrm{d}\Gamma(\bar{B} \to X_s \ell^+\ell^-)}{d\hat{s}}}{\int_{\hat{s}_0}^1 d\hat{s} \frac{\mathrm{d}\Gamma(\bar{B}^0 \to X_u \ell\nu)}{d\hat{s}}}$ [Ligeti et al]

$b \to s\ell\ell$: inclusive

- Standard OPE with many asterisks:

$$\Gamma\left[\bar{B} \to X_s \ell^+ \ell^-\right] = \Gamma\left[\bar{b} \to X_s \ell^+ \ell^-\right] + O\left(\frac{\Lambda^2_{QCD}}{m_b^2}, \frac{\Lambda^3_{QCD}}{m_b^3}, \frac{\Lambda^2_{QCD}}{m_c^2}, \ldots\right)$$

* At low-q^2 experimental upper cuts on M_X are required to eliminate various backgrounds:



- Note that $M_X^2 \simeq \Lambda_{\rm QCD} m_b$ and use SCET:

$$\frac{d^2\Gamma(B \to X_s \ell\ell)}{dq^2 dM_X^2} \sim F(q^2) \otimes \frac{d^2\Gamma(b \to X_s \ell\ell)}{dq^2 d\hat{M}_X^2}$$

hadronic — q^2 dependent Shape Function — partonic

- Normalize to the semileptonic rate with the same M_X cut:
$\Gamma^{\rm cut}_{\rm low}(B \to X_s \ell\ell)/\Gamma^{\rm cut}_{\rm low}(B \to X_u \ell\nu)$

$b \to s\ell\ell$: inclusive

- State of art SM predictions [Huber, Hurth, Jenkins, EL, Qin, Vos, 2007.04191]

$$\mathcal{B}[1,6]_{ee} = (1.78 \pm 0.08_{\rm scale} \pm 0.02_{m_t} \pm 0.04_{C,m_c} \pm 0.02_{m_b} \pm 0.01_{\alpha_s} \pm 0.002_{\rm CKM} \pm 0.03_{\rm BR_{sl}} \pm 0.01_{\lambda_2} \pm 0.09_{\rm resolved}) \cdot 10^{-6}$$
$$= 1.78\,(1 \pm 7.5\%) \cdot 10^{-6}$$

essentially irreducible

$$\mathcal{B}[>14.4]_{ee} = (2.04 \pm 0.28_{\rm scale} \pm 0.02_{m_t} \pm 0.03_{C,m_c} \pm 0.19_{m_b} \pm 0.002_{\rm CKM} \pm 0.03_{\rm BR_{sl}} \pm 0.01_{\alpha_s} \pm 0.13_{\lambda_2} \pm 0.57_{\rho_1} \pm 0.54_{f_{u,s}}) \cdot 10^{-7}$$
$$= 2.04\,(1 \pm 46\%) \cdot 10^{-7}$$

OPE break down

$$\mathcal{R}(14.4)_{ee} = (2.25 \pm 0.12_{\rm scale} \pm 0.03_{m_t} \pm 0.02_{C,m_c} \pm 0.01_{m_b} \pm 0.01_{\alpha_s} \pm 0.20_{\rm CKM} \pm 0.02_{\lambda_2} \pm 0.14_{\rho_1} \pm 0.08_{f_u^0+f_s} \pm 0.12_{f_u^0-f_s}) \cdot 10^{-3}$$
$$= 2.25\,(1 \pm 14\%) \cdot 10^{-3}$$

local power corrections under control

- Dominant uncertainty at low-q^2 are scale and resolved: very hard to improve
- Larger scale uncertainties at high-q^2 are connected to the OPE breakdown
- Branching ratio at high-q^2 has enormous uncertainties from $1/m_b^3$ power corrections
- Both scale and power corrections uncertainties are brought under control when considering the ratio $\mathcal{R}(s_0)$

$b \to s\ell\ell$: inclusive

$\alpha_s(M_z) = 0.1181(11)$	$m_e = 0.51099895$ MeV
$\alpha_e(M_z) = 1/127.955$	$m_\mu = 105.65837$ MeV
$s_W^2 \equiv \sin^2\theta_W = 0.2312$	$m_\tau = 1.77686$ GeV
$\lvert V_{ts}^* V_{tb}/V_{cb}\rvert^2 = 0.96403(87)$ [118]	$\overline{m}_c(\overline{m}_c) = 1.275(25)$ GeV
$\lvert V_{ts}^* V_{tb}/V_{ub}\rvert^2 = 123.5(5.3)$ [118]	$m_b^{1S} = 4.691(37)$ GeV [119, 120]
$\lvert V_{td}^* V_{tb}/V_{cb}\rvert^2 = 0.04195(78)$ [118]	$\lvert V_{us}^* V_{ub}/(V_{ts}^* V_{tb})\rvert = 0.02022(44)$ [118]
$\lvert V_{td}^* V_{tb}/V_{ub}\rvert^2 = 5.38(26)$ [118]	$\arg\left[V_{us}^* V_{ub}/(V_{ts}^* V_{tb})\right] = 115.3(1.3)^\circ$ [118]
$\mathcal{B}(B \to X_c e\bar{\nu})_{\rm exp} = 0.1065(16)$ [121]	$\lvert V_{ud}^* V_{ub}/(V_{td}^* V_{tb})\rvert = 0.420(10)$
$m_B = 5.2794$ GeV	$\arg\left[V_{ud}^* V_{ub}/(V_{td}^* V_{tb})\right] = -88.3(1.4)^\circ$
$M_Z = 91.1876$ GeV	$m_{t,{\rm pole}} = 173.1(0.9)$ GeV
$M_W = 80.379$ GeV	$C = 0.568(7)(10)$ [122]
$\mu_b = 5^{+5}_{-2.5}$ GeV	$\mu_0 = 120^{+120}_{-60}$ GeV
$f_{\rm NV} = (0.02 \pm 0.16)$ GeV^3	$\lambda_2^{\rm eff} = 0.130(21)$ GeV^2 [48]
$f_{\rm V} - f_{\rm NV} = (0.041 \pm 0.052)$ GeV^3	$\lambda_1 = -0.267(90)$ GeV^2 [48]
$[\delta f]_{SU(3)} = (0 \pm 0.04)$ GeV^3	$\rho_1 = 0.038(70)$ GeV^3 [48]
$[\delta f]_{SU(2)} = (0 \pm 0.004)$ GeV^3	

λ_1 and ρ_1 have been extracted from $B \to X_c \ell \nu$ moments [Gambino, Healey, Turczyk, 1606.06174]

Weak annihilation contributions are defined as:

$$f_q^{0,\pm} \equiv \frac{1}{2m_B}\langle B^{0,\pm}\,|\,Q_1^q - Q_2^q\,|\,B^{0,\pm}\rangle$$

$$Q_1^q = \bar{h}_v \gamma_\mu (1-\gamma_5) q \; \bar{q}\gamma^\mu (1-\gamma_5) h_v \,,$$

$$Q_2^q = \bar{h}_v (1-\gamma_5) q \; \bar{q}(1+\gamma_5) h_v \,.$$

In the isospin SU(3) limit there are only two WA matrix elements:

$$f_{\rm V} \equiv f_u^\pm \overset{SU(2)}{=} f_d^0$$

$$f_{\rm NV} \equiv f_u^0 \overset{SU(2)}{=} f_d^\pm \overset{SU(3)}{=} f_s^0 \overset{SU(2)}{=} f_s^\pm$$

$f_{\rm V}$ and $f_{\rm NV}$ extracted from $D_{(s)}$ decays and rescaled by $m_B f_B^2/(m_D f_D^2)$ [Gambino, Healey, Turczyk, 1606.06174]

SU(N) breaking (δf) is an order of magnitude estimate.

$b \to s\ell\ell$: inclusive lattice wish list

- Is it possible to study Weak Annihilation matrix elements on the lattice?
- B-mixing matrix elements:

$$\langle \bar{B}^0 | (\bar{b}_L \gamma_\mu d_L)(\bar{b}_L \gamma^\mu d_L) | B^0 \rangle \Longrightarrow f_B^2 \, B$$

- Weak annihilation (Valence):

$$\langle B^0 | (\bar{d}_L \gamma_\mu b_L)(\bar{b}_L \gamma^\mu d_L) | B^0 \rangle \Longrightarrow f_d^0 \equiv f_V$$

$$\langle B^+ | (\bar{u}_L \gamma_\mu b_L)(\bar{b}_L \gamma^\mu u_L) | B^+ \rangle \Longrightarrow f_u^+ \overset{SU(2)}{=} f_V$$

- Weak annihilation (Non Valence):

$$\langle B^0 | (\bar{u}_L \gamma_\mu b_L)(\bar{b}_L \gamma^\mu u_L) | B^0 \rangle \Longrightarrow f_u^0 \equiv f_{NV} \overset{SU(2)}{=} f_d^\pm \overset{SU(3)}{=} f_s^0 \overset{SU(2)}{=} f_s^+$$

$b \to s\ell\ell$: inclusive

- Something was swept under the rug…
- SF needed for extrapolation in m_{X_s} and to improve the EvtGen Monte Carlo event generator which is the heart of Belle, BaBar and Belle II analyses.
  [EvtGen: Ryd, Lange, Kuznetsova, Versille, Rotondo, Kirkby, Wuerthwein, Ishikawa; Maintained by J. Back, M. Kreps and T. Latham at University of Warwick]
- Hadronic spectrum is based on the Fermi motion implementation presented in Ali, Hiller, Handoko, Morozumi hep-ph/9609449:

$$\frac{d\Gamma_B}{ds\,du\,dp} = \int du' \frac{m_b(p)^2}{m_B} p \left[\frac{4}{\sqrt{\pi} p_F^3} \exp(-p^2/p_F^2)\right] \left(u'^2 + 4 m_b(p)^2 s\right)^{-1/2} \left[\frac{d\Gamma_b}{ds\,du}\right]_{m_b \to m_b(p)}$$

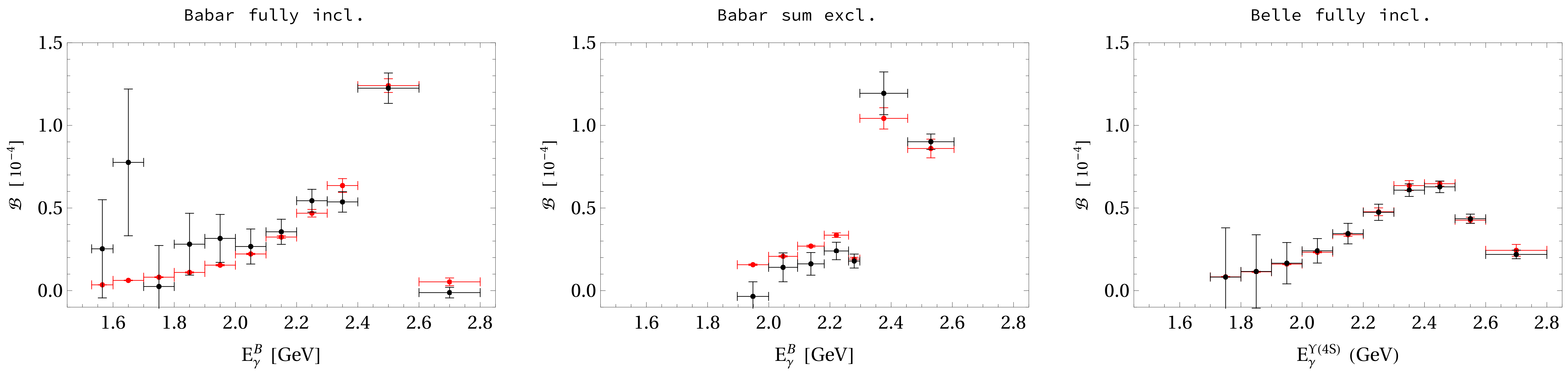
parton level with momentum dependent b mass

- We need to urgently update the code!
- Complete triple differential rate at $O(\alpha_s)$
  [T. Huber, T. Hurth, J. Jenkins, EL to appear]

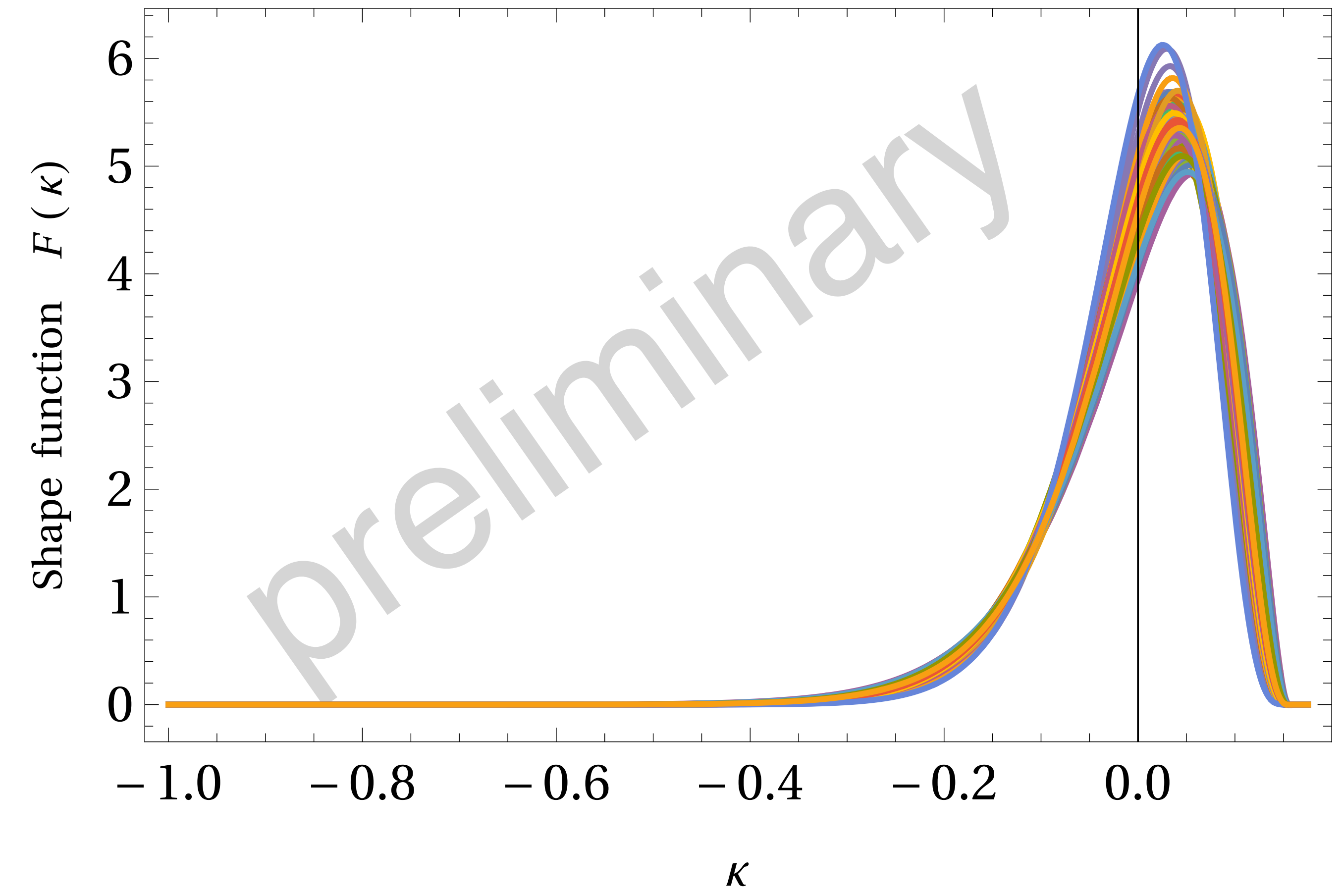
```
{
  pb = _calcprob->FermiMomentum(_pf);

  // effective b-quark mass
  mb = mB*mB + _mq*_mq - 2.0*mB*sqrt(pb*pb + _mq*_mq);
  if ( mb>0. && sqrt(mb)-_ms  < 2.0*ml ) mb= -10.;
}
mb = sqrt(mb);
```

