A window into B anomalies: $B \to D^*\ell\nu$ at nonzero recoil from LQCD

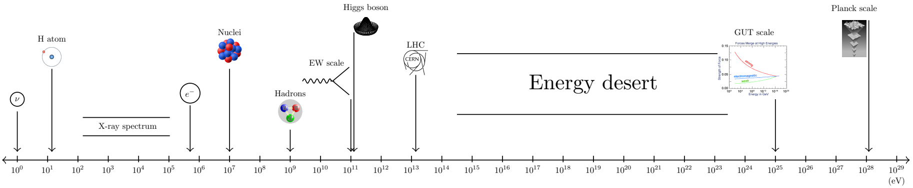
Alejandro Vaquero

Universidad de Zaragoza

Sep 12th, 2022

The Standard Model (SM)

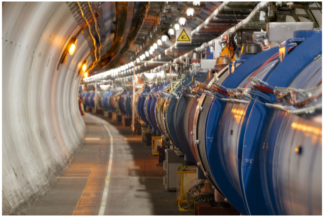
- The Standard Model is (arguably) the most successful theory describing nature we have ever had
- The theory is not completely satisfactory
  - Situation similar to that at the end of the XIX century
- The SM can explain phenomena in a large range of scales



- Yet there is a region where we expect the SM to fail
- The SM is regarded as an effective theory at low energies (low means $E \lesssim v_{EW} \approx 0.1 - 1$ TeV)

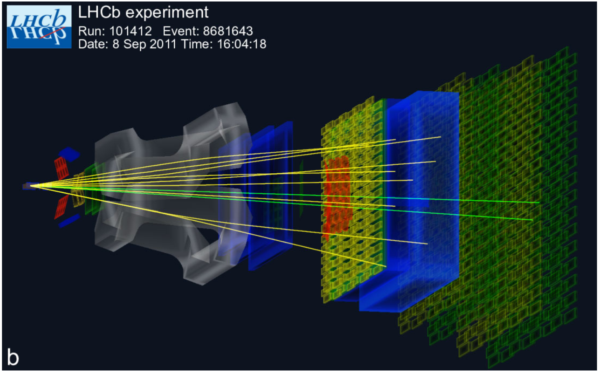
Where to look for new physics?

Energy frontier



Cosmology frontier

Intensity frontier

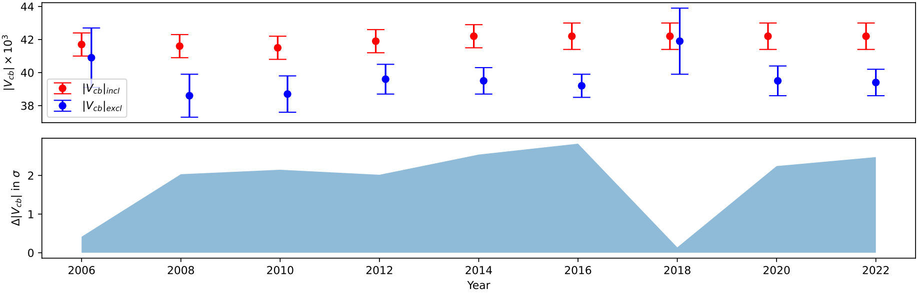


- High expectations with the LHC
- Intensity frontier becoming increasingly important

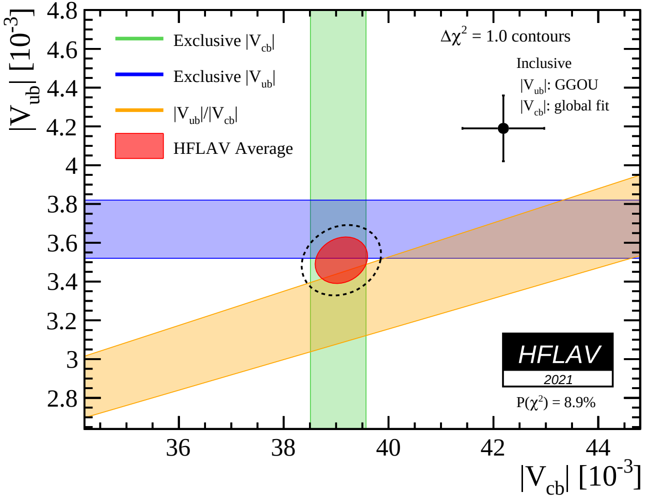
The CKM matrix

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & \boldsymbol{V_{cb}} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- Matrix must be unitary (preserve the norm)
- Tensions have been there for a long time
- Evolution of the tensions according to PDG

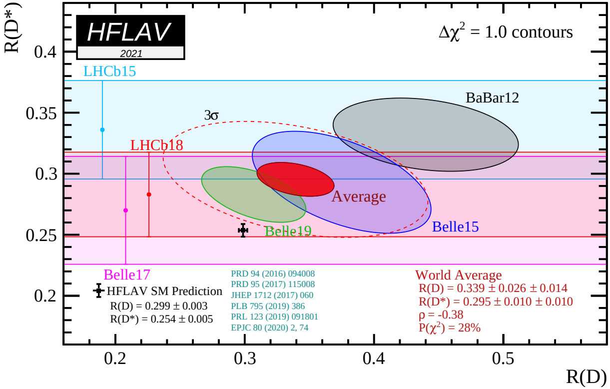


Break: Reminder of $|V_{ub}|$ vs $|V_{cb}|$



Current status of $|V_{ub}|$ vs $|V_{cb}|$ (HFLAV 2021)

$$R\left(D^{(*)}\right) = \frac{\mathcal{B}\left(B \to D^{(*)}\tau\nu_\tau\right)}{\mathcal{B}\left(B \to D^{(*)}\ell\nu_\ell\right)}$$



- Current $\approx 3\sigma$ tension with the SM

The V_{cb} matrix element: Measurement from exclusive processes

$$\underbrace{\frac{d\Gamma}{dw}\left(\bar{B}\rightarrow D^{*}\ell\bar{\nu}_{\ell}\right)}_{\text{Experiment}}=\underbrace{\frac{G_F^2 m_B^5}{48\pi^2}(w^2-1)^{\frac{1}{2}}P(w)\left|\eta_{ew}\right|^2}_{\text{Known factors}}\underbrace{\left|\mathcal{F}(w)\right|^2}_{\text{Theory}}\left|V_{cb}\right|^2$$

- The amplitude $\mathcal{F}$ must be calculated in the theory
  - Extremely difficult task, QCD is non-perturbative
- Can use effective theories (HQET) to say something about $\mathcal{F}$
  - Separate light (non-perturbative) and heavy degrees of freedom as $m_Q \rightarrow \infty$
  - $\lim_{m_Q\rightarrow\infty}\mathcal{F}(w)=\xi(w)$, which is the Isgur-Wise function
  - **We don't know what $\xi(w)$ looks like, but we know $\xi(1)=1$**
  - At large (but finite) mass $\mathcal{F}(w)$ receives corrections $O\left(\alpha_s, \frac{\Lambda_{QCD}}{m_Q}\right)$
- Reduction in the phase space $(w^2-1)^{\frac{1}{2}}$ limits experimental results at $w\approx 1$
  - Need to extrapolate $\left|V_{cb}\right|^2\left|\eta_{ew}\mathcal{F}(w)\right|^2$ to $w=1$
  - This extrapolation is done using well established parametrizations