$b \to s\ell\ell$: inclusive lattice wish list

- The $b \to s\ell\ell$ Shape Functions (q^2 dependent) are connected to the $b \to s\gamma$ one



- A Neural Network fit yields:
  [Gambino, EL, Misiak, Schacht, to appear]



- Factorization theorem:

$$\frac{d\Gamma}{dE_\gamma} = \Gamma_0 \sum_{i \le j=1}^{8} C_i^{\text{eff}\,*}(\mu_b) C_j^{\text{eff}}(\mu_b) \int_{-\infty}^{\lambda} d\kappa\, F(\kappa,\mu) W_{ij}^{pert}(\xi-\kappa,\mu,\mu_b)$$

- Similar analysis done by SIMBA
  [Bernlochner, Lacker, Ligeti, Stewart, F. Tackmann, K. Tackmann]

$b \to s\ell\ell$: inclusive

- Is it possible to gain information on B meson Shape Function on the lattice?
- The leading Shape Function is defined as:

$$S(\omega, \mu) = \frac{1}{2m_B} \langle B | \bar{h}_v \, \delta(iD_+ - m_B + m_b + \omega) \, h_v | B \rangle$$

- Note that the first few moments of the Shape function are directly related to the matrix elements of $1/m_b^2$ and $1/m_b^3$ operators:

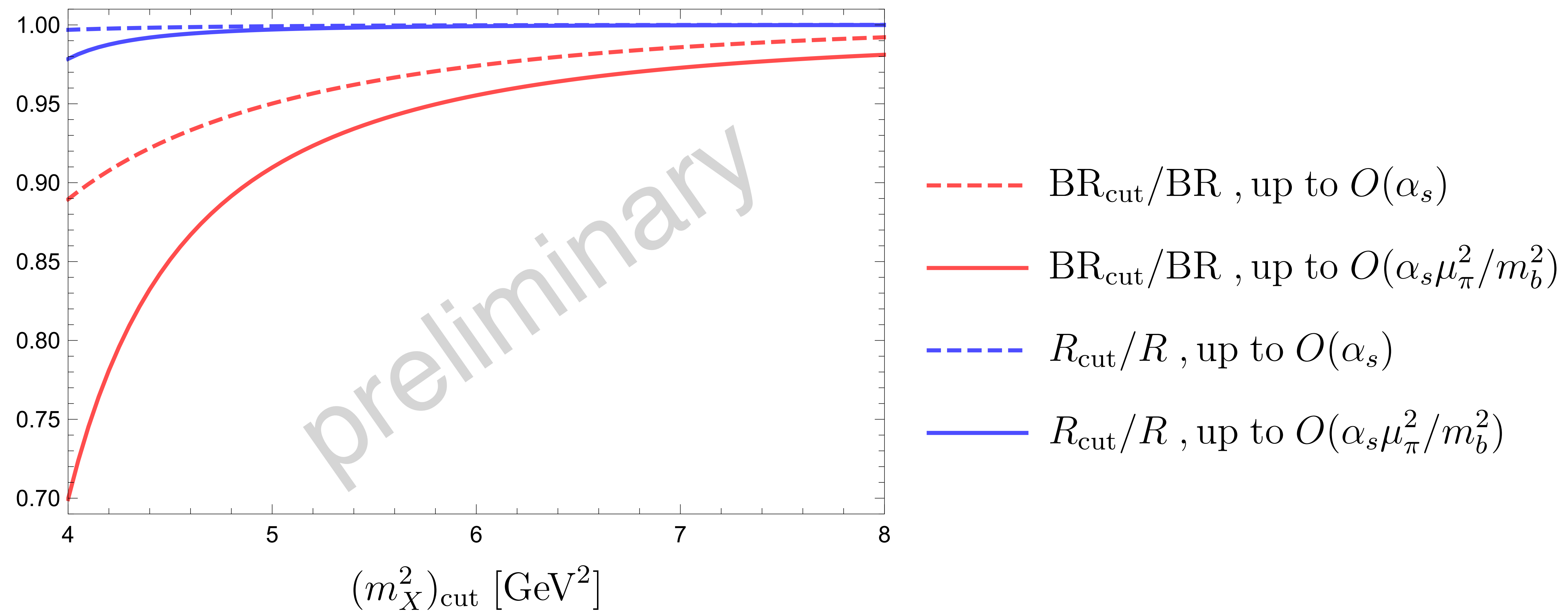
$$\lambda_1 \equiv \frac{1}{2m_B} \langle B | \bar{h}_v (iD)^2 h_v | B \rangle$$

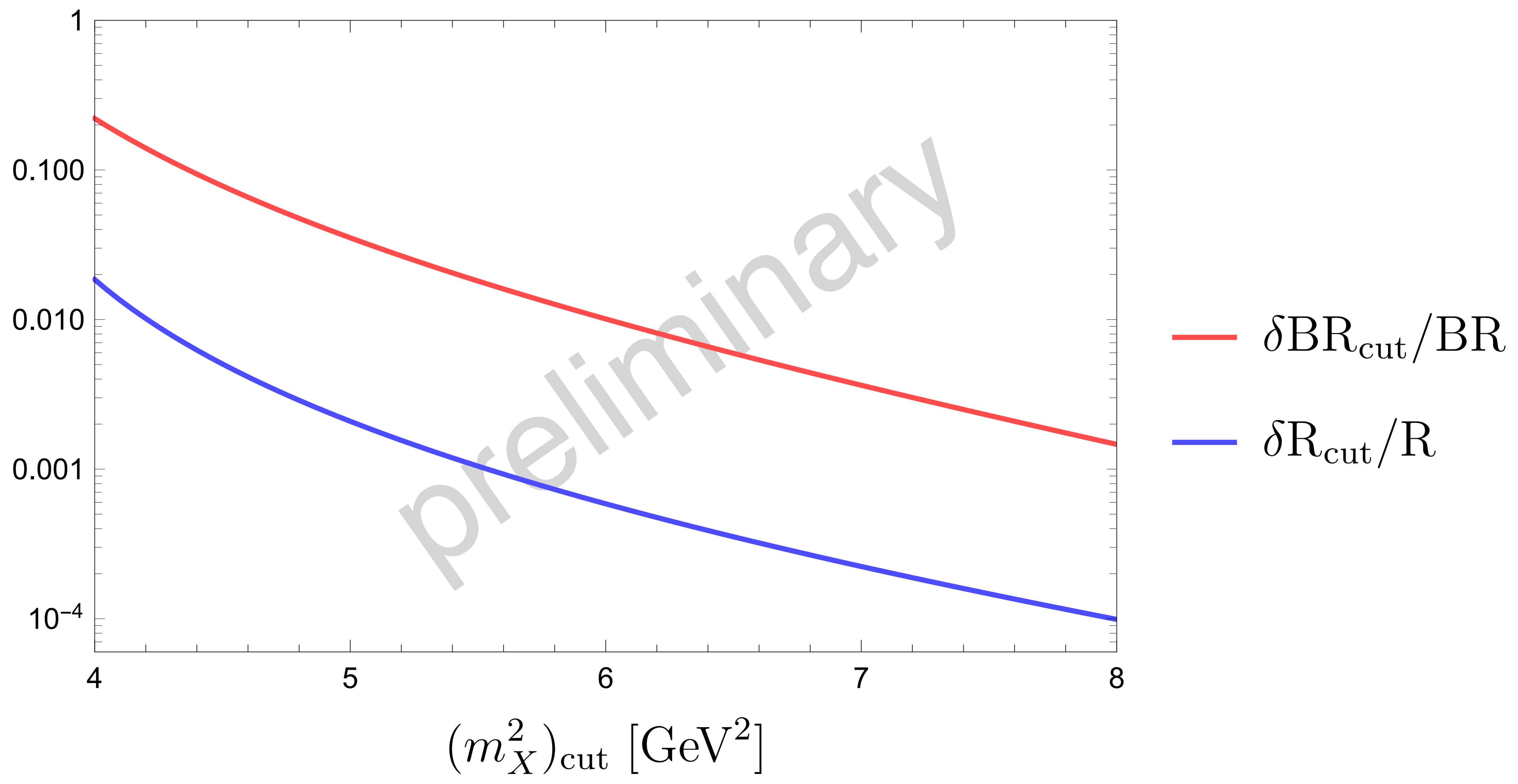
$$\rho_1 \equiv \frac{1}{2m_B} \langle B | \bar{h}_v iD_\mu (iv \cdot D) iD^\mu h_v | B \rangle$$

$$\rho_2 \equiv \frac{1}{6m_B} \langle B | \bar{h}_v iD^\mu (iv \cdot D) iD^\nu h_v (-i\sigma_{\mu\nu}) | B \rangle$$

$b \to s\ell\ell$: inclusive

- It is possible to bypass the need for the Shape Function by normalizing the $B \to X_s \ell\ell$ rate to the $b \to X_u \ell\nu$ one with the same cut on m_X
  [Lee, Ligeti, Stewart, Tackmann, hep-ph/0512191]
  [Huber, Hurth, Jenkins, EL, to appear]
- The OPE depends on the correct q^2 but has partonic kinematics (i.e. b-quark decay). This implies that partonic and hadronic m_X can be very different!



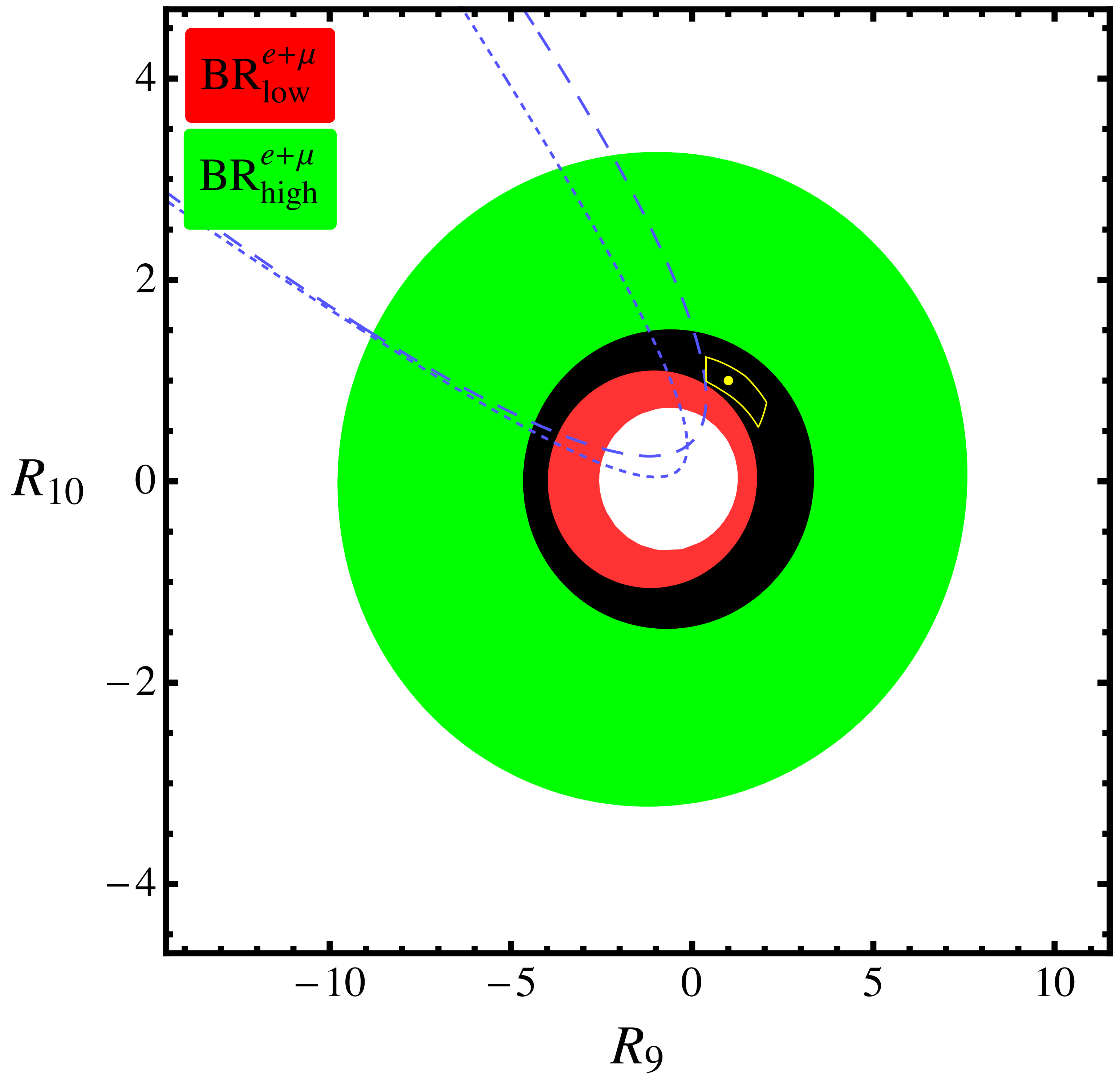


- Cancellation due to near universality of collinear and soft divergences

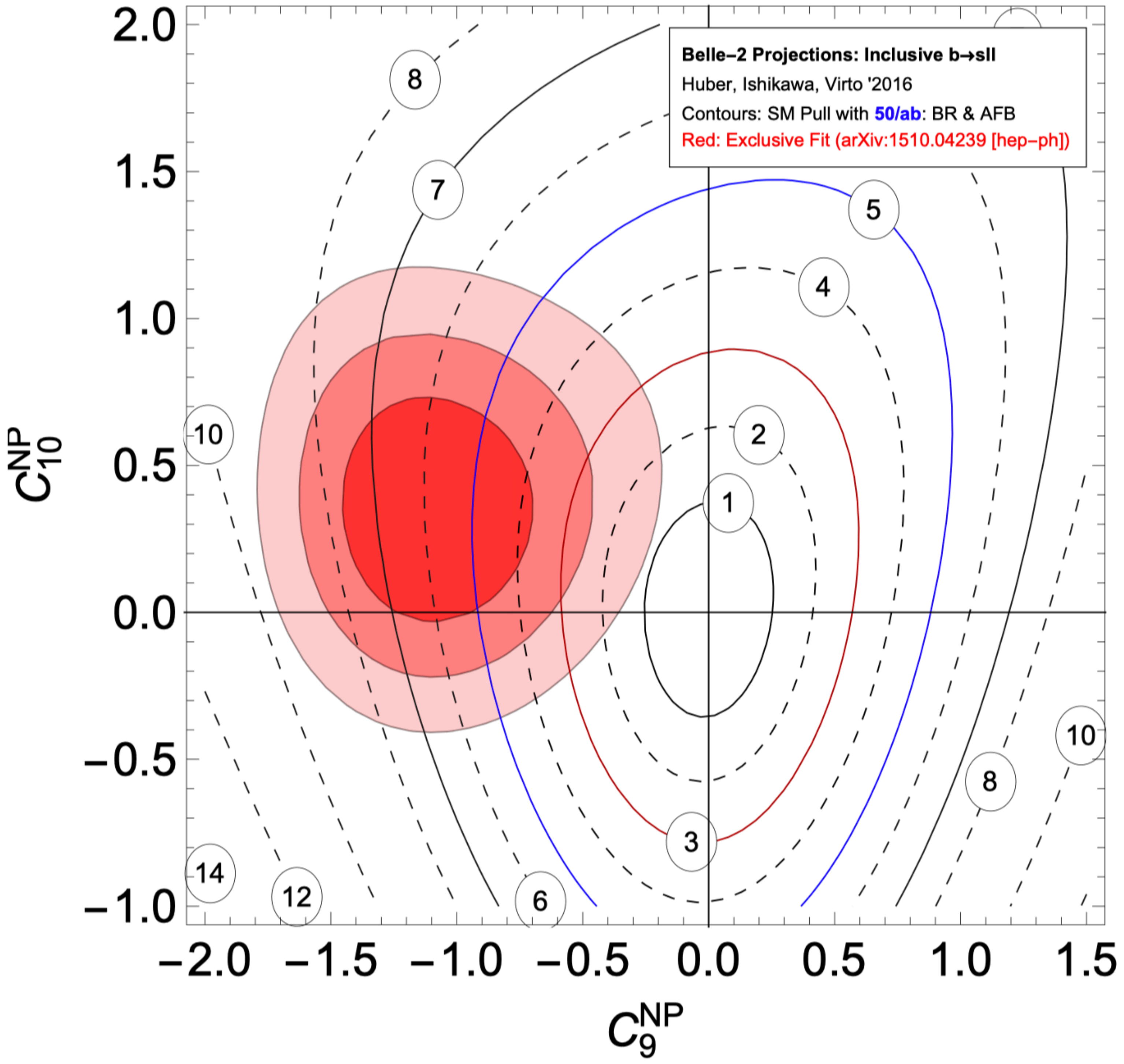
$b \to s\ell\ell$: inclusive

- Inclusive and exclusive modes nicely complement each other and can successfully establish new physics

[Huber, Hurth, Jenkins, EL, Qin, Vos, 1908.07507]



[Ishikawa, Virto, Huber,1808.1056]



- Irreducible uncertainties in inclusive and exclusive modes are independent
- Data will come from two very different experiments
- Need full 50 ab^{-1} from Belle II

$b \to s\ell\ell$: inclusive

- In models with extra Higgses and vectorlike quarks it is easy to evade all existing constraints and generate sufficiently large contributions to C_9^μ and C_{10}^μ
- The interesting phenomenology arises from Yukawa interactions between vectorlike quarks and Higgs doublets which control the mixing between heavy and SM fermions but have negligible impact on the large masses of the former.

SM — mixing in Yukawa interactions

$$\mathcal{L}_{\text{Mass}}^{\text{VLQ}} = \underbrace{-y_d^{ij}\bar{q}_L^i d_R^j H_d - y_u^{ij}\bar{q}_L^i u_R^j H_u}_{\text{SM}} \underbrace{-\lambda_B^i \bar{q}_L^i B_R H_d - \lambda_Q^j \bar{Q}_L^i d_R^j H_d - \kappa_T^i \bar{q}_L^i T_R H_u - \kappa_Q^j \bar{Q}_L^i u_R^j H_u}_{\text{mixing in Yukawa interactions}}$$

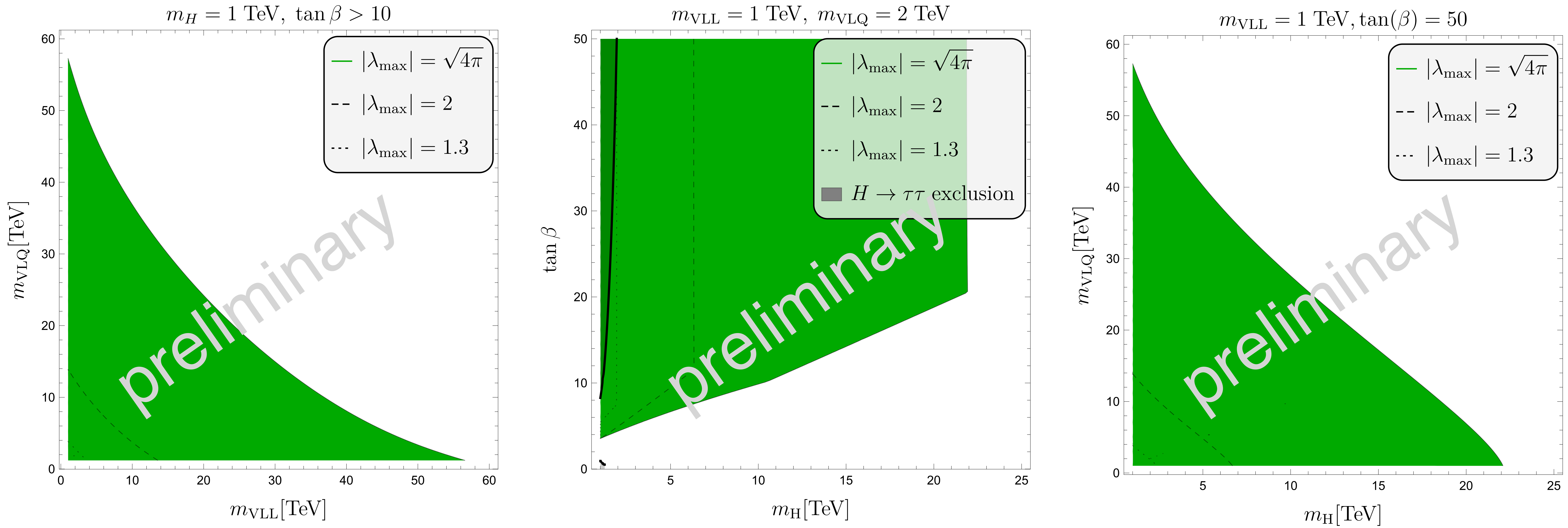
VLQ masses:
$$-M_Q \bar{Q}_L Q_R - M_T \bar{T}_L T_R - M_B \bar{B}_L B_R$$

VLQ Yukawas:
$$-\lambda \bar{Q}_L B_R H_d - \bar{\lambda} H_d^\dagger \bar{B}_L Q_R - \kappa \bar{Q}_L T_R H_u - \bar{\kappa} H_u^\dagger \bar{T}_L Q_R + \text{h.c.}$$

[Dermisek, EL, Shin,1509.04292, 1512.07837, 1608.00662, 1901.03709, 1907.07188]
[Dermisek, EL, McGinnis, Shin, 2005.07222, 2105.10790]

$b \to s\ell\ell$: inclusive

- In models with extra Higgses and vectorlike quarks it is easy to evade all existing constraints and generate sufficiently large contributions to C_9^μ and C_{10}^μ



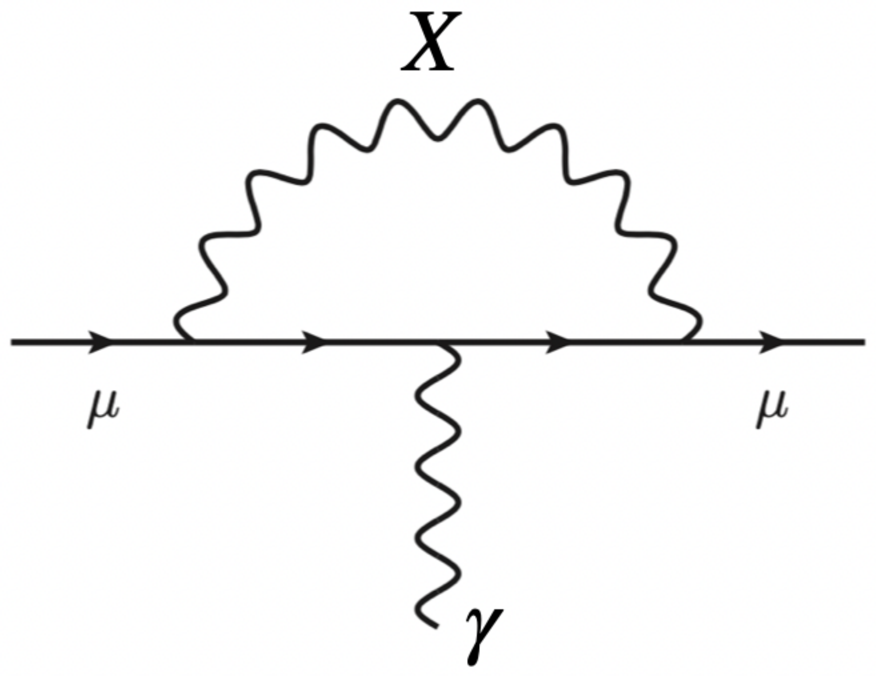
[Dermisek, EL, McGinnis, Bowser, to appear]

- The outer edges of the allowed regions are easily above the reach of LHC@14TeV

$(g-2)_\mu$

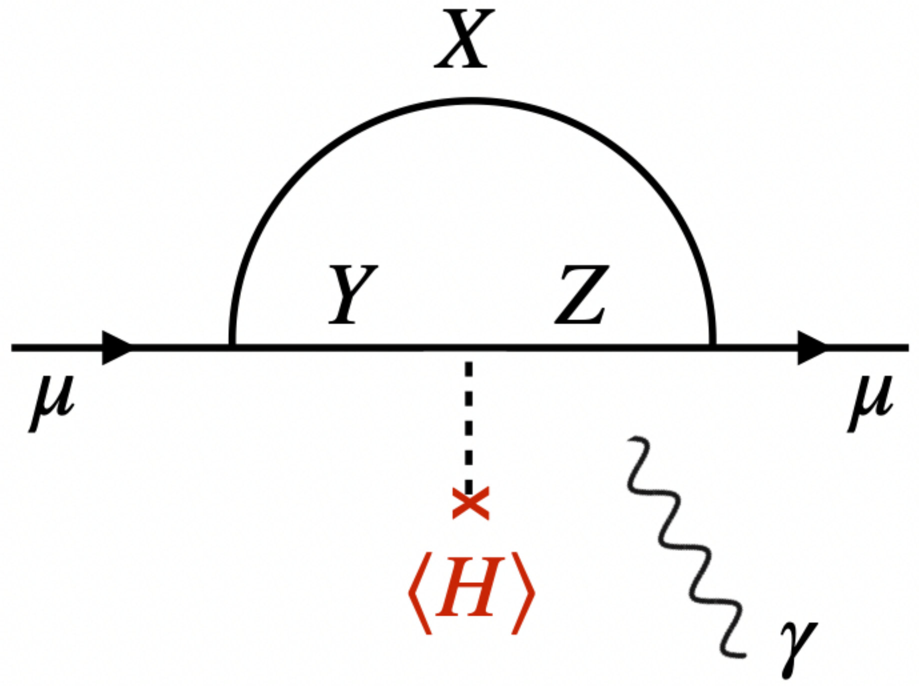
- For instance, in models with vectorlike leptons and extra Higgses the contributions to $(g-2)_\mu$ are chiral enhanced:

Typical NP contribution



$$\Delta a_\mu \simeq \frac{\lambda_{NP}^2}{16\pi^2}\frac{m_\mu^2}{m_{NP}^2}$$

Mass enhanced NP contribution

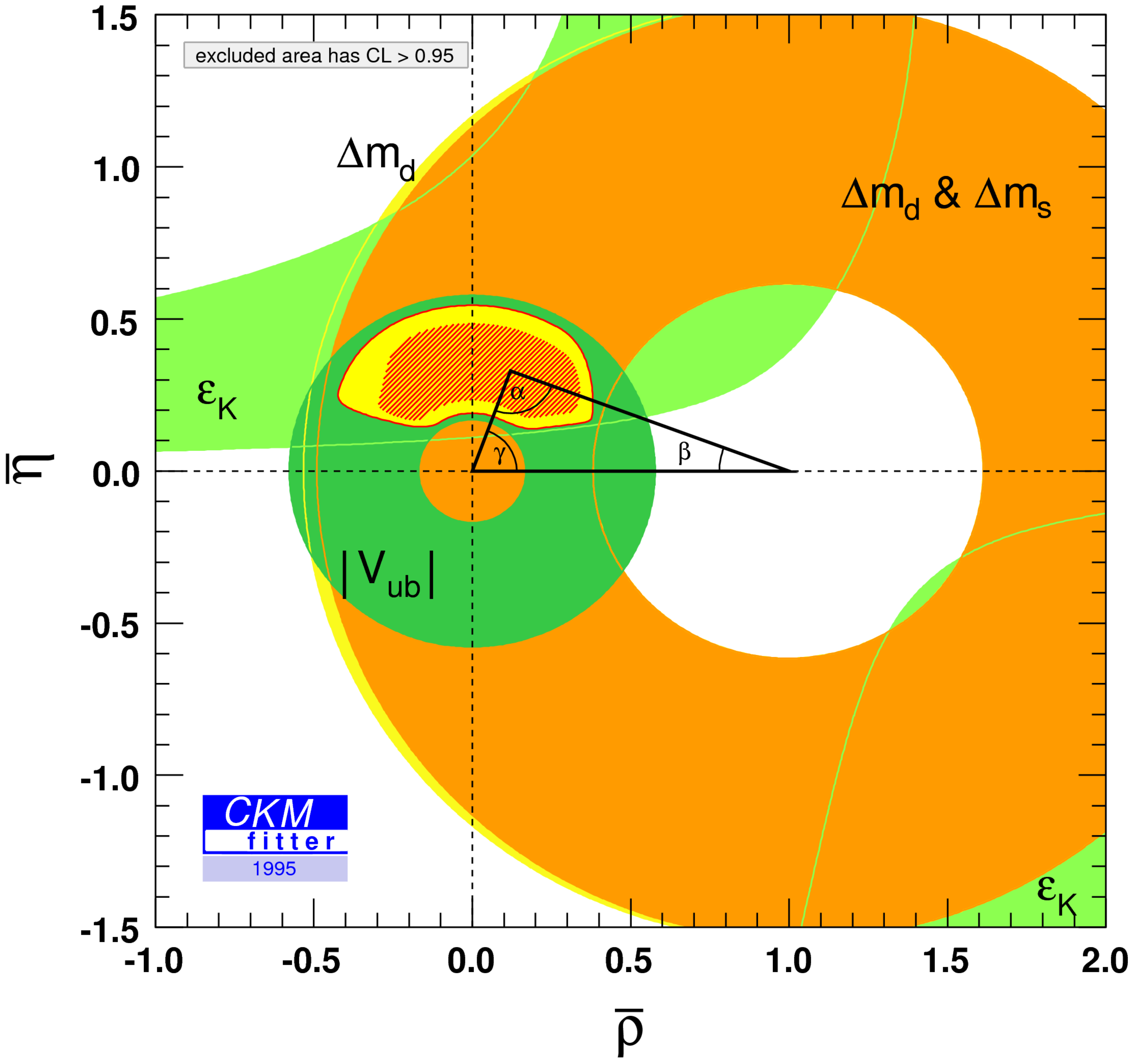


$$\Delta a_\mu \simeq \frac{\lambda_{NP}^3}{16\pi^2}\frac{m_\mu v}{m_{NP}^2}$$

- The current discrepancy (WP20) points to very heavy new physics (~10 TeV) and correlations with other observables (μ-EDM, $h \to \mu\mu$, …)
[Dermisek, Hermanek, McGinnis, Yoon, 2205.14243]