The V_{cb} matrix element: Calculating $R(D^*)$

$$\underbrace{\frac{d\Gamma}{dw}\left(\bar{B} \to D^* \ell \bar{\nu}_\ell\right)}_{\text{Experiment}} = \left[\underbrace{K_1(w, m_\ell)}_{\text{Known factors}} \underbrace{\left|\mathcal{F}(w)\right|^2}_{\text{Theory}} + \underbrace{K_2(w, m_\ell)}_{\text{Known factors}} \underbrace{\left|\mathcal{F}_2(w)\right|^2}_{\text{Theory}}\right] \times \left|V_{cb}\right|^2$$

- The amplitudes $\mathcal{F}$,$\mathcal{F}_2$ must be calculated in the theory
- Since $K_2(w, 0) = 0$, $\mathcal{F}_2$ only contributes significantly with the τ
- Knowing these amplitudes, one can extract $|V_{cb}|$ from experiment
  - It is possible to extract $R(D^*)$ without experimental data!

$$R(D^*) = \frac{\int_1^{w_{\text{Max},\tau}} dw \left[K_1(w, m_\tau)\left|\mathcal{F}(w)\right|^2 + K_2(w, m_\tau)\left|\mathcal{F}_2(w)\right|^2\right] \times \cancel{\left|V_{cb}\right|^2}}{\int_1^{w_{\text{Max}}} dw \left[K_1(w, 0)\left|\mathcal{F}(w)\right|^2\right] \times \cancel{\left|V_{cb}\right|^2}}$$

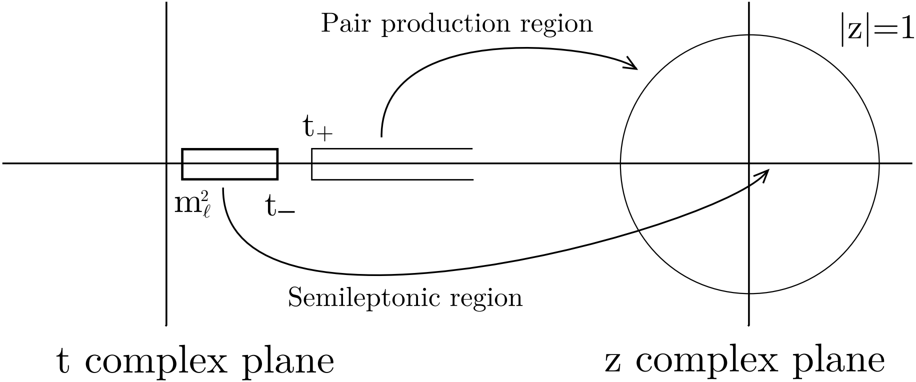
- $|V_{cb}|$ cancels out

The V_{cb} matrix element: The parametrization issue

Most parametrizations perform an expansion in the z parameter

$$\frac{1+z}{1-z} = \sqrt{\frac{t_+ - t}{t_+ - t_-}}, \qquad z = \frac{\sqrt{w+1} - \sqrt{2N}}{\sqrt{w+1} + \sqrt{2N}}$$

with $t_\pm = (m_B \pm m_{D^*})^2\,, \quad t = (p_B - p_{D^*})^2\,, \quad w = v_B \cdot v_{D^*}$



Semileptonic B decays on the lattice: Parametrizations

- Boyd-Grinstein-Lebed (BGL) *Phys. Rev. Lett. 74 (1995) 4603-4606* *Phys.Rev. D56 (1997) 6895-6911* *Nucl.Phys. B461 (1996) 493-511*

$$f_X(w) = \frac{1}{B_{f_X}(z)\phi_{f_X}(z)} \sum_{n=0}^{\infty} a_n z^n$$

  - B_{f_X} Blaschke factors, includes contributions from the poles
  - ϕ_{f_X} is called *outer function* and must be computed for each form factor
  - Weak unitarity constraints $\sum_n |a_n|^2 \leq 1$

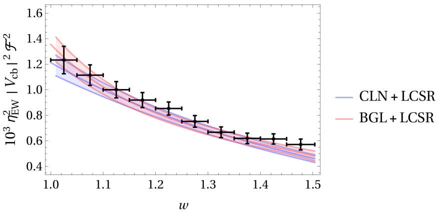
- Caprini-Lellouch-Neubert (CLN) *Nucl. Phys. B530 (1998) 153-181*

$$F(w) \propto 1 - \rho^2 z + cz^2 - dz^3, \quad \text{with } c = f_c(\rho),\, d = f_d(\rho)$$

  - Relies strongly on HQET, spin symmetry and (old) inputs
  - Tightly constrains $F(w)$: four independent parameters, one relevant at $w = 1$
- Current consensus: abandon CLN
  - Spiritual sucessors of CLN Bernlochner *et al.* *Phys.Rev.D 95 (2017) 115008, Phys.Rev.D 97 (2018) 059902*

Bordone, Gubernari, Jung, Straub, Van Dyk... *Eur.Phys.J.C 80 (2020) 74, Eur.Phys.J.C 80 (2020) 347, JHEP 01 (2019) 009*

The V_{cb} matrix element: The parametrization issue



From *Phys. Lett. B769 (2017) 441-445* using Belle data from arXiv:1702.01521 and the Fermilab/MILC'14 value at zero recoil

- CLN seems to underestimate the slope at low recoil
- The BGL value of $|V_{cb}|$ is compatible with the inclusive one

$$|V_{cb}| = 41.7 \pm 2.0 (\times 10^{-3})$$

- Latest Belle dataset and Babar analysis seem to contradict this picture
  - From Babar's paper PRL **123**, 091801 (2019) **BGL is compatible with CLN and far from the inclusive value**
  - Belle's paper PRD **100**, 052007 (2019) finds **similar results in its last revision**
- The discrepancy inclusive-exclusive is not well understood
- Data at $w \gtrsim 1$ is **urgently needed** to settle the issue
- Experimental measurements perform badly at low recoil

We would benefit enormously from a high precision lattice calculation at $w \gtrsim 1$

Break: Heavy quarks in Lattice QCD

- Heavy quark treatment in Lattice QCD
  - For light quarks ($m_l \lesssim \Lambda_{QCD}$), leading discretization errors $\sim \alpha_s^k (a\Lambda_{QCD})^n$
  - For heavy quarks ($m_Q > \Lambda_{QCD}$), discretization errors grow as $\sim \alpha_s^k (am_Q)^n$
    - In this work $am_c \sim 0.15 - 0.6$, but $am_b > 1$
- Need special actions and ETs to describe the bottom quark
  - Relativistic HQ actions (this work $\rightarrow$ FermiLab)
  - Non-Relativistic QCD (NRQCD)
- If the action is improved enough, one can treat the bottom as a light quark
  - Highly improved action AND small lattice spacing
  - Use unphysical values for m_b and extrapolate

The discretization errors needn't disappear **as long as we keep them under control**

Calculating $|V_{cb}|$ on the lattice: Formalism

- Form factors

$$\frac{\langle D^*(p_{D^*},\epsilon^\nu)|\,\mathcal{V}^\mu\,|\bar{B}(p_B)\rangle}{2\sqrt{m_B\,m_{D^*}}} = \frac{1}{2}\epsilon^{\nu *}\varepsilon^{\mu\nu}_{\rho\sigma}v_B^\rho v_{D^*}^\sigma \boldsymbol{h_V}(w)$$

$$\frac{\langle D^*(p_{D^*},\epsilon^\nu)|\,\mathcal{A}^\mu\,|\bar{B}(p_B)\rangle}{2\sqrt{m_B\,m_{D^*}}} =$$

$$\frac{i}{2}\epsilon^{\nu *}\left[g^{\mu\nu}(1+w)\,\boldsymbol{h_{A_1}}(w) - v_B^\nu\left(v_B^\mu \boldsymbol{h_{A_2}}(w) + v_{D^*}^\mu \boldsymbol{h_{A_3}}(w)\right)\right]$$