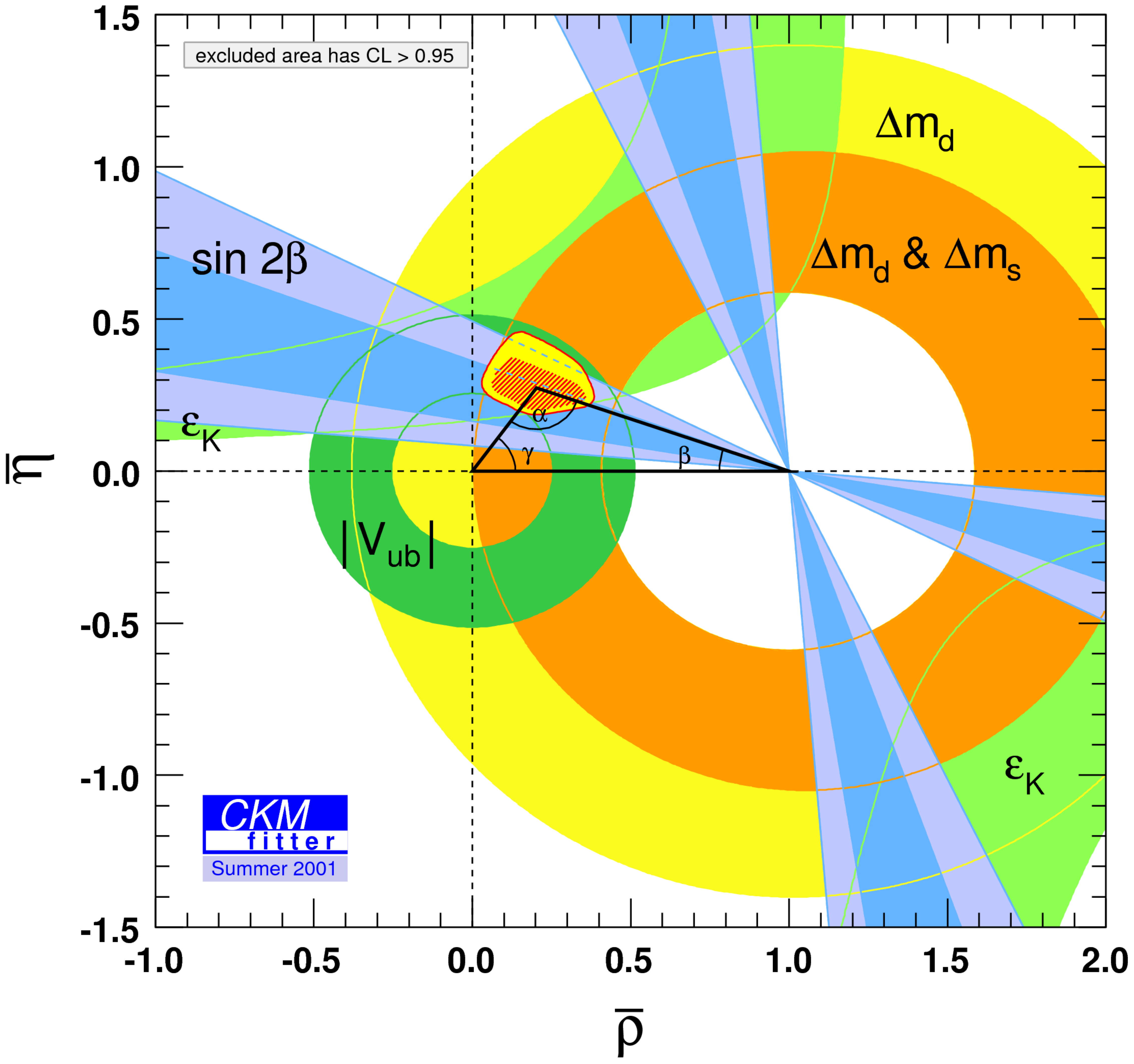
Unitarity Triangle fits through the years

- 1995: before BaBar and Belle



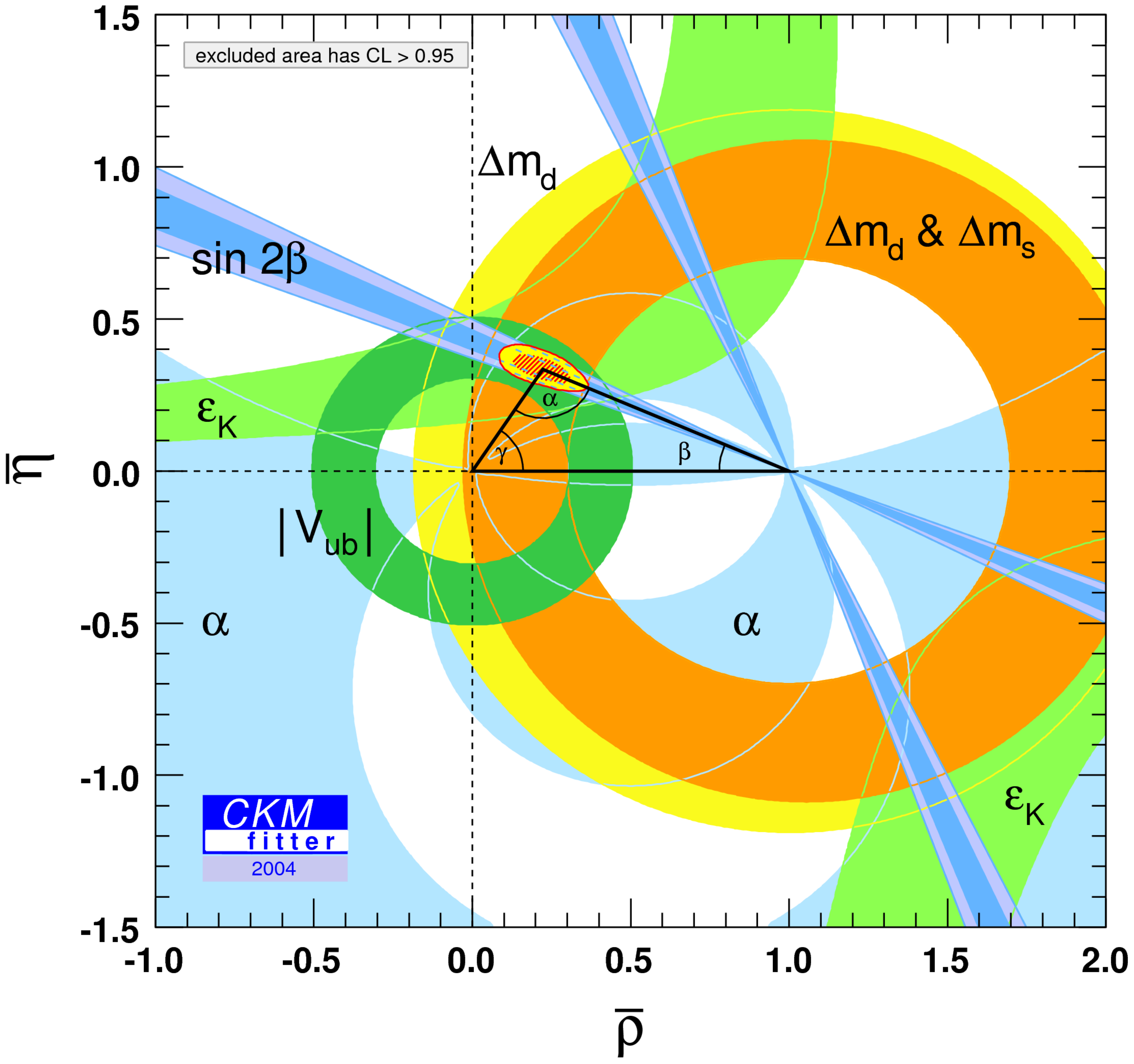
Unitarity Triangle fits through the years

- 2001: first B-factories results



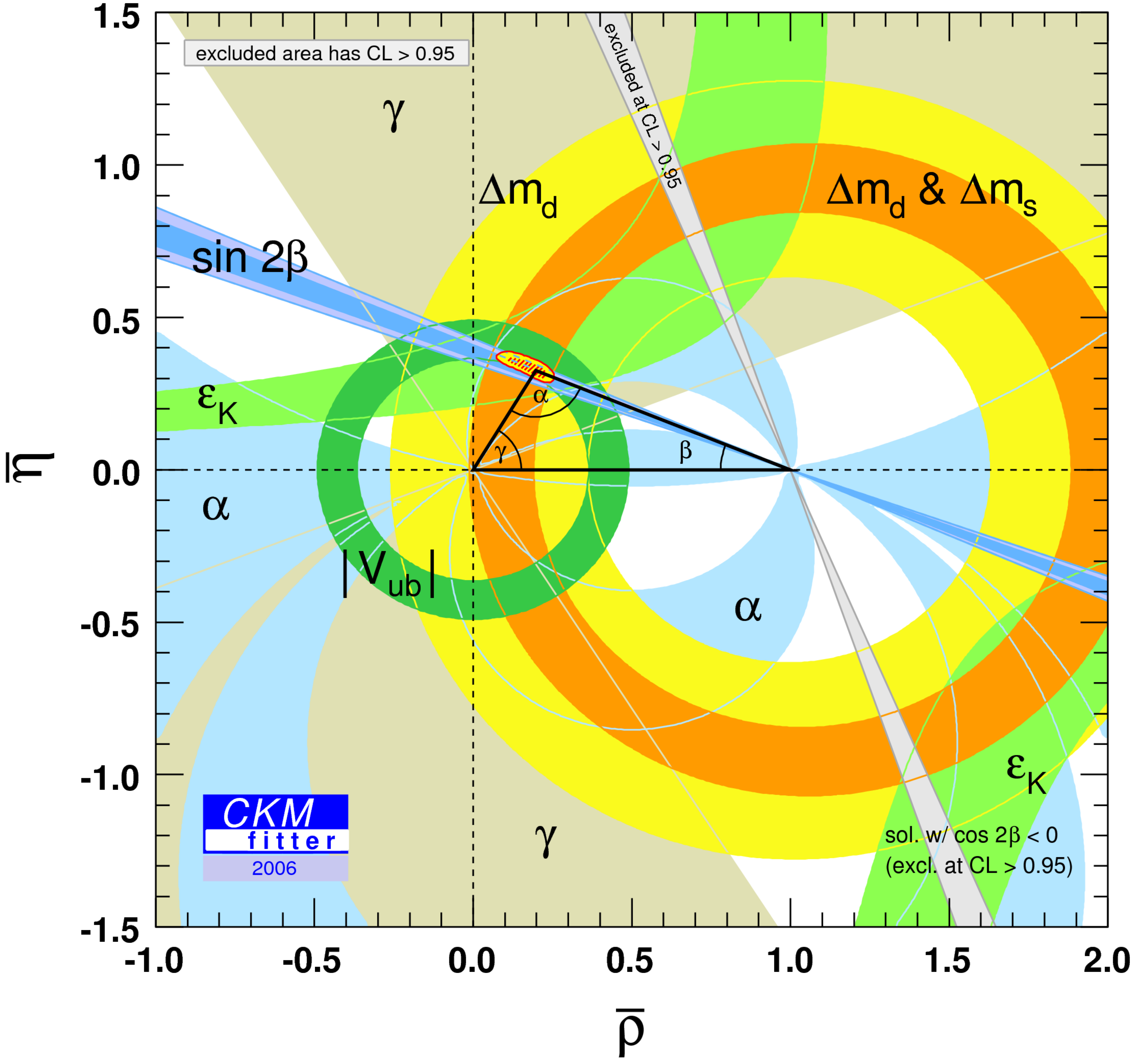
Unitarity Triangle fits through the years

- 2004



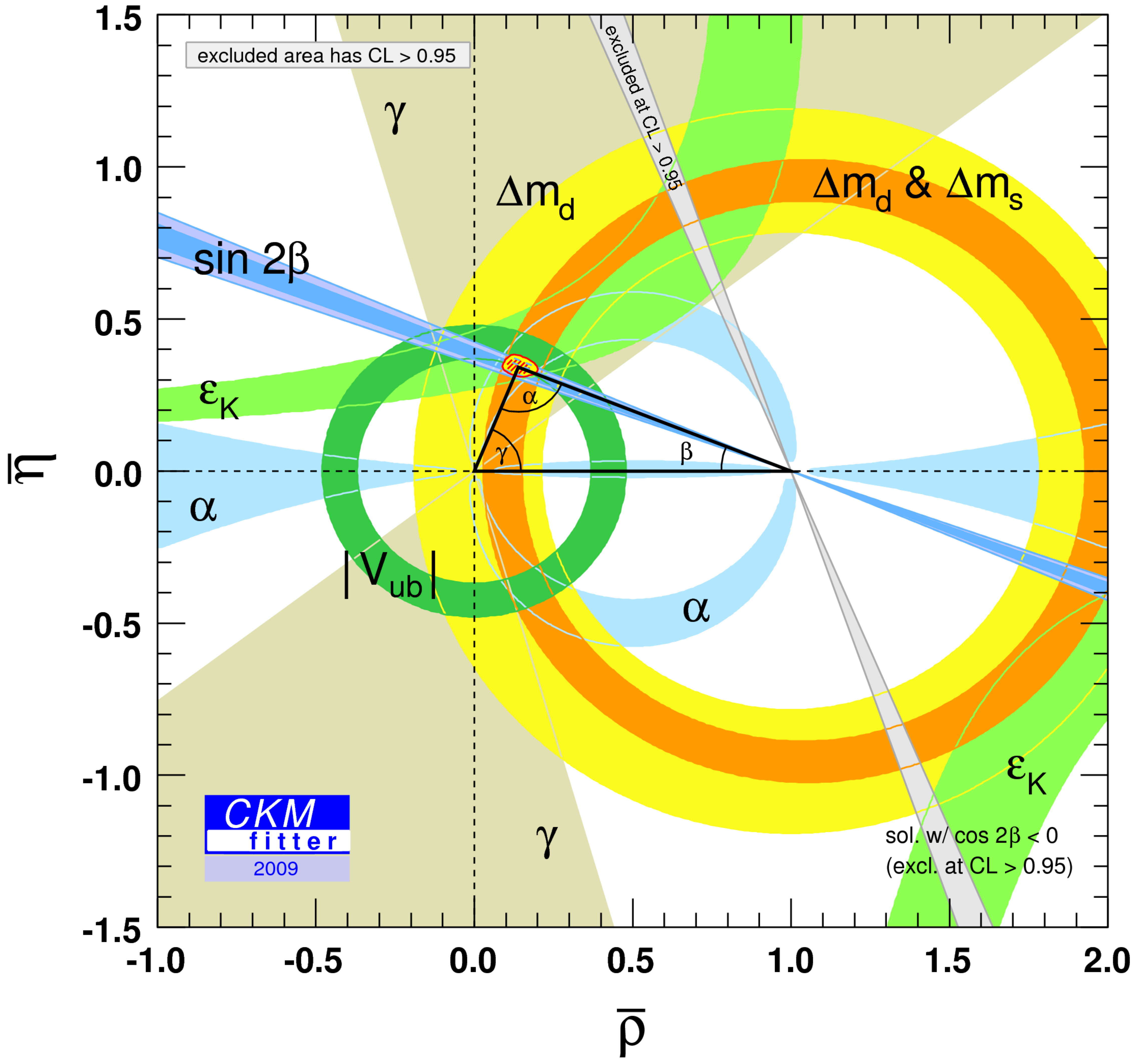
Unitarity Triangle fits through the years

- 2006: ΔM_{B_s} at Tevatron



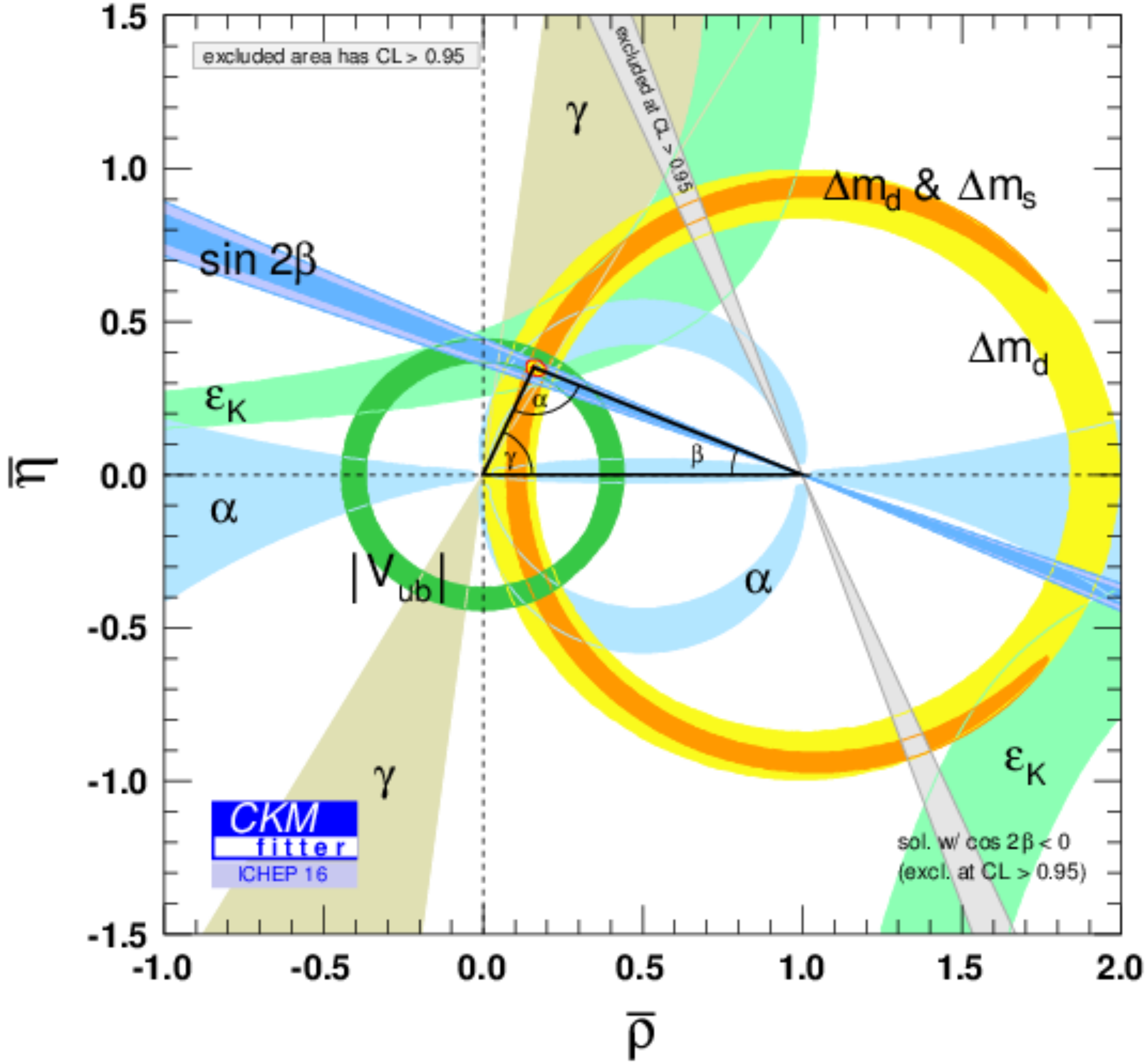
Unitarity Triangle fits through the years