- $\mathcal{V}$ and $\mathcal{A}$ are the vector/axial currents in the continuum
- The h_X enter in the definition of $\mathcal{F}$
- We can calculate $h_{A_{1,2,3},V}$ directly from the lattice

Calculating $|V_{cb}|$ on the lattice: Formalism

- Helicity amplitudes

$$H_\pm = \sqrt{m_B\, m_{D^*}}(w+1)\left(\boldsymbol{h_{A_1}}(w) \mp \sqrt{\frac{w-1}{w+1}}\boldsymbol{h_V}(w)\right)$$

$$H_0 = \sqrt{m_B\, m_{D^*}}(w{+}1)m_B\left[(w-r)\boldsymbol{h_{A_1}}(w) - (w-1)\left(r\,\boldsymbol{h_{A_2}}(w) + \boldsymbol{h_{A_3}}(w)\right)\right]/\sqrt{q^2}$$

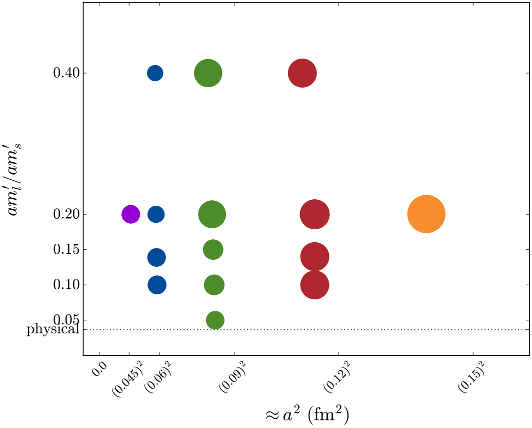
$$H_S = \sqrt{\frac{w^2-1}{r(1+r^2-2wr)}}\left[(1+w)\boldsymbol{h_{A_1}}(w) + (wr-1)\boldsymbol{h_{A_2}}(w) + (r-w)\boldsymbol{h_{A_3}}(w)\right]$$

- Form factor in terms of the helicity amplitudes

$$\chi(w)\,|\mathcal{F}|^2 = \frac{1-2wr+r^2}{12 m_B\, m_{D^*}\,(1-r)^2}\left(H_0^2(w) + H_+^2(w) + H_-^2(w)\right)$$

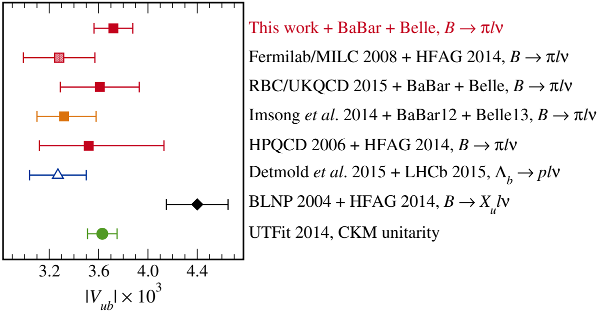
Introduction: Available data and simulations

- Using 15 $N_f = 2 + 1$ MILC ensembles of sea asqtad quarks
- The heavy quarks are treated using the Fermilab action

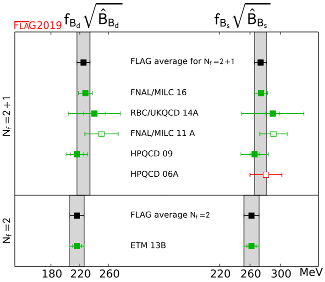


Introduction: The asqtad ensembles

- The asqtad data is being superseded by newer data with improved actions
  - 2^{nd} generation $N_f = 2+1+1$ HISQ and Fermilab charm/bottom quarks
  - 3^{rd} generation $N_f = 2+1+1$ HISQ and a HISQ bottom quark
- Some results from the asqtad ensembles are still competitive today



PRD**92**, (2015) 014024, arXiv:1503.07839



PRD**93**, (2016) 113016, arXiv:1602.03560

This is the last analysis done with asqtad data

Analysis: Extracting the form factors

Calculated ratios

$$\frac{\langle D^*(p)|\,\mathbf{V}\,|D^*(0)\rangle}{\langle D^*(p)|\,V_4\,|D^*(0)\rangle} \to x_f, \qquad w = \frac{1+x_f^2}{1-x_f^2}$$

$$\frac{\left\langle D^*(p_\perp,\varepsilon_\parallel)\right|\mathbf{A}\left|\bar{B}(0)\right\rangle \left\langle \bar{B}(0)\right|\mathbf{A}\left|D^*(p_\perp,\varepsilon_\parallel)\right\rangle}{\langle D^*(0)|\,V_4\,|D^*(0)\rangle \left\langle \bar{B}(0)\right| V_4 \left|\bar{B}(0)\right\rangle}^{*} \to R_{A_1}^2, \qquad h_{A_1} = \left(1-x_f^2\right) R_{A_1}$$

$$\frac{\left\langle D^*(p_\perp,\varepsilon_\perp)\right|\mathbf{V}\left|\bar{B}(0)\right\rangle}{\left\langle D^*(p_\perp,\varepsilon_\parallel)\right|\mathbf{A}\left|\bar{B}(0)\right\rangle} \to X_V, \qquad h_V = \frac{2}{\sqrt{w^2-1}} R_{A_1} X_V$$

$$\frac{\left\langle D^*(p_\parallel,\varepsilon_\parallel)\right|\mathbf{A}\left|\bar{B}(0)\right\rangle}{\left\langle D^*(p_\perp,\varepsilon_\parallel)\right|\mathbf{A}\left|\bar{B}(0)\right\rangle} \to R_1, \qquad h_{A_3} = \frac{2}{w^2-1} R_{A_1}\left(w - R_1\right)$$

$$\frac{\left\langle D^*(p_\perp,\varepsilon_\parallel)\right| A_4 \left|\bar{B}(0)\right\rangle}{\left\langle D^*(p_\perp,\varepsilon_\parallel)\right|\mathbf{A}\left|\bar{B}(0)\right\rangle} \to R_0,$$

$$h_{A_2} = \frac{2}{w^2-1} R_{A_1}\left(wR_1 - \sqrt{w^2-1}R_0 - 1\right)$$

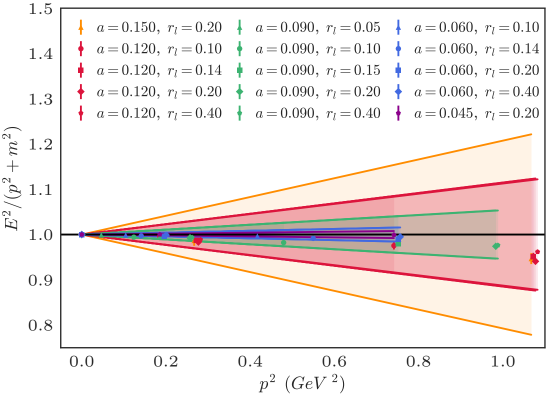
* Phys.Rev. **D**66, 01503 (2002)