- 2009: end of B-factories



Unitarity Triangle fits through the years

- 2016: LHCb arrives (γ) and lattice QCD big impact on B mixing



Unitarity Triangle fits through the years

- 2021: State-of-art

2016

$|V_{ub}| = 3.98(23) \times 10^{-3}$

$|V_{cb}| = 41.80(72) \times 10^{-3}$

$\hat{B}_K = 0.757(13)$

$\Downarrow$

2021

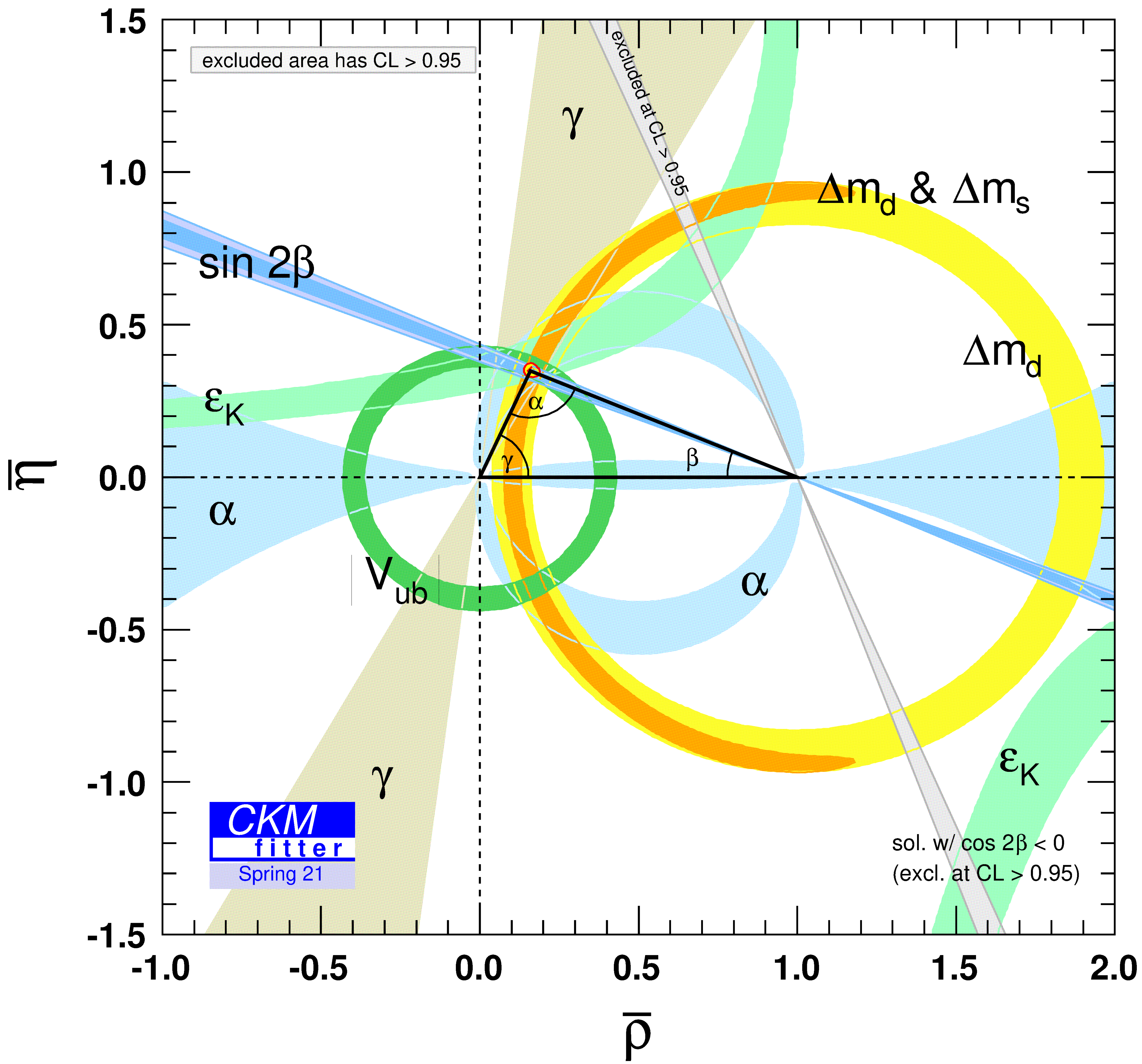
$|V_{ub}| = 3.88(23) \times 10^{-3}$

$|V_{cb}| = 41.15(56) \times 10^{-3}$

$\hat{B}_K = 0.757(13)$

Improved perturbative calculation of ε_K

[Brod, Gorbahn, Stamou, 1911.06822]



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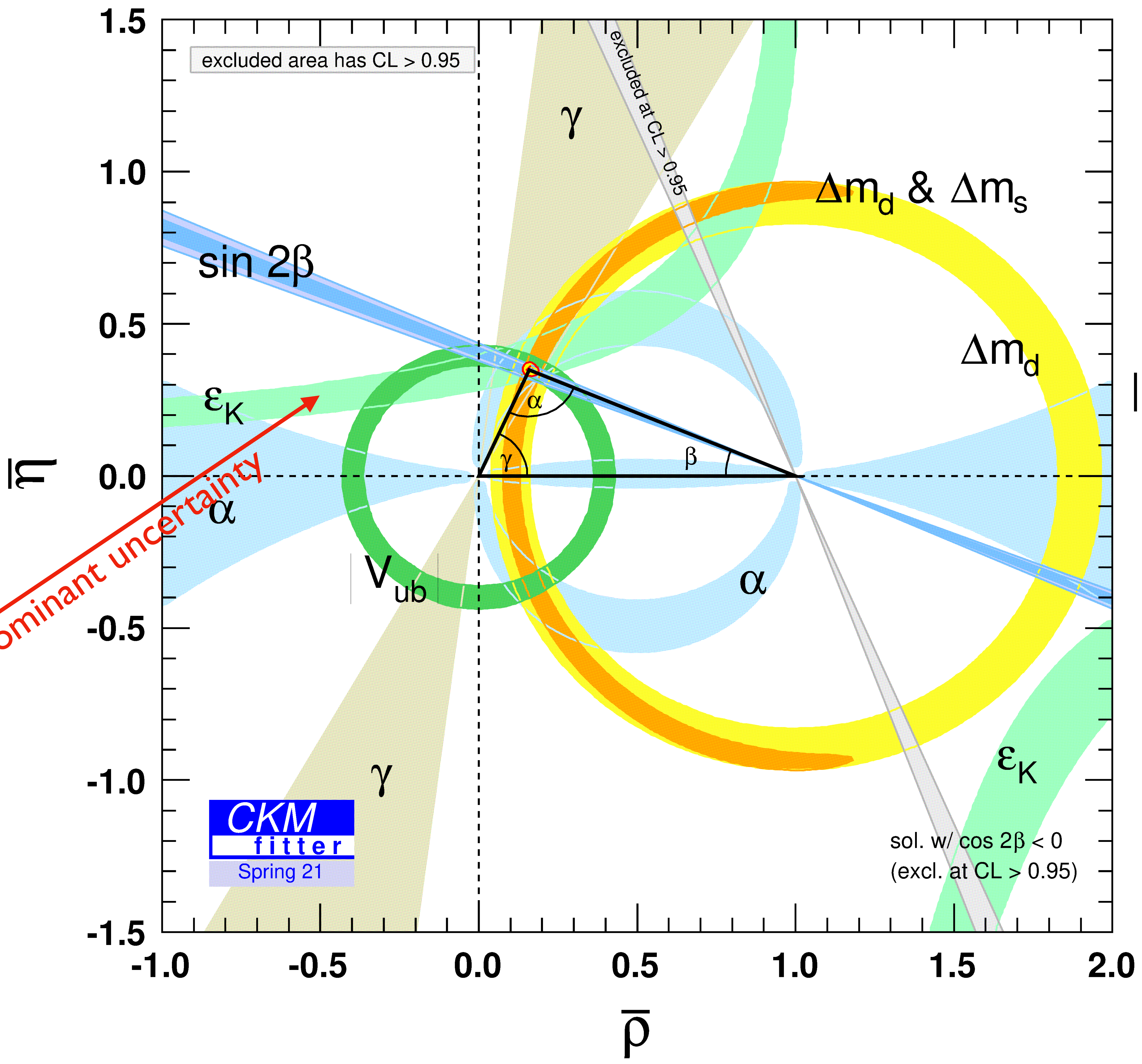
Improved perturbative calculation of ε_K
[Brod, Gorbahn, Stamou, 1911.06822]

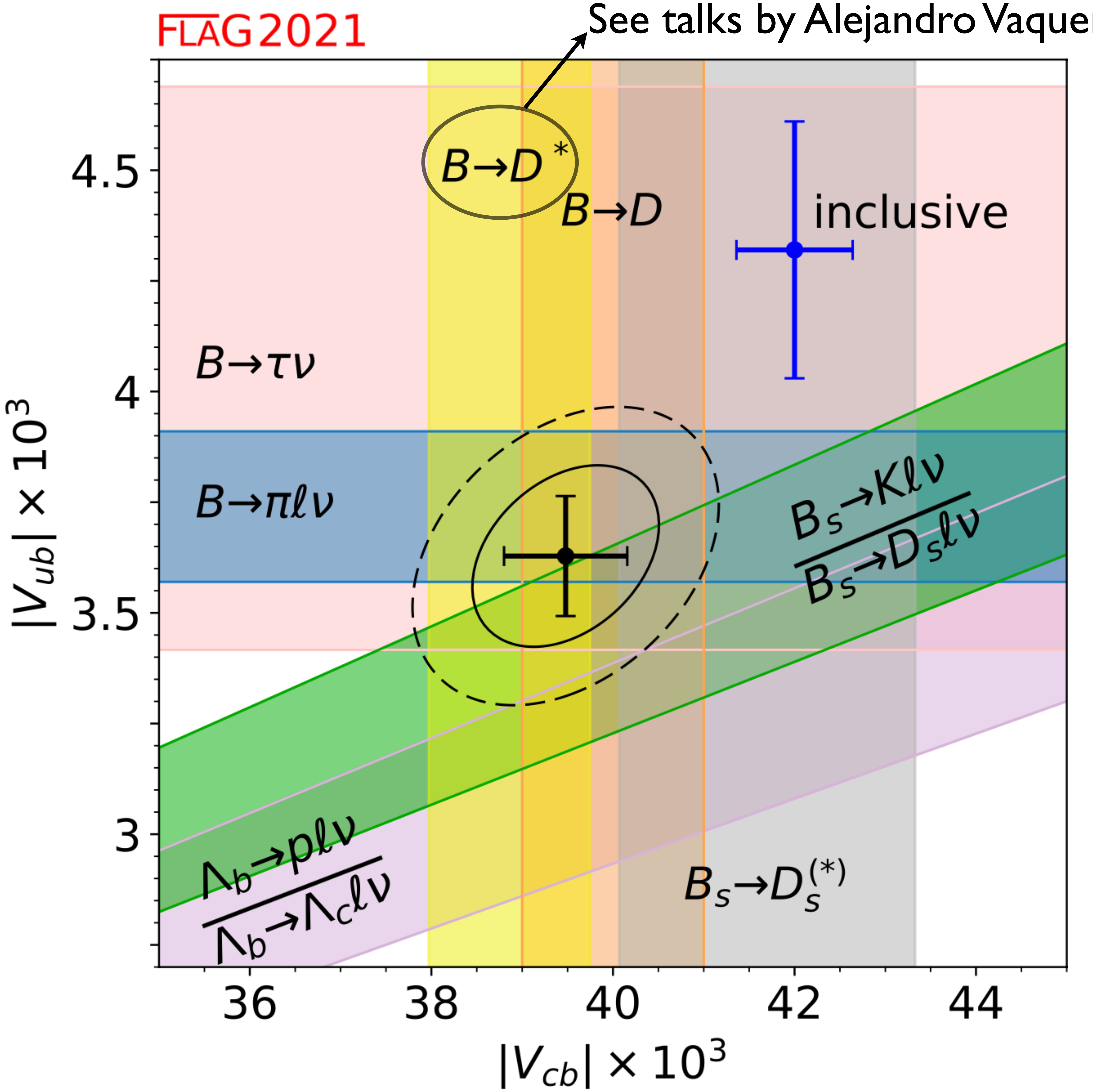
dominant uncertainty



$$|\varepsilon_K|_{\rm SM} = \left(2.161 \pm 0.153_{\rm param} \pm 0.076_{\rm latt} \pm 0.065_{\rm pert}\right) \times 10^{-3}$$

Semileptonic B decays

See talks by Alejandro Vaquero and Guido Martinelli



$$|V_{cb}| = \begin{cases} (39.48 \pm 0.68) \times 10^{-3} & \text{excl } (1.7\%) \\ (42.00 \pm 0.64) \times 10^{-3} & \text{incl } (1.5\%) \\ (40.82 \pm 1.26) \times 10^{-3} & \text{comb } (3.1\%) \end{cases}$$

PDG rescaling factor = 2.7

$$|V_{ub}| = \begin{cases} (3.63 \pm 0.14) \times 10^{-3} & \text{excl } (3.9\%) \\ (4.32 \pm 0.29) \times 10^{-3} & \text{incl } (6.7\%) \\ (3.76 \pm 0.27) \times 10^{-3} & \text{comb } (7.2\%) \end{cases}$$