Analysis: Systematics in the two-point function fits

- Heavy quark discretization effects break the dispersion relation
- The Fermilab action uses tree-level matching, discretization errors $O(\alpha\, m)$

$$a^2E^2(p_\mu) = (am_1)^2 + \frac{m_1}{m_2}(\mathbf{p}a)^2 + \frac{1}{4}\left[\frac{1}{(am_2)^2} - \frac{am_1}{(am_4)^3}\right](a^2\mathbf{p}^2)^2 - \frac{am_1w_4}{3}\sum_{i=1}^{3}(ap_i)^4 + O(p_i^6)$$

- Deviations from the continuum expression measure the size of the discretization errors
- As long as the discretization errors are within expected bounds, this is all right
- Data for B meson only at rest $\longrightarrow$ Ok in the past



Analysis: Current renormalization

- In the coefficients of the terms of our effective theory a dependence arises with the scale (i.e. a)
- The renormalization tries to account for the right dependence
- The scheme we employ is called *Mostly non-perturbative renormalization* of results

$$Z_{V^{1,4},A^{1,4}} = \underbrace{\rho_{V^{1,4},A^{1,4}}}_{\text{Perturbative factor}} \times \underbrace{\sqrt{Z_{V_{bb}} Z_{V_{cc}}}}_{\text{Non-perturbative piece}}$$

- The (relatively large) non-perturbative piece cancels in our ratios
- The (close to one) perturbative piece (matching factor ρ) is calculated at one-loop level for $w = 1$ and $m_c = 0$
- The errors for $w \neq 1$ and $m_c \neq 0$ are estimated and added to the factor
- We calculate ρ_{A_1} and ratios of ρ_X/ρ_{A_1} for the other form factors
- ρ_{A_1} is **blinded** during analysis, hence all the form factors are multiplied by the same blinding factor
- The results shown here are **unblinded**

Analysis: Chiral-continuum fits

- Our data represents the form factors at non-zero a and unphysical m_π
- Extrapolation to the physical pion mass described by EFTs
  - The EFT describe the a and the m_π dependence
- Functional form explicitly known

$$h_{A_1}(w) = \underbrace{\left[1 + \frac{X_{A_1}(\Lambda_\chi)}{m_c^2} + \frac{g^2_{D^*D\pi}}{48\pi^2 f_\pi^2 r_1^2}\text{logs}_{\text{SU3}}(a, m_l, m_s, \Lambda_{QCD})\right.}_{\text{NLO }\chi\text{PT + HQET}}$$

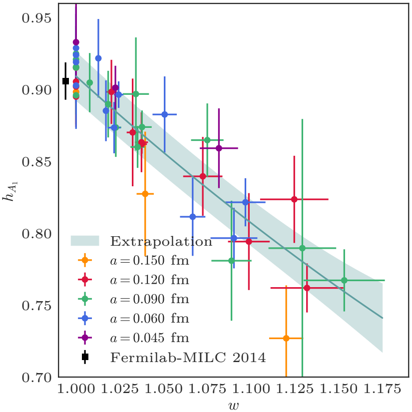
$$\underbrace{+c_1 x_l + c_{a1} x_{a^2}}_{\text{NLO }\chi\text{PT}} \underbrace{-\rho^2_{A_1}(w-1) + k_{A_1}(w-1)^2}_{w\text{ dependence}} \underbrace{\left. + c_2 x_l^2 + c_{a2} x_{a^2}^2 + c_{a,m} x_l x_{a^2}\right]}_{\text{NNLO }\chi\text{PT}} \times$$

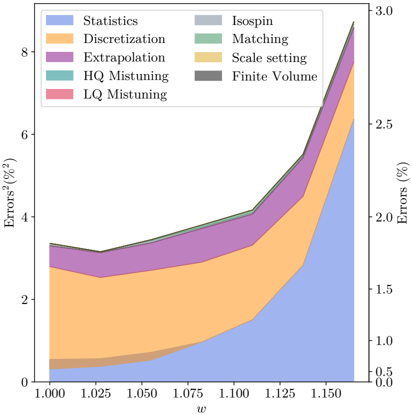
$$\underbrace{\left(1 + \beta_{11}^{A_1}\alpha_s a \Lambda_{\text{QCD}} + \cancel{\beta_{02}^{A_1} a^2 \Lambda^2_{\text{QCD}}} + \beta_{03}^{A_1} a^3 \Lambda^3_{\text{QCD}}\right)}_{\text{HQ discretization errors}}$$

with

$$x_l = B_0 \frac{m_l}{(2\pi f_\pi)^2}, \qquad x_{a^2} = \left(\frac{a}{4\pi f_\pi r_1^2}\right)^2$$

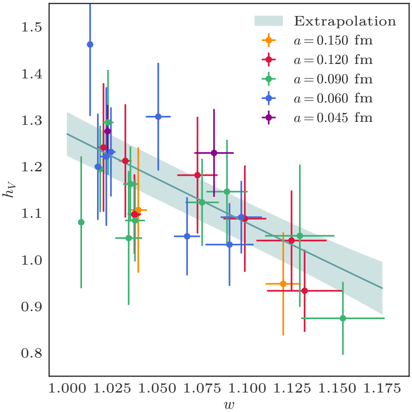
Analysis: Chiral-continuum fits

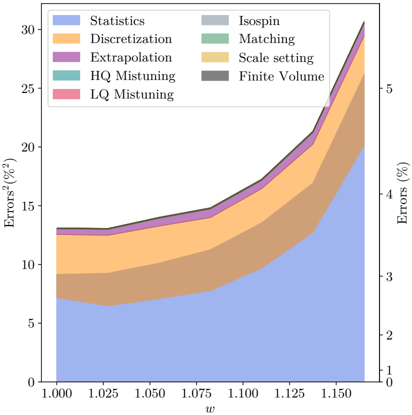




- Combined fit $\chi^2/\text{dof} = 85.2/92$
- $h_{A_1}(1) = 0.909(17)$

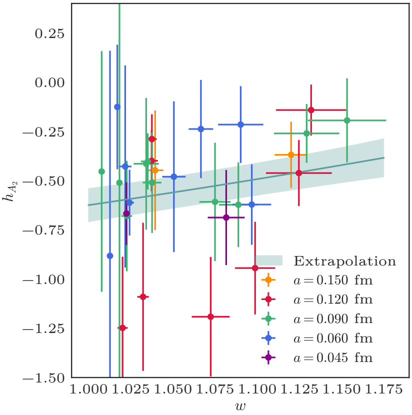
Analysis: Chiral-continuum fits

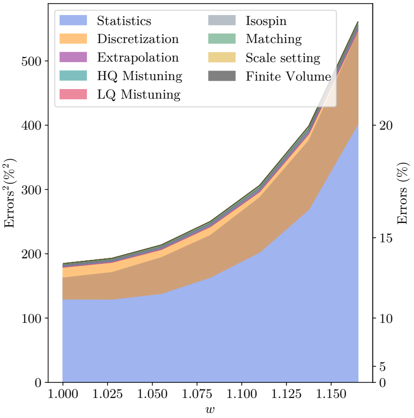




- Combined fit $\chi^2/\text{dof} = 85.2/92$
- $h_V(1) = 1.270(48)$

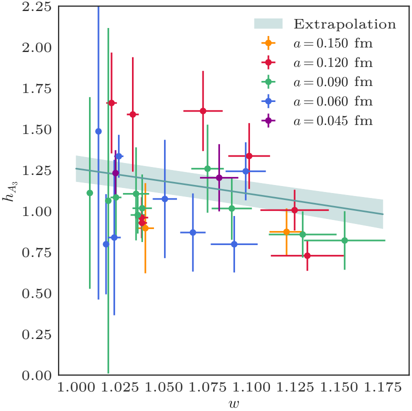
Analysis: Chiral-continuum fits

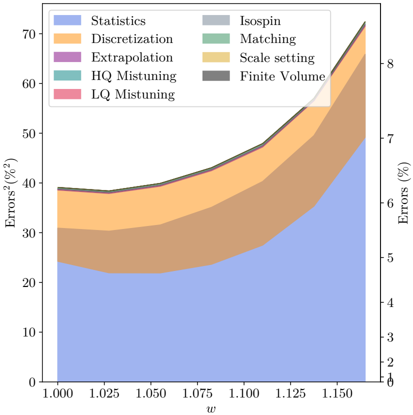




- Combined fit $\chi^2/\text{dof} = 85.2/92$
- $h_{A_2}(1) = -0.624(85)$

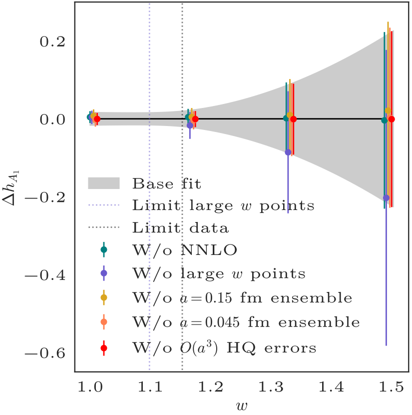
Analysis: Chiral-continuum fits

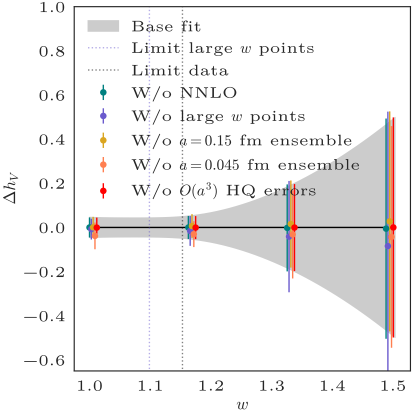




- Combined fit $\chi^2/\text{dof} = 85.2/92$
- $h_{A_3}(1) = 1.259(79)$

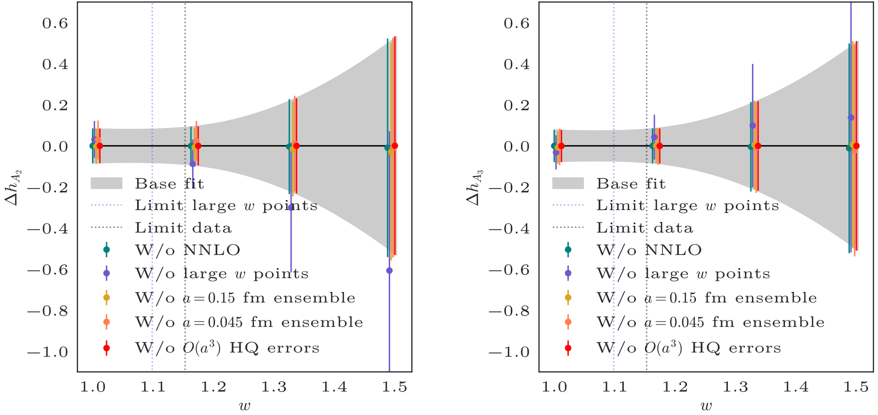
Results: Stability of chiral-continuum fits





	Base	W/o NNLO	W/o large w	W/o $a = 0.15$ fm
χ^2/dof	$\mathbf{85.2/92}$	86.0/107	71.1/75	79.4/86
		W/o $a = 0.045$ fm	W/o HQ $O(a^3)$	
χ^2/dof		81.6/86	85.3/99	

Results: Stability of chiral-continuum fits



	Base	W/o NNLO	W/o large w	W/o $a = 0.15$ fm
χ^2/dof	**85.2/92**	86.0/107	71.1/75	79.4/86
		W/o $a = 0.045$ fm	W/o HQ $O(a^3)$	
χ^2/dof		81.6/86	85.3/99	

Analysis: z-Expansion

- The BGL expansion is performed on different (more convenient) form factors

Phys.Lett. **B**769, 441 (2017), Phys.Lett. **B**771, 359 (2017)

$$g = \frac{h_V(w)}{\sqrt{m_B\, m_{D^*}}} = \frac{1}{\phi_g(z) B_g(z)} \sum_j a_j z^j$$

$$f = \sqrt{m_B\, m_{D^*}}(1+w)h_{A_1}(w) = \frac{1}{\phi_f(z) B_f(z)} \sum_j b_j z^j$$

$$\mathcal{F}_1 = \sqrt{q^2} H_0 = \frac{1}{\phi_{\mathcal{F}_1}(z) B_{\mathcal{F}_1}(z)} \sum_j c_j z^j$$

$$\mathcal{F}_2 = \frac{\sqrt{q^2}}{m_{D^*}\sqrt{w^2-1}} H_S = \frac{1}{\phi_{\mathcal{F}_2}(z) B_{\mathcal{F}_2}(z)} \sum_j d_j z^j$$

- Constraint $\mathcal{F}_1(z=0) = (m_B - m_{D^*}) f(z=0)$
- Constraint $(1+w)m_B^2(1-r)\mathcal{F}_1(z=z_{\text{Max}}) = (1+r)\mathcal{F}_2(z=z_{\text{Max}})$
- BGL (weak) unitarity constraints

$$\sum_j a_j^2 \leq 1, \qquad \sum_j b_j^2 + c_j^2 \leq 1, \qquad \sum_j d_j^2 \leq 1$$

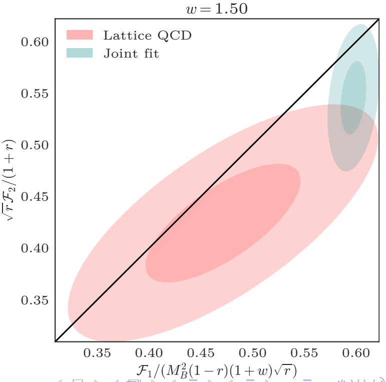
Constraints

- The constraint at zero recoil is used to remove a coefficient of the BGL expansion
- Neither the constraint at maximum recoil nor the unitarity constraints are imposed

How many coefficients in the BGL z-expansion?

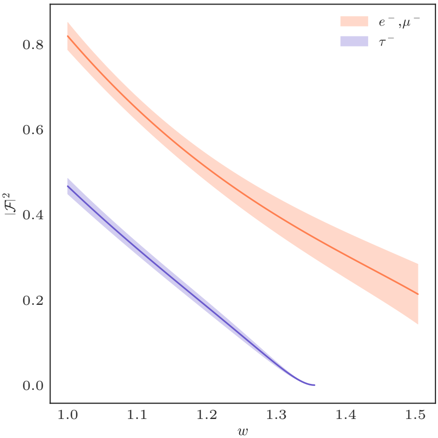
Phys.Rev. **D100 (2019), 013005**

- Add coefficients until
  - We exhaust the degrees of freedom
  - The error is saturated
  - Quadratic/cubic coefficients constrained with priors $0(1)$
- Compared linear/quadratic/cubic fits
  - Agreement in the low order coefficients
  - Quadratic saturates error