PDG rescaling factor = 2.1

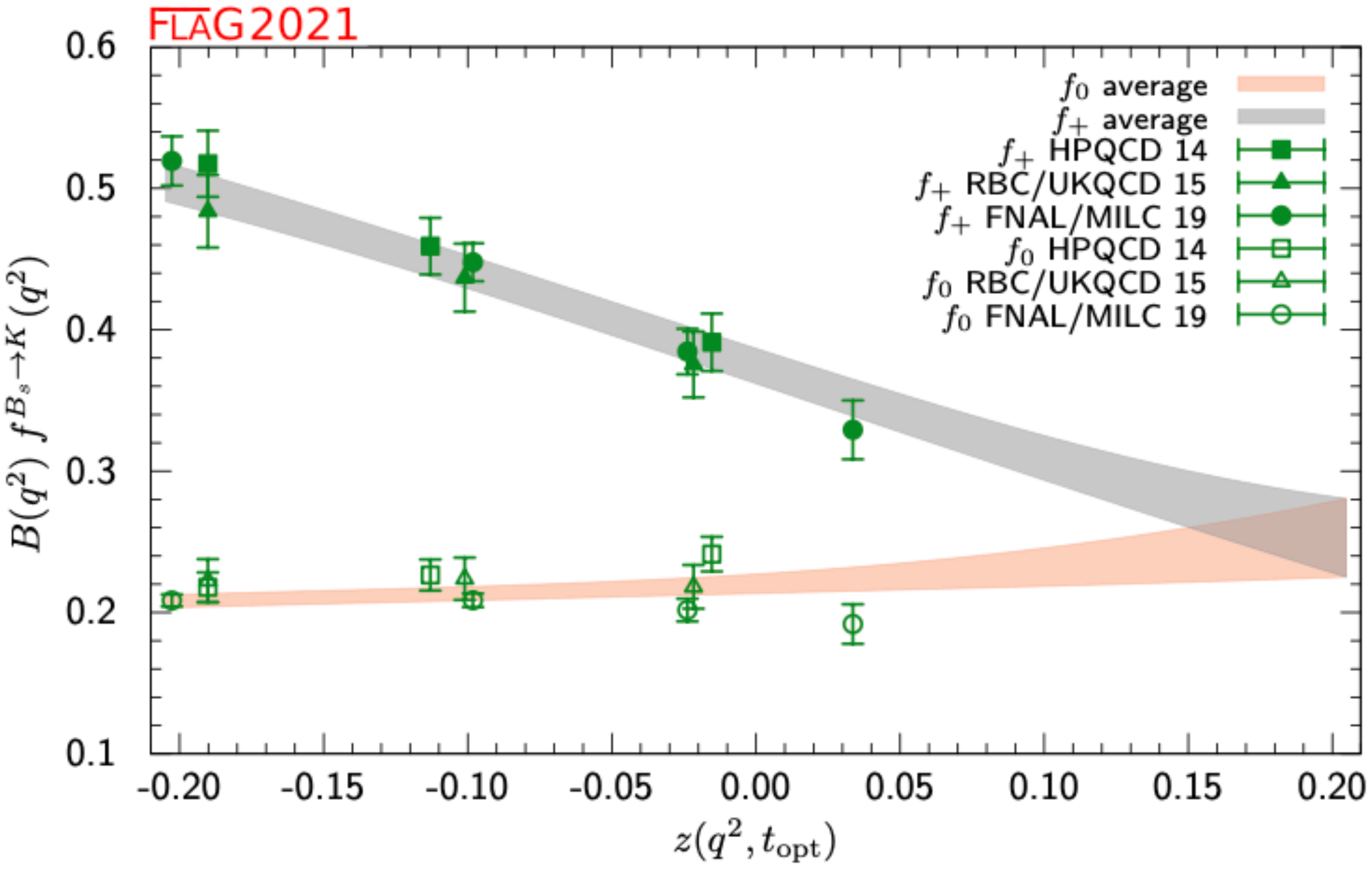
- Exclusive-inclusive tension in V_{cb} and V_{ub} is disturbing
- PDG and CKMfitter almost completely ignore these tensions when producing averages!

CKMfitter: $|V_{cb}| = 41.15(56) \times 10^{-3}$

$|V_{ub}| = 3.88(23) \times 10^{-3}$

A Comment on V_{ub}/V_{cb} from $B_s \to K$

- FLAG combined $B_s \to K$ form factors:



$$\frac{1}{|V_{ub}|^2}\int_{q^2_{\rm min}=m_\mu^2}^{7\ \rm GeV^2} \frac{d\Gamma(B_s\to K^-\mu^+\nu_\mu)}{dq^2} = (2.26\pm 0.38)\ \rm ps^{-1}$$

$$\frac{1}{|V_{ub}|^2}\int_{7\ \rm GeV^2}^{q^2_{\rm max}=(m_{B_s}-m_K)^2} \frac{d\Gamma(B_s\to K^-\mu^+\nu_\mu)}{dq^2} = (4.02\pm 0.31)\ \rm ps^{-1}$$

$$\frac{|V_{ub}|}{|V_{cb}|}(\rm low) = 0.0819\pm 0.0072_{\rm lat.}\pm 0.0029_{\rm exp}$$

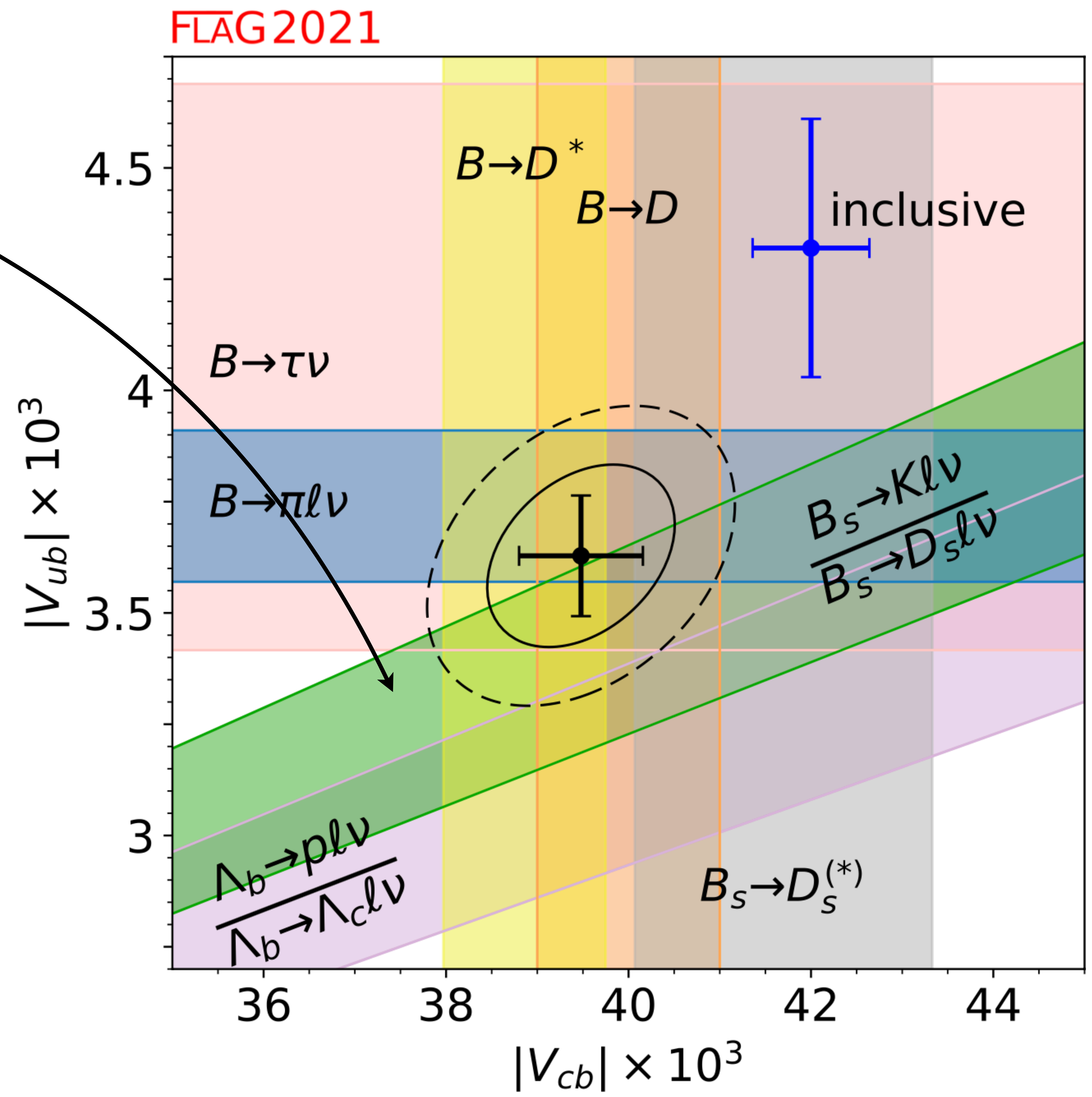
$$\frac{|V_{ub}|}{|V_{cb}|}(\rm high) = 0.0860\pm 0.0037_{\rm lat.}\pm 0.0038_{\rm exp}$$

A Comment on V_{ub}/V_{cb} from $B_s \to K$

- FLAG $|V_{ub}/V_{cb}|$ from $B_s \to K$ (only high-q^2):

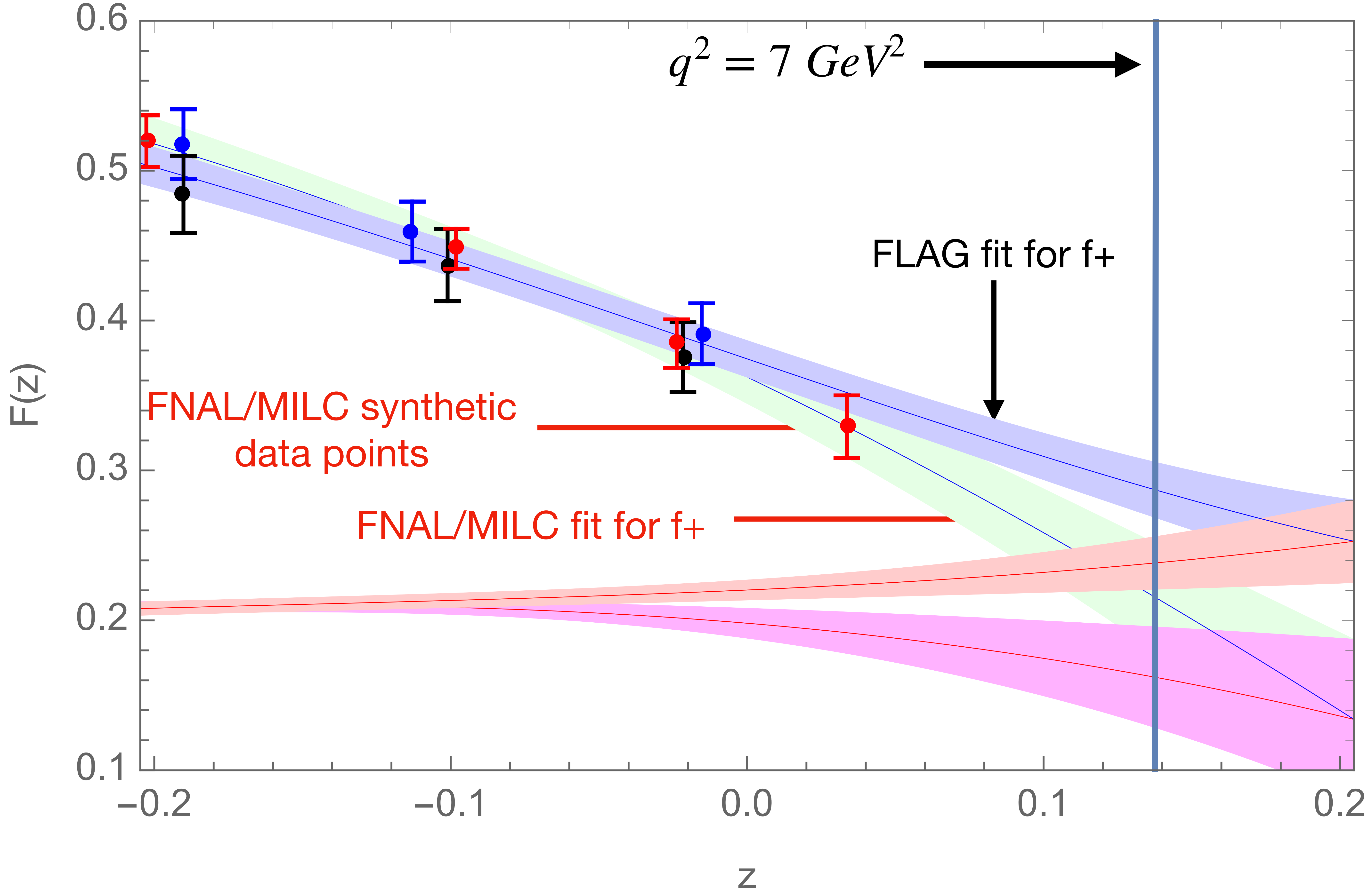
$$\frac{|V_{ub}|}{|V_{cb}|}(\text{low}) = 0.0819 \pm 0.0072_{\text{lat.}} \pm 0.0029_{\text{exp}}$$

$$\frac{|V_{ub}|}{|V_{cb}|}(\text{high}) = 0.0860 \pm 0.0037_{\text{lat.}} \pm 0.0038_{\text{exp}}$$



A Comment on V_{ub}/V_{cb} from $B_s \to K$

- Using only FNAL/MILC:

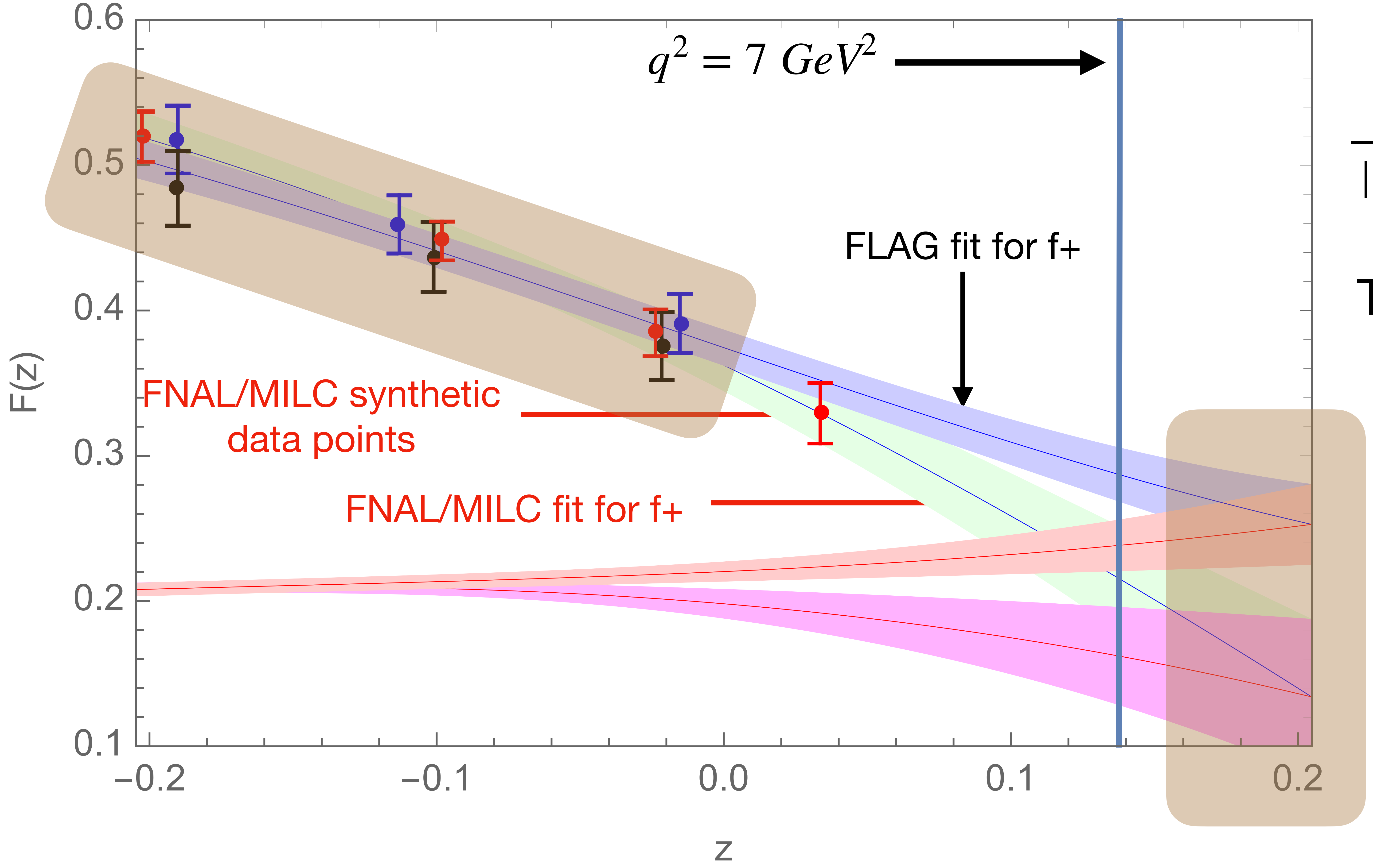


$$\frac{1}{|V_{ub}|^2}\int_{7\ \mathrm{GeV}^2}^{q^2_{\max}} \frac{d\Gamma(B_s \to K^-\mu^+\nu_\mu)}{dq^2} = (3.32 \pm 0.49)\ \mathrm{ps}^{-1}$$

This is the value quoted by LHCb

A Comment on V_{ub}/V_{cb} from $B_s \to K$

- Using only FNAL/MILC:



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This is the value quoted by LHCb

The inclusion of data points from different lattice collaborations lead to huge extrapolation differences even though the points are essentially compatible with the three highest q^2 FNAL/MILC ones

Semileptonic B decays

- Are we reaching the end of the rope with perturbative calculations in B physics?
  - Dyson series is asymptotic and there are indications that convergence of $b \to (u,c)\ell\nu$ calculations at three-loops is already questionable
  - Local power corrections are under control but required accuracy needs m_b^{-3} and m_b^{-4} operators whose matrix elements are essentially unknown beyond order of magnitude estimates
  - Non-local power corrections are vexing: SCET for exclusive $b \to s\ell\ell$, Shape function effects for $B \to X_u\ell\nu$ and $B \to X_s\ell\ell$ (though some help might come from $b \to s\gamma$)
  - Newer catastrophe: resolved contributions to $B \to X_s(\gamma, \ell\ell)$
  - Waiting for Godot: quark-hadron duality violation?
- A lot of room for improvement (both on the experimental and lattice QCD side) in the exclusive determination of $|V_{ub}|$ and $|V_{cb}|$.