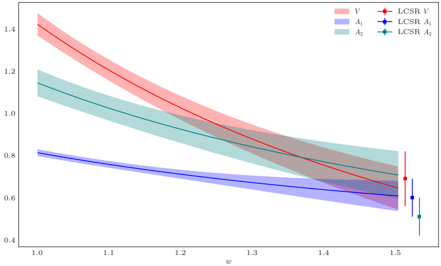


Results: Decay amplitude and form factors

Lattice prediction for the decay amplitude



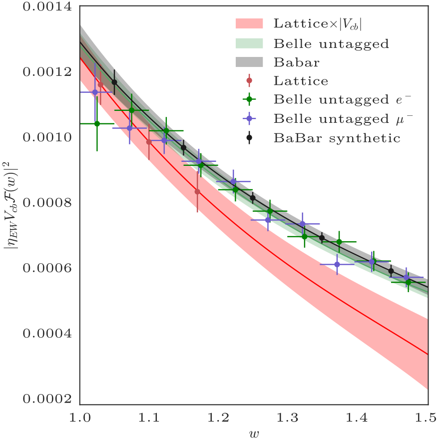
Comparison with LCSR



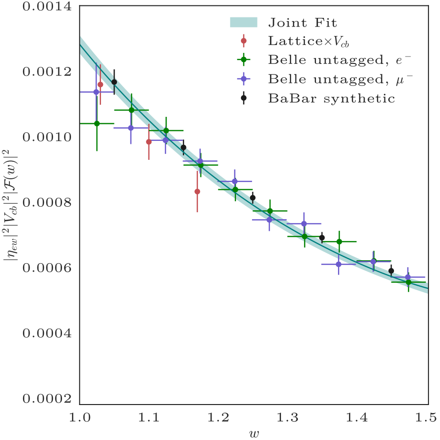
JHEP 01 (2019) 150

- Combined fit $\chi^2/\mathrm{dof} = 0.63$
- Good agreement for A_1, V
- Reasonable agreement for A_2

Separate fits



Joint fit



Fit	Lattice	Exp	Lat + Belle	Lat + BaBar	Lat + Exp
χ^2/dof	$0.63/1$	$104/76$	$111/79$	$8.50/4$	$126/84$

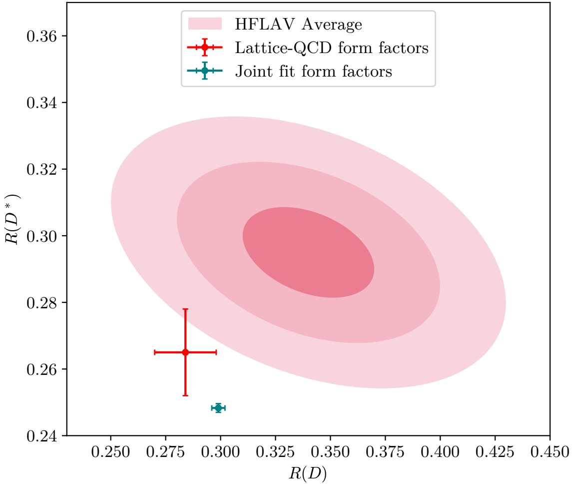
Unblinded, final result $|V_{cb}| = 38.40(78) \times 10^{-3}$

Results: $R(D^*)$ in context

No constraint $w_{\mathbf{Max}}$**:** $R(D^*)_{\mathbf{Lat}} = 0.265(13)$ $\quad R(D^*)_{\mathbf{Lat+Exp}} = 0.2484(13)$

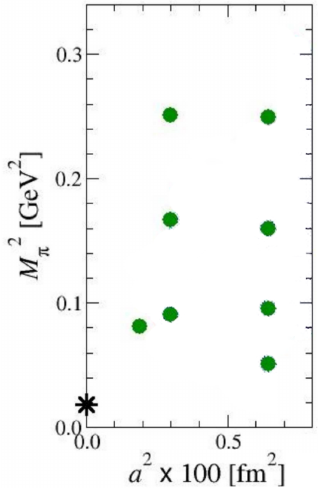
W/ constraint $w_{\mathbf{Max}}$**:** $R(D^*)_{\mathbf{Lat}} = 0.274(10)$ $\quad R(D^*)_{\mathbf{Lat+Exp}} = 0.2492(12)$

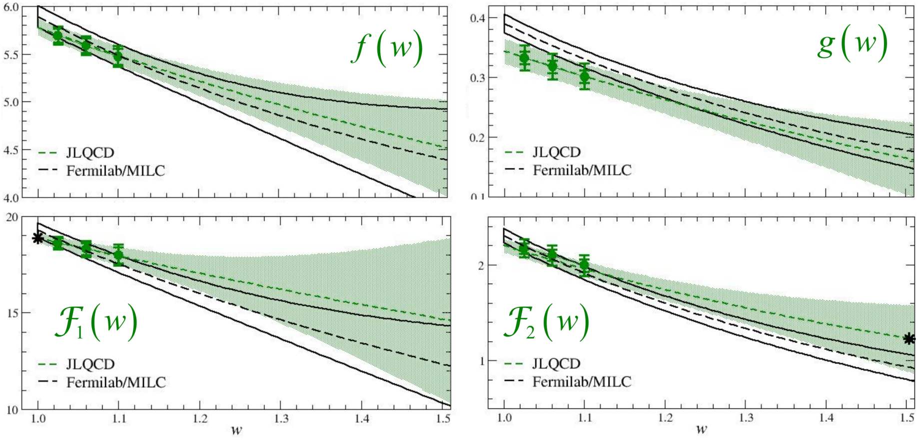
Phys.Rev.D**92** (2015), 034506; Phys.Rev.D**100** (2019), 052007; Phys.Rev.D**103** (2021), 079901; Phys.Rev.Lett. **123** (2019), 091801



Other results: JLQCD

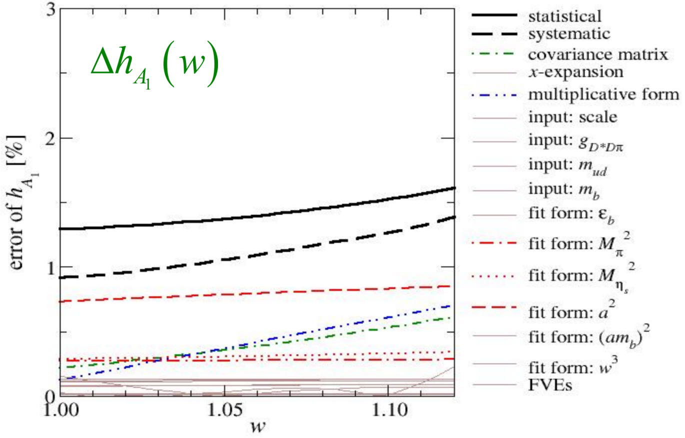
- Using 8 $N_f = 2 + 1$ ensembles of sea DW quarks
- The heavy quarks use the same DW action
  - Simulations at unphysical b masses
  - Requires extrapolation
  - Easier and more precise renormalization
- m_π is as small as 230 MeV
  - Stable D^*





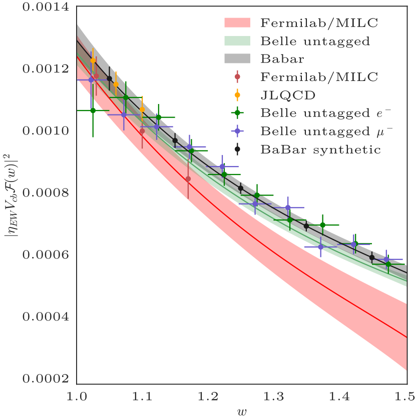
- Milder slope than Fermilab/MILC, but reasonable agreement