  [See Alejandro Vaquero's talk]

Kaon Unitarity Triangle

- The current UT fit is dominated by B observables
- It is interesting to isolate constraints involving Kaon physics:
  [EL, Soni,1508.01801]

$$|\varepsilon_K| = \kappa_\varepsilon C_\varepsilon \hat{B}_K |V_{cb}|^2 \lambda^2 \bar{\eta} \left[|V_{cb}|^2 (1-\bar{\rho})\, \eta_{tt}\, \mathcal{S}(x_t) - \eta_{ut}\, \mathcal{S}(x_c, x_t) \right]$$

$$\varepsilon'_K/\varepsilon_K = \frac{i\omega_+ e^{i(\delta_2-\delta_0)}}{\sqrt{2}\varepsilon_K} \left[\frac{\mathrm{Im}(A_2^{\mathrm{emp}})}{\mathrm{Re}(A_2^{(0)})} - \frac{\mathrm{Im}(A_0^{(0)})}{\mathrm{Re}(A_0^{(0)})}(1-\hat{\Omega}_{\mathrm{eff}}) \right]$$

Isospin breaking corrections to $\mathrm{Im}(A_0)$ *and* $\mathrm{Im}(\mathrm{A}_2)$

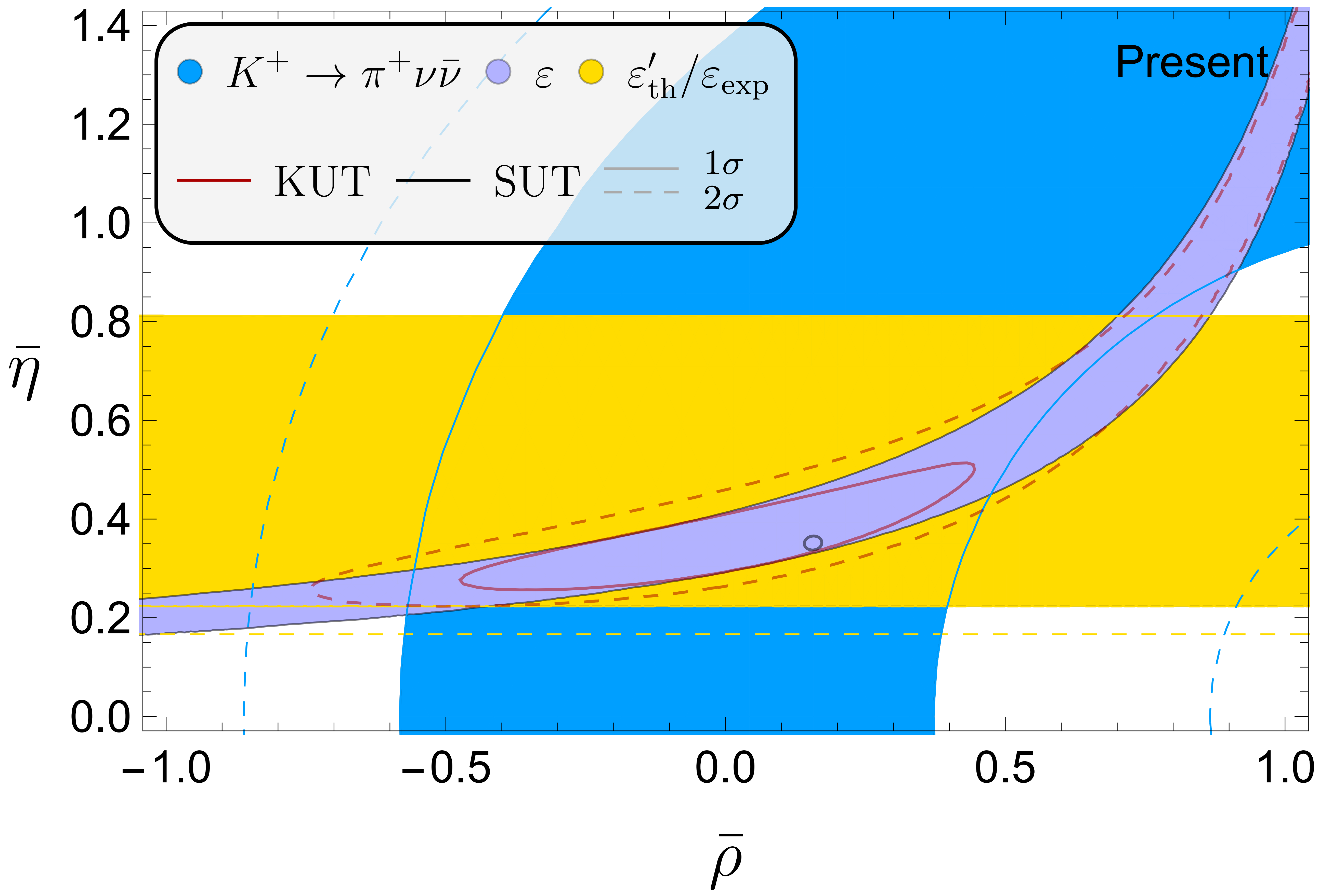
$$\mathrm{BR}(K^+ \to \pi^+ \nu\bar{\nu}) = \kappa_+ (1+\Delta_{\mathrm{EM}}) \left[\left(\frac{\mathrm{Im}(V_{td}V_{ts}^*)}{\lambda^5} X(x_t) \right)^2 + \left(\frac{\mathrm{Re}(V_{cd}V_{cs}^*)}{\lambda} \left[P_c^{SD}(X) + \delta P_{c,u}\right] + \frac{\mathrm{Re}(V_{td}V_{ts}^*)}{\lambda^5} X(x_t) \right)^2 \right]$$

$$\mathrm{BR}(K_L \to \pi^0 \nu\bar{\nu}) = \kappa_L \left(\frac{\mathrm{Im}(V_{td}V_{ts}^*)}{\lambda^5} X(x_t) \right)^2$$

Kaon Unitarity Triangle

- KUT: present status

[EL, Soni, to appear]



- Dominant non-parametric uncertainties:

$$\mathrm{ImA}_2 = -8.34(1.03) \times 10^{-13}$$

$$\mathrm{ImA}_0 = -6.98(0.62)(1.44) \times 10^{-11}\ \mathrm{GeV}$$

[RBC/UKQCD, 2004.09440]

$$\hat{\Omega}_{\rm eff} = (17.0 \pm 9.1) \times 10^{-2}$$

[Cirigliano, Gisbert, Pich, Rodriguez-Sanchez, 2004.09440]

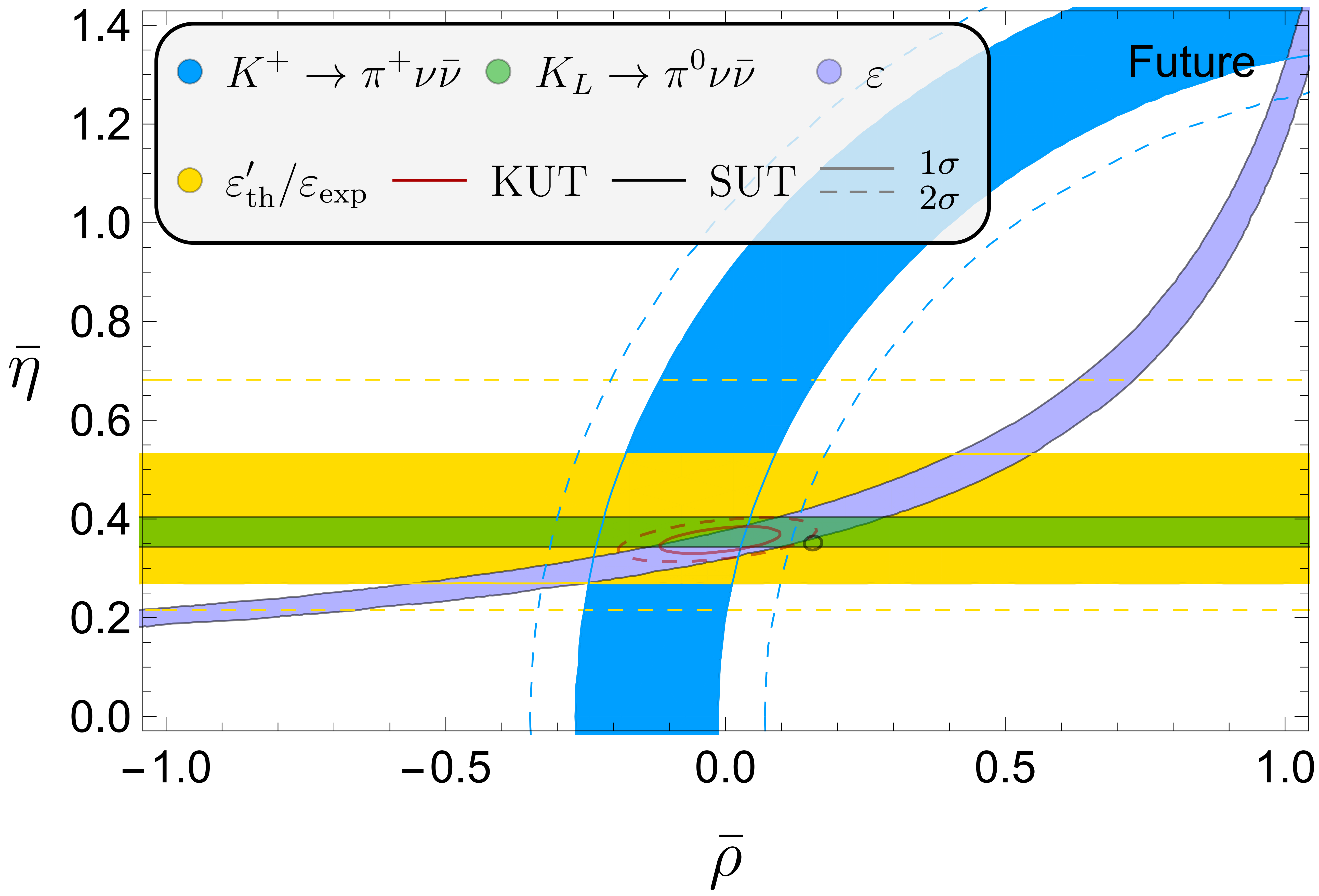
$$\delta P_{c,u} = 0.04 \pm 0.02$$

[Isidori, Mescia, Smith, hep-ph/0503107]

Kaon Unitarity Triangle

- KUT: Projections

[EL, Soni, to appear]



- Reduce uncertainties at $\varepsilon'_{\rm exp}$ level:

$\delta{\rm ImA}_2 : 12\,\% \to 10\,\%$

$\delta{\rm ImA}_0 : 22\,\% \to 10\,\%$

[See Masaaki talk at Lattice2022]

$\delta(\varepsilon'/\varepsilon)_{\rm th} \sim \delta(\varepsilon'/\varepsilon)_{\rm exp}$

- Isospin Breaking effects on the lattice:

$\delta\hat{\Omega}_{\rm eff} : 54\,\% \to ?$

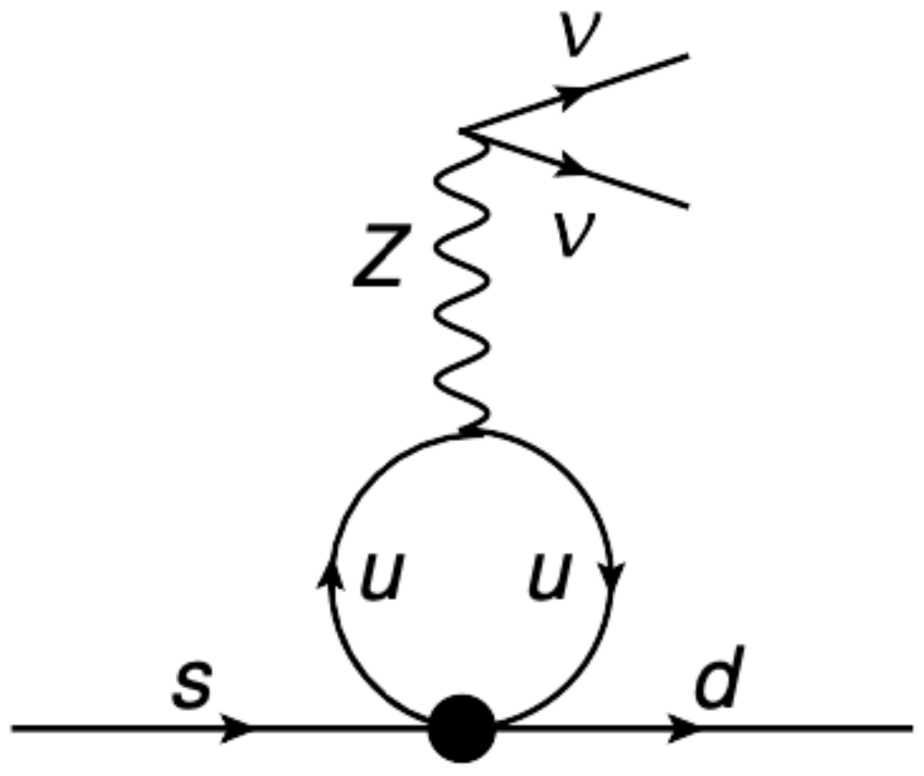
$\delta P_{c,u} :\ 50\,\% \to ?$

[Isidori, Martinelli, Turchetti, hep-lat/0506026]

- Assume projected results from NA62 (100 events at SM rate) and KOTO (measure SM at 10% level)