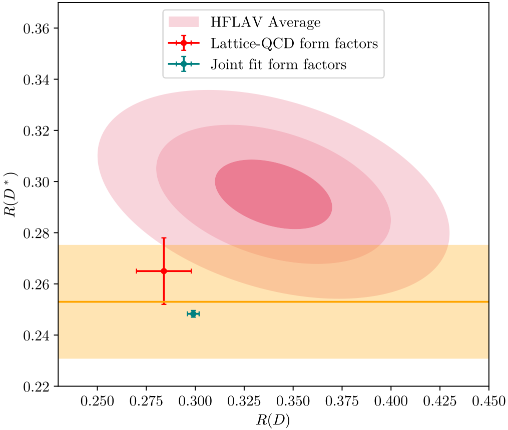
Semileptonic B decays on the lattice: JLQCD



- Discretization errors dominate the systematic contributions
- Statistical errors are the largest contribution in most ff

Other results: JLQCD

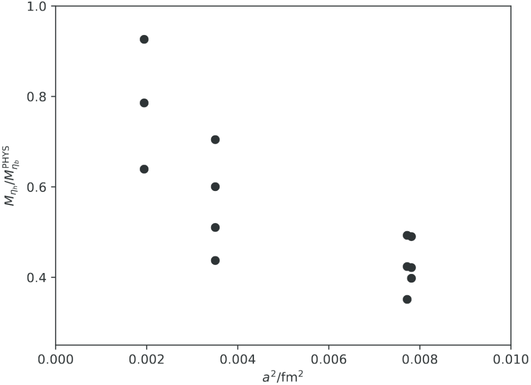




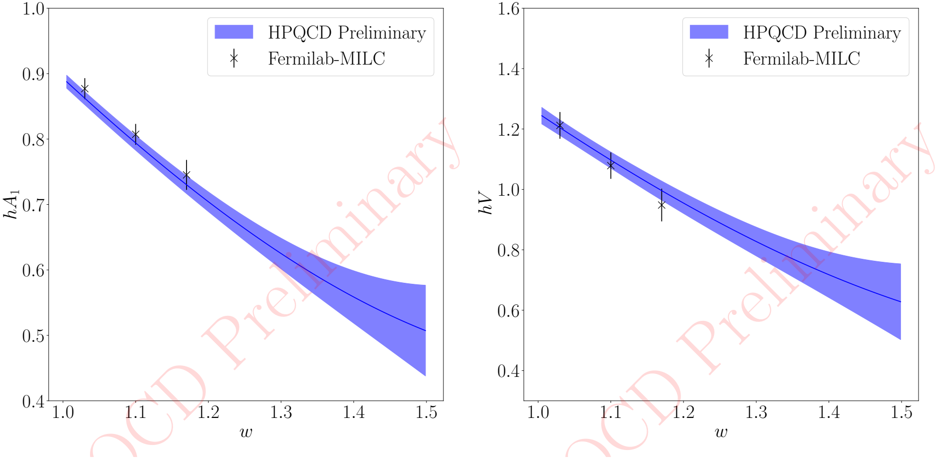
- Fit to Belle dataset, no Coulomb factor
- Combined fit $\chi^2/\text{dof} = 0.94$

Other results: HPQCD

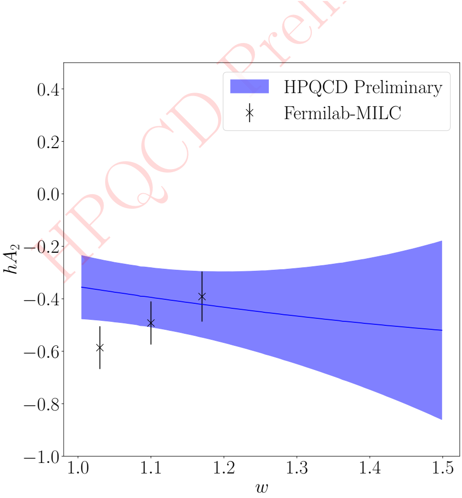
- Using 4 $N_f = 2+1+1$ MILC ensembles of sea HISQ quarks
- The b quark uses the HISQ action and unphysical masses
- m_π ranges from 330 MeV to 129 MeV

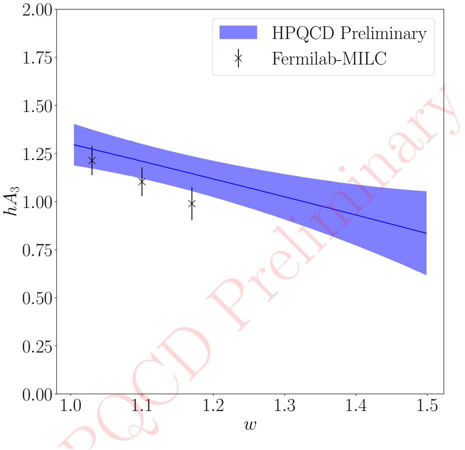


Other results: HPQCD



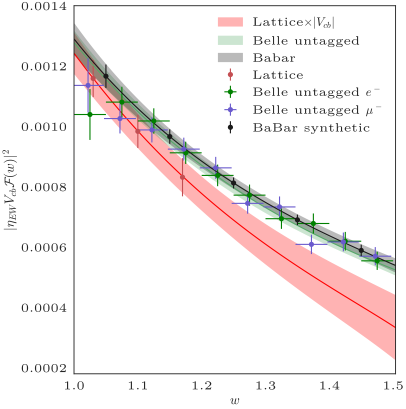
Other results: HPQCD



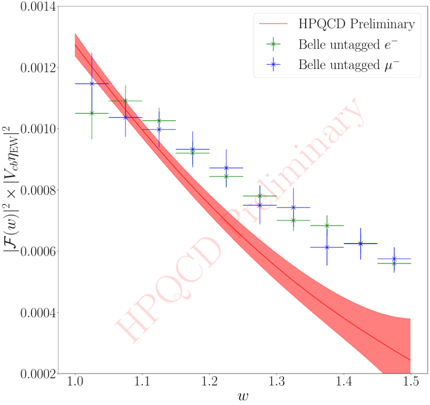


Other results: HPQCD

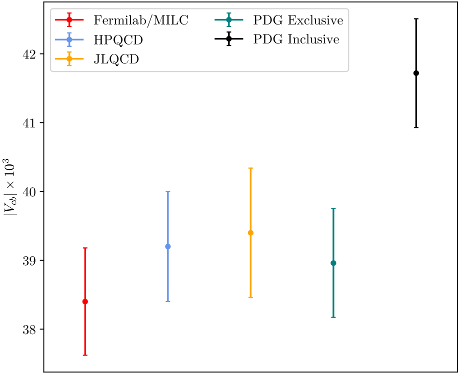
Fermilab/MILC

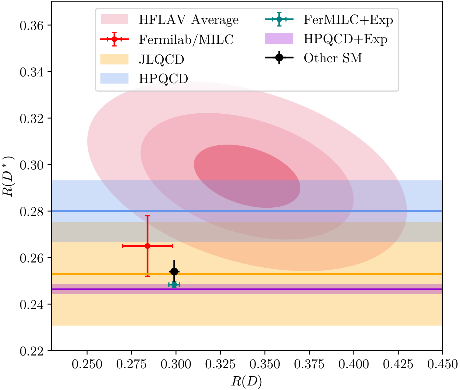


HPQCD



Comparison of results





$$|V_{cb}|^{\mathrm{JLQCD}} = 39.40(94) \times 10^{-3}$$

$$|V_{cb}|^{\mathrm{HPQCD}} = 39.2(8) \times 10^{-3}$$

$$|V_{cb}|^{\mathrm{FerMILC}} = 38.17(85) \times 10^{-3}$$

$$|V_{cb}|^{\mathrm{Excl}} = 38.96(79) \times 10^{-3}$$

$$|V_{cb}|^{\mathrm{Incl}} = 41.72(79) \times 10^{-3}$$

$$R(D^*)^{\mathrm{JLQCD}} = 0.253(22)$$

$$R(D^*)^{\mathrm{HPQCD}} = 0.280(13)$$

$$R(D^*)^{\mathrm{FerMILC}} = 0.265(13)$$

$$R(D^*)^{\mathrm{HFLAV}} = 0.295(14)$$

$$R(D^*)^{\mathrm{SM}} = 0.254(5)$$

Conclusions

- Great progress in both theoretical and experimental fronts
- Didn't cast any light onto the anomalies
  - The inclusive-exclusive tension in the determination of $|V_{cb}|$ remains unsolved
  - The situation of $R(D^*)$ is still unclear
- The next years are going to be critical to figure out what's going on

Stay tuned!

Thank you for your attention

BACKUP SLIDES

Analysis: Heavy quark mistuning corrections

- The simulations are run at approximate physical values of m_c, m_b
- After the runs the differences between the calculated and the physical masses is corrected non-perturbatively
  - The Fermilab action uses the kinetic mass m_2 to compute these corrections
  - $m_1 \to m_2$ as $a \to 0$

Correction process

1. For a particular ensemble correlators are computed at different m_c, m_b
2. All the ratios are calculated for the new values of the heavy quark masses, and the form factors are extracted
3. The derivative of combinations of the form factors with respect to the heavy quark masses is fitted to a suitable function
4. All the form factors are corrected using these results

Shifts are small in most cases, but add a small correlation among all data points

Analysis: Comparison with an improved CLN

- CLN is much more constraining than BGL, using only 4 fit parameters
- We can relax the constraints by allowing errors in the coefficients
  - We take into account the full correlation between ρ^2, c_{A_1} and d_{A_1}
- Update HQET relations between the form factors JHEP 11 (2017) 061

$$h_{A_1}(w) = h_{A_1}(1)\left[1 - 8\rho^2 z + \left(64c_{A_1} - 16\rho^2\right) z^2 + \left(512 d_{A_1} + 256 c_{A_1} - 6\rho^2\right) z^3\right]$$

$$R_0^{\rm CLN}(w) = 1.25(35) - 0.183(77)\,(w-1) + 0.063(23)\,(w-1)^2$$

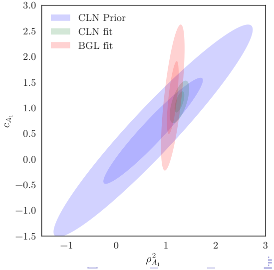
$$R_1^{\rm CLN}(w) = 1.28(36) - 0.101(51)\,(w-1) + 0.066(24)\,(w-1)^2$$

$$R_2^{\rm CLN}(w) = 0.744(44) + 0.128(38)\,(w-1) - 0.079(19)\,(w-1)^2$$

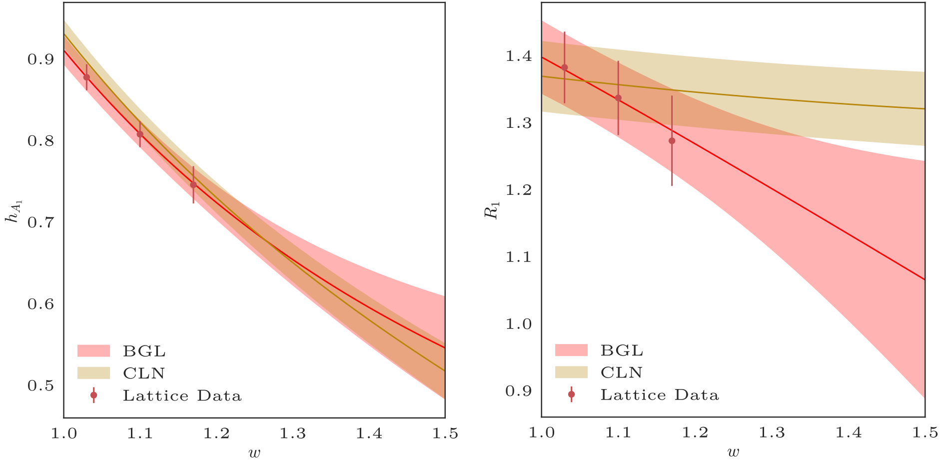
$$R_0^{\rm CLN}(w) = \frac{\sqrt{r}\mathcal{F}_2(w)}{(1+r)h_{A_1}(w)}$$

$$R_1^{\rm CLN}(w) = \frac{h_V(w)}{h_{A_1}(w)}$$

$$R_2^{\rm CLN}(w) = \frac{\frac{m_{D^*}}{m_B}h_{A_2}(w) + h_{A_3}(w)}{h_{A_1}(w)}$$

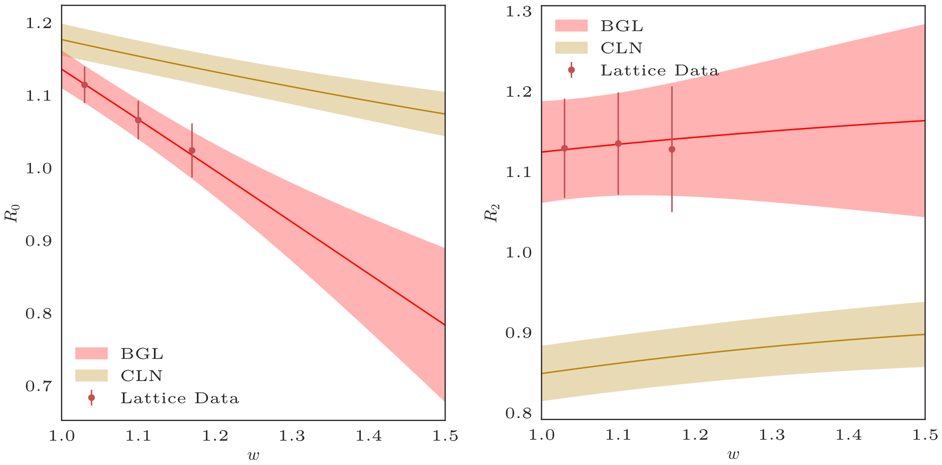


Analysis: Comparison with an improved CLN



- Lattice only $p - \text{value} = \sim O(10^{-5})$
- Predictions for h_{A_1} and R_1^{CLN} look fine

Analysis: Comparison with an improved CLN



- Lattice only $p-\text{value} =\sim O(10^{-5})$
- Predictions for R_0^{CLN} and R_2^{CLN} show tensions

Analysis: The recoil parameter w

- The recoil parameter is measured dynamically
- In the lab frame (B meson at rest)

$$w^2 = 1 + v_{D^*}^2$$

- Ratio of three point functions

$$X_f(p) = \frac{\langle D^*(p)|\,\mathbf{V}\,|D^*(0)\rangle}{\langle D^*(p)|\,V_4\,|D^*(0)\rangle} = \frac{\mathbf{v}_{D^*}}{w+1}$$

- From here

$$w(p) = \frac{1+\mathbf{x}_f^2}{1-\mathbf{x}_f^2}$$

- Alternatively one can use the dispersion relation

Analysis: The recoil parameter w

- Different methods to calculate the recoil parameter
  - In this analysis, we choose the ratio (more conservative)
  - The difference in the final result for the form factors, $|V_{cb}|$ and $R(D^*)$ is not significative