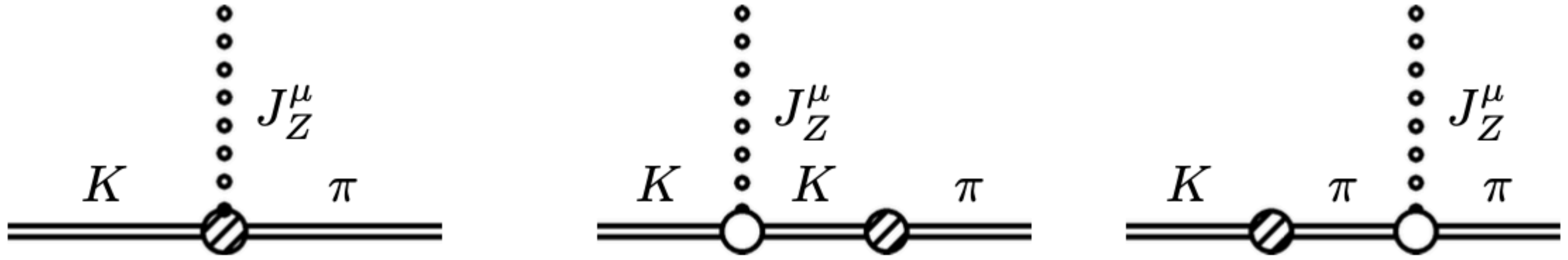
Long distance effects in $K^+ \to \pi^+ \nu \bar{\nu}$

- Dominant source of non-parametric uncertainty on $K^+ \to \pi^+ \nu \bar{\nu}$ originates from up-quark loops:

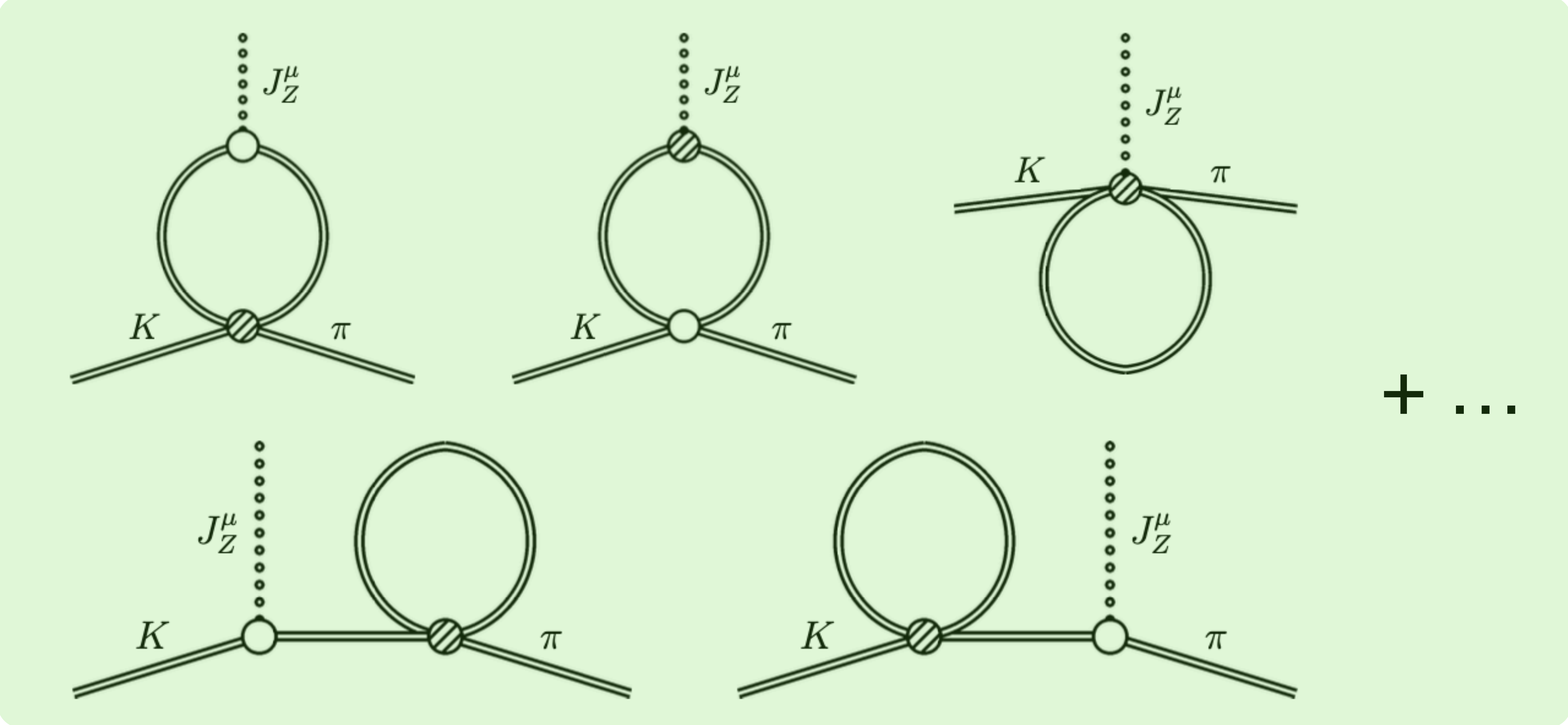


Partial calculation: estimate for $\delta P_{c,u}$ is tree level with 50% uncertainty:
$\delta P_{c,u} = 0.04 \pm 0.02$

- Present estimates obtained by matching the Weak Effective Hamiltonian onto ChiPT.
  [Isidori, Mescia, Smith, hep-ph/0503107]
- Tree level:

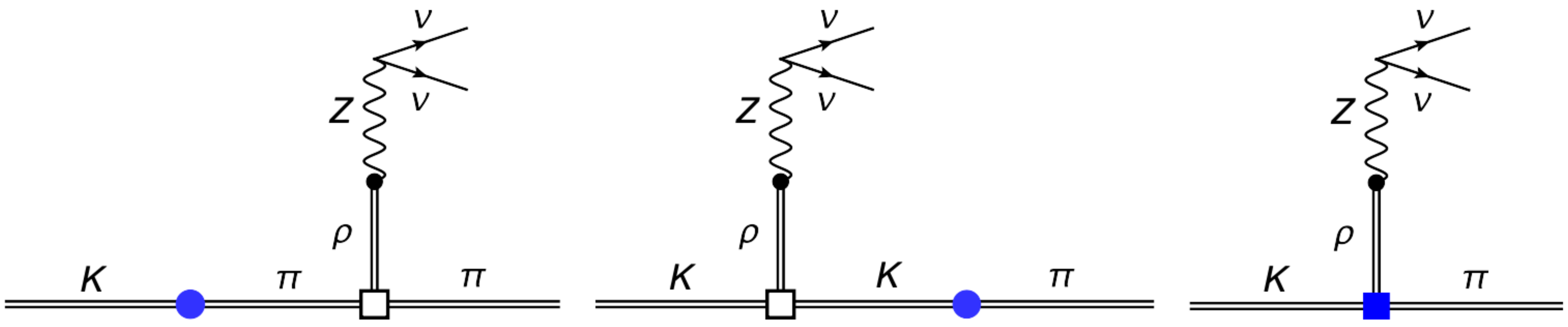


- One loop:



Long distance effects in $K^+ \to \pi^+ \nu \bar{\nu}$

- Extend ChiPT to include vector mesons and use ρ exchange at tree-level to capture dominant pion loop effects [Ecker, Gasser, Leutwyler, Pich, Rafael]
- Matching can be achieved by writing all possible operators and using some approximate techniques (e.g. weak deformation model or factorization) to calculate the Wilson coefficients.
  [EL, Soni, to appear]
- The long distance contributions to $K^+ \to \pi^+ \nu \bar{\nu}$ are then given by:

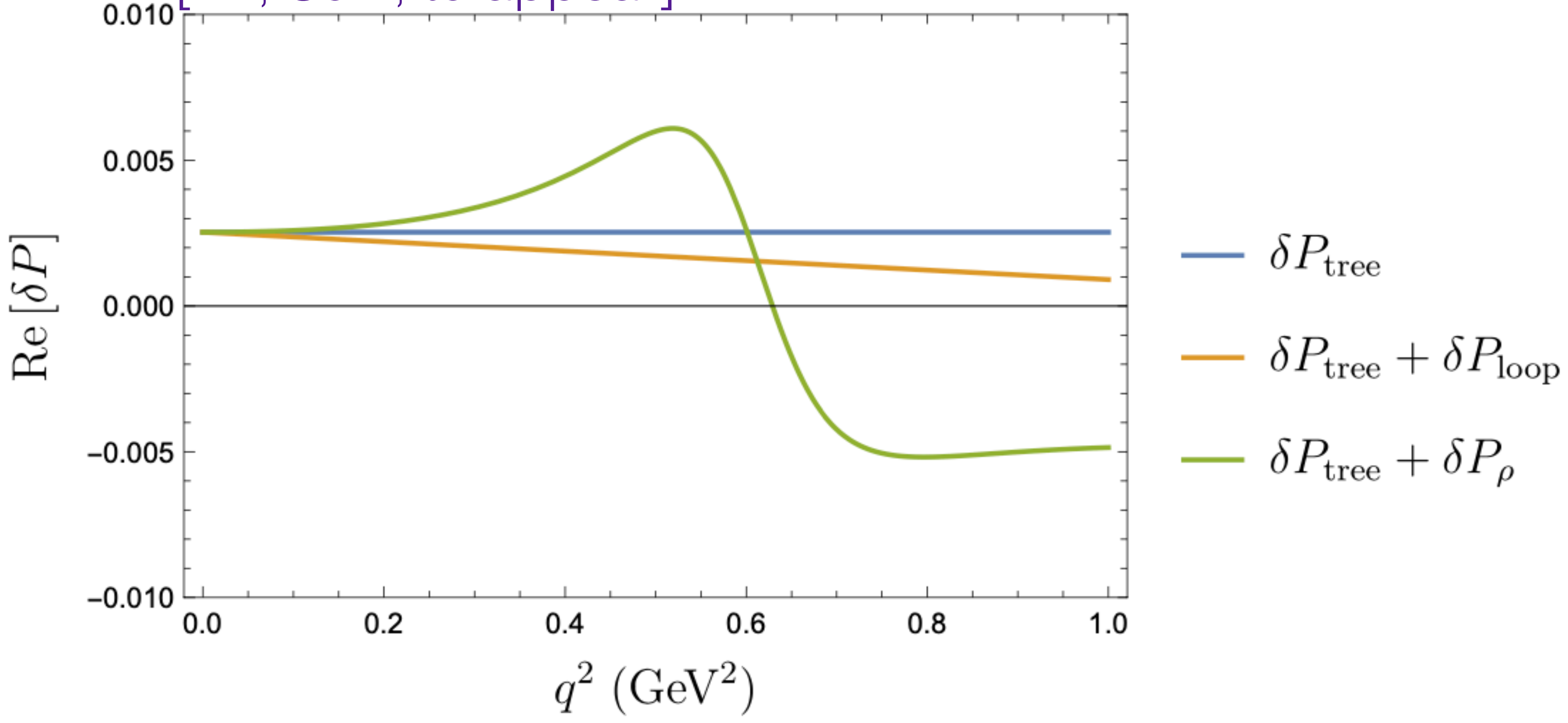


- No new free parameters are needed

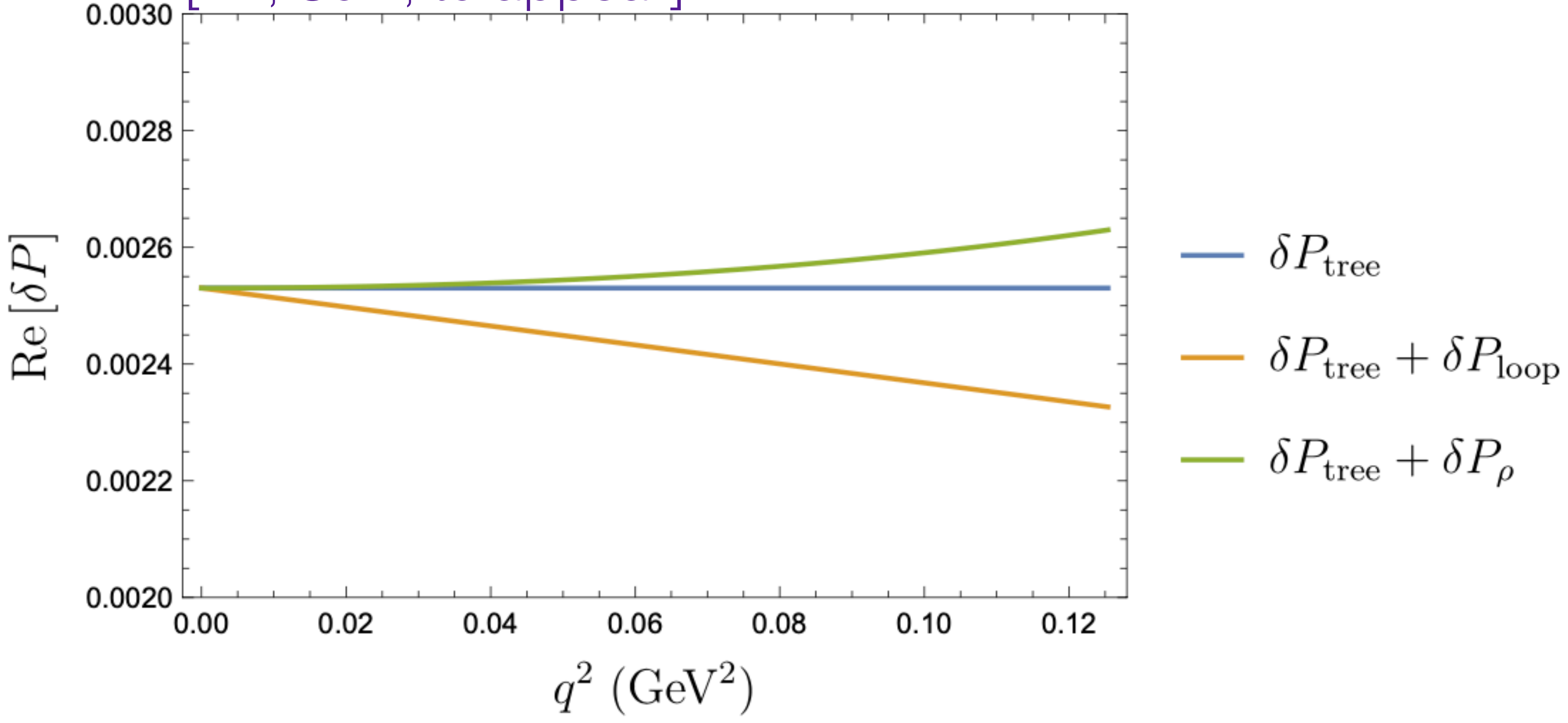
Long distance effects in $K^+ \to \pi^+ \nu \bar{\nu}$

- Preliminary numerical results seem to suggest that corrections to tree-level are smaller than previous estimate suggest:

[EL, Soni, to appear]



[EL, Soni, to appear]



- Given the phenomenological importance of this decay it would be important to have these effects calculated from first principles on the lattice.

[Isidori, Martinelli, Turchetti, hep-lat/0506026]
[Christ, Feng, Portelli, Sachrajda, 1605.04442]
[Christ, Feng, Juttner, Lqwson, Portelli, Sachrajda, PoS (CD15)033]
[Bai, Christ, Feng, Lawson, Portelli, Sachrajda, 1806.11520]

Conclusions

- B-anomalies
  - Exclusive modes:
    - A lot of room for progress on the experimental (LHCb, Belle II) and theoretical (lattice FFs and LCDAs) side
  - Inclusive modes:
    - Progress can be made by a simultaneous analysis of $B \to X_s \gamma$, $B \to X_u \ell \nu$ and $B \to X_s \ell \ell$ to reduce Shape Function uncertainties
    - Calculation of HQET matrix elements on lattice?
- Semileptonic decays and extraction of V_{ub} and V_{cb}
  - Inclusive seems to be close to its endgame
  - A lot of room in various exclusive b-hadron semileptonic decays (FFs from lattice)
- Kaon physics (ε', $K \to \pi \nu \bar{\nu}$)
  - Need to bring uncertainty on $\pi\pi$ matrix elements to the 10% level and include IB corrections
  - Calculate long distance contributions to $K^+ \to \pi^+ \nu \bar{\nu}$ on the lattice